1 The Model

Our framework is the standard two-sector model of a small open economy embellished with multiple types of public sector debt and multiple tax and spending variables. The country produces a traded good q_x and a non-traded good q_n from private capital k, labor L, and government-supplied infrastructure z. Besides these domestically produced goods, agents can import a traded good for consumption c_m and machines $\mathfrak{m}_{mm}$ to produce factories (private capital) and infrastructure (public capital). All quantity variables except labor are detrended by $(1+g)^t$, where g is the exogenous long-run growth rate of real GDP.[1] A composite good produced abroad is the numeraire, with the associated consumer price index (CPI) denoted by P_t^*, which is assumed to be equal to one for simplicity. Since the time horizon for the DSA is about 20 years, the model abstracts from money and all nominal rigidities.[2]

We lay out the model in stages, starting with the specification of technology.

1.1 Firms

1.1.1 Technology

In each sector j, the representative firm use Cobb-Douglas technologies to convert labor $L_{j,t}$, private capital $k_{j,t-1}$, and effectively productive infrastructure z_{t-1}^e, which is a public good, into output:[3]

$$q_{x,t} = A_{x,t}\left(z_{t-1}^e\right)^{\psi_x}\left(k_{x,t-1}\right)^{\alpha_x}\left(L_{x,t}\right)^{1-\alpha_x}, \tag{1}$$

and

$$q_{n,t} = A_{n,t}\left(z_{t-1}^e\right)^{\psi_n}\left(k_{n,t-1}\right)^{\alpha_n}\left(L_{n,t}\right)^{1-\alpha_n}. \tag{2}$$

The firm productivities are expressed as

$$A_{x,t} = a_x\left(\frac{q_{x,t-1}^I}{\bar{q}_x^I}\right)^{\sigma_x}\left(k_{x,t-1}^I\right)^{\xi_x} \qquad \text{and} \qquad A_{n,t} = a_n\left(\frac{q_{n,t-1}^I}{\bar{q}_n^I}\right)^{\sigma_n}\left(k_{n,t-1}^I\right)^{\xi_n}$$

[1] In the long run all variables, including real GDP, grow at the same exogenous growth rate g. However, in the short to medium term, significant public and private capital accumulation, resulting from scaling up investment, implies that the growth rate of the economy can go above g.

[2] We are currently working on a version that includes money and nominal price rigidities along the lines of the model in Berg et al. (2010b).

[3] We assume Cobb-Douglas technologies but, to some extent, we do not expect significant changes in our results by considering CES technologies.

and feature sector-specific externalities of two types, with variables with the superindex I denoting sectoral quantities: a "static" externality associated with private capital accumulation—$\left(k_{j,t-1}^{I}\right)^{\xi_j}$ for $j = x, n$ —as in Arrow (1962); and a "learning-by-doing" externality that depends on the deviations of the lagged sector output from the (initial) steady state—$\left(\frac{q_{j,t-1}^{I}}{\bar{q}_j^{I}}\right)^{\sigma j}$ for $j = x, n$. When the latter externality is greater in the traded sector than that in the non-traded sector, then it can capture the notion of Dutch-disease as in Berg et al. (2010b), where a decline in the traded sector imposes an economic cost through a sectoral loss in total-factor productivity (TFP).

Factories and infrastructure are built by combining one imported machine with a_j ($j = k, z$) units of a non-traded input (e.g., construction). The supply prices of private capital and infrastructure are thus

$$P_{k,t} = P_{mm,t} + a_k P_{n,t}, \tag{3}$$

and

$$P_{z,t} = P_{mm,t} + a_z P_{n,t}, \tag{4}$$

where P_n is the (relative) price of the non-traded good and P_{mm} is the (relative) price of imported machinery.

1.1.2 Factor Demands

Competitive profit-maximizing firms equate the marginal value product of each input to its factor price. This yields the input demand equations

$$P_{n,t}(1-\alpha_n)\frac{q_{n,t}}{L_{n,t}} = w_t, \tag{5}$$

$$P_{x,t}(1-\alpha_x)\frac{q_{x,t}}{L_{x,t}} = w_t, \tag{6}$$

$$P_{n,t}\alpha_n\frac{q_{n,t}}{k_{n,t-1}} = r_{n,t}, \tag{7}$$

and

$$P_{x,t}\alpha_x\frac{q_{x,t}}{k_{x,t-1}} = r_{x,t}, \tag{8}$$

where w is the wage and r_j is the rental earned by capital in sector j. Labor is intersectorally mobile, so the same wage appears in (5) and (6). Capital is sector-specific, but r_x differs from r_n only on the transition path. After adjustment is complete and k_x and k_n have settled at their equilibrium levels, the rentals are equal.

1.2 Consumers

There are two types of private agents, savers and non-savers, with the former and the latter distinguished by the superscripts $\mathfrak{s}$ and $\mathfrak{h}$, respectively. Labor supply of savers is fixed at $L^{\mathfrak{s}}$ while that of non-savers is $L^{\mathfrak{h}} = aL^{\mathfrak{s}}$ with $a > 0$. The two types of agents consume the domestic traded good $c^i_{x,t}$, the foreign traded good $c^i_{m,t}$, and the domestic non-traded good $c^i_{n,t}$ for $i = \mathfrak{s}, \mathfrak{h}$. These goods are combined into a CES basket

$$c^i_t = \left[\rho_x^{\frac{1}{\epsilon}} \left(c^i_{x,t}\right)^{\frac{\epsilon-1}{\epsilon}} + \rho_m^{\frac{1}{\epsilon}} \left(c^i_{m,t}\right)^{\frac{\epsilon-1}{\epsilon}} + (\rho_n)^{\frac{1}{\epsilon}} \left(c^i_{n,t}\right)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \qquad \text{for } i = \mathfrak{s}, \mathfrak{h} \quad (9)$$

where ρ_x, ρ_m, and ρ_n are CES distribution parameters and ϵ is the intratemporal elasticity of substitution. In addition $\rho_n = 1 - \rho_x - \rho_m$.

The (relative) CPI associated with the basket (9) is $P_t = \left[\rho_x P_{x,t}^{1-\epsilon} + \rho_m P_{m,t}^{1-\epsilon} + \rho_n P_{n,t}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$, while the demand functions for each good can be expressed as

$$c^i_{j,t} = \rho_j \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} c^i_t \qquad \text{for} \quad j = x, m, n \quad \text{and} \quad i = \mathfrak{s}, \mathfrak{h}.$$

Savers can invest i_x and i_n amounts in private capital that depreciates at the rate δ, pay user fees charged for infrastructure services according to μz^e, can buy domestic bonds b—which cannot be bought in any market by foreigners—and pay a real interest rate r, and can contract foreign debt b^* that charges an exogenous real interest rate r^*. They solve the intertemporal problem

$$Max \sum_{t=0}^{\infty} \beta^t \frac{(c^{\mathfrak{s}}_t)^{1-1/\tau}}{1-1/\tau},$$

subject to

$$\begin{aligned} P_t b^{\mathfrak{s}}_t - b^{\mathfrak{s}*}_t \;=\;& r_{x,t} k^{\mathfrak{s}}_{x,t-1} + r_{n,t-1} k^{\mathfrak{s}}_{n,t-1} + w_t L^{\mathfrak{s}}_t + \frac{\mathcal{R}_t}{1+a} + \frac{\mathcal{T}_t}{1+a} - \frac{1+r^*_{t-1}}{1+g} b^{\mathfrak{s}*}_{t-1} + \frac{1+r_{t-1}}{1+g} P_t b^{\mathfrak{s}}_{t-1} \\ & - P_{k,t}\left(i^{\mathfrak{s}}_{x,t} + i^{\mathfrak{s}}_{n,t} + AC^{\mathfrak{s}}_{x,t} + AC^{\mathfrak{s}}_{n,t}\right) - P_t c^{\mathfrak{s}}_t (1+h_t) - \mu z^e_{t-1} - \mathcal{P}^{\mathfrak{s}}_t - \Phi^{\mathfrak{s}}_t, \qquad (10) \end{aligned}$$

$$(1+g) k^{\mathfrak{s}}_{x,t} = i^{\mathfrak{s}}_{x,t} + (1-\delta) k^{\mathfrak{s}}_{x,t-1}, \qquad (11)$$

and

$$(1+g)k_{n,t}^{\mathfrak{s}} = i_{n,t}^{\mathfrak{s}} + (1-\delta)k_{n,t-1}^{\mathfrak{s}}, \tag{12}$$

where $\beta = 1/[(1+\varrho)(1+g)^{(1-\tau)/\tau}]$ is the discount factor; ϱ is the pure time preference rate; τ is the intertemporal elasticity of substitution; δ is the depreciation rate; $\mathcal{R}$ are remittances; $\mathcal{T}$ are (net) transfers; h denotes the consumption value added tax (VAT); and $\Phi^{\mathfrak{s}}$ are profits from domestic firms. Remittances and transfers are proportional to the agent's share in aggregate employment. Observe that in the budget constraint (10), the trend growth rate appears in several places in (10)-(12), reflecting the fact that some variables are dated at t and others at t-1 and that P_t multiplies $b_t^{\mathfrak{s}}$ and $b_{t-1}^{\mathfrak{s}}$because domestic bonds are indexed to the price level.[4] Also note that there are adjustment costs incurred in changing the capital stock—$AC_{j,t}^{\mathfrak{s}} \equiv \frac{v}{2}\left(\frac{i_{j,t}^{\mathfrak{s}}}{k_{j,t-1}^{\mathfrak{s}}} - \delta - g\right)^2 k_{j,t-1}^{\mathfrak{s}}$, for $j = x, n$ and with $v > 0$—and portfolio adjustment costs associated with foreign liabilities—$\mathcal{P}_t^{\mathfrak{s}} \equiv \frac{\eta}{2}(b_t^{\mathfrak{s}*} - \bar{b}^{\mathfrak{s}*})^2$, where $\bar{b}^*$ is the (initial) steady-state value of the private foreign liabilities.[5]

The choice variables in the optimization problem are $c_t^{\mathfrak{s}}$, $b_t^{\mathfrak{s}}$, $b_t^{\mathfrak{s}*}$, $i_{j,t}^{\mathfrak{s}}$, and $k_{j,t}^{\mathfrak{s}}$ for $j = x, n$. Routine manipulations of the first-order conditions deliver:

$$c_t^{\mathfrak{s}} = c_{t+1}^{\mathfrak{s}} \left(\beta \frac{1+r_t}{1+g} \frac{1+h_t}{1+h_{t+1}}\right)^{-\tau}, \tag{13}$$

$$(1+r_t)\frac{P_{t+1}}{P_t} = \frac{1+r_t^*}{\left[1-\eta(b_t^{\mathfrak{s}*} - \bar{b}^{\mathfrak{s}*})\right]}, \tag{14}$$

$$\frac{r_{x,t+1}}{P_{k,t+1}} + 1 - \delta + v\Upsilon_{x,t+1}^{\mathfrak{s}} \left(\frac{i_{x,t+1}^{\mathfrak{s}}}{k_{x,t}^{\mathfrak{s}}} + 1 - \delta\right) - \frac{v}{2}\left(\Upsilon_{x,t+1}^{\mathfrak{s}}\right)^2 = (1+r_t)\frac{P_{t+1}}{P_t}\frac{P_{k,t}}{P_{k,t+1}}\left(1 + v\Upsilon_{x,t}^{\mathfrak{s}}\right), \tag{15}$$

and

$$\frac{r_{n,t+1}}{P_{k,t+1}} + 1 - \delta + v\Upsilon_{n,t+1}^{\mathfrak{s}} \left(\frac{i_{n,t+1}^{\mathfrak{s}}}{k_{n,t}^{\mathfrak{s}}} + 1 - \delta\right) - \frac{v}{2}\left(\Upsilon_{n,t+1}^{\mathfrak{s}}\right)^2 = (1+r_t)\frac{P_{t+1}}{P_t}\frac{P_{k,t}}{P_{k,t+1}}\left(1 + v\Upsilon_{n,t}^{\mathfrak{s}}\right), \tag{16}$$

[4] The convention for detrending the capital stocks differs from that for other variables. Because $K_{j,t-1}^{\mathfrak{s}}$—the capital stock before detrending—is the capital stock in use at time t, we define $k_{j,t-1}^{\mathfrak{s}} \equiv K_{j,t-1}^{\mathfrak{s}}/(1+g)^t$. Under this convention, $\bar{\imath}_j^{\mathfrak{s}} = (\delta+g)\bar{k}_j^{\mathfrak{s}}$ in the long run—as required for the capital stock to grow at the trend growth rate g.

[5] For simplicity, we assume that adjustment costs are zero when the capital stock grows at the trend growth rate g. This ensures that adjustment costs are zero across steady states as in models that ignore trend growth.

where $\Upsilon^{\mathfrak{s}}_{j,t} = \left(\frac{i^{\mathfrak{s}}_{j,t}}{k^{\mathfrak{s}}_{j,t-1}} - \delta - g\right)$ for $j = x, n$. Each of these equations admits a straightforward intuitive interpretation. Equation (13) is a slightly irregular Euler equation in which the slope of the consumption path depends on the real interest rate adjusted for trend growth and on changes in the VAT. The other three equations are arbitrage conditions. Equation (14) equalizes the real interest rate on domestic bonds to the real interest rate on foreign private debt, adjusted by portfolio costs. Similarly, equations (15) and (16) require the return on capital in each sector, net of marginal adjustment costs, to equal the real interest rate.

In our modelling decisions, we balance realism and flexibility in introducing portfolio adjustment costs to capture different degrees of integration of the private sector into world capital markets. Equation (14) implicitly defines a private demand for foreign debt, which can be explicitly expressed as:

$$\eta(b^{\mathfrak{s}*}_t - \bar{b}^{\mathfrak{s}*}) = 1 - \frac{1 + r^*_t}{(1 + r_t)\frac{P_{t+1}}{P_t}}.$$

In this equation, the value of η controls the degree of capital mobility. For some emerging market economies, a low η may be appropriate reflecting an open capital account. Elastic capital flows then keep the domestic rate close to the foreign rate. In LICs, where η is comparatively big, the capital account is fairly closed, and the private sector has limited capacity to borrow from abroad.[6]

In addition, we assume that on its foreign debt the private sector pays a constant premium $\mathfrak{u}$ over the interest rate that the government pays on external commercial debt r_{dc}, so

$$r^*_t = r_{dc,t} + \mathfrak{u}.$$

This specification together with (14) allows us to match the low capital and investment ratios observed in LICs. The reason is that at the steady state the domestic interest rate r and the foreign rate r^* are equal, meaning that $\bar{r} = \bar{r}_{dc} + \mathfrak{u}$. Thus even if $\bar{r}_{dc}$ is low, by picking an appropriate $\mathfrak{u}$, we can obtain a somewhat high domestic real interest rate $\bar{r}$ and, consequently, a high return on private capital. But because of decreasing returns, this high return implies realistically that the ratios of the capital stock and investment to GDP must be low at the initial steady-state equilibrium.

[6]From a technical point of view, the portfolio costs also help to ensure stationarity of $b^{\mathfrak{s}*}_t$. See Schmitt-Grohé and Uribe (2003) for alternative methods to ensure stationarity of net foreign assets.

Non-savers have the same utility function as that of savers and consume all of their income from wages, remittances, and transfers each period. The non-saver's budget constraint then reads

$$(1+h_t)P_t c_t^{\mathfrak{h}} = w_t L^{\mathfrak{h}} + \frac{a}{1+a}(\mathcal{R}_t + \mathcal{T}_t). \tag{17}$$

In specifying this part of the model we have aimed for realism combined with flexibility and generality. The realism of hand-to-mouth consumers (non-savers) is indisputable given that in LICs a substantial portion of households fall into this category.[7] From the modeling perspective their inclusion allows us to break Ricardian equivalence.

We aggregate across both types of households, so $x_t = x_t^{\mathfrak{s}} + x_t^{\mathfrak{h}}$ for $x_t = c_t$, $c_{l,t}$, L_t, b_t^*, b_t, $i_{j,t}$, $k_{j,t}$, $AC_{j,t}$, $\mathcal{P}_t$, Φ_t, and the subindices $l = x, n, m$ and $j = x, n$. Bear in mind that $b_t^{\mathfrak{h}*} = b_t^{\mathfrak{h}} = i_{j,t}^{\mathfrak{h}} = k_{j,t}^{\mathfrak{h}} = AC_{j,t}^{\mathfrak{h}} = \mathcal{P}_t^{\mathfrak{h}} = \Phi_t^{\mathfrak{h}} = 0$ for $j = x, n$.

1.3 The Government

1.3.1 Infrastructure, Public Investment and Efficiency

Casual observation and indirect empirical evidence support the conjecture in Hulten (1996) and Pritchett (2000): often the productivity of infrastructure is high but the return on public investment very low for the simple reason that a good deal of public investment *spending* does not increase the stock of productive capital. Measurement error introduced by equating growth of productive capital with net investment can explain why estimated TFP growth is zero or negative in many LDCs, as suggested by Pritchett (2000); and why empirical studies generally find a much stronger positive relationship between growth and physical indicators of infrastructure than between growth and capital stock series calculated via the perpetual inventory method, as reviewed by Straub (2008).[8]

[7] For instance the 2009 Steadman Survey finds that 62 percent of Ugandans do not have access to financial services.

[8] To understand the role of efficiency, it may be useful to imagine that all the available public investment projects at a given point in time are ranked from highest to lowest rate of return. In an efficient investment process, an additional dollar is spent on the best available project. It is possible, though, because of incompetence, corruption, or imperfect information, that a government may choose worse projects. A lower efficiency is a measure of the degree of deviation from the optimal process. A complementary way to think about efficiency is simply that a fraction of spending is simply wasted, e.g. misclassified as investment when it in fact just covers transfers to civil servants.

We thus allow for inefficiencies in public capital creation. Public investment i_z produces additional infrastructure z according to:

$$(1+g)z_t = (1-\delta)z_{t-1} + i_{z,t}, \tag{18}$$

but some of the newly built infrastructure may not be economically valuable productive infrastructure, since *effectively productive* capital z_t^e, which is actually used in technologies (1) and (2), evolves according to:

$$z_t^e = \bar{s}\bar{z} + s(z_t - \bar{z}), \quad \text{with} \quad \bar{s} \in [0,1] \quad \text{and} \quad s \in [0,1], \tag{19}$$

where $\bar{s}$ and s are parameters of efficiency at and off steady state, and $\bar{z}$ is public capital at the (initial) steady state.

Note that by combining equations (18) and (19), we obtain

$$(1+g)z_t^e = (1-\delta)z_{t-1}^e + s(i_{z,t} - \bar{\imath}_z) + \bar{s}\bar{\imath}_z, \tag{20}$$

where $\bar{\imath}_z = (\delta+g)\bar{z}$ is the public investment at the (initial) steady state. This is the same specification for public investment inefficiencies as that of Berg et al. (2010b), and it is similar to the one in Agenor (2010). Since $s \in [0,1]$, this specification makes clear that *one* dollar of additional public investment $(i_{z,t} - \bar{\imath}_z)$ does not translate into *one* dollar of effectively productive capital (z_t^e). For the simulations below, we assume $\bar{s} = 0.6$ and $s = 0.6$. These values are slightly higher that the estimates of Arestoff and Hurlin (2006) for some emerging economies. In principle, the public investment management quality index of Dabla-Norris et al. (2011) could help calibrate this parameter.

1.3.2 Fiscal Adjustment and the Public Sector Budget Constraint

The government spends on transfers, debt service, and infrastructure investment. It collects revenue from the consumption VAT and from user fees for infrastructure services, which are expressed as a fixed multiple/fraction f of recurrent costs, that is $\mu = f\delta P_{zo}$. When revenues fall short of expenditures, the resulting deficit is financed through domestic borrowing $\Delta b_t = b_t - b_{t-1}$, external concessional borrowing $\Delta d_t = d_t - d_{t-1}$, or external commercial borrowing $\Delta d_{c,t} = d_{c,t} - d_{c,t-1}$ viz.:

$$\begin{aligned} P_t\Delta b_t + \Delta d_{c,t} + \Delta d_t \quad &= \quad \frac{r_{t-1}-g}{1+g}P_t b_{t-1} + \frac{r_{d,t-1}-g}{1+g}d_{t-1} + \frac{r_{dc,t-1}-g}{1+g}d_{c,t-1} \\ &\quad + P_{z,t}\mathbb{I}_{z,t} + \mathcal{T}_t - h_t P_t c_t - \mathcal{G}_t - \mathcal{N}_t - \mu z_{t-1}^e, \end{aligned} \tag{21}$$

where d, dc, $\mathcal{G}$, and $\mathcal{N}$ denote concessional debt, external commercial debt, grants, and natural resource revenues (if any);[9] and r_d and r_{dc} are the real interest rates (in dollars) on concessional and commercial loans. The interest rate on concessional loans is assumed to be constant $r_{d,t} = r_d$, while the interest rate on external commercial debt incorporates a risk premium that depends on the deviations of the external public debt to GDP ratio $\left(\frac{d_t+d_{c,t}}{y_t}\right)$ from its (initial) steady-state value $\left(\frac{\bar{d}+\bar{d}_c}{\bar{y}}\right)$. That is,

$$r_{dc,t} = r^f + \upsilon_g e^{\eta_g\left(\frac{d_t+d_{c,t}}{y_t} - \frac{\bar{d}+\bar{d}_c}{\bar{y}}\right)}, \tag{22}$$

where r^f is a risk-free world interest rate and $y_t = P_{x,t}q_{x,t} + P_{n,t}q_{n,t}$ is GDP. Van der Ploeg and Venables (2011) provide positive estimates for η_g. But note by setting $\upsilon_g > 0$ and $\eta_g = 0$, our specification embeds the case of an exogenous risk premium that does depend on public debt.

The term $P_{z,t}\mathbb{I}_{z,t}$ in the budget constraint (21) corresponds to public investment outlays including costs overruns associated with absorptive capacity constraints. It is defined as

$$\mathbb{I}_{z,t} = \mathcal{H}_t(i_{z,t} - \bar{\imath}_z) + \bar{\imath}_z.$$

Because skilled administrators are in scarce supply in LICs, ambitious public investment programs are often plagued by poor planning, weak oversight, and myriad coordination problems, all of which contribute to large cost overruns during the implementation phase.[10] To capture this, we multiply new investment $(i_{z,t} - \bar{\imath}_z)$ by $\mathcal{H}_t = \left(1 + \frac{i_{z,t}}{z_{t-1}} - \delta - g\right)^{\phi}$, where $\phi \geq 0$ determines the severity of the the absorptive capacity—or "bottleneck"—constraint in the public sector. The constraint affects only implementation costs for new projects: in a steady state, $\left(1 + \frac{\bar{\imath}_z}{\bar{z}} - \delta - g\right)^{\phi} = 1$ as $\bar{\imath}_z = (\delta + g)\bar{z}$.

Policy makers accept all concessional loans proffered by official creditors. The borrowing and amortization schedule for these loans is fixed exogenously. Given the paths for public investment and concessional borrowing,

[9] We model natural resources revenues as a net foreign transfer, following Dagher et al. (2012). As such the measure of GDP used below corresponds to non-oil GDP. For a more comprehensive analysis of oil production, foreign investment, and managing natural resources in LICs see Berg et al. (2012).

[10] Development agencies report that cost overruns of 35% and more are common for new projects in Africa. The most important factor by far is inadequate competitive bidding for tendered contracts. See Foster and Briceno-Garmendia (2010).

the fiscal gap *before* policy adjustment ($\mathfrak{Gap}_t$) can be defined as:

$$\mathfrak{Gap}_t = \frac{1+r_d}{1+g}d_{t-1}-d_t+\frac{r_{dc,t-1}-g}{1+g}dc_{t-1}+\frac{r_{t-1}-g}{1+g}P_t b_{t-1}+P_{z,t}\mathbb{I}_t+\mathcal{T}_o-h_o P_t c_t-\mathcal{G}_t-\mathcal{N}_t-\mu z^e_{t-1}. \tag{23}$$

That is, $\mathfrak{Gap}_t$ corresponds to expenditures (including interest rate payments on debt) less revenues and concessional borrowing, when transfers and taxes are kept at their *initial* levels $\mathcal{T}_o$and h_o, respectively. Using this definition, we can rewrite the budget constraint (21), in any given year, as:

$$\mathfrak{Gap}_t = P_t\Delta b_t + \Delta d_{c,t} + (h_t - h_o)P_t c_t - (\mathcal{T}_t - \mathcal{T}_o). \tag{24}$$

In the short/medium run, this gap $\mathfrak{Gap}_t$ in (24) can be covered by domestic and/or external commercial borrowing $P_t\Delta b_t + \Delta d_{c,t}$, tax adjustments $(h_t - h_o)P_t c_t$, and/or transfers adjustments $-(\mathcal{T}_t - \mathcal{T}_o)$. For the sake of comparing different borrowing schemes, in the experiments below, we will focus on cases where part of this gap can be filled with either external commercial loans *or* domestic loans, but not with both at the same time.

Debt sustainability requires, however, that the VAT and transfers eventually adjust to cover the entire gap (i.e., $P_t\Delta b_t + \Delta d_{c,t} = 0$). We let policy makers divide the burden of adjustment (net windfall when $\mathfrak{Gap}_t < 0$) between transfers cuts and tax increases. The adjustments, defined below according to some reaction functions, have as part of their targets the following debt-stabilizing values for transfers and the VAT

$$h_t^{\text{target}} = h_o + (1-\lambda)\frac{\mathfrak{Gap}_t}{P_t c_t} \tag{25}$$

and

$$\mathcal{T}_t^{\text{target}} = \mathcal{T}_o - \lambda\mathfrak{Gap}_t, \tag{26}$$

where λ is a policy parameter that splits the fiscal adjustment between taxes and transfers and therefore satisfies $0 \leq \lambda \leq 1$. For the extreme case of $\lambda = 0$ (respectively $\lambda = 1$) all the adjustment falls on taxes (respectively transfers).

Taxes and transfers are defined according to the following reaction functions:

$$h_t = Min\left\{h_t^r, h^u\right\} \tag{27}$$

and

$$\mathcal{T}_t = Max\left\{\mathcal{T}_t^r, \mathcal{T}^l\right\}, \tag{28}$$

where h^u is a ceiling on taxes, $\mathcal{T}^l$ is a floor for transfers, and h_t^r and $\mathcal{T}_t^r$ are determined by the fiscal rules

$$h_t^r = h_{t-1}+\lambda_1(h_t^{\text{target}}-h_{t-1})+\lambda_2\frac{(x_{t-1}-x^{\text{target}})}{y_t}, \quad \text{with} \quad \lambda_1, \lambda_2 > 0, \tag{29}$$

and

$$\mathcal{T}_t^r = \mathcal{T}_{t-1} + \lambda_3(\mathcal{T}_t^{\text{target}} - \mathcal{T}_{t-1}) - \lambda_4(x_{t-1} - x^{\text{target}}), \quad \text{with} \quad \lambda_3, \lambda_4 > 0, \tag{30}$$

with $y = P_n q_n + P_x q_x$ and $x = b$ or d_c, depending on whether the rules respond to domestic debt or commercial debt. The target for debt x^{target} is given exogenously. The ceiling h^u on taxes and the floor $\mathcal{T}^l$ on transfers can also have discrete jumps over time, as we show below.[11] This allows us to model staggered tax and transfers structures.

Given the targets, the reaction functions defined in (27)-(30) together with the budget constraint (24) embody the core policy dilemma. Fiscal adjustment is painful, especially when administered suddenly in large doses. The government would prefer therefore to phase-in tax increases and expenditure cuts slowly ($\lambda_1 > 0$ and $\lambda_3 > 0$). Since fiscal adjustment is gradual, the debt instrument that varies endogenously to satisfy the government budget constraint may rise above its target level in the time it takes h_t and $\mathcal{T}_t$ to reach h_t^{target} and $\mathcal{T}_t^{\text{target}}$. When this happens, the transition path includes a phase in which $\mathcal{T}_t < \mathcal{T}_t^{\text{target}}$ and/or $h_t > h_t^{\text{target}}$ to generate the fiscal surpluses needed to pay down the debt. In addition, and despite the fact that the rules respond to debt (i.e., $\lambda_2 > 0$ and $\lambda_4 > 0$), if the government moves too slowly (i.e., λ_1 and λ_3 are too low), or if the bounds h^u and $\mathcal{T}^r$ constrain adjustment too much, interest payments will rise faster than revenue net of transfers, causing the debt to grow explosively. Large debt-financed increases in public investment are undeniably risky—the economy converges to a stationary equilibrium only if policy makers win the race against time.

1.4 Market-Clearing Conditions and External Debt Accumulation

Flexible wages and prices ensure that demand continuously equals supply in the labor market

$$L_x + L_n = L, \tag{31}$$

where the labor supply $L = L^{\mathfrak{s}} + L^{\mathfrak{h}}$ is fixed.

In the non-tradables market, after aggregating across types of consumers, we obtain

$$q_{n,t} = \rho_n \left(\frac{P_{n,t}}{P_t}\right)^{-\epsilon} c_t + a_k \left(i_{x,t} + i_{n,t} + AC_{x,t} + AC_{n,t}\right) + a_z \mathbb{I}_{z,t}. \tag{32}$$

[11] For instance, to introduce discrete jumps in the cap on taxes we can respecify the rule as $h_t = Min\{h_t^r, h_t^u\}$, where $h_t^u = h_o^u + \Delta h_t^u$, h_o^u is the initial cap and Δh_t^u is a discrete jump (shock) at time t, which can be temporary or permanent.

The first term of the right-hand side of (32) is the demand for non-traded consumer goods, while the second and third terms link public and private investment to orders for new capital goods.

Finally, aggregating across consumers and adding the public and private sector budget constraints produce the accounting identity that growth in the country's net foreign debt equals the difference between national spending and national income:

$$\begin{aligned} d_t - d_{t-1} + d_{c,t} - d_{c,t-1} + b_t^* - b_{t-1}^* \quad = \quad & \frac{r_d - g}{1+g} d_{t-1} + \frac{r_{dc,t-1} - g}{1+g} d_{c,t-1} + \frac{r_{t-1}^* - g}{1+g} b_{t-1}^* \\ & + \mathcal{P}_t + P_{z,t} \mathbb{I}_{z,t} + P_{k,t} \left(i_{x,t} + i_{n,t} + AC_{x,t} + AC_{n,t} \right) \\ & + P_t c_t - P_{n,t} q_{n,t} - P_{x,t} q_{x,t} - \mathcal{R}_t - \mathcal{G}_t - \mathcal{N}_t. \end{aligned} \tag{33}$$

Equation (33) includes extra terms that reflect the impact of trend growth on real interest costs and the contributions of remittances, grants, and natural resource related foreign transfers to gross income. The textbook identity emerges when $g = \mathcal{R}_t = \mathcal{G}_t = \mathcal{N}_t = 0$.