1 Appendix

Letting η_t^s , η_t^k and λ_t be the Lagrangian multipliers on equations (??), (??) and (??) respectively, the first order conditions are given by:

$$c_t : [c_t - \varkappa c_{t-1}]^{-\gamma} - \varkappa\beta(1+\mathfrak{g})[c_{t+1} - \varkappa c_t]^{-\gamma} = \lambda_t, \tag{1}$$

$$g_t : [g_t - \varkappa g_{t-1}]^{-\gamma} - \varkappa\beta(1+\mathfrak{g})[g_{t+1} - \varkappa g_t]^{-\gamma} = \lambda_t, \tag{2}$$

$$i_t^k : e_k\eta_t^k = \lambda_t, \tag{3}$$

$$i_t^s : e_s\eta_t^s = \lambda_t, \tag{4}$$

$$d_t : \lambda_t = \beta\lambda_{t+1}\left\{\begin{array}{c} 1 + r^* + \frac{\pi}{\rho_1^2}\left[\exp\left[\rho_1(d_t - \bar{d} - \psi V_t)\right] - \rho_2(d_t - \bar{d} - \psi V_t) - \rho_3\right] \\ +\frac{\pi}{\rho_1^2}\left[\rho_1 \exp\left[\rho_1(d_t - \bar{d} - \psi V_t)\right] - \rho_2\right] d_t \end{array}\right\}, \tag{5}$$

$$k_t : \lambda_t = \beta(1+\mathfrak{g})\lambda_{t+1}\frac{\left[e_k\theta_k\frac{y_{t+1}^n}{k_t} + (1-\delta_k) - e_k\frac{\phi_k}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 + e_k\phi_k\left(\frac{k_{t+1}}{k_t} - 1\right)\frac{k_{t+1}}{k_t}\right]}{\left[(1+\mathfrak{g}) + e_k\phi_k\left(\frac{k_t}{k_{t-1}} - 1\right)\right]}, \tag{6}$$

$$s_t : \lambda_t = \beta(1+\mathfrak{g})\lambda_{t+1}\frac{\left[e_s\theta_s\frac{y_{t+1}^n}{s_t} + (1-\delta_s) - e_s\frac{\phi_s}{2}\left(\frac{s_{t+1}}{s_t} - 1\right)^2 + e_s\phi_s\left(\frac{s_{t+1}}{s_t} - 1\right)\frac{s_{t+1}}{s_t}\right]}{\left[(1+\mathfrak{g}) + e_s\phi_s\left(\frac{s_t}{s_{t-1}} - 1\right)\right]}, \tag{7}$$

$$\eta_t^s : (1+\mathfrak{g})s_{t+1} = e_s i_t^s + (1-\delta_s)s_t, \tag{8}$$

$$\eta_t^k : (1+\mathfrak{g})k_{t+1} = e_k i_t^k + (1-\delta_k)k_t, \tag{9}$$

$$\lambda_t : (1+\mathfrak{g})d_t = (1+r_{t-1})d_{t-1} + c_t + i_t^k + AC_t^k + g_t + i_t^s + AC_t^s - y_t^n - y_t^o - T_t. \tag{10}$$

At steady-state, we find that:

$$[c(1-\varkappa)]^{-\gamma}[1-\varkappa\beta(1+\mathfrak{g})]=\lambda, \tag{11}$$

$$\kappa[g(1-\varkappa)]^{-\gamma}[1-\varkappa\beta(1+\mathfrak{g})]=\lambda, \tag{12}$$

$$e_k\eta^k=\lambda, \tag{13}$$

$$e_s\eta^s=\lambda, \tag{14}$$

$$1=\beta\left[1+r^*+\frac{\pi}{\rho_1^2}[1-\rho_3+(\rho_1-\rho_2)d]\right], \tag{15}$$

$$1=\beta\left[e_k\theta_k\frac{y^n}{k}+(1-\delta_k)\right], \tag{16}$$

$$1=\beta\left[e_s\theta_s\frac{y^n}{s}+(1-\delta_s)\right], \tag{17}$$

$$s(\mathfrak{g}+\delta_s)=e_si^s, \tag{18}$$

$$k(\mathfrak{g}+\delta_k)=e_ki^k, \tag{19}$$

$$(\mathfrak{g}-r)d+y^n+y^o+T=c+i^k+g+i^s. \tag{20}$$

Combining (11) and (12):

$$\kappa=\left(\frac{g}{c}\right)^{\gamma} \tag{21}$$