

# fn noisy\_top\_k

Michael Shoemate

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This proof resides in “**contrib**” because it has not completed the vetting process.

This document proves soundness of `noisy_top_k` in `mod.rs` at commit `e62b0aa2` (outdated<sup>1</sup>). `noisy_top_k` noisily selects the index of the best score from a vector of input scores  $k$  times.

## 1 Hoare Triple

### Preconditions

#### Compiler-Verified

*Types consistent with pseudocode.*

#### Caller-Verified

- Each item of `x` is finite.

### Pseudocode

```
1 def noisy_top_k(
2     x: list[TIA],
3     scale: RBig,
4     k: usize,
5     negate: bool,
6     replacement: bool,
7 ):
8     sign = Sign.from_(negate)
9     scale = scale.into_rational()
10
11    y = [x_i.into_rational() * sign for x_i in x] # 
12    return peel_permute_and_flip(y, scale, k, replacement)
```

### Postcondition

**Theorem 1.1.** • If replacement is set, returns a sample from  $\mathcal{M}_{EM}$  (as defined in MS2023 Definition 4), otherwise returns a sample from  $\mathcal{M}_{PF}$  (as defined in MS2023 Lemma 1),  $k$  times by peeling, where  $scale = \frac{2\Delta}{\epsilon}$ .

- Errors are data-independent, except for exhaustion of entropy.

<sup>1</sup>See new changes with `git diff e62b0aa2..76ab1a9 rust/src/measurements/noisy_top_k/mod.rs`

*Proof.* By the precondition that each element in  $x$  is finite, the conversion into rational is infallible.

By the postcondition of `peel_permute_and_flip`, and the potential negation on line 11, the postcondition is satisfied.

The only source of error is due to entropy exhaustion. □