`fn accuracy_to_discrete_gaussian_scale`

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This document contains materials associated with `accuracy_to_discrete_gaussian_scale`.

By `discrete_gaussian_scale_to_accuracy`, the relationship between α, a and $scale$, is:

$$1 - \alpha = \frac{\sum_{y=0}^{a-1} (1 + 1[y \neq 0]) e^{-(y/s)^2/2}}{\sum_{z \in \mathbb{Z}} e^{-(z/s)^2/2}}$$

A closed-form expression for s doesn't exist, so we use a numerical approach by a binary search. A loose upper bound is provided by `accuracy_to_gaussian_scale`. The binary search finds the smallest s such that

$$\alpha \leq 1 - \frac{\sum_{y=0}^{a-1} (1 + 1[y \neq 0]) e^{-(y/s)^2/2}}{\sum_{z \in \mathbb{Z}} e^{-(z/s)^2/2}}$$