## fn accuracy\_to\_discrete\_laplacian\_scale

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This document contains materials associated with accuracy\_to\_discrete\_laplacian\_scale. By discrete\_laplacian\_scale\_to\_accuracy, the relationship between  $\alpha$ , a and scale, is:

$$\alpha = 2\frac{e^{(1-a)/s}}{e^{1/s} + 1}$$

A closed-form expression for s doesn't exist, so we use a numerical approach by a binary search. A loose upper bound is provided by accuracy\_to\_laplacian\_scale. The binary search finds the smallest s such that

$$\alpha \le 2\frac{e^{(1-a)/s}}{e^{1/s}+1}$$