# fn make_select_private_candidate

Michael Shoemate

This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of `make_select_private_candidate` in `mod.rs` at commit f5bb719 (outdated[1]).

`make_select_private_candidate` returns a Measurement that returns a release from `measurement` whose score is above `threshold`, or may fail and return nothing.

## 1 Hoare Triple

### Precondition

#### Compiler-verified

- Generic `DI` (input domain) is a type with trait `Domain`.
- Generic `MI` (input metric) is a type with trait `Metric`.
- `MetricSpace` is implemented for `(DI, MI)`. Therefore `MI` is a valid metric on `DI`.
- Argument `measurement` is a measurement whose output metric is `MaxDivergence`, and releases a tuple `(f64, TO)`, where `TO` is some arbitrary type.
- Argument `stop_probability` is of type `f64`.
- Argument `threshold` is of type `f64`.

#### User-verified

None

### Pseudocode

```
def make_select_private_candidate(
    measurement: Measurement,
    stop_probability: float,
    threshold: float,
) -> Measurement:
    if not 0 <= stop_probability < 1: #
        raise "stop_probability must be in [0, 1)"

    if not threshold.is_finite():
        raise "threshold must be finite"

    scale = None
    if stop_probability > 0.0:
```

[1] See new changes with `git diff f5bb719..9d49603 rust/src/combinators/select_private_candidate/mod.rs`

```
        ln_cp = (1.0).neg_inf_sub(stop_probability).inf_ln()
        scale = ln_cp.recip().neg().into_rational() #

    def function(arg):
        remaining_iterations = None
        if scale is not None:
            remaining_iterations = UBig.ONE + sample_geometric_exp_fast(scale) #

        while True:
            score, output = measurement(arg)

            if score >= threshold:
                return score, output

            if remaining_iterations is not None:
                remaining_iterations -= UBig.ONE
                if remaining_iterations == UBig.ZERO:
                    return None

    return Measurement(
        input_domain=measurement.input_domain,
        input_metric=measurement.input_metric,
        output_measure=measurement.output_measure,
        function=function,
        privacy_map=lambda d_in: measurement.map(d_in).inf_mul(2),
    )
```

### Postcondition

For every setting of the input parameters `(measurement, stop_probability, threshold, DI, MI, TO)` to `make_select_private_candidate` such that the given preconditions hold, `make_select_private_candidate` raises an error (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of members $x$ and $x'$ in `input_domain`, `invoke`$(x)$ and `invoke`$(x')$ either both return the same error or neither return an error.

2. (Privacy guarantee). For every pair of members $x$ and $x'$ in `input_domain` and for every pair (`d_in`, `d_out`), where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for
`output_measure`, if $x, x'$ are `d_in`-close under `input_metric`, `privacy_map`(`d_in`) does not raise an error, and `privacy_map`(`d_in`) $=$ `d_out`, then `function`$(x)$, `function`$(x')$ are `d_out`-close under `output_measure`.

## 2 Proofs

This section shows that the pseudocode implements Theorem 3.1 in [1], where `stop_probability` is $\gamma$ and `threshold` is $\tau$.

**Theorem 2.1.** The function pseudocode is equivalent to Algorithm 1 in [1].

*Proof.* `scale` is $-1/\ln(1-\gamma)$, with arithmetic conservatively rounded down. The rounding down corresponds to higher effective stop probability. A higher effective stop probability results in a lower effective privacy usage $\epsilon_0$. This means the advertised privacy loss is a conservative overestimate.

Since $\gamma$ is restricted to $[0, 1)$ by 6, scale is non-negative. Therefore in 20, the precondition of `sample_geometric_exp_fast` is satisfied. In Algorithm 1, the mechanism is invoked once before potentially terminating, so one is added to the sample. `remaining_iterations`, a sample from the shifted geometric distribution, is then equivalent to a count of the number of coin flips made until sampling one heads, as is used in Algorithm 1.

We then only run as many iterations as has been sampled, as in Algorithm 1. In the case where $T$ is infinity and $\gamma$ is zero, then the algorithm only terminates when score exceeds threshold.

If the measurement releases a score of NaN, then this is effectively treated as a score below negative infinity, as NaN will never compare greater than the threshold.

Otherwise, the rest of the algorithm is evidently equivalent. □

**Theorem 2.2.** `make_select_private_candidate` is consistent with Theorem 3.1 of [1].

*Proof.* Since there is no limit on iterations, $\epsilon_0$ is zero. By 2.1, the function is equivalent to Algorithm 1, so we can claim parts a-e of Theorem 3.1. Since `measurement` is a valid measurement, then we know that it satisfies $\epsilon_1$-DP. The privacy map returns $2\epsilon_1 + \epsilon_0$ with arithmetic rounded conservatively up, which is consistent with part b in Theorem 3.1. □

**(Privacy guarantee.)** By 2.2, assuming correctness of Theorem 3.1 part b in [1], then for every pair of elements $x, x' \in$ `input_domain` and every $d_{MI}(x, x') \leq$ `d_in` with `d_in` $\geq 0$, if $x, x'$ are `d_in`-close then `function`$(x)$, `function`$(x')$ are `privacy_map`(`d_in`)-close under `output_measure` (the Max-Divergence).

# References

[1] Jingcheng Liu and Kunal Talwar. Private selection from private candidates, 2018.