

# fn make\_row\_by\_row\_fallible

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of `make_row_by_row_fallible` in `mod.rs` at commit `f5bb719` (outdated<sup>1</sup>).

`make_row_by_row_fallible` returns a Transformation that applies a user-specified function to each record in the input dataset. The function is permitted to return a data-independent error.

## Vetting History

- Pull Request #562

## 1 Hoare Triple

### Precondition

- DI (input domain) is a type with trait `RowByRowDomain<D0>`. This trait provides a way to apply a map function to each record in the input dataset to retrieve a dataset that is a member of the output domain, of type D0. The trait further implies that `DatasetDomain` is also implemented for DI.
- D0 (output domain) is a type with trait `DatasetDomain`. `DatasetDomain` is used to define the type of the row domain.
- M (metric) is a type with trait `DatasetMetric`. `DatasetMetric` is used to restrict the set of valid metrics to those which measure distances between datasets.
- `MetricSpace` is implemented for (DI, M). Therefore M is a valid metric on DI.
- `MetricSpace` is implemented for (D0, M).
- `row_function` has no side-effects.
- If the input to `row_function` is a member of `input_domain`'s row domain, then the output is a member of `output_row_domain`, or a data-independent error.

### Pseudocode

```
1 def make_row_by_row_fallible(
2     input_domain: DI,
3     input_metric: M,
4     output_row_domain: D0,
5     # a function from input domain's row type to output domain's row type
6     row_function: Callable([[DI_RowDomain_Carrier], D0_RowDomain_Carrier])
7 ) -> Transformation:
```

<sup>1</sup>See new changes with git diff f5bb719..eb7ad44 rust/src/transformations/manipulation/mod.rs

```

9   # where .translate is defined by the RowByRowDomain trait
10  output_domain = input_domain.translate(output_row_domain)
11
12  def function(data: DI_Carrier) -> DO_Carrier:
13      # where .apply_rows is defined by the RowByRowDomain trait
14      return DI.apply_rows(data, row_function)
15
16  stability_map = new_stability_map_from_constant(1) #
17
18  return Transformation(
19      input_domain, output_domain, function,
20      input_metric, input_metric, stability_map)

```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`input_domain`, `input_metric`, `output_domain`, `row_function`, `DI`, `DO`, `M`) to `make_row_by_row` such that the given preconditions hold, `make_row_by_row` raises an error (at compile time or run time) or returns a valid transformation. A valid transformation has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Appropriate output domain). For every member  $x$  in `input_domain`, `function(x)` is in `output_domain` or raises a data-independent runtime error.
3. (Stability guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_metric`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `stability_map(d_{in})` does not raise an error, and `stability_map(d_{in}) = d_{out}`, then `function(x), function(x')` are  $d_{out}$ -close under `output_metric`.

## 2 Proofs

*Proof. (Part 1 – appropriate output domain).* By the definition of `RowByRowDomain`, `DI.apply_rows(data, row_function)` returns a dataset in `input_domain.translate(output_row_domain)`, if `row_function` is a mapping between `input_domain`'s row domain to `output_row_domain`. This is satisfied by the precondition on `row_function`. Thus, for all settings of input arguments, the function returns a dataset in the output domain.  $\square$

Before proceeding with proving the validity of the stability map, we first provide a lemma.

**Lemma 2.1.** Let  $f$  denote the `row_function`. For any choice  $u, v$  of input arguments in the input domain, and any choice  $M$  for which `DatasetMetric` is implemented for,  $d_M([f(u_1), f(u_2), \dots], [f(v_1), f(v_2), \dots]) \leq d_M([u_1, u_2, \dots], [v_1, v_2, \dots])$ .

*Proof.* Assume WLOG that any source of randomness is fixed when  $f$  is computed on  $u$  vs  $v$ . Given this assumption, and the precondition that  $f$  has no side-effects, if  $u_i = v_i$ , then  $f(u_i) = f(v_i)$ . That is, the row function cannot increase the distance between corresponding rows in any adjacent dataset. On the other hand, it is possible for  $f(u_i) = f(v_i)$ , even if  $u_i \neq v_i$ . For example, if  $f$  is a constant function, then  $f(u_i) = f(v_i)$  for all  $i$ . Therefore, by any of the metrics that `DatasetMetric` is implemented for,  $f$  can only make datasets more similar.  $\square$

*Proof. (Part 2 – stability map).* Take any two elements  $u, v$  in the `input_domain` and any pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$

has the associated type for `output_metric`. Assume  $u, v$  are  $d_{in}$ -close under `input_metric` and that  $\text{stability\_map}(d_{in}) \leq d_{out}$ .

$$\begin{aligned}
d_M(\text{function}(u), \text{function}(v)) &= d_M([f(u_1), f(u_2), \dots], [f(v_1), f(v_2), \dots]) && \text{since D0 is a DatasetDomain} \\
&\leq d_M([u_1, u_2, \dots], [v_1, v_2, \dots]) && \text{by 2.1} \\
&= d_M(u, v) && \text{since DI is a DatasetDomain} \\
&= d_{in} && \text{by the first assumption} \\
&\leq T0.inf\_cast(d_{in}) && \text{by InfCast} \\
&\leq T0.one().inf\_mul(T0.inf\_cast(d_{in})) && \text{by InfMul} \\
&= \text{stability\_map}(d_{in}) && \text{by pseudocode line 16} \\
&\leq d_{out} && \text{by the second assumption}
\end{aligned}$$

It is shown that `function(u)`, `function(v)` are  $d_{out}$ -close under `output_metric`.  $\square$