

# CompositionMeasure for Approximate<ZeroConcentratedDivergence>

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of **CompositionMeasure** for **Approximate<ZeroConcentratedDivergence>** in **mod.rs** at **commit f5bb719** (outdated<sup>1</sup>).

## 1 Hoare Triple

### Precondition

### Compiler-Verified

Types matching pseudocode.

### Caller-Verified

None

### Pseudocode

```
1 class CompositionMeasure(ApproximateZeroConcentratedDivergence):
2     def composability( #
3         self, adaptivity: Adaptivity
4     ) -> Composability:
5         return Composability.Sequential
6
7     def compose(self, d_mids: Vec[Self_Distance]) -> Self_Distance:
8         rho_g, del_g = 0.0, 0.0
9         for rho_i, del_i in d_mids:
10             rho_g = rho_g.inf_add(rho_i)
11             del_g = del_g.inf_add(del_i)
12         return rho_g, del_g
```

### Postcondition

**Theorem 1.1.** `composability` returns `Ok(out)` if the composition of a vector of privacy parameters `d_mids` is bounded above by `self.compose(d_mids)` under `adaptivity` `adaptivity` and `out-composability`. Otherwise returns an error.

*Proof.* By the postcondition of **InfAdd** we have that  $\sum_i d\_mids_i \leq \text{compose}(d\_mids)$ , where the summation is applied independently to rhos and deltas, and the comparison applies to both the global rho and global delta.

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<sup>1</sup>See new changes with `git diff f5bb719..712ba41 rust/src/combinators/sequential_composition/mod.rs`

| Adaptivity     | Sequential        | Concurrent |
|----------------|-------------------|------------|
| Non-Adaptive   | Lemma 8.2[BS16]   | None       |
| Adaptive       | Lemma 8.2[BS16]   | None       |
| Fully-Adaptive | Theorem 1[WRRW23] | None       |

This table is reflected in the implementation of `composability` on line 2.

□

## References

- [BS16] Mark Bun and Thomas Steinke. Concentrated differential privacy: Simplifications, extensions, and lower bounds, 2016.
- [WRRW23] Justin Whitehouse, Aaditya Ramdas, Ryan Rogers, and Zhiwei Steven Wu. Fully adaptive composition in differential privacy, 2023.