

fn make_stable_truncate

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of `make_stable_truncate` in `mod.rs` at commit `f5bb719` (outdated¹).

1 Hoare Triple

Precondition

Caller Verified

None

Function

```
1 def make_stable_truncate(  
2     input_domain: DslPlanDomain,  
3     input_metric: FrameDistance[SymmetricIdDistance],  
4     plan: DslPlan,  
5 ) -> Transformation[  
6     DslPlanDomain,  
7     DslPlanDomain,  
8     FrameDistance[SymmetricIdDistance],  
9     FrameDistance[SymmetricDistance],  
10 ]:  
11     # the identifier is protected from changes, so we can use the identifier from the input  
12     # metric  
13     # instead of the identifier from the middle_metric to match truncations  
14     input, truncations, truncation_bounds = match_truncations(  
15         plan, input_metric[0].identifier  
16     )  
17     if truncations.is_empty():  
18         return ValueError("failed to match truncation")  
19  
20     t_prior = input.make_stable(input_domain, input_metric)  
21     middle_domain, middle_metric = t_prior.output_space()  
22  
23     for bound in truncation_bounds:  
24         for key in bound.by:  
25             # raises if the key is not infallible row-by-row  
26             make_stable_expr( #  
27                 WildExprDomain(  
28                     columns=middle_domain.series_domains,
```

¹See new changes with `git diff f5bb719..9817d0ef rust/src/transformations/make_stable_lazyframe/truncate/mod.rs`

```

29         context=Context.RowByRow,
30     ),
31     PartitionDistance(middle_metric[0]),
32     key,
33 )
34
35 output_domain = middle_domain.clone()
36 for truncation in truncations: #
37     output_domain = truncate_domain(output_domain, truncation)
38
39 def function(plan: DslPlan) -> DslPlan:
40     for truncation in truncations:
41         match truncation:
42             case Truncation.Filter(predicate):
43                 plan = DslPlan.Filter(
44                     input=plan,
45                     predicate=predicate,
46                 )
47
48             case Truncation.GroupBy(keys, aggs):
49                 plan = DslPlan.GroupBy(
50                     input=plan,
51                     keys=keys,
52                     aggs=aggs,
53                     apply=None,
54                     maintain_order=False,
55                     options=GroupbyOptions.default(),
56                 )
57     return plan
58
59 def stability_map(id_bounds: Bounds) -> Bounds:
60     #
61     total_num_ids = id_bounds.get_bound({}).per_group
62
63     # each truncation is used to derive row bounds
64     new_bounds = []
65     for truncation_bound in truncation_bounds: #
66         # each truncation is used to derive row bounds
67         new_bounds.append(
68             truncate_id_bound( #
69                 id_bounds.get_bound(truncation_bound.by), #
70                 truncation_bound,
71                 total_num_ids,
72             )
73         )
74     return Bounds(new_bounds)
75
76 t_truncate = Transformation.new(
77     middle_domain,
78     output_domain,
79     Function.new(function),
80     middle_metric,
81     FrameDistance(SymmetricDistance),
82     StabilityMap.new_fallible(stability_map),
83 )
84 return t_prior >> t_truncate

```

Postcondition

Theorem 1.1. For every setting of the input parameters (`input_domain`, `input_metric`, `plan`) to `make_stable_truncate` such that the given preconditions hold, `make_stable_truncate` raises an exception (at compile time or run time) or returns a valid transformation. A valid transformation has the following properties:

1. (Appropriate output domain). For every element x in `input_domain`, `function(x)` is in `output_domain` or raises a data-independent runtime exception.
2. (Stability guarantee). For every pair of elements x, x' in `input_domain` and for every pair (d_in, d_out) , where d_in has the associated type for `input_metric` and d_out has the associated type for `output_metric`, if x, x' are d_in -close under `input_metric`, `stability_map(d_in)` does not raise an exception, and `stability_map(d_in) ≤ d_out`, then `function(x), function(x')` are d_out -close under `output_metric`.

Appropriate Output Domain. By line 26, the grouping keys are stable row-by-row transformations of the data, therefore the preconditions of 36 are satisfied. By the postcondition of 36, for every element x in `input_domain`, `function(x)` is in `output_domain` or raises a data-independent runtime exception. \square

Stability guarantee. By line 60, `total_num_ids` is the total number of ids an individual may contribute to a dataset:

$$d_{\text{SymId}}(\text{function}(x), \text{function}(x')) \leq \text{total_num_ids}. \quad (1)$$

By the postcondition of `match_truncations`, for each `truncation_bound` on line 65,

$$\begin{aligned} \max_{id} ||d_{\text{Sym}}(\text{function}(x)_{id,g}, \text{function}(x')_{id,g})||_{\infty} &\leq \text{truncation.per_group}, \\ \max_{id} ||d_{\text{Sym}}(\text{function}(x)_{id,g}, \text{function}(x')_{id,g})||_0 &\leq \text{truncation.num_groups}, \end{aligned}$$

where g denotes the group when partitioned by `truncation_bound.by`.

The preconditions of 68 are satisfied on line 69 (`id_bound.by` is equal to `truncation.by`), so by the postcondition of `truncate_id_bound`,

$$\begin{aligned} ||d_{\text{Sym}}(\text{function}(x)_g, \text{function}(x')_g)||_{\infty} &\leq \text{row_bound.per_group}, \\ ||d_{\text{Sym}}(\text{function}(x)_g, \text{function}(x')_g)||_0 &\leq \text{row_bound.num_groups}, \end{aligned}$$

where `row_bound` denotes the return value.

For each truncation on line 65, `truncate_id_bound` on line 68 computes upper bounds on the resulting distance between adjacent datasets. All acquired bounds are valid upper bounds on the distance between the two datasets, by the postcondition of `match_truncations`, that each truncation does not invalidate the truncation bounds of the previous truncations.

It is shown that for every pair of elements x, x' in `input_domain` and for every pair (d_in, d_out) , where d_in has the associated type for `input_metric` and d_out has the associated type for `output_metric`, if x, x' are d_in -close under `input_metric`, `stability_map(d_in)` does not raise an exception, and `stability_map(d_in) ≤ d_out`, then `function(x), function(x')` are d_out -close under `output_metric`. \square