

# MakeNoise<AtomDomain<T>, AbsoluteDistance<T>, M0> for ConstantTimeGeometric<T>

Michael Shoemate

This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of `MakeNoise` over scalars for `ConstantTimeGeometric` in `mod.rs` at commit [f5bb719](#) (outdated<sup>1</sup>).

The intuition of this implementation is that a vector-valued mechanism can be used to privatize a scalar-valued input, by transforming the input into a singleton vector, applying the vector mechanism, and then unpacking the resulting singleton vector.

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

`MakeNoise` is parameterized as follows:

- DI is of type `AtomDomain<T>`
- MI is of type `AbsoluteDistance<T>`
- M0 implements trait `Measure`

The following trait bounds are also required:

- Generic T implements trait `Integer`
- Generic M0 implements trait `Measure`
- Type `usize` implements trait `ExactIntCast<T>`
- Type `RBig` implements trait `TryFrom<T>`
- Type `ZExpFamily<1>` implements trait `NoisePrivacyMap<L1Distance<RBig>, M0>`. This bound requires that it must be possible to construct a privacy map for the combination of `ZExpFamily<1>` noise distribution, distance type and privacy measure. Since the `ConstantTimeGeometric` distribution is equivalent to `ZExpFamily<1>`, maps built for `ZExpFamily<1>` can be used for `ConstantTimeGeometric`.

### User-Verified

None

---

<sup>1</sup>See new changes with `git diff f5bb719..b6fd53d rust/src/measurements/noise/distribution/geometric/mod.rs`

## Pseudocode

```
1 # analogous to impl MakeNoise<AtomDomain<T>, AbsoluteDistance<T>, MO> for
  ConstantTimeGeometric<T> in Rust
2 class ConstantTimeGeometric:
3     def make_noise(
4         self, input_space: tuple[AtomDomain[T], AbsoluteDistance[QI]]
5     ) -> Measurement[AtomDomain[T], T, AbsoluteDistance[QI], MO]:
6         t_vec = make_vec(input_space) #
7         m_noise = self.make_noise(t_vec.output_space()) #
8
9     return t_vec >> m_noise >> then_index_or_default(0) #
```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`self`, `input_space`, `T`, `MO`) to `make_noise` such that the given preconditions hold, `make_noise` raises an error (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Privacy guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_measure`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `privacy_map(d_{in})` does not raise an error, and `privacy_map(d_{in}) = d_{out}`, then `function(x), function(x')` are  $d_{out}$ -close under `output_measure`.

*Proof.* Neither constructor `make_vec` nor `MakeNoise.make_noise` have manual preconditions, and the postconditions guarantee a valid transformation and valid measurement, respectively. `then_index_or_default` also does not have preconditions, and its postcondition guarantees that it returns a valid postprocessor.

The chain of a valid transformation, valid measurement and valid postprocessor is a valid measurement. □