impl SelectionMeasure for RangeDivergence

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1 Hoare Triple

Precondition

Compiler-verified

- Associated Type `RV`
  - `RV` must implement trait `InverseCDF`.
- Method `random_variable` *Types consistent with pseudocode.*
- Method `privacy_map` *Types consistent with pseudocode.*

Caller-verified

- Associated Type `RV` (*no caller verified preconditions*)
- Method `random_variable`
  - `scale` is positive (*cannot be null due to `FBig` dtype*).
- Method `privacy_map`
  - `d_in` is non-null and non-negative.
  - `scale` is non-null and non-negative.

Pseudocode

```python
class RangeDivergence(SelectionMeasure):
    ONE_SHOT = True
    RV = GumbelRV

    @staticmethod
    def random_variable(shift: FBig, scale: FBig) -> GumbelRV:
        return GumbelRV(shift=shift, scale=scale)

    @staticmethod
    def privacy_map(d_in: f64, scale: f64, k: usize) -> f64:
        if d_in < 0:
            raise ValueError("input distance must be non-negative")

        if scale.is_zero():
            return f64.INFINITY

        return d_in.inf_div(scale).inf_mul(f64.inf_cast(k))
```

Postcondition

Theorem 1.1. The implementation is consistent with all associated items in the `SelectionMeasure` trait.

1. Associated Type `RV`
2. Method `random_variable`
3. Method `privacy_map`

Definition 1.2. A random variable follows the Gumbel distribution if it has density

$$f(x) = \frac{1}{\beta} e^{-e^{-e^{-z}} - z} \tag{1}$$

where $z = \frac{x-\mu}{\beta}$, μ is the location parameter and β is the scale parameter.

Proof of valid associated type: `RV`. The associated type `RV` is defined as `GumbelRV`, which represents a random variable following the Gumbel distribution 1.2. The compiler verifies that `GumbelRV` implements the `InverseCDF` trait. □

Proof of valid method: `random_variable`. By the precondition on `scale` being positive, `random_variable` returns a valid instance of `GumbelRV`. □

Proof of valid method: `privacy_map`. By Lemma 4.2 of [2], $\mathcal{M}^k_{\text{Gumbel}}(x)$ is equal in distribution to the peeling exponential mechanism, which is the k-fold composition of the exponential mechanism. Proposition 2 of [1] shows that the exponential mechanism satisfies the RangeDivergence privacy guarantee. □

References

[1] Jinshuo Dong, David Durfee, and Ryan Rogers. Optimal differential privacy composition for exponential mechanisms and the cost of adaptivity. *CoRR*, abs/1909.13830, 2019.

[2] David Durfee and Ryan Rogers. Practical differentially private top-k selection with pay-what-you-get composition. *CoRR*, abs/1905.04273, 2019.