CompositionMeasure for MaxDivergence

Michael Shoemate

This proof resides in "contrib" because it has not completed the vetting process.

Proves soundness of the implementation of CompositionMeasure for MaxDivergence in mod.rs at commit f5bb719 (outdated¹).

1 Hoare Triple

Precondition

Compiler-Verified

Types matching pseudocode.

Caller-Verified

None

Pseudocode

```
class CompositionMeasure(MaxDivergence):
    def composability( #
        self, adaptivity: Adaptivity
    ) -> Composability:
        if matches(adaptivity, Adaptivity.FullyAdaptive):
            raise "fully-adaptive composition is not currently supported for max-divergence"
        return Composability.Concurrent

def compose(self, d_mids: Vec[Self_Distance]) -> Self_Distance:
        d_out = 0.0
    for d_mid in d_mids:
        d_out = d_out.inf_add(d_mid)
    return d_out
```

Postcondition

Theorem 1.1. composability returns Ok(out) if the composition of a vector of privacy parameters d_mids is bounded above by self.compose(d_mids) under adaptivity adaptivity and out-composability. Otherwise returns an error.

Proof. By the postcondition of InfAdd we have that $\sum_i d_{mids_i} \leq compose(d_{mids})$.

Adaptivity	Sequential	Concurrent
Non-Adaptive	Theorem 1[DMNS06]	Theorem 1.8[VW21]
Adaptive	Theorem 1[DMNS06]	Theorem $1.8[VW21]$
Fully-Adaptive	None	None

¹See new changes with git diff f5bb719..05d7909 rust/src/combinators/sequential_composition/mod.rs

This table is reflected in the implementation of composability on line 2.

References

[DMNS06] Cynthia Dwork, Frank McSherry, Kobbi Nissim, and Adam Smith. Calibrating noise to sensitivity in private data analysis. In Shai Halevi and Tal Rabin, editors, *Theory of Cryptography*, pages 265–284, Berlin, Heidelberg, 2006. Springer Berlin Heidelberg.

[VW21] Salil Vadhan and Tianhao Wang. Concurrent composition of differential privacy, 2021.