

# fn sample\_geometric\_exp\_slow

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This proof resides in “contrib” because it has not completed the vetting process.

Proves soundness of `fn sample_geometric_exp_slow` in `mod.rs` at commit `0be3ab3e6` (outdated<sup>1</sup>). This proof is an adaptation of subsection 5.2 of [CKS20].

## Vetting history

- Pull Request #519

## 1 Hoare Triple

### Precondition

$x \in \mathbb{Q} \wedge x > 0$

### Pseudocode

```
1 def sample_geometric_exp_slow(x) -> int:
2     k = 0
3     while True:
4         if sample_bernoulli_exp(x): #
5             k += 1
6         else:
7             return k
```

### Postcondition

For any setting of the input parameter  $x$  such that the given preconditions hold, `sample_geometric_exp_slow` either returns `Err(e)` due to a lack of system entropy, or `Ok(out)`, where `out` is distributed as  $Geometric(1 - \exp(-x))$ .

## 2 Proof

Assume the preconditions are met.

**Lemma 2.1.** `sample_geometric_exp_slow` only returns `Err(e)` when there is a lack of system entropy.

*Proof.* The preconditions on  $x$  satisfy the preconditions on `sample_bernoulli_exp`, so by its definition, it only returns an error if there is a lack of system entropy. The only source of errors is from this function, therefore `sample_geometric_exp_slow` only returns `Err(e)` when there is a lack of system entropy.  $\square$

<sup>1</sup>See new changes with `git diff 0be3ab3e6..5da95cf rust/src/traits/samplers/cks20/mod.rs`

**Theorem 2.2.** [CKS20] If the outcome of `sample_geometric_exp_slow` is `Ok(out)`, then `out` is distributed as  $Geometric(1 - \exp(-x))$ . That is,  $P[\text{out} = k] = \exp(-x)(1 - \exp(-x))^k$

*Proof.* The distribution of the  $i^{th}$  boolean returned on line 4 is  $B_i \sim Bernoulli(\exp(x))$ , because the preconditions on `x` satisfy the preconditions for `sample_bernoulli_exp`.

$$\begin{aligned}
P[\text{out} = k] &= P[B_1 = B_2 = \dots = B_k = \perp \wedge B_{k+1} = \top] \\
&= P[B_{k+1} = \top] \prod_{i=1}^k P[B_i = \perp] && \text{All } B_i \text{ are independent.} \\
&= \exp(-x)(1 - \exp(-x))^k
\end{aligned}$$

□

*Proof.* 1 holds by 2.1 and 2.2.

□

## References

[CKS20] Clément L. Canonne, Gautam Kamath, and Thomas Steinke. The discrete gaussian for differential privacy. *CoRR*, abs/2004.00010, 2020.