fn exponential_top_k

Michael Shoemate

August 29, 2025

This proof resides in **"contrib"** because it has not completed the vetting process.

This document proves soundness of `exponential_top_k` in `mod.rs` at commit e62b0aa2 (outdated[1]). `exponential_top_k` noisily selects the index of the best score from a vector of input scores k times without replacement.

1 Hoare Triple

Preconditions

Compiler-Verified

Types consistent with pseudocode.

Caller-Verified

- Each item of `x` is finite.

Pseudocode

```
def exponential_top_k(x: list[TIA], scale: RBig, k: usize, negate: bool):
    sign = Sign.from_(negate)
    scale = scale.into_rational()

    y = [x_i.into_rational() * sign for x_i in x]  #
    return peel_permute_and_flip(y, scale, k)
```

Postcondition

Theorem 1.1.
- Returns the index of the top element z_i, where each $z_i \sim \text{Exp}(\text{shift} = y_i, \text{scale} = \texttt{scale})$, and each $y_i = -x_i$ if `negate`, else $y_i = x_i$, k times with removal.
- Errors are data-independent, except for exhaustion of entropy.

Proof. By the precondition that each element in x is finite, the conversion into rational is infallible.

By the postcondition of `peel_permute_and_flip`, and the potential negation on line 5, the postcondition is satisfied.

The only source of error is due to entropy exhaustion. □

[1] See new changes with `git diff e62b0aa2..b0ed5c8 rust/src/measurements/noisy_top_k/exponential/mod.rs`