fn binary_search_by_mut

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1 Hoare Triple

Precondition

Compiler-verified

Types consistent with pseudocode.

Caller-verified

- The predicate `pred` is monotonic over `data`.
- `pred` may mutate its argument, but not change its true value used for comparisons.

Pseudocode

```
def binary_search_by_mut(
    x: list[T],
    f: Callable[[T], Literal["less", "greater"]],
) -> int:
    size = x.len()
    if size == 0:  #
        return 0

    base = 0 #

    while size > 1:
        half = size // 2
        mid = base + half

        cmp = f(x[mid])
        base = base if "greater" == cmp else mid

        size -= half

    cmp = f(x[base])
    return base + int(cmp == "less")
```

Postcondition

Theorem 1.1. Returns the index i of the first element in x that is less than $f(x_i)$, or an error if the comparator fails.

Proof. Let n be the length of the list `x`.

We perform a binary search on `x` using the comparator `f`. The binary search algorithm works by repeatedly dividing the search interval in half. If the value of the comparator at the midpoint is less, we narrow the interval to the lower half; otherwise, we narrow it to the upper half.

On line 6, the algorithm terminates early if the list is empty, to avoid an out-of-bounds error. On line 9, `base` is 0 and `size` is n, spanning the entire list.

1. **Initialization**:
   - At the start, `base` is 0 and `size` is n.
   - The loop invariant holds because `base` is 0 and `high` is n.
2. **Loop Invariant**:
   - At the start of each iteration, `base` is the smallest index such that all indices i before `base` satisfy `f(x[i]) == "less"`.
   - At the start of each iteration, `base + size` is the smallest index such that all indices i from `base + size` onwards satisfy `f(x[i]) != "less"`.
3. **Termination**: The loop terminates when the width of the interval is two (`size` is one).

The function determines whether to return the index of the first or second element in the length-2 interval by evaluating `f(x[base])`. By the loop invariant, the index returned is the first element in the list that satisfies the predicate `f(x[i]) == "less"`. □