

# fn make\_float\_to\_bigint

Michael Shoemate

This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of `make_float_to_bigint` in `mod.rs` at commit f5bb719 (outdated<sup>1</sup>).

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

- Generic T implements trait `Float`
- Const-generic P is of type `usize`
- Generic QI implements trait `Number`
- Type `RBig` implements traits `TryFrom<T>` and `TryFrom<QI>`. This is for fallible exact casting to rationals from floats in the function and input sensitivity in the privacy map.
- Type `i32` implements trait `ExactIntCast<T as FloatBits>::Bits>`, This requirement means that the raw bits of T can be exactly cast to an `i32`.

#### User-Verified

None

### Pseudocode

```
1 def make_float_to_bigint(
2     input_space: tuple[VectorDomain[AtomDomain[T]], LpDistance[P, QI]], k: i32
3 ) -> Transformation[
4     VectorDomain[AtomDomain[T]],
5     VectorDomain[AtomDomain[IBig]],
6     LpDistance[P, QI],
7     LpDistance[P, RBig],
8 ]:
9     input_domain, input_metric = input_space
10    if input_domain.element_domain.nullable():
11        raise "input_domain may not contain NaN elements"
12    size = input_domain.size
13    rounding_distance = get_rounding_distance(k, size, T) #
```

<sup>1</sup>See new changes with `git diff f5bb719..efc395b rust/src/measurements/noise/nature/float/mod.rs`

```

16     def elementwise_function(x_i): #
17         x_i = RBig.try_from(x_i).unwrap_or(RBig.ZERO) #
18         return find_nearest_multiple_of_2k(x_i, k) #
19
20     def stability_map(d_in):
21         try:
22             d_in = RBig.try_from(d_in)
23         except Exception:
24             raise f"d_in ({d_in}) must be finite"
25         return x_mul_2k(d_in + rounding_distance, -k) #
26
27     return Transformation.new(
28         input_domain,
29         VectorDomain( #
30             element_domain=AtomDomain.default(IBig),
31             size=size,
32         ),
33         Function.new(lambda x: [elementwise_function(x_i) for x_i in x]),
34         input_metric,
35         LpDistance.default(),
36         StabilityMap.new_fallible(stability_map),
37     )

```

## Postcondition

### Theorem 1.1.

**Theorem 1.2.** For every setting of the input parameters (`input_space`, `k`, `T`, `P`, `QI`) to `make_float_to_bigint` such that the given preconditions hold, `make_float_to_bigint` raises an error (at compile time or run time) or returns a valid transformation. A valid transformation has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Appropriate output domain). For every member  $x$  in `input_domain`, `function(x)` is in `output_domain` or raises a data-independent runtime error.
3. (Stability guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair `(d_in, d_out)`, where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for `output_metric`, if  $x, x'$  are `d_in`-close under `input_metric`, `stability_map(d_in)` does not raise an error, and `stability_map(d_in) = d_out`, then `function(x), function(x')` are `d_out`-close under `output_metric`.

*Proof.* In the definition of the function on line 16, `RBig.try_from` is infallible when the input is non-nan making the function infallible. Line 14 checks that  $k$  is not `i32.MIN`, which satisfies the precondition for `find_nearest_multiple_of_2k` on line 18, and ensures that negation is well-defined on line 25. There are no other sources of error in the function, so the function cannot raise data-dependent errors.

The function also always returns a vector of IBigs, of the same length as the input, meaning the output of the function is always a member of the output domain, as defined on line 29.

The stability argument breaks down into three parts:

- The casting from float to rational on line 17 is 1-stable, because the real values of the numbers remain un-changed, meaning the distance between adjacent inputs always remains the same.

- The rounding on line 18 can cause an increase in the sensitivity equal to  $n^{1/p} \cdot (2^k - 2^{k_{min}})$ .

$$\max_{x \sim x'} d_{Lp}(f(x), f(x')) \quad (1)$$

$$= \max_{x \sim x'} |\text{round}_k(x) - \text{round}_k(x')|_p \quad (2)$$

$$\leq \max_{x \sim x'} |(x + 2^{k-1}) - (x' - 2^{k-1} + 2^{k_{min}})|_p \quad (3)$$

$$\leq \max_{x \sim x'} |x - x'|_p + |1_n \cdot (2^k - 2^{k_{min}})|_p \quad (4)$$

$$= \max_{x \sim x'} d_{Lp}(x, x') + n^{1/p} \cdot (2^k - 2^{k_{min}}) \quad (5)$$

$$= \mathbf{d\_in} + n^{1/p} \cdot (2^k - 2^{k_{min}}) \quad (6)$$

This increase in the sensitivity is reflected on line 25, which, by the postcondition of `get_rounding_distance`, returns the maximum increase in sensitivity due to rounding, matching the above analysis.

- The discarding of the denominator on line 18 is  $2^k$ -stable, as the denominator is  $2^k$ . This increase in sensitivity is also reflected on line 25, where the sensitivity is multiplied by a factor of  $2^{-k}$ .

For every pair of elements  $x, x'$  in `input_domain` and for every pair  $(\mathbf{d\_in}, \mathbf{d\_out})$ , where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for `output_metric`, if  $x, x'$  are `d_in`-close under `input_metric`, `stability_map(d_in)` does not raise an exception, and `stability_map(d_in) ≤ d_out`, then `function(x), function(x')` are `d_out`-close under `output_metric`.  $\square$