

CompositionMeasure for Approximate<ZeroConcentratedDivergence>

Michael Shoemate

This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of **CompositionMeasure** for **Approximate<ZeroConcentratedDivergence>** in **mod.rs** at commit [f5bb719](#) (outdated¹).

1 Hoare Triple

Precondition

Compiler-Verified

Types matching pseudocode.

Caller-Verified

None

Pseudocode

```
1 class CompositionMeasure(ApproximateZeroConcentratedDivergence):
2     def composability( # 
3         self, adaptivity: Adaptivity
4     ) -> Composability:
5         return Composability.Sequential
6
7     def compose(self, d_mids: Vec[Self_Distance]) -> Self_Distance:
8         rho_g, del_g = 0.0, 0.0
9         for rho_i, del_i in d_mids:
10             rho_g = rho_g.inf_add(rho_i)
11             del_g = del_g.inf_add(del_i)
12     return rho_g, del_g
```

Postcondition

Theorem 1.1. `composability` returns `Ok(out)` if the composition of a vector of privacy parameters `d_mids` is bounded above by `self.compose(d_mids)` under `adaptivity` adaptivity and `out`-composability. Otherwise returns an error.

Proof. By the postcondition of `InfAdd` we have that $\sum_i d_{mids_i} \leq \text{compose}(d_{mids})$, where the summation is applied independently to rhos and deltas, and the comparison applies to both the global rho and global delta.

¹See new changes with `git diff f5bb719..28f53a3 rust/src/combinators/sequential_composition/mod.rs`

| Adaptivity | Sequential | Concurrent |
|----------------|-------------------------------------|------------|
| Non-Adaptive | Lemma 8.2[BS16] | None |
| Adaptive | Lemma 8.2[BS16] | None |
| Fully-Adaptive | Theorem 1[WRRW23] | None |

This table is reflected in the implementation of `composability` on line 2.

□

References

- [BS16] Mark Bun and Thomas Steinke. Concentrated differential privacy: Simplifications, extensions, and lower bounds, 2016.
- [WRRW23] Justin Whitehouse, Aaditya Ramdas, Ryan Rogers, and Zhiwei Steven Wu. Fully adaptive composition in differential privacy, 2023.