fn sample_bernoulli_exp

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This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of sample_bernoulli_exp in mod.rs at commit 0be3ab3e6 (outdated[1]).

fn sample_bernoulli_exp returns a sample from the $Bernoulli(exp(-x))$ distribution for some rational, non-negative, finite x. This proof is an adaptation of subsection 5.1 of [CKS20].

Vetting history

- Pull Request #519

1   Hoare Triple

Precondition

$x \in \mathbb{Q} \wedge x > 0$

Pseudocode

```
1  def sample_bernoulli_exp(x) -> bool:
2      while x >= 1:
3          if sample_bernoulli_exp1(1):   #
4              x -= 1
5          else:
6              return False
7      return sample_bernoulli_exp1(x)    #
```

Postcondition

For any setting of the input parameters x such that the given preconditions hold, sample_bernoulli_exp either returns Err(e) due to a lack of system entropy, or Ok(out), where out is distributed as $Bernoulli(exp(-x))$.

2   Proof

Assume the preconditions are met.

Lemma 2.1. sample_bernoulli_exp only returns Err(e) when there is a lack of system entropy.

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[1]See new changes with `git diff 0be3ab3e6..1963666 rust/src/traits/samplers/cks20/mod.rs`

Proof. In all invocations of `sample_bernoulli_exp1`, the argument passed satisfies its definition preconditions, by the preconditions on `x` and function logic. Thus, by its definition, `sample_bernoulli_exp1` only returns an error when there is a lack of system entropy. The only source of errors in `sample_bernoulli_exp` is from the invocation of `sample_bernoulli_exp1`. Therefore `sample_bernoulli_exp` only returns `Err(e)` when there is a lack of system entropy. □

Lemma 2.2. `out` is distributed as $Bernoulli(exp(-x))$.

Proof. For $0 \leq i \leq \lfloor x \rfloor$, let b_i denote the ith outcome of `sample_bernoulli_exp1` on line 3. By the definition of `sample_bernoulli_exp1`, under the established conditions and preconditions, each B_i is distributed as $Bernoulli(exp(-1))$. Let c denote the outcome of `sample_bernoulli_exp1` on line 7. Similarly as before, C is distributed $Bernoulli(exp(-(x - \lfloor x \rfloor)))$.

$$P[\mathtt{out} = \top] = P[B_1 = B_2 = ... = B_{\lfloor x \rfloor} = C = \top] \qquad \text{out is only } \top \text{ if } \forall i, B_i = \top \text{ and } C = \top$$

$$= \prod_{i=1}^{\lfloor x \rfloor} P[B_i = \top]P[C = \top] \qquad \text{all } B_i \text{ and } C \text{ are independent}$$

$$= exp(-1)^{\lfloor x \rfloor} exp(\lfloor x \rfloor - x)$$

$$= exp(-x)$$

Therefore, `out` is distributed as $Bernoulli(exp(-x))$. □

Proof. 1 holds by 2.1 and 2.2. □

References

[CKS20] Clément L. Canonne, Gautam Kamath, and Thomas Steinke. The discrete gaussian for differential privacy. *CoRR*, abs/2004.00010, 2020.