fn compute_score

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This proof resides in "contrib" because it has not completed the vetting process.

Proves soundness of compute_score in mod.rs at commit f5bb719 (outdated¹). compute_score returns a score for each candidate passed in, where the score is the distance between the candidate and the ideal alpha-quantile.

Vetting History

• Pull Request #456

1 Hoare Triple

Precondition

- TIA (input atom type) is a type with trait PartialOrd.
- x is non-null
- candidates is strictly increasing
- alpha_numer / alpha_denom is in [0,1]
- size_limit * alpha_denom does not overflow

Function

```
def compute_score(
      x: list[TIA],
      candidates: list[TIA],
      alpha_num: usize,
      alpha_den: usize,
      size_limit: usize
  ) -> list[usize]:
      x = list(sorted(x))
10
      num_lt = [0] * len(candidates)
11
      num_eq = [0] * len(candidates)
12
13
      {\tt count\_lt\_eq\_recursive} (
14
15
           num_lt, # mutated in-place
           num_eq, # mutated in-place
```

¹See new changes with git diff f5bb719..3d4a262 rust/src/transformations/quantile_score_candidates/mod.rs

Postcondition

Each element in the return value corresponds to the score of the candidate at the same index in candidates:

compute_score(
$$X, c, \alpha_{num}, \alpha_{den}, l$$
) = $|\alpha_{den} \cdot \min(\#(X < c), l), \alpha_{num} \cdot \min(|X| - \#(X = c), l)|$ (1)

2 Proof

By the preconditions on $compute_count$, and by sorting x, the preconditions on $count_lt_eq_recursive$ are satisfied.

By the definition of $count_{lt_eq_recursive}$, num_{lt} contains the number of values in x less than c, for each candidate, and similarly for num_{eq} .

Then the score is evaluated for each candidate in a loop. The scoring function cannot overflow or return null because the risk of overflow is protected by the preconditions. The function is also completely nonnegative due to the use of abs_diff.