fn make_is_equal

Sílvia Casacuberta

This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of `fn make_is_equal` in `mod.rs` at commit 0db9c6036 (outdated[1]). The transformation checks if each element in a vector dataset is equivalent to a given value, returning a vector of booleans.

Vetting History

- Pull Request #512

1 Hoare Triple

Precondition

To ensure the correctness of the output, we require the following preconditions:

- Type `TIA` must have trait `PartialEq`.
- Type `M` must have trait `DatasetMetric`.
- `MetricSpace` is implemented for `(VectorDomain<AtomDomain<TIA>>, M)`. Therefore `M` is a valid metric on `VectorDomain<AtomDomain<TIA>>`.
- `MetricSpace` is implemented for `(VectorDomain<AtomDomain<bool>>, M)`.

Pseudocode

```
def make_is_equal(
    input_domain: VectorDomain[AtomDomain[TIA]],
    input_metric: M,
    value: TIA
): #
    output_row_domain = atom_domain(T=bool)

    def is_equal(arg: TA) -> TA: #
        return value == arg

    return make_row_by_row( #
        input_domain,
        input_metric,
        output_row_domain,
        is_equal
    )
```

[1]See new changes with `git diff 0db9c6036..c3c9a76 rust/src/transformations/manipulation/mod.rs`

Postconditions

Theorem 1.1. For every setting of the input parameters `(input_domain, input_metric, value)` to `make_is_equal` such that the given preconditions hold, `make_is_equal` raises an error (at compile time or run time) or returns a valid transformation. A valid transformation has the following properties:

1. (Data-independent runtime errors). For every pair of members x and x' in `input_domain`, `invoke`(x) and `invoke`(x') either both return the same error or neither return an error.

2. (Appropriate output domain). For every member x in `input_domain`, `function`(x) is in `output_domain` or raises a data-independent runtime error.

3. (Stability guarantee). For every pair of members x and x' in `input_domain` and for every pair (`d_in`, `d_out`), where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for
`output_metric`, if x, x' are `d_in`-close under `input_metric`, `stability_map(d_in)` does not raise an error, and `stability_map(d_in)` $=$ `d_out`, then `function`(x), `function`(x') are `d_out`-close under `output_metric`.

2 Proof

Lemma 2.1. The invocation of `make_row_by_row` (line 11) satisfies its preconditions.

Proof. The preconditions of `make_is_equal` and pseudocode definition (line 5) ensure that the type preconditions of `make_row_by_row` are satisfied. The remaining preconditions of `make_row_by_row` are:

- `row_function` has no side-effects.
- If the input to `row_function` is a member of `input_domain`'s row domain, then the output is a member of `output_row_domain`.

The first precondition is satisfied by the definition of `is_equal` (line 8) in the pseudocode.

For the second precondition, assume the input is a member of `input_domain`'s row domain. By the definition of `PartialEq`, the output of `is_equal` is boolean, and `AtomDomain<bool>` includes all booleans. Therefore, the output is a member of `output_row_domain`. □

We now prove the postcondition of `make_is_equal`.

Proof. By 2, the preconditions of `make_row_by_row` are satisfied. Thus, by the definition of `make_row_by_row`, the output is a valid transformation. □