`fn make_laplace_threshold`

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This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of the implementation of `make_laplace_threshold` in `mod.rs` at commit f5bb719 (outdated[1]).

Thresholded noise mechanisms may be parameterized along many different axes:

- key dtype: i8, i16, i32, i64, u8, u16, u32, u64, f32, f64, UBig, IBig, RBig
- metric dtype: i8, i16, i32, i64, u8, u16, u32, u64, f32, f64, UBig, IBig, RBig
- measure: max divergence, zero concentrated divergence
- distribution: laplace, gaussian

All parameterizations reduce to a single core mechanism that perturbs a signed big integers with noise sampled from the appropriate discrete distribution, and then thresholds and shuffles the result.

The implementation of this function constructs a random variable denoting the noise distribution to add, and then dispatches to the `MakeNoiseThreshold<DI, MI, MO>` trait which constructs the core mechanism and wraps it in pre-processing transformations and post-processors to match the desired parameterization.

1 Hoare Triple

Precondition

Compiler-Verified

- generic `DI` implements trait `NoiseDomain`
- generic `MI` implements trait `Metric`
- generic `MO` implements trait `Measure`
- `DI_Atom` implements trait `Nature`. This trait encodes the relationship between the atomic data type and the type of the noise distribution that is compatible with it: `DI_Atom_RV2`. In Rust, this corresponds to the (ugly) `<DI::Atom as Nature>::RV<2>` type.
- `DI_Atom_RV2` implements trait `MakeNoiseThreshold`. That is, it must be possible to build the mechanism from this new equivalent distribution.
- type `(DI, MI)` implements trait `MetricSpace`

User-Verified

None

[1] See new changes with `git diff f5bb719..66159cf7 rust/src/measurements/noise_threshold/distribution/laplace/mod.rs`

Pseudocode

```
def make_laplace_threshold(
    input_domain: DI,
    input_metric: MI,
    scale: f64,
    threshold: DI_Atom,
    k: Option[i32],
) -> Measurement[DI, DI_Carrier, MI, MO]:
    return DiscreteLaplace(scale, k).make_noise_threshold(
        (input_domain, input_metric), threshold
    )
```

Postcondition

Theorem 1.1.

Theorem 1.2. For every setting of the input parameters (`input_domain, input_metric, scale, threshold, k, DI, MI, MO`) to `make_laplace_threshold` such that the given preconditions hold, `make_laplace_threshold` raises an exception (at compile time or run time) or returns a valid measurement. A valid measurement has the following property:

1. (Privacy guarantee). For every pair of elements x, x' in `input_domain` and for every pair (`d_in`, `d_out`), where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for `output_measure`, if x, x' are `d_in`-close under `input_metric`, `privacy_map`(`d_in`) does not raise an exception, and `privacy_map`(`d_in`) $\leq$ `d_out`, then `function`(x), `function`(x') are `d_out`-close under `output_measure`.

Proof. We first construct a random variable `DiscreteLaplace` representing the desired noise distribution. Since `MakeNoiseThreshold.make_noise_threshold` has no preconditions, the postcondition follows, which matches the postcondition for this function. □