fn peel_permute_and_flip

Michael Shoemate

September 24, 2025

This proof resides in "contrib" because it has not completed the vetting process.

This document proves soundness of $peel_permute_and_flip$ in mod.rs at commit e62b0aa2 (outdated¹). $peel_permute_and_flip$ noisily selects the index of the greatest score from a vector of input scores k times without replacement.

1 Hoare Triple

Preconditions

Types consistent with pseudocode.

Pseudocode

```
def peel_permute_and_flip(x: list[RBig], scale: RBig, k: usize):
      natural_order = []
      sorted_order = BTreeSet.new()
      for _ in range(min(k, x.len())):
          index = permute_and_flip(x, scale) #
          x.remove(index)
          # map index on modified x back to original x (postprocessing)
10
          for del_ in sorted_order: # |
               if del_ <= index:</pre>
11
                   index += 1
               else:
                   break
14
15
           sorted_order.insert(index)
16
17
          natural_order.push(index)
18
      return natural_order
```

Postcondition

Theorem 1.1. Returns the index of the max element z_i , where each $z_i \sim \text{Exp}(\text{shift} = x_i, \text{scale} = \text{scale})$, k times without replacement.

Proof. The pseudocode applies $permute_and_flip$ on line 6 up to k times (no greater than the number of scores), each time removing the selected score on line 7.

Since the index in subsequent selections may be offset by an earlier removal, a data-independent post-processing of the index is performed on line 10.

¹See new changes with git diff e62b0aa2..d809578 rust/src/measurements/noisy_top_k/exponential/mod.rs