

fn noisy_top_k

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This proof resides in “**contrib**” because it has not completed the vetting process.

This document proves soundness of `noisy_top_k` in `mod.rs` at commit `e62b0aa2` (outdated¹). `noisy_top_k` noisily selects the index of the best score from a vector of input scores k times.

1 Hoare Triple

Preconditions

Compiler-Verified

Types consistent with pseudocode.

Caller-Verified

- Each item of `x` is finite.

Pseudocode

```
1 def noisy_top_k(  
2     x: list[TIA],  
3     scale: RBig,  
4     k: usize,  
5     negate: bool,  
6     replacement: bool,  
7 ):  
8     sign = Sign.from_(negate)  
9     scale = scale.into_rational()  
10  
11     y = [x_i.into_rational() * sign for x_i in x] #  
12     return peel_permute_and_flip(y, scale, k, replacement)
```

Postcondition

Theorem 1.1. • If `replacement` is set, returns a sample from \mathcal{M}_{EM} (as defined in MS2023 Definition 4), otherwise returns a sample from \mathcal{M}_{PF} (as defined in MS2023 Lemma 1), k times by peeling, where $\text{scale} = \frac{2 \cdot \Delta}{\epsilon}$.

- Errors are data-independent, except for exhaustion of entropy.

¹See new changes with `git diff e62b0aa2..ac4a2d7 rust/src/measurements/noisy_top_k/mod.rs`

Proof. By the precondition that each element in x is finite, the conversion into rational is infallible.

By the postcondition of `peel_permute_and_flip`, and the potential negation on line 11, the postcondition is satisfied.

The only source of error is due to entropy exhaustion. □