fn get_margin

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This proof resides in "contrib" because it has not completed the vetting process.

Proves soundness of get_margin in mod.rs at commit f5bb719 (outdated¹).

get_margin returns a Margin for a given set of grouping columns (by) whose descriptors are no more restrictive than what is known in FrameDomain.

1 Hoare Triple

Precondition

Compiler-verified

Types matching pseudocode.

Human-verified

None

Pseudocode

```
from math import prod
  def get_margin(domain: FrameDomain, by: set[Expr]) -> Margin:
      margin = next( #
          (m for m in domain.margins if m.by == by),
          Margin.by(by)
      subset_margins = [ #
10
          margin
          for margin in domain.margins
11
12
          if margin.by.issubset(by)
13
14
      margin.max_length = min( #
15
          m.max_length
16
17
          for m in subset_margins
          if m.max_length is not None
18
19
20
      all_max_groups = [ #
21
22
          (m.by, m.max_groups)
          for m in domain.margins
23
          if m.max_groups is not None
25
```

¹See new changes with git diff f5bb719..82b8bdd rust/src/domains/polars/frame/mod.rs

```
max_groups_covering = find_min_covering(grouping_columns, all_max_groups)
26
27
       if max_groups_covering is not None:
          margin.max_groups = prod(v for _, v in max_groups_covering)
28
29
      all_invariants = ( #
30
          m.invariant
31
           for m in domain.margins
          if by.issubset(m.by)
33
34
      margin.invariant = max(all_invariants, key={None: 0, "Keys": 1, "Lengths": 2}.get)
35
36
      return margin
```

Postcondition

Returns a Margin that describes properties of members of domain when grouped by by.

2 Proofs

Proof. On line 4, margin is either a valid margin descriptor for by, by the definition of domain, or it is the default margin, which is a valid margin descriptor for all potential datasets.

We now update descriptors based on other information available in domain.

On line 9, subset_margins is the subset of margins spanned by by. Then 15 assigns the smallest known descriptors over any margin spanning a subset of the grouping columns.

If max_length is known about a coarser data grouping (when grouped by fewer columns), then these descriptors still apply to a finer data grouping, as group length can only decrease when more finely splitting data. Therefore max_length remains valid after mutation.

Line 21 retrieves all known max_groups descriptors. There are no manual preconditions to find_min_covering, therefore we claim the postcondition, that the output is a covering for by.

The number of groups can be no greater than the cardinality of the cartesian product of the keys of each grouping column. Therefore the code finds a set of max_groups descriptors that covers the grouping columns, and then updates margin to the product of the covering.

On an aside, for utility, while the covering found may not be the smallest, the greedy algorithm will always choose a singleton cover if it is available, therefore this update to the descriptor cannot increase the descriptor.

Finally, on line 30, all_invariants contains invariants for any margin finer than by. If group keys and/or lengths are known for a finer grouping, then they are also valid for a coarser grouping. Therefore invariants is updated to the strongest invariant for any grouping as fine or finer than by.

Since the initial margin (4) was valid, and all updates have also been shown to be valid, get_margin returns a Margin that describes properties of members of domain when grouped by by.