fn make_gaussian_threshold

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> This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of the implementation of `make_gaussian_threshold` in `mod.rs` at commit f5bb719 (outdated[1]).

Thresholded noise mechanisms may be parameterized along many different axes:

- key dtype: i8, i16, i32, i64, u8, u16, u32, u64, f32, f64, UBig, IBig, RBig

- metric dtype: i8, i16, i32, i64, u8, u16, u32, u64, f32, f64, UBig, IBig, RBig

- measure: max divergence, zero concentrated divergence

- distribution: laplace, gaussian

All parameterizations reduce to a single core mechanism that perturbs a signed big integers with noise sampled from the appropriate discrete distribution, and then thresholds and shuffles the result.

The implementation of this function constructs a random variable denoting the noise distribution to add, and then dispatches to the `MakeNoiseThreshold<DI, MI, MO>` trait which constructs the core mechanism and wraps it in pre-processing transformations and post-processors to match the desired parameterization.

1 Hoare Triple

Precondition

Compiler-Verified

- generic DI implements trait `NoiseDomain`

- generic MI implements trait `Metric`

- generic MO implements trait `Measure`

- generic DiscreteGaussian implements trait `MakeNoiseThreshold`

- type (DI, MI) implements trait `MetricSpace`

User-Verified

None

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[1]See new changes with `git diff f5bb719..8d95338f rust/src/measurements/noise_threshold/distribution/gaussian/mod.rs`

Pseudocode

```
1  def make_gaussian_threshold(
2      input_domain: DI,
3      input_metric: MI,
4      scale: f64,
5      threshold: DI_Atom,
6      k: Option[i32],
7  ) -> Measurement[DI, DI_Carrier, MI, MO]:
8      return DiscreteGaussian(scale, k).make_noise_threshold(
9          (input_domain, input_metric), threshold
10      )
```

Postcondition

Theorem 1.1.

Theorem 1.2. For every setting of the input parameters (`input_domain, input_metric, scale, threshold, k, DI, MI, MO`) to `make_gaussian_threshold` such that the given preconditions hold, `make_gaussian_threshold` raises an exception (at compile time or run time) or returns a valid measurement. A valid measurement has the following property:

1. (Privacy guarantee). For every pair of elements x, x' in `input_domain` and for every pair (d_in, d_out), where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for `output_measure`, if x, x' are `d_in`-close under `input_metric`, `privacy_map(d_in)` does not raise an exception, and `privacy_map(d_in)` $\leq$ `d_out`, then $\text{function}(x), \text{function}(x')$ are `d_out`-close under `output_measure`.

Proof. We first construct a random variable `DiscreteGaussian` representing the desired noise distribution. Since `MakeNoiseThreshold.make_noise_threshold` has no preconditions, the postcondition follows, which matches the postcondition for this function. □