MakeNoise<DI, MI, MO> for DiscreteGaussian

Michael Shoemate

This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of the implementation of MakeNoise for DiscreteGaussian in mod.rs at commit f5bb719 (outdated[1]).

This is an intermediary compile-time layer whose purpose is to dispatch to either the integer or floating-point variations of the mechanism, depending on the type of data in the input domain.

It does this through the use of the Nature trait, which has concrete implementations for each possible input type. This layer makes interior layers simpler to work with, and does not have privacy implications. It also makes make_gaussian easier to call, by simplifying the type signature.

1 Hoare Triple

Precondition

Compiler-Verified

MakeNoise is parameterized as follows:

- DI implements trait Domain
- MI implements trait Metric
- MO implements trait Measure

The following trait bounds are also required:

- (DI, MI) implements trait MetricSpace
- DI_Atom implements trait Nature. This trait encodes the relationship between the atomic data type and the type of the noise distribution that is compatible with it: DI_Atom_RV2. In Rust, this corresponds to the (ugly) <DI::Atom as Nature>::RV<2> type.
- DI_Atom_RV2 implements trait MakeNoise. That is, it must be possible to build the mechanism from this new equivalent distribution.

User-Verified

None

[1] See new changes with `git diff f5bb719..b7e402f7 rust/src/measurements/noise/distribution/gaussian/mod.rs`

Pseudocode

```
# analogous to impl MakeNoise<DI, MI, MO> for DiscreteGaussian in Rust
class DiscreteGaussian:
    def make_noise(self, input_space: tuple[DI, MI]) -> Measurement[DI, DI_Carrier, MI, MO]:
        # an equivalent random variable specific to the atom dtype
        rv_nature = DI_Atom.new_distribution(self.scale, self.k) #
        # build a measurement sampling from this equivalent distribution
        return rv_nature.make_noise(input_space) #
```

Postcondition

Theorem 1.1.

Theorem 1.2. For every setting of the input parameters (`self, input_space, DI, MI, MO`) to `make_noise` such that the given preconditions hold, `make_noise` raises an exception (at compile time or run time) or returns a valid measurement. A valid measurement has the following property:

1. (Privacy guarantee). For every pair of elements x, x' in `input_domain` and for every pair (`d_in`, `d_out`), where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for `output_measure`, if x, x' are `d_in`-close under `input_metric`, `privacy_map`(`d_in`) does not raise an exception, and `privacy_map`(`d_in`) $\leq$ `d_out`, then `function`(x), `function`(x') are `d_out`-close under `output_measure`.

Proof. On line 7, `make_noise` has no preconditions, so irregardless of any prior logic, the postcondition of `make_noise` follows that the output is a valid measurement. □

The complexity in the type system here is designed to be free of privacy implications, to help simplify the core, privacy-sensitive implementation.