

# fn sample\_geometric\_exp\_slow

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Proves soundness of `fn sample_geometric_exp_slow` in `mod.rs` at commit `1f9230c` (outdated<sup>1</sup>). This proof is adapted from subsection 5.2 of [CKS20].

## 1 Hoare Triple

### Precondition

#### Compiler-verified

Argument `x` is of type `RBig`, a bignum rational

#### User-verified

$x > 0$

### Pseudocode

```
1 def sample_geometric_exp_slow(x) -> int:
2     k = 0
3     while True:
4         if sample_bernoulli_exp(x): #
5             k += 1
6         else:
7             return k
```

### Postcondition

**Theorem 1.1.** For any setting of the input parameter `x` such that the given preconditions hold, `sample_geometric_exp_slow` either returns `Err(e)` due to a lack of system entropy, or `Ok(out)`, where `out` is distributed as  $\text{Geometric}(1 - \exp(-x))$ .

**Definition 1.2.** If  $K \sim \text{Geometric}(p)$ , then for  $k \in \{0, 1, \dots\}$

$$\Pr[K = k] = (1 - p)^k \cdot p. \quad (1)$$

**Definition 1.3.** If  $B \sim \text{Bernoulli}(p)$ , then for  $b \in \{\top, \perp\}$

$$\Pr[B = b] = \begin{cases} p & b = \top \\ 1 - p & b = \perp \end{cases} \quad (2)$$

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<sup>1</sup>See new changes with `git diff 1f9230c..f99639a rust/src/traits/samplers/cks20/mod.rs`

## 2 Proof

Assume the preconditions are met.

**Lemma 2.1.** `sample_geometric_exp_slow` only returns `Err(e)` when there is a lack of system entropy.

*Proof.* The preconditions on `x` satisfy the preconditions on `sample_bernoulli_exp`, so by its definition, it only returns an error if there is a lack of system entropy. The only source of errors is from this function, therefore `sample_geometric_exp_slow` only returns `Err(e)` when there is a lack of system entropy.  $\square$

**Theorem 2.2.** [CKS20] If the outcome of `sample_geometric_exp_slow` is `Ok(out)`, then `out` is distributed as  $\text{Geometric}(1 - \exp(-x))$ .

*Proof.* The distribution of the  $i^{th}$  boolean returned on line 4 is  $B_i \sim \text{Bernoulli}(\exp(-x))$ , because the preconditions on `x` satisfy the preconditions for `sample_bernoulli_exp`.

$$\begin{aligned} \Pr[\text{out} = k] &= \Pr[B_1 = B_2 = \dots = B_k = \top \wedge B_{k+1} = \perp] \\ &= \Pr[B_{k+1} = \perp] \prod_{i=1}^k \Pr[B_i = \top] && \text{All } B_i \text{ are independent.} \\ &= (1 - \exp(-x))^k \exp(-x)^k \end{aligned}$$

By Definition 1.2, setting  $p = 1 - \exp(-x)$ , then `out`  $\sim \text{Geometric}(1 - \exp(-x))$ .  $\square$

*Proof of Theorem 1.1.* Holds by 2.1 and 2.2.  $\square$

## References

- [CKS20] Clément L. Canonne, Gautam Kamath, and Thomas Steinke. The discrete gaussian for differential privacy. *CoRR*, abs/2004.00010, 2020.