CompositionMeasure for Approximate<ZeroConcentratedDivergence>

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> This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of the implementation of `CompositionMeasure` for `Approximate<ZeroConcentratedDivergence>` in mod.rs at commit f5bb719 (outdated[1]).

1 Hoare Triple

Precondition

Compiler-Verified

Types matching pseudocode.

Caller-Verified

None

Pseudocode

```
1  class CompositionMeasure ( ApproximateZeroConcentratedDivergence ) :
2      def composability (   #
3          self , adaptivity : Adaptivity
4      ) -> Composability :
5          return Composability . Sequential
6
7      def compose ( self , d_mids : Vec [ Self_Distance ]) -> Self_Distance :
8          rho_g , del_g = 0.0 , 0.0
9          for rho_i , del_i in d_mids :
10             rho_g = rho_g . inf_add ( rho_i )
11             del_g = del_g . inf_add ( del_i )
12         return rho_g , del_g
```

Postcondition

Theorem 1.1. `composability` returns `Ok(out)` if the composition of a vector of privacy parameters `d_mids` is bounded above by `self.compose(d_mids)` under `adaptivity` adaptivity and `out`-composability. Otherwise returns an error.

Proof. By the postcondition of `InfAdd` we have that $\sum_i \texttt{d\_mids}_i \leq \texttt{compose(d\_mids)}$, where the summation is applied independently to rhos and deltas, and the comparison applies to both the global rho and global delta.

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[1]See new changes with `git diff f5bb719..5c077eb rust/src/combinators/sequential_composition/mod.rs`

Adaptivity	Sequential	Concurrent
Non-Adaptive	Lemma 8.2[BS16]	None
Adaptive	Lemma 8.2[BS16]	None
Fully-Adaptive	Theorem 1[WRRW23]	None

This table is reflected in the implementation of `composability` on line 2.

□

References

[BS16]   Mark Bun and Thomas Steinke. Concentrated differential privacy: Simplifications, extensions, and lower bounds, 2016.

[WRRW23] Justin Whitehouse, Aaditya Ramdas, Ryan Rogers, and Zhiwei Steven Wu. Fully adaptive composition in differential privacy, 2023.