

# fn peel\_permute\_and\_flip

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This proof resides in “**contrib**” because it has not completed the vetting process.

This document proves soundness of `peel_permute_and_flip` in `mod.rs` at commit `e62b0aa2` (outdated<sup>1</sup>). `peel_permute_and_flip` noisily selects the index of the greatest score from a vector of input scores  $k$  times without replacement.

## 1 Hoare Triple

### Preconditions

*Types consistent with pseudocode.*

### Pseudocode

```
1 def peel_permute_and_flip(x: list[RBig], scale: RBig, k: usize):
2     natural_order = []
3     sorted_order = BTreeSet.new()
4
5     for _ in range(min(k, x.len())):
6         index = permute_and_flip(x, scale) #
7         x.remove(index) #
8
9         # map index on modified x back to original x (postprocessing)
10        for del_ in sorted_order: # |
11            if del_ <= index:
12                index += 1
13            else:
14                break
15
16        sorted_order.insert(index)
17        natural_order.push(index)
18
19    return natural_order
```

### Postcondition

**Theorem 1.1.** Returns the index of the max element  $z_i$ , where each  $z_i \sim \text{Exp}(\text{shift} = x_i, \text{scale} = \text{scale})$ ,  $k$  times without replacement.

*Proof.* The pseudocode applies `permute_and_flip` on line 6 up to  $k$  times (no greater than the number of scores), each time removing the selected score on line 7.

Since the index in subsequent selections may be offset by an earlier removal, a data-independent post-processing of the index is performed on line 10.  $\square$

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<sup>1</sup>See new changes with `git diff e62b0aa2..6e2747d rust/src/measurements/noisy_top_k/exponential/mod.rs`