fn make_int_to_bigint

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This proof resides in "contrib" because it has not completed the vetting process.

Proves soundness of the implementation of make_int_to_bigint in mod.rs at commit f5bb719 (out-dated¹).

1 Hoare Triple

Precondition

Compiler-Verified

• Generic T implements trait SaturatingCast<IBig>

User-Verified

None

Pseudocode

```
def make_int_to_bigint(
      input_space: tuple[VectorDomain[AtomDomain[T]], LpDistance[P, QI]],
  ) -> Transformation[
      VectorDomain[AtomDomain[T]],
      VectorDomain[AtomDomain[IBig]],
      LpDistance[P, QI],
      LpDistance[P, RBig],
  ]:
8
      input_domain, input_metric = input_space
10
      def stability_map(d_in):
11
12
              return RBig.try_from(d_in)
13
          except Exception:
14
              raise f"d_in ({d_in}) must be finite"
15
16
17
      return Transformation.new(
          input_domain,
18
          VectorDomain(
19
               element_domain=AtomDomain.default(IBig),
20
               size=input_domain.size,
21
22
          Function.new(lambda x: [IBig.from_(x_i) for x_i in x]),
23
          input_metric,
          LpDistance.default(),
25
          StabilityMap.new_fallible(stability_map),
26
```

¹See new changes with git diff f5bb719...de8a6b rust/src/measurements/noise/nature/integer/mod.rs

Postcondition

Theorem 1.1.

Theorem 1.2. For every setting of the input parameters (T) to make_int_to_bigint such that the given preconditions hold, make_int_to_bigint raises an error (at compile time or run time) or returns a valid transformation. A valid transformation has the following properties:

- 1. (Data-independent runtime errors). For every pair of members x and x' in input_domain, invoke(x) and invoke(x') either both return the same error or neither return an error.
- 2. (Appropriate output domain). For every member x in input_domain, function(x) is in output_domain or raises a data-independent runtime error.
- 3. (Stability guarantee). For every pair of members x and x' in input_domain and for every pair (d_in,d_out), where d_in has the associated type for input_metric and d_out has the associated type for output_metric, if x,x' are d_in-close under input_metric, stability_map(d_in) does not raise an

output_metric, if x, x' are d_in-close under input_metric, stability_map(d_in) does not raise an error, and stability_map(d_in) = d_out, then function(x), function(x') are d_out-close under output_metric.

Proof. By the definition of the function on line ??, and since IBig.from is infallible, the function is infallible, meaning that the function cannot raise data-dependent errors. The function also always returns a vector of IBigs, of the same length as the input, meaning the output of the function is always a member of the output domain, as defined on line ??. Finally, the function is 1-stable, because the real values of the numbers remain un-changed, meaning the distance between adjacent inputs always remains the same, satisfying the stability property.