

# fn make\_count

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of `make_count` in `mod.rs` at commit `f5bb719` (outdated<sup>1</sup>).

`make_count` returns a Transformation that computes a count of the number of records in a vector. The length of the vector, of type `usize`, is exactly casted to a user specified output type `T0`. If the length is too large to be represented exactly by `T0`, the cast saturates at the maximum value of type `T0`.

## Vetting History

- Pull Request #513

## 1 Hoare Triple

### Precondition

- `TIA` (atomic input type) is a type with trait `Primitive`. `Primitive` implies `TIA` has the trait bound:
  - `CheckNull` so that `TIA` is a valid atomic type for `AtomDomain`
- `T0` (output type) is a type with trait `Number`. `Number` further implies `T0` has the trait bounds:
  - `InfSub` so that the output domain is compatible with the output metric
  - `CheckNull` so that `T0` is a valid atomic type for `AtomDomain`
  - `ExactIntCast` for casting a vector length index of type `usize` to `T0`. `ExactIntCast` further implies `T0` has the trait bound:
    - \* `ExactIntBounds`, which gives the `MAX_CONSECUTIVE` value of type `T0`
  - `One` provides a way to retrieve `T0`’s representation of 1
  - `DistanceConstant` to satisfy the preconditions of `new_stability_map_from_constant`

### Pseudocode

```
1 def make_count(
2     input_domain: VectorDomain[AtomDomain[TIA]],
3     input_metric: SymmetricDistance
4 ):
5     output_domain = AtomDomain(T0) #
6
7     def function(data: Vec[TIA]) -> T0: #
8         size = input_domain.size(data) #
9         try: #
10             return T0.exact_int_cast(size) #
```

<sup>1</sup>See new changes with git diff f5bb719..0c327cc rust/src/transformations/count/mod.rs

```

11     except FailedCast:
12         return TO.MAX_CONSECUTIVE #
13
14     output_metric = AbsoluteDistance(TO)
15
16     stability_map = new_stability_map_from_constant(TO.one()) #
17
18     return Transformation(
19         input_domain, output_domain, function,
20         input_metric, output_metric, stability_map)

```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`TIA`, `TO`) to `make_count` such that the given pre-conditions hold, `make_count` raises an error (at compile time or run time) or returns a valid transformation. A valid transformation has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Appropriate output domain). For every member  $x$  in `input_domain`, `function(x)` is in `output_domain` or raises a data-independent runtime error.
3. (Stability guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_metric`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `stability_map(d_{in})` does not raise an error, and `stability_map(d_{in}) = d_{out}`, then `function(x), function(x')` are  $d_{out}$ -close under `output_metric`.

## 2 Proofs

*Proof. (Part 1 – appropriate output domain).* The `output_domain` is `AtomDomain(TO)`, so it is sufficient to show that `function` always returns non-null values of type `TO`. By the definition of the `ExactIntCast` trait, `TO.exact_int_cast` always returns a non-null value of type `TO` or raises an exception. If an exception is raised, the function returns `TO.MAXIMUM_CONSECUTIVE`, which is also a non-null value of type `TO`. Thus, in all cases, the function (from line 9) returns a non-null value of type `TO`.  $\square$

Before proceeding with proving the validity of the stability map, we provide a couple lemmas.

**Lemma 2.1.**  $|\text{function}(u) - \text{function}(v)| \leq |\text{len}(u) - \text{len}(v)|$ , where `len` is an alias for `input_domain.size`.

*Proof.* By `CollectionDomain`, we know `size` on line 8 is of type `usize`, so it is non-negative and integral. Therefore, by the definition of `ExactIntCast`, the invocation of `TO.exact_int_cast` on line 10 can only fail if the argument is greater than `TO.MAX_CONSECUTIVE`. In this case, the value is replaced with `TO.MAX_CONSECUTIVE`. Therefore,  $\text{function}(u) = \min(\text{len}(u), c)$ , where  $c = \text{TO.MAX_CONSECUTIVE}$ . We use this equality to prove the lemma:

$$\begin{aligned} |\text{function}(u) - \text{function}(v)| &= |\min(\text{len}(u), c) - \min(\text{len}(v), c)| \\ &\leq |\text{len}(u) - \text{len}(v)| \end{aligned} \quad \text{since clamping is stable}$$

$\square$

**Lemma 2.2.** For vector  $v$  with each element  $\ell \in v$  drawn from domain  $\mathcal{X}$ ,  $\text{len}(v) = \sum_{z \in \mathcal{X}} h_v(z)$ .

*Proof.* Every element  $\ell \in v$  is drawn from domain  $\mathcal{X}$ , so summing over all  $z \in \mathcal{X}$  will sum over every element  $\ell \in v$ . Recall that the definition of `SymmetricDistance` states that  $h_v(z)$  will return the number of occurrences of value  $z$  in vector  $v$ . Therefore,  $\sum_{z \in \mathcal{X}} h_v(z)$  is the sum of the number of occurrences of each unique value; this is equivalent to the total number of items in the vector.

Since `CollectionDomain` is implemented for `VectorDomain<AtomDomain<TIA>`, we depend on the correctness of the implementation Conditioned on the correctness of the implementation of `CollectionDomain` for `VectorDomain<AtomDomain<TIA>`, the variable `size` is of type `usize` containing the number of elements in `arg`. Therefore,  $\sum_{z \in \mathcal{X}} h_v(z)$  is equivalent to `size`.  $\square$

*Proof. (Part 2 – stability map).* Take any two elements  $u, v$  in the `input_domain` and any pair  $(d\_in, d\_out)$ , where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for `output_metric`. Assume  $u, v$  are `d_in`-close under `input_metric` and that  $\text{stability\_map}(d\_in) \leq d\_out$ . These assumptions are used to establish the following inequality:

$$\begin{aligned}
|\text{function}(u) - \text{function}(v)| &\leq |\text{len}(u) - \text{len}(v)| && \text{by 2.1} \\
&= \left| \sum_{z \in \mathcal{X}} h_u(z) - \sum_{z \in \mathcal{X}} h_v(z) \right| && \text{by 2.2} \\
&= \left| \sum_{z \in \mathcal{X}} (h_u(z) - h_v(z)) \right| && \text{by algebra} \\
&\leq \sum_{z \in \mathcal{X}} |h_u(z) - h_v(z)| && \text{by triangle inequality} \\
&= d_{Sym}(u, v) && \text{by } \text{SymmetricDistance} \\
&\leq d\_in && \text{by the first assumption} \\
&\leq \text{T0.inf\_cast}(d\_in) && \text{by InfCast} \\
&\leq \text{T0.one().inf\_mul}(\text{T0.inf\_cast}(d\_in)) && \text{by InfMul} \\
&= \text{stability\_map}(d\_in) && \text{by pseudocode line 16} \\
&\leq d\_out && \text{by the second assumption}
\end{aligned}$$

It is shown that `function(u), function(v)` are `d_out`-close under `output_metric`.  $\square$