

impl TopKMeasure for MaxDivergence

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This proof resides in “**contrib**” because it has not completed the vetting process.

This document proves soundness of **TopKMeasure** for **MaxDivergence** in **mod.rs** at commit **e62b0aa2** (outdated¹).

1 Hoare Triple

Precondition

Compiler-verified

- Associated Const **REPLACEMENT** = **false**
- Method **privacy_map**
Types consistent with pseudocode.

Caller-verified

- Method **privacy_map**
 - **d_in** is non-null and positive.
 - **scale** is non-null and positive.

Pseudocode

```
1 # MaxDivergence
2 REPLACEMENT = False
3
4 def privacy_map(d_in: f64, scale: f64) -> f64:
5     return d_in.inf_div(scale)
```

Postcondition

Theorem 1.1. The implementation is consistent with the associated items in the **TopKMeasure** trait.

1. Method **privacy_map**: For any x, x' where $d_{\text{in}} \geq d_{\text{Range}}(x, x')$, return $d_{\text{out}} \geq D_{\text{self}}(f(x), f(x'))$, where $f(x) = \text{noisy_top_k}(x = x, k = 1, \text{scale} = \text{scale}, \text{replacement} = \text{Self} :: \text{REPLACEMENT})$.

Proof. Since **Self::REPLACEMENT** is false, then by the postcondition of **noisy_top_k**, **noisy_top_k** returns a sample from \mathcal{M}_{PF} . In the case that scores are not monotonic, by [1] Theorem 1, \mathcal{M}_{PF} satisfies ϵ -DP, because the range distance is equal to $2 \cdot \Delta$. Otherwise in the case that scores are monotonic, by [1] Remark 1, \mathcal{M}_{PF} satisfies $\epsilon/2$ -DP, but the range distance is equal to Δ , thus satisfying ϵ -DP. \square

¹See new changes with `git diff e62b0aa2..9b02cf48 rust/src/measurements/noisy_top_k/mod.rs`

References

- [1] Ryan McKenna and Daniel Sheldon. Permute-and-flip: A new mechanism for differentially private selection, 2020.