

impl SelectionMeasure for RangeDivergence

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1 Hoare Triple

Precondition

Compiler-verified

- Associated Constant `ONE_SHOT`
 - `ONE_SHOT` is true if the measure supports one-shot top- k composition.
- Associated Type `RV`
 - `RV` must implement trait `InverseCDF`.
- Method `random_variable` *Types consistent with pseudocode.*
- Method `aprivacy_map` *Types consistent with pseudocode.*

Caller-verified

- Method `random_variable`
 - `scale` is positive (*cannot be null due to `FBig` dtype*).
- Method `privacy_map`
 - `d_in` is non-null and non-negative.
 - `scale` is non-null and non-negative.

Pseudocode

```
1 class RangeDivergence(SelectionMeasure):
2     ONE_SHOT = True
3     RV = GumbelRV
4
5     @staticmethod
6     def random_variable(shift: FBig, scale: FBig) -> GumbelRV:
7         return GumbelRV(shift=shift, scale=scale)
8
9     @staticmethod
10    def privacy_map(d_in: f64, scale: f64, k: usize) -> f64:
11        if d_in < 0:
12            raise ValueError("input distance must be non-negative")
13
14        if scale.is_zero():
15            return f64.INFINITY
16
17        return d_in.inf_div(scale).inf_mul(f64.inf_cast(k))
```

Postcondition

Theorem 1.1. The implementation is consistent with all associated items in the **SelectionMeasure** trait.

1. Associated Constant `ONE_SHOT`
2. Associated Type RV
3. Method `random_variable`
4. Method `privacy_map`

Proof of valid associated constant: `ONE_SHOT`. Since the proof of `privacy_map` only holds if k is less than two, `ONE_SHOT` is defined to be false. \square

Definition 1.2. A random variable follows the Exponential distribution if it has density

$$f(x) = \frac{1}{\beta} e^{-z} \tag{1}$$

where $z = \frac{x-\mu}{\beta}$, μ is the shift (location) parameter and β is the scale parameter.

Proof of valid associated type: `RV`. The associated type `RV` is defined as `ExponentialRV`, which represents a random variable following the Exponential distribution 1.2. The compiler verifies that `ExponentialRV` implements the **InverseCDF** trait. \square

Proof of valid method: `random_variable`. By the precondition on `scale` being positive, `random_variable` returns a valid instance of **ExponentialRV**. \square

Proof of valid method: `privacy_map`. To be merged from: <https://github.com/opensp/opensp/pull/1678/files> \square