## fn accuracy\_to\_discrete\_gaussian\_scale

Michael Shoemate

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This document contains materials associated with accuracy\_to\_discrete\_gaussian\_scale. By discrete\_gaussian\_scale\_to\_accuracy, the relationship between  $\alpha$ , a and scale, is:

$$1 - \alpha = \frac{\sum_{y=0}^{a-1} (1 + 1[y \neq 0]) e^{-(y/s)^2/2}}{\sum_{z \in \mathbb{Z}} e^{-(z/s)^2/2}}$$

A closed-form expression for s doesn't exist, so we use a numerical approach by a binary search. A loose upper bound is provided by accuracy\_to\_gaussian\_scale. The binary search finds the smallest s such that

$$\alpha \le 1 - \frac{\sum_{y=0}^{a-1} (1 + 1[y \ne 0]) e^{-(y/s)^2/2}}{\sum_{z \in \mathbb{Z}} e^{-(z/s)^2/2}}$$