

# CompositionMeasure for Approximate<MaxDivergence>

Michael Shoemate

This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of **CompositionMeasure** for **Approximate<MaxDivergence>** in **mod.rs** at commit **f5bb719** (outdated<sup>1</sup>).

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

Types matching pseudocode.

#### Caller-Verified

None

### Pseudocode

```
1 class CompositionMeasure(ApproximateMaxDivergence):
2     def composability( #
3         self, adaptivity: Adaptivity
4     ) -> Composability:
5         if matches(adaptivity, Adaptivity.FullyAdaptive):
6             raise "fully-adaptive composition is not currently supported for max-divergence"
7         return Composability.Concurrent
8
9     def compose(self, d_mids: Vec[Self_Distance]) -> Self_Distance:
10        eps_g, del_g = 0.0, 0.0
11        for eps_i, del_i in d_mids:
12            eps_g = eps_g.inf_add(eps_i)
13            del_g = del_g.inf_add(del_i)
14        return eps_g, del_g
```

### Postcondition

**Theorem 1.1.** `composability` returns `Ok(out)` if the composition of a vector of privacy parameters `d_mids` is bounded above by `self.compose(d_mids)` under `adaptivity` `adaptivity` and `out-composability`. Otherwise returns an error.

*Proof.* By the postcondition of **InfAdd** we have that  $\sum_i d\_mids_i \leq \text{compose}(d\_mids)$ , where the summation is applied independently to epsilons and deltas, and the comparison applies to both the global epsilon and global delta.

---

<sup>1</sup>See new changes with `git diff f5bb719..6ad9e482 rust/src/combinators/sequential_composition/mod.rs`

Adaptivity	Sequential	Concurrent
Non-Adaptive	Theorem 1[DKM <sup>+</sup> 06]	Theorem 1.3[VZ23]
Adaptive	Theorem 1[DKM <sup>+</sup> 06]	Theorem 1.3[VZ23]
Fully-Adaptive	None	None

This table is reflected in the implementation of `composability` on line 2.

□

## References

- [DKM<sup>+</sup>06] Cynthia Dwork, Krishnaram Kenthapadi, Frank McSherry, Ilya Mironov, and Moni Naor. Our data, ourselves: Privacy via distributed noise generation. In Serge Vaudenay, editor, *Advances in Cryptology - EUROCRYPT 2006*, pages 486–503, Berlin, Heidelberg, 2006. Springer Berlin Heidelberg.
- [VZ23] Salil Vadhan and Wanrong Zhang. Concurrent composition theorems for differential privacy. In *Proceedings of the 55th Annual ACM Symposium on Theory of Computing, STOC '23*, page 507–519. ACM, June 2023.