

# fn make\_int\_to\_bigint\_threshold

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of `make_int_to_bigint_threshold` in `mod.rs` at commit f5bb719 (outdated<sup>1</sup>).

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

- Generic TK implements trait `Hashable`
- Generic TV implements trait `Integer` and `SaturatingCast<IBig>`
- Const-generic P is of type `usize`
- Generic QI implements trait `Number`

#### User-Verified

None

### Pseudocode

```
1 def make_int_to_bigint_threshold(
2     input_space: tuple[
3         MapDomain[AtomDomain[TK], AtomDomain[TV]], LOPInfDistance[P, AbsoluteDistance[QI]]
4     ],
5 ) -> Transformation[
6     MapDomain[AtomDomain[TK], AtomDomain[TV]],
7     MapDomain[AtomDomain[TK], AtomDomain[IBig]],
8     LOPInfDistance[P, AbsoluteDistance[QI]],
9     LOPInfDistance[P, AbsoluteDistance[RBIG]],
10 ]:
11     input_domain, input_metric = input_space
12
13     def stability_map(d_in):
14         10, lp, li = d_in
15         lp = UBig.try_from(lp)
16         li = UBig.try_from(li)
17         return 10, lp, li
18
19     return Transformation.new(
20         input_domain,
```

<sup>1</sup>See new changes with git diff f5bb719..79856e1 rust/src/measurements/noise\_threshold/nature/integer/mod.rs

```

21     MapDomain( #
22         key_domain=input_domain.key_domain,
23         value_domain=AtomDomain.default(IBig),
24     ),
25         Function.new(lambda x: {k: IBig.from_(v) for k, v in x.items()}), #
26         input_metric,
27         LOP1.default(),
28         StabilityMap.new_fallible(stability_map),
29     )

```

## Postcondition

**Theorem 1.1.**

**Theorem 1.2.** For every setting of the input parameters (`input_space`, `TK`, `TV`, `P`, `QI`) to `make_int_to_bigint_threshold` such that the given preconditions hold, `make_int_to_bigint_threshold` raises an error (at compile time or run time) or returns a valid transformation. A valid transformation has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Appropriate output domain). For every member  $x$  in `input_domain`, `function(x)` is in `output_domain` or raises a data-independent runtime error.
3. (Stability guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_metric`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `stability_map(d_{in})` does not raise an error, and `stability_map(d_{in}) = d_{out}`, then `function(x), function(x')` are  $d_{out}$ -close under `output_metric`.

*Proof.* By the definition of the function on line 25, and since `IBig.from` is infallible, the function is infallible, meaning that the function cannot raise data-dependent errors. The function also always returns a hashmap whose keys are un-changed, and values are IBigs, meaning the output of the function is always a member of the output domain, as defined on line 21. Finally, the function is 1-stable, because the real values of the numbers remain un-changed, meaning the distance between adjacent inputs always remains the same, satisfying the stability property.  $\square$