

# MakeNoise<AtomDomain<T>, AbsoluteDistance<QI>, M0> for IntExpFamily<P>

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of **MakeNoise** over scalars for **IntExpFamily** in **mod.rs** at commit **f5bb719** (outdated<sup>1</sup>).

The intuition of this implementation is that a vector-valued mechanism can be used to privatize a scalar-valued input, by transforming the input into a singleton vector, applying the vector mechanism, and then unpacking the resulting singleton vector.

This matches the code and proof for the float case, **MakeNoise<AtomDomain<T>, AbsoluteDistance<QI>, M0> for FloatExpFamily<P>**, except for elementary data type.

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

- Generic **T** implements trait **Integer** and **SaturatingCast<IBig>** The saturating cast is for infallible postprocessing of big ints back to type **T**.
- Const-generic **P** is of type **usize**
- Generic **QI** implements trait **Integer**
- Generic **M0** implements trait **Measure**
- Type **IBig** implements trait **From<T>**. This infallible exact cast is for converting integers to big ints in the preprocessing transformation.
- Type **RBig** implements trait **TryFrom<QI>**. This is for fallible casting from input sensitivity of type **QI** to a rational in the privacy map.
- Type **ZExpFamily<P>** implements trait **NoisePrivacyMap<LpDistance<P, RBig>, M0>**. This bound requires that it must be possible to construct a privacy map for this combination of noise distribution, distance type and privacy measure.

#### User-Verified

None

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<sup>1</sup>See new changes with `git diff f5bb719..dd13d02 rust/src/measurements/noise/nature/integer/mod.rs`

## Pseudocode

```
1 class IntExpFamily:
2     def make_noise(
3         self, input_space: tuple[AtomDomain[T], AbsoluteDistance[QI]]
4     ) -> Measurement[AtomDomain[T], T, AbsoluteDistance[QI], MO]:
5         t_vec = make_vec(input_space) #
6         m_noise = self.make_noise(t_vec.output_space()) #
7
8         return t_vec >> m_noise >> then_index_or_default(0) #
```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`self`, `input_space`, `MO`, `T`, `P`, `QI`) to `make_noise` such that the given preconditions hold, `make_noise` raises an exception (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of elements  $x, x'$  in `input_domain`, `function(x)` returns an error if and only if `function(x')` returns an error.
2. (Privacy guarantee). For every pair of elements  $x, x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_measure`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `privacy_map(d_in)` does not raise an exception, and `privacy_map(d_in) ≤ d_out`, then `function(x), function(x')` are  $d_{out}$ -close under `output_measure`.

*Proof.* Neither constructor `make_vec` nor `MakeNoise.make_noise` have manual preconditions, and the postconditions guarantee a valid transformation and valid measurement, respectively. `then_index_or_default` also does not have preconditions, and its postcondition guarantees that it returns a valid postprocessor.

The chain of a valid transformation, valid measurement and valid postprocessor is a valid measurement.  $\square$