fn noisy_top_k

Michael Shoemate

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This proof resides in **"contrib"** because it has not completed the vetting process.

This document proves soundness of `noisy_top_k` in `mod.rs` at commit e62b0aa2 (outdated[1]). `noisy_top_k` noisily selects the index of the best score from a vector of input scores k times.

1 Hoare Triple

Preconditions

Compiler-Verified

Types consistent with pseudocode.

Caller-Verified

- Each item of `x` is finite.

Pseudocode

```
def noisy_top_k(
    x: list[TIA],
    scale: RBig,
    k: usize,
    negate: bool,
    replacement: bool,
):
    sign = Sign.from_(negate)
    scale = scale.into_rational()

    y = [x_i.into_rational() * sign for x_i in x]  #
    return peel_permute_and_flip(y, scale, k, replacement)
```

Postcondition

Theorem 1.1.

- If replacement is set, returns a sample from $\mathcal{M}_{EM}$ (as defined in MS2023 Definition 4), otherwise returns a sample from $\mathcal{M}_{PF}$ (as defined in MS2023 Lemma 1), k times by peeling, where $\texttt{scale} = \frac{2 \cdot \Delta}{\epsilon}$.
- Errors are data-independent, except for exhaustion of entropy.

[1] See new changes with `git diff e62b0aa2..69115df rust/src/measurements/noisy_top_k/mod.rs`

Proof. By the precondition that each element in x is finite, the conversion into rational is infallible.

By the postcondition of `peel_permute_and_flip`, and the potential negation on line 11, the postcondition is satisfied.

The only source of error is due to entropy exhaustion. □