

# MakeNoiseThreshold<DI, MI, MO> for DiscreteGaussian

Michael Shoemate

This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of **MakeNoiseThreshold** for **DiscreteGaussian** in **mod.rs** at **commit f5bb719** (outdated<sup>1</sup>).

This is an intermediary compile-time layer whose purpose is to dispatch to either the integer or floating-point variations of the mechanism, depending on the type of data in the input domain.

It does this through the use of the **Nature** trait, which has concrete implementations for each possible input type. This layer makes interior layers simpler to work with, and does not have privacy implications. It also makes **make\_gaussian\_threshold** easier to call, by simplifying the type signature.

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

**MakeNoiseThreshold** is parameterized as follows:

- DI implements trait **Domain**
- MI implements trait **Metric**
- MO implements trait **Measure**

The following trait bounds are also required:

- (DI, MI) implements trait **MetricSpace**
- **DI\_Atom** implements trait **Nature**. This trait encodes the relationship between the atomic data type and the type of the noise distribution that is compatible with it: **DI\_Atom\_RV2**. In Rust, this corresponds to the (ugly) `<DI::Atom as Nature>::RV<2>` type.
- **DI\_Atom\_RV2** implements trait **MakeNoiseThreshold**. That is, it must be possible to build the mechanism from this new equivalent distribution.
- type (DI, MI) implements trait **MetricSpace**

#### User-Verified

None

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<sup>1</sup>See new changes with `git diff f5bb719..eb3c9f6 rust/src/measurements/noise_threshold/distribution/gaussian/mod.rs`

## Pseudocode

```
1 # analogous to impl MakeNoiseThreshold<DI, MI, MO> for DiscreteGaussian in Rust
2 class DiscreteGaussian:
3     def make_noise_threshold(self, input_space, threshold) -> Measurement[DI, DI_Carrier, MI
4         , MO]:
5         # an equivalent random variable specific to the atom dtype
6         rv_nature = DI_Atom.new_distribution(self.scale, self.k) #
7         # build a measurement sampling from this equivalent distribution
8         return rv_nature.make_noise_threshold(input_space, threshold) #
```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`self`, `input_space`, `threshold`, `DI`, `MI`, `MO`) to `make_noise_threshold` such that the given preconditions hold, `make_noise_threshold` raises an exception (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of elements  $x, x'$  in `input_domain`, `function(x)` returns an error if and only if `function(x')` returns an error.
2. (Privacy guarantee). For every pair of elements  $x, x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_measure`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `privacy_map(d_in)` does not raise an exception, and `privacy_map(d_in) ≤ d_out`, then `function(x), function(x')` are  $d_{out}$ -close under `output_measure`.

*Proof.* On line 7, `make_noise_threshold` has no preconditions, so irregardless of any prior logic, the postcondition of `make_noise_threshold` follows that the output is a valid measurement.  $\square$

The complexity in the type system here is designed to be free of privacy implications, to help simplify the core, privacy-sensitive implementation.