

# CompositionMeasure for Approximate<ZeroConcentratedDivergence>

Michael Shoemate

This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of **CompositionMeasure** for **Approximate<ZeroConcentratedDivergence>** in **mod.rs** at commit [f5bb719](#) (outdated<sup>1</sup>).

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

Types matching pseudocode.

#### Caller-Verified

None

### Pseudocode

```
1 class CompositionMeasure(ApproximateZeroConcentratedDivergence):
2     def composability( # 
3         self, adaptivity: Adaptivity
4     ) -> Composability:
5         return Composability.Sequential
6
7     def compose(self, d_mids: Vec[Self_Distance]) -> Self_Distance:
8         rho_g, del_g = 0.0, 0.0
9         for rho_i, del_i in d_mids:
10             rho_g = rho_g.inf_add(rho_i)
11             del_g = del_g.inf_add(del_i)
12     return rho_g, del_g
```

### Postcondition

**Theorem 1.1.** `composability` returns `Ok(out)` if the composition of a vector of privacy parameters `d_mids` is bounded above by `self.compose(d_mids)` under `adaptivity` adaptivity and `out`-composability. Otherwise returns an error.

*Proof.* By the postcondition of `InfAdd` we have that  $\sum_i d_{mids_i} \leq \text{compose}(d_{mids})$ , where the summation is applied independently to rhos and deltas, and the comparison applies to both the global rho and global delta.

<sup>1</sup>See new changes with `git diff f5bb719..1b82039 rust/src/combinators/sequential_composition/mod.rs`

| Adaptivity     | Sequential                          | Concurrent |
|----------------|-------------------------------------|------------|
| Non-Adaptive   | Lemma 8.2[ <a href="#">BS16</a> ]   | None       |
| Adaptive       | Lemma 8.2[ <a href="#">BS16</a> ]   | None       |
| Fully-Adaptive | Theorem 1[ <a href="#">WRRW23</a> ] | None       |

This table is reflected in the implementation of `composability` on line 2.

□

## References

- [BS16] Mark Bun and Thomas Steinke. Concentrated differential privacy: Simplifications, extensions, and lower bounds, 2016.
- [WRRW23] Justin Whitehouse, Aaditya Ramdas, Ryan Rogers, and Zhiwei Steven Wu. Fully adaptive composition in differential privacy, 2023.