

# MakeNoise<VectorDomain<AtomDomain<IBig>>, MI, MO> for RV

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of `MakeNoise` for `RV` over vectors of big integers in `mod.rs` at commit [f5bb719](#) (outdated<sup>1</sup>).

This is the core implementation of all variations of the gaussian or laplace mechanism.

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

`MakeNoise` is parameterized as follows:

- `MI` implements trait `Metric`
- `MO` implements trait `Measure`
- `RV` implements trait `Sample`

The following trait bounds are also required:

- `(VectorDomain<AtomDomain<IBig>>, MI)` implements trait `MetricSpace`
- `RV` implements `NoisePrivacyMap<MI, MO>`

#### User-Verified

None

### Pseudocode

```
1 # analogous to impl MakeNoise<VectorDomain<AtomDomain<IBig>>, MI, MO> for RV in Rust
2 class RV:
3     def make_noise(self, input_space) -> Measurement[VectorDomain[AtomDomain[IBig]], Vec[IBig], MI, MO]:
4         input_domain, input_metric = input_space
5         return Measurement.new(
6             input_domain,
7             Function.new_fallible(
8                 lambda x: [self.sample(x_i) for x_i in x]), #
9             input_metric,
```

<sup>1</sup>See new changes with git diff f5bb719..78e659b rust/src/measurements/noise/mod.rs

```

10     MO.default(),
11     self.noise_privacy_map(),  #
12 )

```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`self`, `input_space`, `MI`, `MO`, `RV`) to `make_noise` such that the given preconditions hold, `make_noise` raises an error (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Privacy guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_measure`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `privacy_map(d_in)` does not raise an error, and `privacy_map(d_in) = d_out`, then `function(x), function(x')` are  $d_{out}$ -close under `output_measure`.

*Proof of data-independent errors.* The precondition of `Sample.sample` requires that `self` is a valid distribution. This is satisfied by the postcondition of `NoisePrivacyMap<MI, MO>` on line 11. The postcondition of `Sample.sample` guarantees that the function only ever returns an error independently of the data.  $\square$

For the proof of the privacy guarantee, start by reviewing the postcondition of `NoisePrivacyMap<MI, MO>`, which has an associated function `noise_privacy_map` to be called on line 11.

**Lemma 1.2** (Postcondition of `NoisePrivacyMap`). Given a distribution `self`, returns `Err(e)` if `self` is not a valid distribution. Otherwise the output is `Ok(privacy_map)` where `privacy_map` observes the following:

Define  $\text{function}(x) = x + Z$  where  $Z$  is a vector of iid samples from `self`.

For every pair of elements  $x, x'$  in `VectorDomain<AtomDomain<IBig>>`, and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_measure`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `privacy_map(d_in)` does not raise an exception, and `privacy_map(d_in) ≤ d_out`, then `function(x), function(x')` are  $d_{out}$ -close under `output_measure`.

*Proof of privacy guarantee.* Assuming line 11 does not fail, then the returned privacy map is subject to Theorem 1.2. The privacy guarantee applies when  $\text{function}(x) = x + Z$ , where  $Z$  is a vector of iid samples from `self`. In this case `self` describes the noise distribution.

We argue that `function` is consistent with the function described in Lemma 1.2. Line 8 calls `self.sample(x_i)` on each element in the input vector. The precondition that `self` represents a valid distribution is satisfied by the postcondition of Lemma 1.2; the distribution is valid when the construction of the privacy map does not raise an exception. Since the preconditions for `Sample.sample` are satisfied, the postcondition claims that either an error is returned independently of the input `shift`, or `shift + Z` where  $Z$  is a sample from the distribution defined by `self`. This is consistent with the definition of `function(x)` in the privacy map.

Therefore, the privacy guarantee from Lemma 1.2 applies to the returned measurement.  $\square$