MakeNoiseThreshold<DI, MI, MO> for DiscreteGaussian

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> This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of the implementation of `MakeNoiseThreshold` for `DiscreteGaussian` in mod.rs at commit f5bb719 (outdated[1]).

This is an intermediary compile-time layer whose purpose is to dispatch to either the integer or floating-point variations of the mechanism, depending on the type of data in the input domain.

It does this through the use of the `Nature` trait, which has concrete implementations for each possible input type. This layer makes interior layers simpler to work with, and does not have privacy implications. It also makes `make_gaussian_threshold` easier to call, by simplifying the type signature.

1 Hoare Triple

Precondition

Compiler-Verified

`MakeNoiseThreshold` is parameterized as follows:

- `DI` implements trait `Domain`
- `MI` implements trait `Metric`
- `MO` implements trait `Measure`

The following trait bounds are also required:

- `(DI, MI)` implements trait `MetricSpace`
- `DI_Atom` implements trait `Nature`. This trait encodes the relationship between the atomic data type and the type of the noise distribution that is compatible with it: `DI_Atom_RV2`. In Rust, this corresponds to the (ugly) `<DI::Atom as Nature>::RV<2>` type.
- `DI_Atom_RV2` implements trait `MakeNoiseThreshold`. That is, it must be possible to build the mechanism from this new equivalent distribution.
- type `(DI, MI)` implements trait `MetricSpace`

User-Verified

None

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[1]See new changes with `git diff f5bb719..0cd8942 rust/src/measurements/noise_threshold/distribution/gaussian/mod.rs`

Pseudocode

```
1  # analogous to impl MakeNoiseThreshold<DI, MI, MO> for DiscreteGaussian in Rust
2  class DiscreteGaussian:
3      def make_noise_threshold(self, input_space, threshold) -> Measurement[DI, DI_Carrier, MI
       , MO]:
4          # an equivalent random variable specific to the atom dtype
5          rv_nature = DI_Atom.new_distribution(self.scale, self.k) #
6          # build a measurement sampling from this equivalent distribution
7          return rv_nature.make_noise_threshold(input_space, threshold) #
```

Postcondition

Theorem 1.1. For every setting of the input parameters (`self`, `input_space`, `threshold`, `DI`, `MI`, `MO`) to `make_noise_threshold` such that the given preconditions hold, `make_noise_threshold` raises an error (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of members x and x' in `input_domain`, `invoke`(x) and `invoke`(x') either both return the same error or neither return an error.

2. (Privacy guarantee). For every pair of members x and x' in `input_domain` and for every pair $(\mathtt{d\_in}, \mathtt{d\_out})$, where `d_in` has the associated type for `input_metric` and `d_out` has the associated type for
   `output_measure`, if x, x' are `d_in`-close under `input_metric`, `privacy_map`(`d_in`) does not raise an error, and `privacy_map`(`d_in`) = `d_out`, then `function`(x), `function`(x') are `d_out`-close under `output_measure`.

Proof. On line 7, `make_noise_threshold` has no preconditions, so irregardless of any prior logic, the post-condition of `make_noise_threshold` follows that the output is a valid measurement. □

The complexity in the type system here is designed to be free of privacy implications, to help simplify the core, privacy-sensitive implementation.