

# fn noisy\_top\_k

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This proof resides in “**contrib**” because it has not completed the vetting process.

This document proves soundness of `noisy_top_k` in `mod.rs` at commit `e62b0aa2` (outdated<sup>1</sup>). `noisy_top_k` noisily selects the index of the best score from a vector of input scores  $k$  times.

## 1 Hoare Triple

### Preconditions

#### Compiler-Verified

*Types consistent with pseudocode.*

#### Caller-Verified

- Each item of `x` is finite.

### Pseudocode

```
1 def noisy_top_k(  
2     x: list[TIA],  
3     scale: RBig,  
4     k: usize,  
5     negate: bool,  
6     replacement: bool,  
7 ):  
8     sign = Sign.from_(negate)  
9     scale = scale.into_rational()  
10  
11     y = [x_i.into_rational() * sign for x_i in x] #  
12     return peel_permute_and_flip(y, scale, k, replacement)
```

### Postcondition

**Theorem 1.1.** • If `replacement` is set, returns a sample from  $\mathcal{M}_{EM}$  (as defined in MS2023 Definition 4), otherwise returns a sample from  $\mathcal{M}_{PF}$  (as defined in MS2023 Lemma 1),  $k$  times by peeling, where  $\text{scale} = \frac{2 \cdot \Delta}{\epsilon}$ .

- Errors are data-independent, except for exhaustion of entropy.

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<sup>1</sup>See new changes with `git diff e62b0aa2..356f7ee rust/src/measurements/noisy_top_k/mod.rs`

*Proof.* By the precondition that each element in  $x$  is finite, the conversion into rational is infallible.

By the postcondition of `peel_permute_and_flip`, and the potential negation on line 11, the postcondition is satisfied.

The only source of error is due to entropy exhaustion. □