

# fn accuracy\_to\_discrete\_laplacian\_scale

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This document contains materials associated with `accuracy_to_discrete_laplacian_scale`.  
By `discrete_laplacian_scale_to_accuracy`, the relationship between  $\alpha$ ,  $a$  and  $scale$ , is:

$$\alpha = 2 \frac{e^{(1-a)/s}}{e^{1/s} + 1}$$

A closed-form expression for  $s$  doesn't exist, so we use a numerical approach by a binary search.  
A loose upper bound is provided by `accuracy_to_laplacian_scale`.  
The binary search finds the smallest  $s$  such that

$$\alpha \leq 2 \frac{e^{(1-a)/s}}{e^{1/s} + 1}$$