fn peel_permute_and_flip

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December 6, 2025

> This proof resides in **"contrib"** because it has not completed the vetting process.

This document proves soundness of `peel_permute_and_flip` in `mod.rs` at commit e62b0aa2 (outdated[1]). `peel_permute_and_flip` noisily selects the index of the greatest score from a vector of input scores k times without replacement.

1   Hoare Triple

Preconditions

Types consistent with pseudocode.

Pseudocode

```
1  def peel_permute_and_flip(
2      x: list[RBig],
3      scale: RBig,
4      k: usize,
5      replacement: bool,
6  ):
7      natural_order = []
8      sorted_order = BTreeSet.new()
9
10     for _ in range(min(k, x.len())):
11         index = permute_and_flip(x, scale, replacement)  #
12         x.remove(index)  #
13
14         # map index on modified x back to original x (postprocessing)
15         for del_ in sorted_order:  # |
16             if del_ <= index:
17                 index += 1
18             else:
19                 break
20
21         sorted_order.insert(index)
22         natural_order.push(index)
23
24     return natural_order
```

Postcondition

Theorem 1.1.   • If replacement is set, returns a sample from $\mathcal{M}_{EM}$ (as defined in MS2023 Definition 4), otherwise returns a sample from $\mathcal{M}_{PF}$ (as defined in MS2023 Lemma 1), k times by peeling, where $\texttt{scale} = \frac{2 \cdot \Delta}{\epsilon}$.

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[1] See new changes with `git diff e62b0aa2..f969979 rust/src/measurements/noisy_top_k/mod.rs`

- Errors are data-independent, except for exhaustion of entropy.

Proof. The pseudocode applies `permute_and_flip` on line 11 up to k times (no greater than the number of scores), each time removing the selected score on line 12.

Since the index in subsequent selections may be offset by an earlier removal, a data-independent post-processing of the index is performed on line 15.

The only source of error is due to entropy exhaustion. □