

MakeNoise<DI, MI, MO> for DiscreteGaussian

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of `MakeNoise` for `DiscreteGaussian` in `mod.rs` at commit `f5bb719` (outdated¹).

This is an intermediary compile-time layer whose purpose is to dispatch to either the integer or floating-point variations of the mechanism, depending on the type of data in the input domain.

It does this through the use of the `Nature` trait, which has concrete implementations for each possible input type. This layer makes interior layers simpler to work with, and does not have privacy implications. It also makes `make_gaussian` easier to call, by simplifying the type signature.

1 Hoare Triple

Precondition

Compiler-Verified

`MakeNoise` is parameterized as follows:

- DI implements trait `Domain`
- MI implements trait `Metric`
- MO implements trait `Measure`

The following trait bounds are also required:

- (DI, MI) implements trait `MetricSpace`
- DI_Atom implements trait `Nature`. This trait encodes the relationship between the atomic data type and the type of the noise distribution that is compatible with it: `DI_Atom_RV2`. In Rust, this corresponds to the (ugly) `<DI::Atom as Nature>::RV<2>` type.
- DI_Atom_RV2 implements trait `MakeNoise`. That is, it must be possible to build the mechanism from this new equivalent distribution.

User-Verified

None

¹See new changes with `git diff f5bb719..9fec598 rust/src/measurements/noise/distribution/gaussian/mod.rs`

Pseudocode

```
1 # analogous to impl MakeNoise<DI, MI, MO> for DiscreteGaussian in Rust
2 class DiscreteGaussian:
3     def make_noise(self, input_space: tuple[DI, MI]) -> Measurement[DI, DI_Carrier, MI, MO]:
4         # an equivalent random variable specific to the atom dtype
5         rv_nature = DI_Atom.new_distribution(self.scale, self.k) #
6         # build a measurement sampling from this equivalent distribution
7         return rv_nature.make_noise(input_space) #
```

Postcondition

Theorem 1.1. For every setting of the input parameters (`self`, `input_space`, `DI`, `MI`, `MO`) to `make_noise` such that the given preconditions hold, `make_noise` raises an error (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of members x and x' in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Privacy guarantee). For every pair of members x and x' in `input_domain` and for every pair (d_{in}, d_{out}) , where d_{in} has the associated type for `input_metric` and d_{out} has the associated type for `output_measure`, if x, x' are d_{in} -close under `input_metric`, `privacy_map(d_{in})` does not raise an error, and $\text{privacy_map}(d_{in}) = d_{out}$, then `function(x), function(x')` are d_{out} -close under `output_measure`.

Proof. On line 7, `make_noise` has no preconditions, so irregardless of any prior logic, the postcondition of `make_noise` follows that the output is a valid measurement. \square

The complexity in the type system here is designed to be free of privacy implications, to help simplify the core, privacy-sensitive implementation.