

# MakeNoise<AtomDomain<T>, AbsoluteDistance<QI>, M0> for IntExpFamily<P>

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of `MakeNoise` over scalars for `IntExpFamily` in `mod.rs` at commit `f5bb719` (outdated<sup>1</sup>).

The intuition of this implementation is that a vector-valued mechanism can be used to privatize a scalar-valued input, by transforming the input into a singleton vector, applying the vector mechanism, and then unpacking the resulting singleton vector.

This matches the code and proof for the float case, `MakeNoise<AtomDomain<T>, AbsoluteDistance<QI>, M0> for FloatExpFamily<P>`, except for elementary data type.

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

- Generic T implements trait `Integer` and `SaturatingCast<IBig>` The saturating cast is for infallible postprocessing of big ints back to type T.
- Const-generic P is of type `usize`
- Generic QI implements trait `Integer`
- Generic M0 implements trait `Measure`
- Type `IBig` implements trait `From<T>`. This infallible exact cast is for converting integers to big ints in the preprocessing transformation.
- Type `RBig` implements trait `TryFrom<QI>`. This is for fallible casting from input sensitivity of type QI to a rational in the privacy map.
- Type `ZExpFamily<P>` implements trait `NoisePrivacyMap<LpDistance<P, RBig>, M0>`. This bound requires that it must be possible to construct a privacy map for this combination of noise distribution, distance type and privacy measure.

#### User-Verified

None

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<sup>1</sup>See new changes with `git diff f5bb719..c9d9559 rust/src/measurements/noise/nature/integer/mod.rs`

## Pseudocode

```
1 class IntExpFamily:
2     def make_noise(
3         self, input_space: tuple[AtomDomain[T], AbsoluteDistance[QI]]
4     ) -> Measurement[AtomDomain[T], T, AbsoluteDistance[QI], M0]:
5         t_vec = make_vec(input_space) #
6         m_noise = self.make_noise(t_vec.output_space()) #
7
8     return t_vec >> m_noise >> then_index_or_default(0) #
```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`self`, `input_space`, `M0`, `T`, `P`, `QI`) to `make_noise` such that the given preconditions hold, `make_noise` raises an error (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Privacy guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_measure`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `privacy_map(d_{in})` does not raise an error, and `privacy_map(d_{in}) = d_{out}`, then `function(x), function(x')` are  $d_{out}$ -close under `output_measure`.

*Proof.* Neither constructor `make_vec` nor `MakeNoise.make_noise` have manual preconditions, and the postconditions guarantee a valid transformation and valid measurement, respectively. `then_index_or_default` also does not have preconditions, and its postcondition guarantees that it returns a valid postprocessor.

The chain of a valid transformation, valid measurement and valid postprocessor is a valid measurement. □