

# fn peel\_permute\_and\_flip

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This proof resides in “**contrib**” because it has not completed the vetting process.

This document proves soundness of `peel_permute_and_flip` in `mod.rs` at commit `e62b0aa2` (outdated<sup>1</sup>). `peel_permute_and_flip` noisily selects the index of the greatest score from a vector of input scores  $k$  times without replacement.

## 1 Hoare Triple

### Preconditions

*Types consistent with pseudocode.*

### Pseudocode

```
1 def peel_permute_and_flip(
2     x: list[RBig],
3     scale: RBig,
4     k: usize,
5     replacement: bool,
6 ):
7     natural_order = []
8     sorted_order = BTreeSet.new()
9
10    for _ in range(min(k, x.len())):
11        index = permute_and_flip(x, scale, replacement)  #
12        x.remove(index)  #
13
14        # map index on modified x back to original x (postprocessing)
15        for del_ in sorted_order:  # |
16            if del_ <= index:
17                index += 1
18            else:
19                break
20
21        sorted_order.insert(index)
22        natural_order.push(index)
23
24    return natural_order
```

### Postcondition

**Theorem 1.1.** • If replacement is set, returns a sample from  $\mathcal{M}_{EM}$  (as defined in MS2023 Definition 4), otherwise returns a sample from  $\mathcal{M}_{PF}$  (as defined in MS2023 Lemma 1),  $k$  times by peeling, where  $scale = \frac{2\cdot\Delta}{\epsilon}$ .

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<sup>1</sup>See new changes with `git diff e62b0aa2..db56d1a rust/src/measurements/noisy_top_k/mod.rs`

- Errors are data-independent, except for exhaustion of entropy.

*Proof.* The pseudocode applies `permute_and_flip` on line 11 up to  $k$  times (no greater than the number of scores), each time removing the selected score on line 12.

Since the index in subsequent selections may be offset by an earlier removal, a data-independent post-processing of the index is performed on line 15.

The only source of error is due to entropy exhaustion. □