`fn x_mul_2k`

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This proof resides in **"contrib"** because it has not completed the vetting process.

Proves soundness of the implementation of `x_mul_2k` in `mod.rs` at commit f5bb719 (outdated[1]).

1 Hoare Triple

Precondition

Compiler-Verified

None

User-Verified

$k \neq$ `i32.MIN`

Pseudocode

```
1 def x_mul_2k(x: RBig, k: i32) -> RBig:
2     num, den = x.into_parts()  #
3     if k < 0:
4         den <<= cast(-k, usize)
5     else:
6         num <<= cast(k, usize)
7 
8     return RBig.from_parts(num, den)
```

Postcondition

Theorem 1.1. Return $x \cdot 2^k$.

Proof. Since x is a fraction, line 2 splits x into its numerator and denominator.
Consider two cases:

- If $k < 0$, then multiplying the denominator by 2^-k is equivalent to multiplying by 2^k. Since k is negative, then $-k$ is positive, meaning the cast to `usize` is valid. Shifting $-k$ zeros to the left is equivalent to multipying the denominator by 2^k. Negation of k is well-defined for all values of `i32`, except for `i32.MIN`, which is not allowed by the precondition. Therefore, the result is $x \cdot 2^k$.
- If $k \geq 0$, then we multiply the numerator by 2^k. Since k is positive, the cast to `usize` is valid. Shifting k zeros to the left is equivalent to multipying the numerator by 2^k. Therefore, the result is $x \cdot 2^k$.

In both cases, the result is $x \cdot 2^k$. □

[1] See new changes with `git diff f5bb719..31afb846 rust/src/measurements/noise/nature/float/utilities/mod.rs`