

# fn make\_gaussian\_threshold

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This proof resides in “**contrib**” because it has not completed the vetting process.

Proves soundness of the implementation of `make_gaussian_threshold` in `mod.rs` at commit `f5bb719` (outdated<sup>1</sup>).

Thresholded noise mechanisms may be parameterized along many different axes:

- key dtype: i8, i16, i32, i64, u8, u16, u32, u64, f32, f64, UBig, IBig, RBig
- metric dtype: i8, i16, i32, i64, u8, u16, u32, u64, f32, f64, UBig, IBig, RBig
- measure: max divergence, zero concentrated divergence
- distribution: laplace, gaussian

All parameterizations reduce to a single core mechanism that perturbs a signed big integers with noise sampled from the appropriate discrete distribution, and then thresholds and shuffles the result.

The implementation of this function constructs a random variable denoting the noise distribution to add, and then dispatches to the `MakeNoiseThreshold<DI, MI, MO>` trait which constructs the core mechanism and wraps it in pre-processing transformations and post-processors to match the desired parameterization.

## 1 Hoare Triple

### Precondition

#### Compiler-Verified

- generic DI implements trait `NoiseDomain`
- generic MI implements trait `Metric`
- generic MO implements trait `Measure`
- generic `DiscreteGaussian` implements trait `MakeNoiseThreshold`
- type (DI, MI) implements trait `MetricSpace`

#### User-Verified

None

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<sup>1</sup>See new changes with `git diff f5bb719..b1905ce rust/src/measurements/noise_threshold/distribution/gaussian/mod.rs`

## Pseudocode

```
1 def make_gaussian_threshold(
2     input_domain: DI,
3     input_metric: MI,
4     scale: f64,
5     threshold: DI_Atom,
6     k: Option[i32],
7 ) -> Measurement[DI, DI_Carrier, MI, MO]:
8     return DiscreteGaussian(scale, k).make_noise_threshold(
9         (input_domain, input_metric), threshold
10    )
```

## Postcondition

**Theorem 1.1.** For every setting of the input parameters (`input_domain`, `input_metric`, `scale`, `threshold`, `k`, `DI`, `MI`, `MO`) to `make_gaussian_threshold` such that the given preconditions hold, `make_gaussian_threshold` raises an error (at compile time or run time) or returns a valid measurement. A valid measurement has the following properties:

1. (Data-independent runtime errors). For every pair of members  $x$  and  $x'$  in `input_domain`, `invoke(x)` and `invoke(x')` either both return the same error or neither return an error.
2. (Privacy guarantee). For every pair of members  $x$  and  $x'$  in `input_domain` and for every pair  $(d_{in}, d_{out})$ , where  $d_{in}$  has the associated type for `input_metric` and  $d_{out}$  has the associated type for `output_measure`, if  $x, x'$  are  $d_{in}$ -close under `input_metric`, `privacy_map(d_in)` does not raise an error, and  $\text{privacy\_map}(d_{in}) = d_{out}$ , then `function(x), function(x')` are  $d_{out}$ -close under `output_measure`.

*Proof.* We first construct a random variable `DiscreteGaussian` representing the desired noise distribution. Since `MakeNoiseThreshold.make_noise_threshold` has no preconditions, the postcondition follows, which matches the postcondition for this function.  $\square$