DeepRob

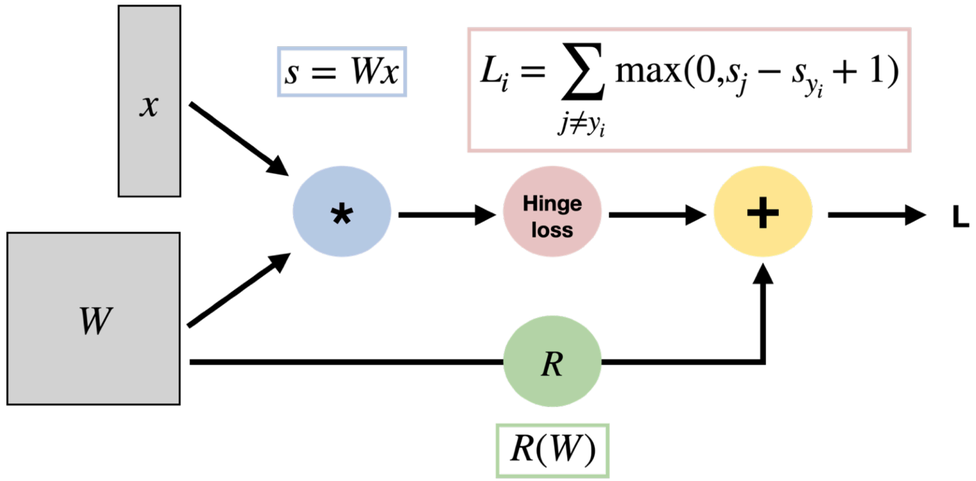
Lecture 7
Convolutional Neural Networks
University of Michigan and University of Minnesota

Project 1 — Reminder

- Instructions and code available on the website
  - Here: deeprob.org/projects/project1/
- Uses Python, PyTorch and Google Colab
- Implement KNN, linear SVM, and linear softmax classifiers
- **Autograder is online and updated**
- **Due Thursday, January 26th 11:59 PM EST**

Recap from Previous Lecture

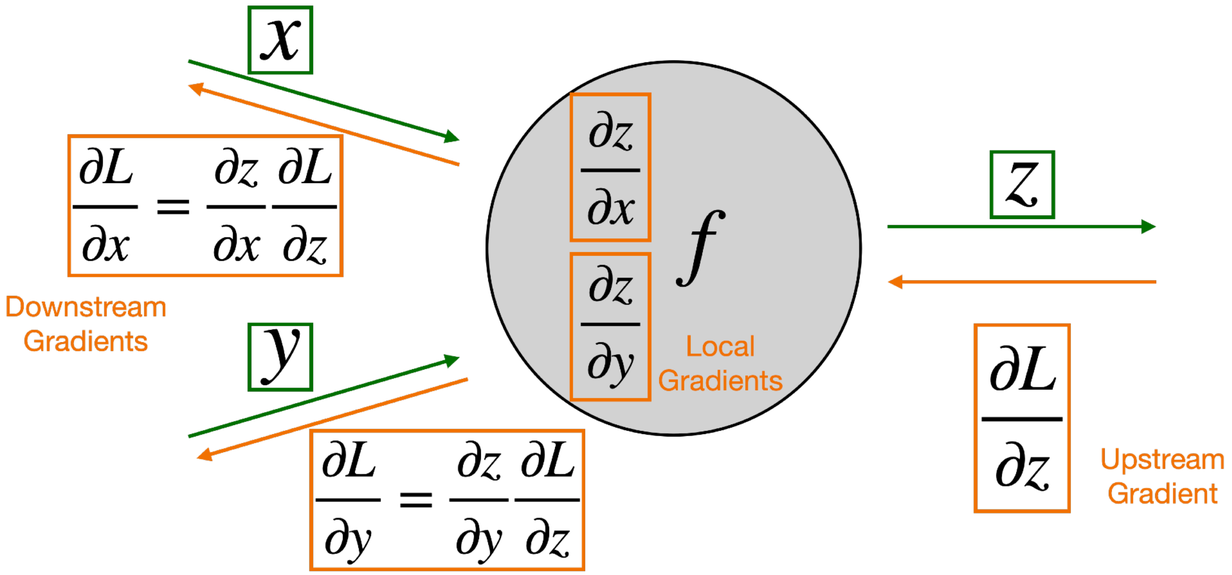
Represent complex expressions as **computational graphs**



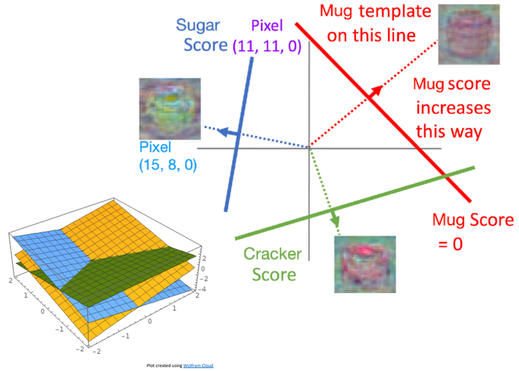
1. Forward pass: Compute outputs

2. Backward pass: Compute gradients

During the backward pass, each node in the graph receives **upstream gradients** and multiplies them by **local gradients** to compute **downstream gradients**

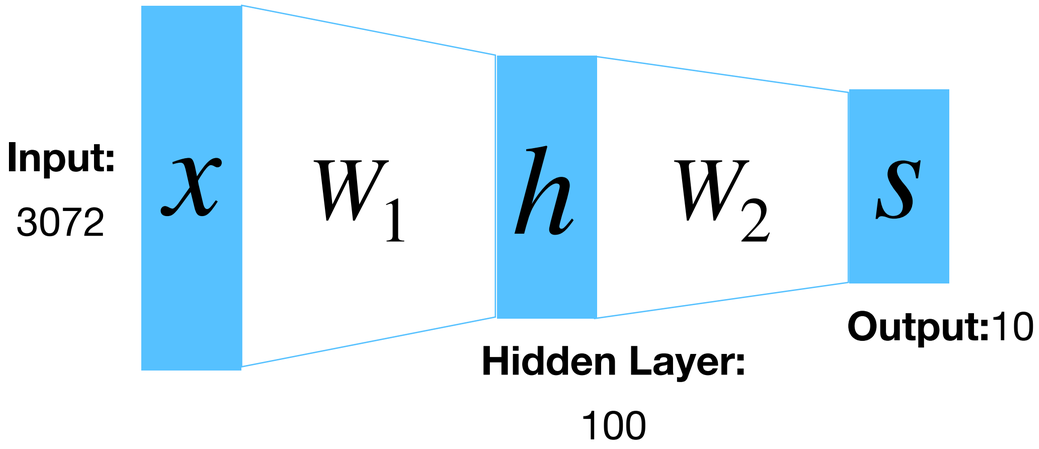


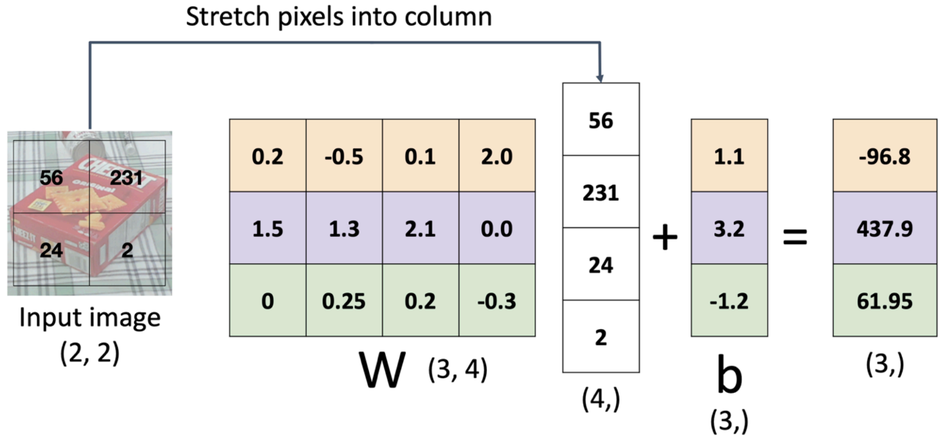
Recap from Previous Lecture



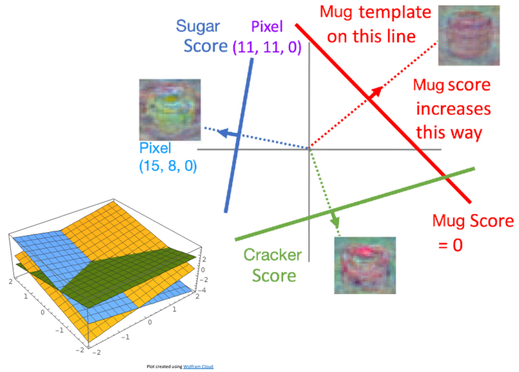
Problem: So far our classifiers don't respect the spatial structure of images!

$$f(x) = W_2 \max(0, W_1 x + b_1) + b_2$$





Recap from Previous Lecture



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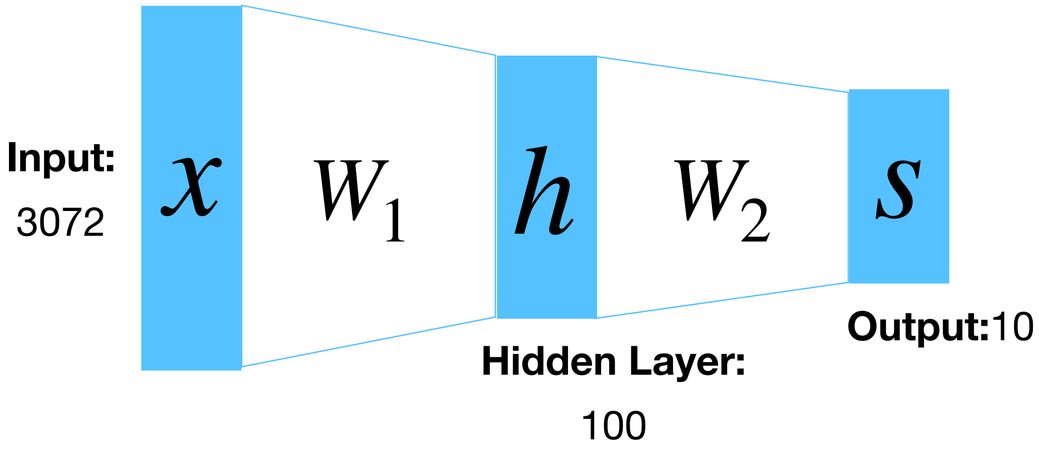
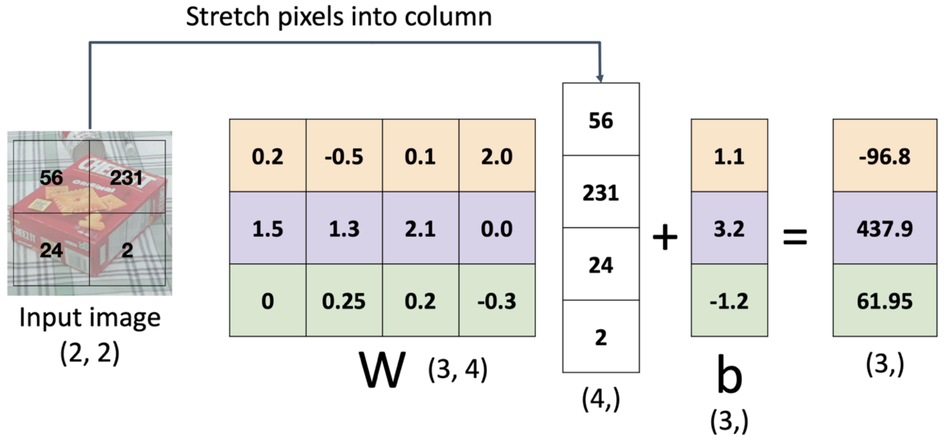


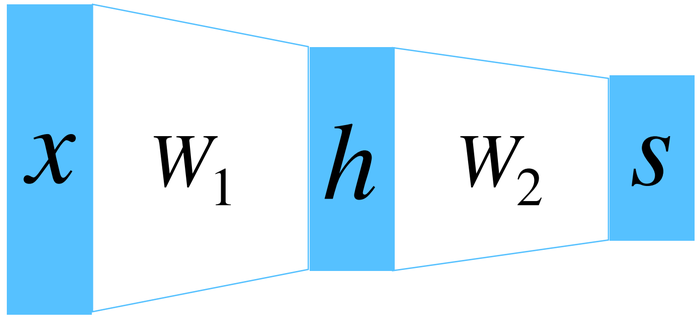
Problem: So far our classifiers don't respect the spatial structure of images!

Solution: Define new computational nodes that operate on images!

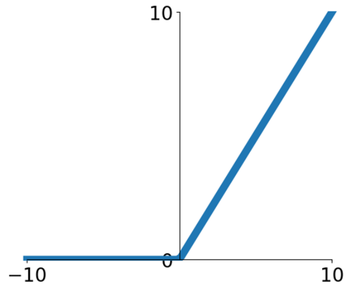


Components of Fully-Connected Networks

Fully-Connected Layers

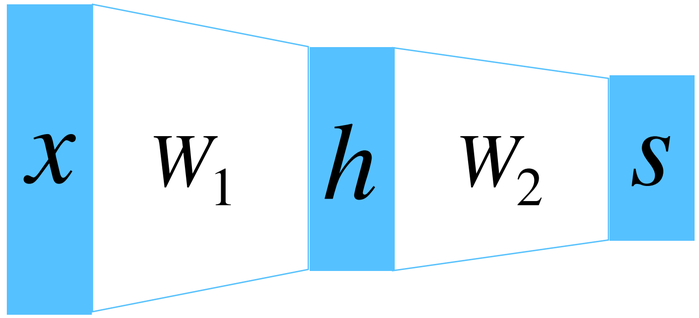


Activation Functions

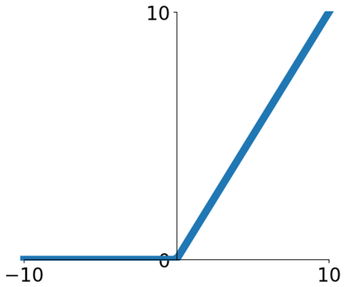


Components of Convolutional Neural Networks

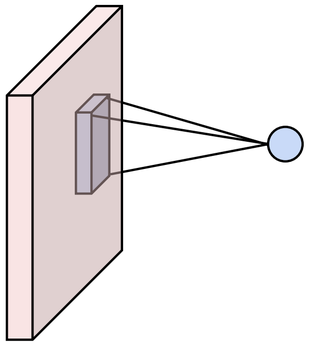
Fully-Connected Layers



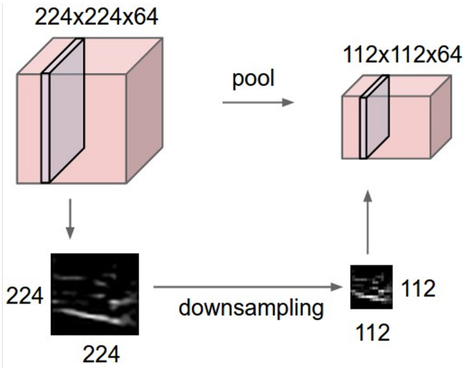
Activation Functions



Convolution Layers



Pooling Layers

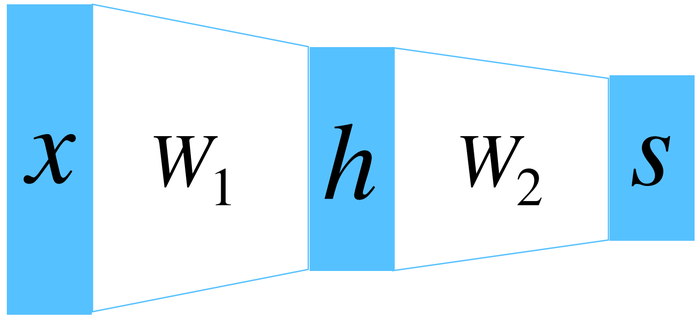


Normalization

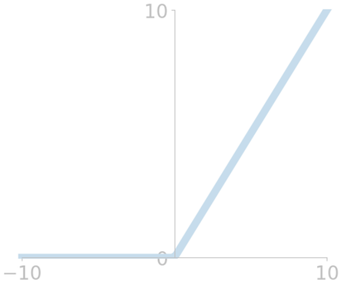
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Components of Convolutional Neural Networks

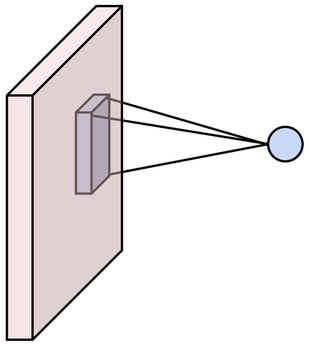
Fully-Connected Layers



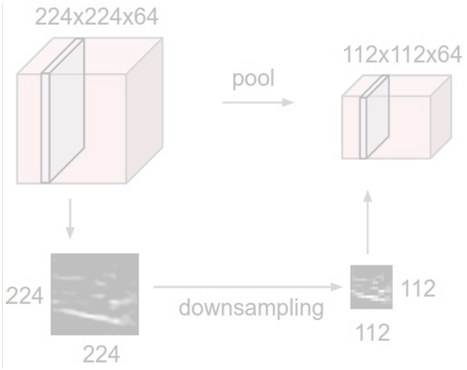
Activation Functions



Convolution Layers



Pooling Layers

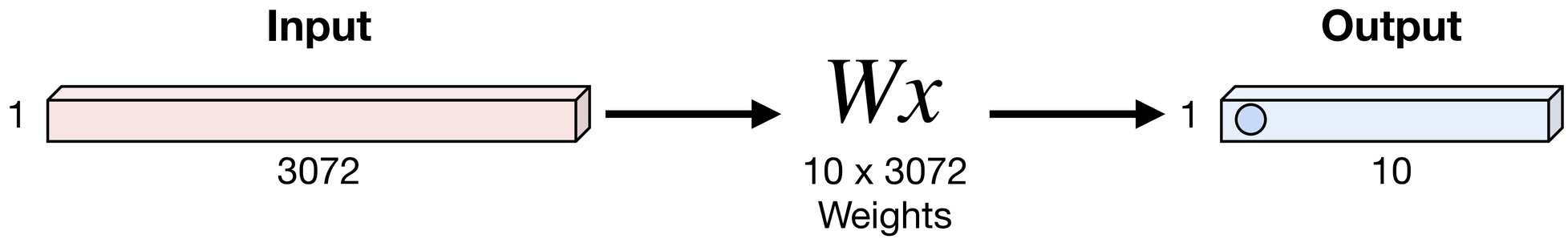


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Fully-Connected Layer

3x32x32 image → stretch to 3072x1



Fully-Connected Layer

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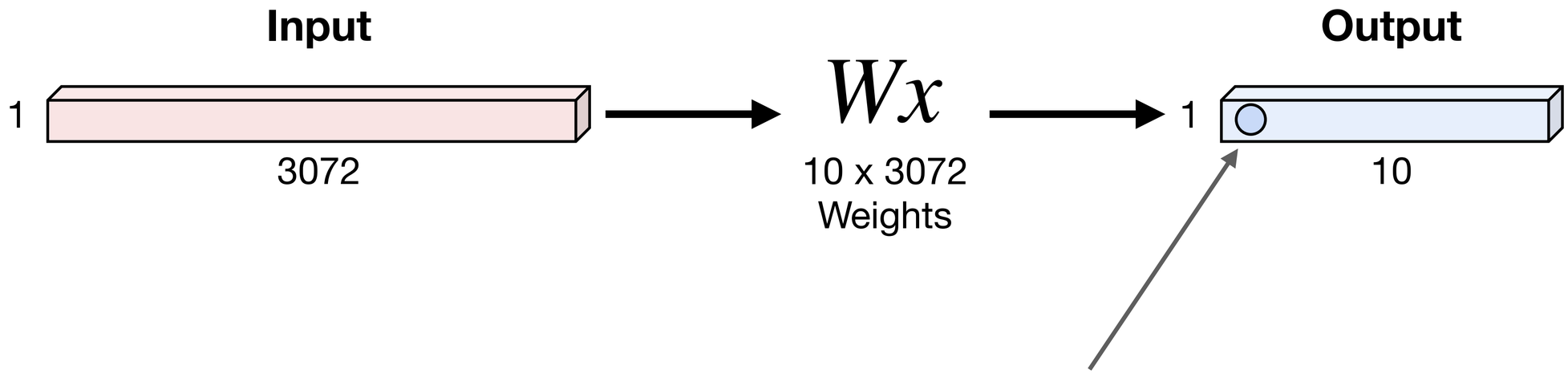
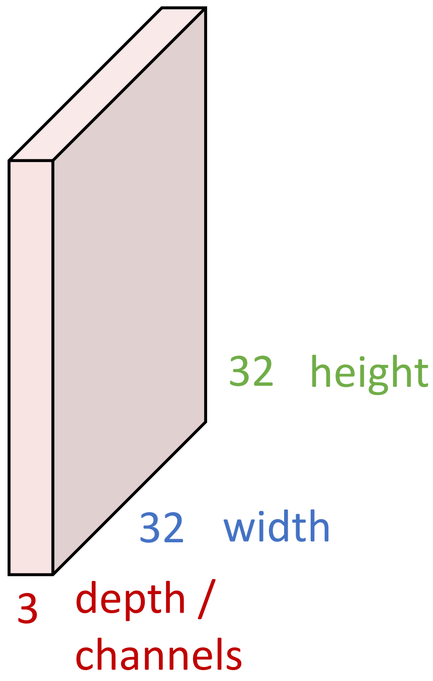


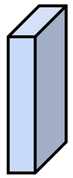
1 number:
The result of taking a dot product between a row of W and the input

Convolution Layer

3x32x32 image: preserve spatial structure

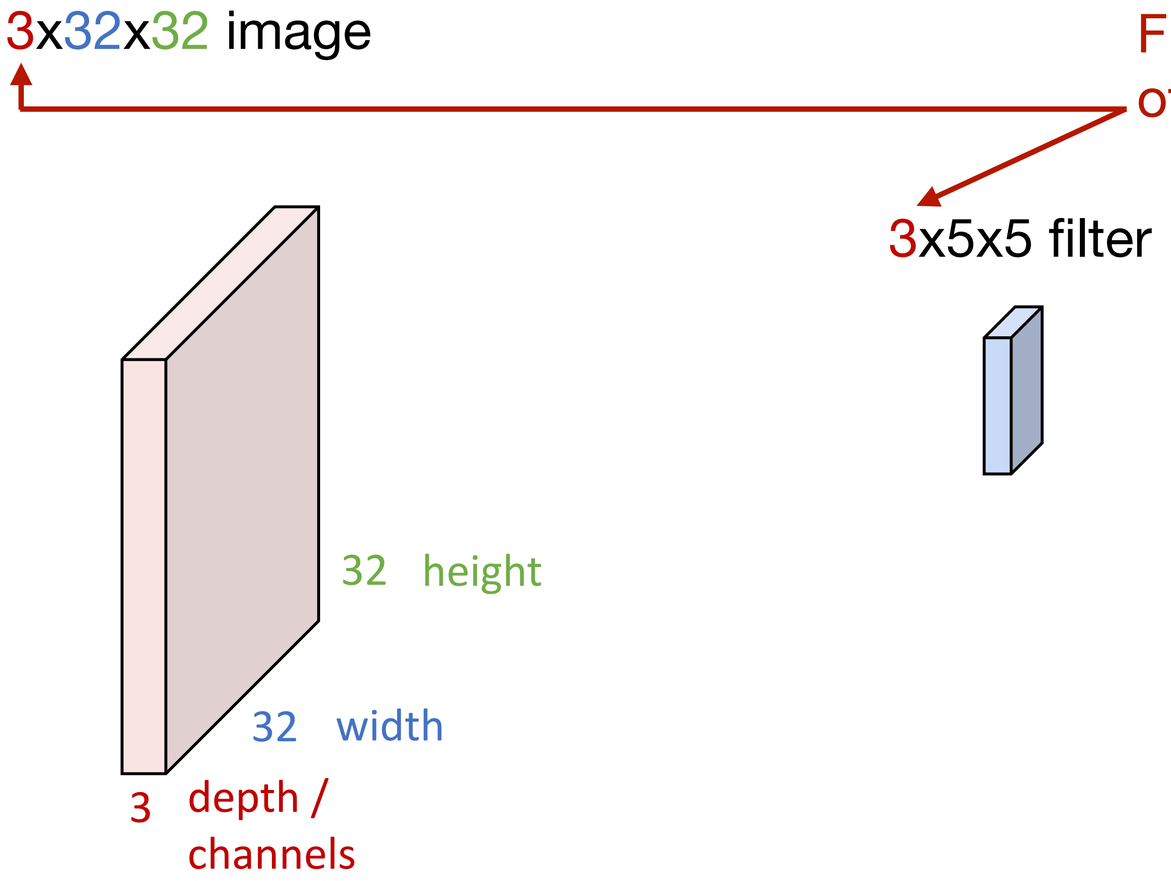


3x5x5 filter



Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

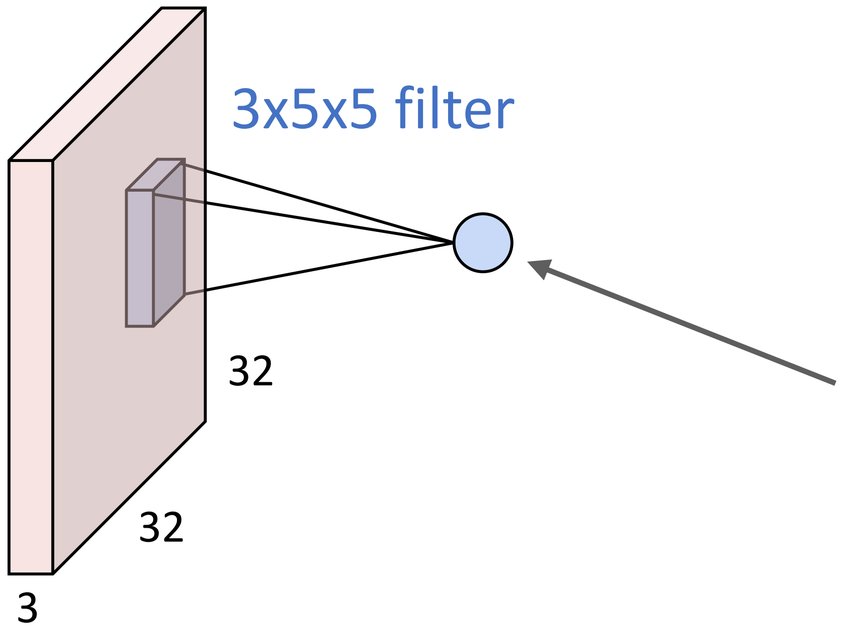
Convolution Layer



Convolve the filter with the image
i.e. “slide over the image spatially, computing dot products”

Convolution Layer

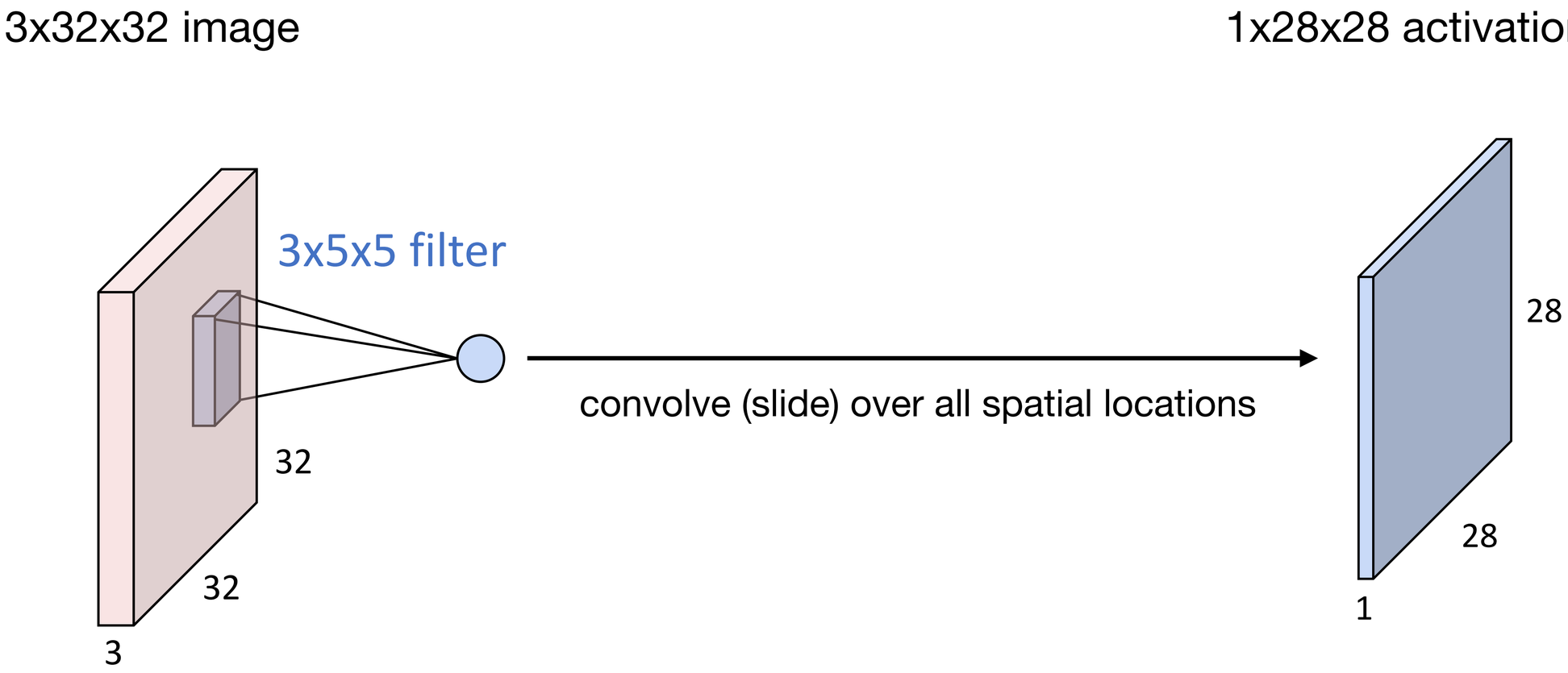
3x32x32 image



1 number:
The result of taking a dot product between the filter and a small 3x5x5 portion of the image
(i.e. 3*5*5=75-dimensional dot product + bias)

$$w^T x + b$$

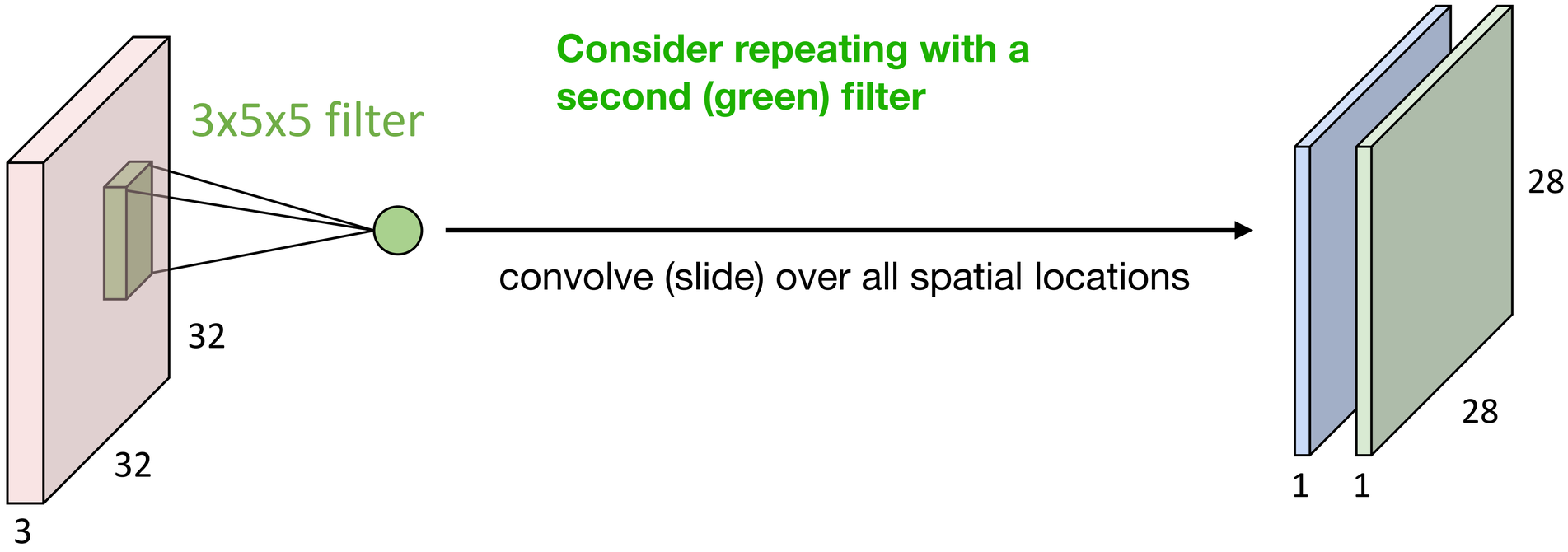
Convolution Layer



Convolution Layer

3x32x32 image

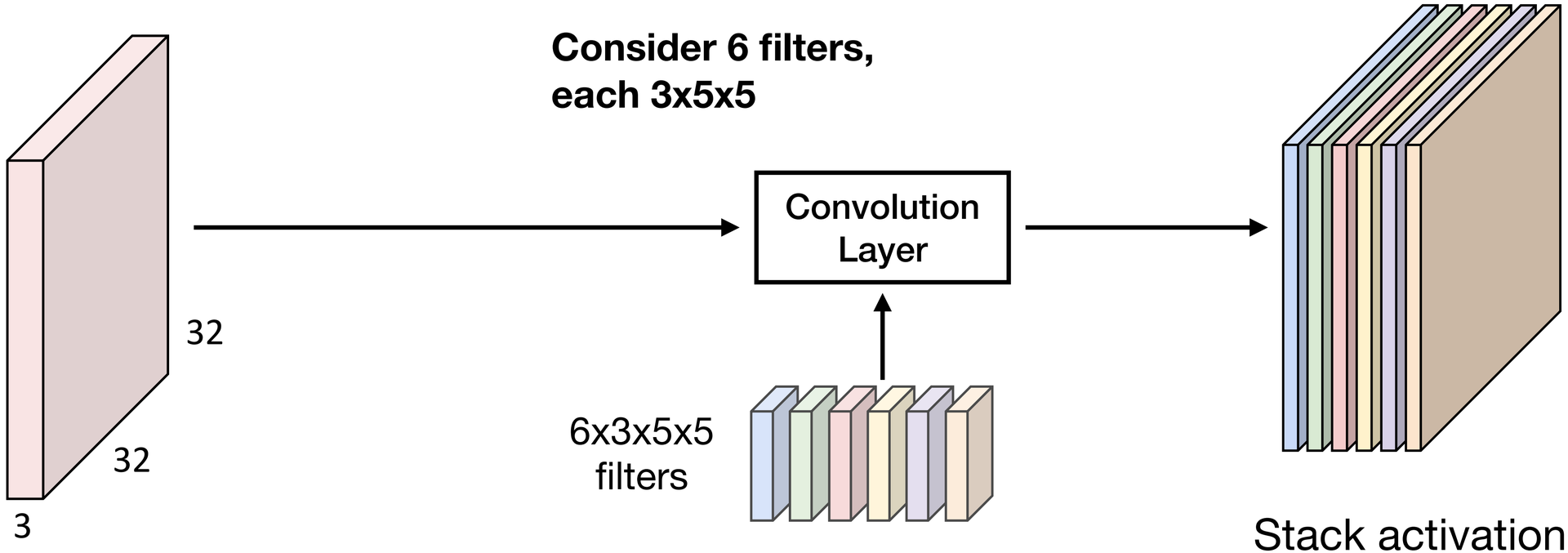
two 1x28x28 activation map



Convolution Layer

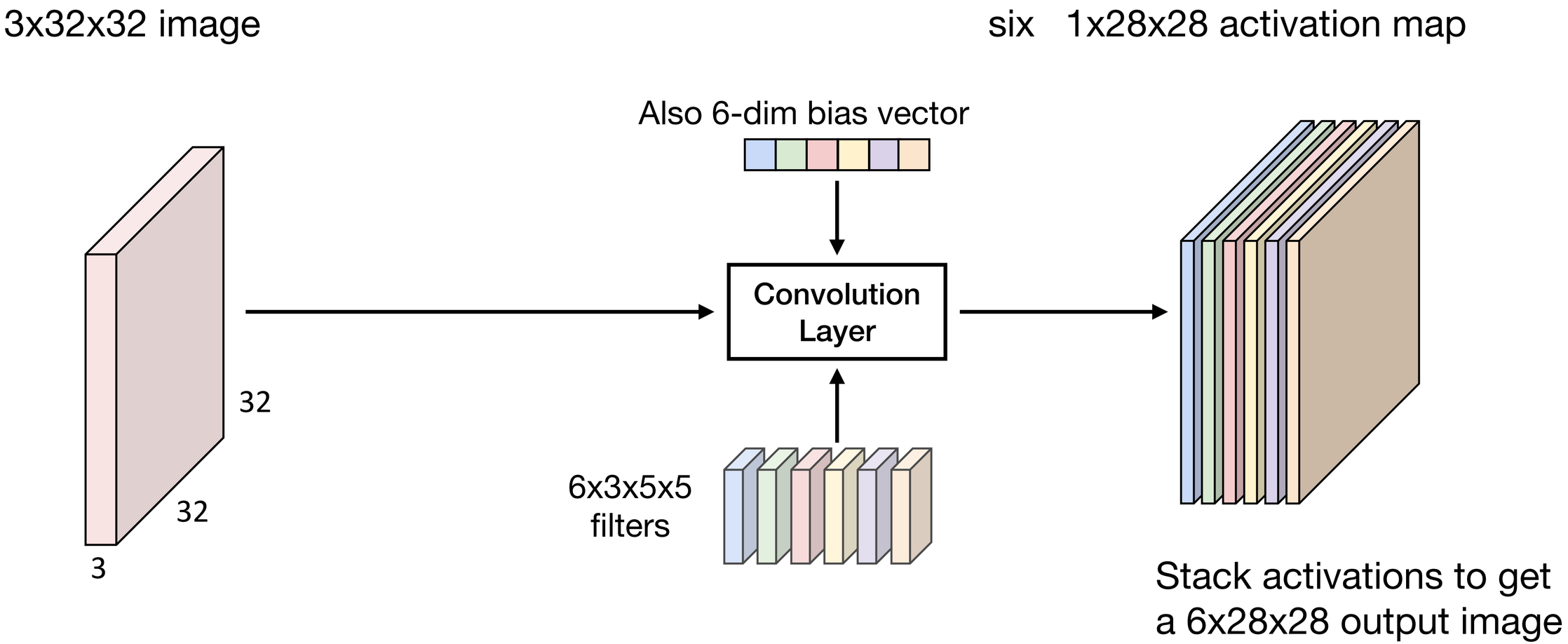
3x32x32 image

six 1x28x28 activation map

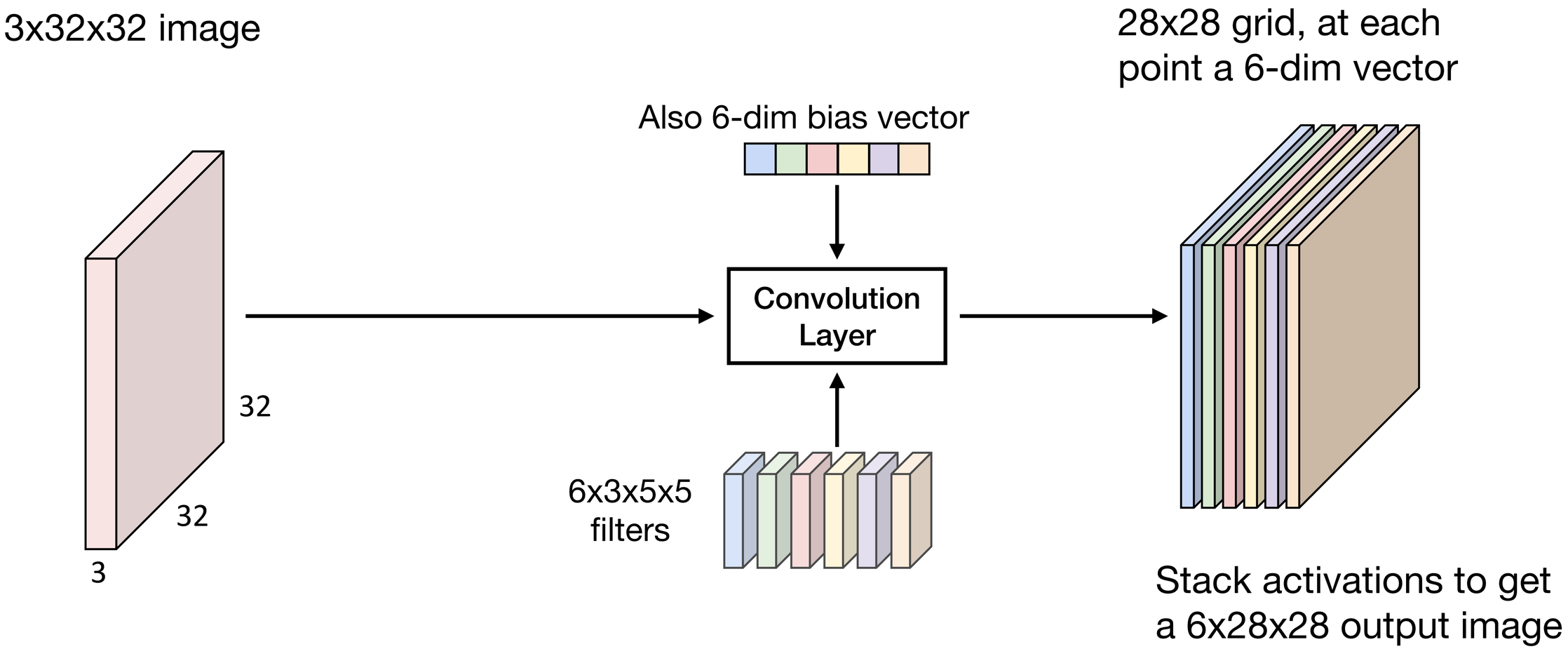


Stack activations to get
a 6x28x28 output image

Convolution Layer

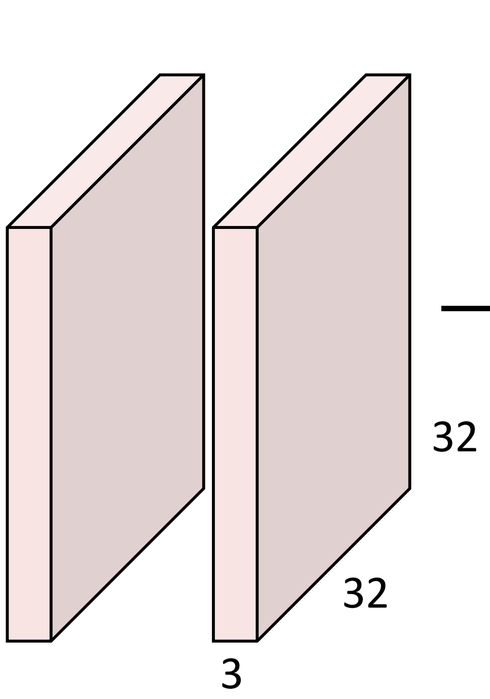


Convolution Layer

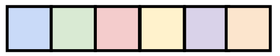


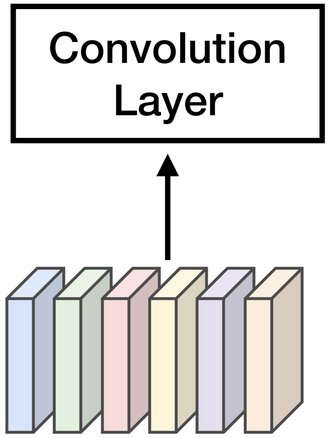
Convolution Layer

2x3x32x32
batch of images

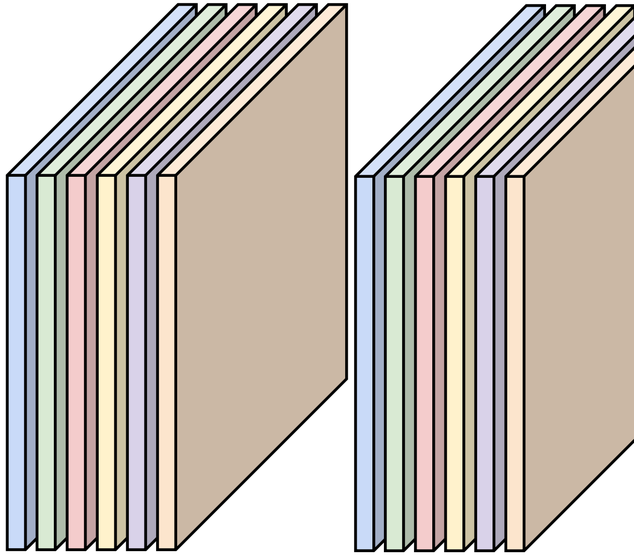


Also 6-dim bias vector

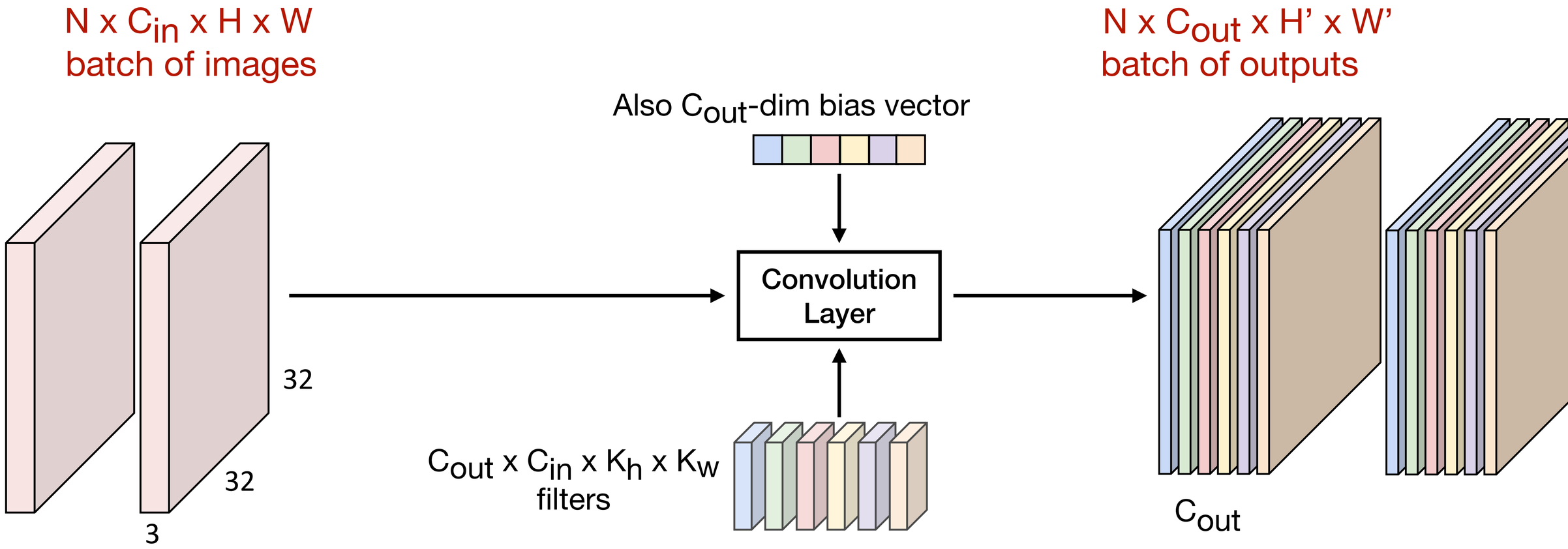




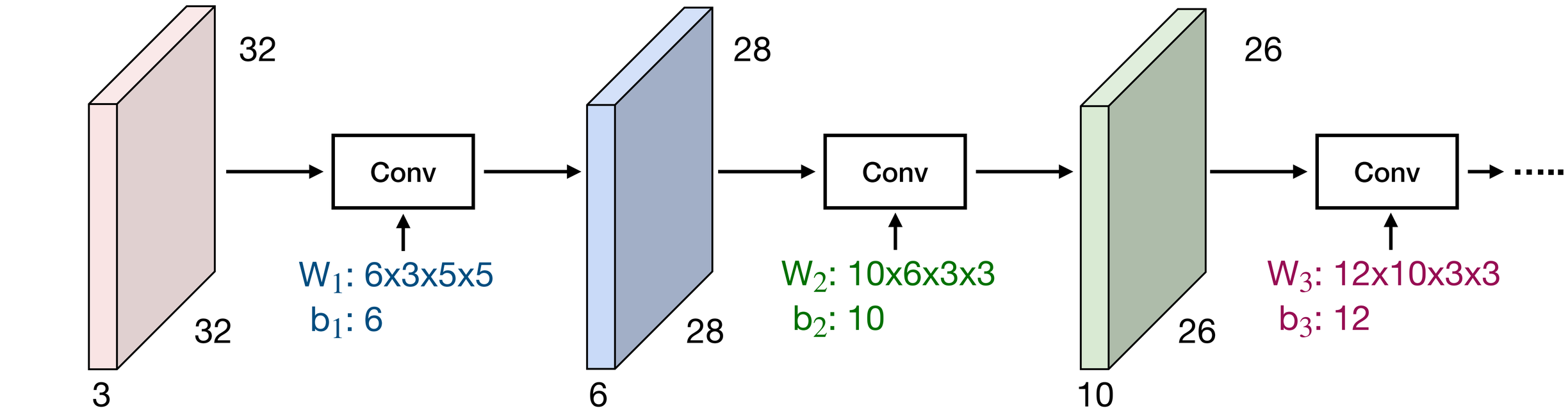
2x6x28x28
batch of outputs



Convolution Layer

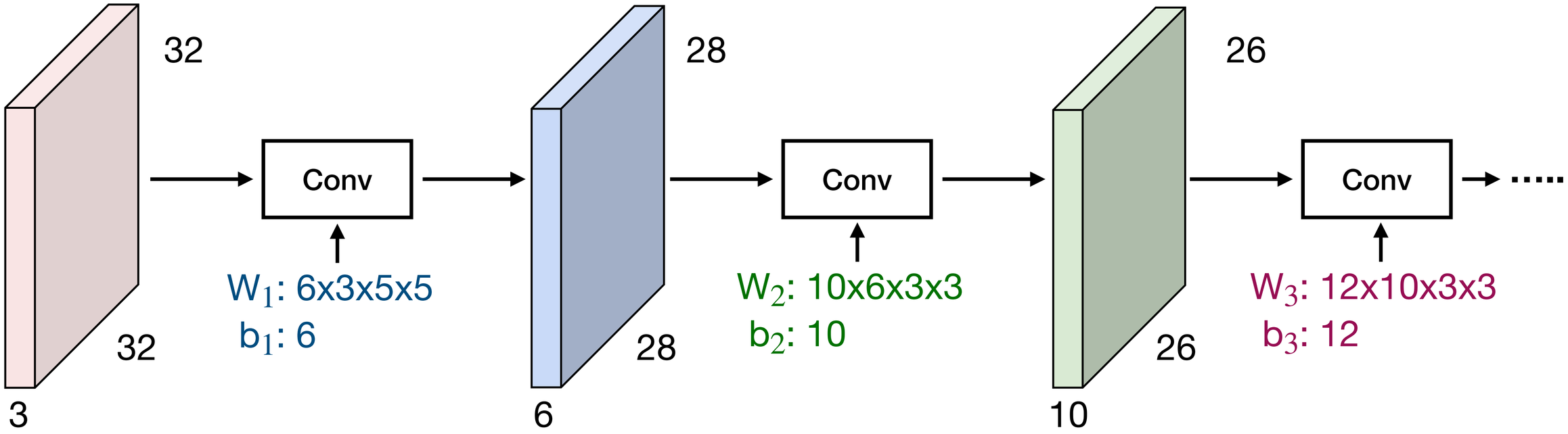


Stacking Convolutions



Stacking Convolutions

Q: What happens if we stack two convolution layers?

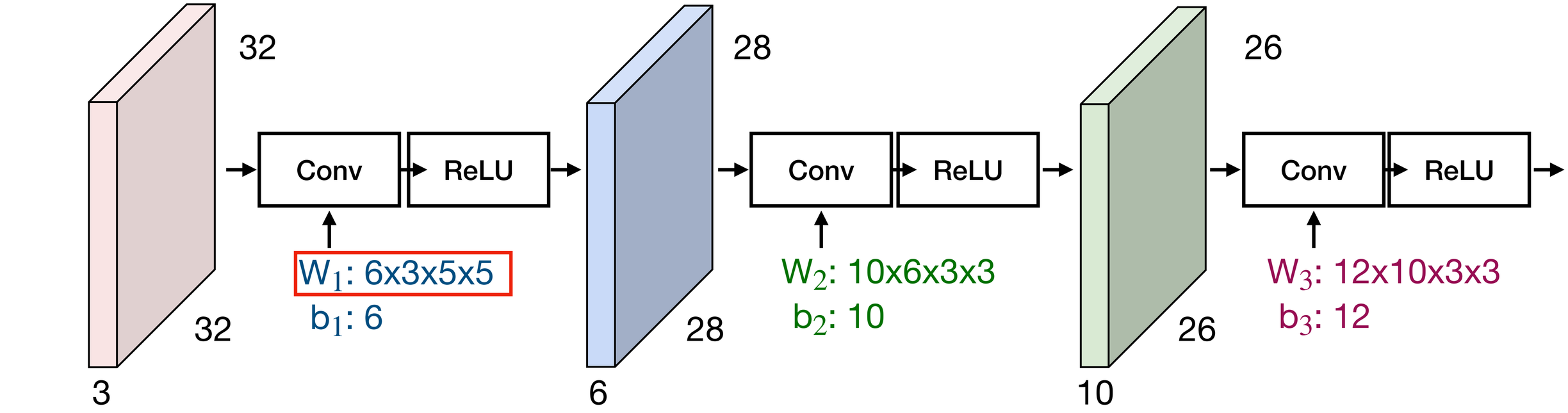


input
N x 3 x 32 x 32

First hidden layer
N x 6 x 28 x 28

Second hidden layer
N x 10 x 28 x 28

What do convolutional filters learn?



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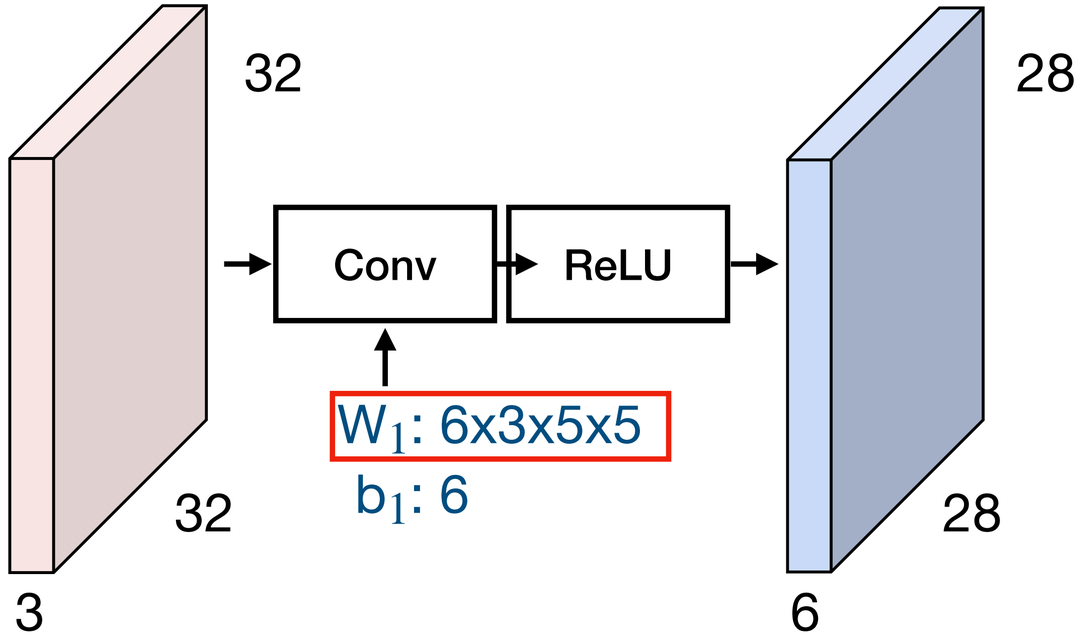
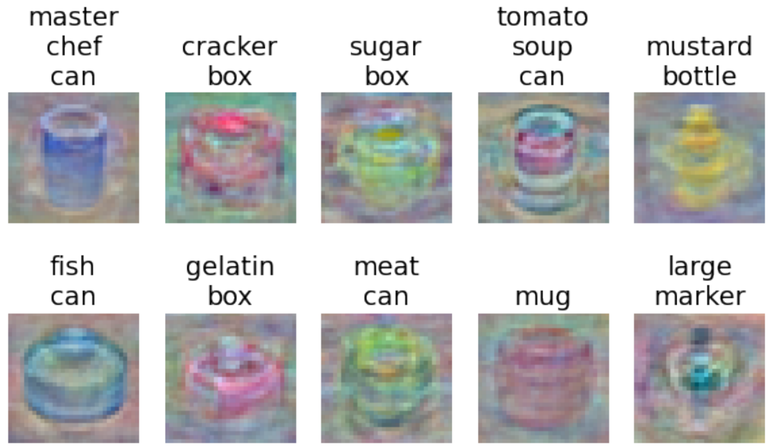


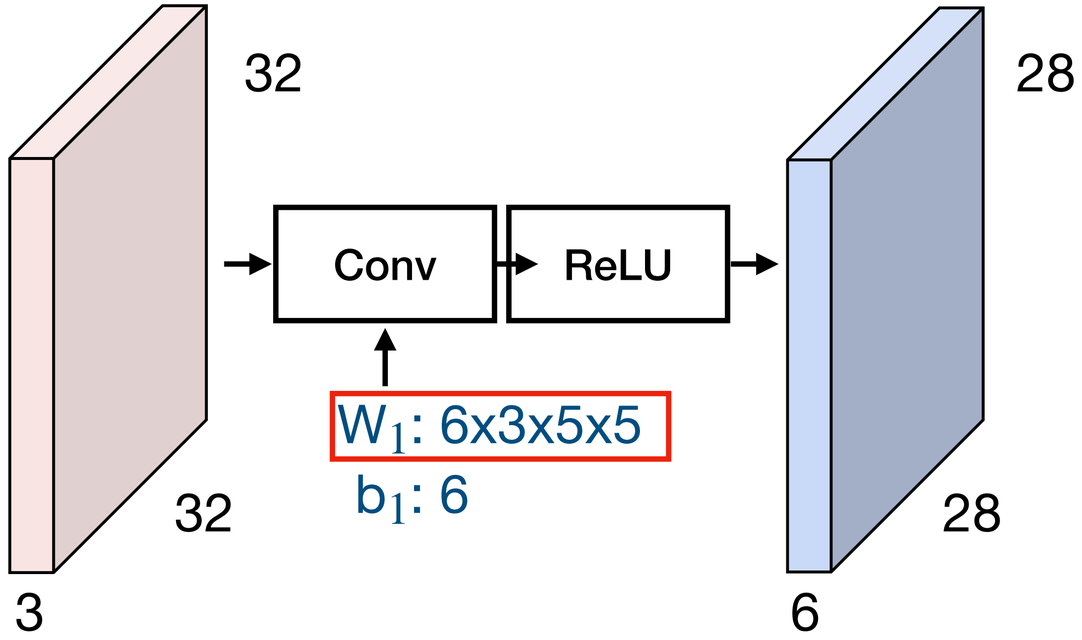
input
N x 3 x 32 x 32

First hidden layer
N x 6 x 28 x 28

Linear classifier: One template per class



What do convolutional filters learn?

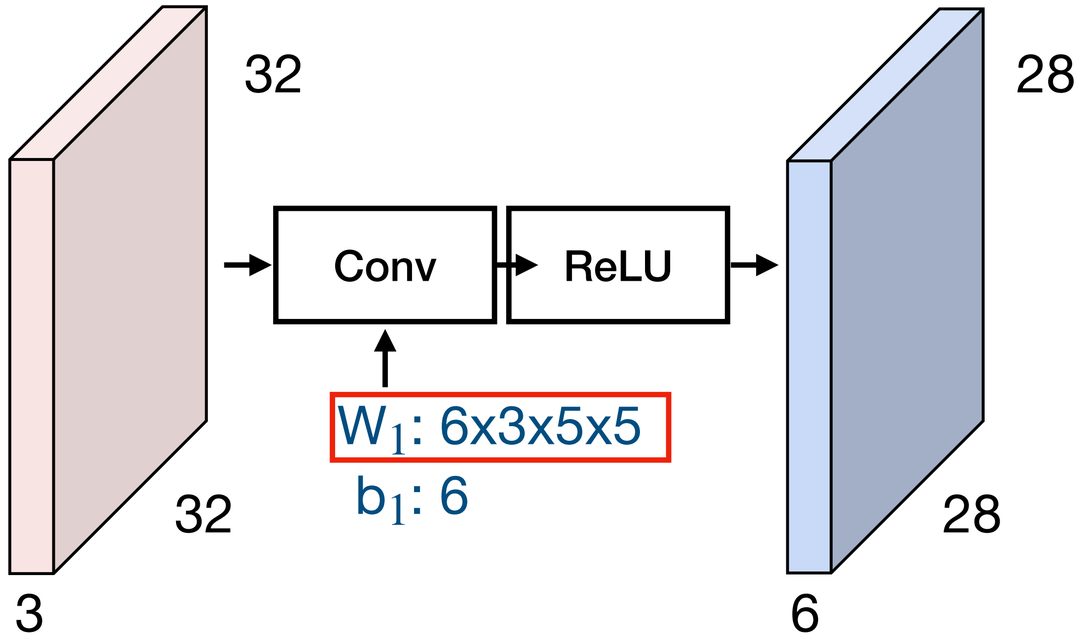


input
N x 3 x 32 x 32

First hidden layer
N x 6 x 28 x 28

MLP: Bank of whole-image templates

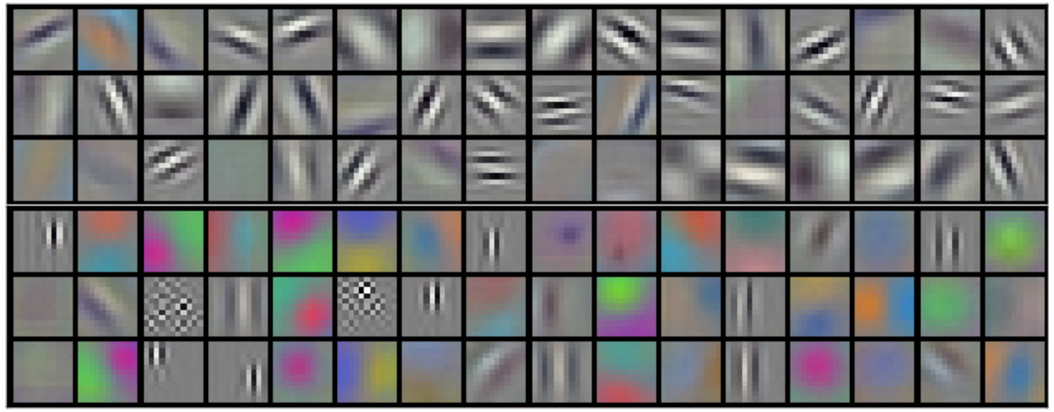
What do convolutional filters learn?



input
N x 3 x 32 x 32

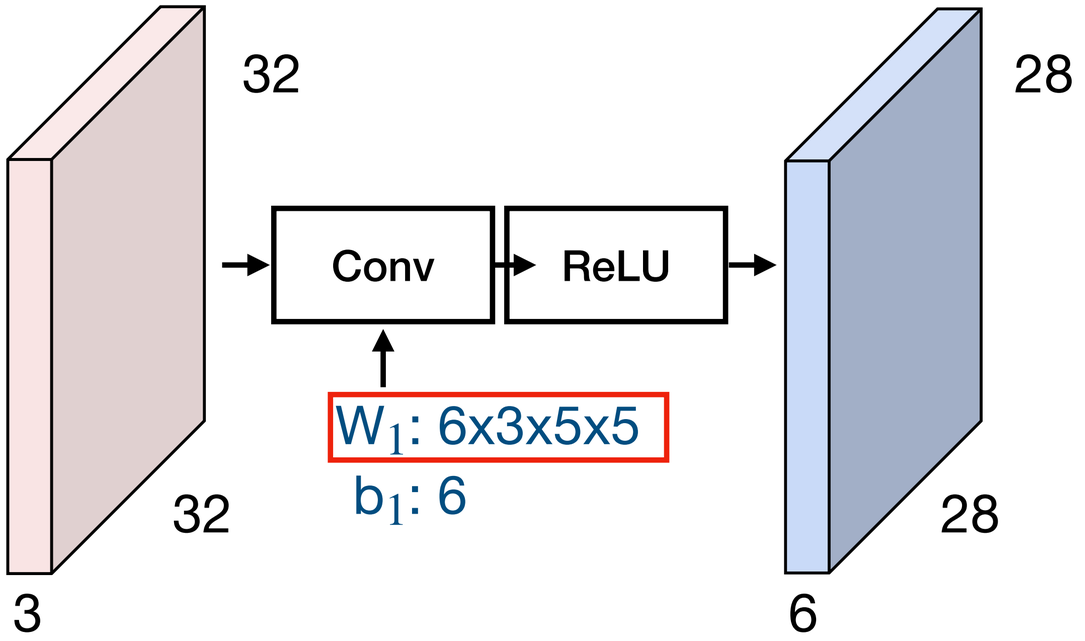
First hidden layer
N x 6 x 28 x 28

First-layer conv filters: local image templates
(often learns oriented edges, opposing colors)



AlexNet: 96 filters, each 3x11x11

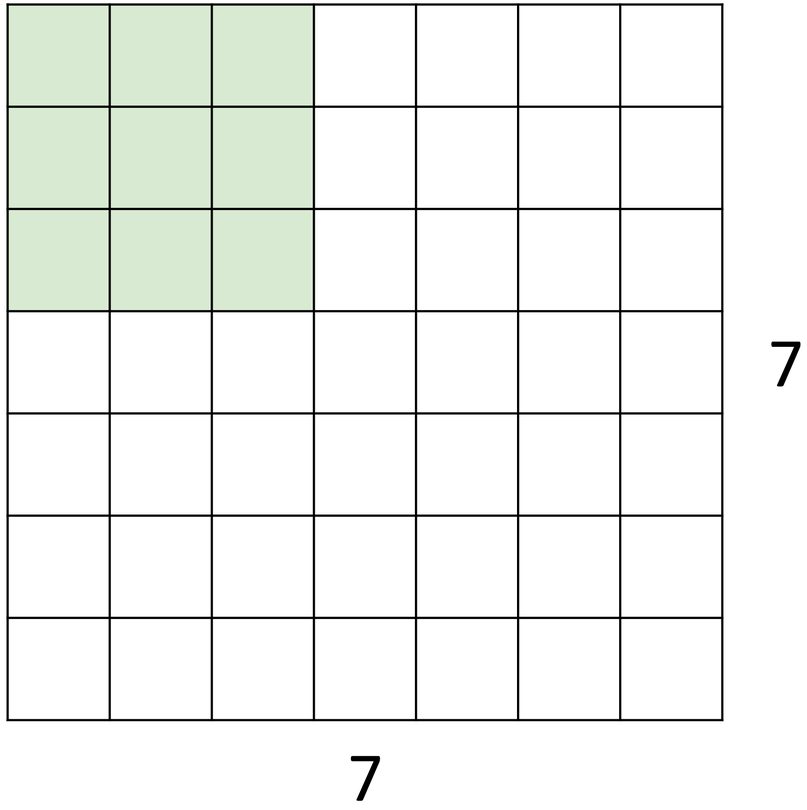
A closer look at spatial dimensions



input
N x 3 x 32 x 32

First hidden layer
N x 6 x 28 x 28

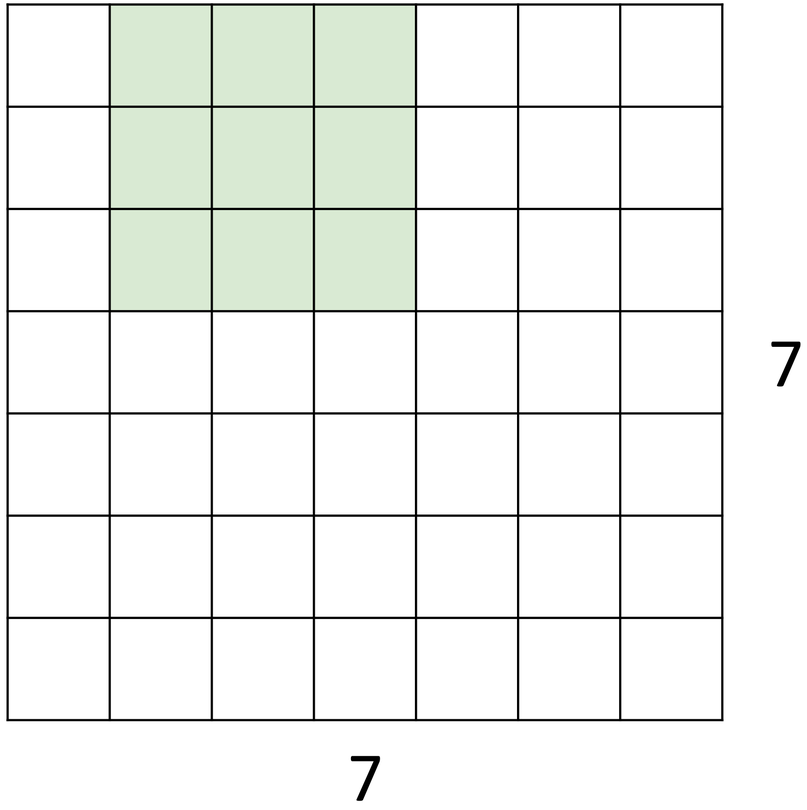
A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

A closer look at spatial dimensions



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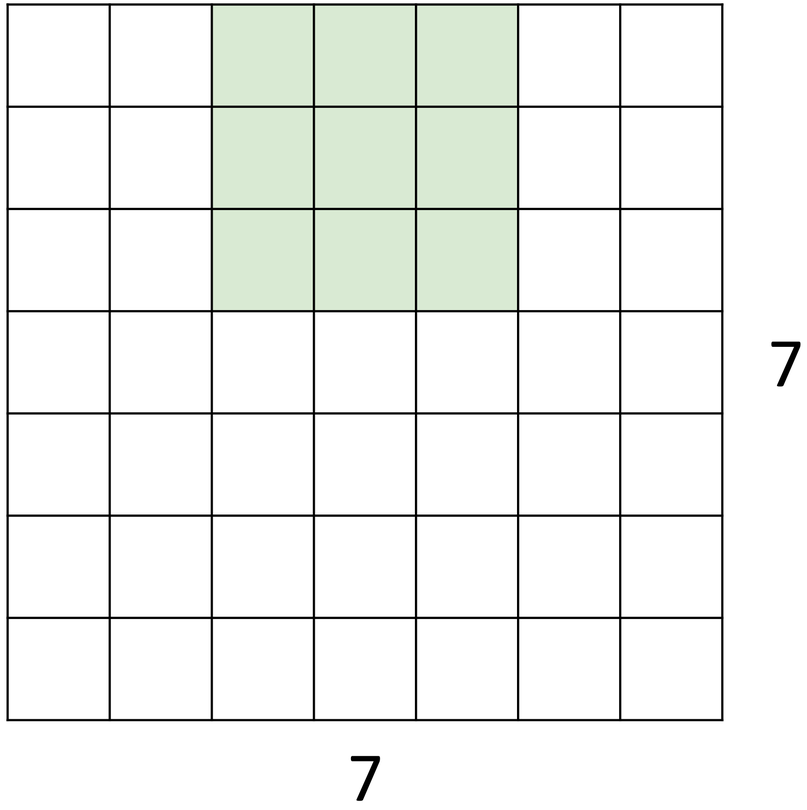
Filter: 3x3

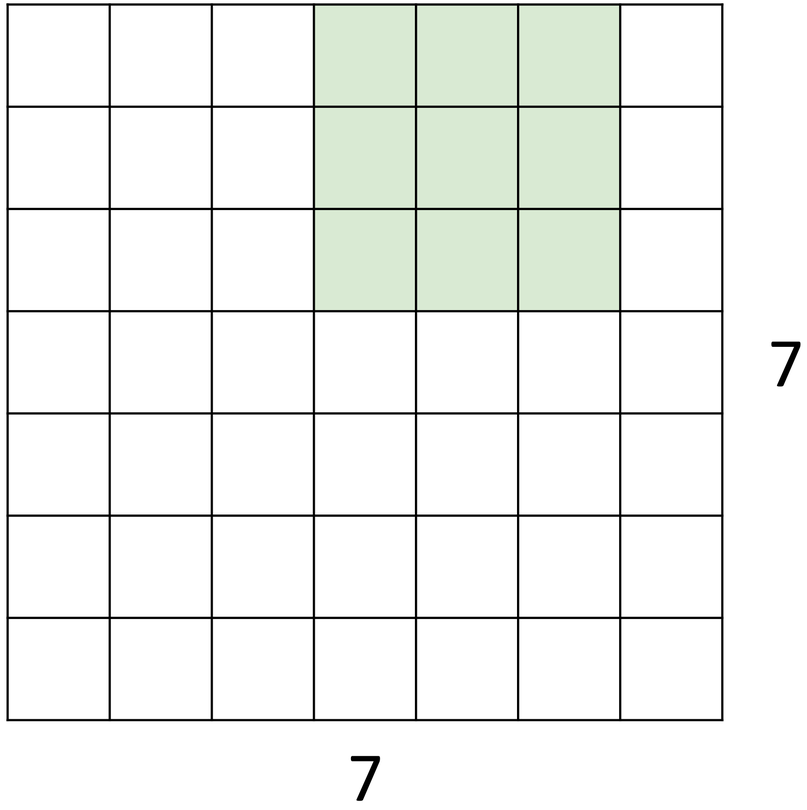
A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

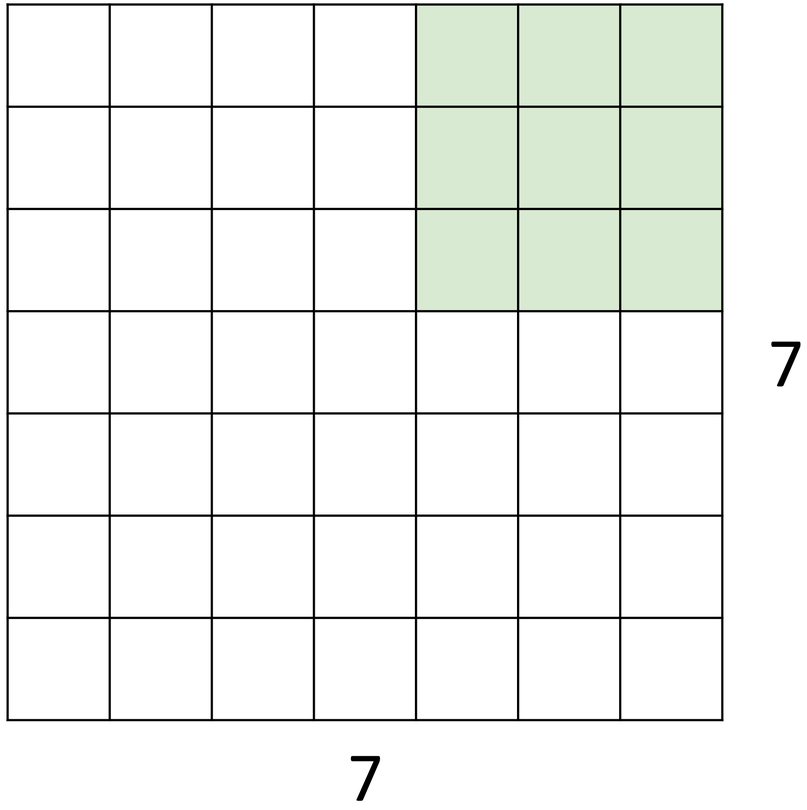
A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

A closer look at spatial dimensions

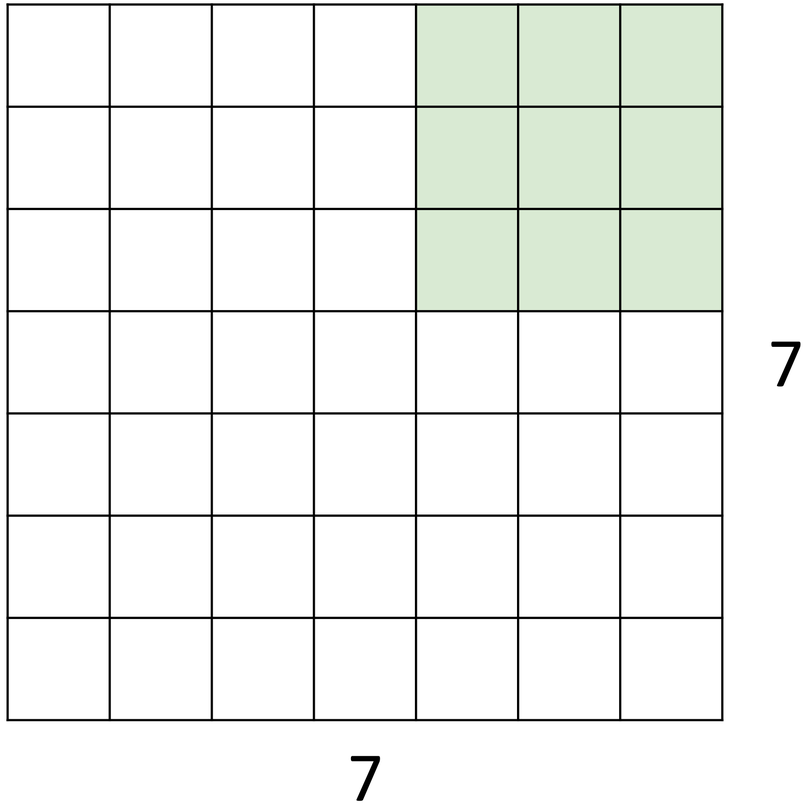


Input: 7x7

Filter: 3x3

Output: 5x5

A closer look at spatial dimensions



Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Output: W – K + 1

Problem: Feature maps “shrink” with each layer!

A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Output: W – K + 1

Problem: Feature maps “shrink” with each layer!

Solution: **padding**
Add zeros around the input

A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

Input: 7x7

Filter: 3x3

Output: 5x5

In general:

Input: W

Filter: K

Padding: P

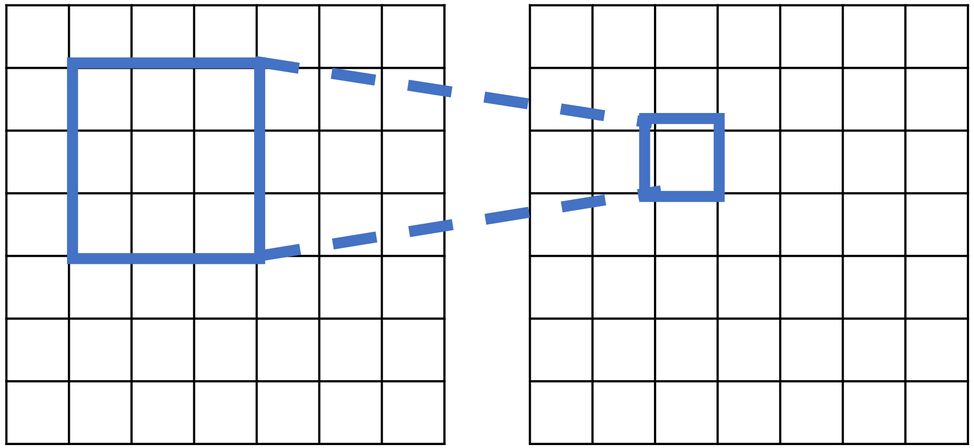
Output: W – K + 1 + 2P

Very common:

Set P = (K – 1) / 2 to make output have same size as input!

Receptive Fields

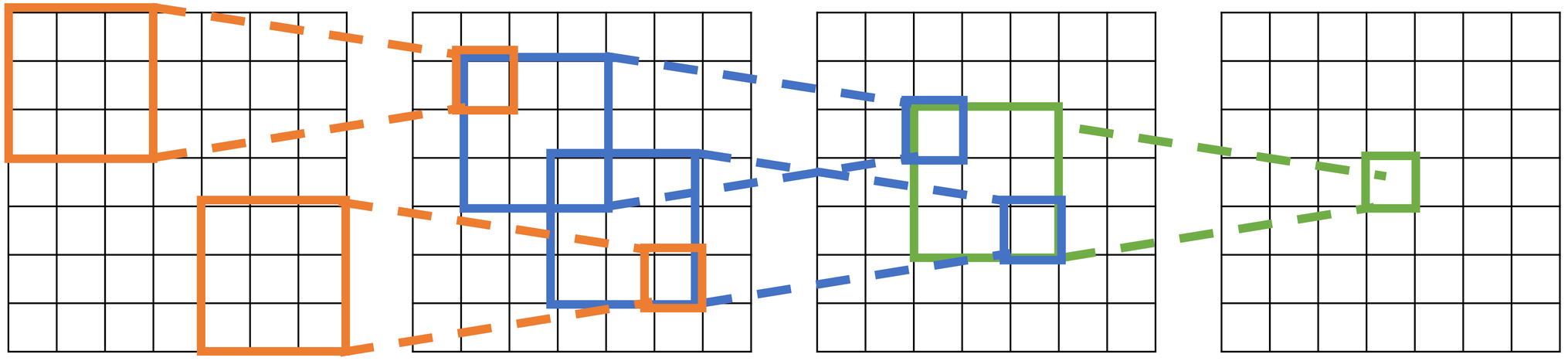
For convolution with kernel size K, each element in the output depends on a K x K **receptive field** in the input



Input Output

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



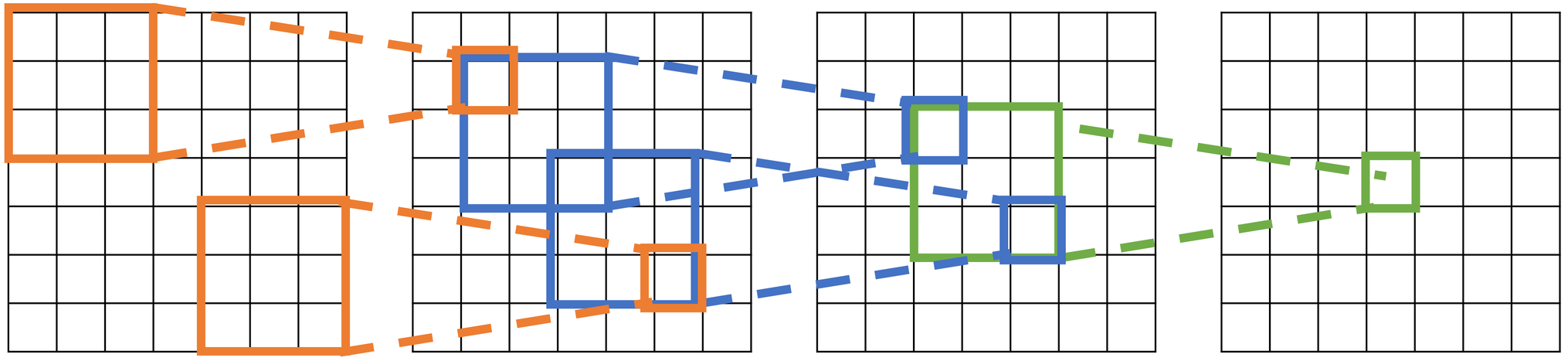
Input

Output

Be careful – ”receptive field in the input” vs “receptive field in the previous layer”
Hopefully clear from context!

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



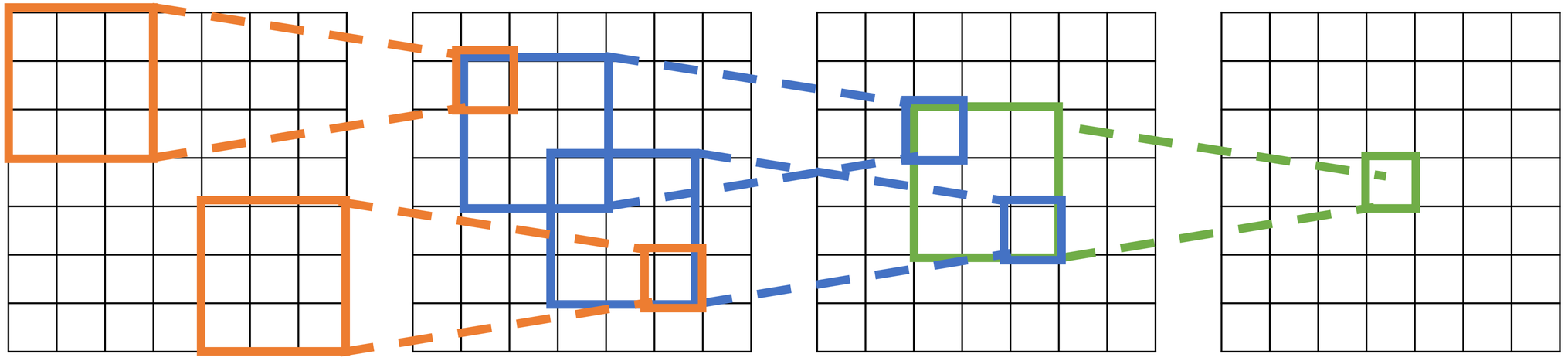
Input

Output

Problem: For large images we need many layers for each output to “see” the whole image image

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



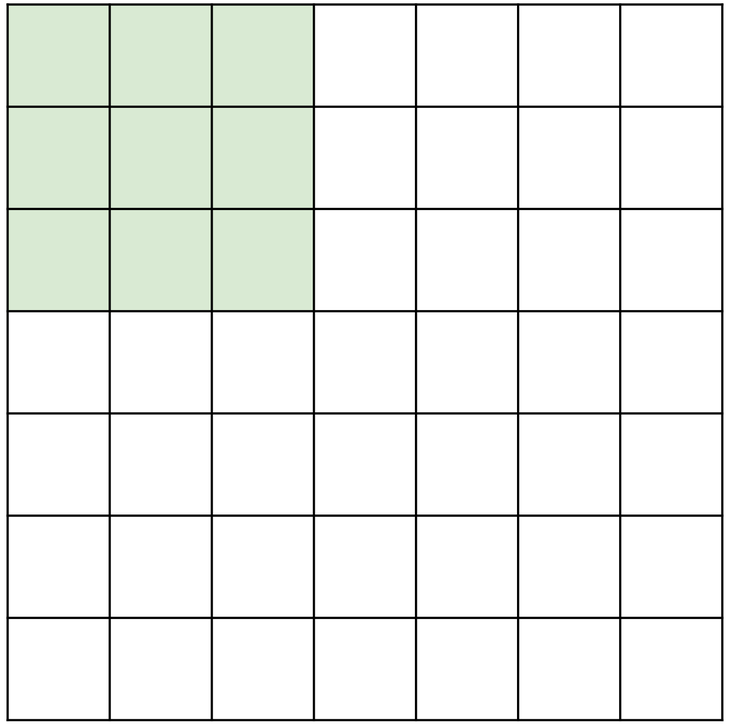
Input

Output

Problem: For large images we need many layers for each output to “see” the whole image image

Solution: Downsample inside the network

Strided Convolution

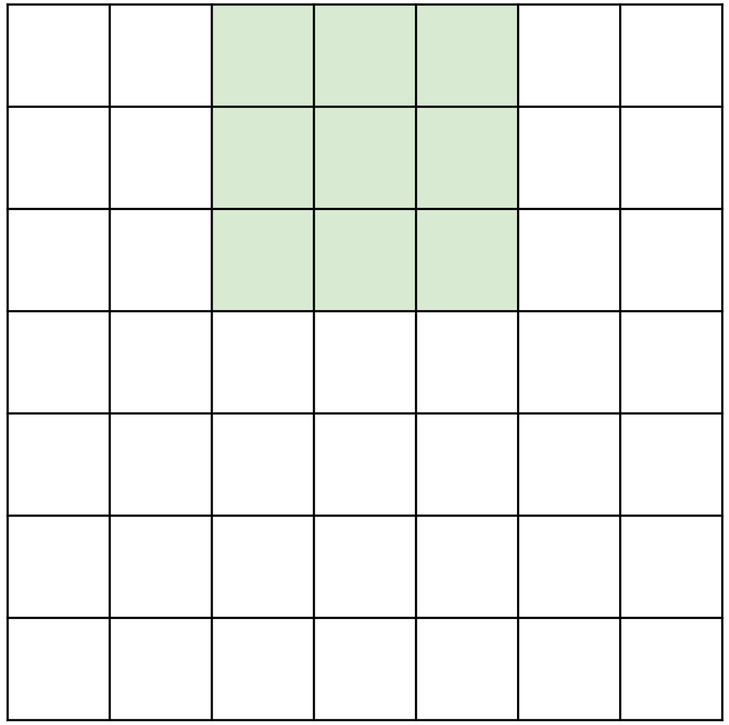


Input: 7x7

Filter: 3x3

Stride: 2

Strided Convolution

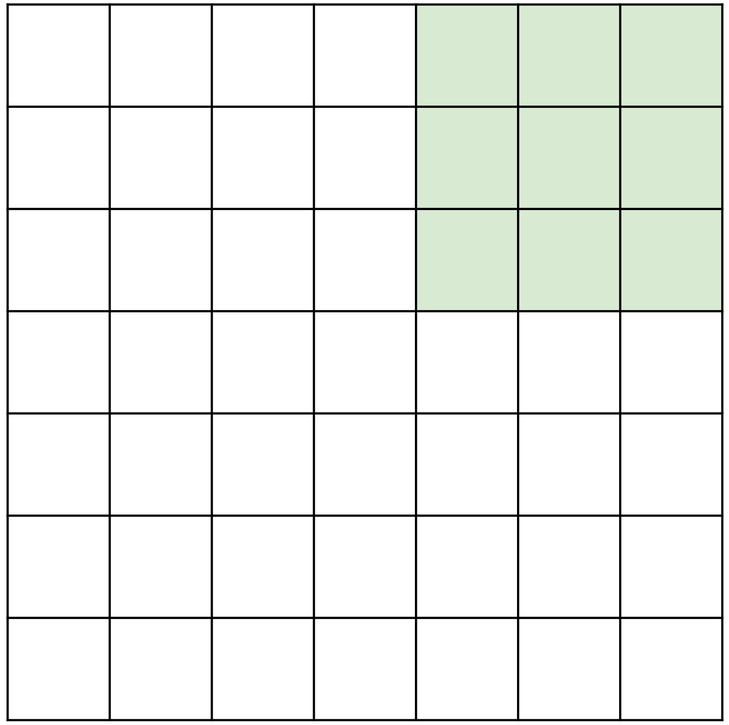


Input: 7x7

Filter: 3x3

Stride: 2

Strided Convolution



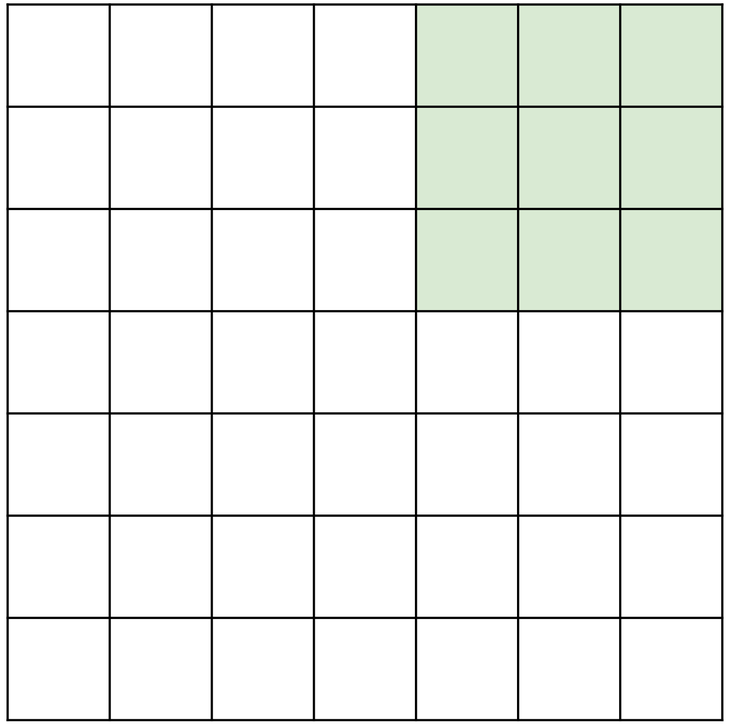
Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

Strided Convolution

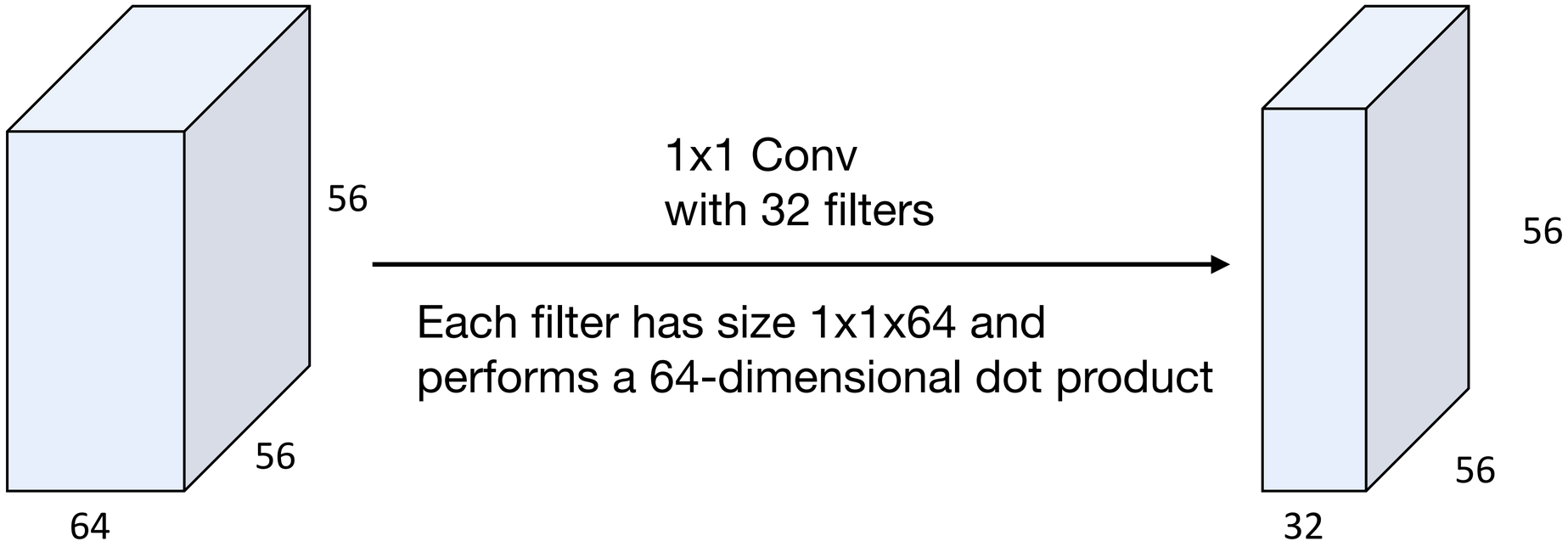


Input: 7x7
Filter: 3x3
Stride: 2

Output: 3x3

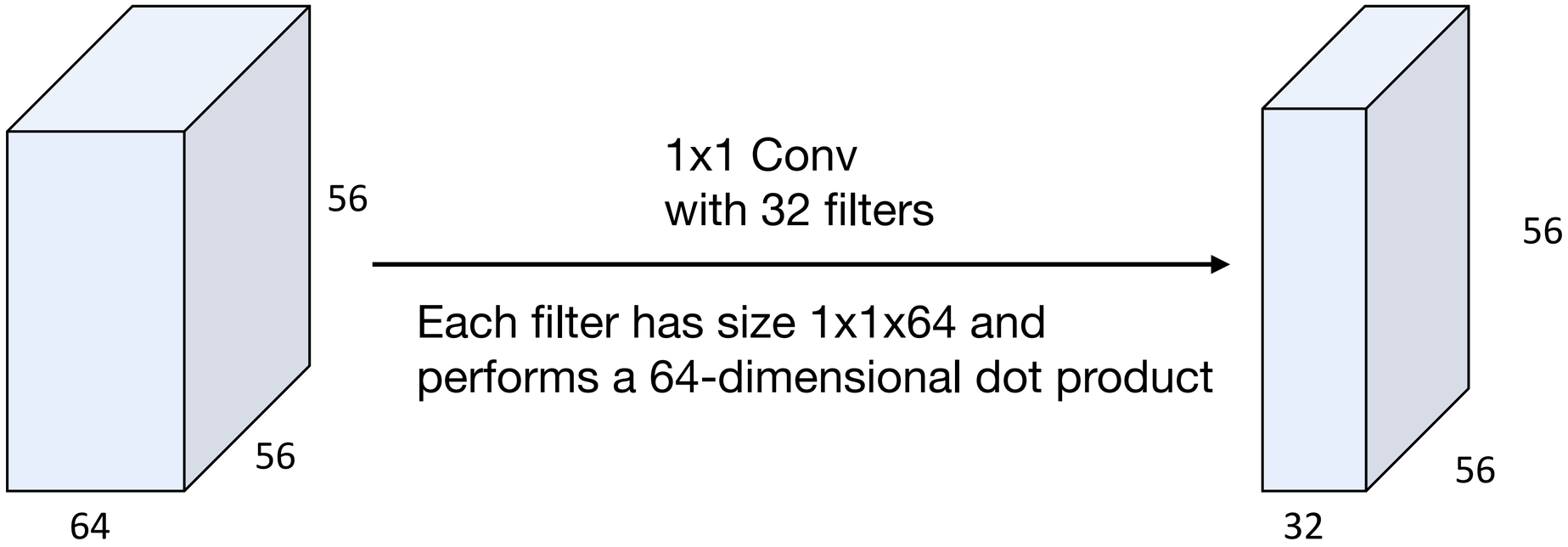
In general:
Input: W
Filter: K
Padding: P
Stride: S
Output: (W – K + 2P) / S + 1

Example: 1x1 Convolution



Stacking 1x1 conv layers gives MLP operating on each input position

Example: 1x1 Convolution



Lin et al., “Network in Network”, ICLR 2014

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W

giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- H' = (H – K + 2P) / S + 1
- W' = (W – K + 2P) / S + 1

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W
giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- H' = (H – K + 2P) / S + 1
- W' = (W – K + 2P) / S + 1

Common settings:

$K_H = K_W$ (Small square filters)

P = (K – 1) / 2 ("Same" padding)

C_{in}, C_{out} = 32, 64, 128, 256 (powers of 2)

K = 3, P = 1, S = 1 (3x3 conv)

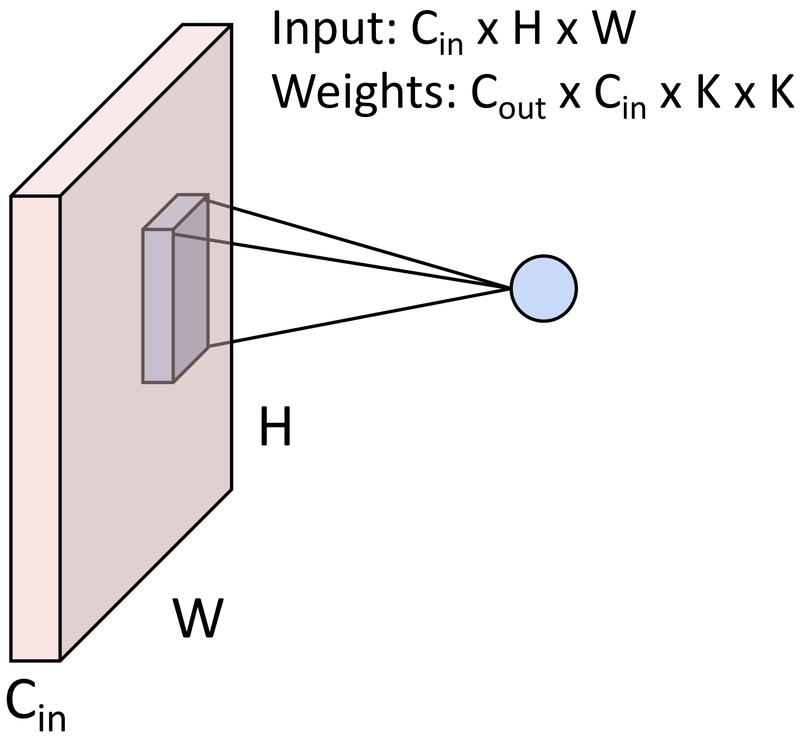
K = 5, P = 2, S = 1 (5x5 conv)

K = 1, P = 0, S = 1 (1x1 conv)

K = 3, P = 1, S = 2 (Downsample by 2)

Other types of convolution

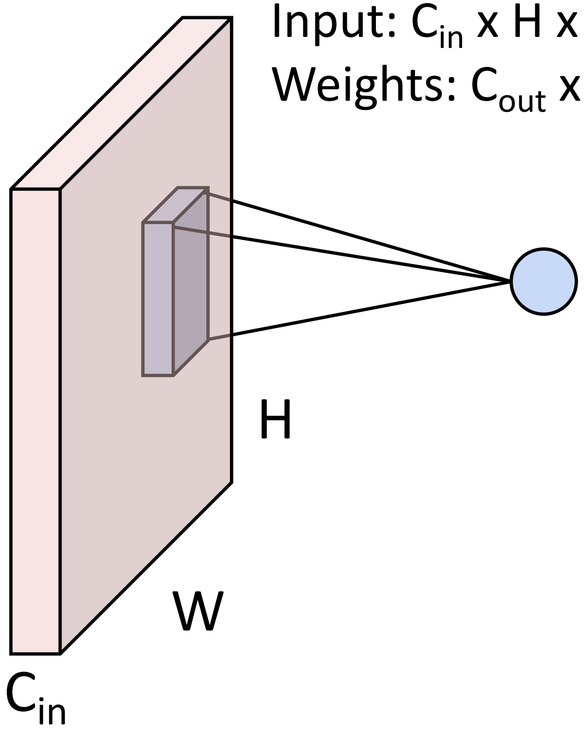
So far: 2D Convolution



Other types of convolution

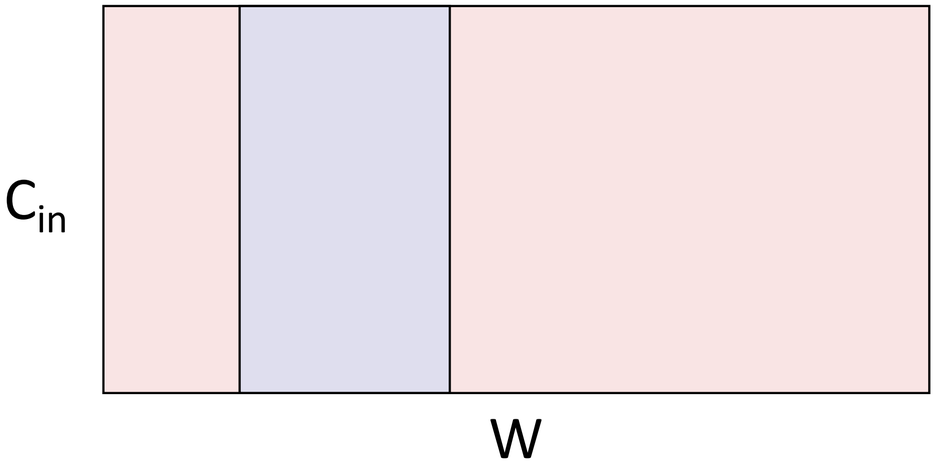
So far: 2D Convolution

Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



1D Convolution

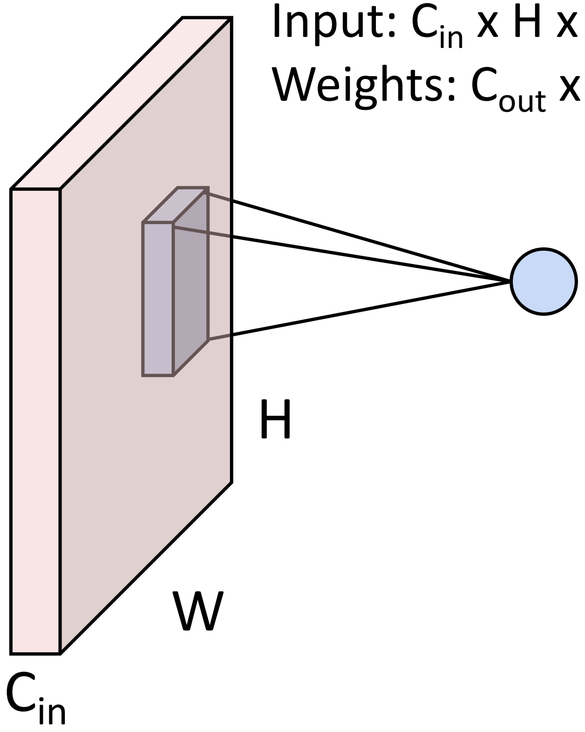
Input: C_{in} x W
Weights: C_{out} x C_{in} x K



Other types of convolution

So far: 2D Convolution

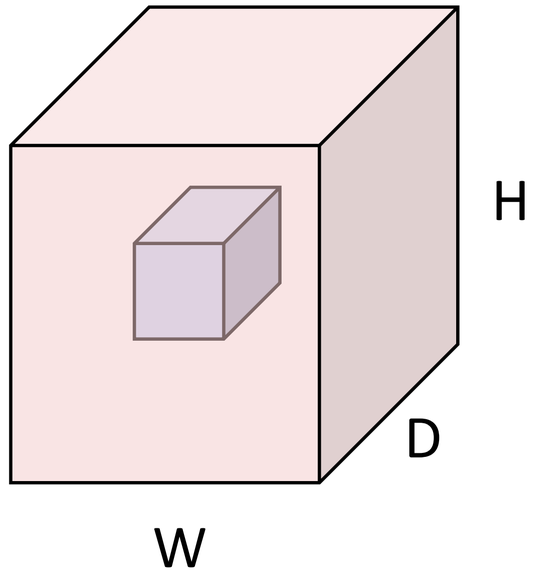
Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



3D Convolution

Input: C_{in} x H x W x D
Weights: C_{out} x C_{in} x K x K x K

C_{in}-dim vector
at each point
in the volume



PyTorch Convolution Layer

Conv2d

CLASS `torch.nn.Conv2d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

PyTorch Convolution Layer

Conv2d

CLASS `torch.nn.Conv2d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

Conv1d

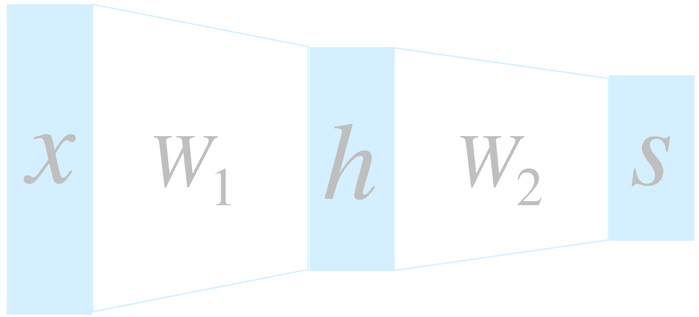
CLASS `torch.nn.Conv1d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

Conv3d

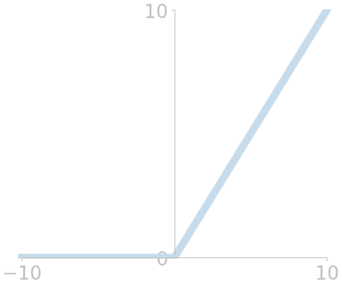
CLASS `torch.nn.Conv3d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

Components of Convolutional Neural Networks

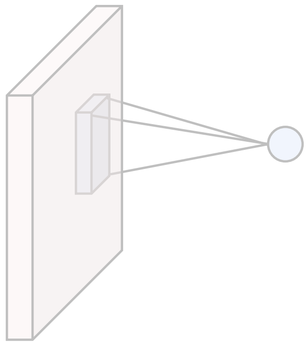
Fully-Connected Layers



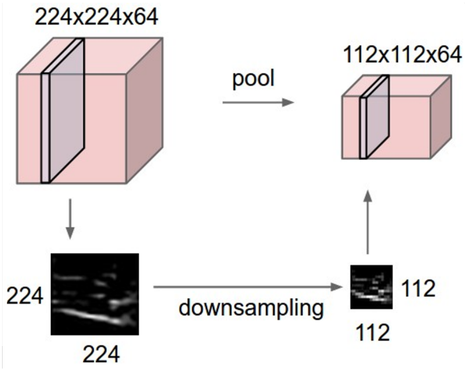
Activation Functions



Convolution Layers



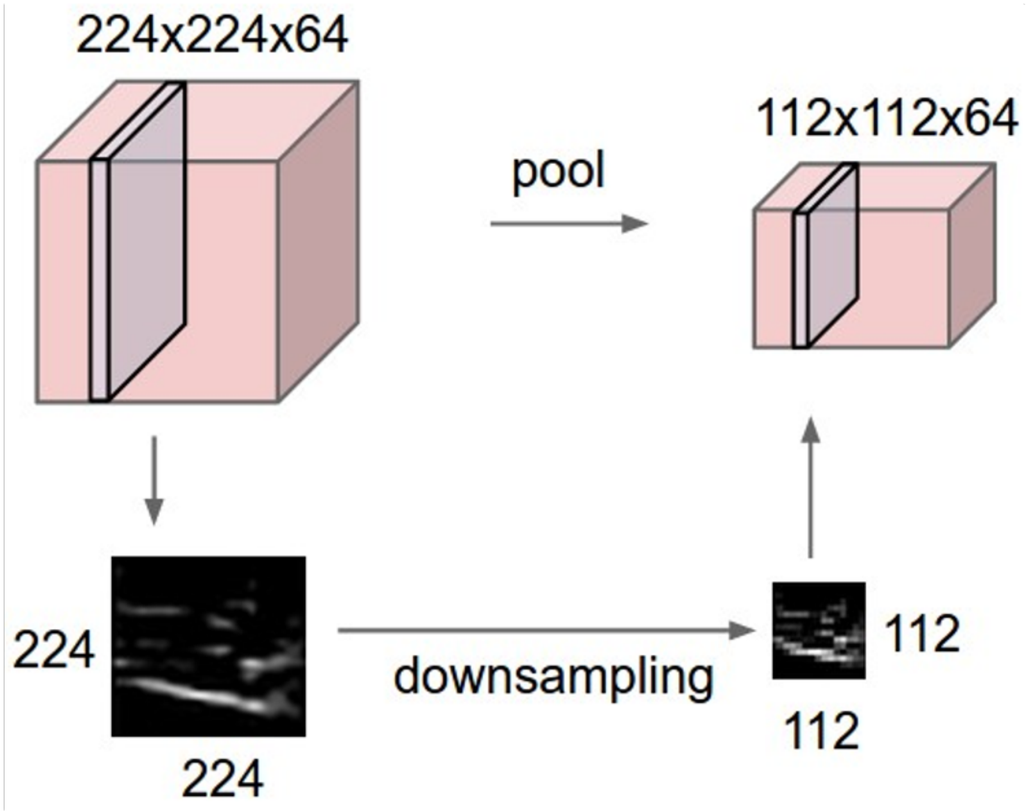
Pooling Layers



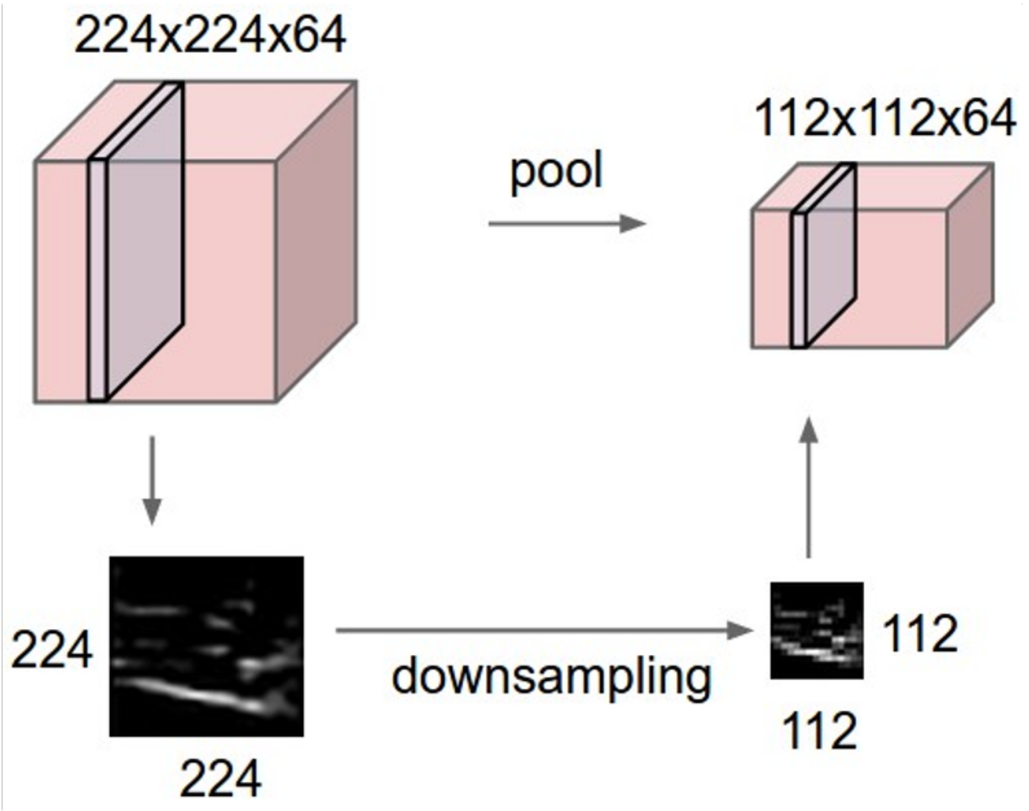
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Pooling Layers: Another way to downsample

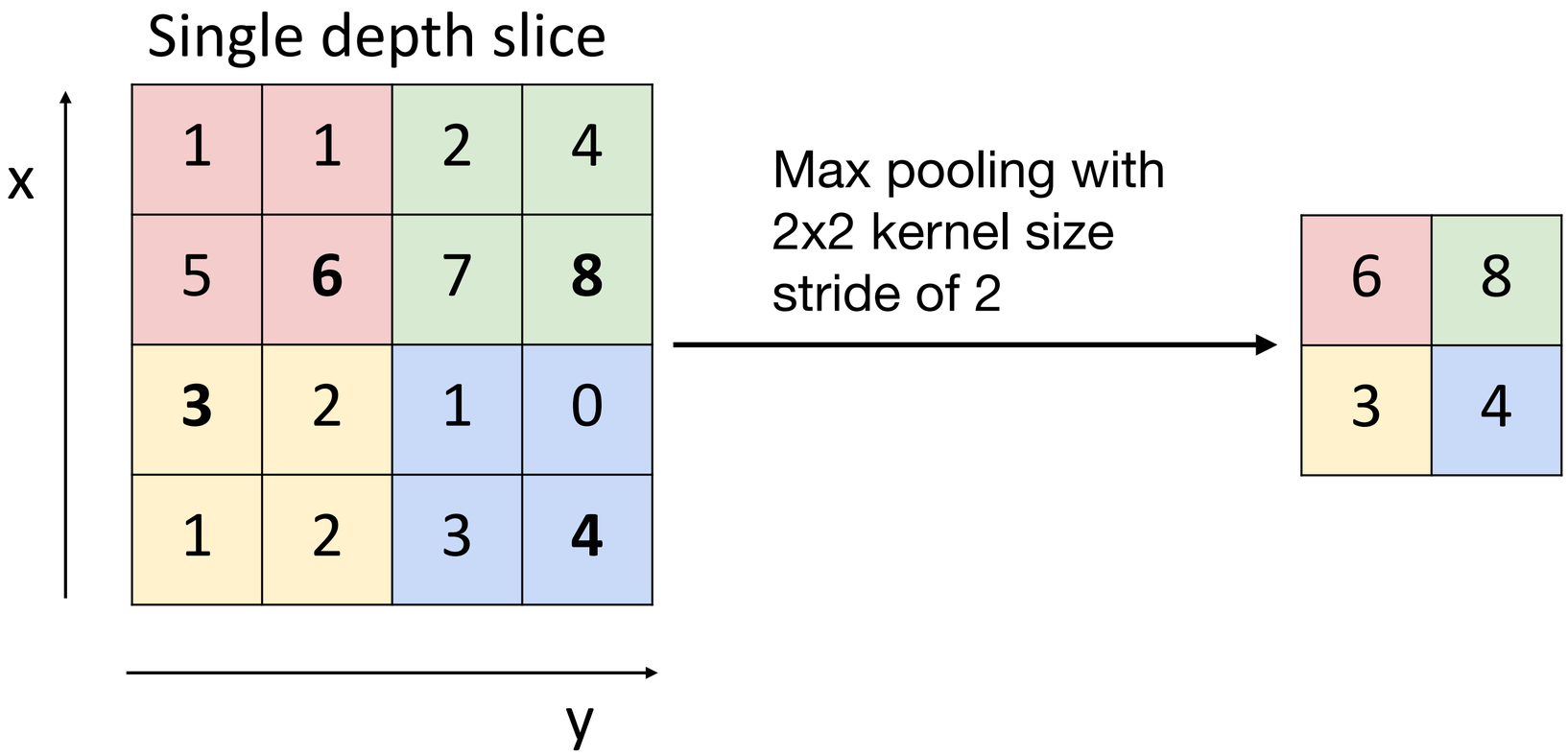


Pooling Layers: Another way to downsample

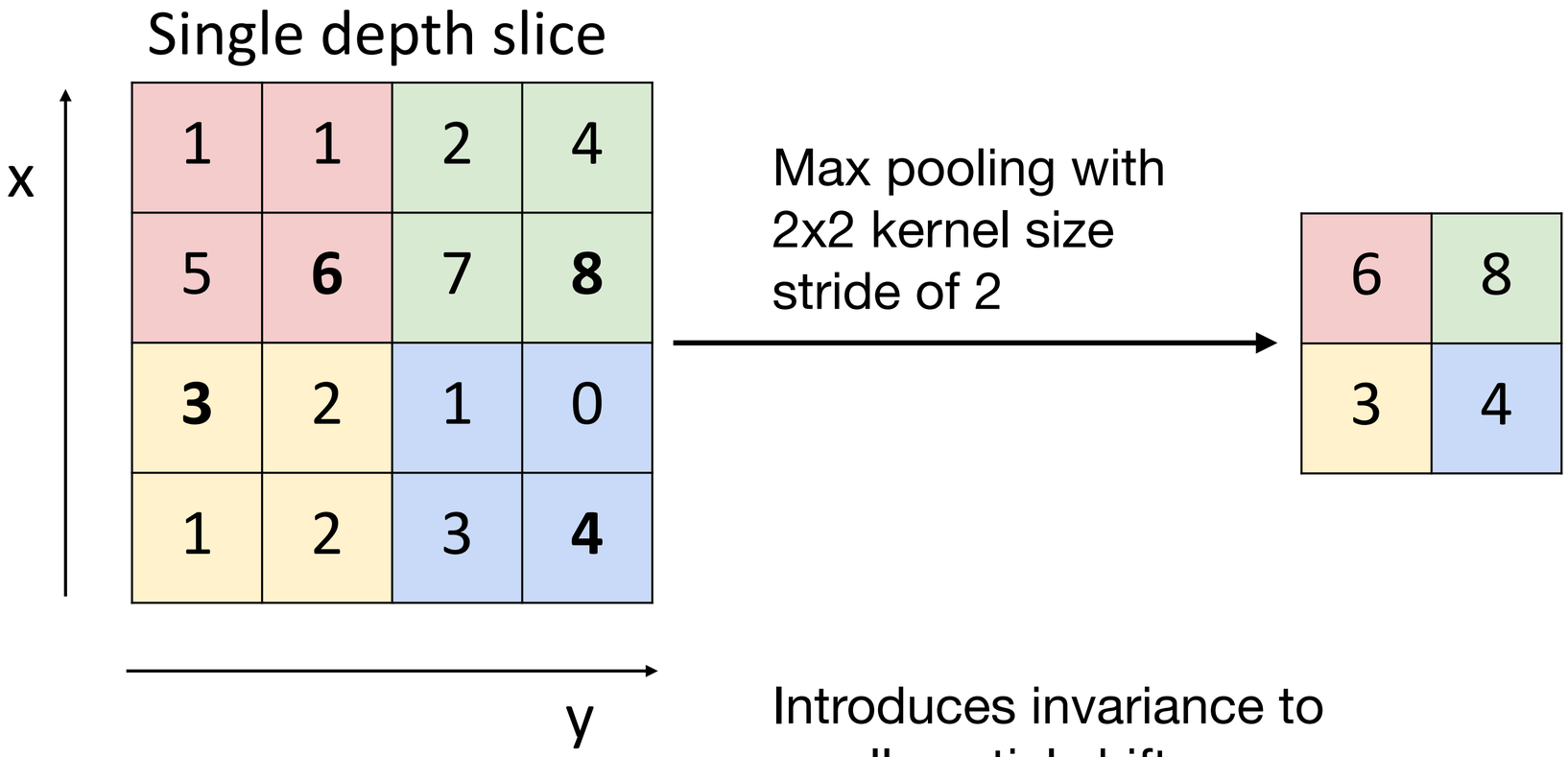


Hyperparameters:
Kernel size
Stride
Pooling function

Max Pooling


Single depth slice
x
y
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
Max pooling with
2x2 kernel size
stride of 2
6 8
3 4

Max Pooling



Introduces invariance to small spatial shifts

No learnable parameters!

Pooling Summary

Input: C x H x W

Hyperparameters:

- Kernel size: K
- Stride: S
- Pooling function (max, avg)

Output: C x H’ x W’ where

- H’ = (H – K) / S + 1
- W’ = (W – K) / S + 1

Learnable parameters: None!

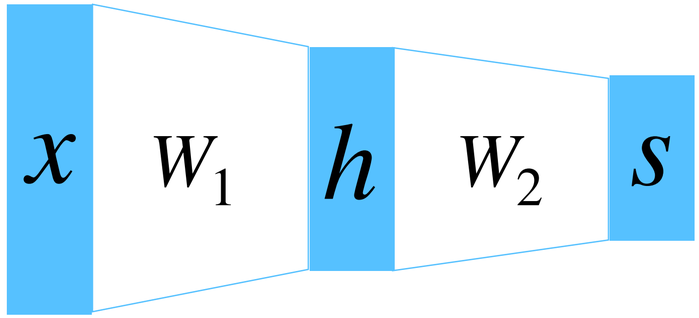
Common settings:

max, K = 2, S = 2

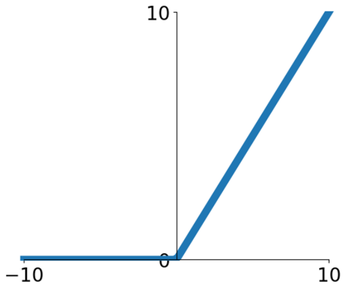
max, K = 3, S = 2 (AlexNet)

Components of Convolutional Neural Networks

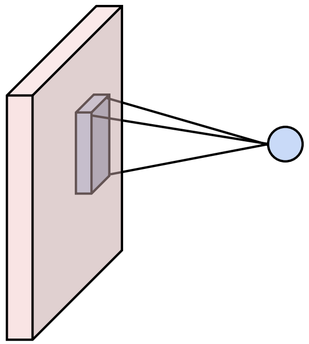
Fully-Connected Layers



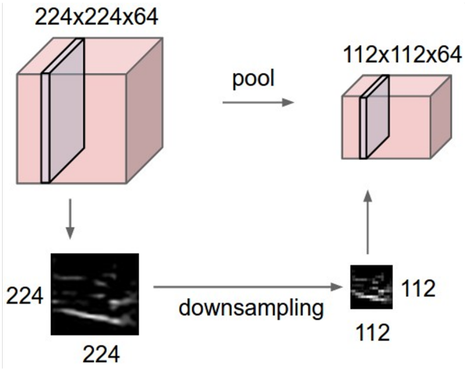
Activation Functions



Convolution Layers



Pooling Layers



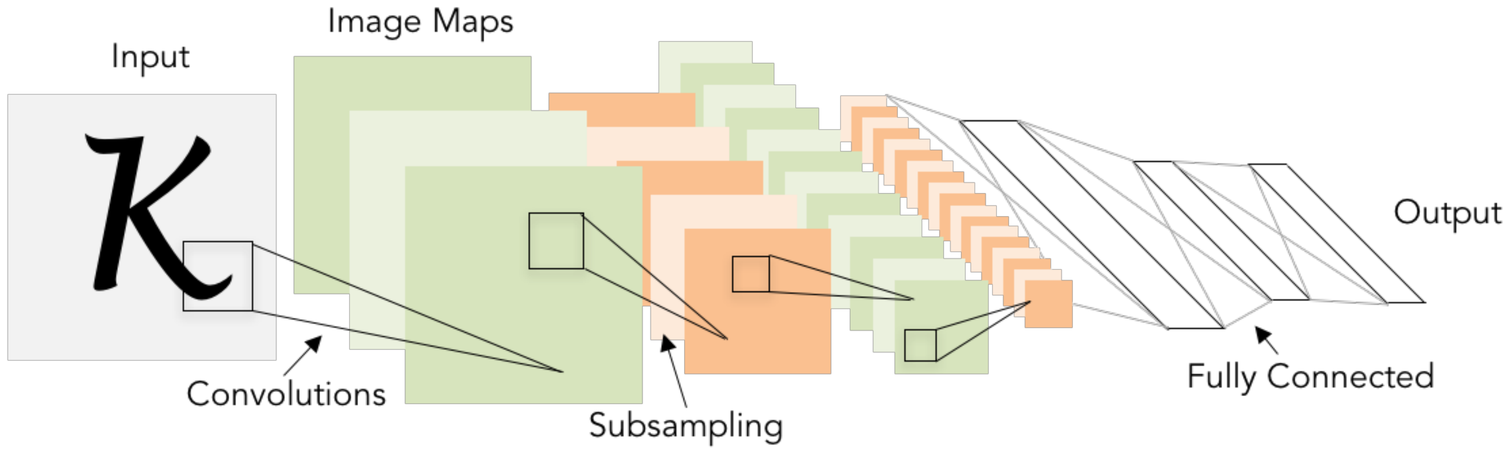
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Convolutional Networks

Classic architecture: [Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

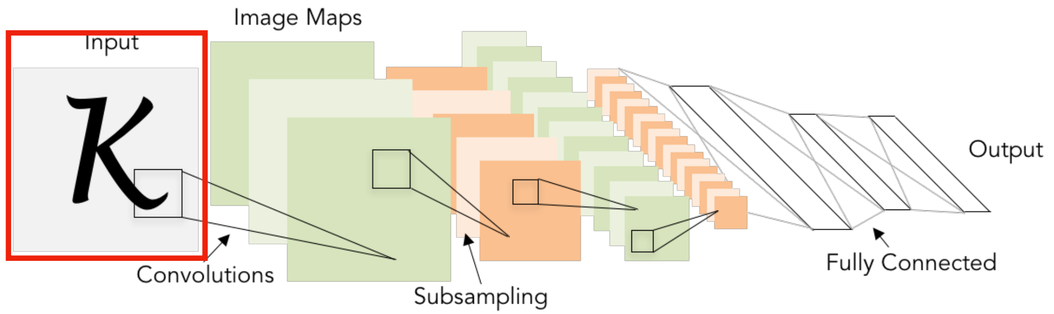
Example: LeNet-5



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

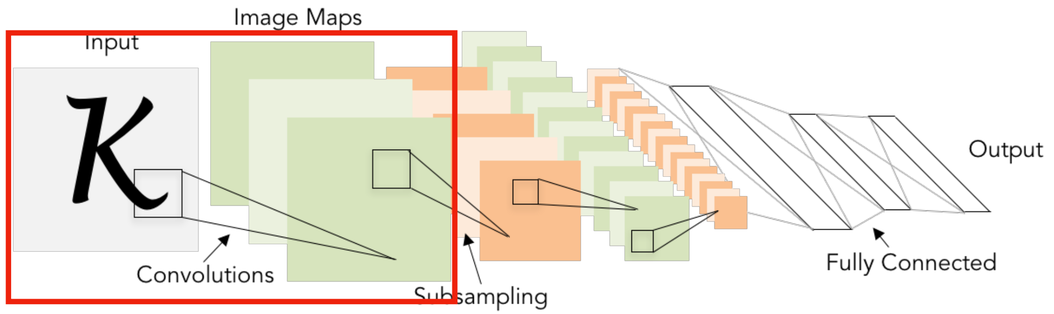
Layer	Output Size	Weight Size
Input	1 x 28 x 28	



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

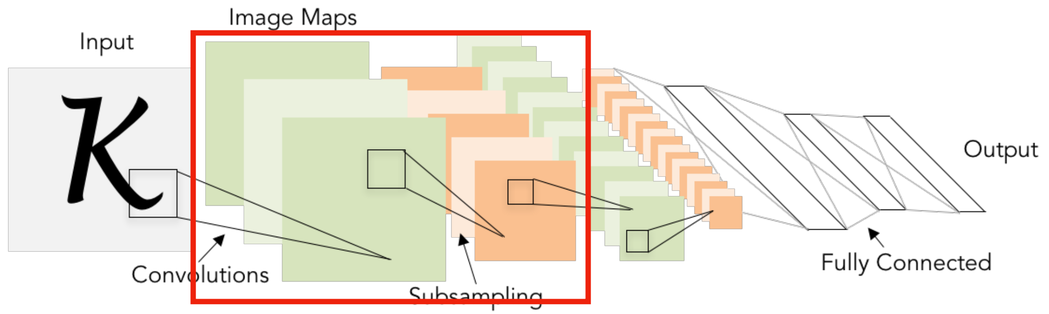
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

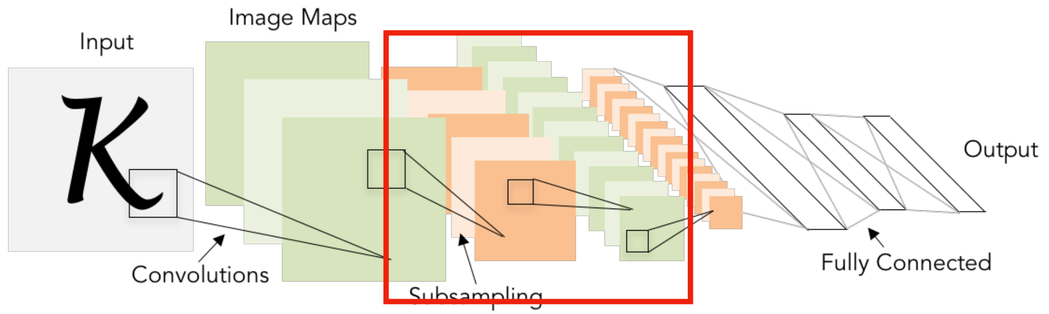
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

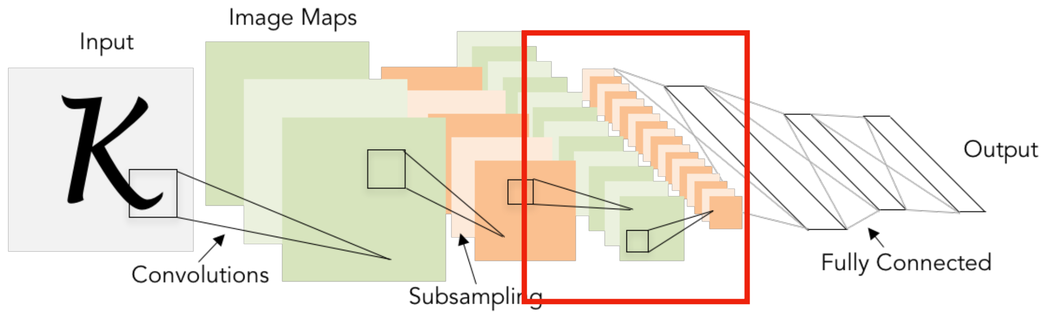
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

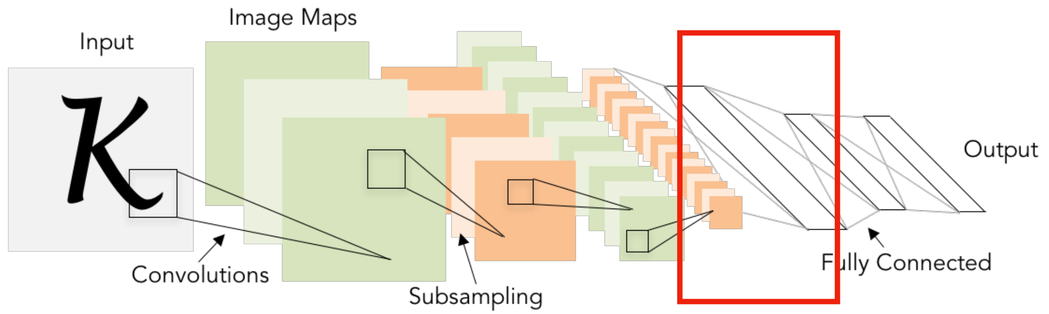
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
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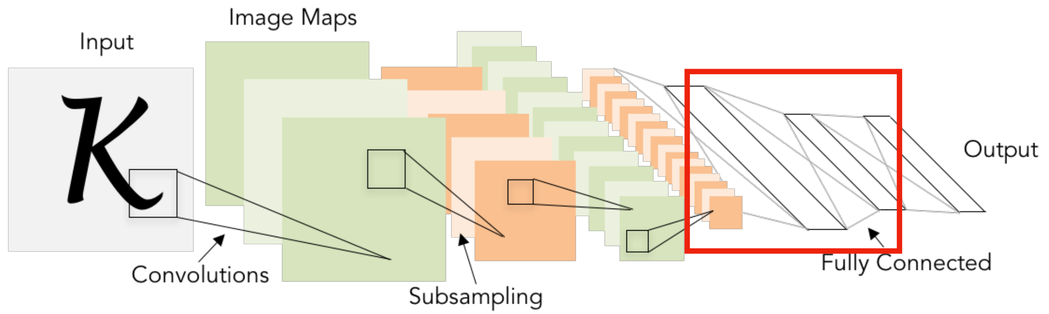
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	



Lecun et al., “Gradient-based learning applied to document recognition”, 1998

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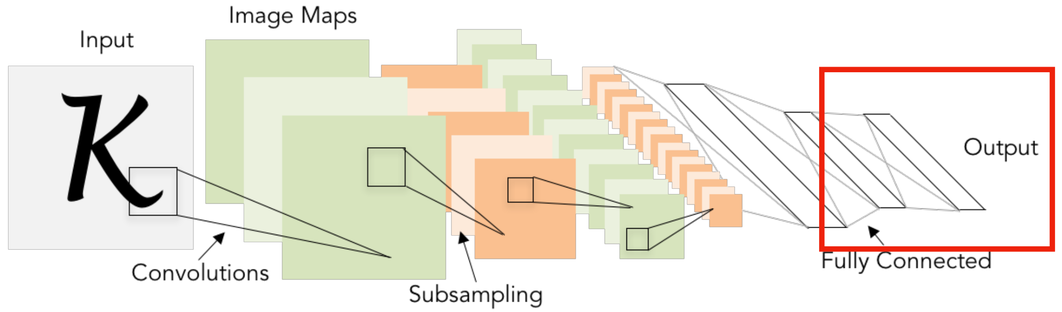
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10



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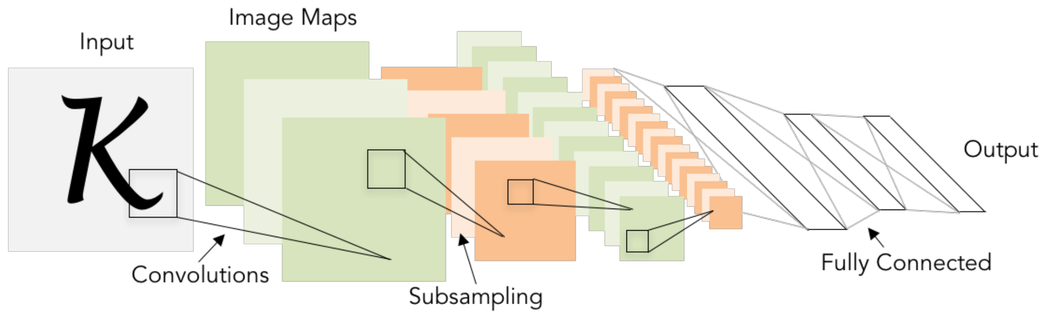

As we progress through the network:

Spatial size decreases
(using pooling or striped convolution)

Number of channels increases
(total “volume” is preserved!)

Lecun et al., “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

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Input	1 x 28 x 28	
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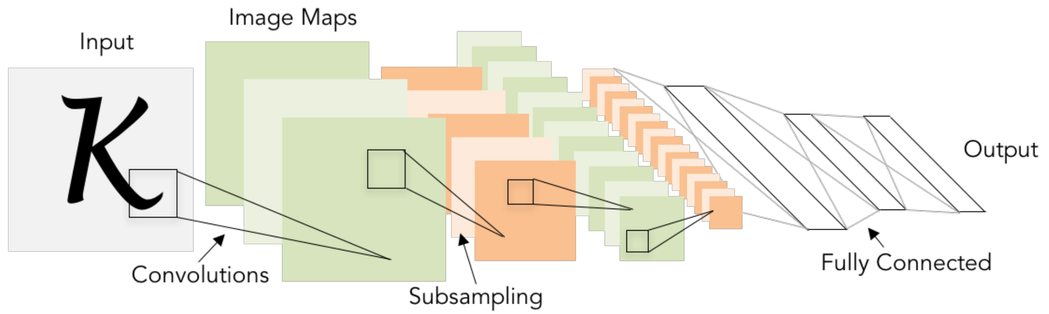


As we progress through the network:

Spatial size decreases
(using pooling or striped convolution)

Number of channels increases
(total “volume” is preserved!)

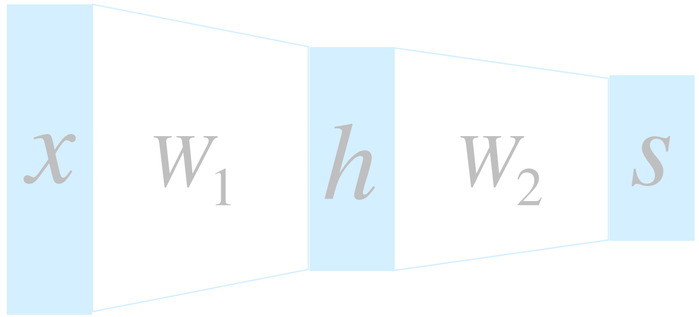
Some modern architectures
break this trend—stay tuned!

Lecun et al., “Gradient-based learning applied to document recognition”, 1998

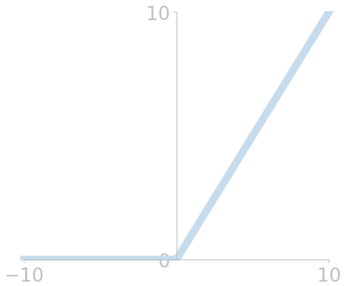
Problem: Deep Networks very hard to train

Components of Convolutional Neural Networks

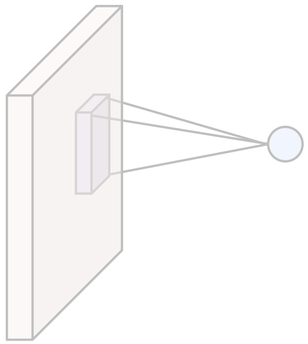
Fully-Connected Layers



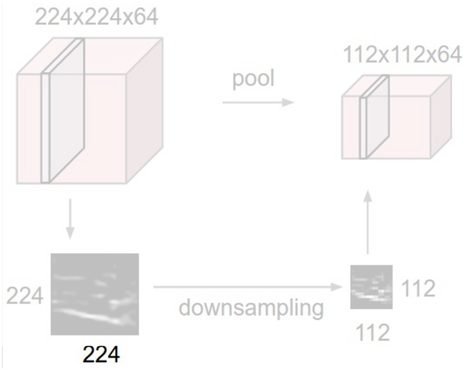
Activation Functions



Convolution Layers



Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Batch Normalization

Idea: “Normalize” the outputs of a layer so they have zero mean and unit variance

Why? Helps reduce “internal covariate shift”, improves optimization results

We can normalize a batch of activations using:

$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

Batch Normalization

Idea: “Normalize” the outputs of a layer so they have zero mean and unit variance

Why? Helps reduce “internal covariate shift”, improves optimization results

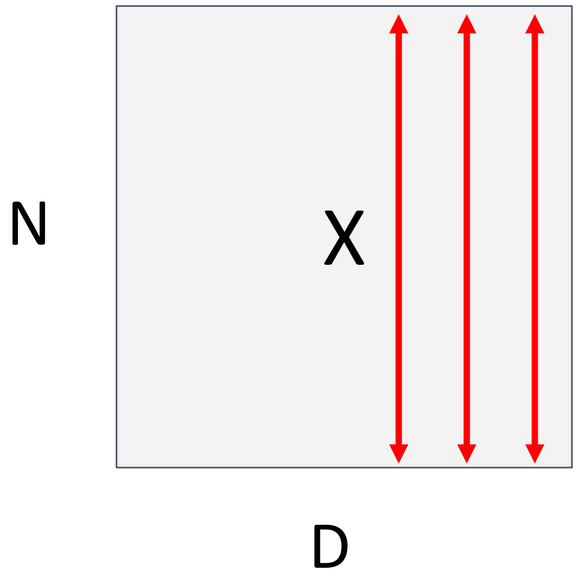
We can normalize a batch of activations using:

$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

This is a **differentiable function**, so we can use it as an operator in our networks and backdrop through it!

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$



$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N} \left(x_{i,j} - \mu_j\right)^2$$

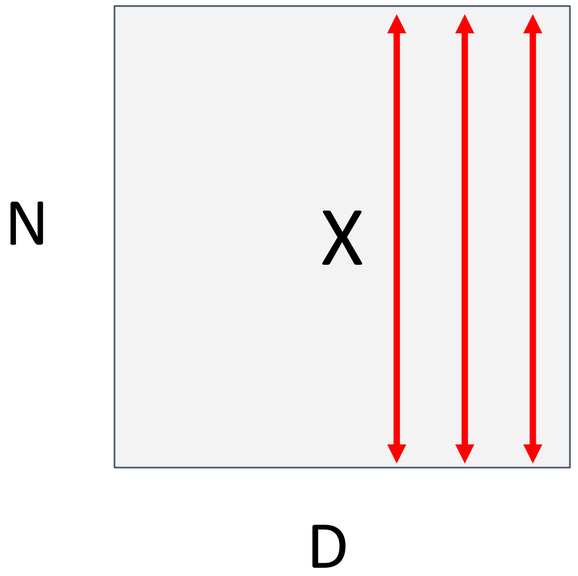
Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$



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Normalized x, Shape is N x D

Problem: What if zero-mean, unit variance is too hard of a constraint?

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^{D}$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N} \left(x_{i,j} - \mu_j\right)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^{D}$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

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Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Problem: Estimates depend on minibatch; can't run layer at test-time!

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$\gamma, \beta \in \mathbb{R}^{D}$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$\mu_j =$ (Running) average of values seen during training — Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training — Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

$\mu_j =$ (Running) average of values seen during training

Per-channel mean, shape is D

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^D$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$\mu_j^{test} = 0$

For each training iteration:

$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$

$\mu_j^{test} = 0.99\, \mu_j^{test} + 0.01\, \mu_j$

(Similar for σ)

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^{D}$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

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Per-channel mean, shape is D

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Per-channel std, shape is D

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Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$\gamma, \beta \in \mathbb{R}^{D}$

During testing batchnorm becomes a linear operator!
Can be fused with the previous fully-connected or conv layer

$\mu_j =$ (Running) average of values seen during training — Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training — Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization for ConvNets

Batch Normalization for
fully-connected networks

$$x : N \times D$$

Normalize ↓

$$\mu, \sigma : 1 \times D$$

$$\gamma, \beta : 1 \times D$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Batch Normalization for
convolutional networks
(Spatial Batchnorm, BatchNorm2D)

$$x : N \times C \times H \times W$$

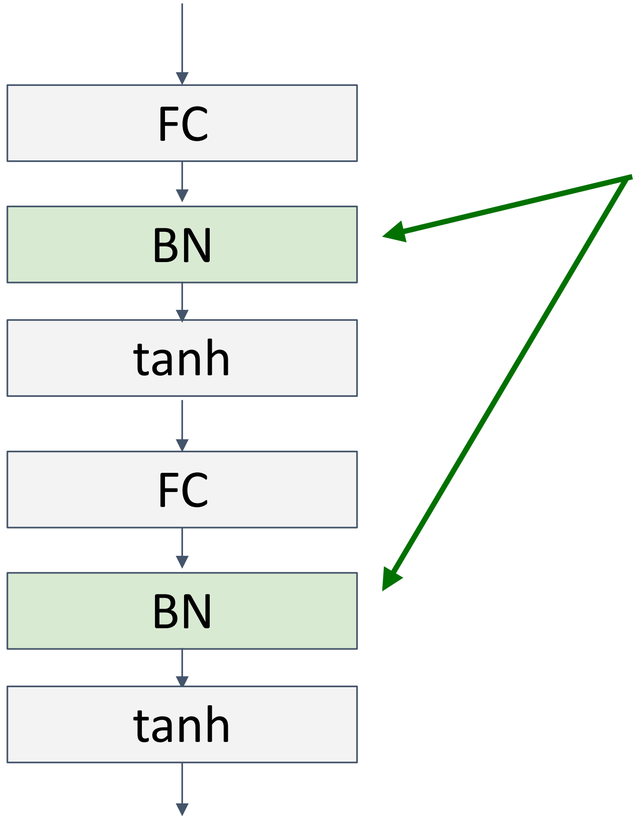
Normalize ↓ ↓ ↓

$$\mu, \sigma : 1 \times C \times 1 \times 1$$

$$\gamma, \beta : 1 \times C \times 1 \times 1$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

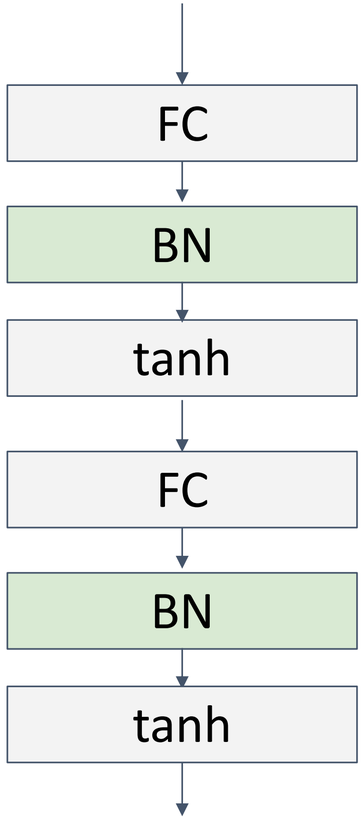
Batch Normalization



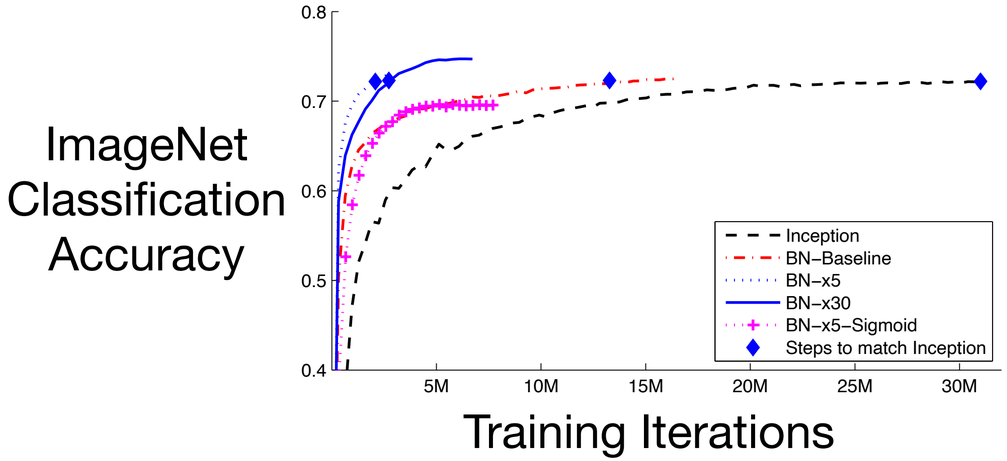
Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity

$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

Batch Normalization

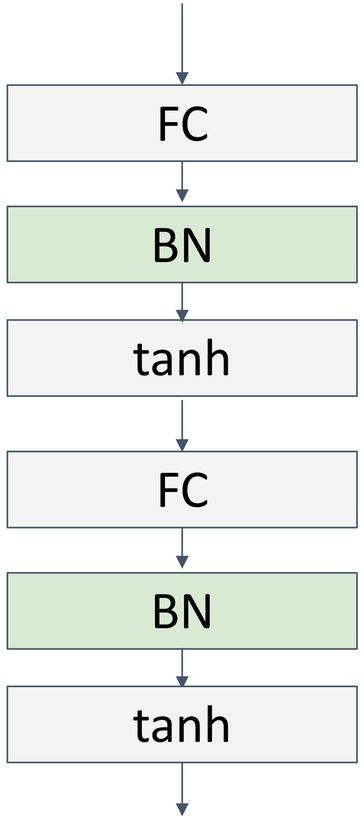


- Makes deep networks much easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv



Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Batch Normalization



- Makes deep networks much easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv

- Not well-understood theoretically (yet)
- Behaves differently during training and testing: very common source of bugs!

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Layer Normalization

Batch Normalization for **fully-connected** networks

$$x : N \times D$$

Normalize

$$\mu, \sigma : 1 \times D$$

$$\gamma, \beta : 1 \times D$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Layer Normalization for fully-connected networks
Same behavior at train and test!
Used in RNNs, Transformers

$$x : N \times D$$

Normalize

$$\mu, \sigma : N \times 1$$

$$\gamma, \beta : 1 \times D$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Instance Normalization

Batch Normalization for convolutional networks

$$x : N \times C \times H \times W$$

Normalize

$$\mu, \sigma : 1 \times C \times 1 \times 1$$

$$\gamma, \beta : 1 \times C \times 1 \times 1$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Instance Normalization for convolutional networks

$$x : N \times C \times H \times W$$

Normalize

$$\mu, \sigma : N \times C \times 1 \times 1$$

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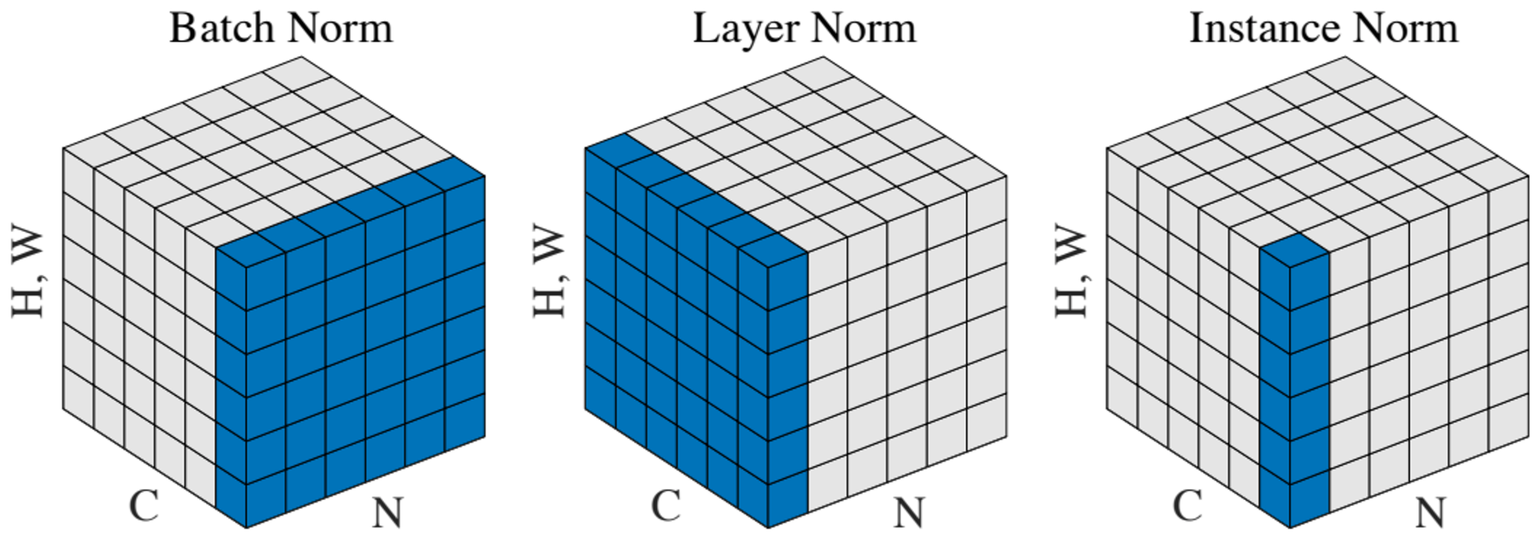
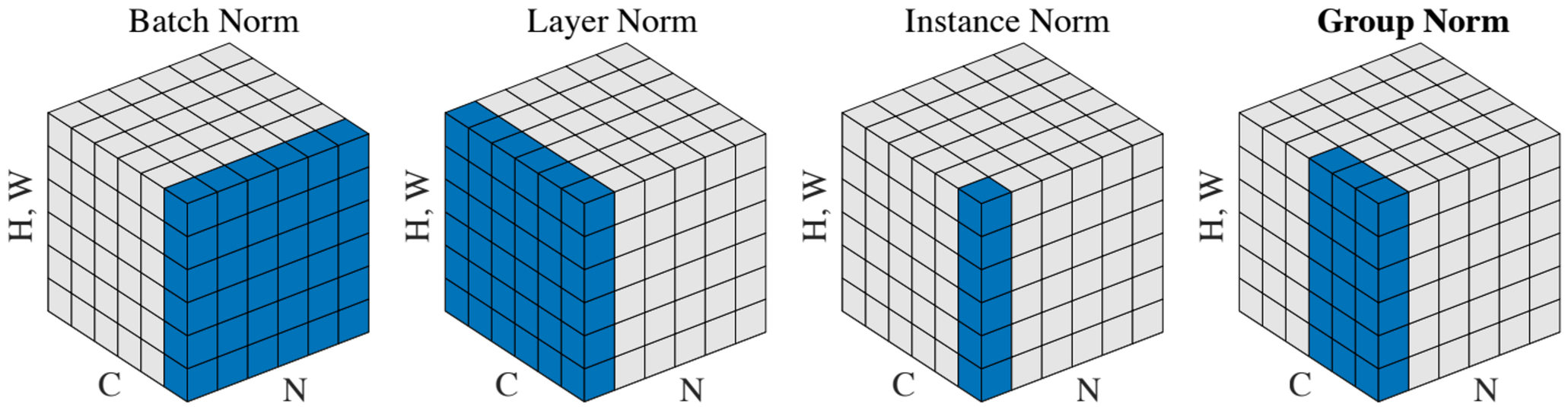
Comparison of Normalization Layers



Wu and He, “Group Normalization”, ECCV 2018

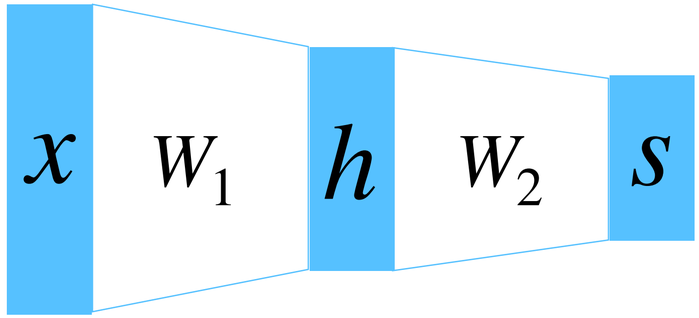
Group Normalization



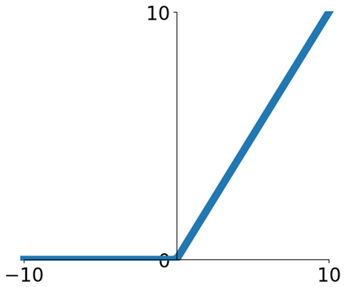
Wu and He, “Group Normalization”, ECCV 2018

Summary: Components of Convolutional Network

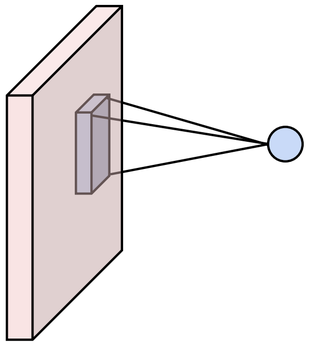
Fully-Connected Layers



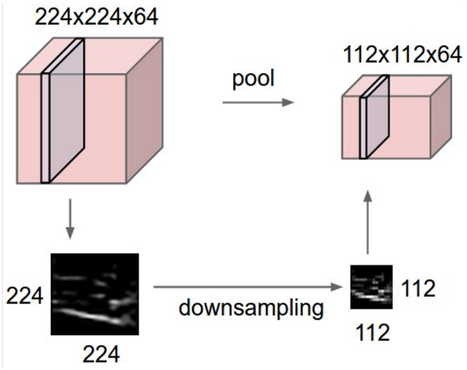
Activation Functions



Convolution Layers



Pooling Layers

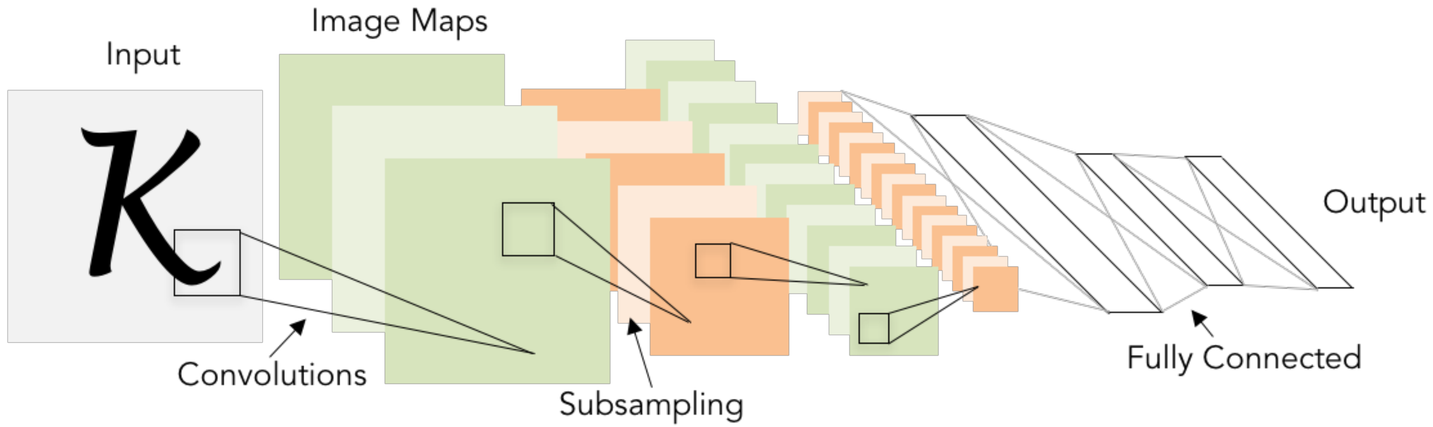


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Summary: Components of Convolutional Network

Problem: What is the right way to combine all these components?



Next time: CNN Architectures

Final Project Overview

- Research-oriented final project
  - Instead of a final exam!

- Objectives
  - Gain experience reading literature
  - Reproduce published results
  - Propose a new idea and test the results!

Final Project Overview

- Research-oriented final project
  - Instead of a final exam!

Can be completed in teams of 1-3 people

- Objectives
  - Gain experience reading literature
  - Reproduce published results
  - Propose a new idea and test the results!

Final Project Deliverables

1. A written paper review
2. In-class paper presentation
3. Reproduce published results
4. Extend results with new idea, technique or dataset
5. Document results in written report

(1) Paper Review and (2) Presentation

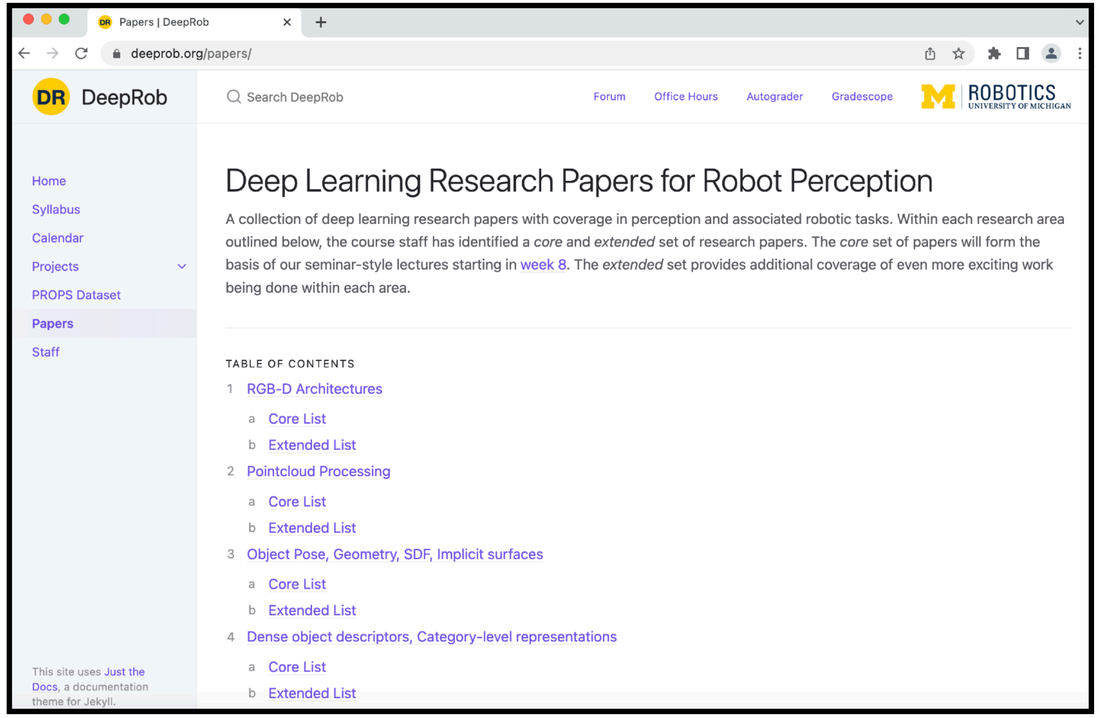
Final project teams will be based on overlapping interest

Students will choose from the ‘core’ list of papers on course website

Each team will be assigned one of the ‘core’ papers to review and present in-class

The 1-page paper review will be due **1-week before** the scheduled presentation

Presentation schedule will be based on paper topic as shown in course calendar



More details on review and presentation criteria in following lectures

(3) Paper Reproduction and (4) Extension

Each team will choose a paper relating
to deep learning and robot perception

Doesn't have to be same paper you presented in class

Then reimplement and reproduce at least one of the
paper's published results **(not necessarily all the results)**

Then, each team will test one of their own ideas!

By extending the paper's model using new architecture or technique or dataset

Your chance to experiment with deep learning and contribute to the field!

More details on reproduction and extension
in following lectures

(5) Project Report

- The final deliverable for your final project
- A 1-2 page paper
  - What problem within robot perception or manipulation?
  - What work has been done in this area?
  - What approach did you investigate?
  - What questions and directions exist for future work?

More details on report in following lectures

Final Project Grading Overview

- Final Project:
  - Paper Review: 3%
  - Paper Presentation: 3%
  - Paper Reproduction: 6%
  - Algorithmic Extension: 6%
  - Written Report: 6%

More details on report in following lectures

DeepRob

Lecture 7
Convolutional Neural Networks
University of Michigan and University of Minnesota