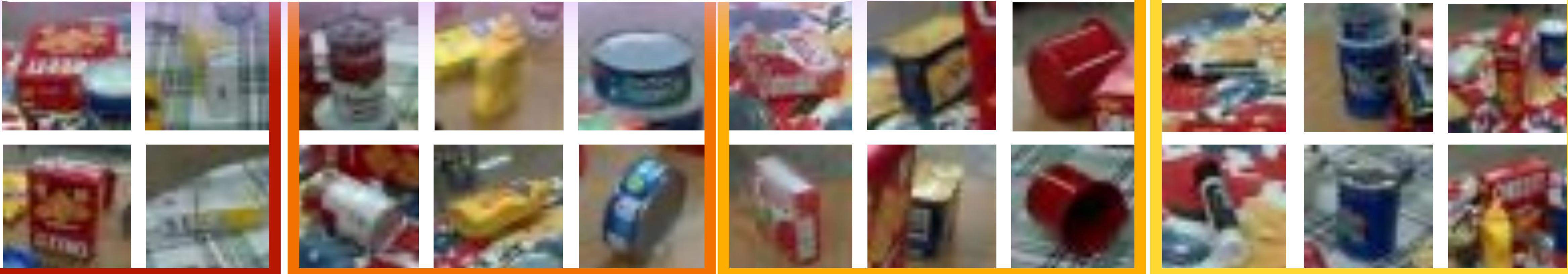
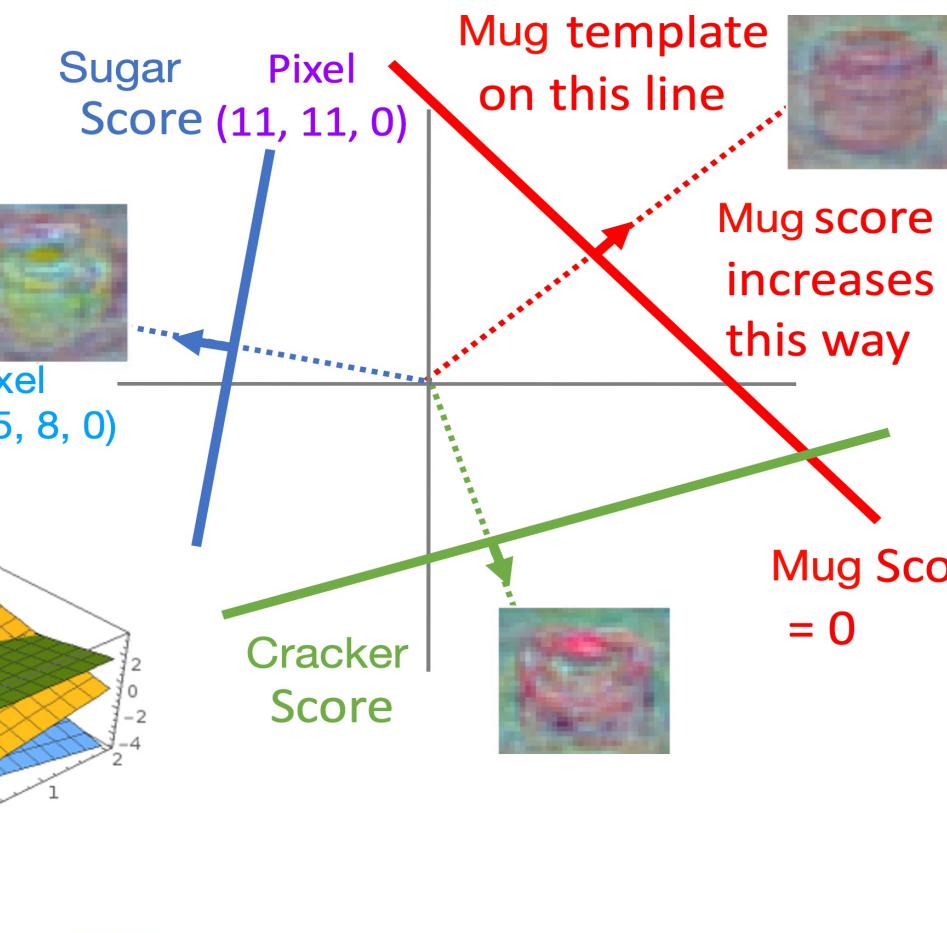




DEEP Rob

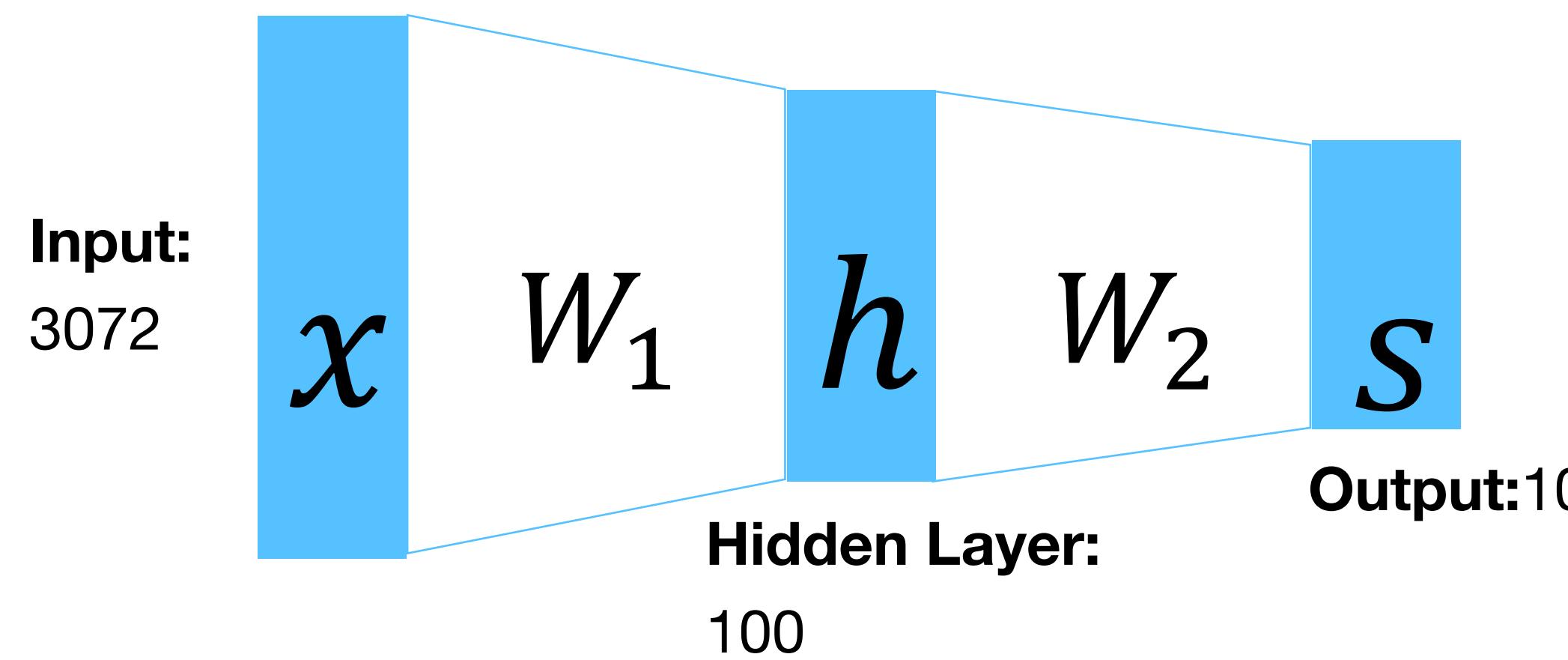
Lecture 6
Convolutional Neural Networks
University of Michigan | Department of Robotics





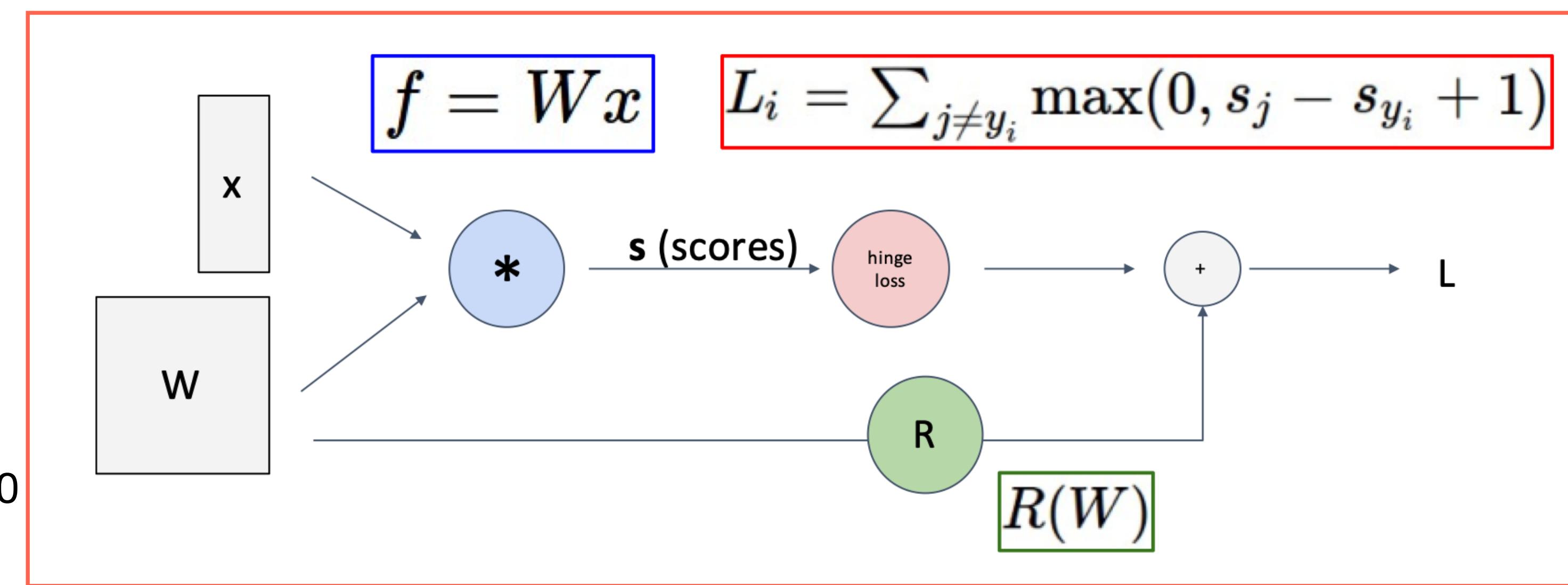
Recap: Backpropagation

$$f(x) = W_2 \max(0, W_1 x + b_1) + b_2$$



Forward pass

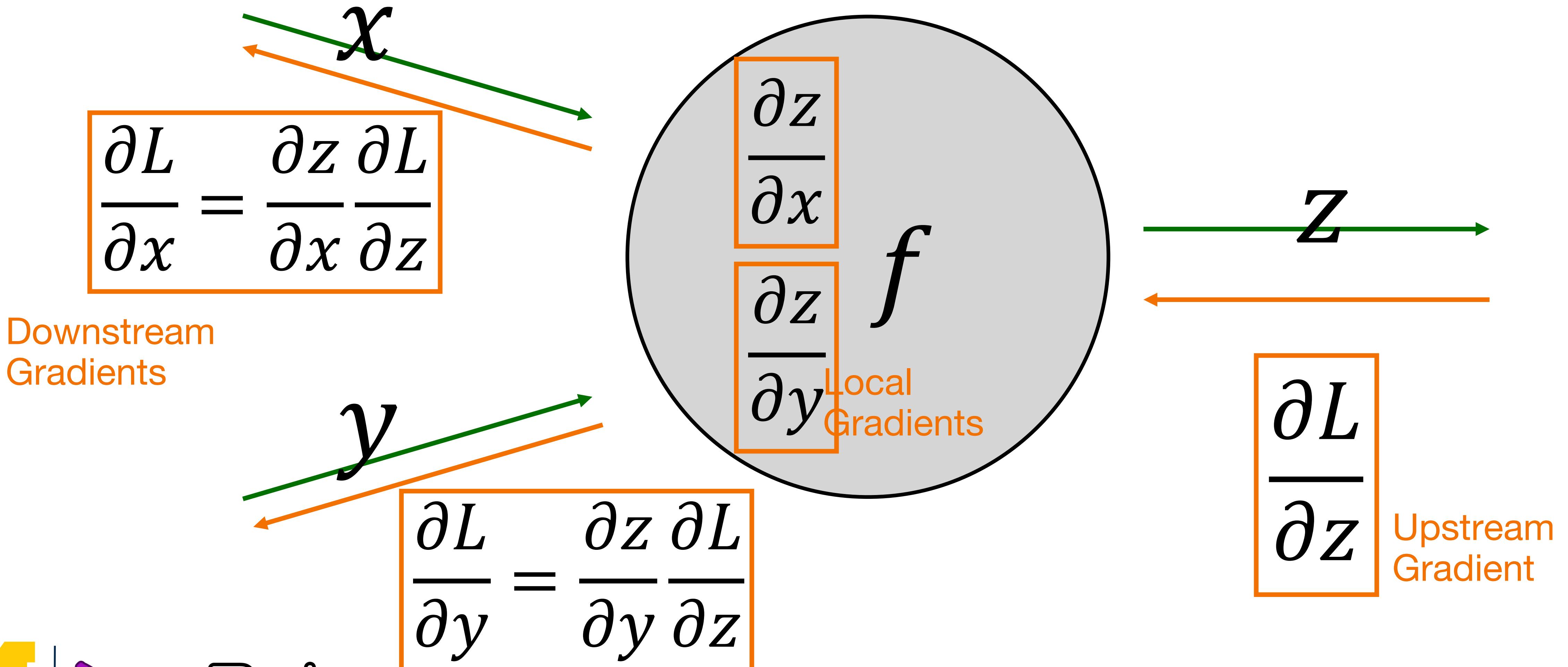
Backward pass
(Backprop)



Computational Graph

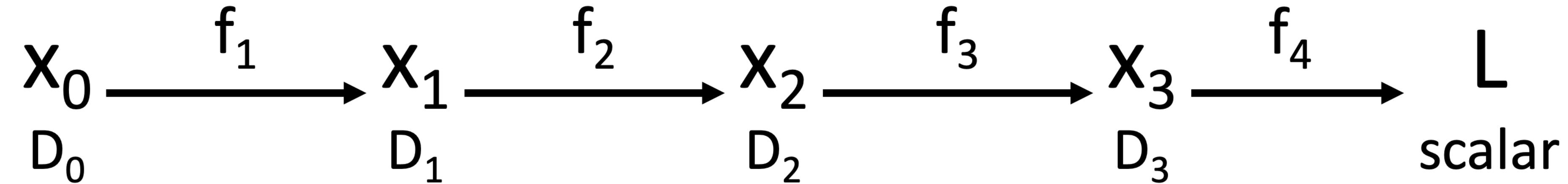


Recap: Backpropagation





Recap: “The Chain Rule”



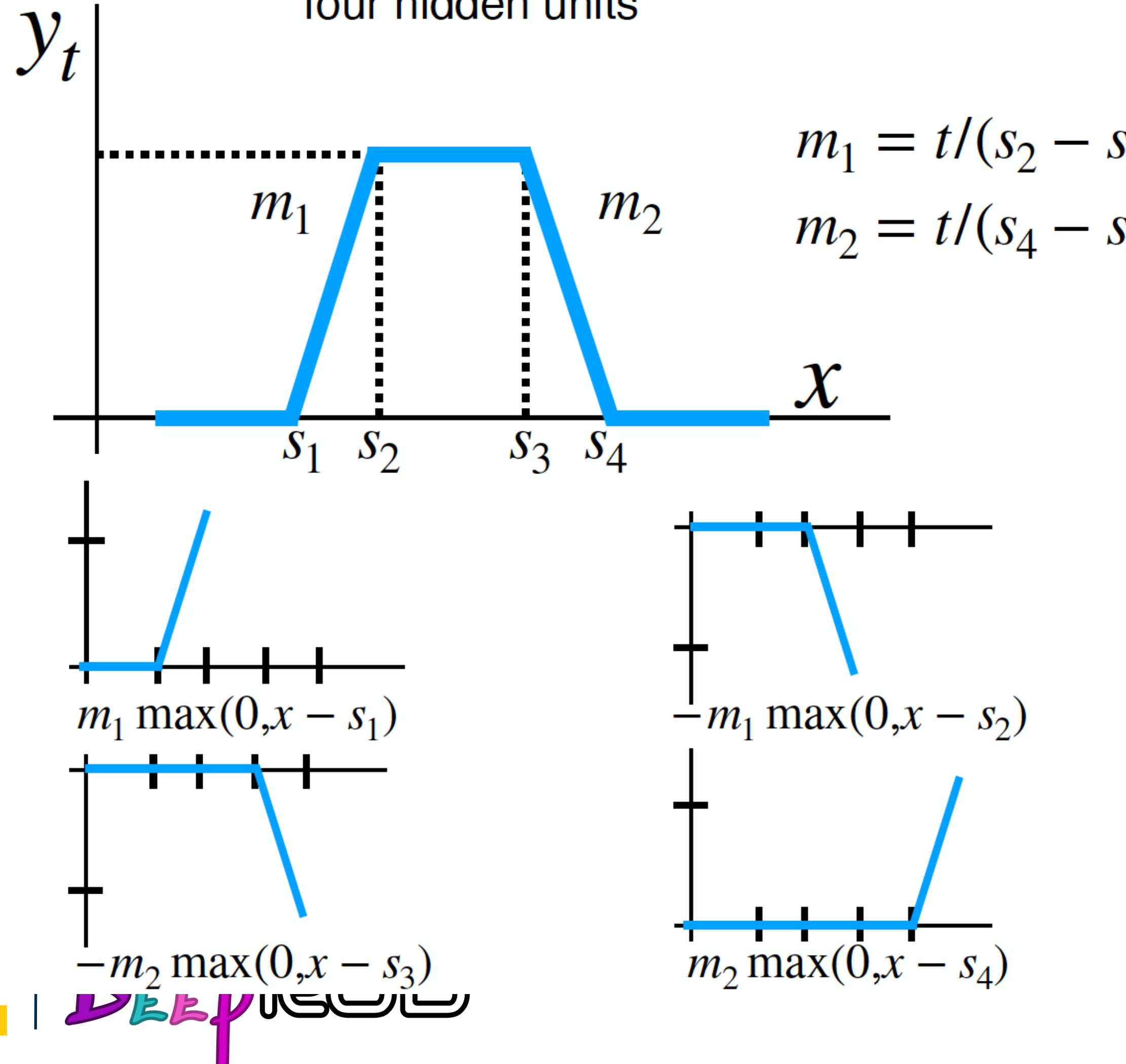
Chain rule

$$\frac{\partial L}{\partial x_0} = \left(\frac{\partial x_1}{\partial x_0} \right) \left(\frac{\partial x_2}{\partial x_1} \right) \left(\frac{\partial x_3}{\partial x_2} \right) \left(\frac{\partial L}{\partial x_3} \right)$$

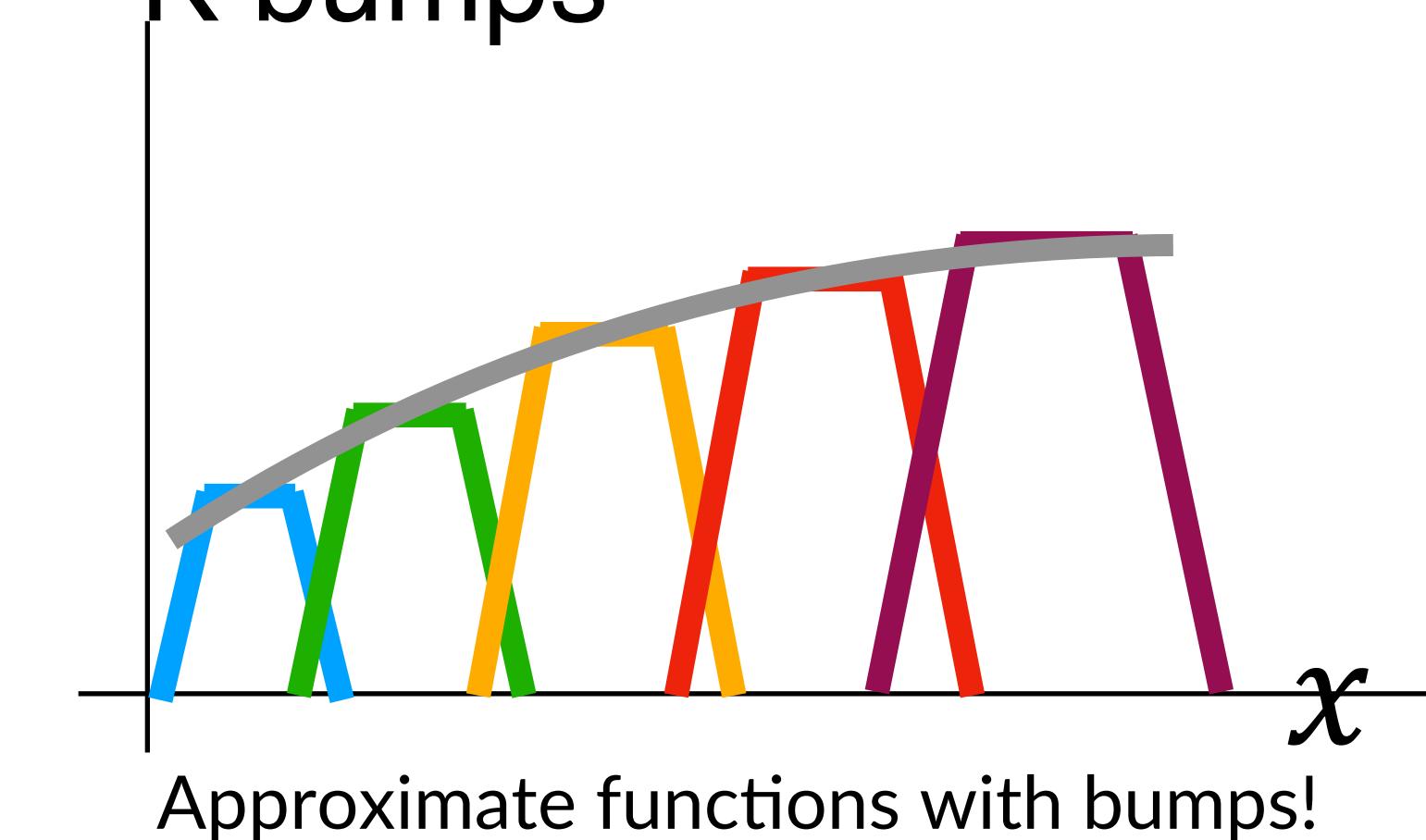


Recap: Universal Approximation

We can build a “bump function” using four hidden units

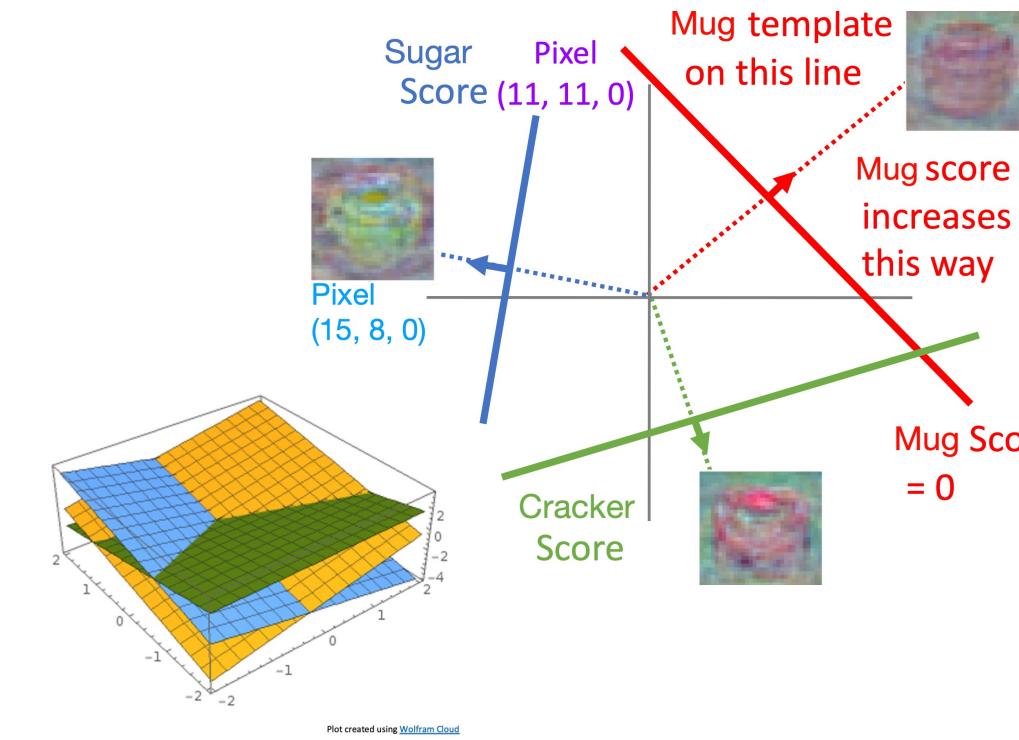


With $4K$ hidden units we can build a sum of K bumps





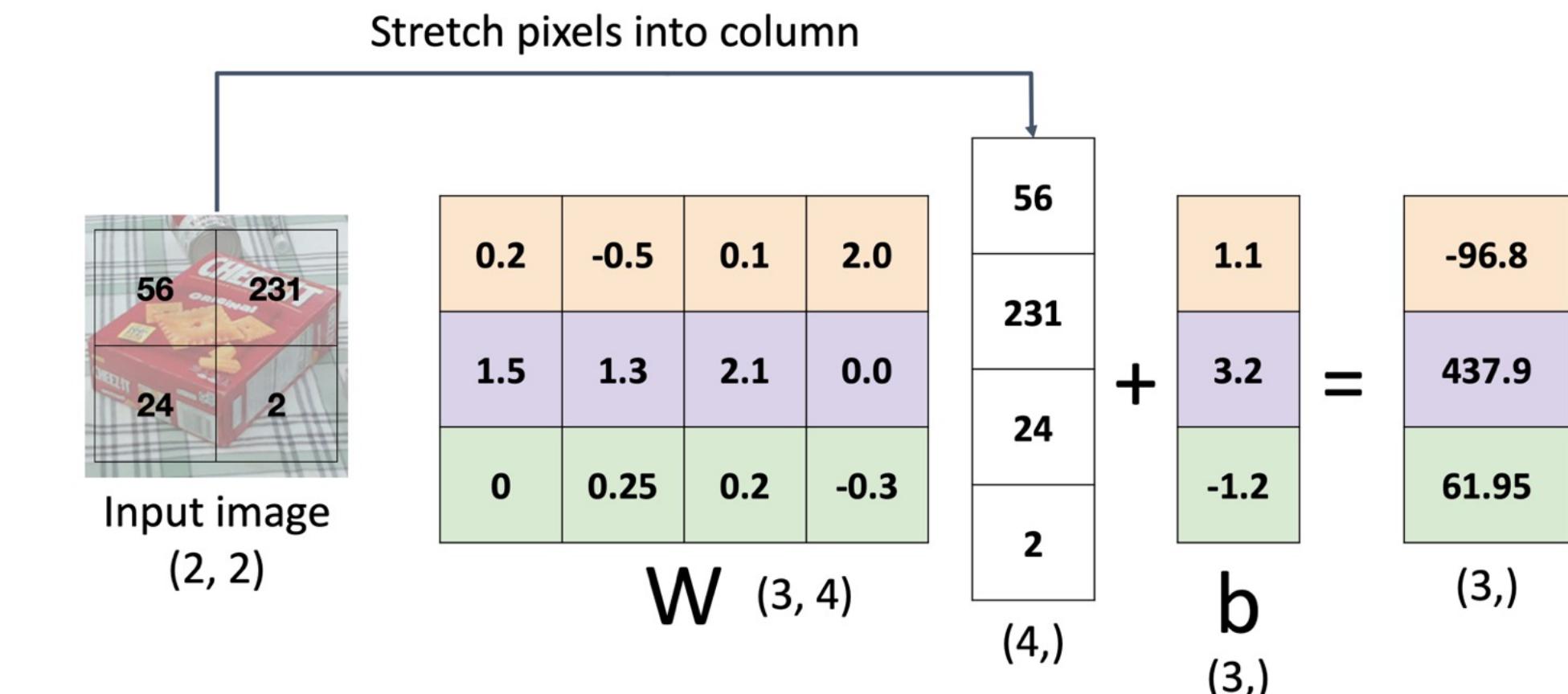
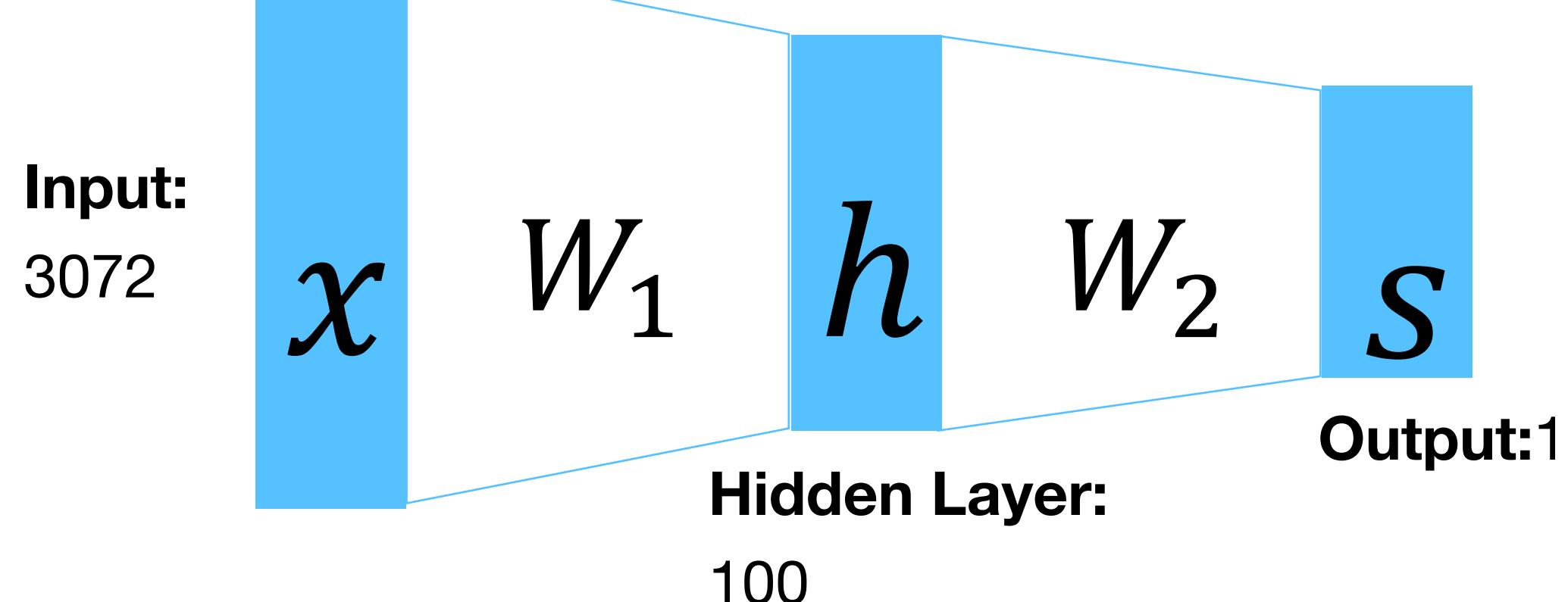
Spatial Structure?



Problem: So far our classifiers don't respect the spatial structure of images!

Solution: Define new computational nodes that operate on images!

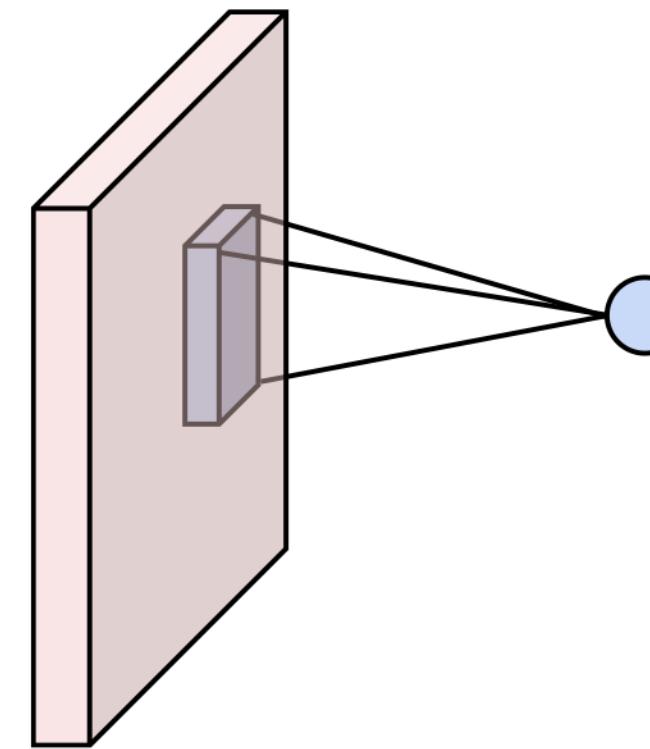
$$f(x) = W_2 \max(0, W_1 x + b_1) + b_2$$



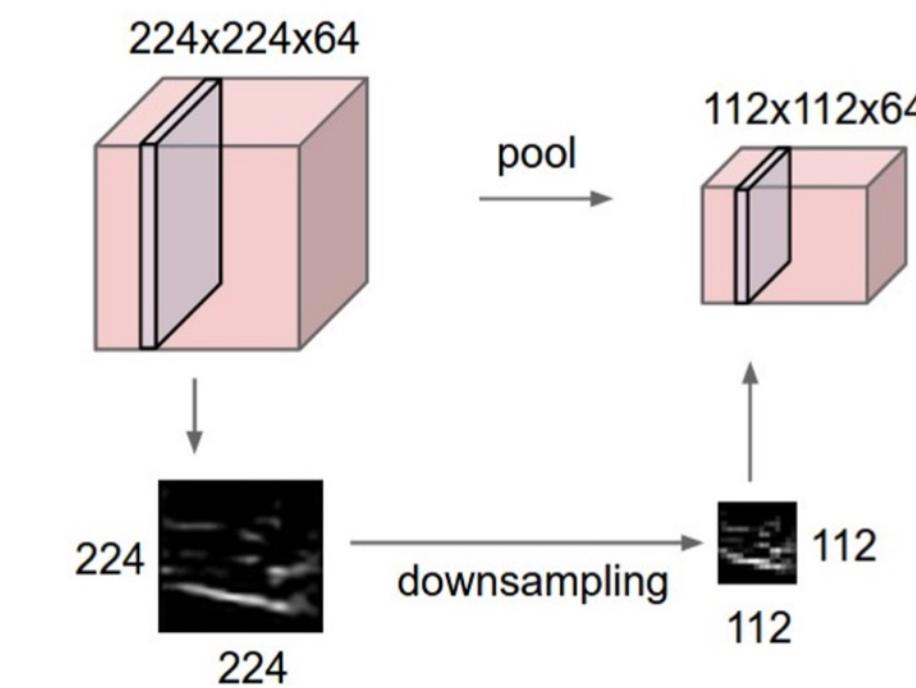


Components of Convolutional Networks

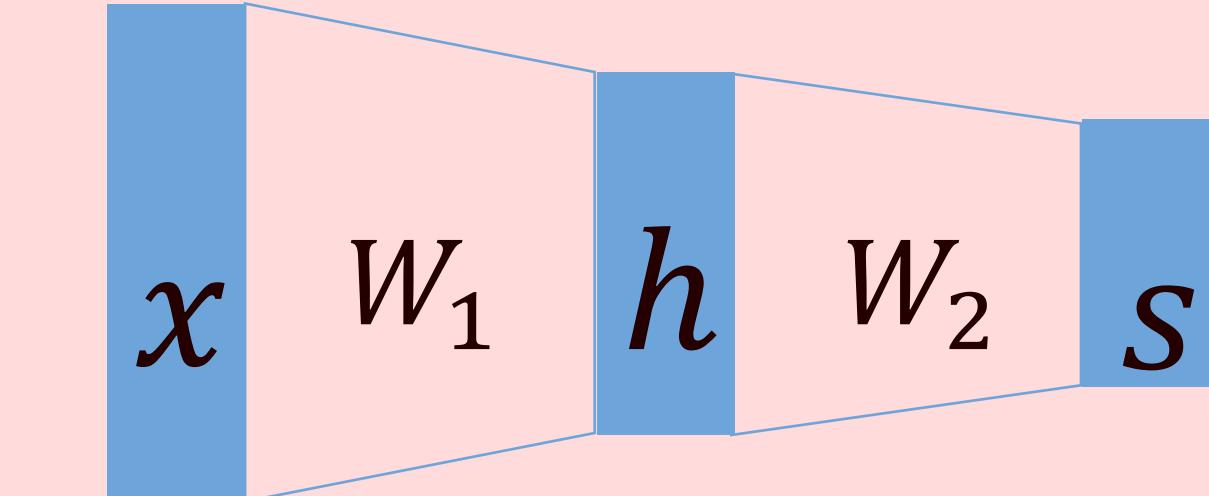
Convolution Layers



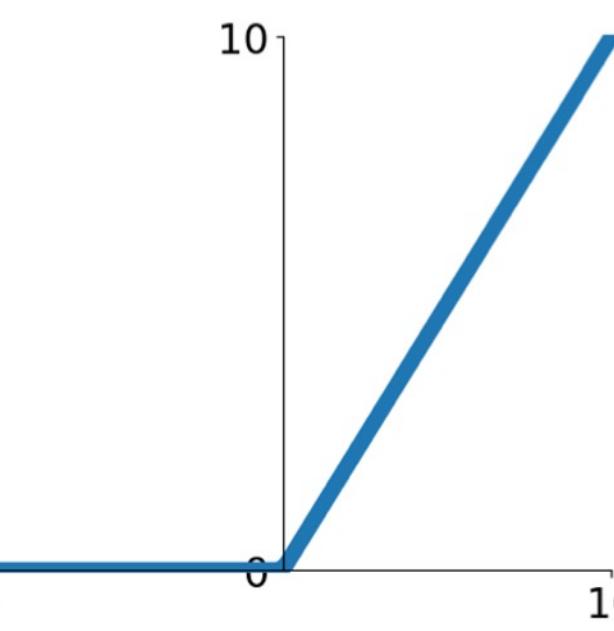
Pooling Layers



Fully-Connected Layers



Activation Function

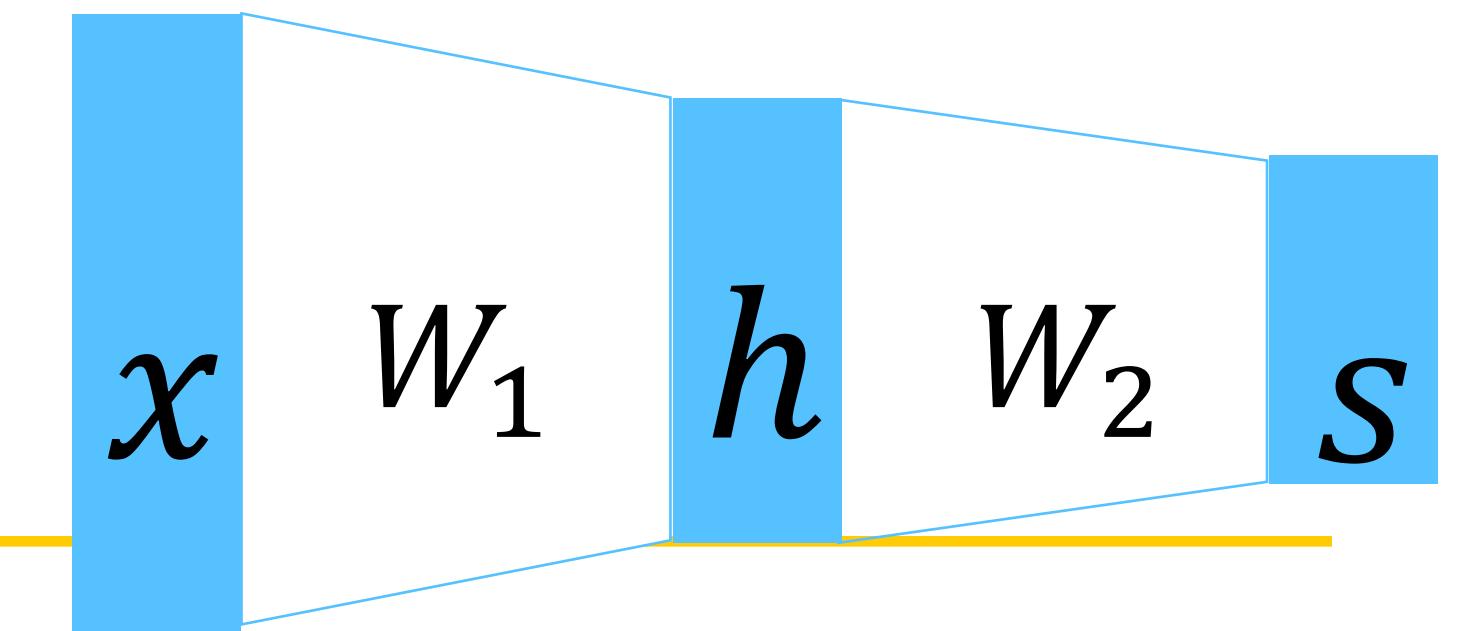


Normalization

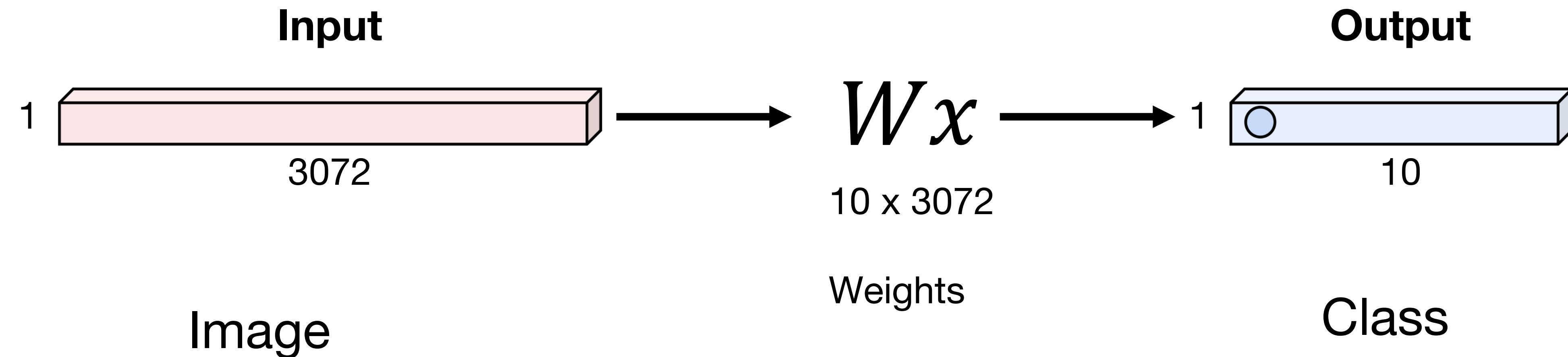
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



Fully-Connected Layer



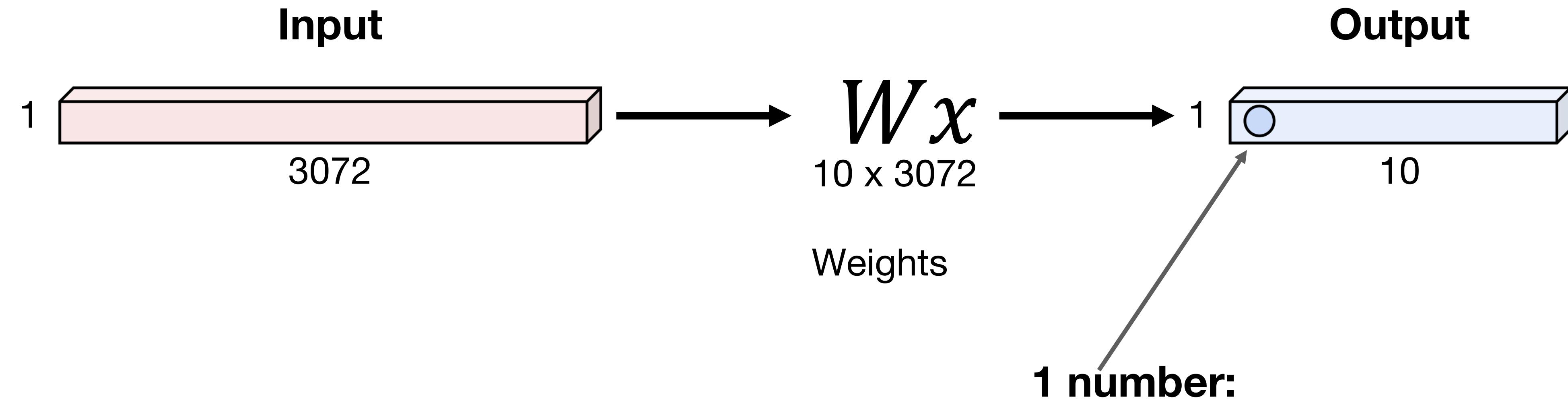
3x32x32 image \longrightarrow stretch to 3072x1





Fully-Connected Layer

3x32x32 image → stretch to 3072x1



The result of taking a dot product between a row of W and the input



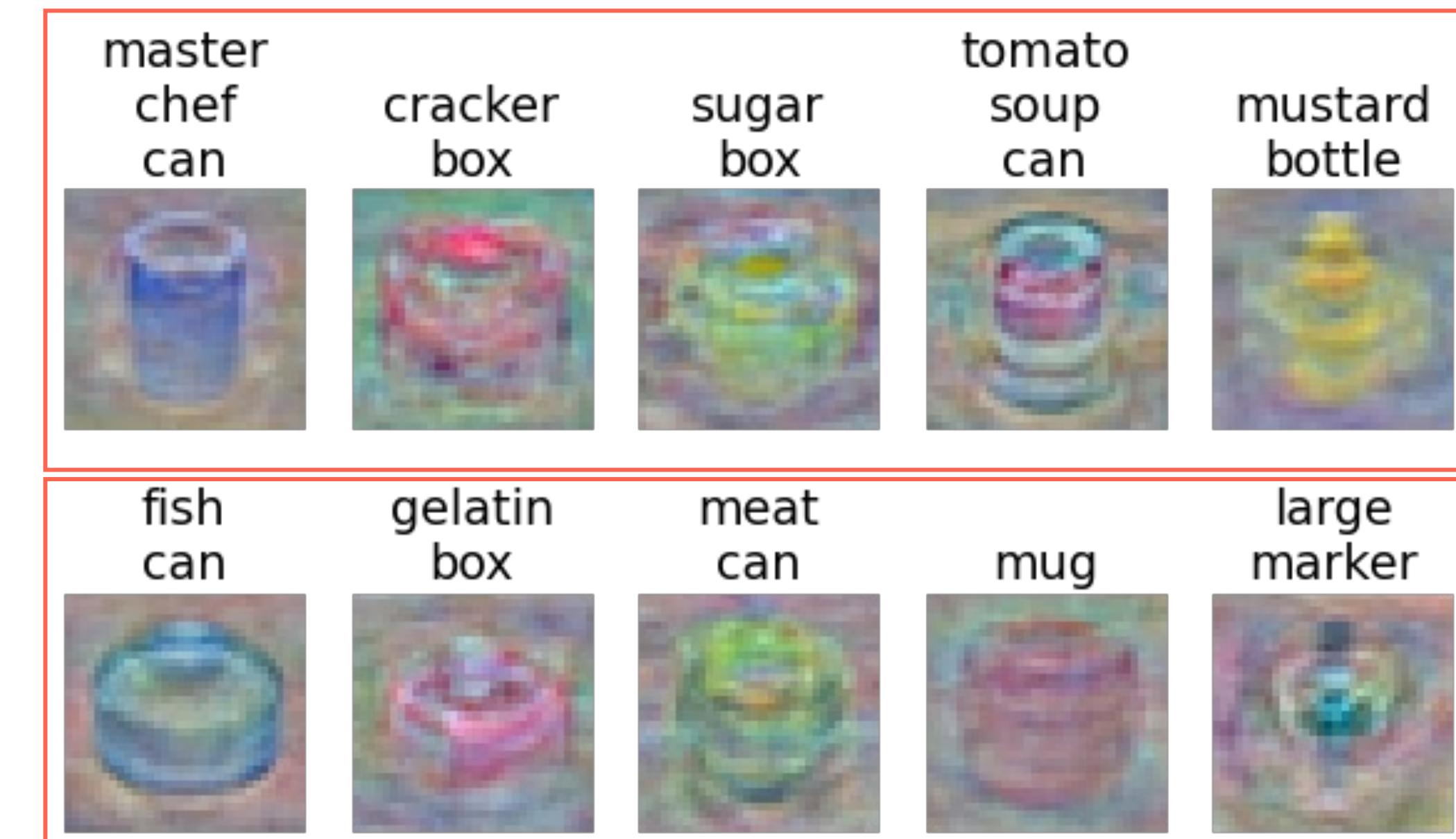
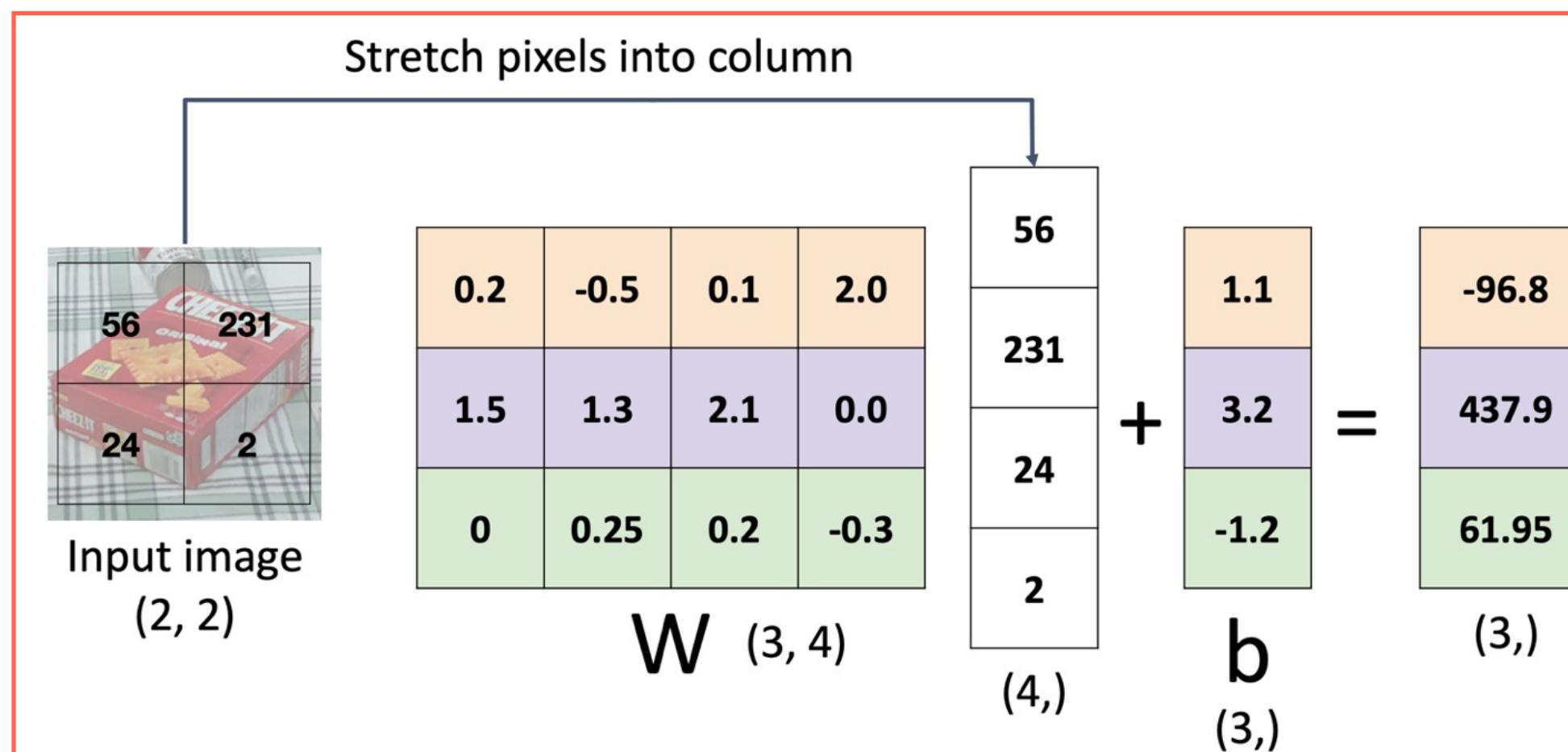
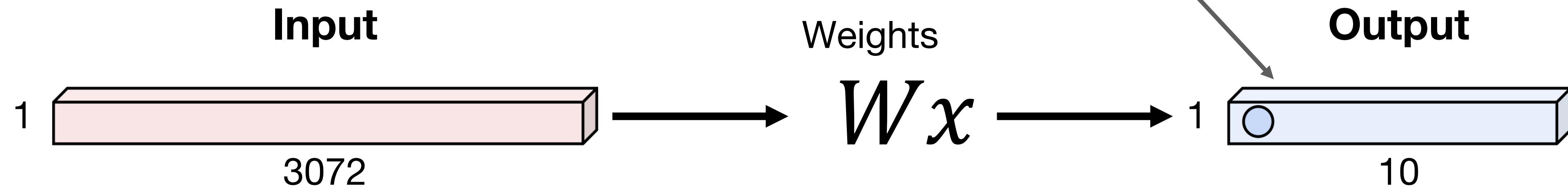
Fully-Connected Layer

3x32x32 image → stretch to 3072x1

10 x 3072

1 number:

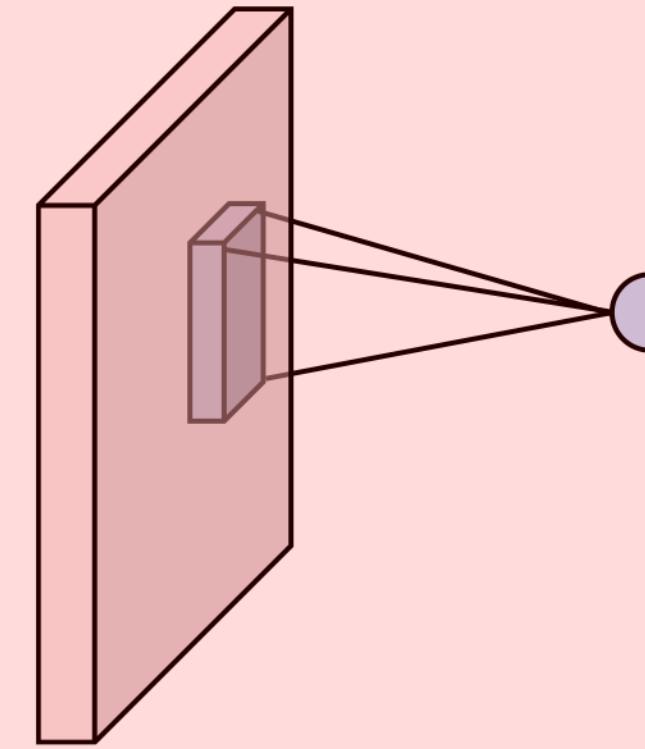
The result of taking a dot product between a row of W and the input



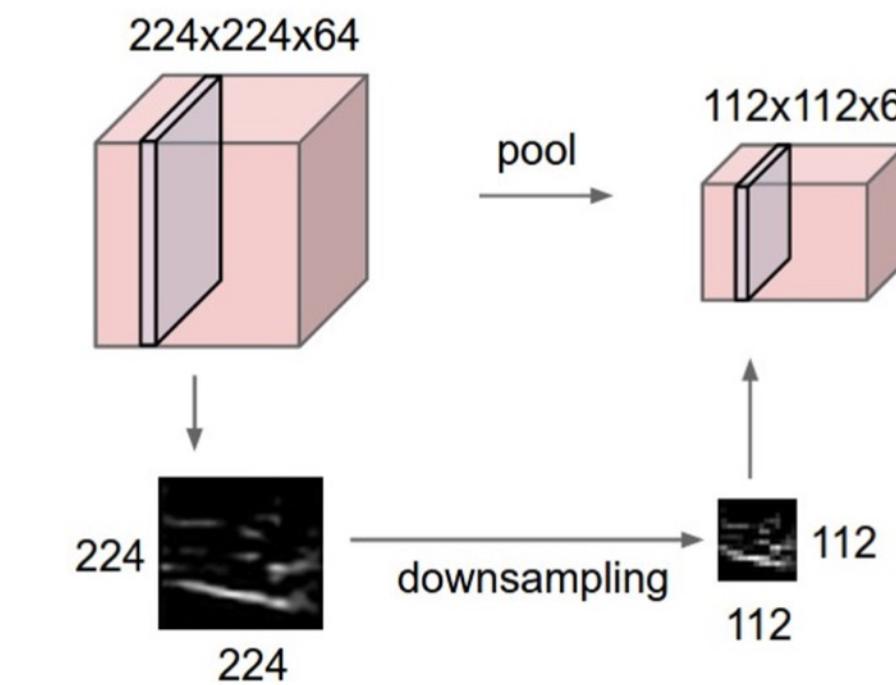


Components of Convolutional Networks

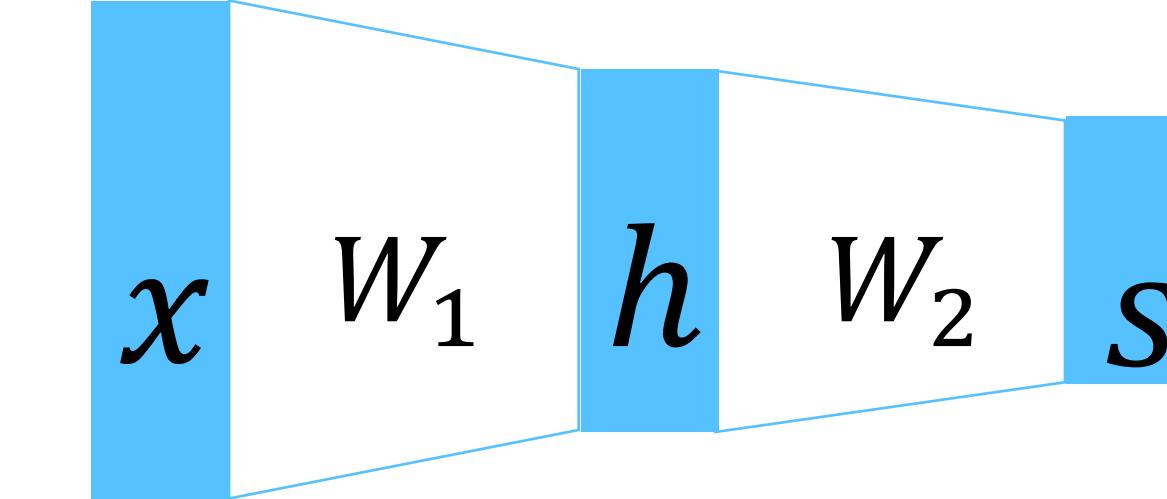
Convolution Layers



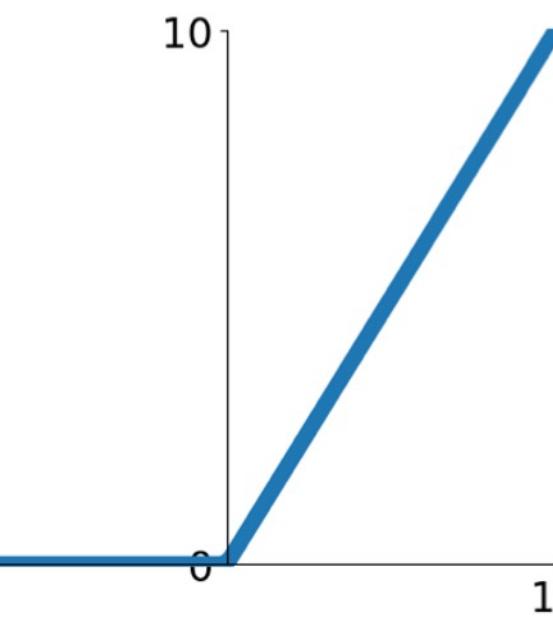
Pooling Layers



Fully-Connected Layers



Activation Function



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



Convolution Operation

| | | | | |
|----------------|----------------|----------------|---|---|
| 3 ₀ | 3 ₁ | 2 ₂ | 1 | 0 |
| 0 ₂ | 0 ₂ | 1 ₀ | 3 | 1 |
| 3 ₀ | 1 ₁ | 2 ₂ | 2 | 3 |
| 2 | 0 | 0 | 2 | 2 |
| 2 | 0 | 0 | 0 | 1 |

| | | |
|------|------|------|
| 12.0 | 12.0 | 17.0 |
| 10.0 | 17.0 | 19.0 |
| 9.0 | 6.0 | 14.0 |

Kernel:

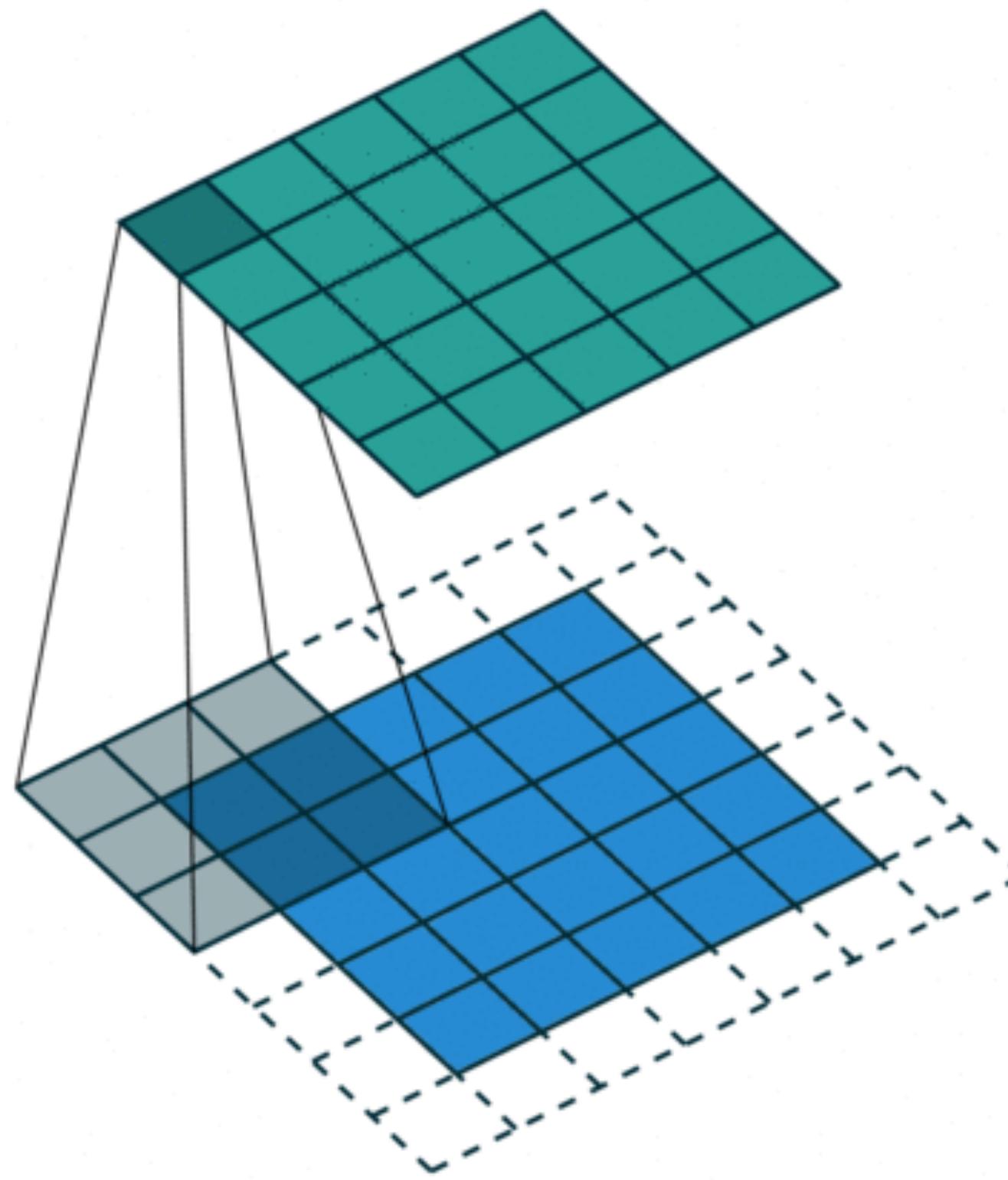
| | | |
|---|---|---|
| 0 | 1 | 2 |
| 2 | 2 | 0 |
| 0 | 1 | 2 |

(3x3)

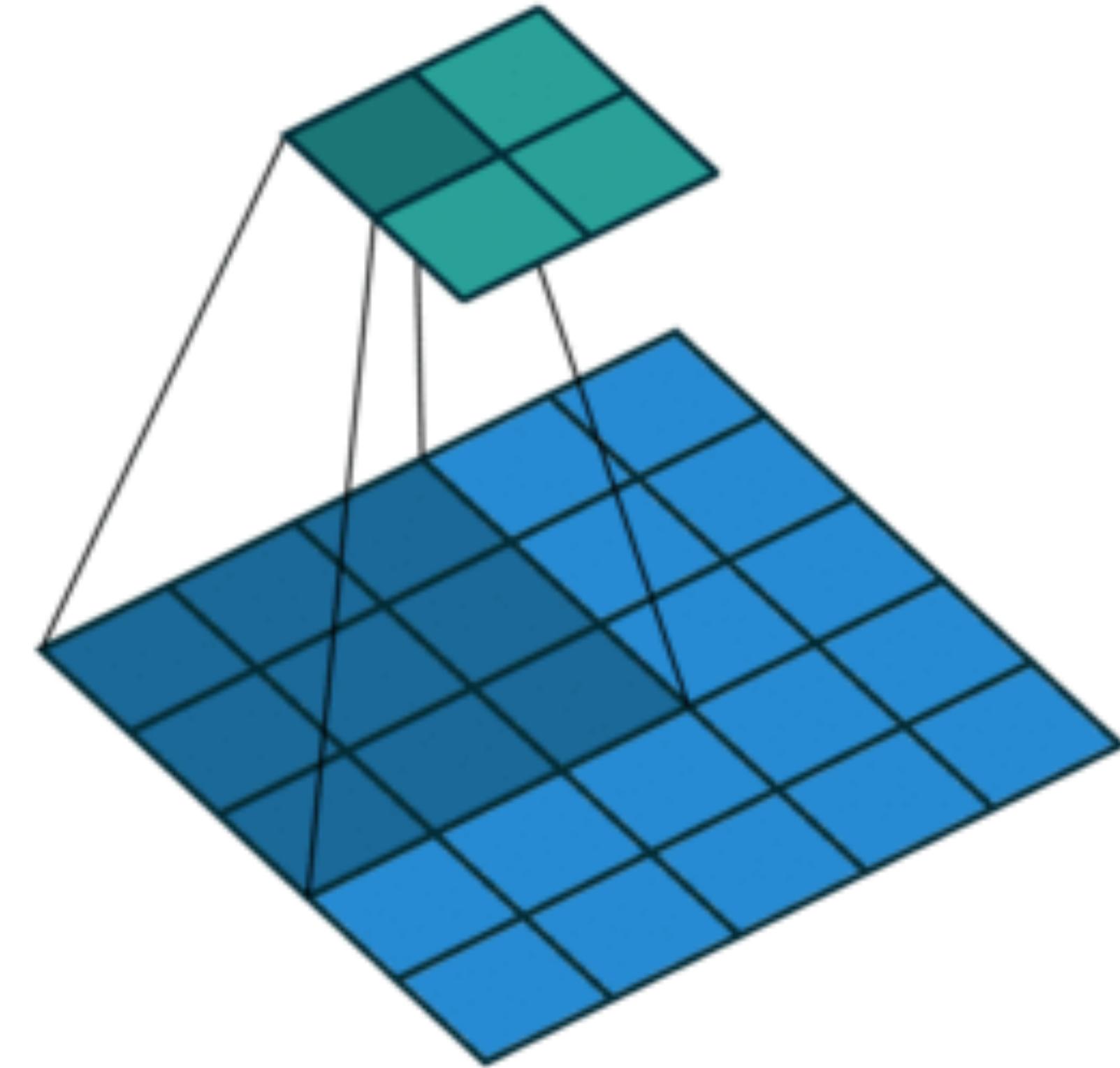
<https://towardsdatascience.com/intuitively-understanding-convolutions-for-deep-learning-1f6f42faee1>



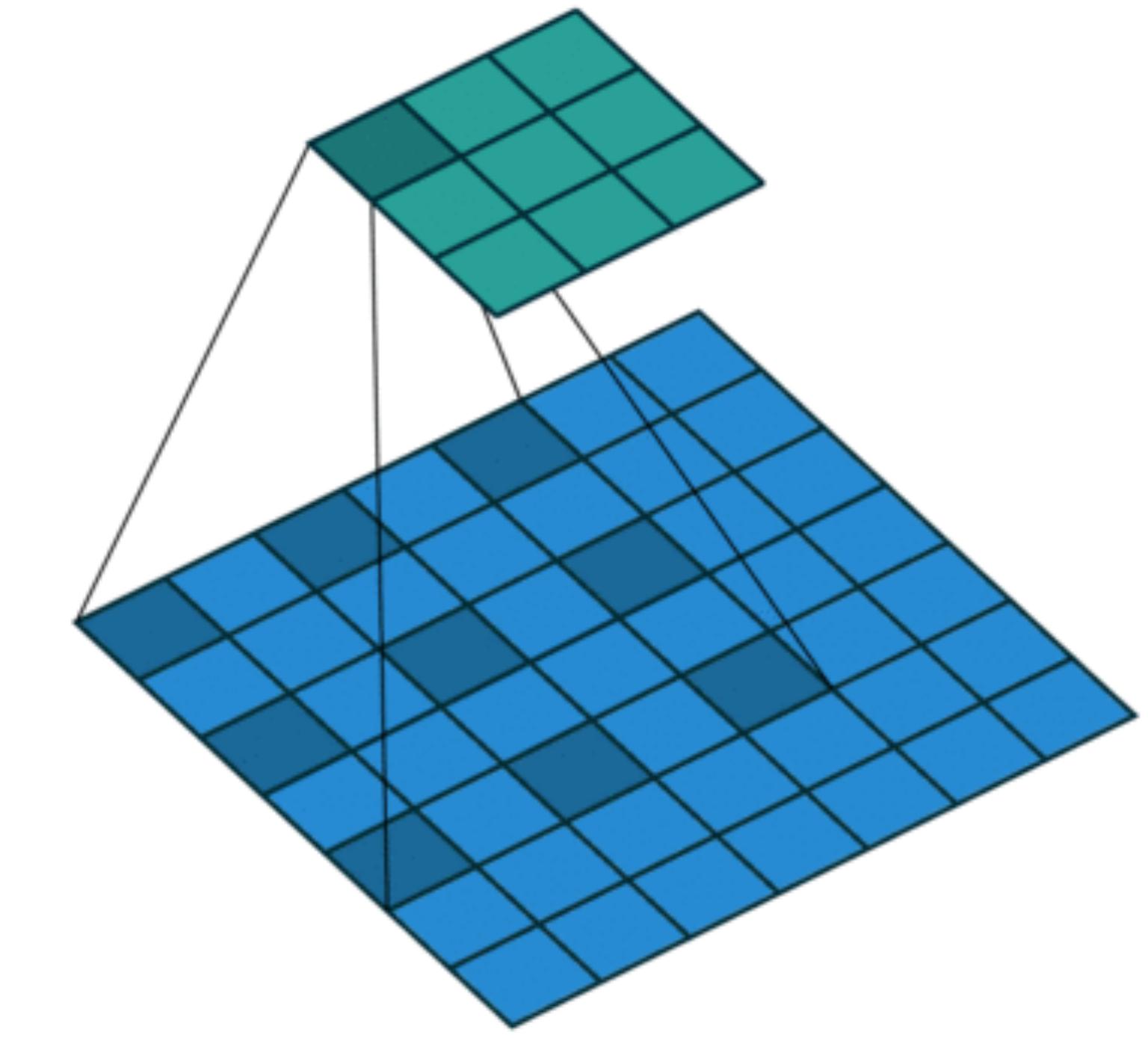
Convolution Operation



Padding



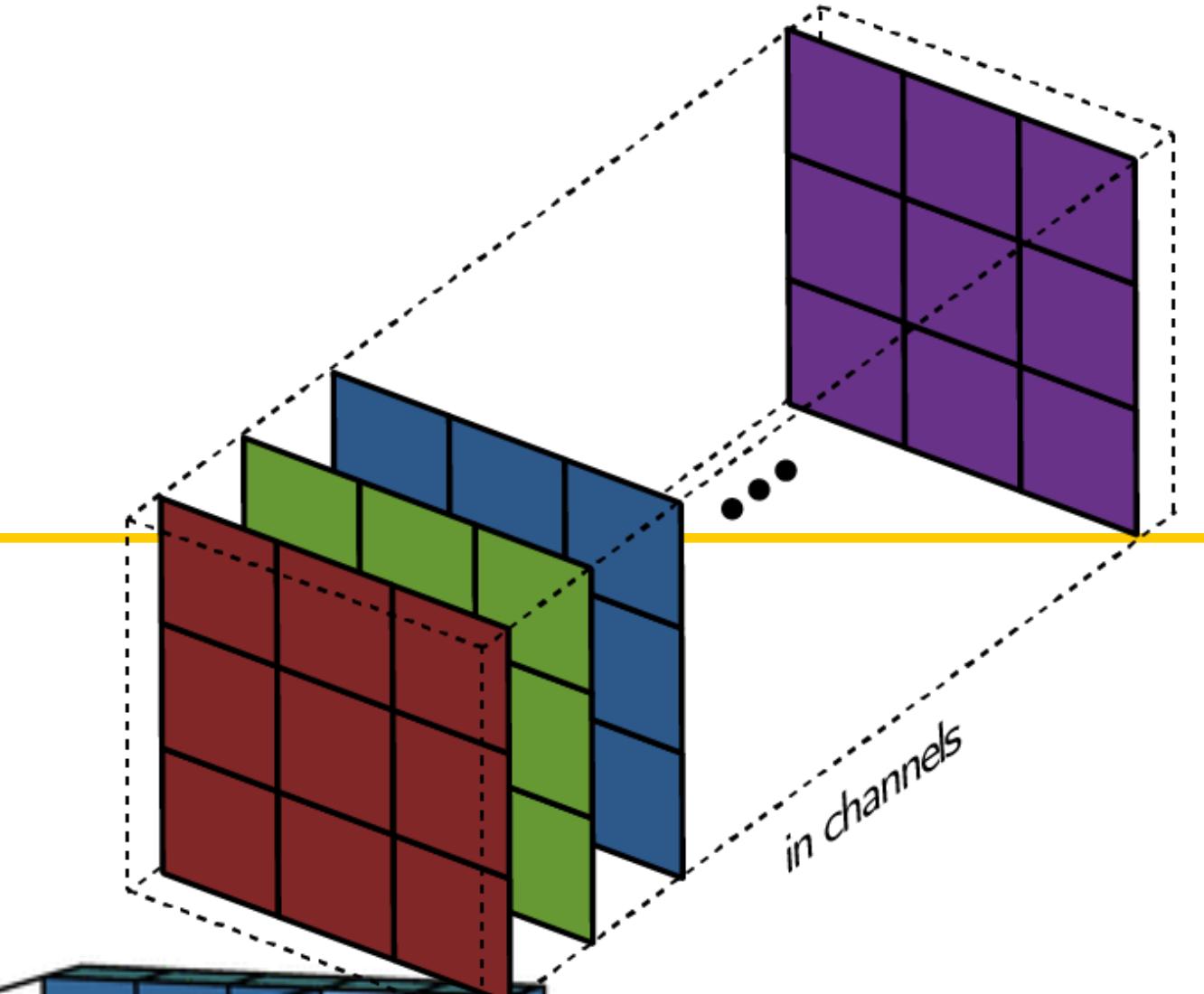
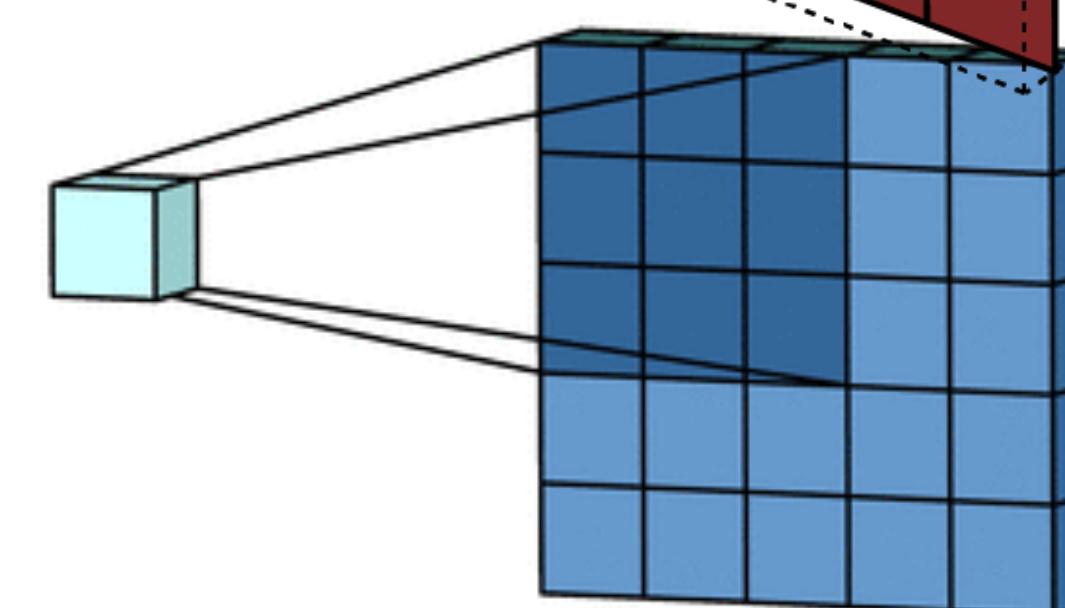
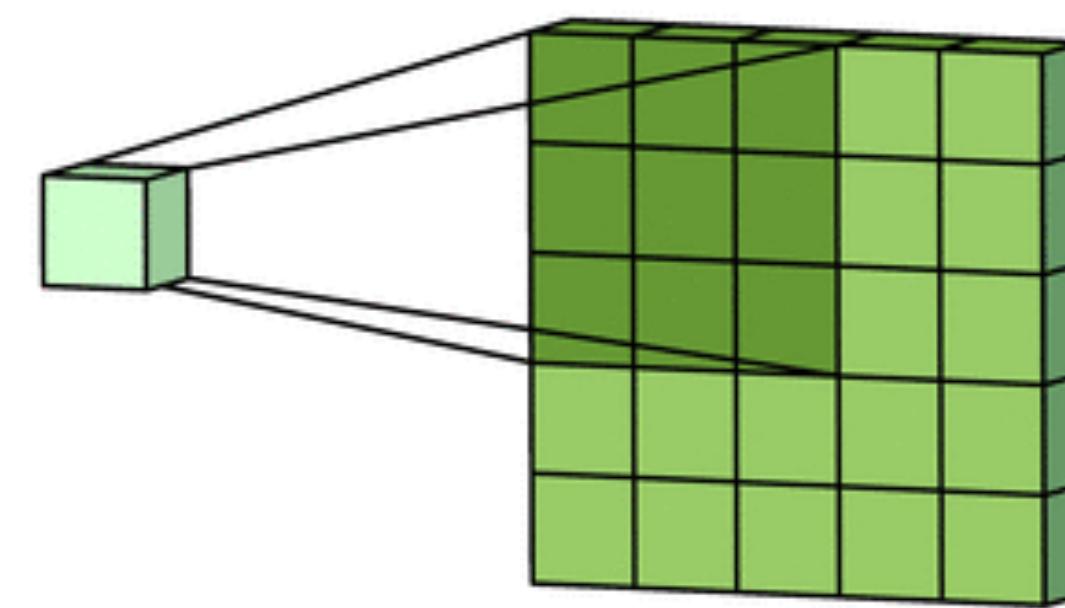
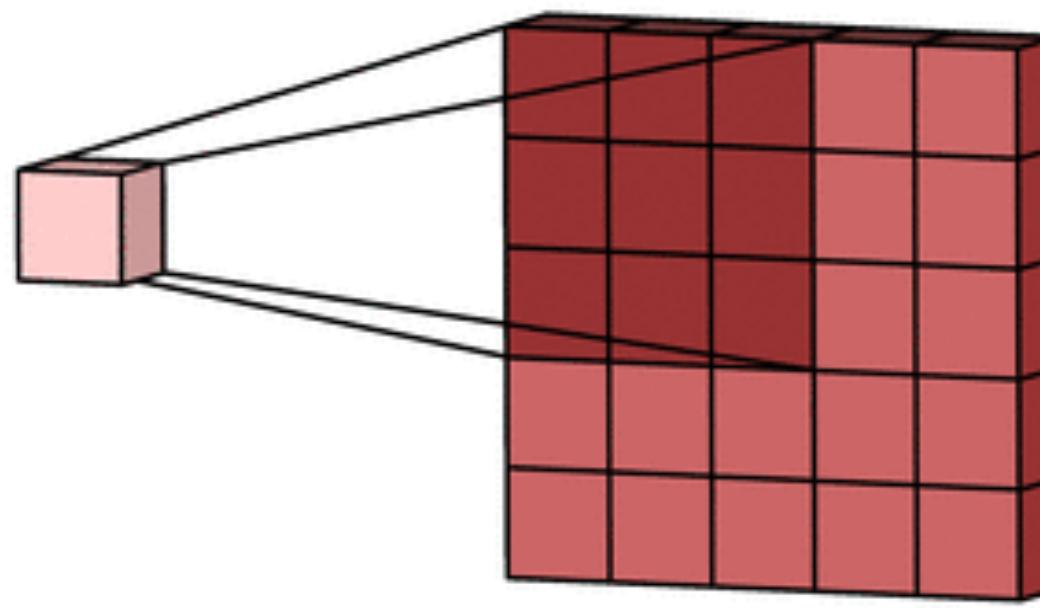
Stride = 2



dilation = 2

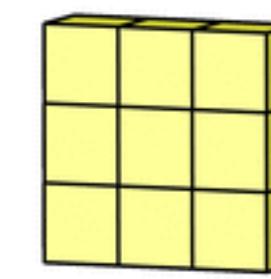
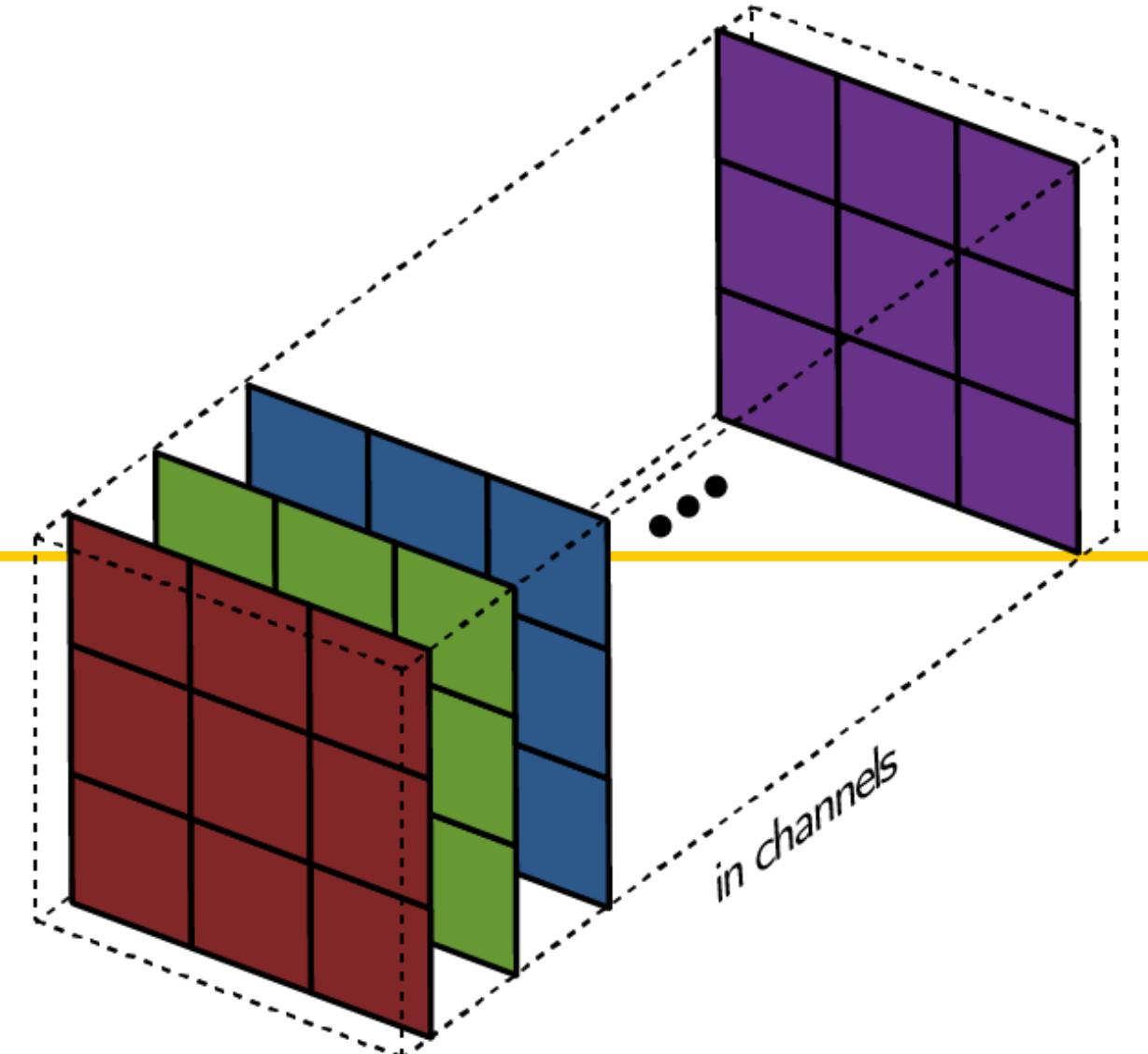
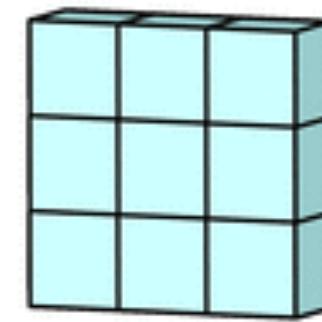
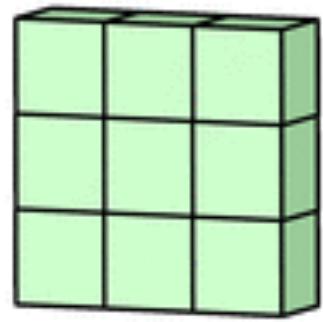
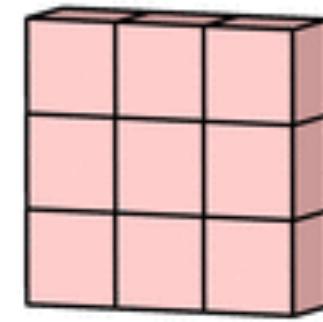


Convolution Filters





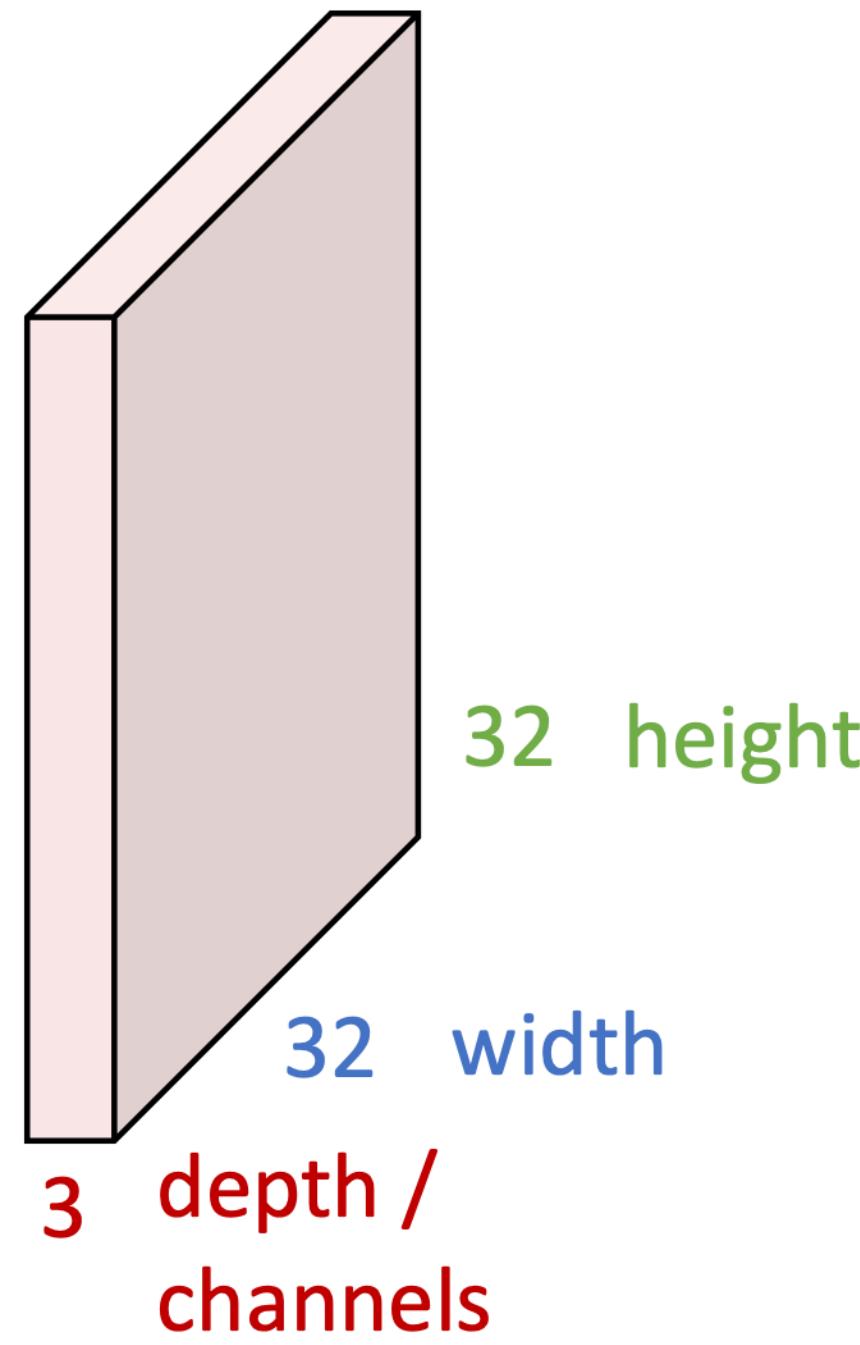
Convolution Filters





Convolution Layer

$3 \times 32 \times 32$ image: preserve spatial structure



$3 \times 5 \times 5$ filter

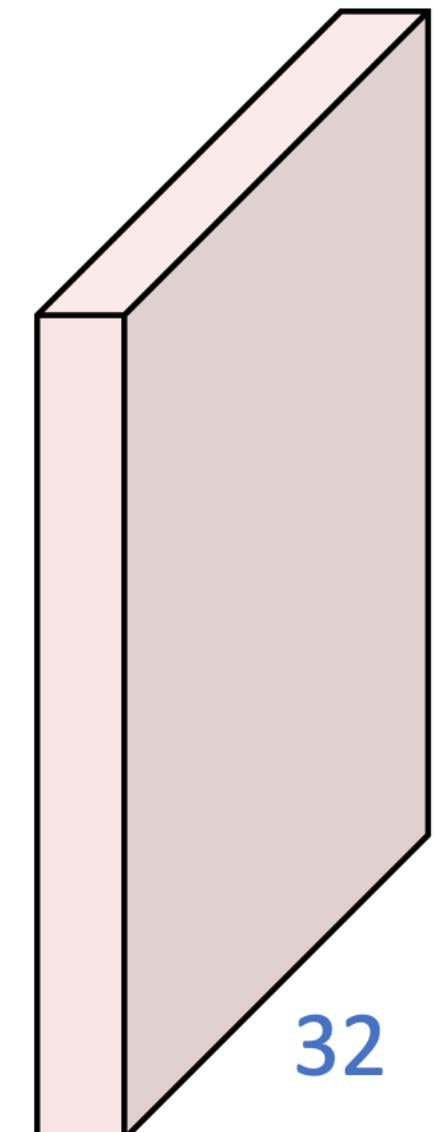


Convolve the filter with the image
i.e., “slide over the image spatially,
computing dot products”



Convolution Layer

$3 \times 32 \times 32$ image



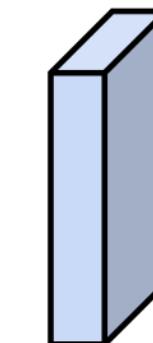
3 depth /
channels

32 width

32 height

Filters always extend the full depth
of the input volume

$3 \times 5 \times 5$ filter



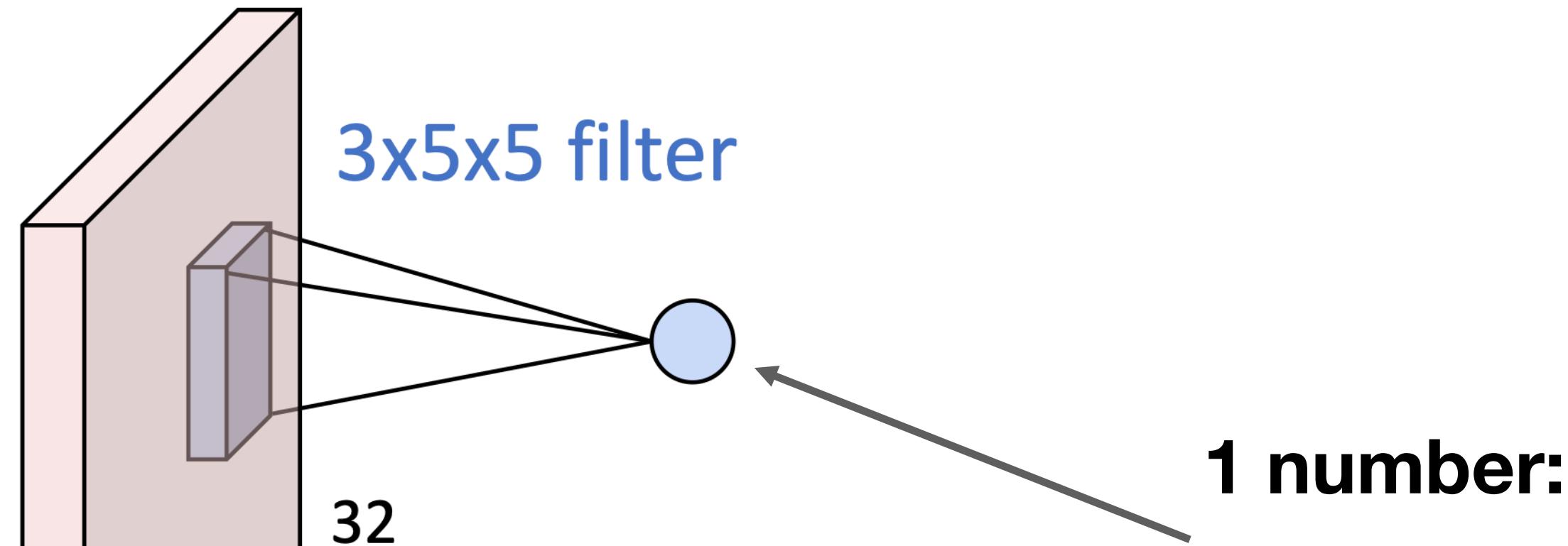
Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”



Convolution Layer

3x32x32 image

$$L_{\text{out}} = (L_{\text{in}} + 2 * \text{padding} - \text{dilation} * (\text{kernel} - 1) - 1) / \text{stride} + 1$$



1 number:

The result of taking a dot product between the filter and a small 3x5x5 portion of the image

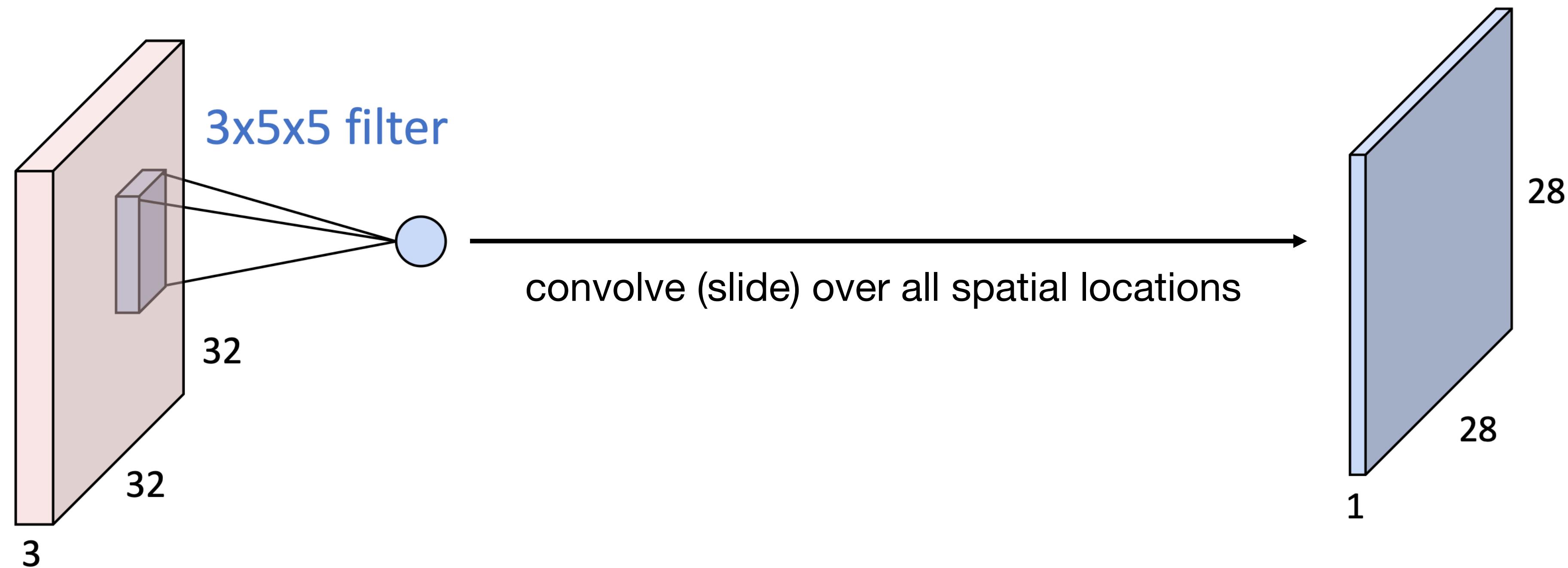
(i.e., $3 \times 5 \times 5 = 75$ -dimensional dot product + bias)
 $w^T x + b$



Convolution Layer

3x32x32 image

1x28x28 activation map

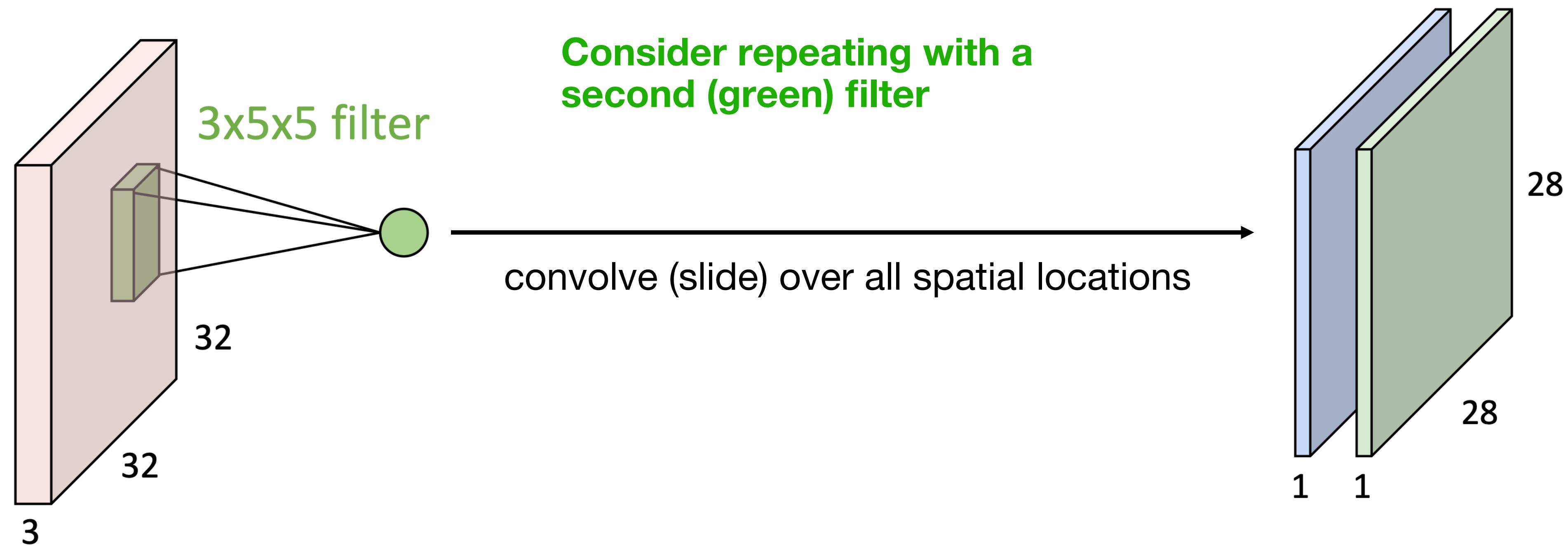




Convolution Layer

3x32x32 image

two 1x28x28 activation map

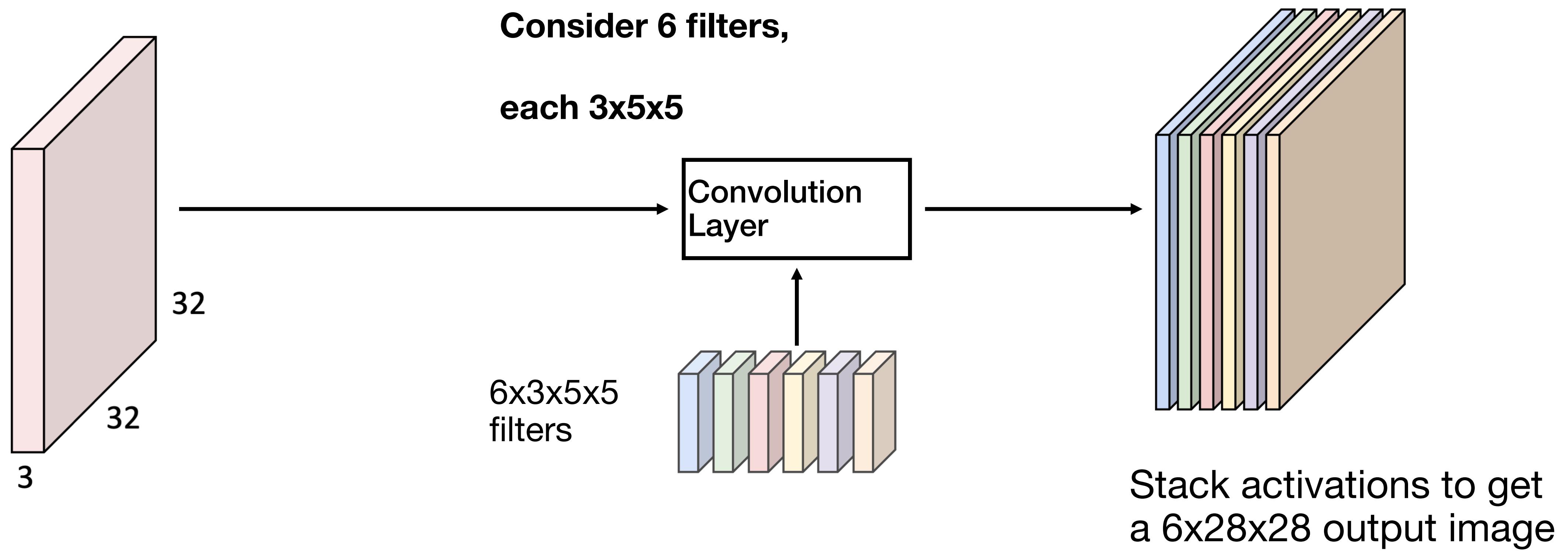




Convolution Layer

3x32x32 image

six 1x28x28 activation map

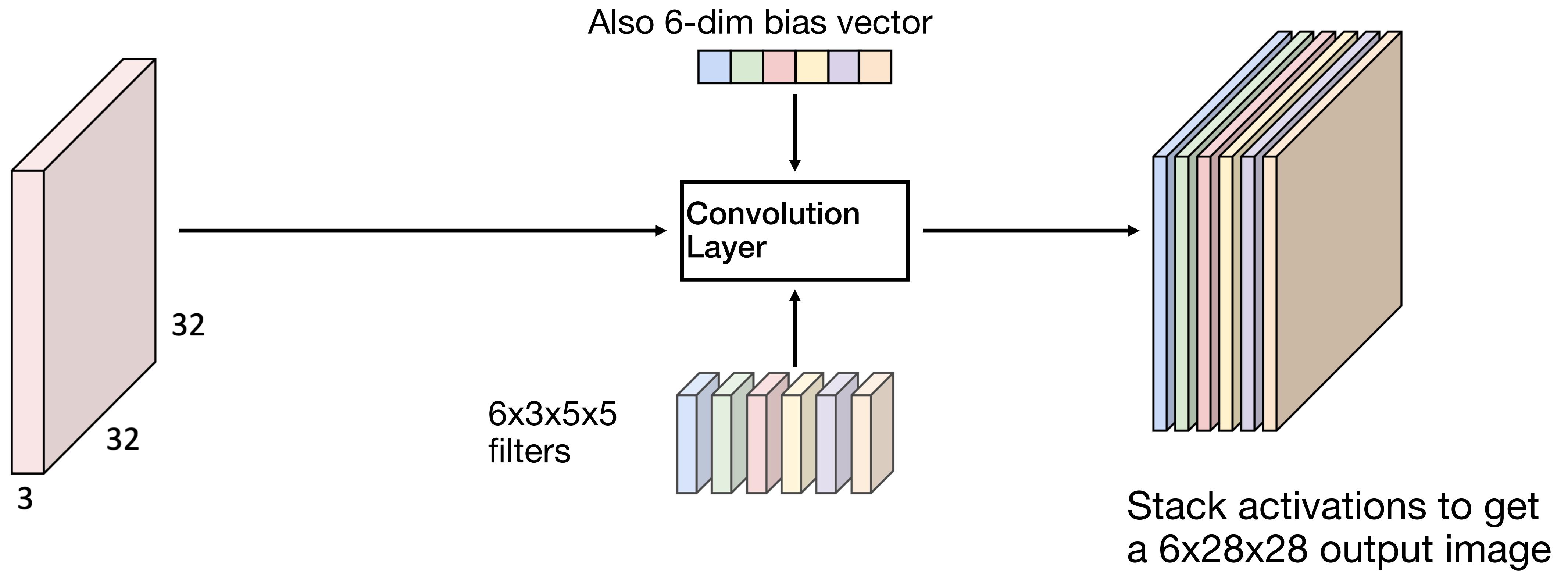




Convolution Layer

3x32x32 image

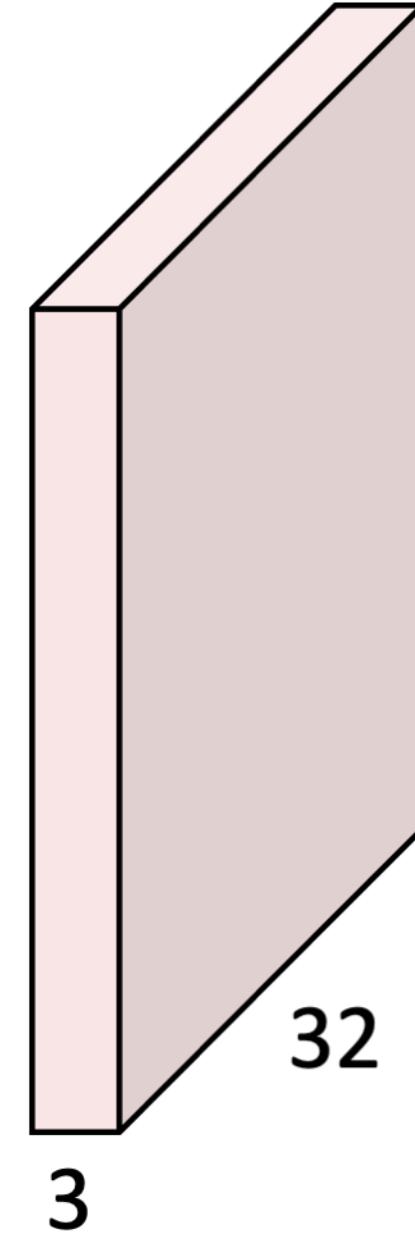
six 1x28x28 activation map





Convolution Layer

3x32x32 image



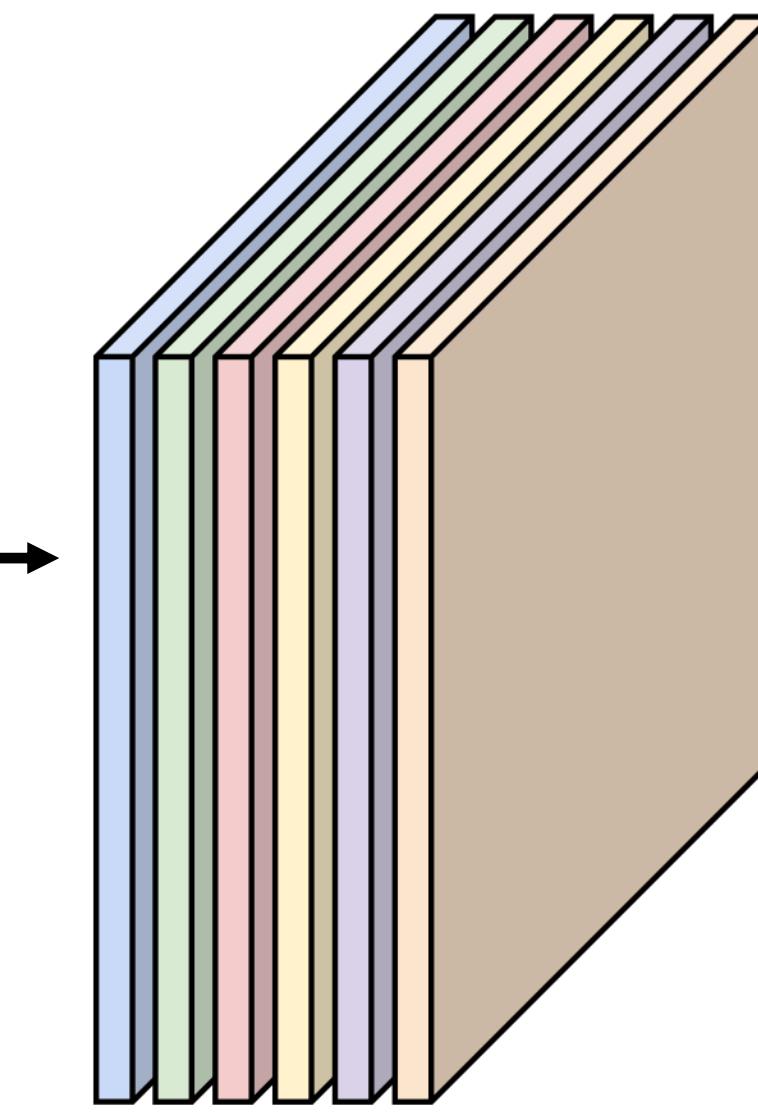
Also 6-dim bias vector



Convolution
Layer

6x3x5x5
filters

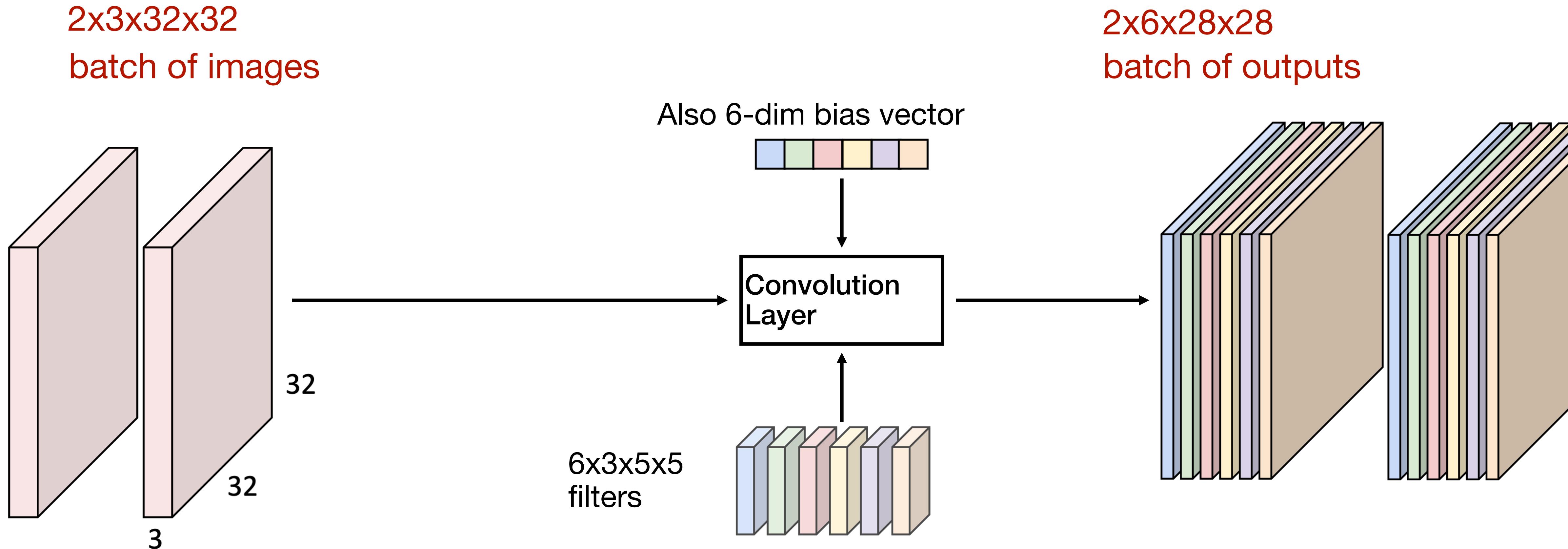
28x28 grid, at each
point a 6-dim vector



Stack activations to get
a 6x28x28 output image

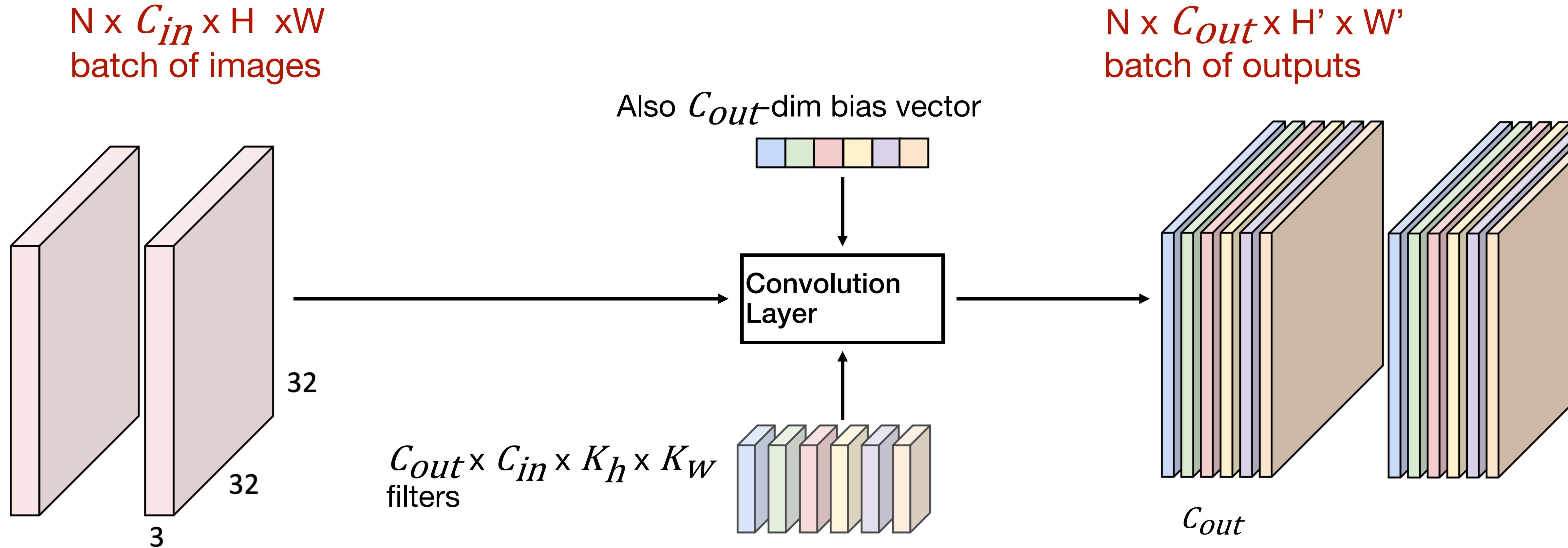


Convolution Layer



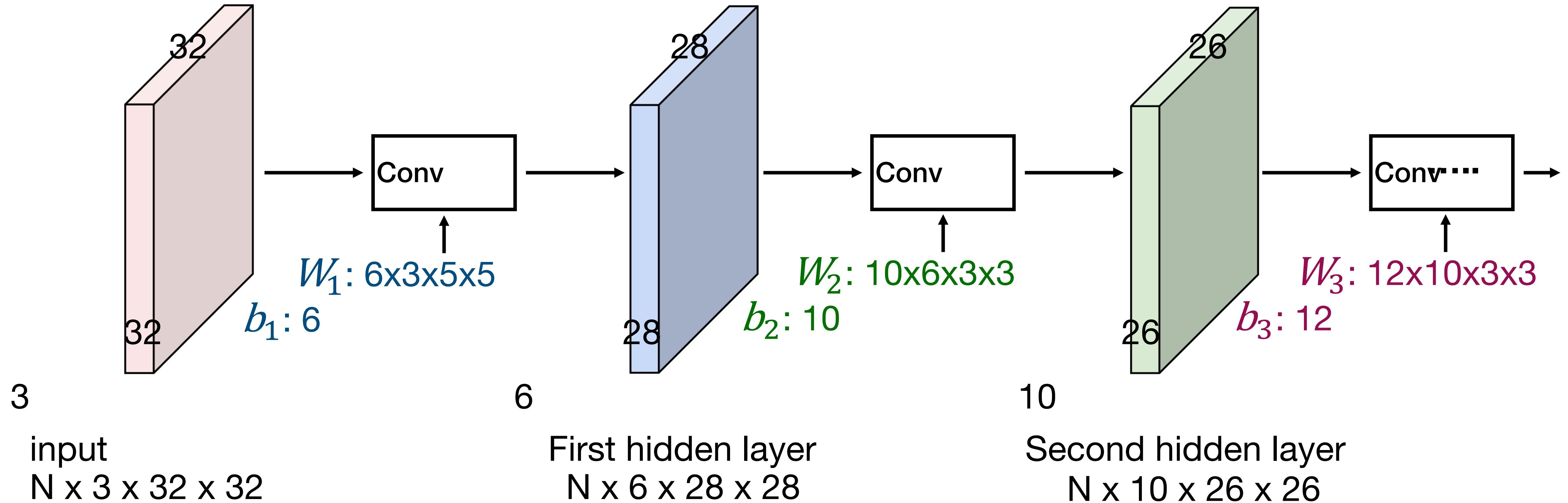


Convolution Layer: General dimensions





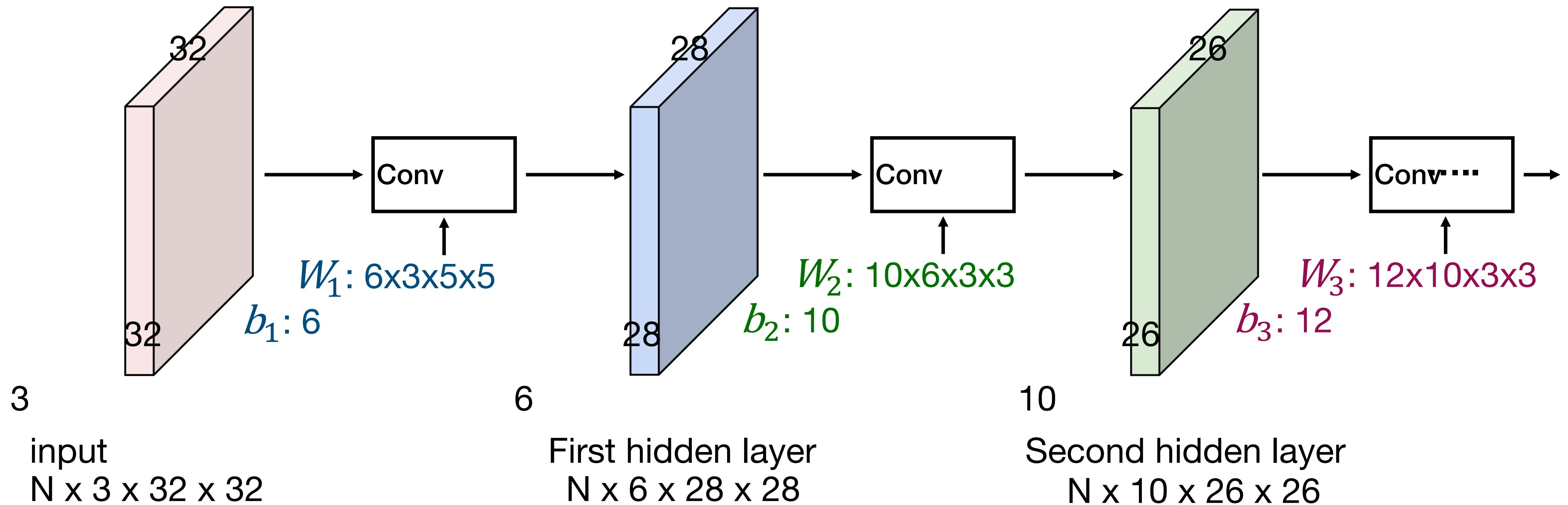
Stacking Convolutions





Stacking Convolutions

Q: What happens if we stack
two convolution layers?

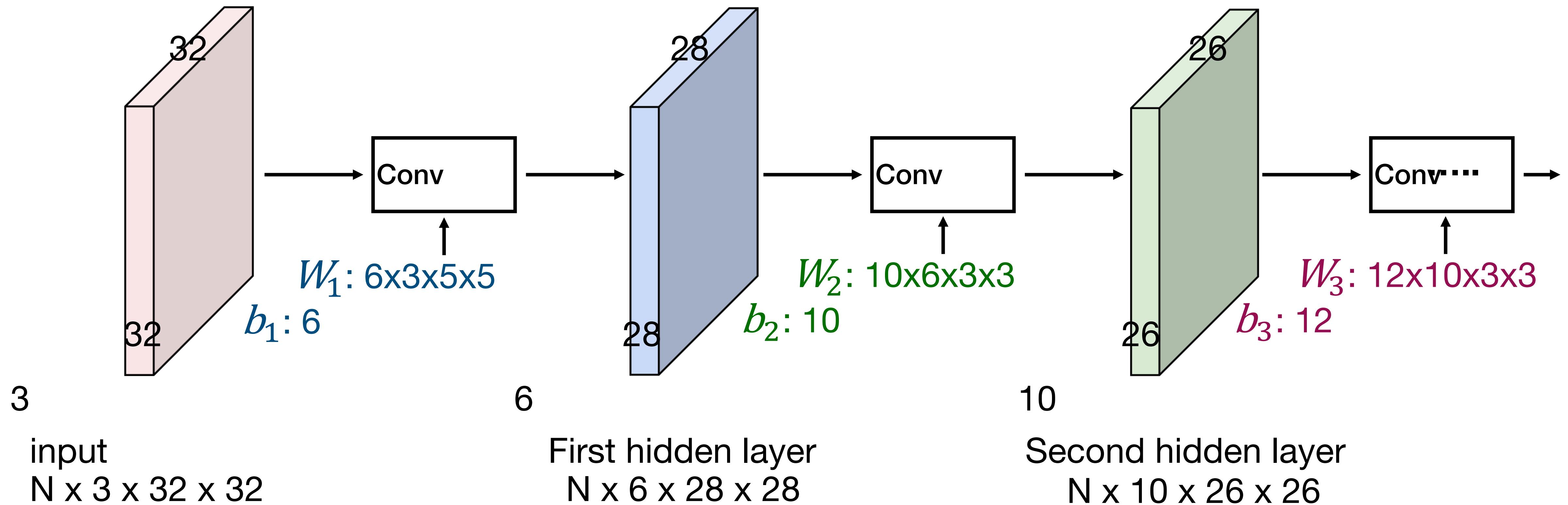




Stacking Convolutions

Q: What happens if we stack two convolution layers?

(Recall $y = W_2 W_1 x$ is a linear classifier)



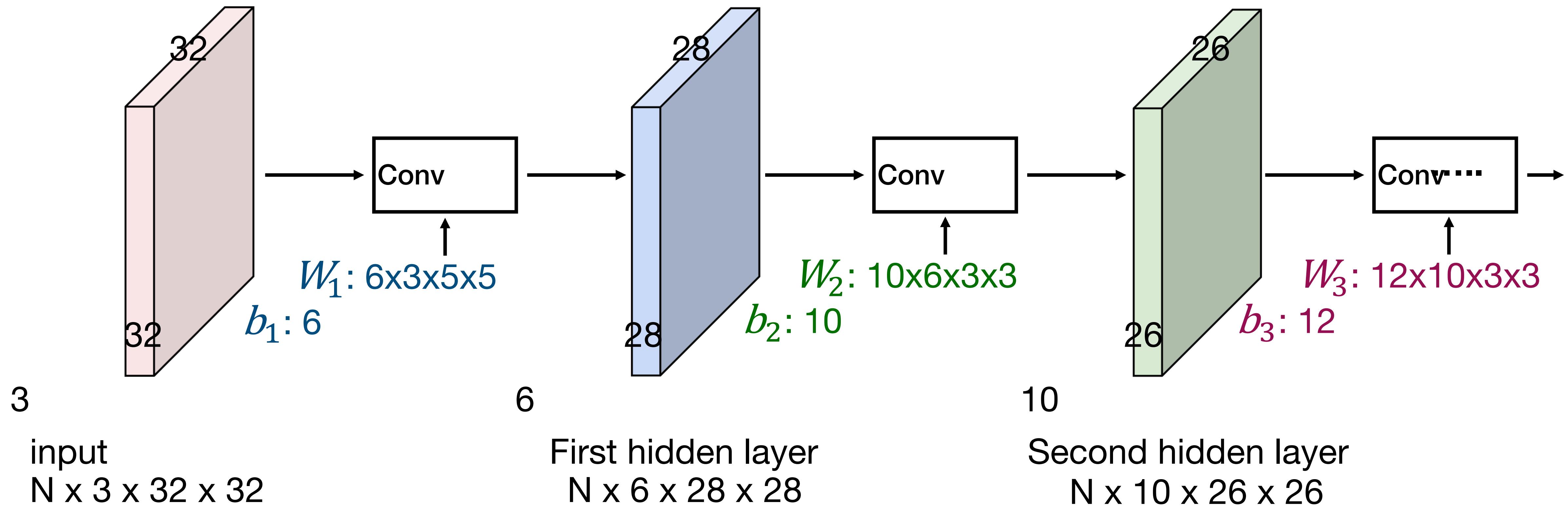


Stacking Convolutions

Q: What happens if we stack two convolution layers?

(Recall $y = W_2 W_1 x$ is a linear classifier)

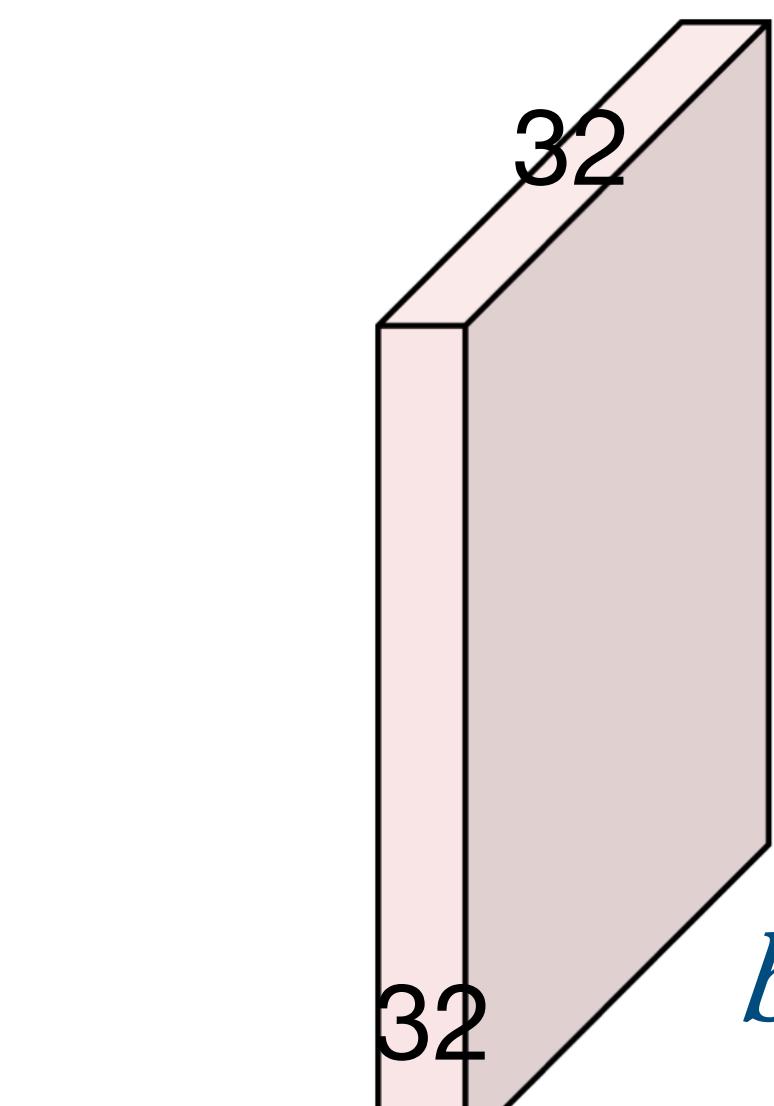
A: We get another convolution!



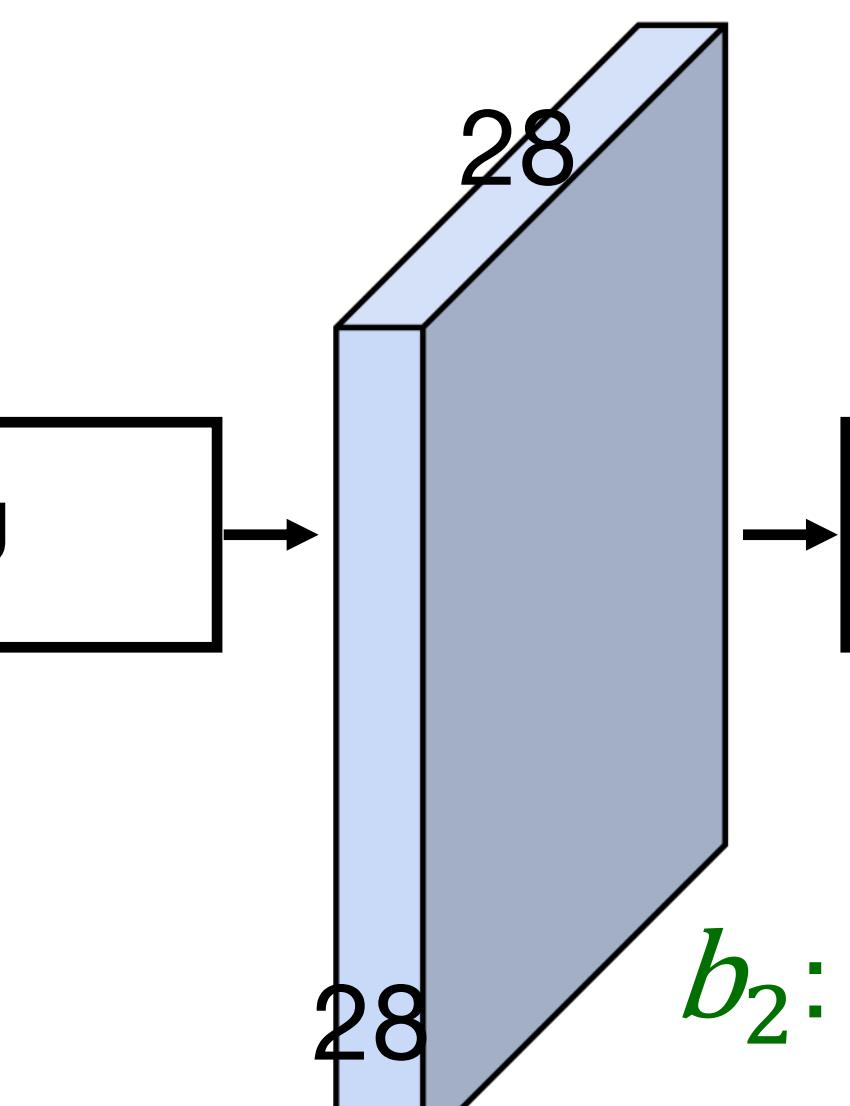


Stacking Convolutions: insert activation function

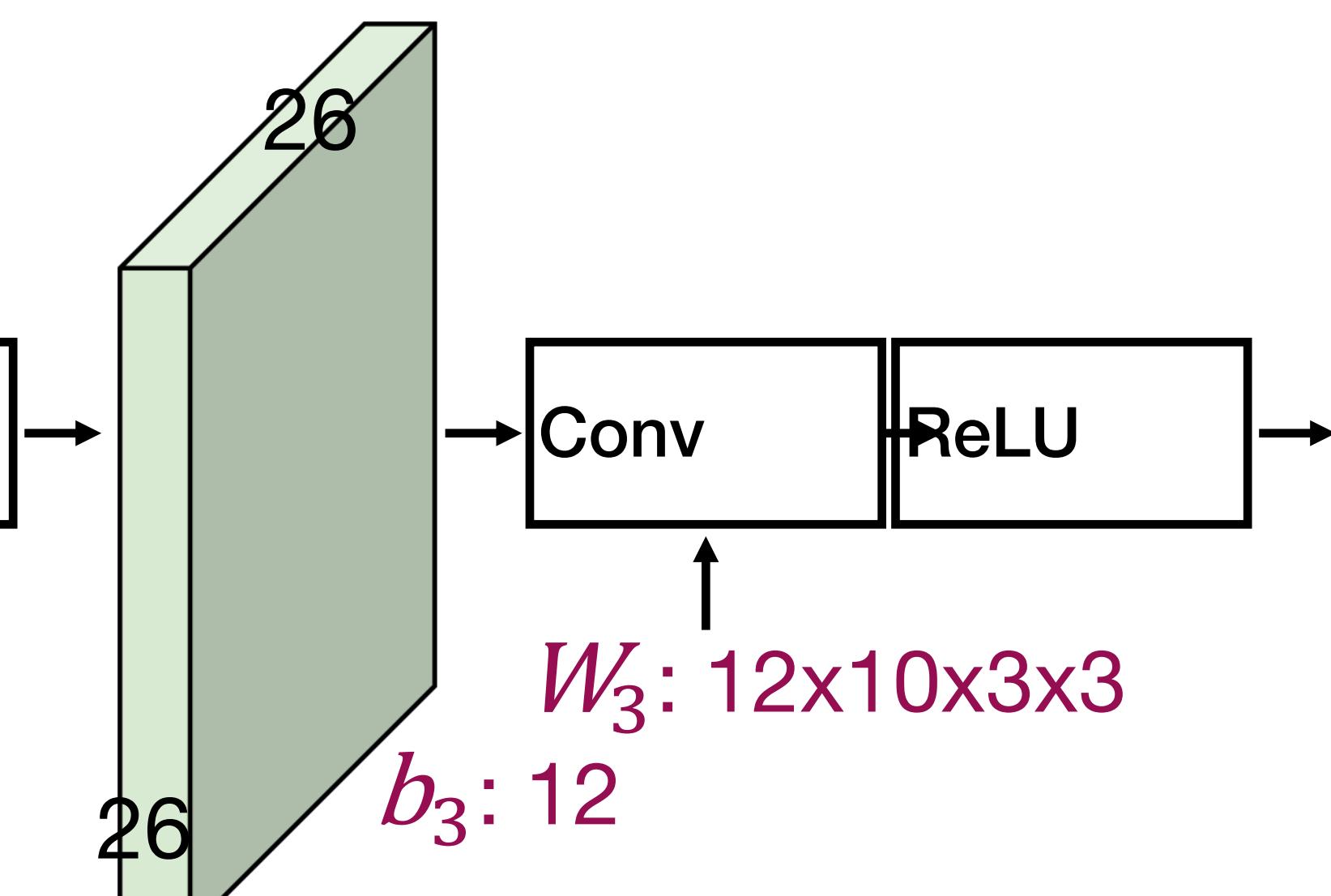
Q: What happens if we stack two convolution layers?



(Recall $y = W_2 W_1 x$ is a linear classifier)



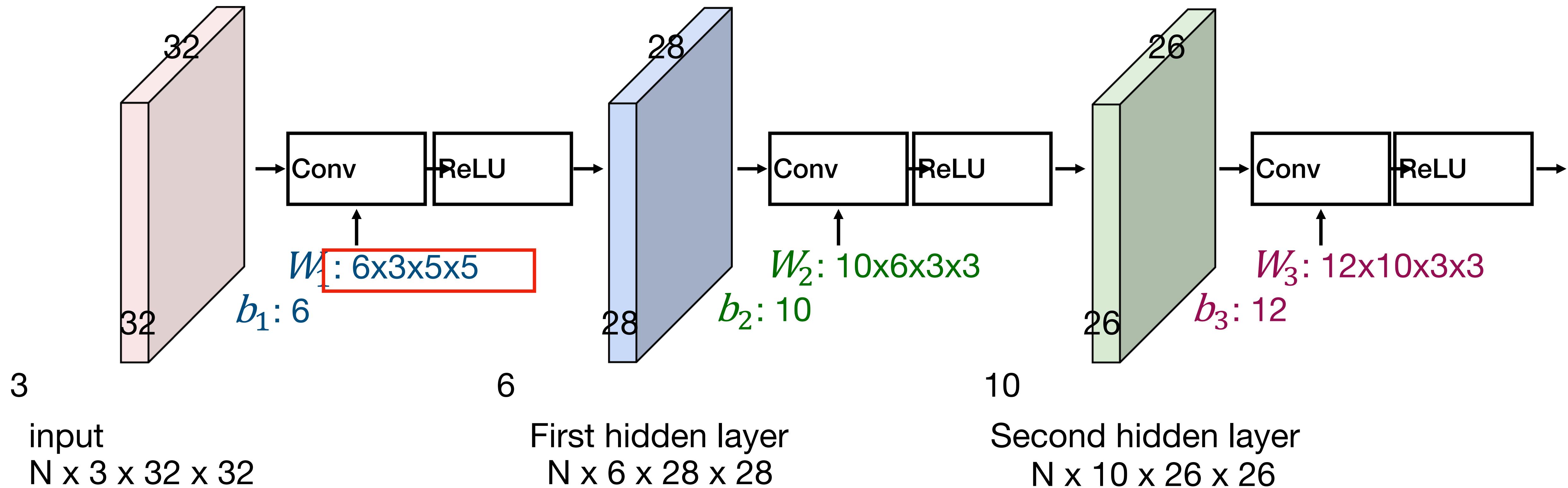
A: We get another convolution!



Non-linear relationships

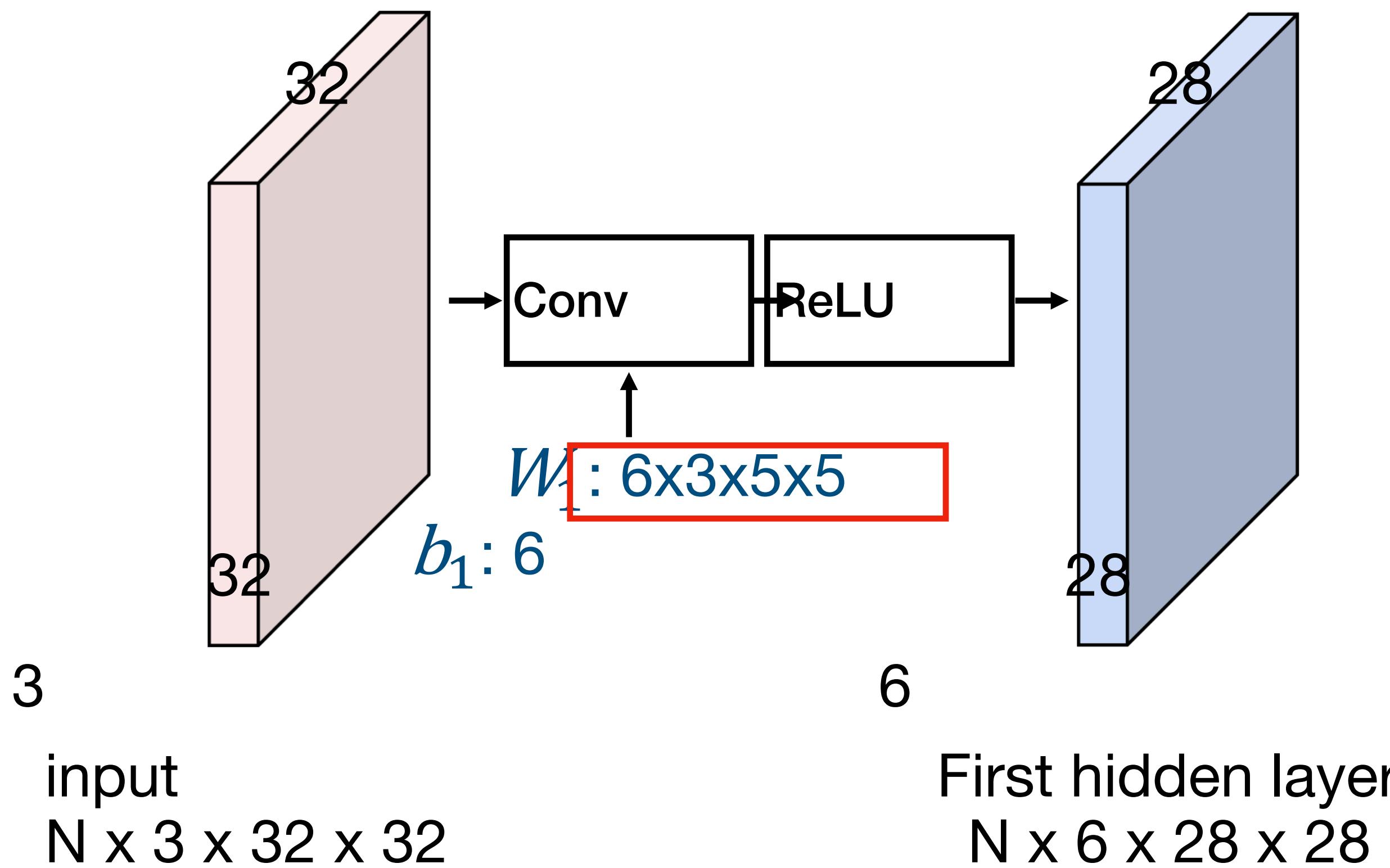


What do convolutional filters learn?

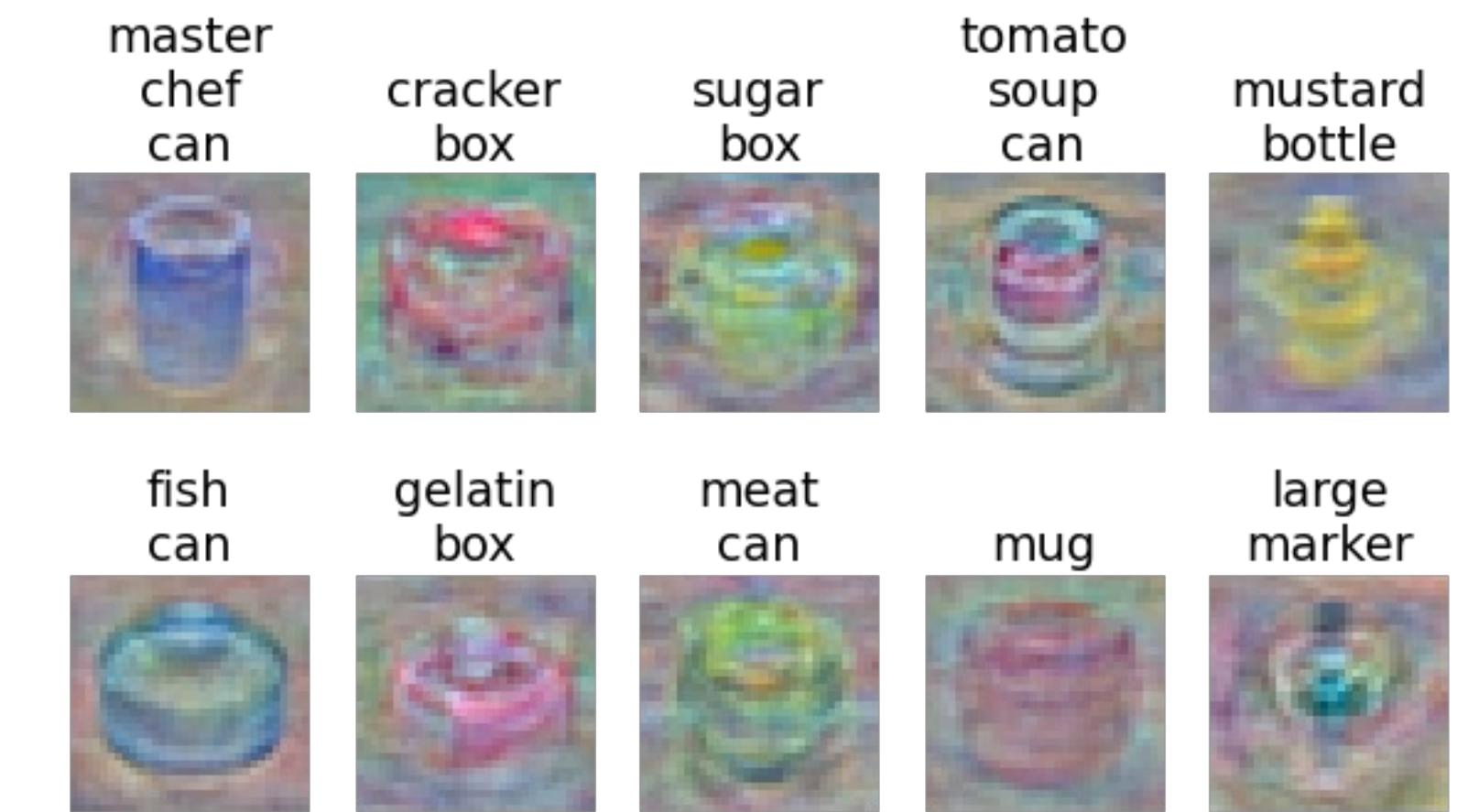




What do convolutional filters learn?

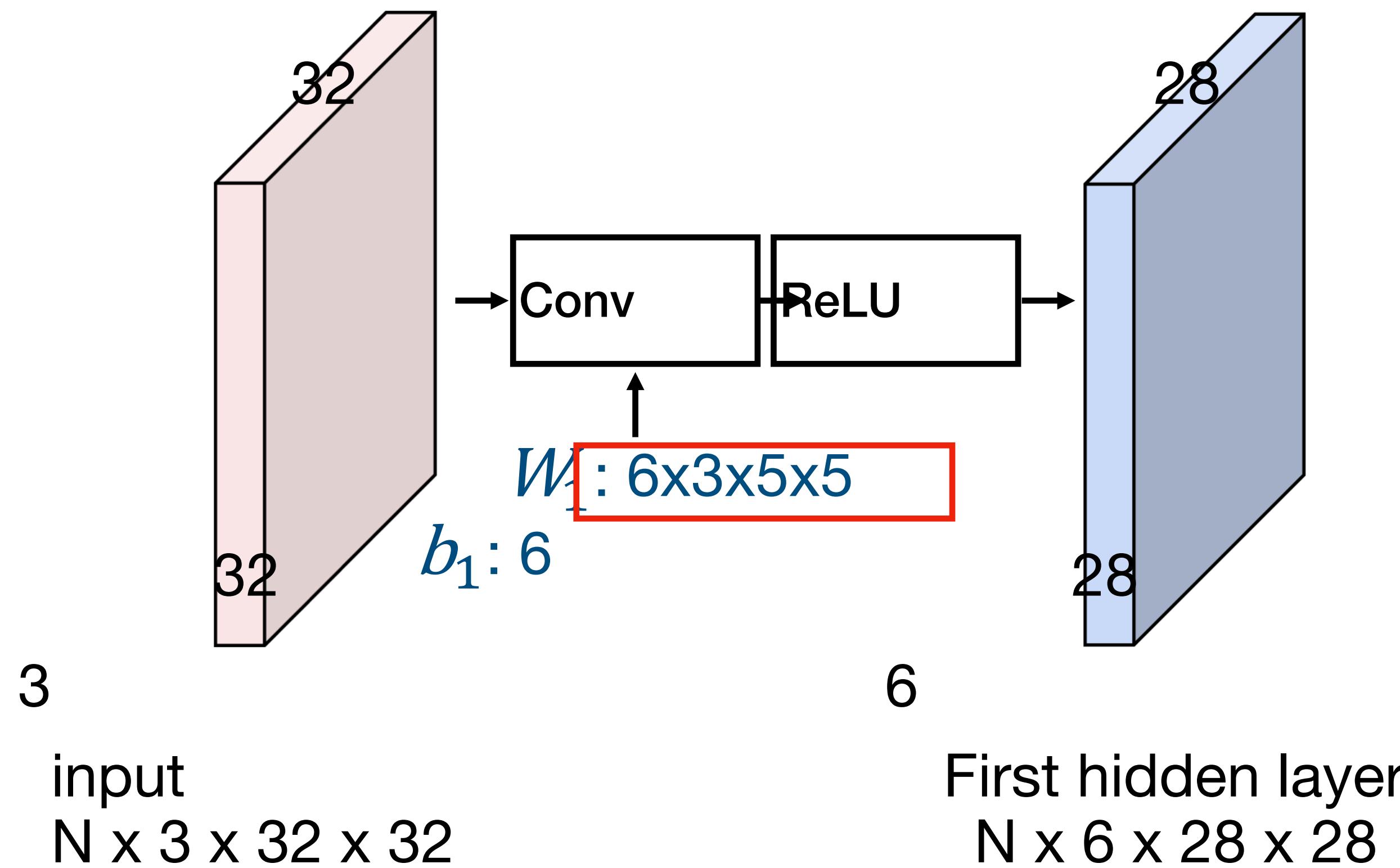


Linear classifier: One template per class





What do convolutional filters learn?



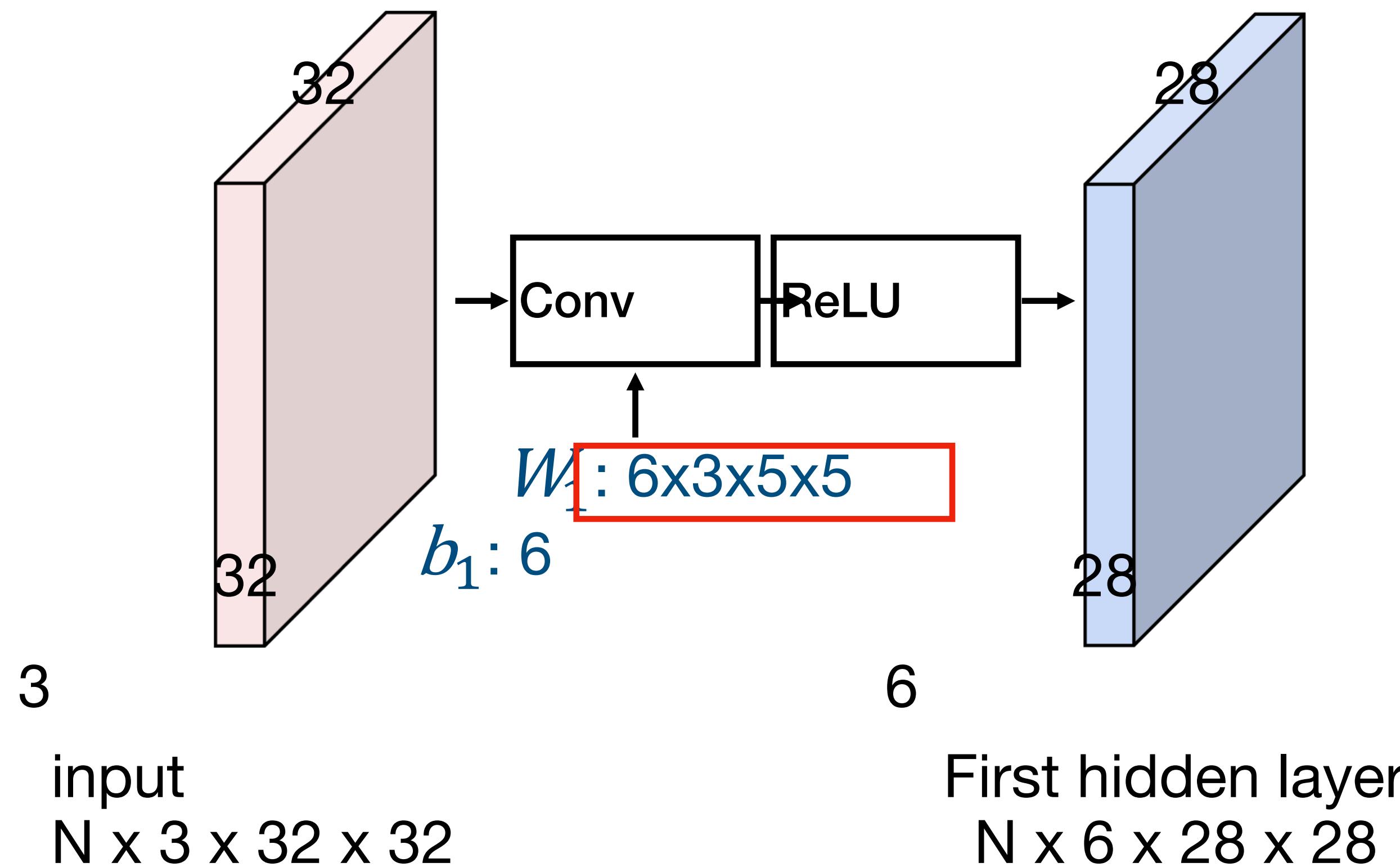
MLP: Bank of whole-image templates



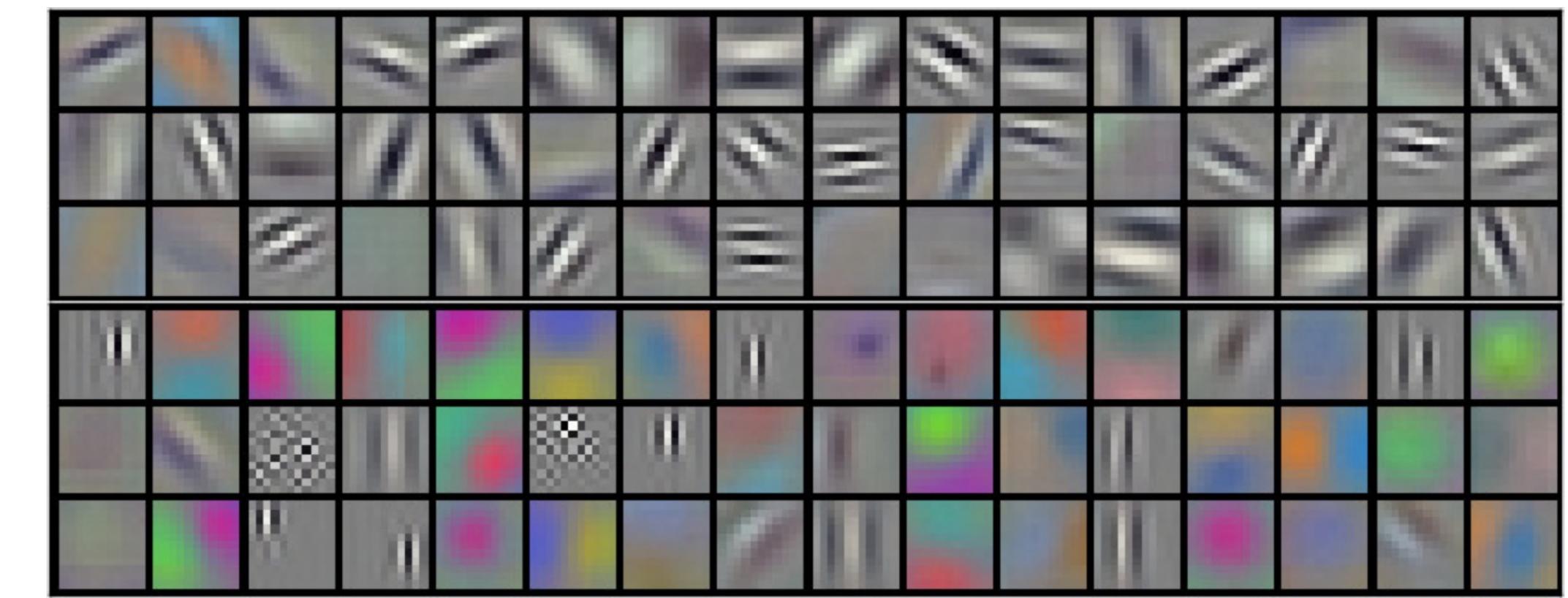
* Global wrt. the entire image



What do convolutional filters learn?



First-layer conv filters: local image templates
(often learns oriented edges, opposing colors)



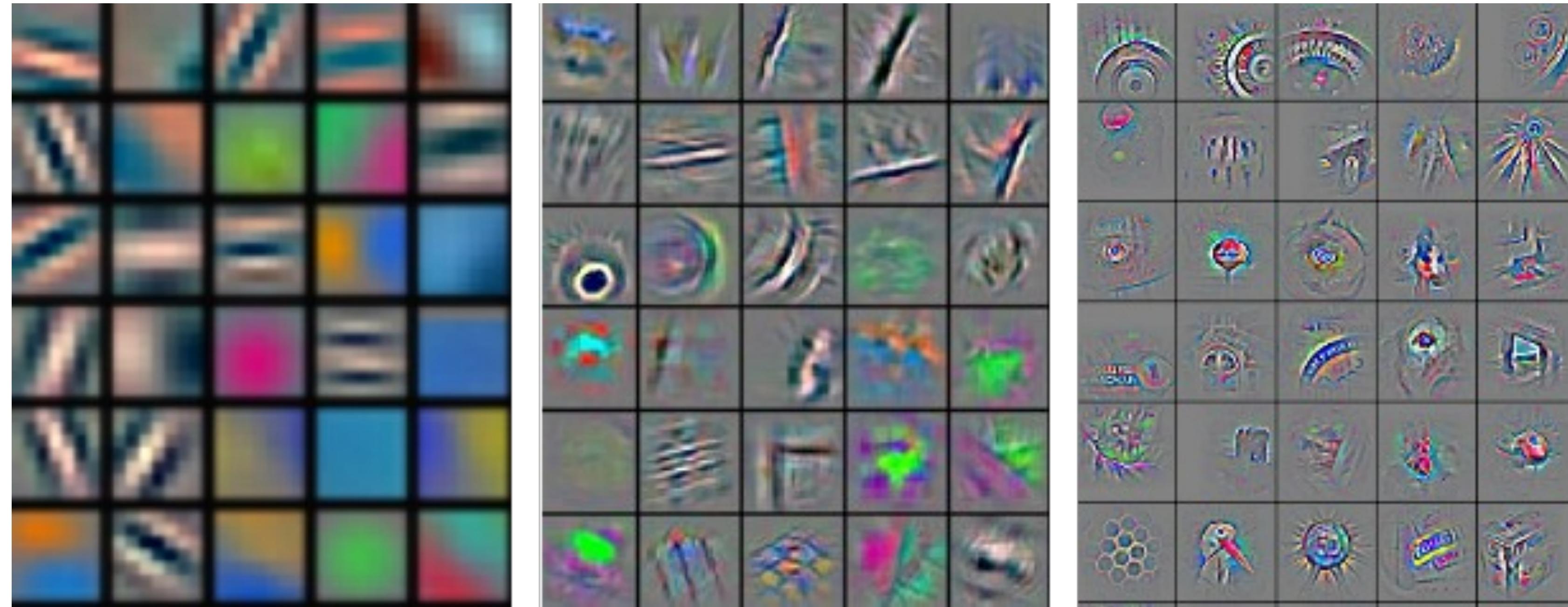
[AlexNet](#): 96 filters, each $3 \times 11 \times 11$

* Local



What do convolutional filters learn?

Feature visualization

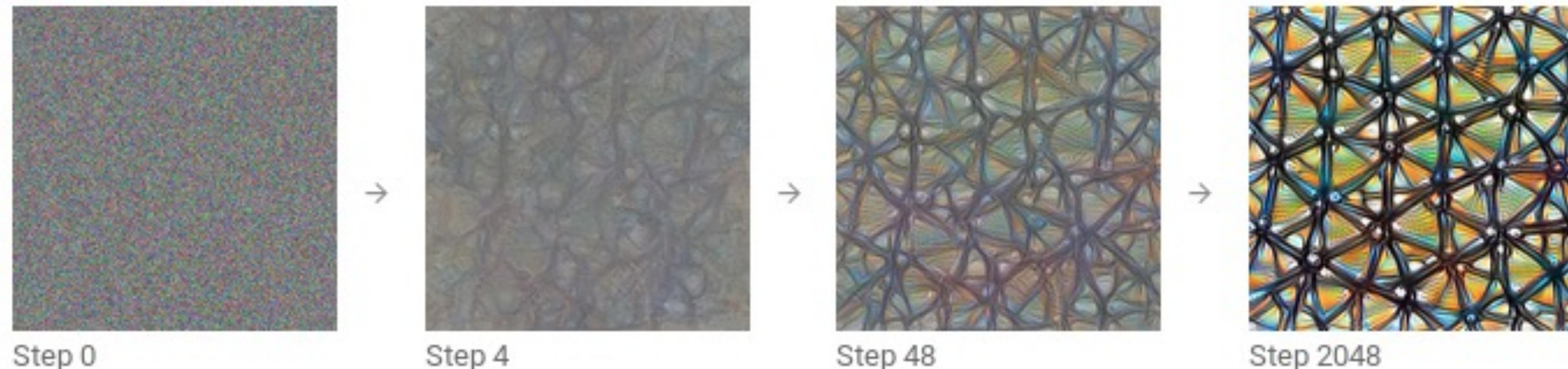
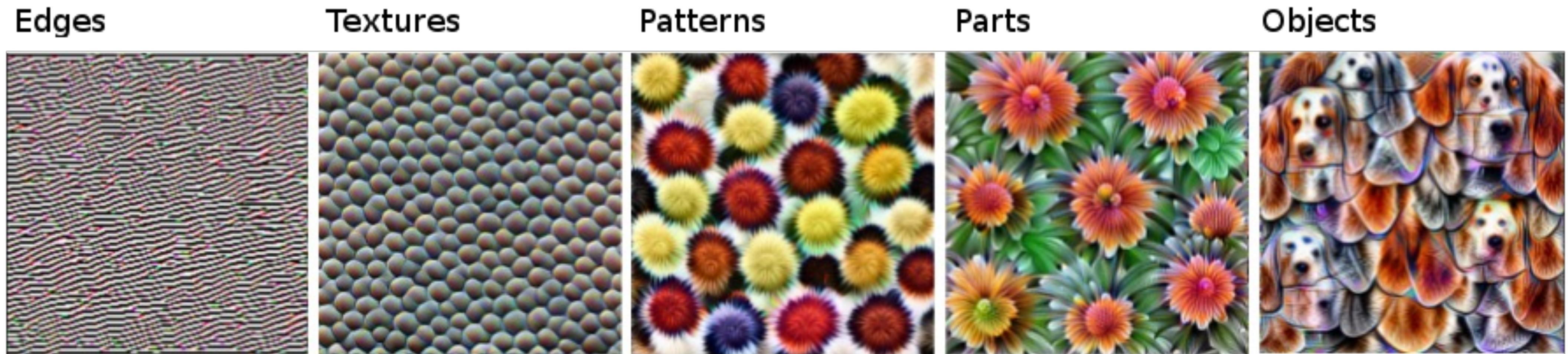




What do convolutional filters learn?

Feature visualization

distill.pub





What do convolutional filters learn?

Activation mask

<https://christophm.github.io/interpretable-ml-book/cnn-features.html>

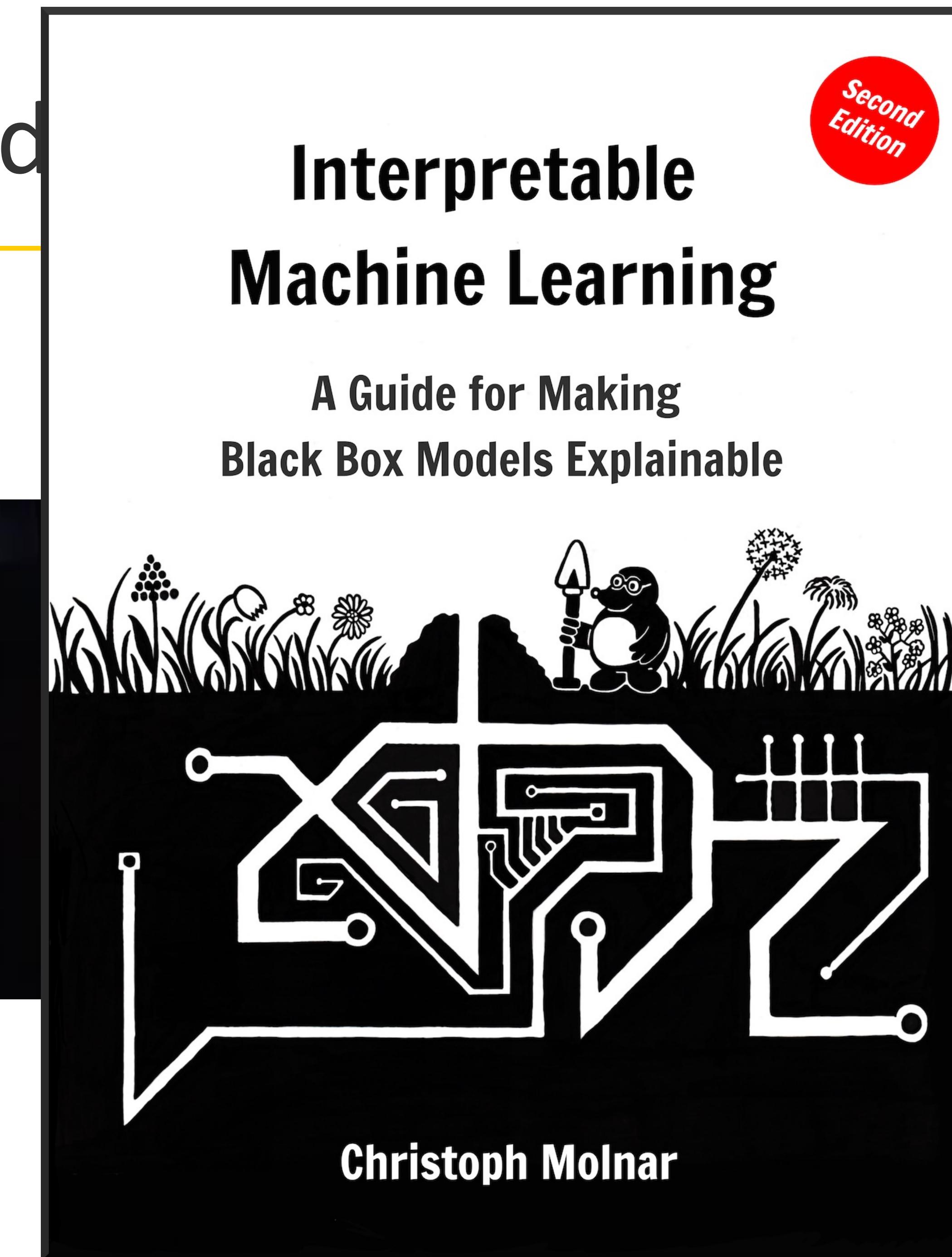
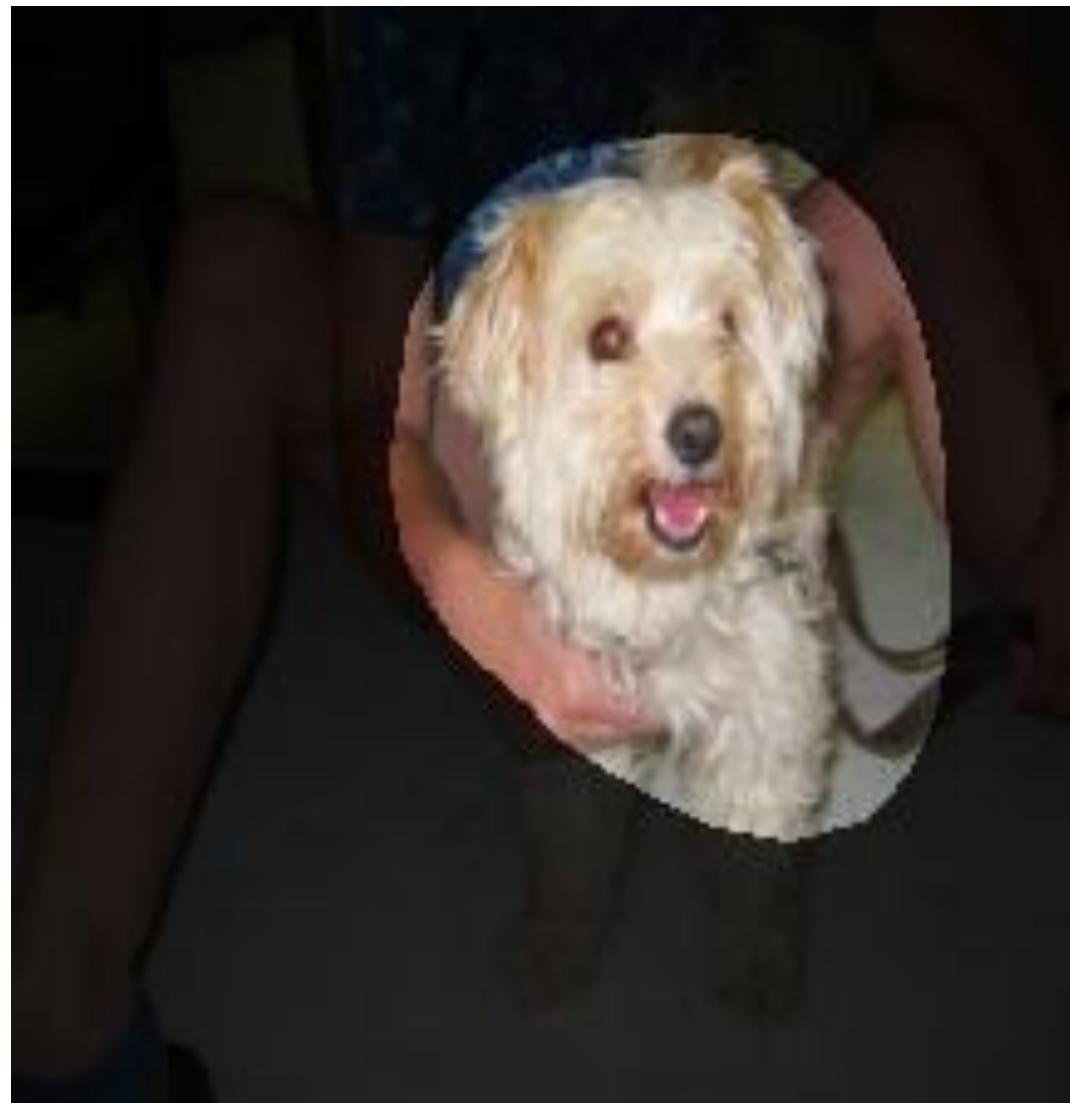


- = Human annotated ground truth
- = Top activated area
- = Area of Intersection
- = Area of Union



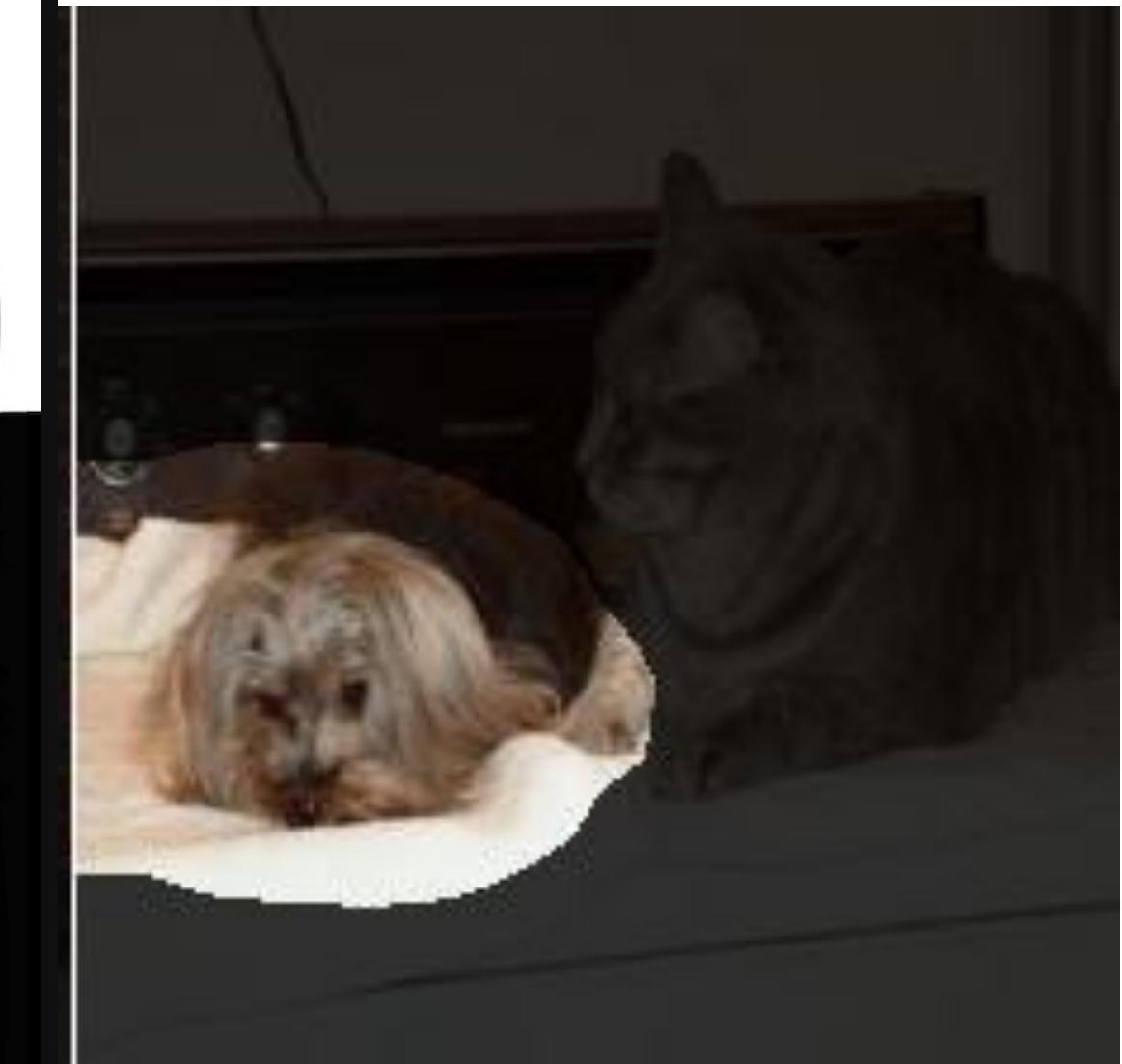
What do

Activation mask



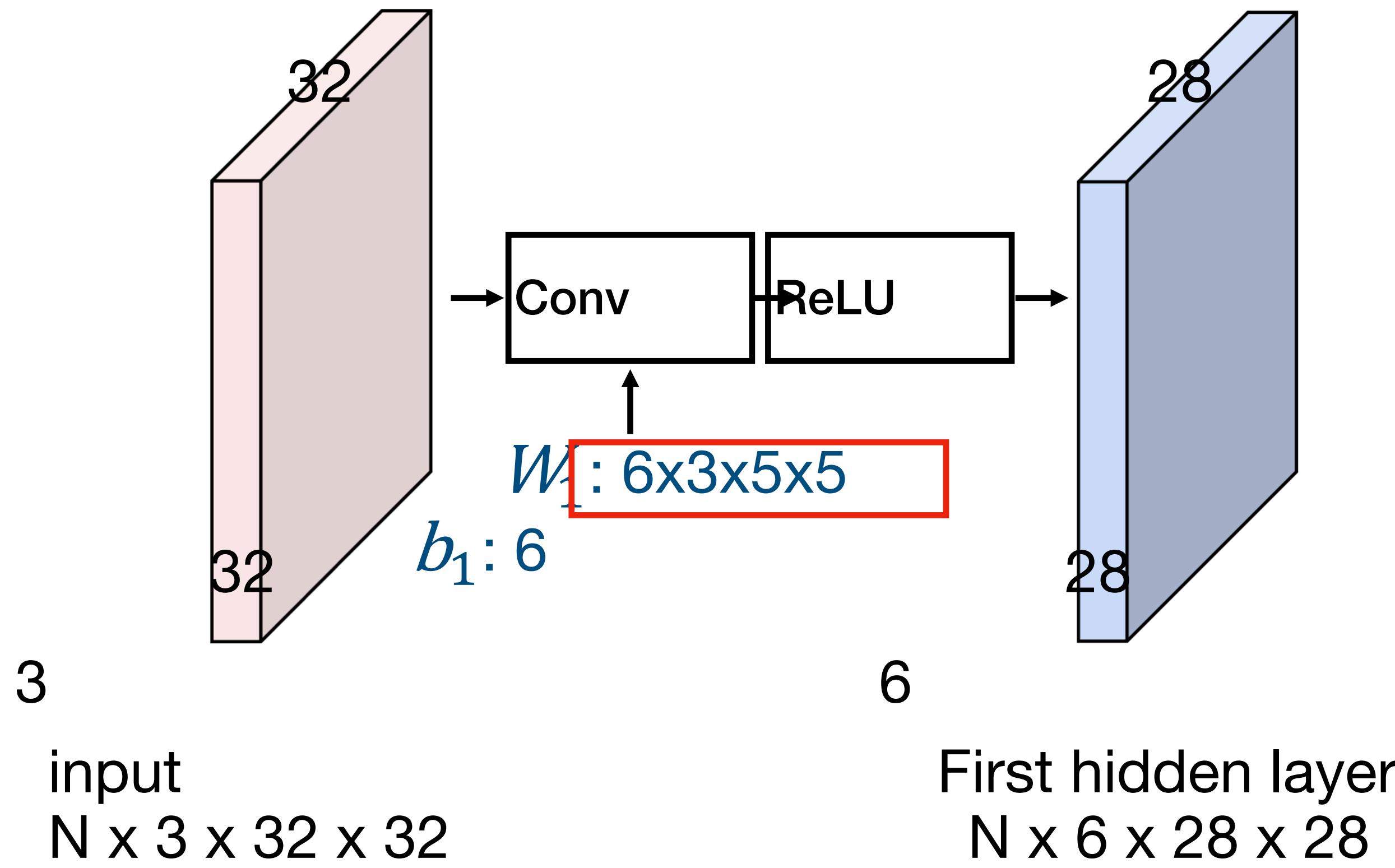
learn?

<https://github.com/interpretable-ml-book/cnn-features.html>



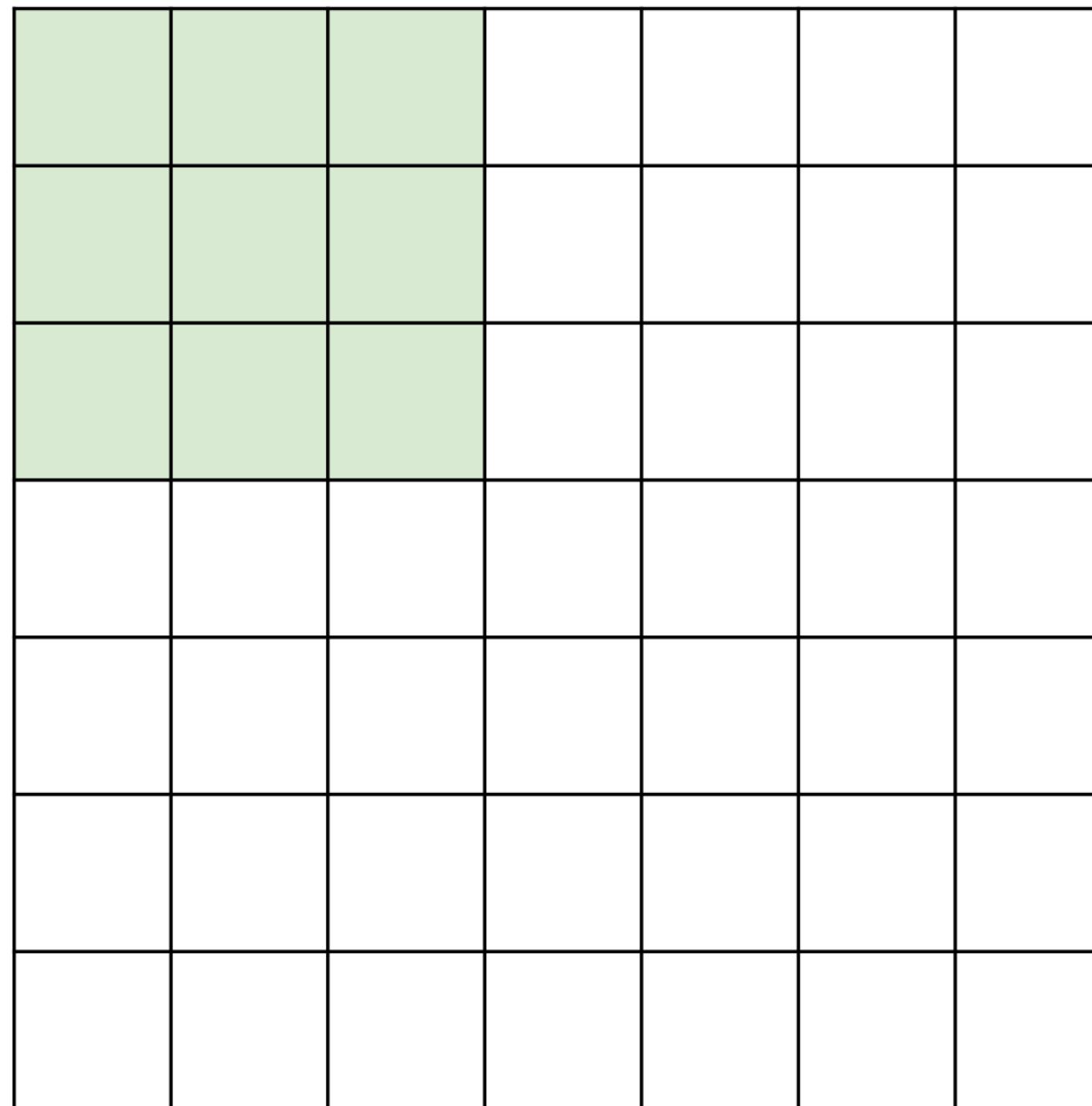


A closer look at spatial dimensions





A closer look at spatial dimensions



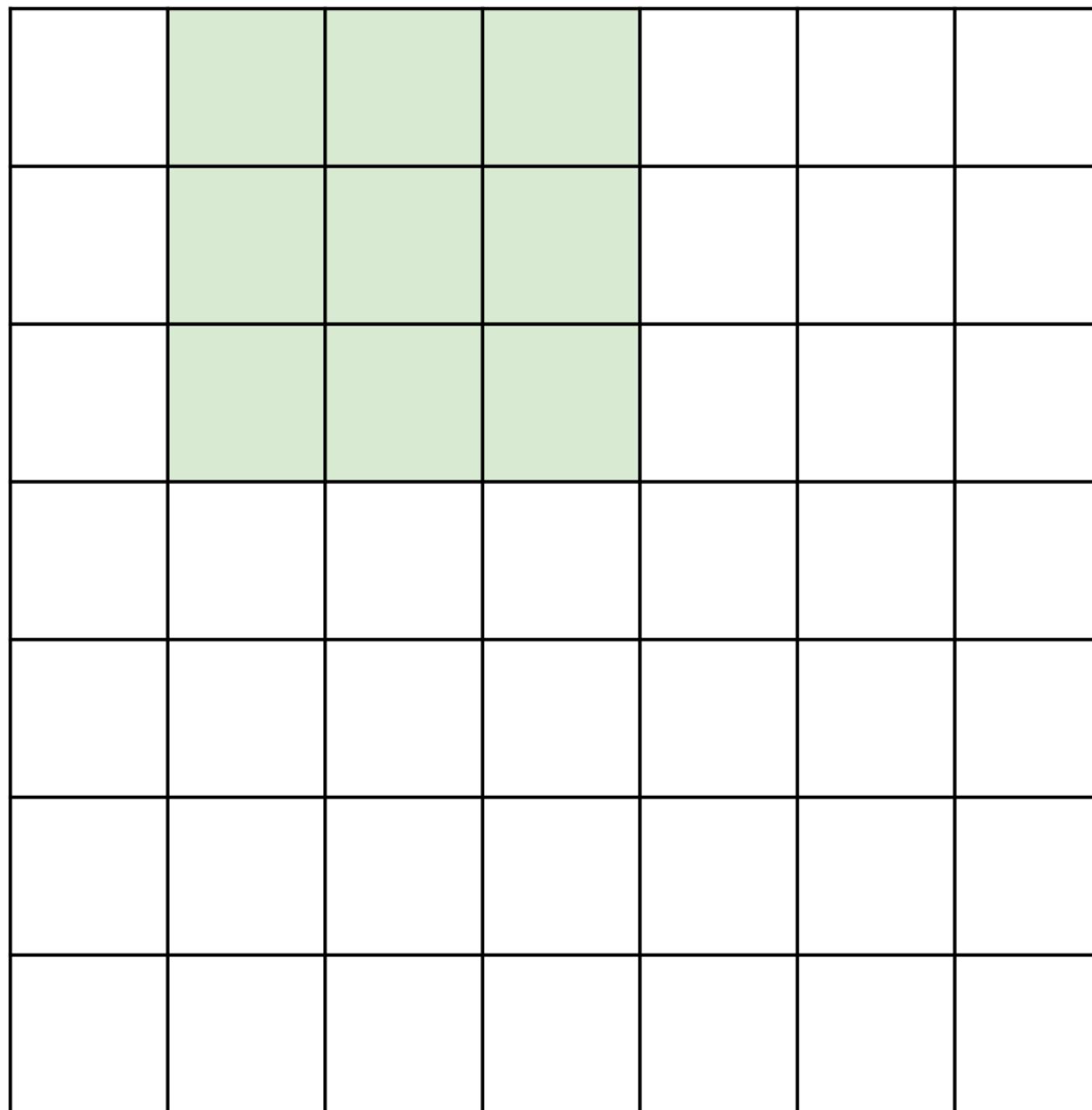
7

Input: 7x7
Filter: 3x3

7



A closer look at spatial dimensions



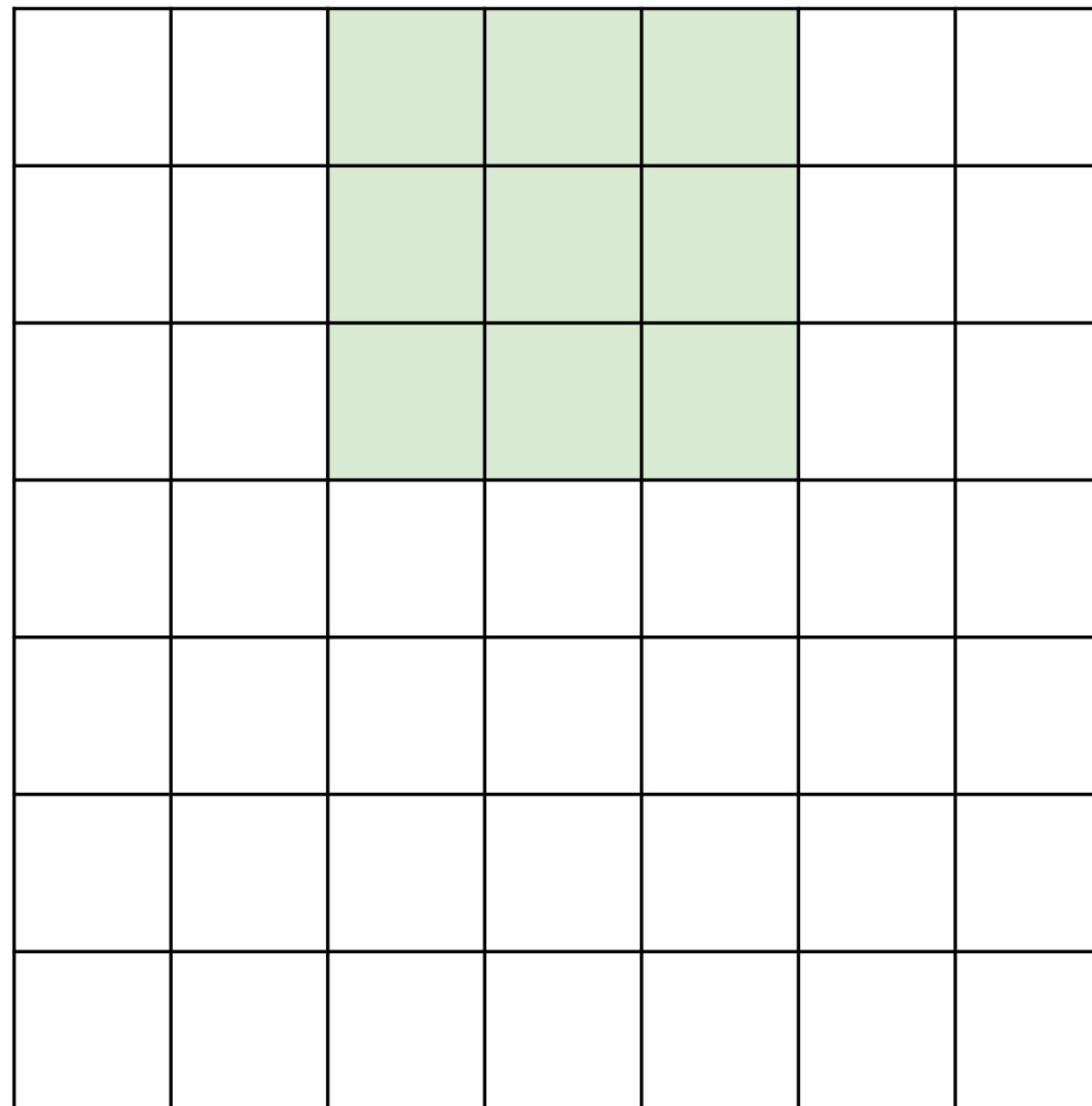
7

Input: 7x7
Filter: 3x3

7



A closer look at spatial dimensions



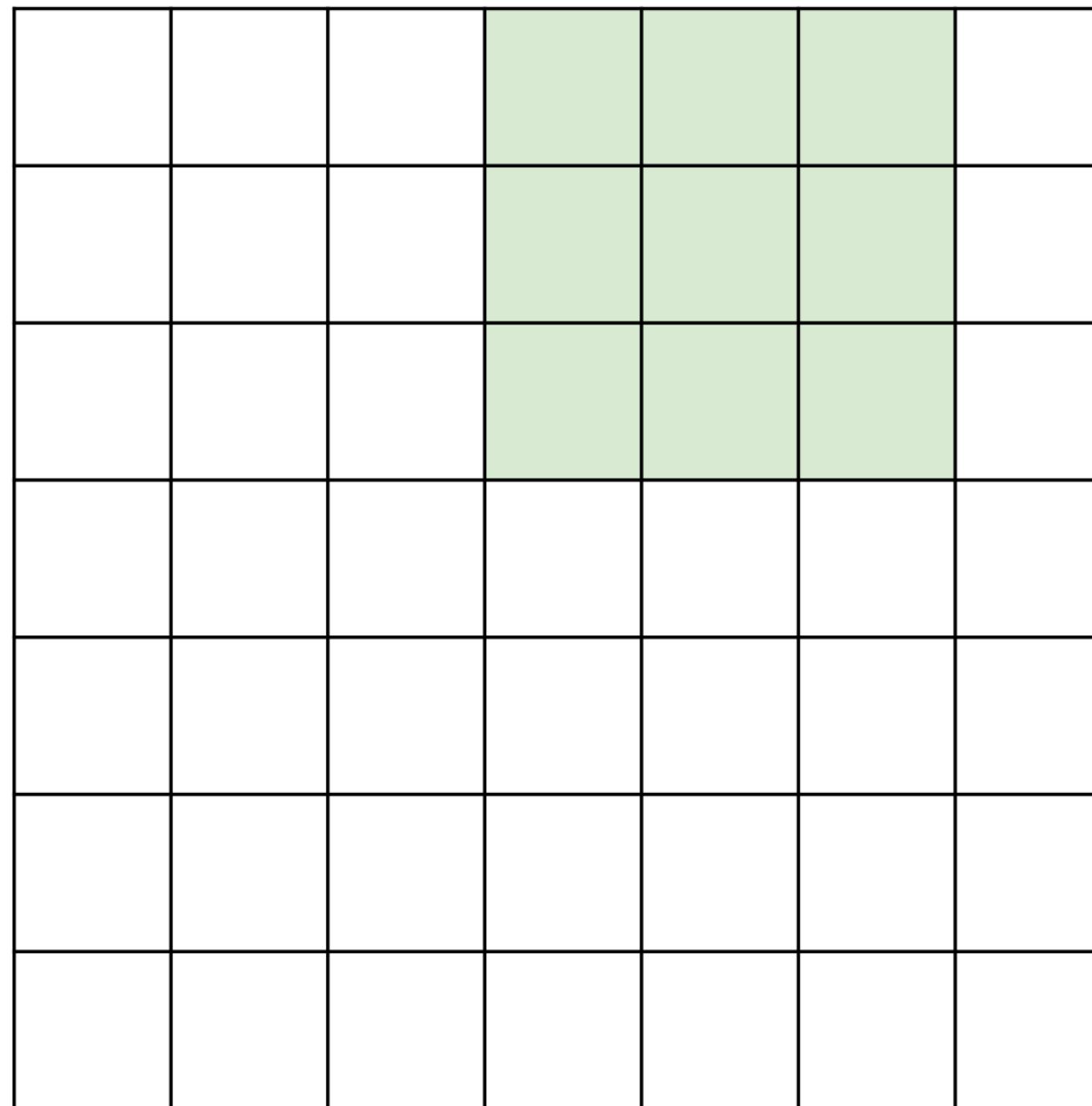
7

Input: 7x7
Filter: 3x3

7



A closer look at spatial dimensions



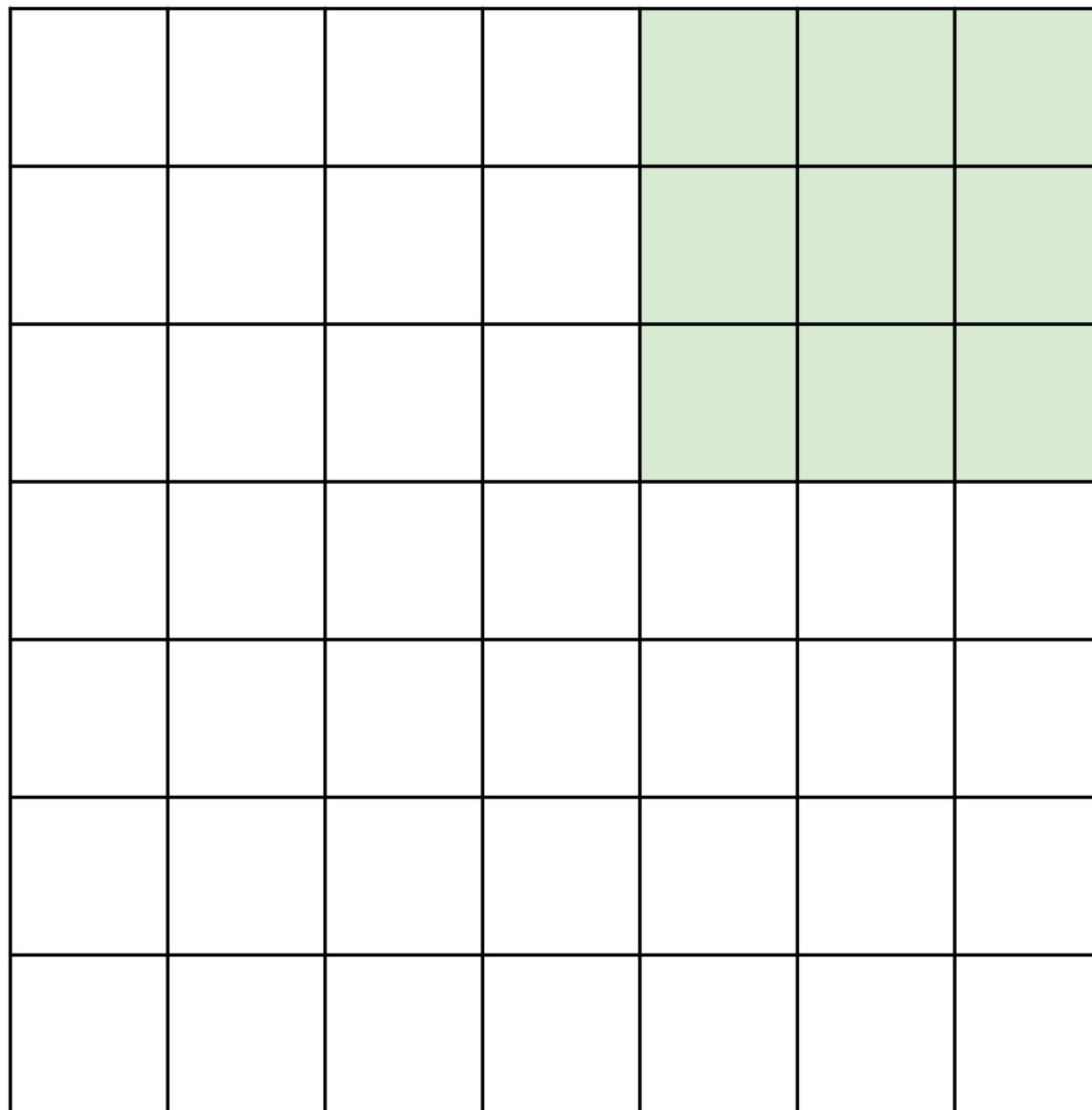
7

7

Input: 7x7
Filter: 3x3



A closer look at spatial dimensions



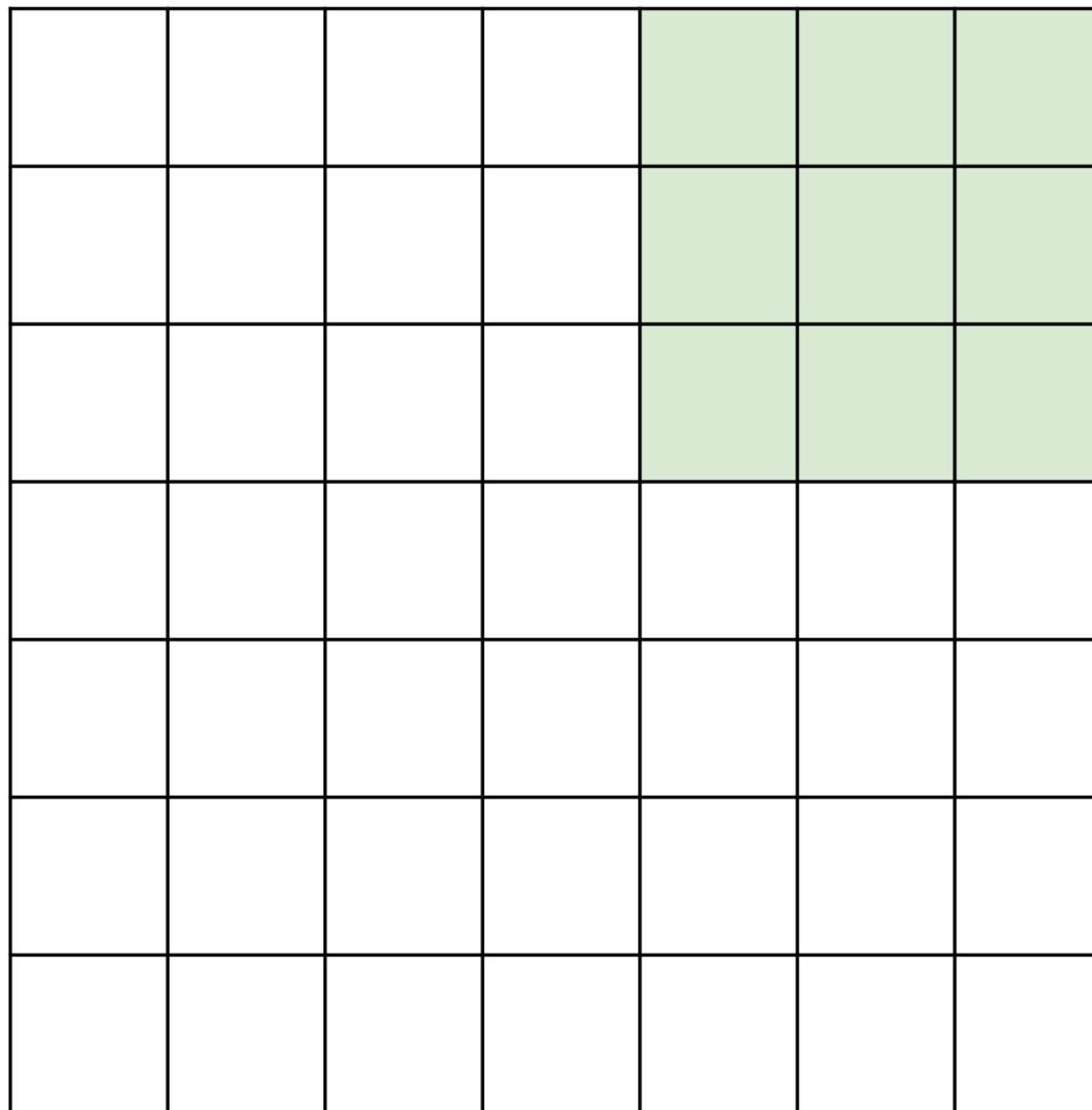
7

7

Input: 7x7
Filter: 3x3
Output: 5x5



A closer look at spatial dimensions



7

Input: 7x7
Filter: 3x3
Output: 5x5

In general: **Problem:** Feature
Input: W maps “shrink”
Filter: K with each layer!
Output: $W - K + 1$



A closer look at spatial dimensions

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Input: 7x7

Filter: 3x3

Output: 5x5

In general:

Input: W

Filter: K

Output: $W - K + 1$

Problem: Feature
maps “shrink”
with each layer!

Solution: padding

Add zeros around the input



A closer look at spatial dimensions

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | | | | | | | | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Input: 7x7

Filter: 3x3

Output: 5x5

In general:

Input: W

Filter: K

Padding: P

Output: $W - K + 1 + 2P$

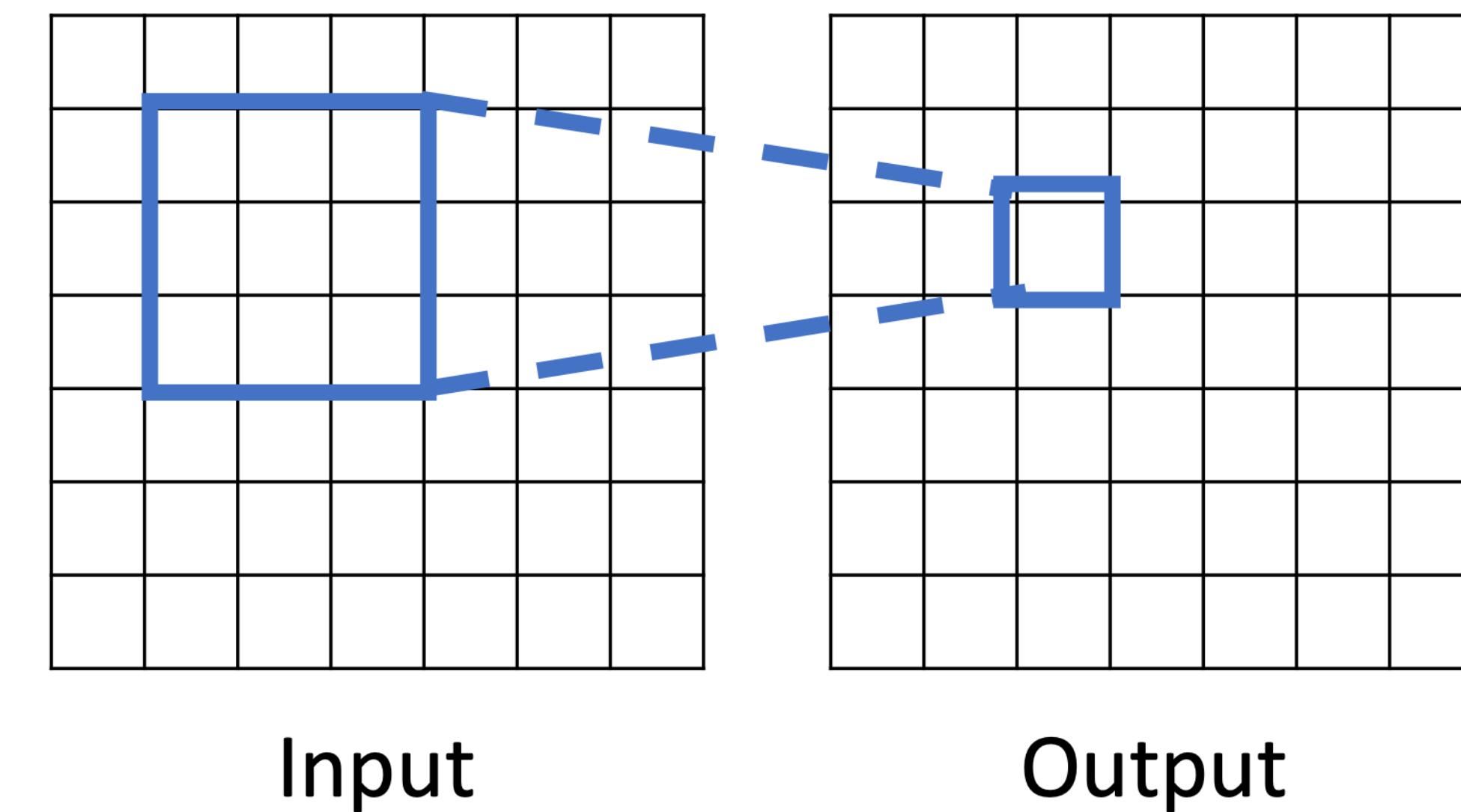
Very common:

Set $P = (K - 1) / 2$ to
make output have
same size as input!



Receptive Fields

For convolution with kernel size K, each element in the output depends on a $K \times K$ **receptive field** in the input



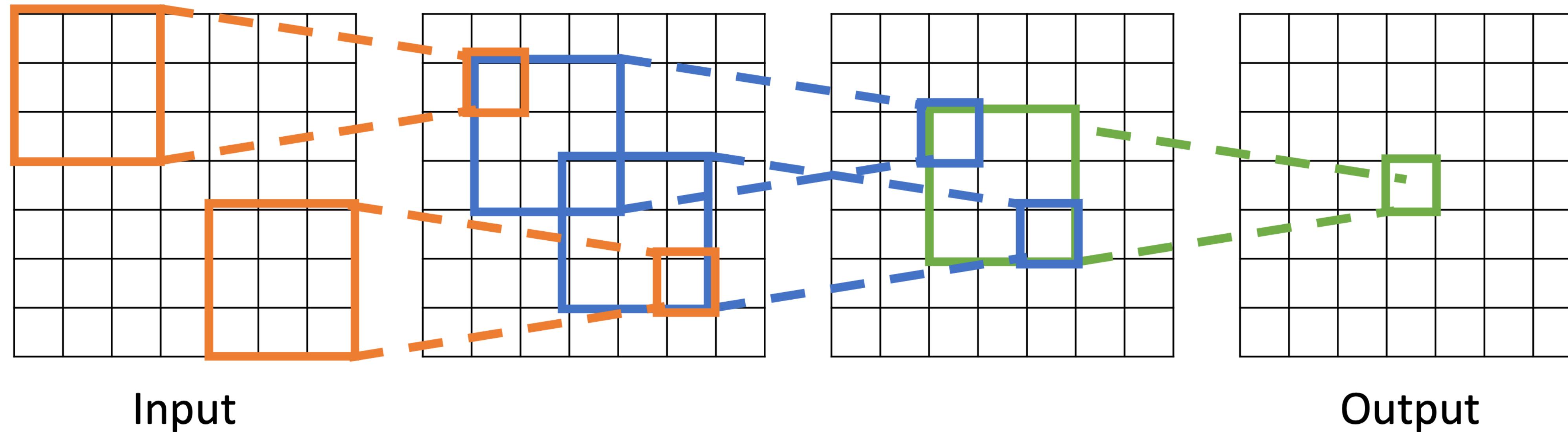
Input

Output



Receptive Fields

Each successive convolution adds $K - 1$ to the receptive field size
With L layers the receptive field size is $1 + L * (K - 1)$

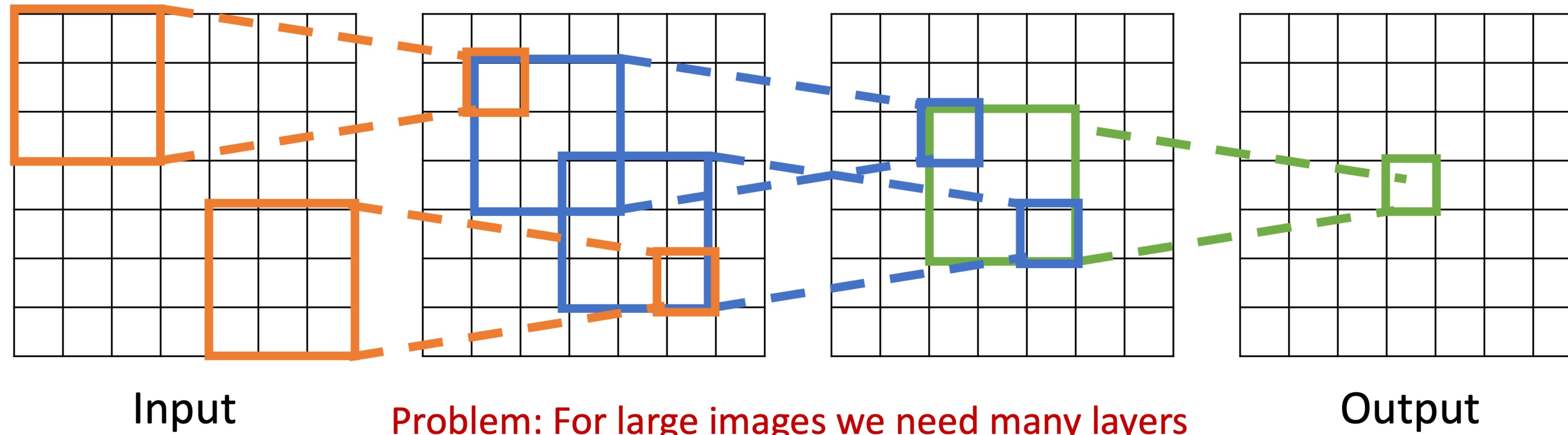


Be careful – “receptive field in the input” vs “receptive field in the previous layer”
Hopefully clear from context!



Receptive Fields

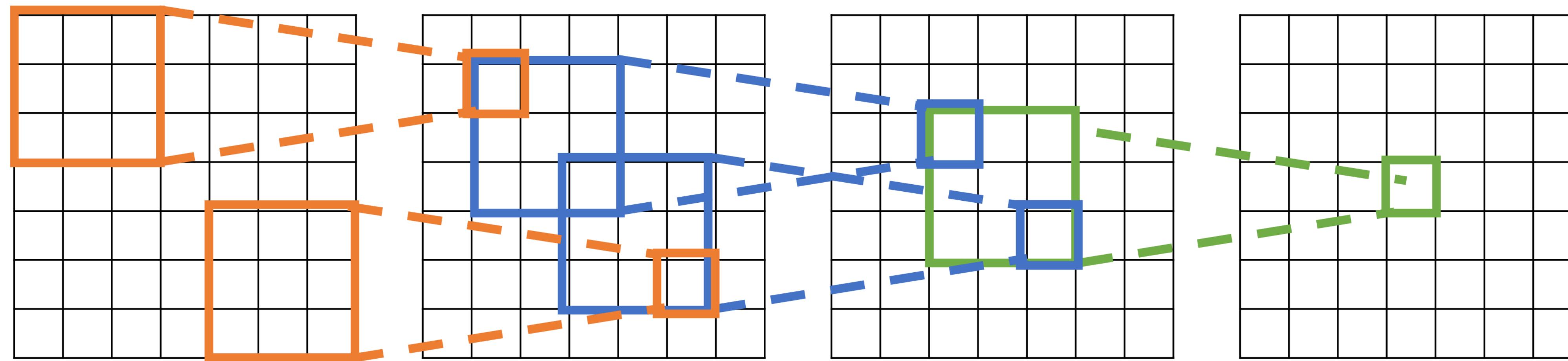
Each successive convolution adds $K - 1$ to the receptive field size
With L layers the receptive field size is $1 + L * (K - 1)$





Receptive Fields

Each successive convolution adds $K - 1$ to the receptive field size
With L layers the receptive field size is $1 + L * (K - 1)$



Input

Problem: For large images we need many layers
for each output to “see” the whole image

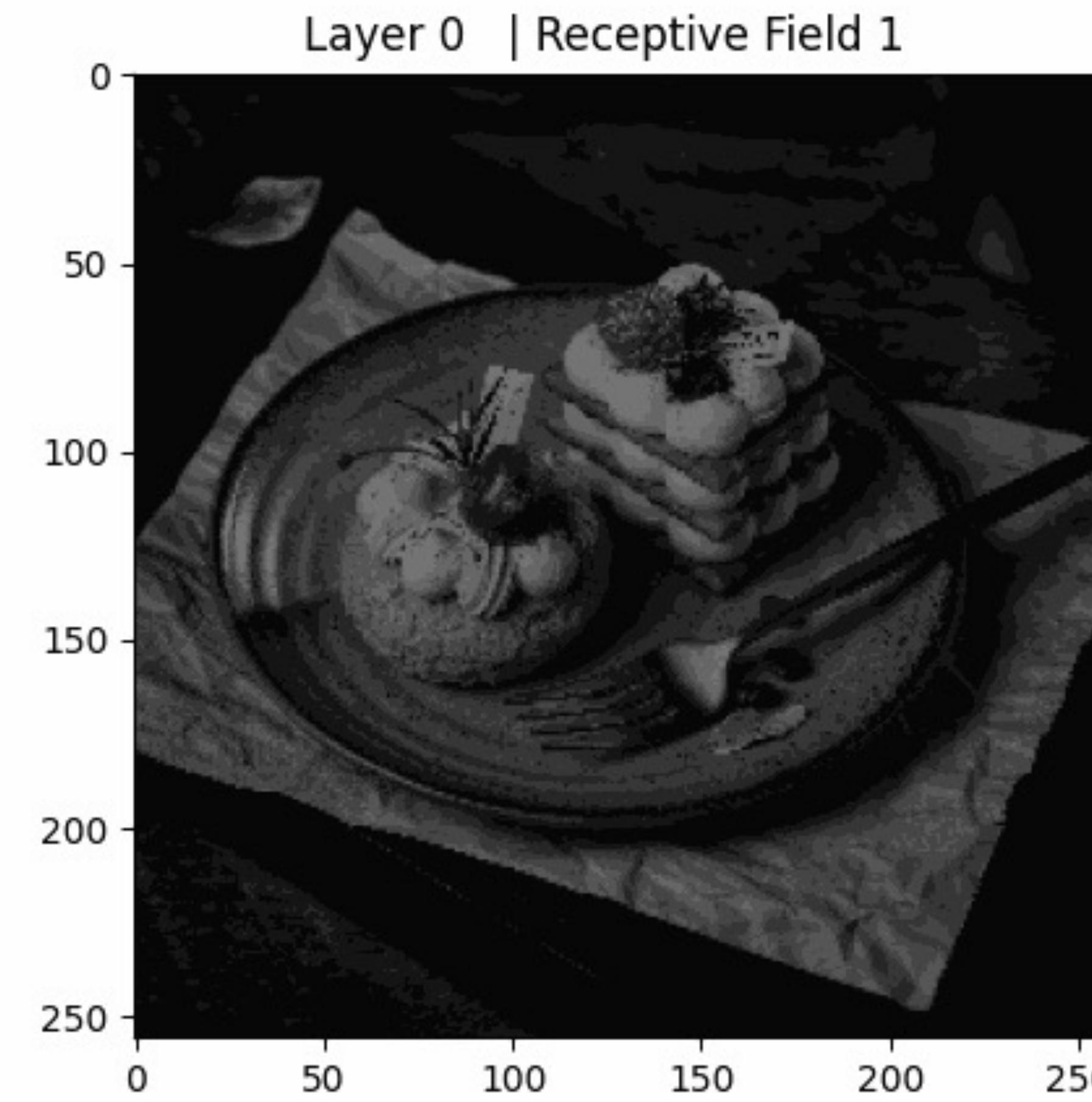
Output

Solution: Downsample inside the network



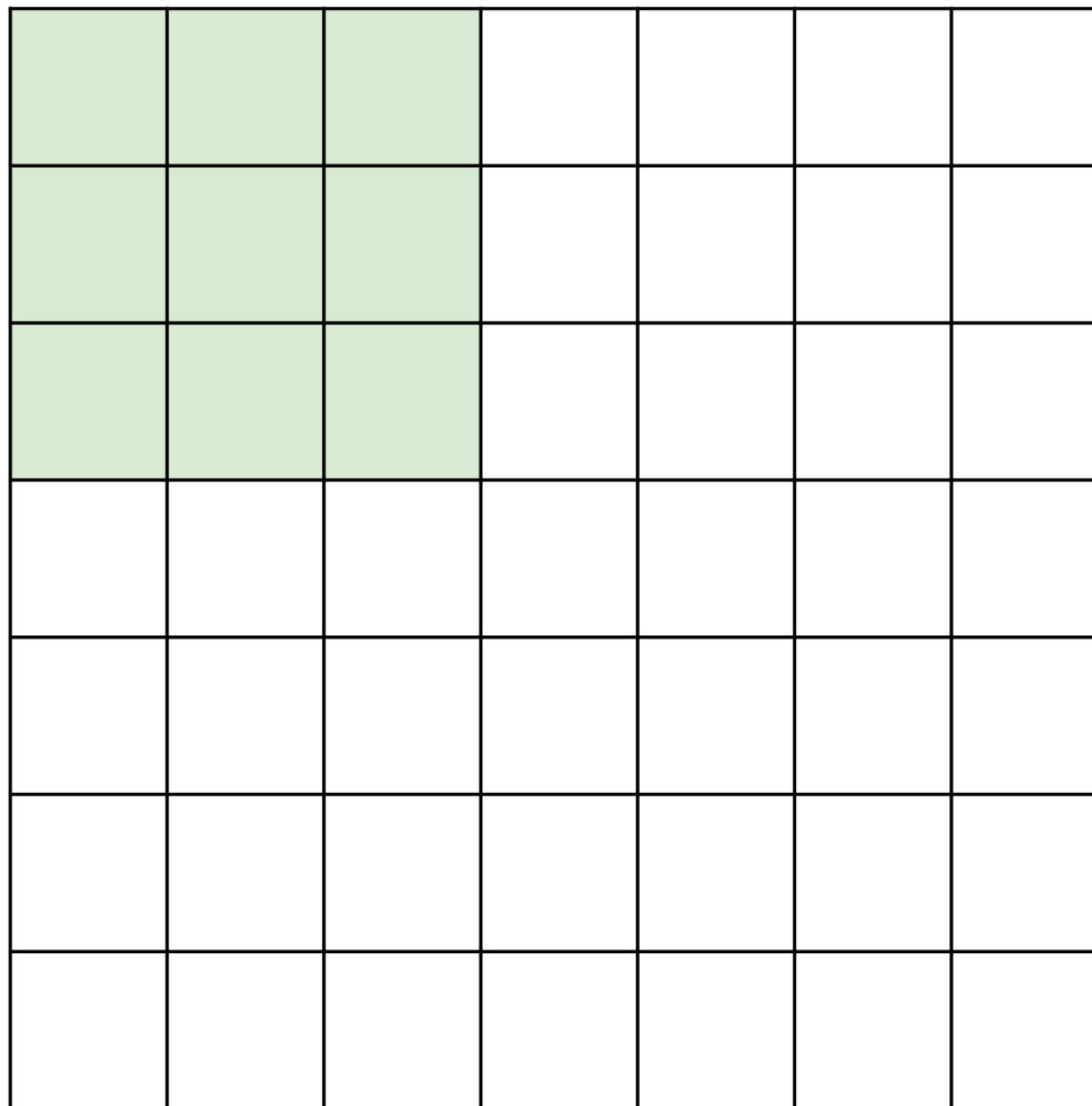
Receptive Fields

<https://github.com/Fangyh09/pytorch-receptive-field>





Strided Convolution



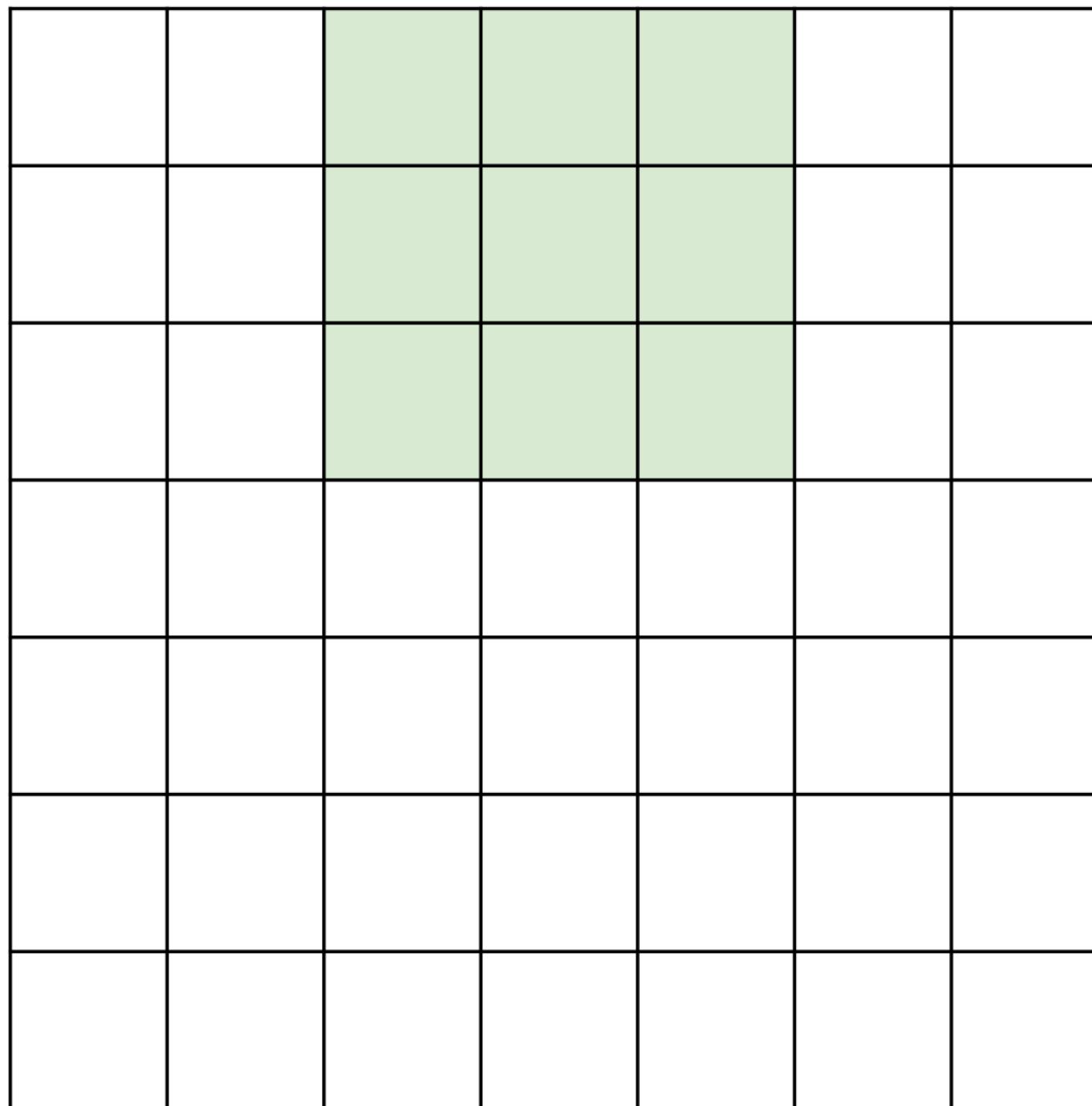
Input: 7x7

Filter: 3x3

Stride: 2



Strided Convolution



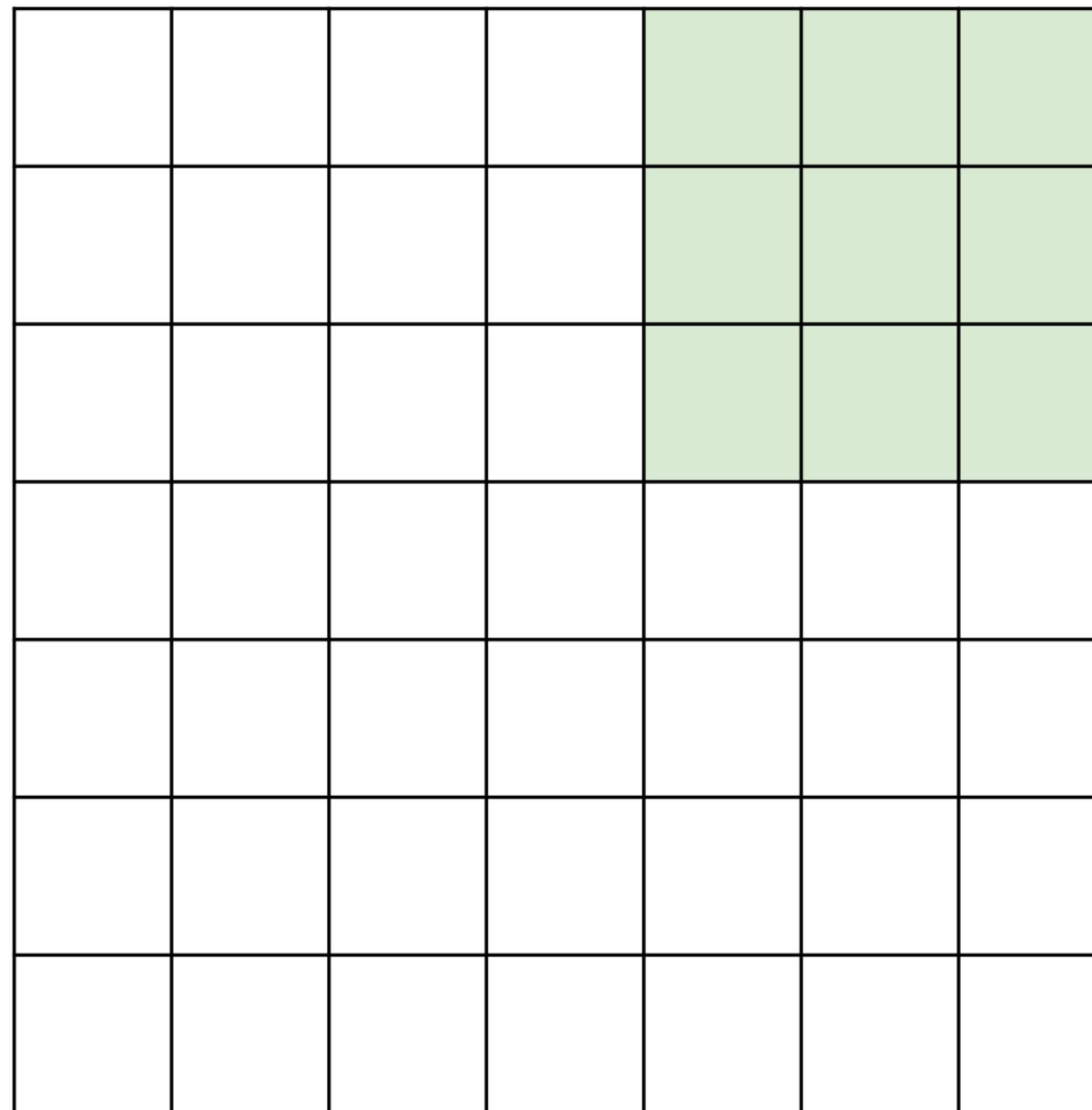
Input: 7x7

Filter: 3x3

Stride: 2



Strided Convolution



Input: 7x7

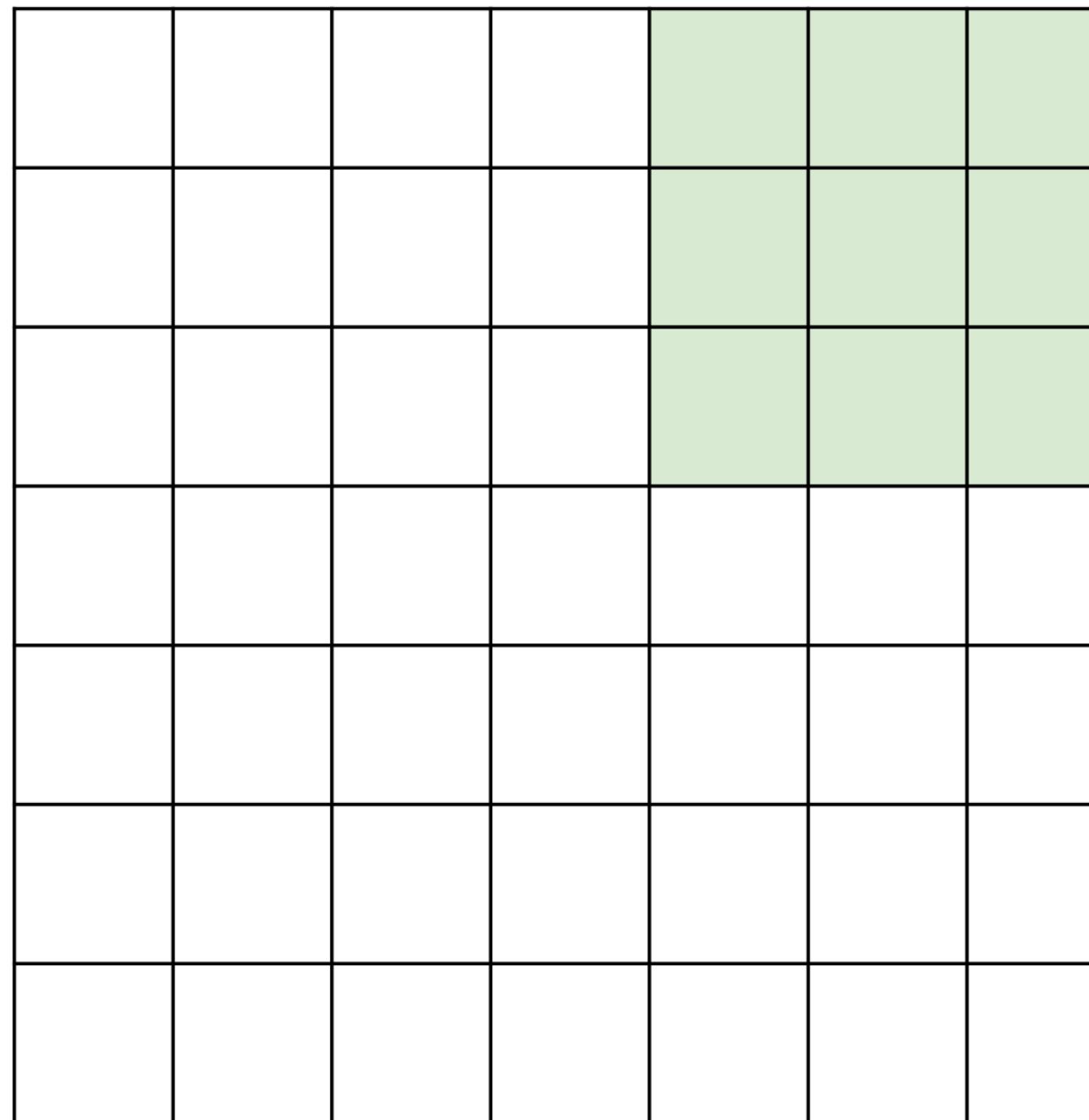
Filter: 3x3

Stride: 2

Output: 3x3



Strided Convolution



Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

In general:

Input: W

Filter: K

Padding: P

Stride: S

Output: $(W - K + 2P) / S + 1$



Dilated Convolution

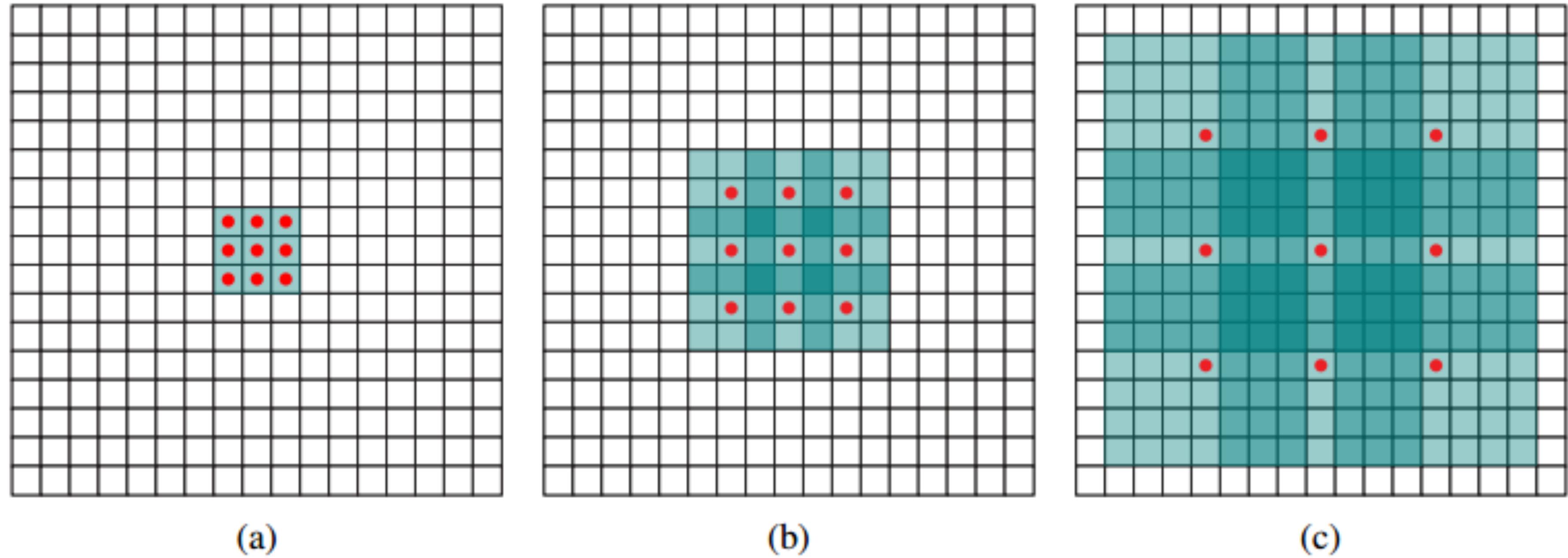
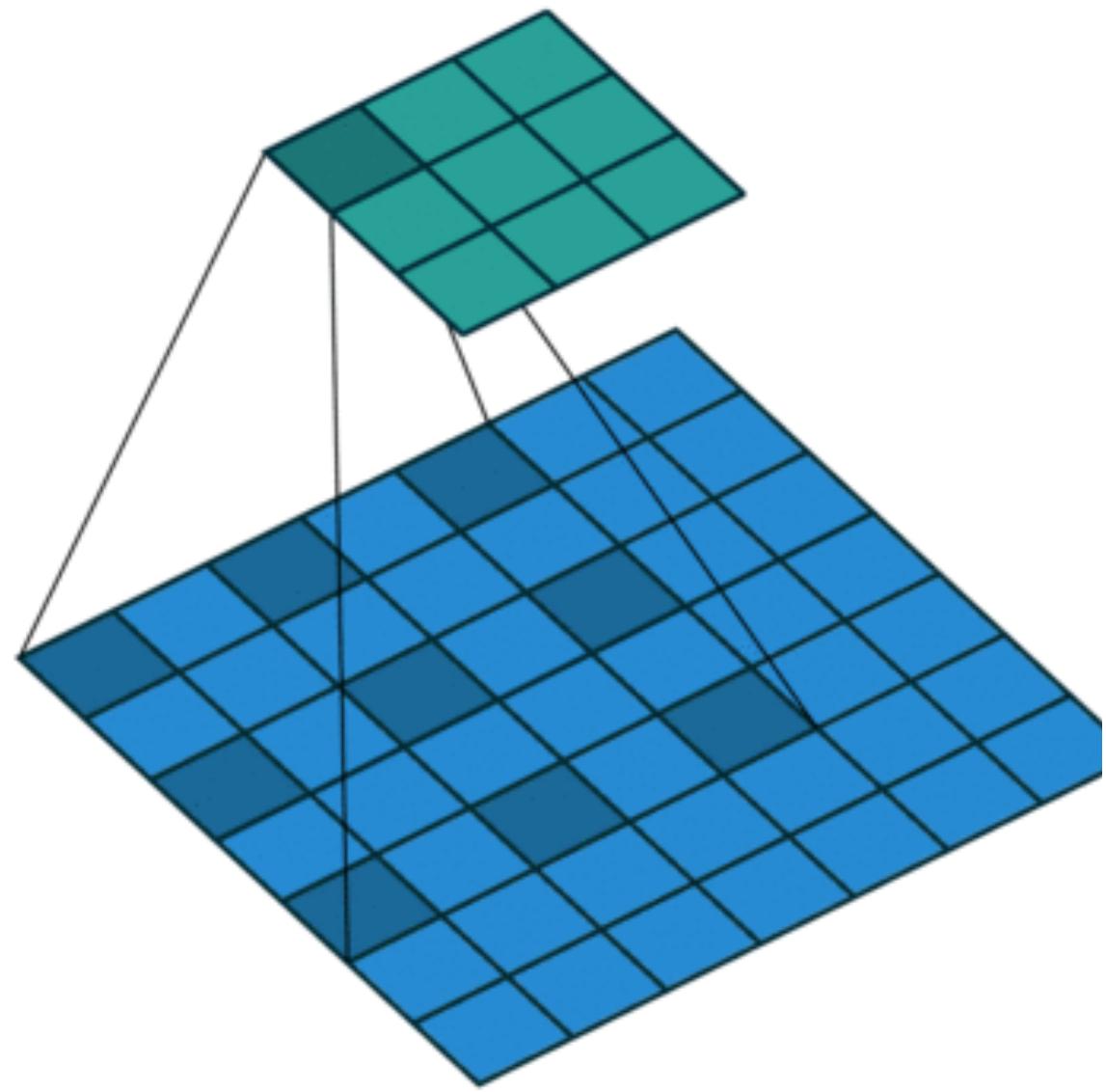


Figure 1: Systematic dilation supports exponential expansion of the receptive field without loss of resolution or coverage. (a) F_1 is produced from F_0 by a 1-dilated convolution; each element in F_1 has a receptive field of 3×3 . (b) F_2 is produced from F_1 by a 2-dilated convolution; each element in F_2 has a receptive field of 7×7 . (c) F_3 is produced from F_2 by a 4-dilated convolution; each element in F_3 has a receptive field of 15×15 . The number of parameters associated with each layer is identical. The receptive field grows exponentially while the number of parameters grows linearly.

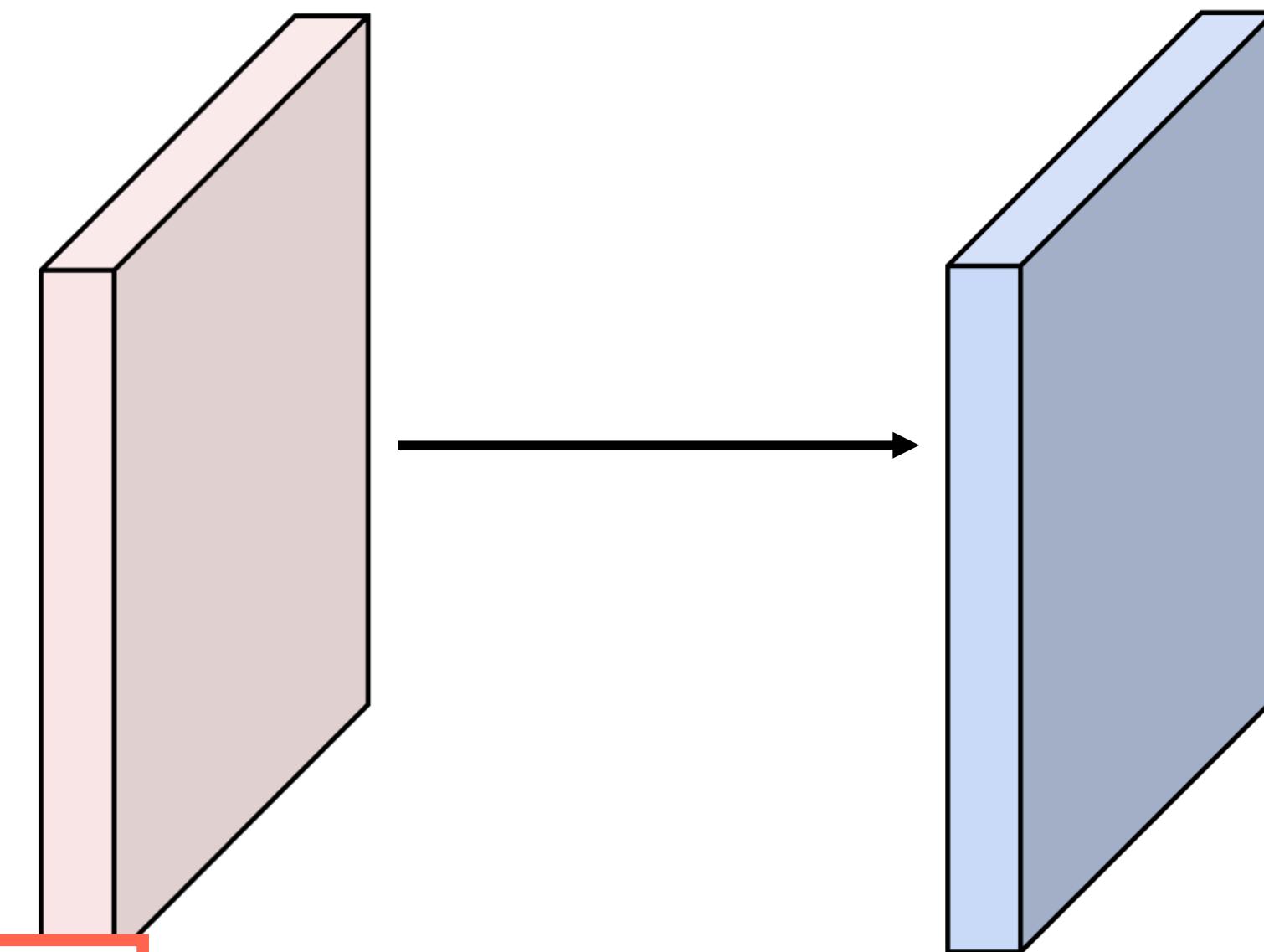


Convolution Example

Input volume: $3 \times 32 \times 32$

10 5x5 filters with stride 1, pad 2

Q: What is the output volume size?



Input: W
Filter: K
Padding: P
Stride: S
Output: $(W - K + 2P) / S + 1$



Convolution Example

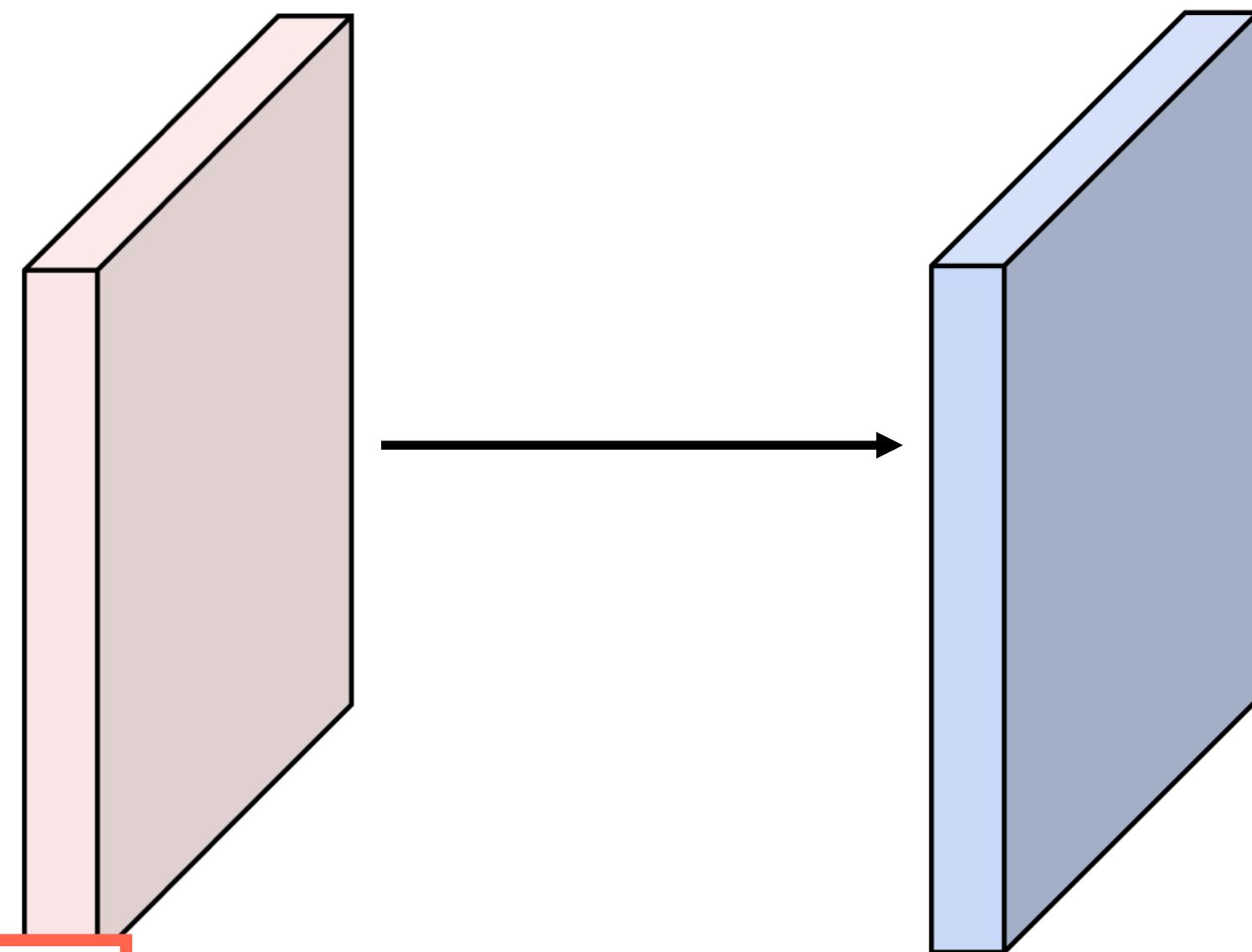
Input volume: $3 \times 32 \times 32$

10 5x5 filters with stride 1, pad 2

Q: What is the output volume size?

$$(32 - 5 + 2 * 2) / 1 + 1 = 32 \text{ spatially}$$

So, 10 x 32 x 32 output



Input: W
Filter: K
Padding: P
Stride: S
Output: $(W - K + 2P) / S + 1$



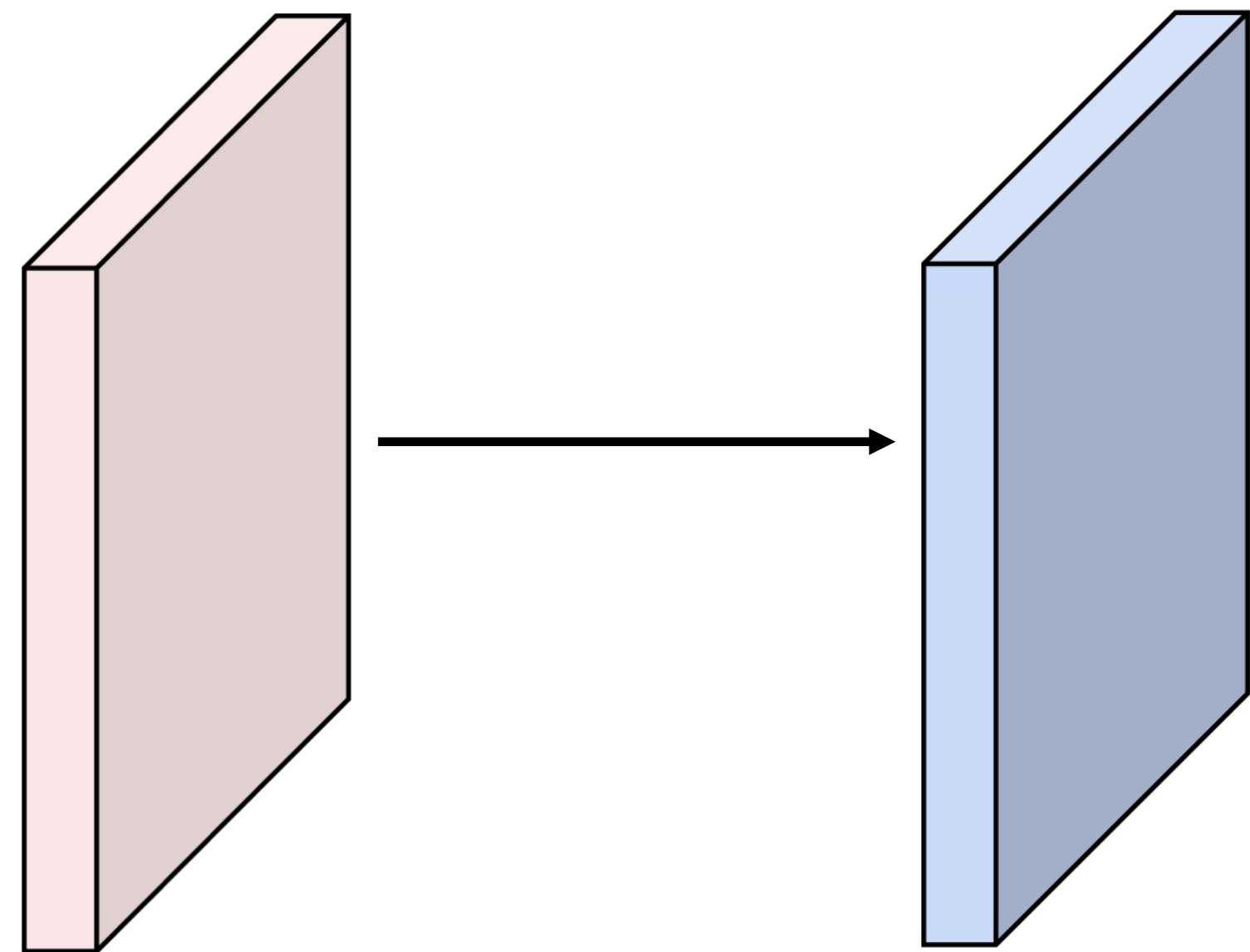
Convolution Example

Input volume: $3 \times 32 \times 32$

10 5x5 filters with stride 1, pad 2

Output volume size: $10 \times 32 \times 32$

Q: What is the number of learnable parameters?





Convolution Example

Input volume: $3 \times 32 \times 32$

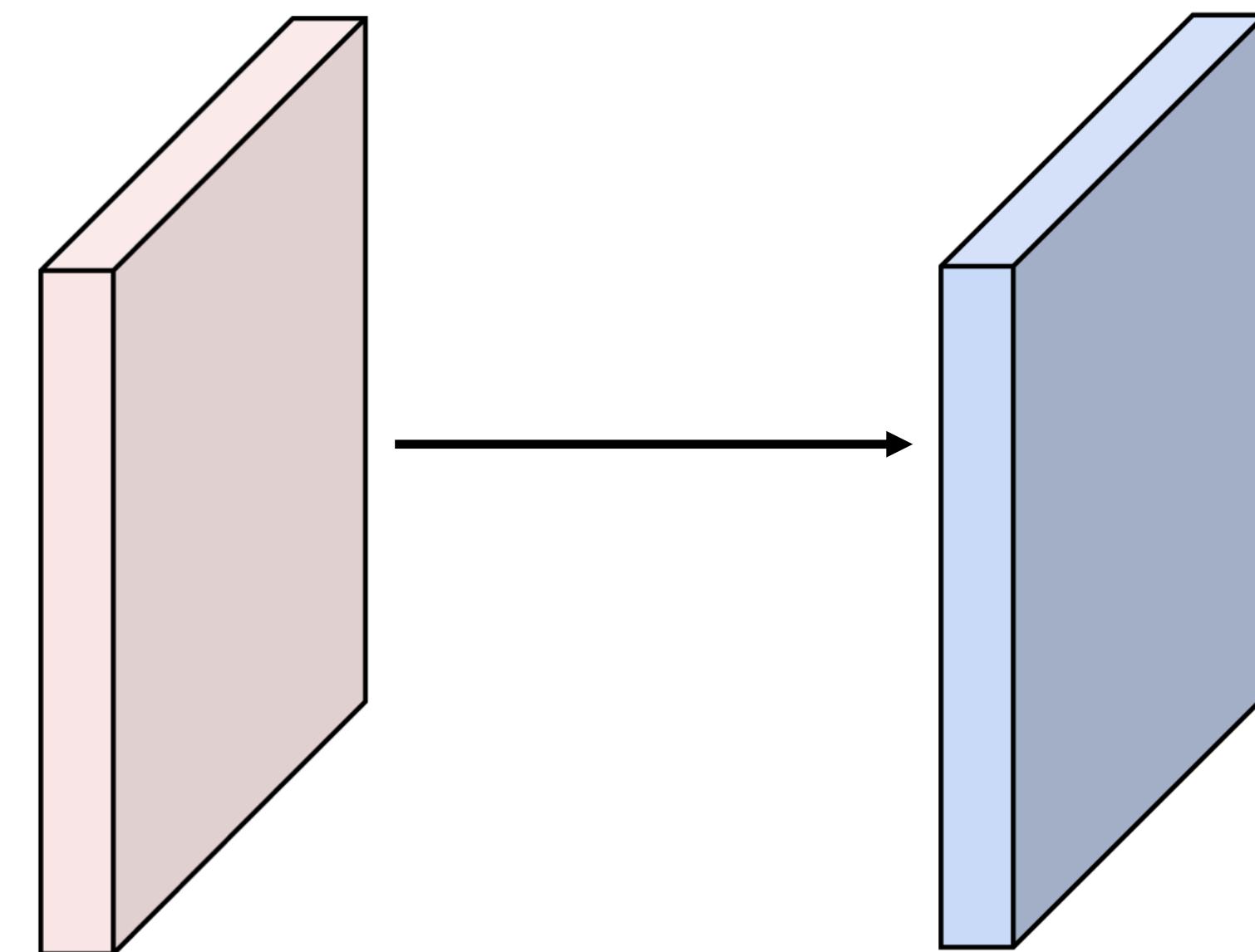
10 5x5 filters with stride 1, pad 2

Output volume size: $10 \times 32 \times 32$

Q: What is the number of learnable parameters?

Parameters per filter: $(3 \times 5 \times 5) + 1 = 76$

10 filters, so total is $10 \times 76 = 760$





Convolution Example

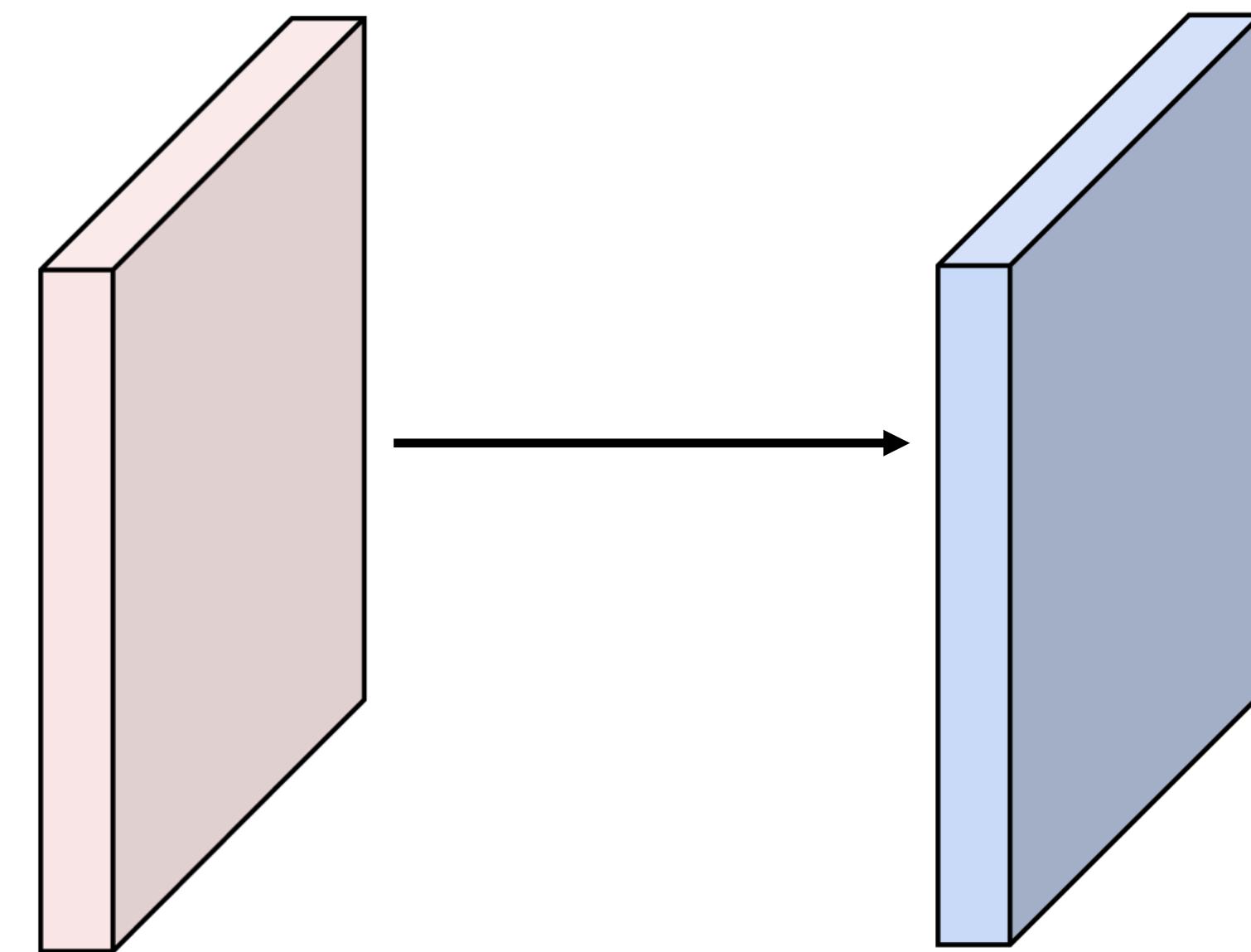
Input volume: $3 \times 32 \times 32$

10 5x5 filters with stride 1, pad 2

Output volume size: $10 \times 32 \times 32$

Number of learnable parameters: 760

Q: What is the number of multiply-add operations?





Convolution Example

Input volume: $3 \times 32 \times 32$

10 5×5 filters with stride 1, pad 2

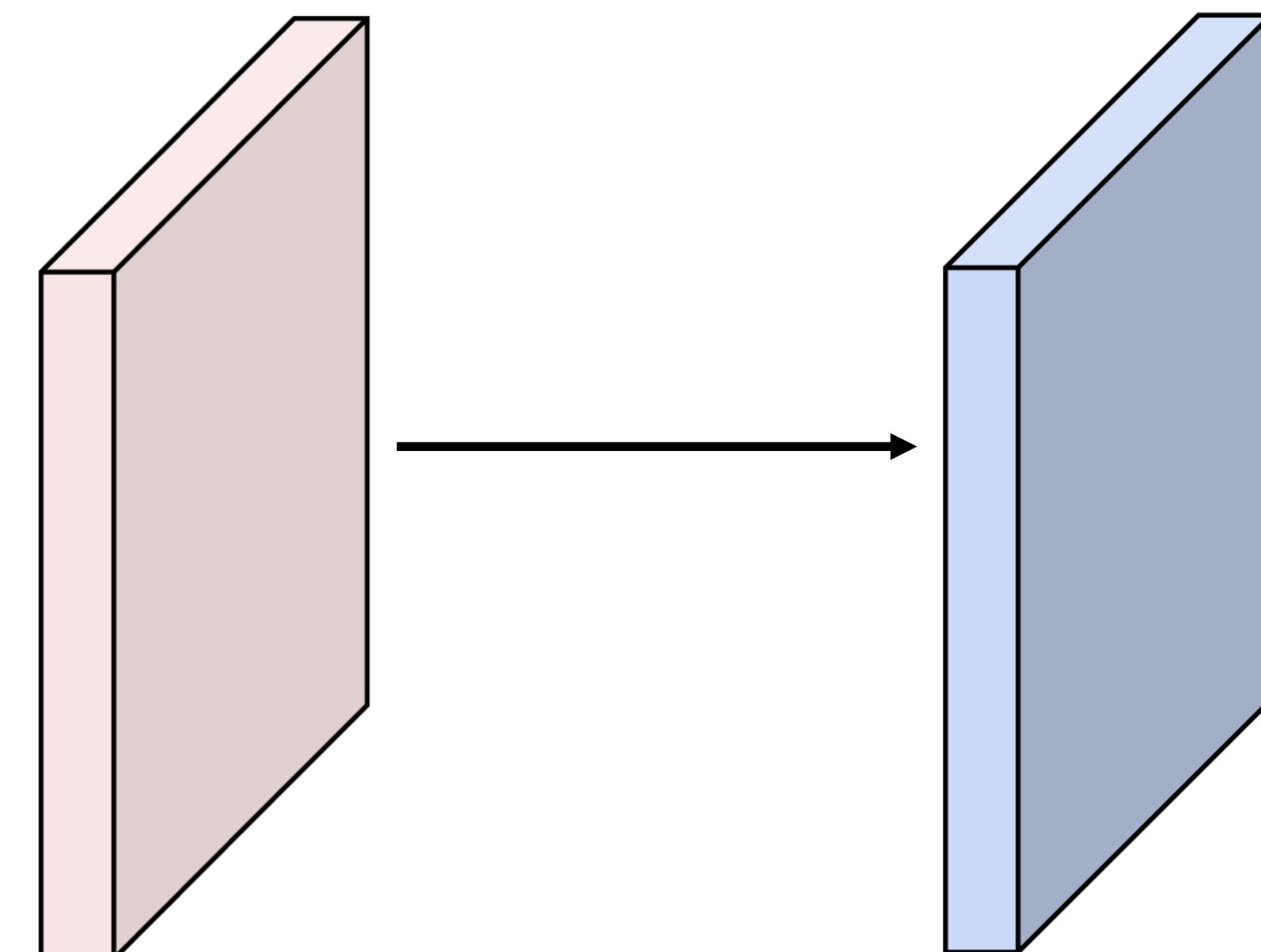
Output volume size: $10 \times 32 \times 32$

Number of learnable parameters: 760

Q: What is the number of multiply-add operations?

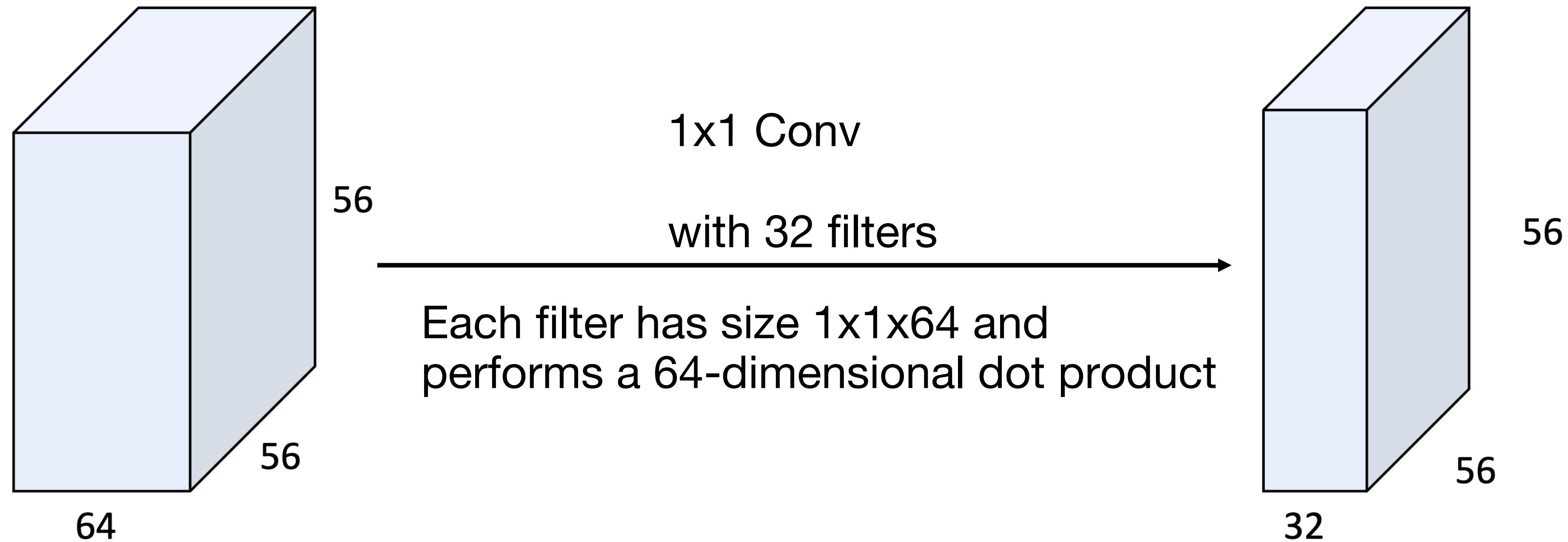
$10 \times 32 \times 32 = 10,240$ outputs, each from inner product

of two $3 \times 5 \times 5$ tensors, so total = $75 * 10,240 = \underline{\underline{768,000}}$



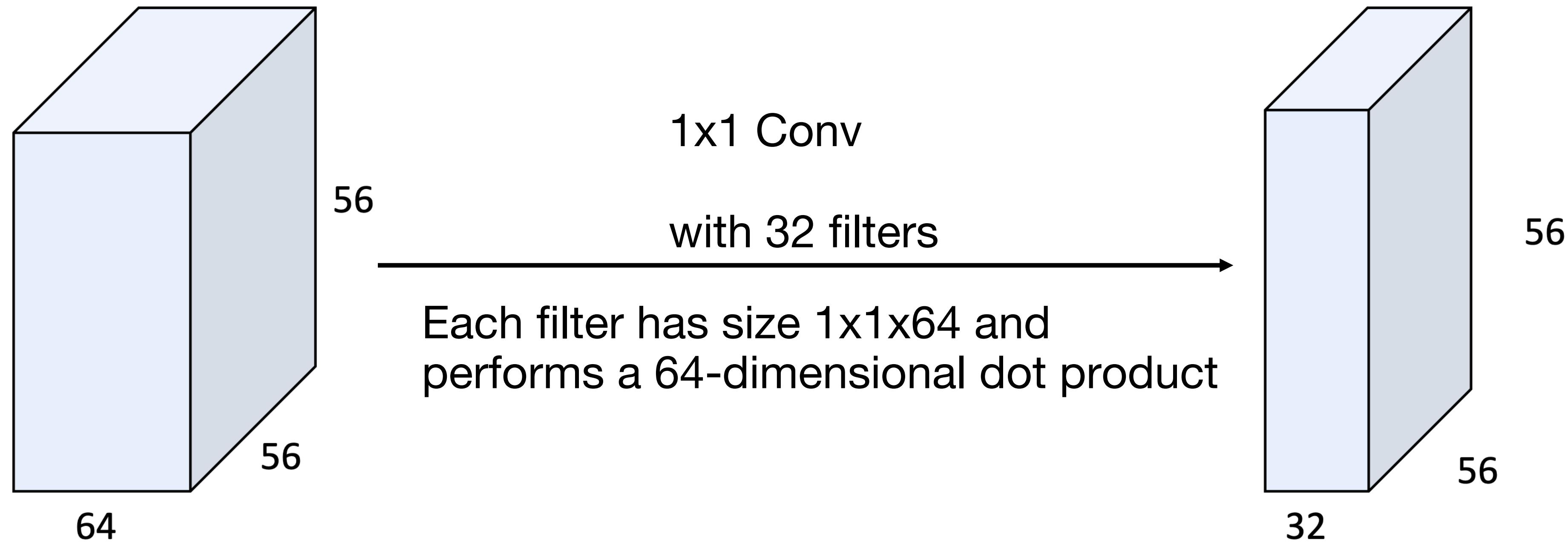


Example: 1x1 Convolution





Example: 1x1 Convolution



Stacking 1x1 conv layers gives MLP
operating on each input position



Convolution Summary

Input: $C_{in} \times H \times W$

Hyperparameters:

- **Kernel size:** $K_H \times K_W$
- **Number filters:** C_{out}
- **Padding:** P
- **Stride:** S

Weight matrix: $C_{out} \times C_{in} \times K_H \times K_W$

giving C_{out} filters of size $C_{in} \times K_H \times K_W$

Bias vector: C_{out}

Output size: $C_{out} \times H' \times W'$ where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$



Convolution Summary

Input: $C_{in} \times H \times W$

Hyperparameters:

- **Kernel size:** $K_H \times K_W$
- **Number filters:** C_{out}
- **Padding:** P
- **Stride:** S

Weight matrix: $C_{out} \times C_{in} \times K_H \times K_W$
giving C_{out} filters of size $C_{in} \times K_H \times K_W$

Bias vector: C_{out}

Output size: $C_{out} \times H' \times W'$ where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$

Common settings:

$K_H = K_W$ (Small square filters)

$P = (K - 1) / 2$ ("Same" padding)

$C_{in}, C_{out} = 32, 64, 128, 256$ (powers of 2)

$K = 3, P = 1, S = 1$ (3x3 conv)

$K = 5, P = 2, S = 1$ (5x5 conv)

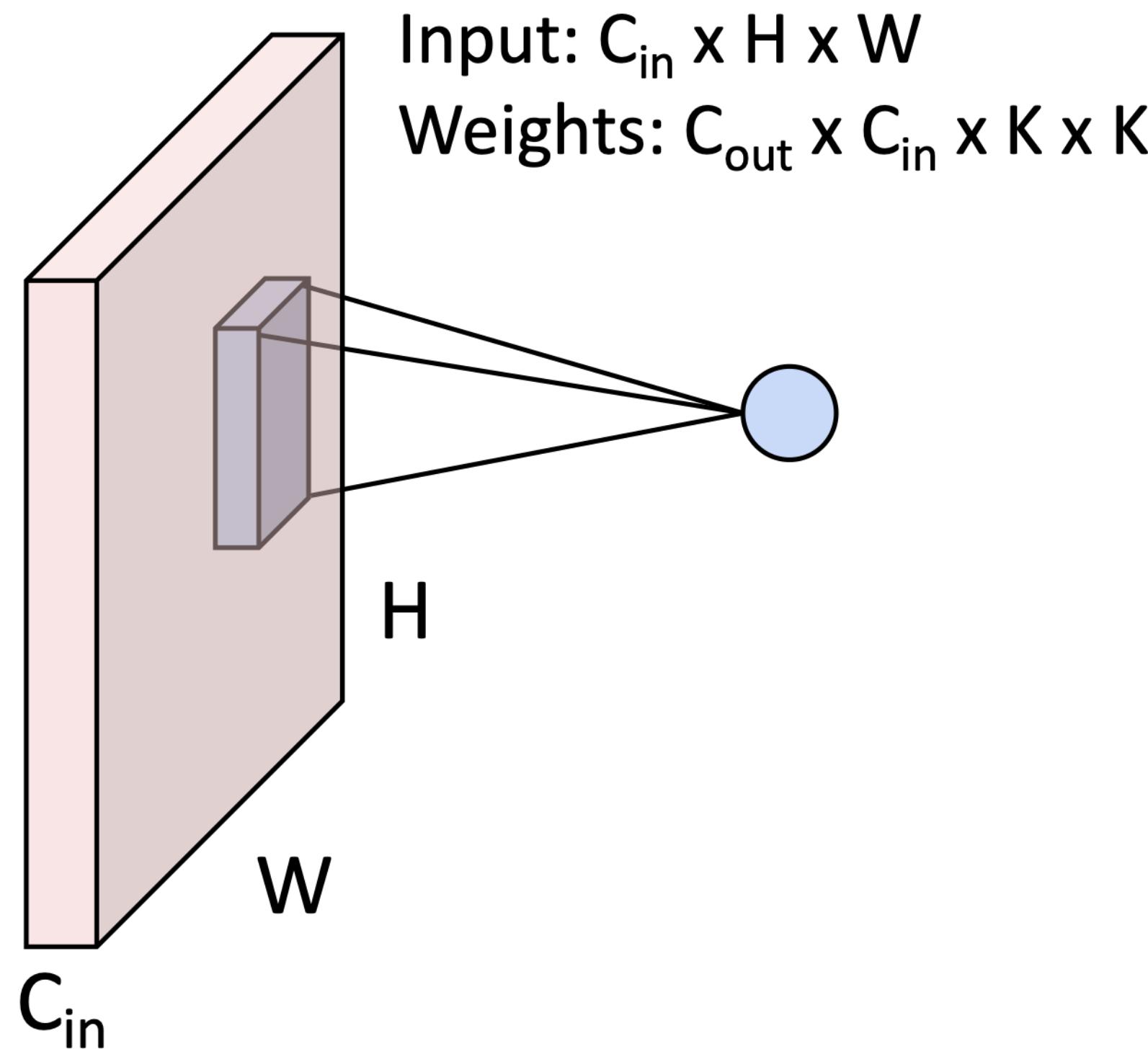
$K = 1, P = 0, S = 1$ (1x1 conv)

$K = 3, P = 1, S = 2$ (Downsample by 2)



Other types of convolution

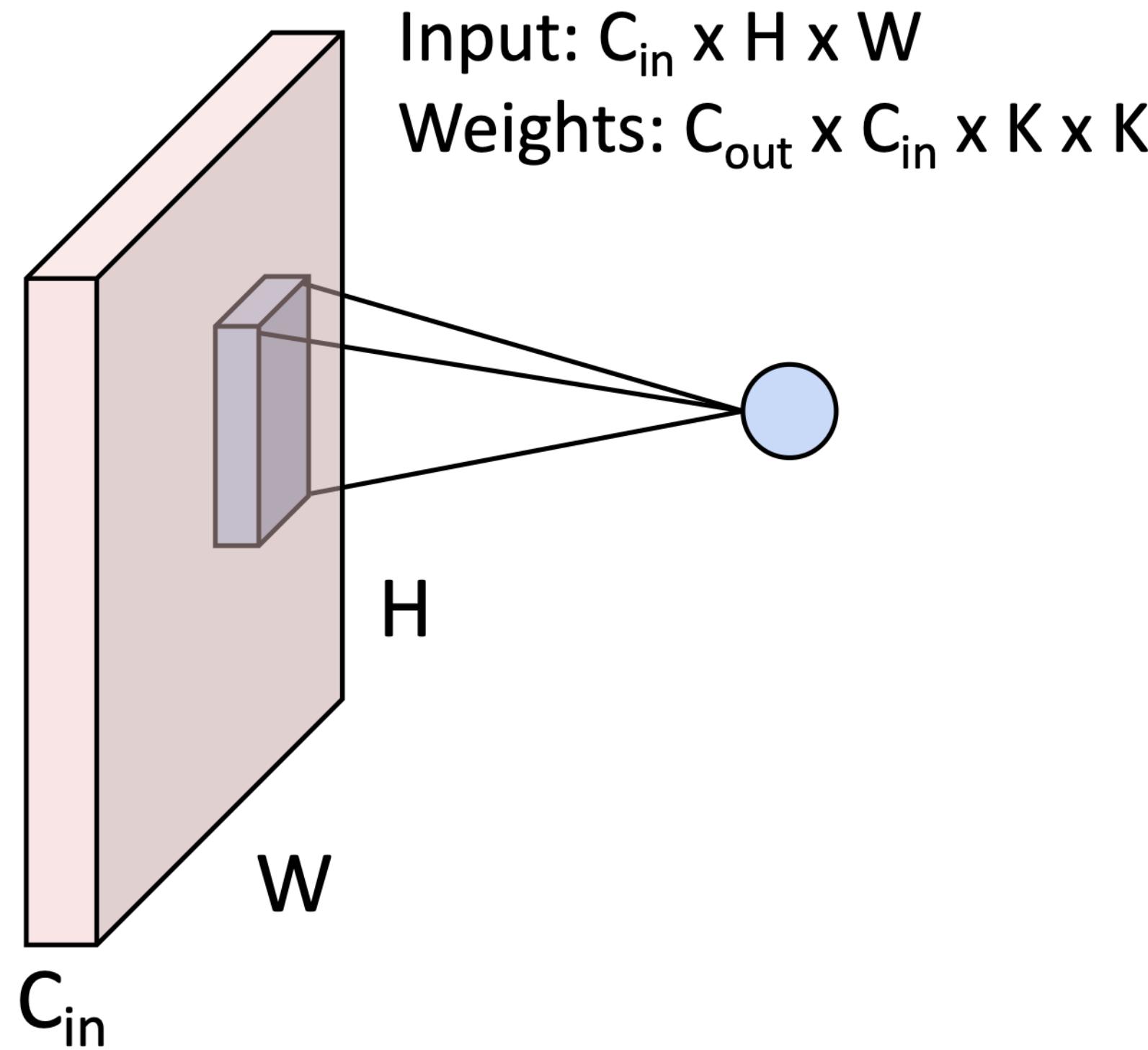
So far: 2D Convolution



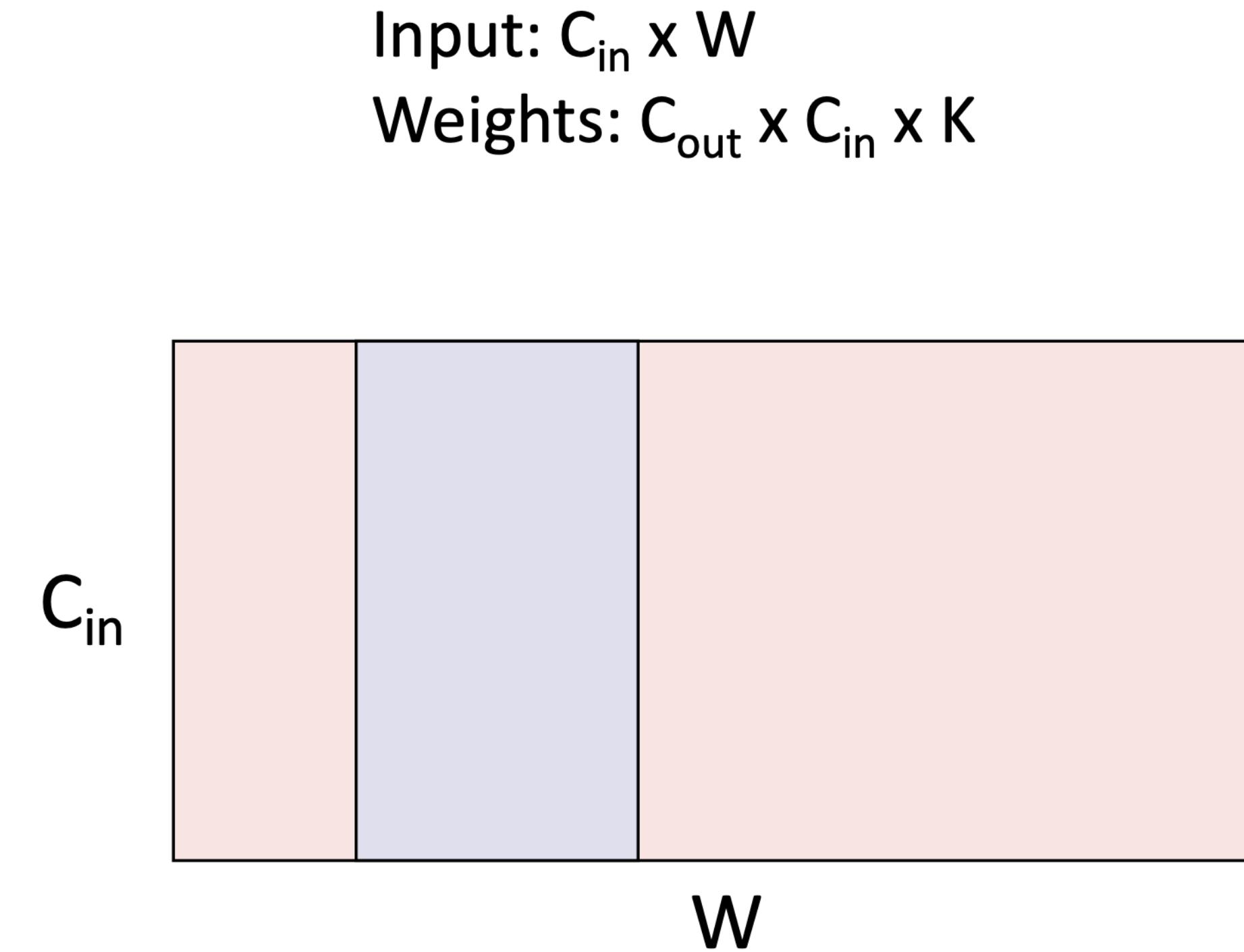


Other types of convolution

So far: 2D Convolution



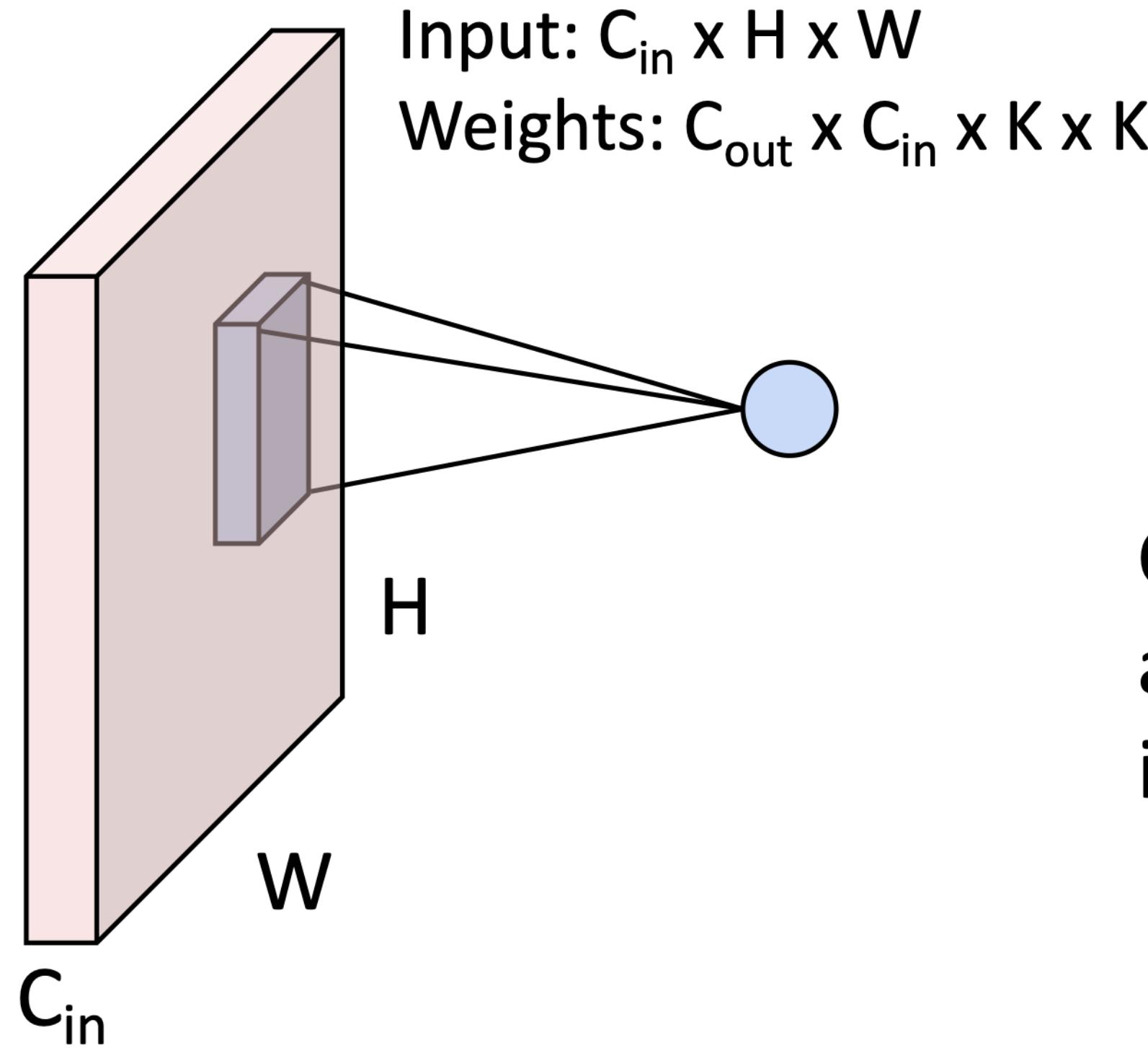
1D Convolution





Other types of convolution

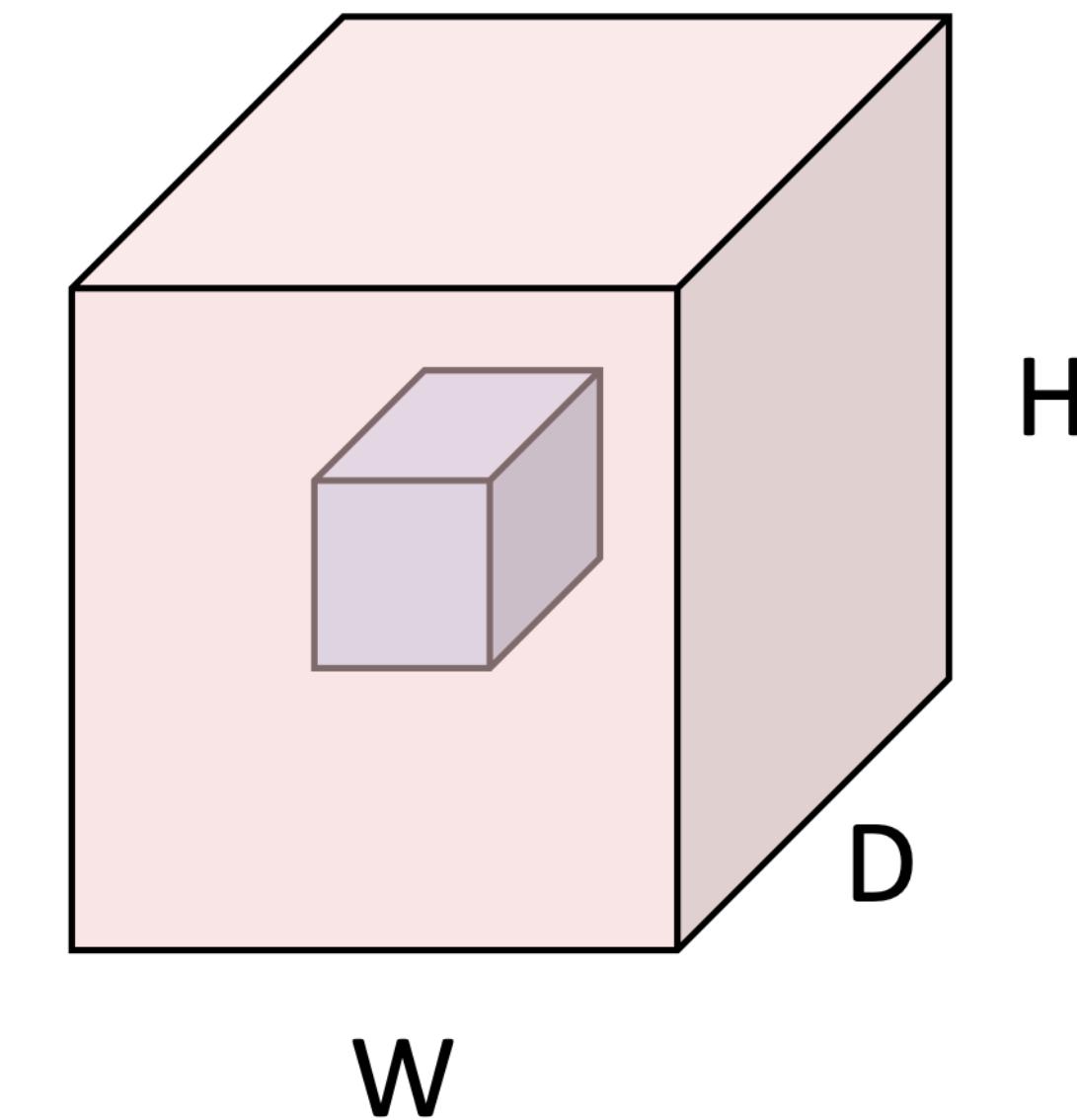
So far: 2D Convolution



C_{in} -dim vector
at each point
in the volume

3D Convolution

Input: $C_{in} \times H \times W \times D$
Weights: $C_{out} \times C_{in} \times K \times K \times K$





PyTorch Convolution Layer

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')`

[SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$



PyTorch Convolution Layer

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')`

[SOURCE]

Conv1d

CLASS `torch.nn.Conv1d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')`

[SOURCE] ↗

Conv3d

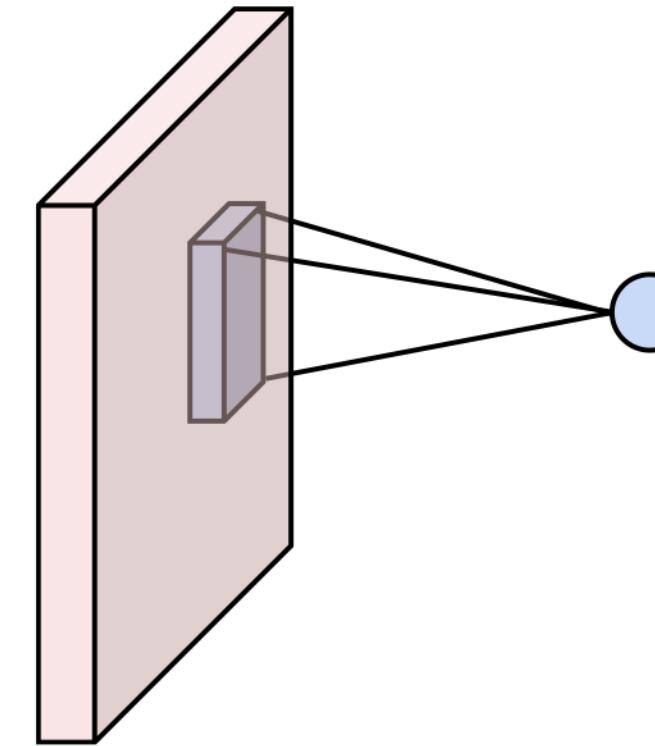
CLASS `torch.nn.Conv3d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')`

[SOURCE]

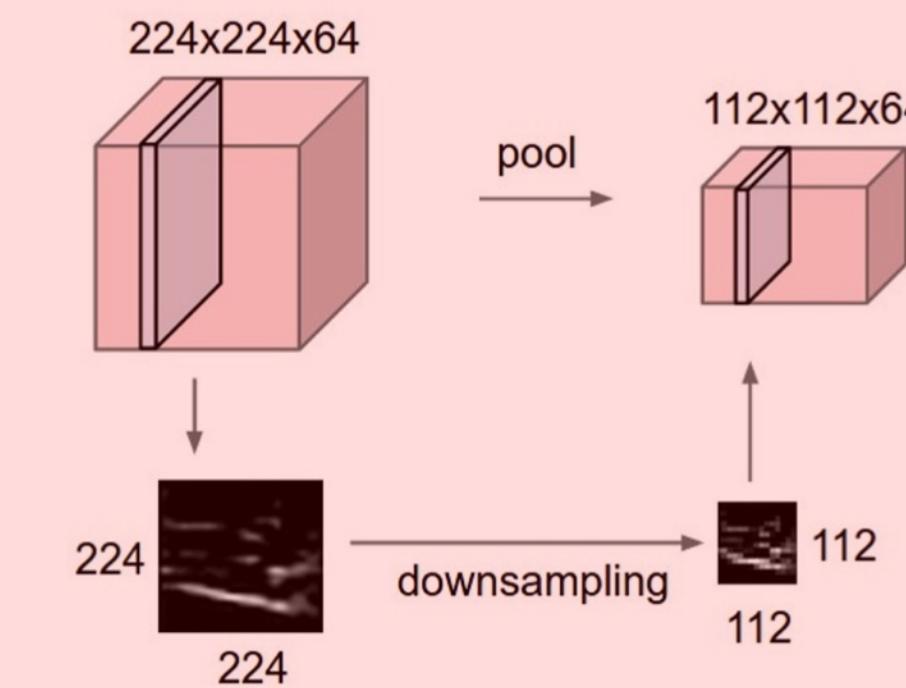


Components of Convolutional Networks

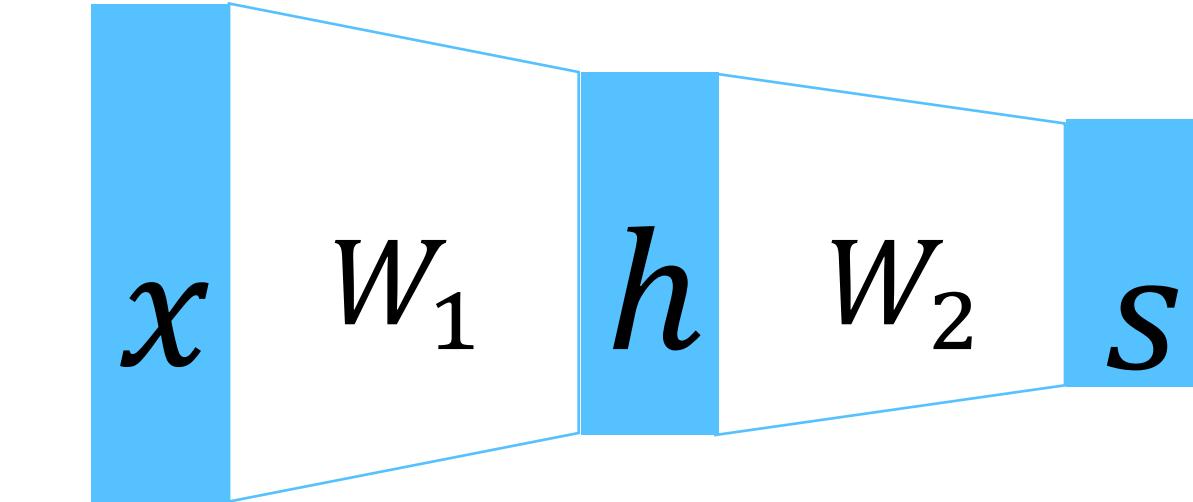
Convolution Layers



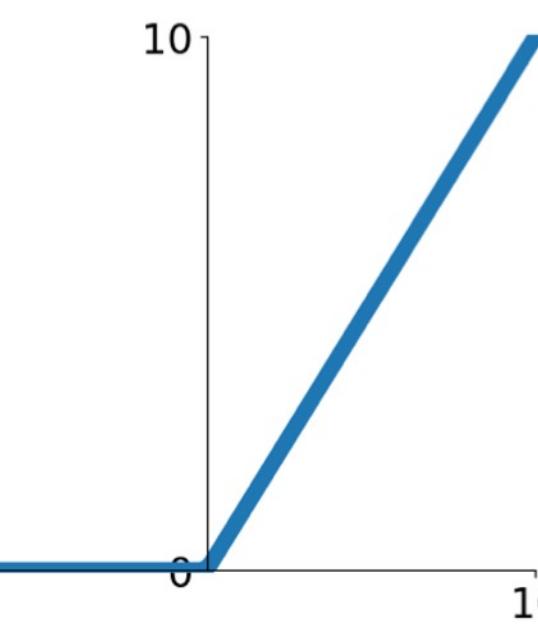
Pooling Layers



Fully-Connected Layers



Activation Function

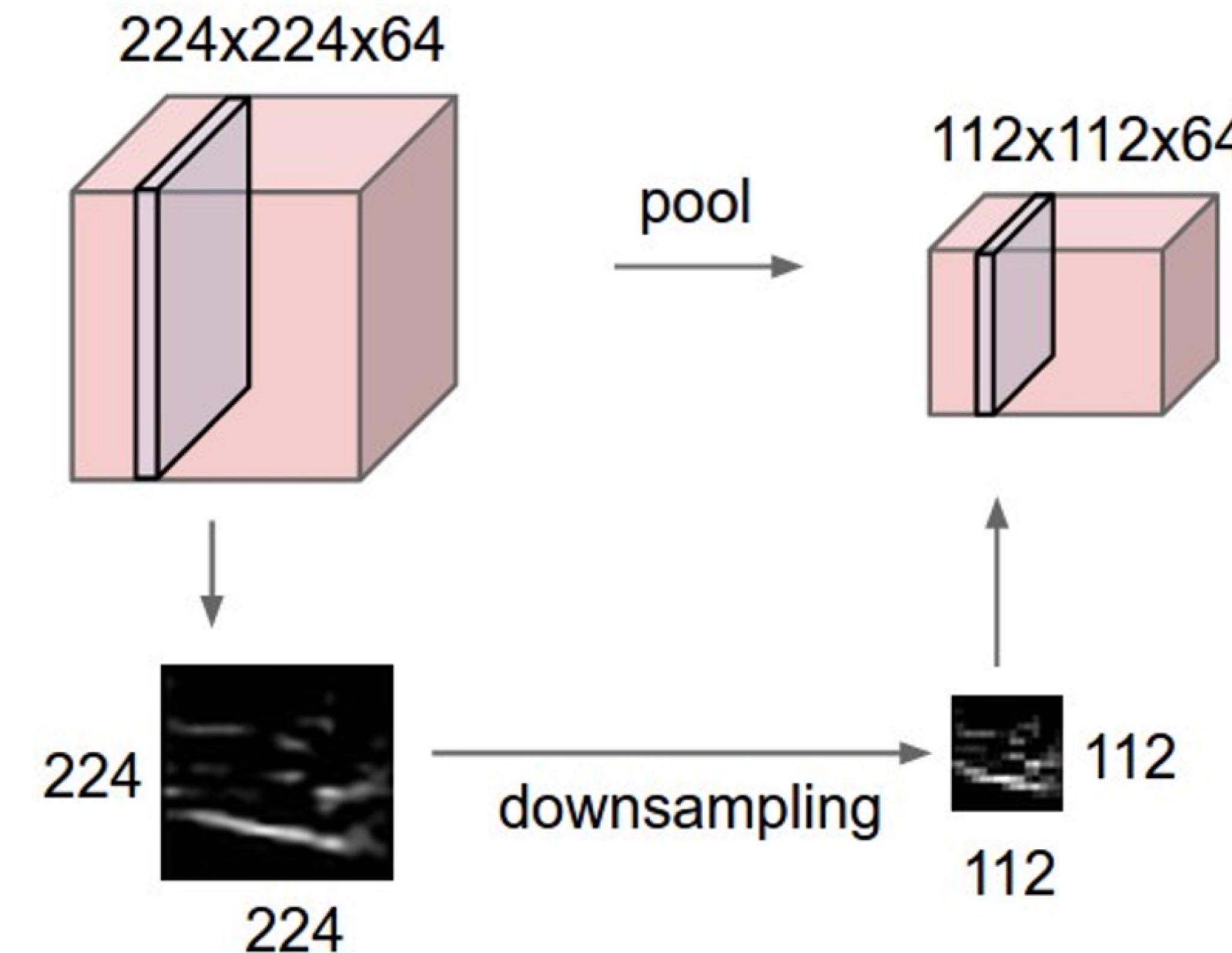


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



Pooling Layers: Another way to downsample



Hyperparameters:

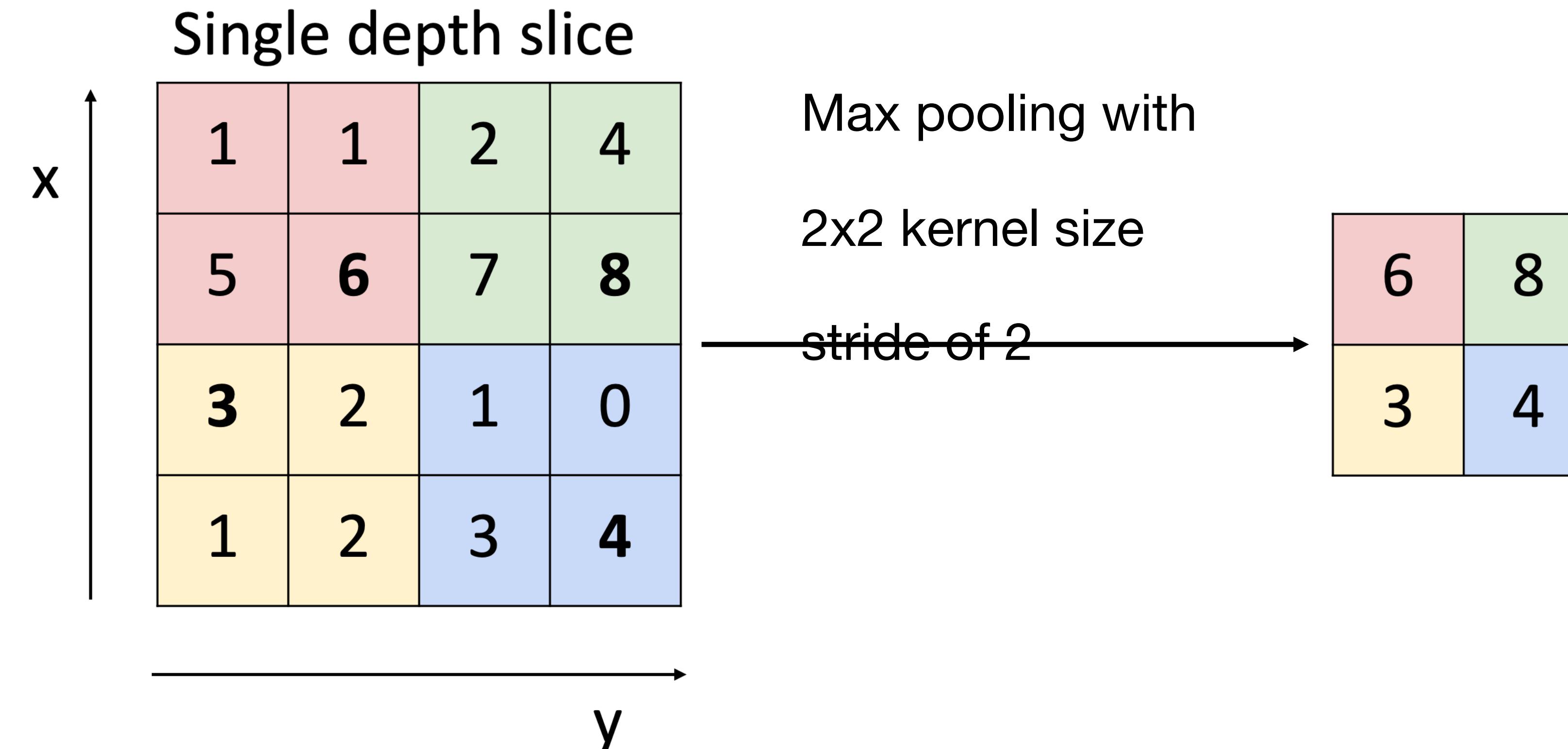
Kernel size

Stride

Pooling function

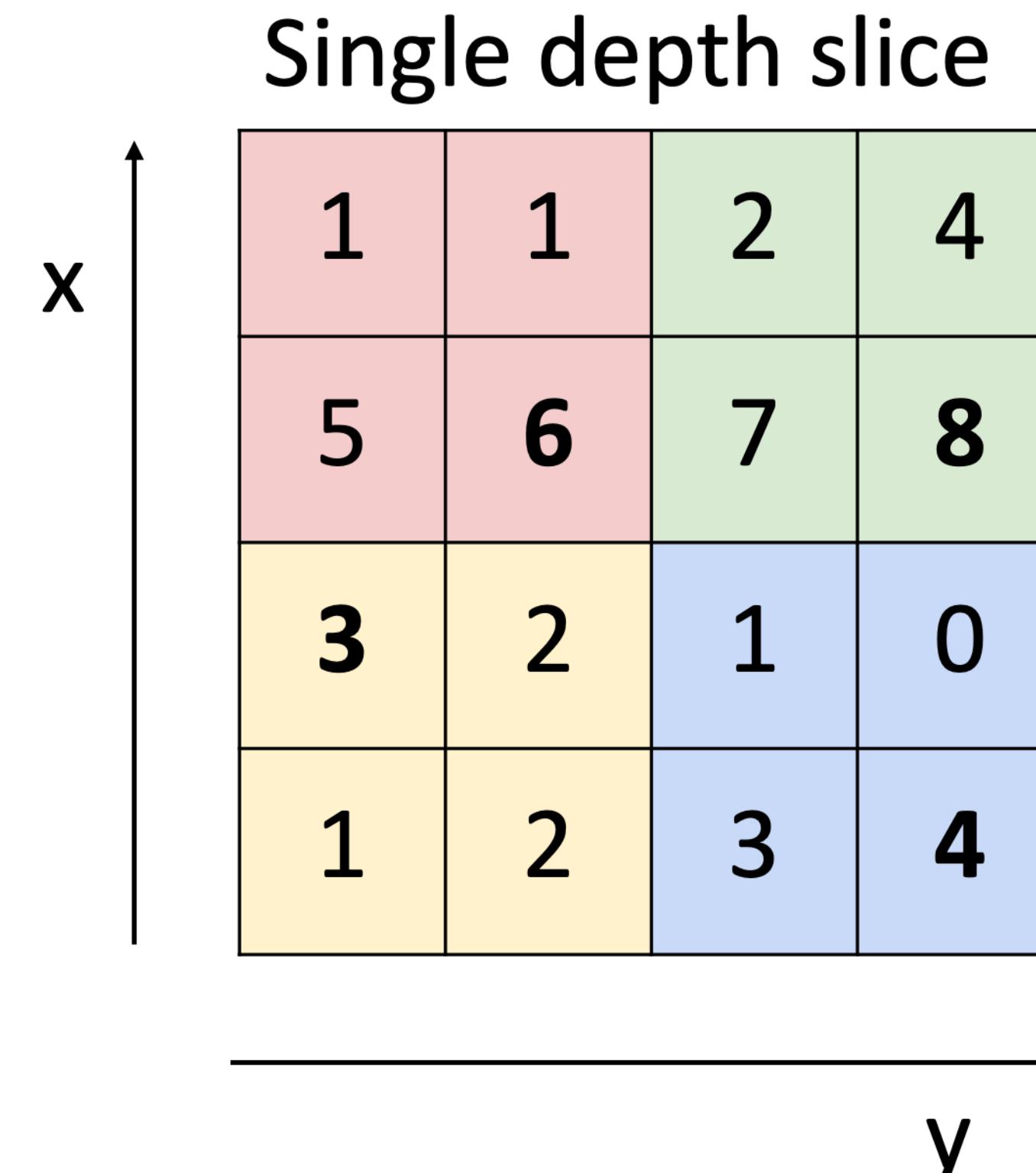


Max Pooling





Max Pooling



Max pooling with
2x2 kernel size
stride of 2

| | |
|---|---|
| 6 | 8 |
| 3 | 4 |

Introduces invariance to
small spatial shifts

No learnable parameters!



Pooling Summary

Input: $C \times H \times W$

Hyperparameters:

- Kernel size: K
- Stride: S
- Pooling function (max, avg)

Common settings:

max, $K = 2, S = 2$

max, $K = 3, S = 2$ (AlexNet)

Output: $C \times H' \times W'$ where

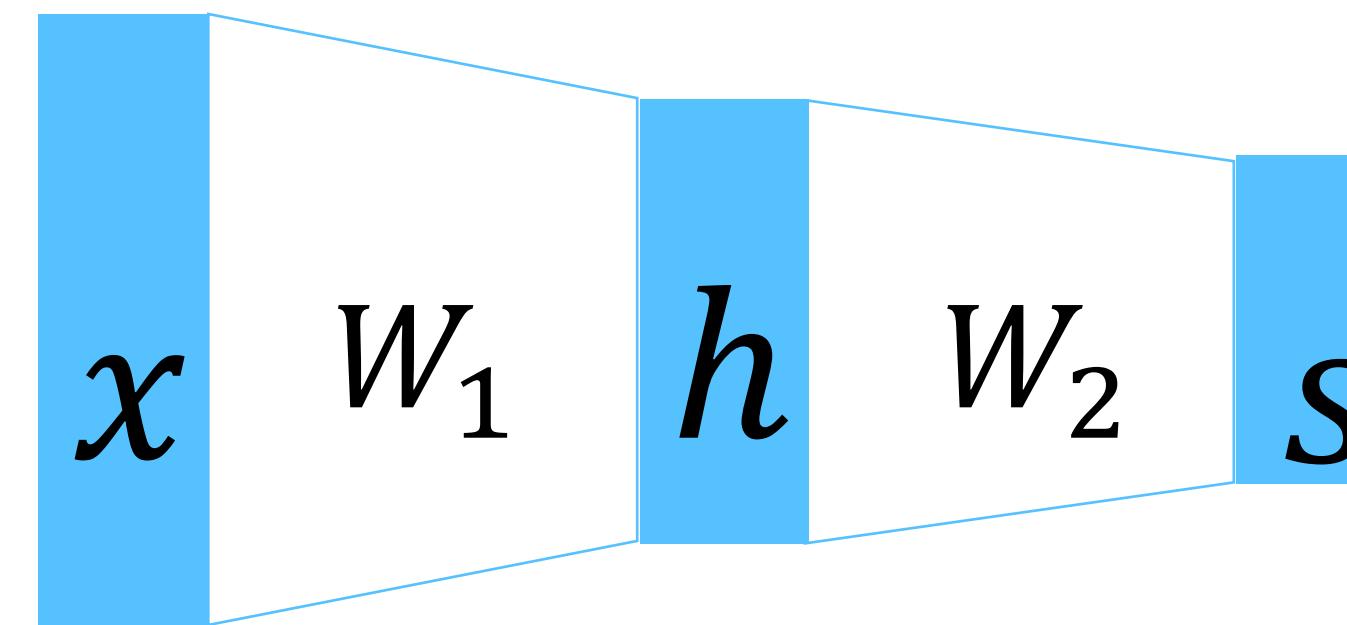
- $H' = (H - K) / S + 1$
- $W' = (W - K) / S + 1$

Learnable parameters: None!

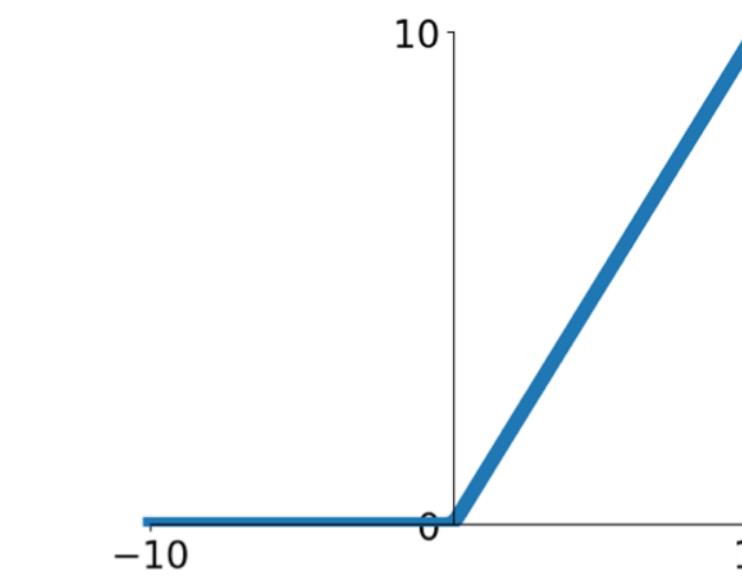


Components of Convolutional Neural Networks

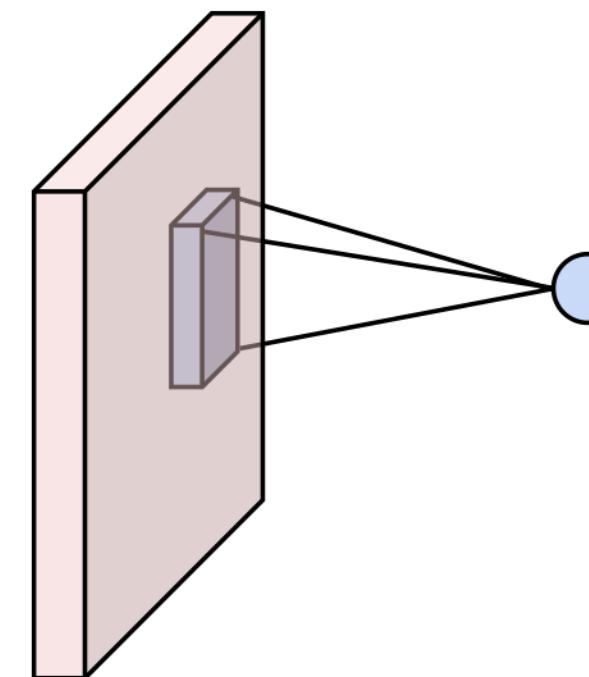
Fully-Connected Layers



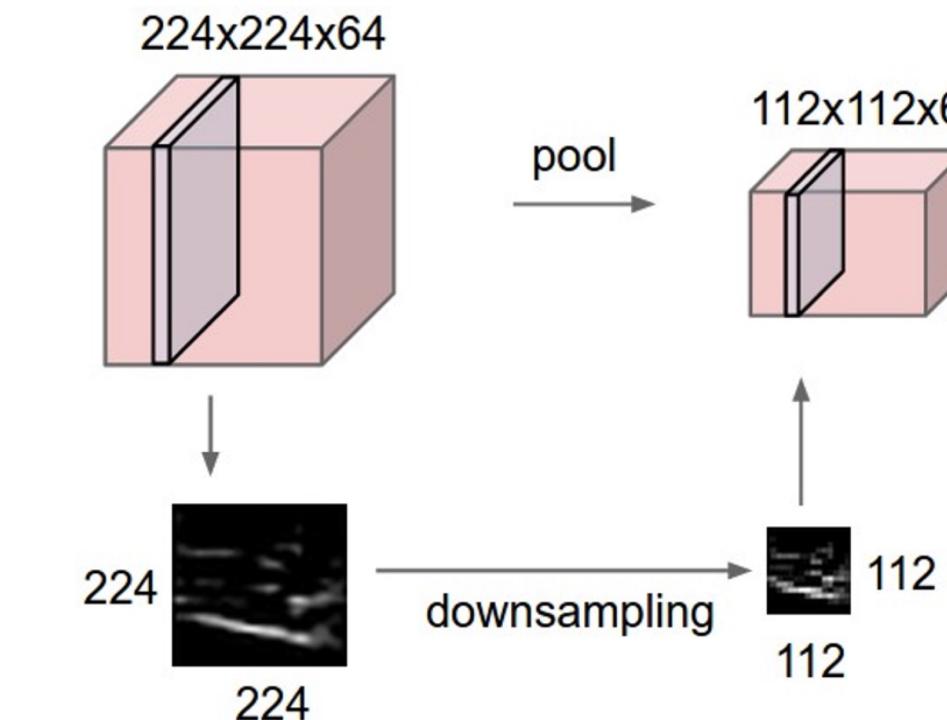
Activation Functions



Convolution Layers



Pooling Layers



Normalization

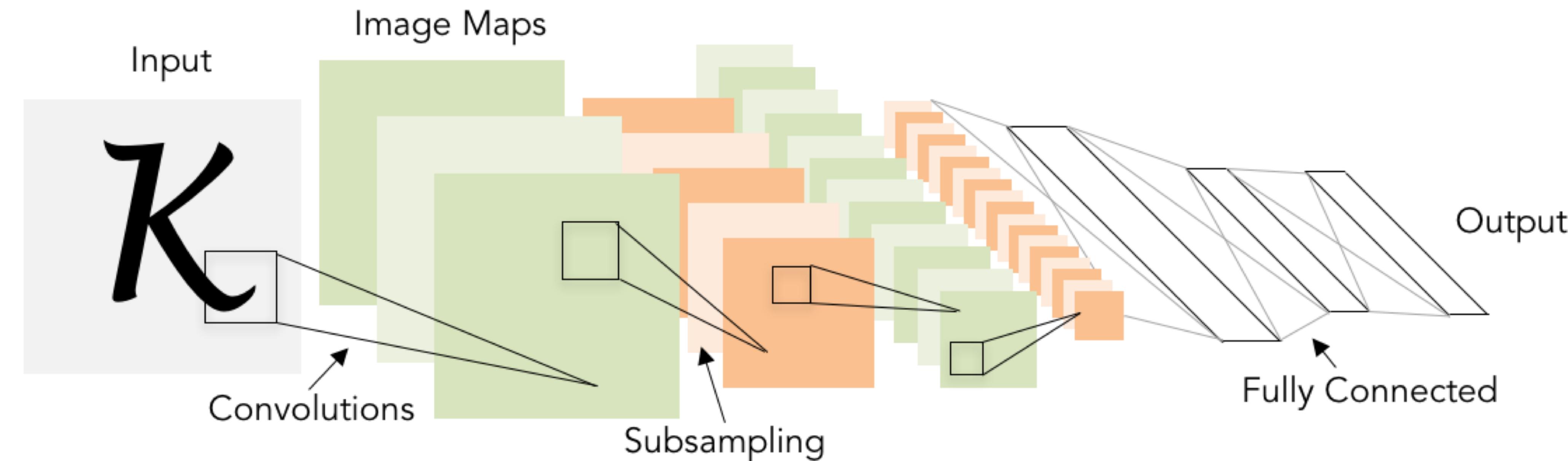
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$



Convolutional Neural Networks

Classic architecture: [Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

Example: LeNet-5

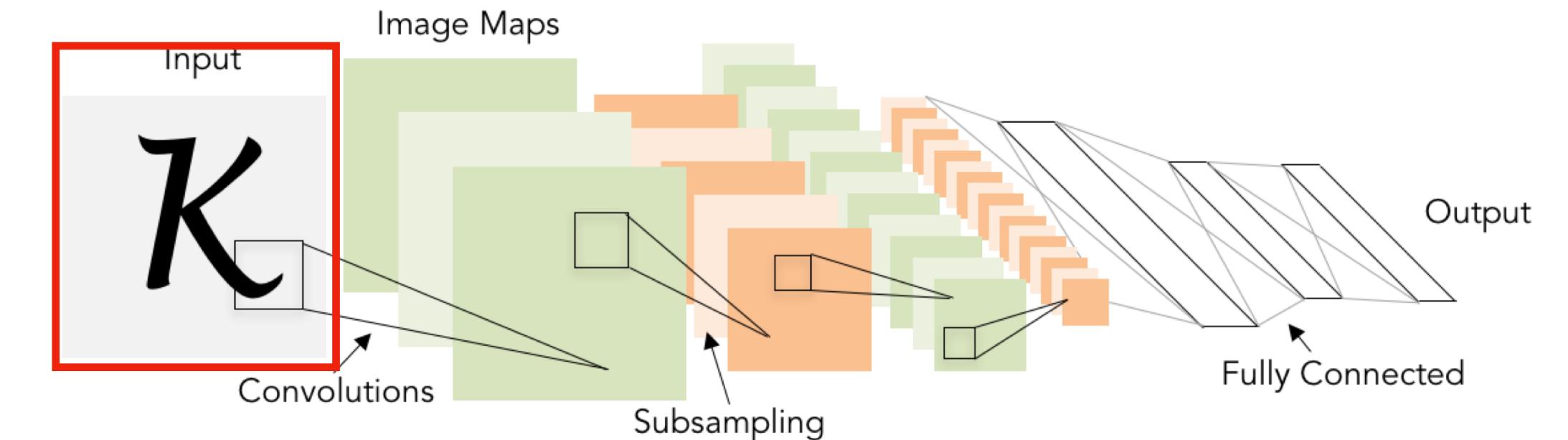


Lecun et al., "Gradient-based learning applied to document recognition", 1998



Example: LeNet-5

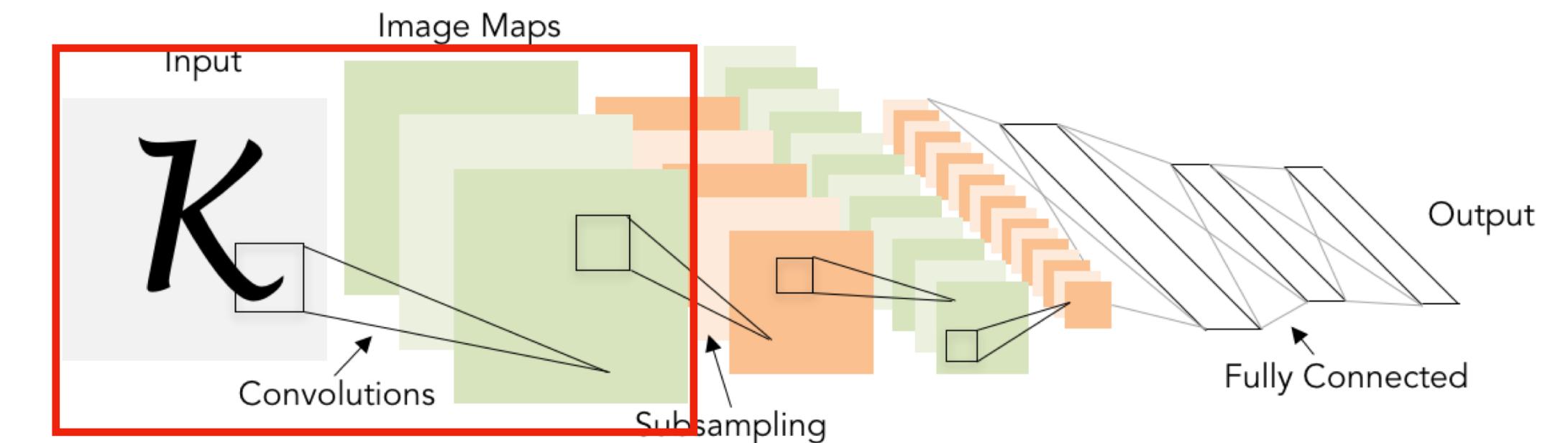
| Layer | Output Size | Weight Size |
|-------|-------------|-------------|
| Input | 1 x 28 x 28 | |





Example: LeNet-5

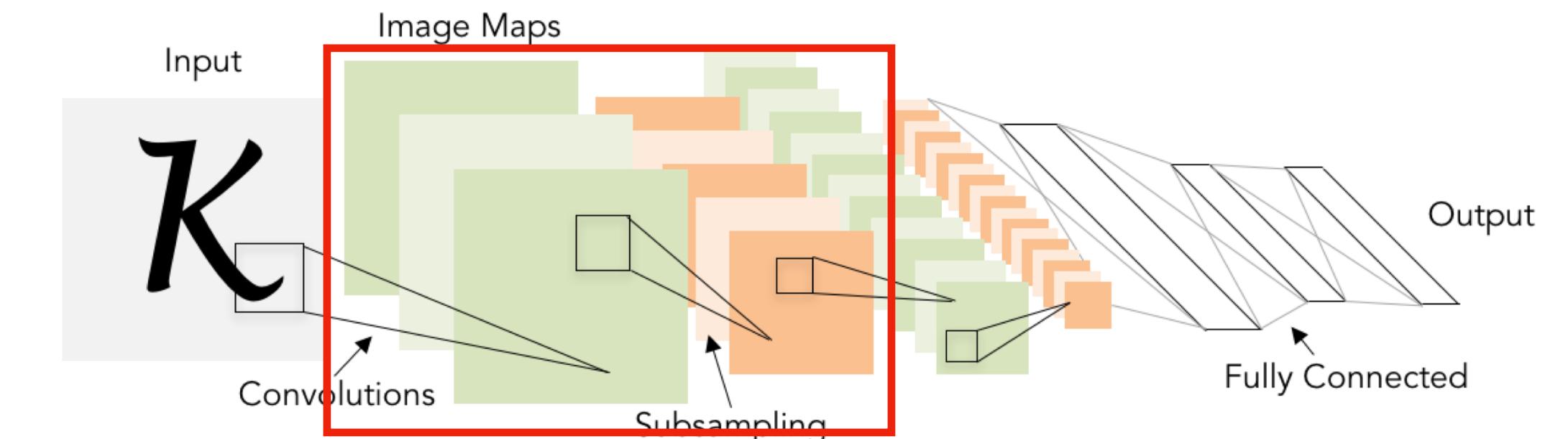
| Layer | Output Size | Weight Size |
|-----------------------------------------------|--------------------------|---------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20$, $K=5$, $P=2$, $S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |





Example: LeNet-5

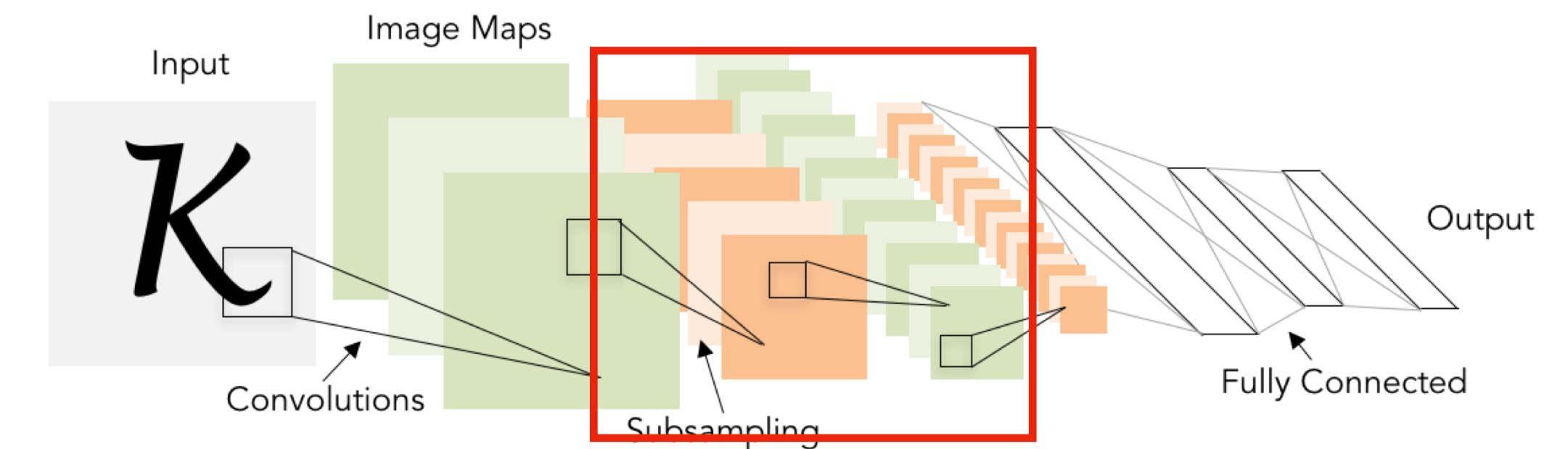
| Layer | Output Size | Weight Size |
|--------------------------------------|--------------------------|---------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20, K=5, P=2, S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |
| MaxPool($K=2, S=2$) | $20 \times 14 \times 14$ | |





Example: LeNet-5

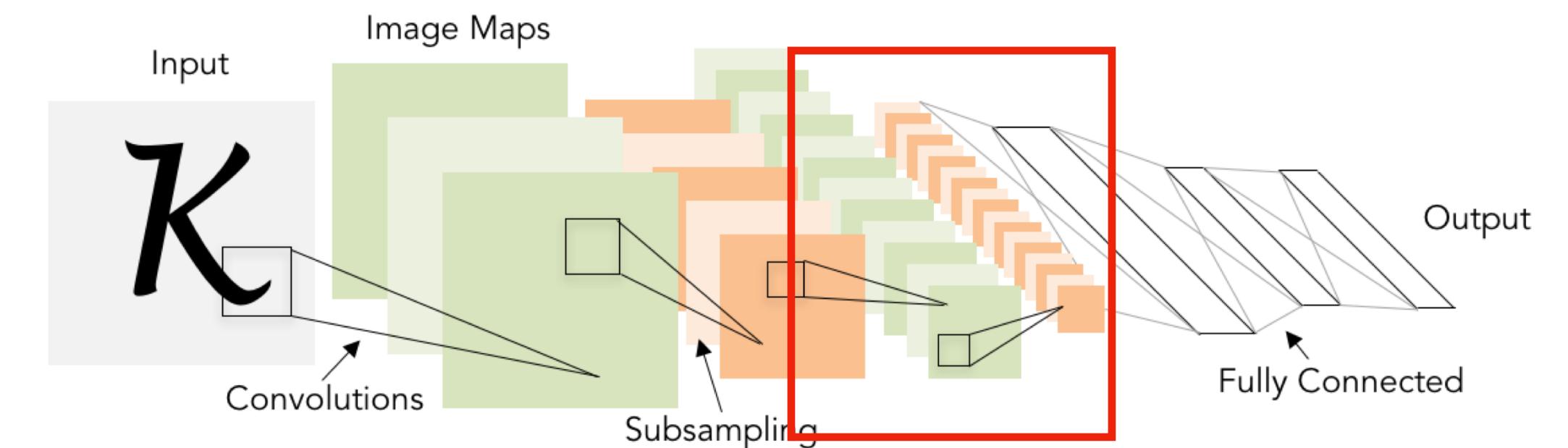
| Layer | Output Size | Weight Size |
|--------------------------------------|--------------------------|----------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20, K=5, P=2, S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |
| MaxPool($K=2, S=2$) | $20 \times 14 \times 14$ | |
| Conv ($C_{out}=50, K=5, P=2, S=1$) | $50 \times 14 \times 14$ | $50 \times 20 \times 5 \times 5$ |
| ReLU | $50 \times 14 \times 14$ | |





Example: LeNet-5

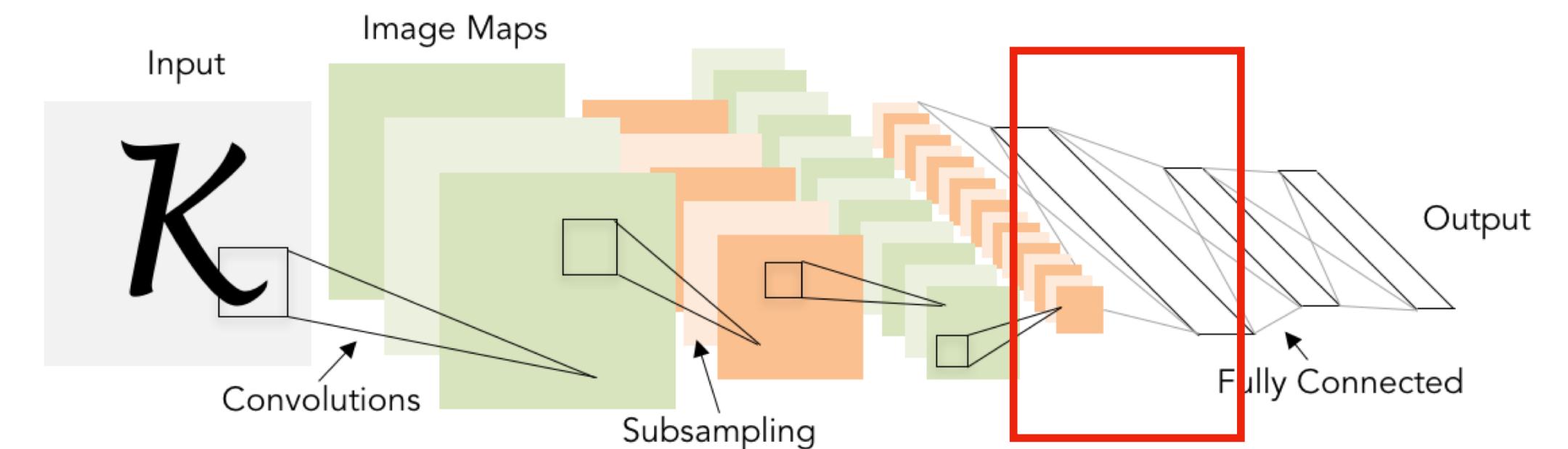
| Layer | Output Size | Weight Size |
|--------------------------------------|--------------------------|----------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20, K=5, P=2, S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |
| MaxPool($K=2, S=2$) | $20 \times 14 \times 14$ | |
| Conv ($C_{out}=50, K=5, P=2, S=1$) | $50 \times 14 \times 14$ | $50 \times 20 \times 5 \times 5$ |
| ReLU | $50 \times 14 \times 14$ | |
| MaxPool($K=2, S=2$) | $50 \times 7 \times 7$ | |





Example: LeNet-5

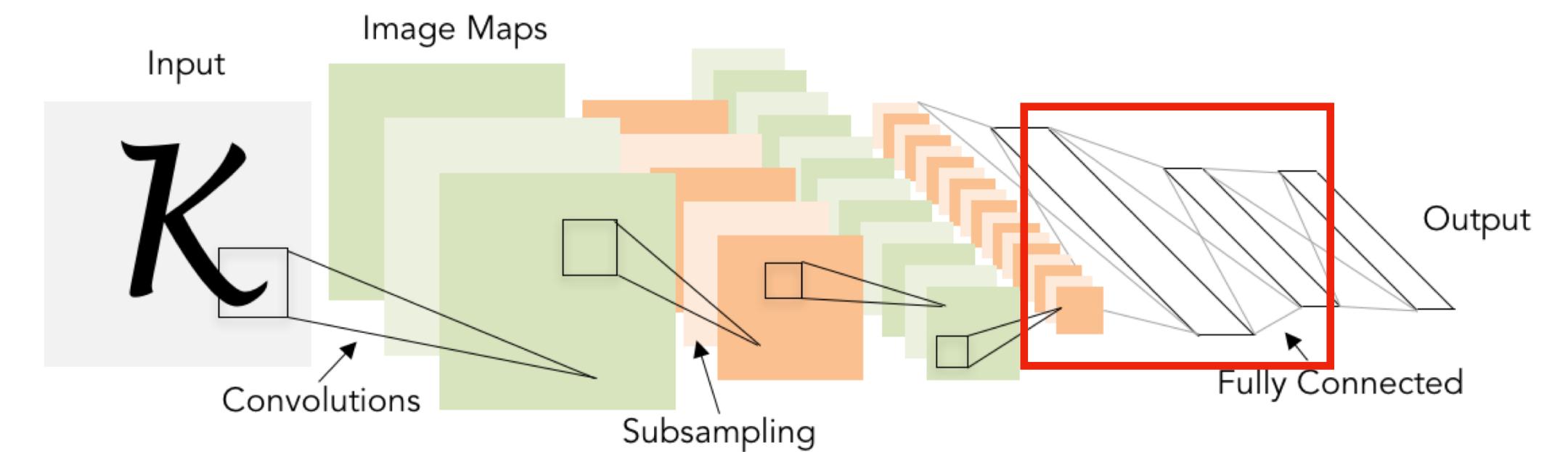
| Layer | Output Size | Weight Size |
|-----------------------------------------------|--------------------------|----------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20$, $K=5$, $P=2$, $S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |
| MaxPool($K=2$, $S=2$) | $20 \times 14 \times 14$ | |
| Conv ($C_{out}=50$, $K=5$, $P=2$, $S=1$) | $50 \times 14 \times 14$ | $50 \times 20 \times 5 \times 5$ |
| ReLU | $50 \times 14 \times 14$ | |
| MaxPool($K=2$, $S=2$) | $50 \times 7 \times 7$ | |
| Flatten | 2450 | |





Example: LeNet-5

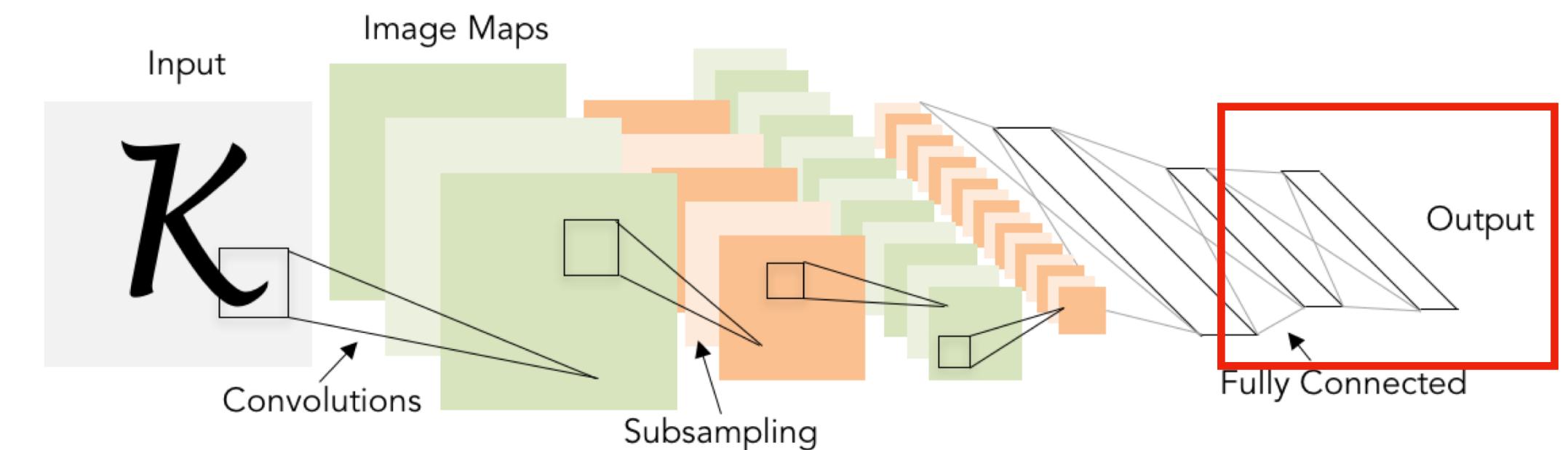
| Layer | Output Size | Weight Size |
|-----------------------------------------------|--------------------------|----------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20$, $K=5$, $P=2$, $S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |
| MaxPool($K=2$, $S=2$) | $20 \times 14 \times 14$ | |
| Conv ($C_{out}=50$, $K=5$, $P=2$, $S=1$) | $50 \times 14 \times 14$ | $50 \times 20 \times 5 \times 5$ |
| ReLU | $50 \times 14 \times 14$ | |
| MaxPool($K=2$, $S=2$) | $50 \times 7 \times 7$ | |
| Flatten | 2450 | |
| Linear ($2450 \rightarrow 500$) | 500 | 2450×500 |
| ReLU | 500 | |





Example: LeNet-5

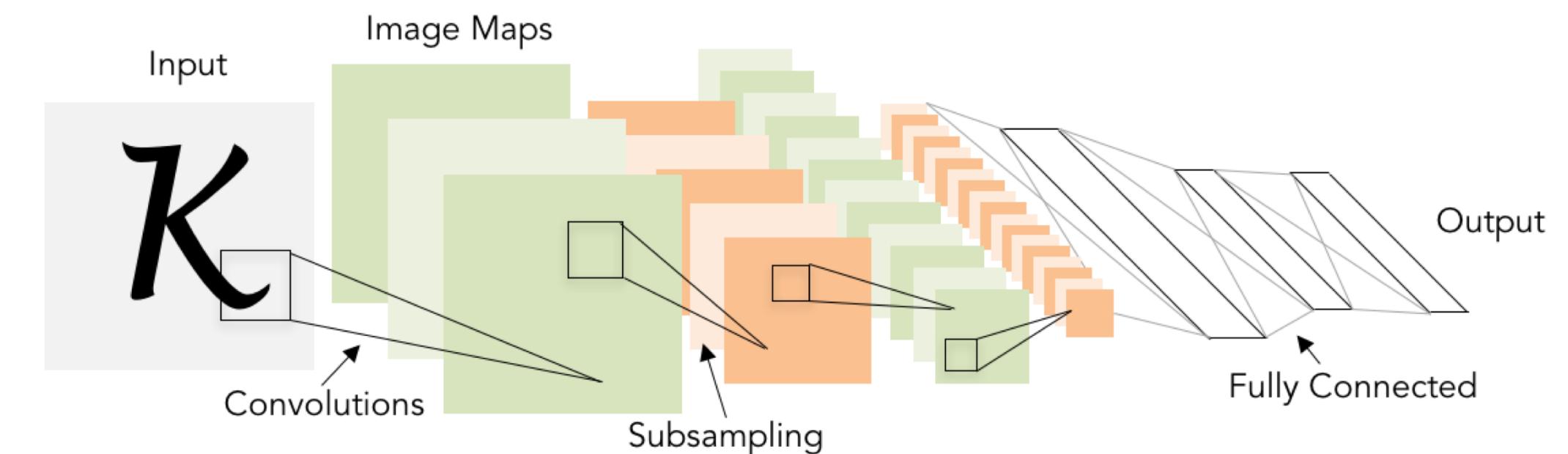
| Layer | Output Size | Weight Size |
|--------------------------------------|--------------------------|----------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20, K=5, P=2, S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |
| MaxPool($K=2, S=2$) | $20 \times 14 \times 14$ | |
| Conv ($C_{out}=50, K=5, P=2, S=1$) | $50 \times 14 \times 14$ | $50 \times 20 \times 5 \times 5$ |
| ReLU | $50 \times 14 \times 14$ | |
| MaxPool($K=2, S=2$) | $50 \times 7 \times 7$ | |
| Flatten | 2450 | |
| Linear ($2450 \rightarrow 500$) | 500 | 2450×500 |
| ReLU | 500 | |
| Linear ($500 \rightarrow 10$) | 10 | 500×10 |





Example: LeNet-5

| Layer | Output Size | Weight Size |
|--------------------------------------|--------------------------|----------------------------------|
| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20, K=5, P=2, S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
| ReLU | $20 \times 28 \times 28$ | |
| MaxPool($K=2, S=2$) | $20 \times 14 \times 14$ | |
| Conv ($C_{out}=50, K=5, P=2, S=1$) | $50 \times 14 \times 14$ | $50 \times 20 \times 5 \times 5$ |
| ReLU | $50 \times 14 \times 14$ | |
| MaxPool($K=2, S=2$) | $50 \times 7 \times 7$ | |
| Flatten | 2450 | |
| Linear (2450 -> 500) | 500 | 2450×500 |
| ReLU | 500 | |
| Linear (500 -> 10) | 10 | 500×10 |

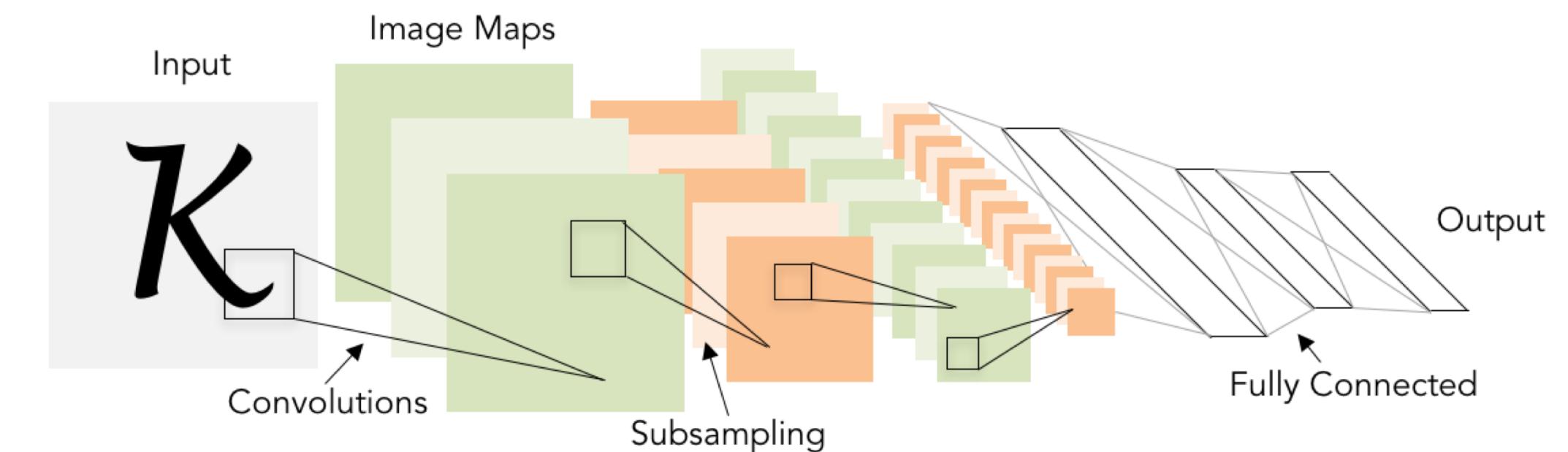


As we progress through the network:
Spatial size decreases
(using pooling or striped convolution)
Number of channels increases
(total “volume” is preserved!)



Example: LeNet-5

| Layer | Output Size | Weight Size |
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| Input | $1 \times 28 \times 28$ | |
| Conv ($C_{out}=20, K=5, P=2, S=1$) | $20 \times 28 \times 28$ | $20 \times 1 \times 5 \times 5$ |
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As we progress through the network:
Spatial size **decreases**

(using pooling or striped convolution)
Number of channels **increases**

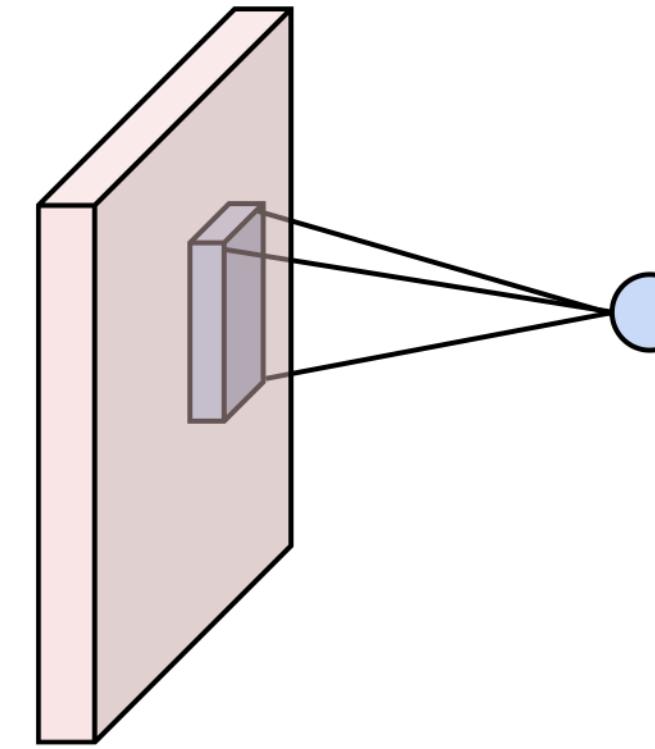
(total “volume” is preserved!)

Some modern architectures
break this trend—stay tuned!

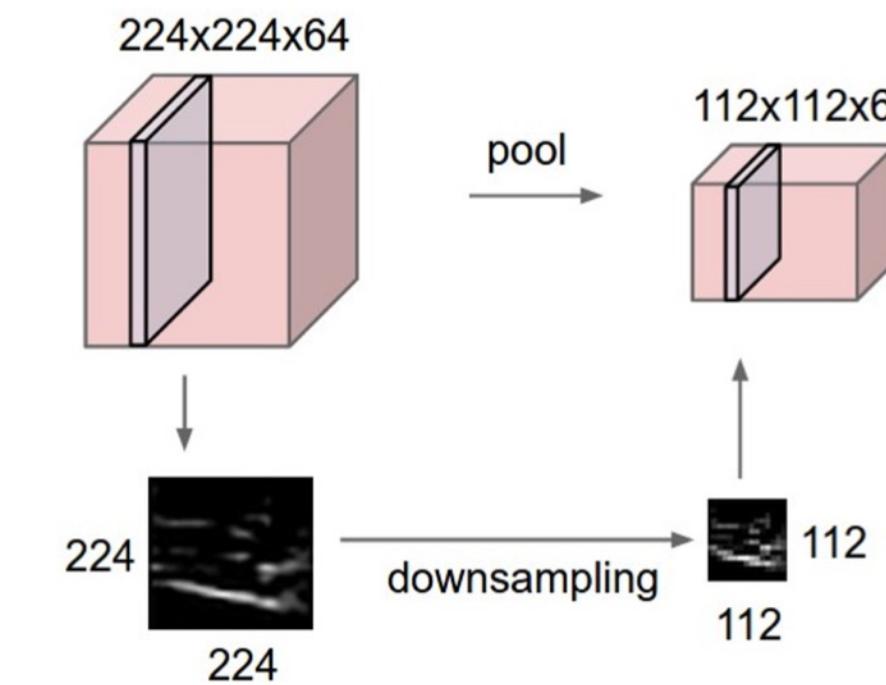


Components of Convolutional Networks

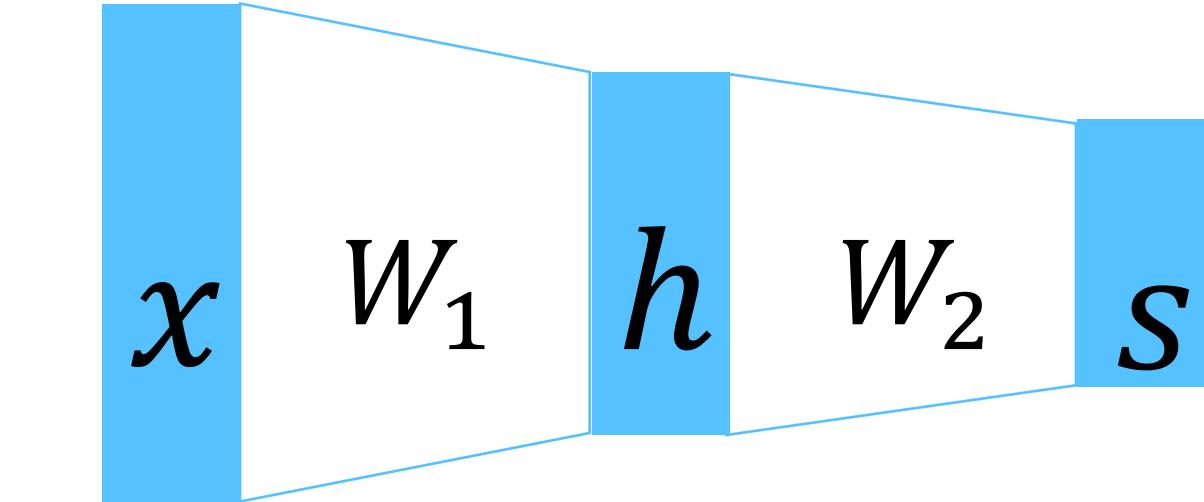
Convolution Layers



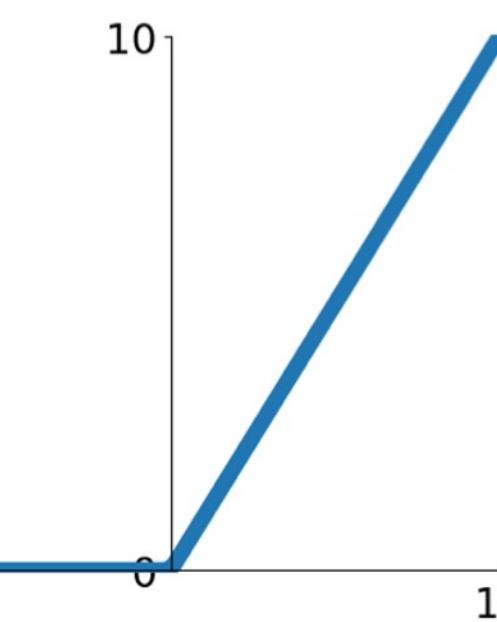
Pooling Layers



Fully-Connected Layers



Activation Function



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

Problem:
Deep
Networks
very hard to
train



Batch Normalization

Idea: “Normalize” the outputs of a layer so they have zero mean and unit variance

Why? Helps reduce “internal covariate shift”, improves optimization results

We can normalize a batch of activations using:

$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015



Batch Normalization

Idea: “Normalize” the outputs of a layer so they have zero mean and unit variance

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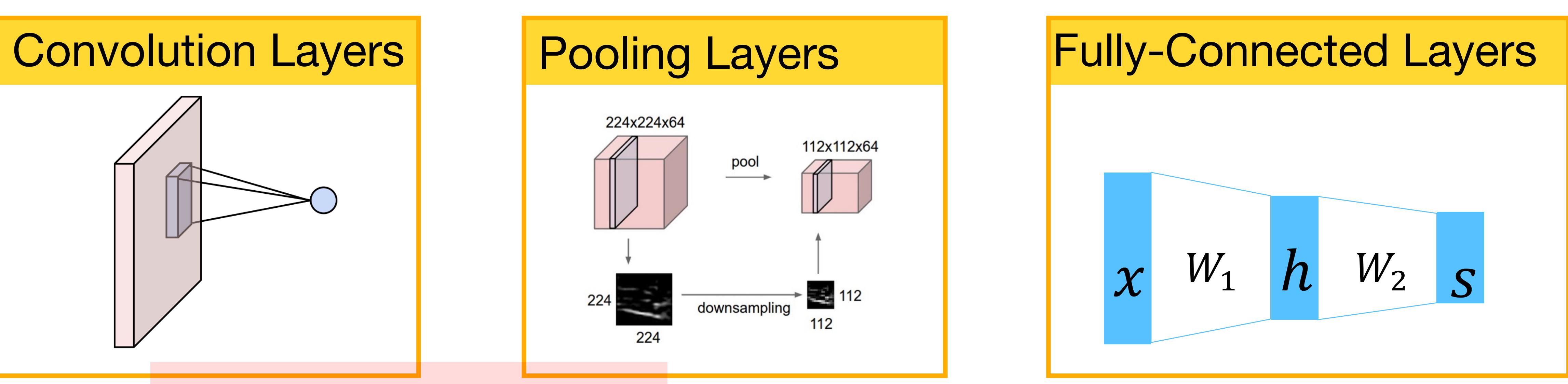
We can normalize a batch of activations using:

$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

This is a **differentiable function**, so we can use it as an operator in our networks and backdrop through it!



Components of Convolutional Networks



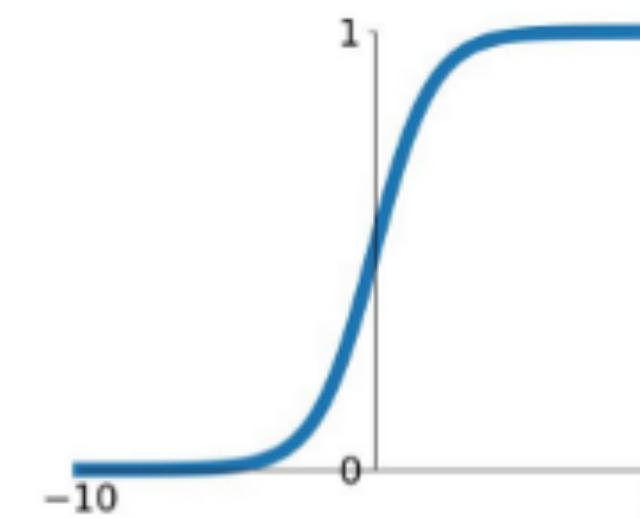
Problem:
Deep
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Activation Functions

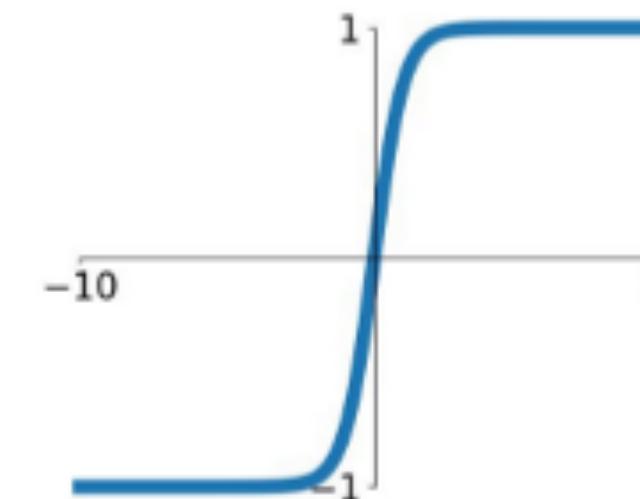
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



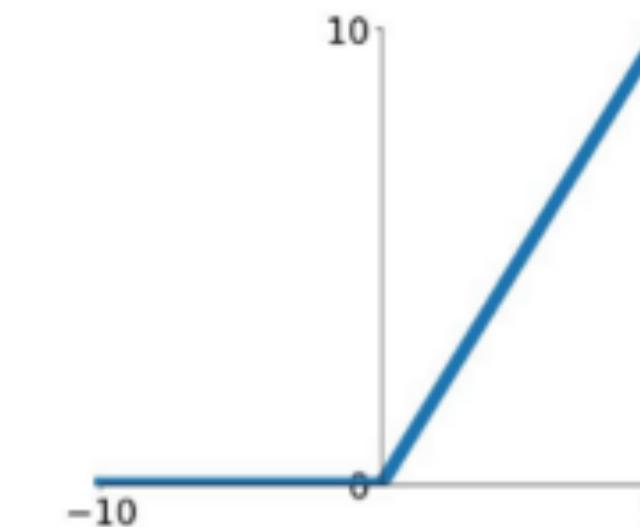
tanh

$$\tanh(x)$$



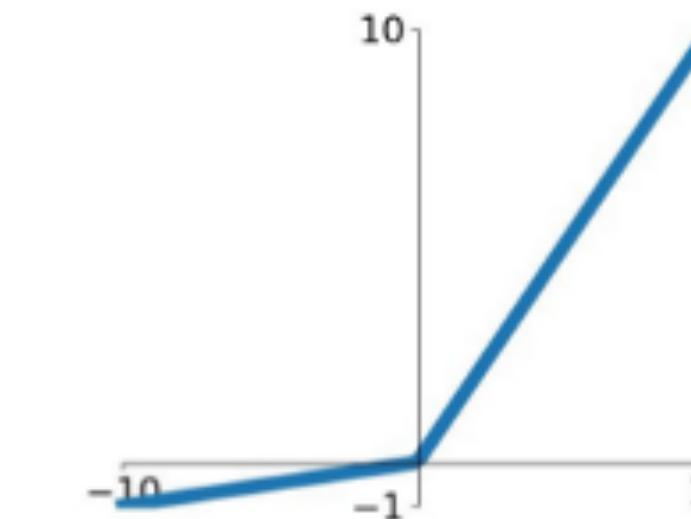
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

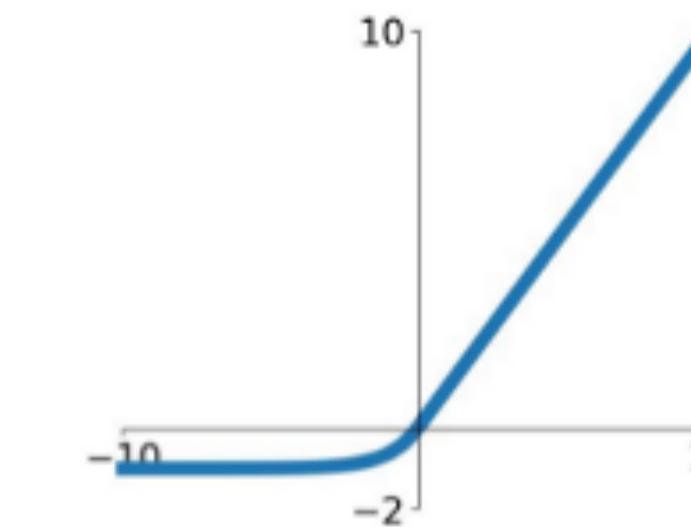


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

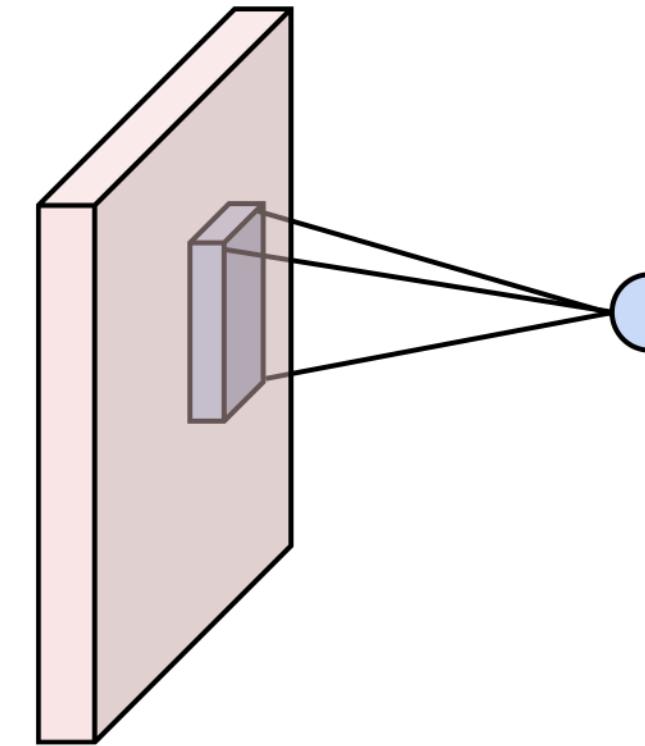
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



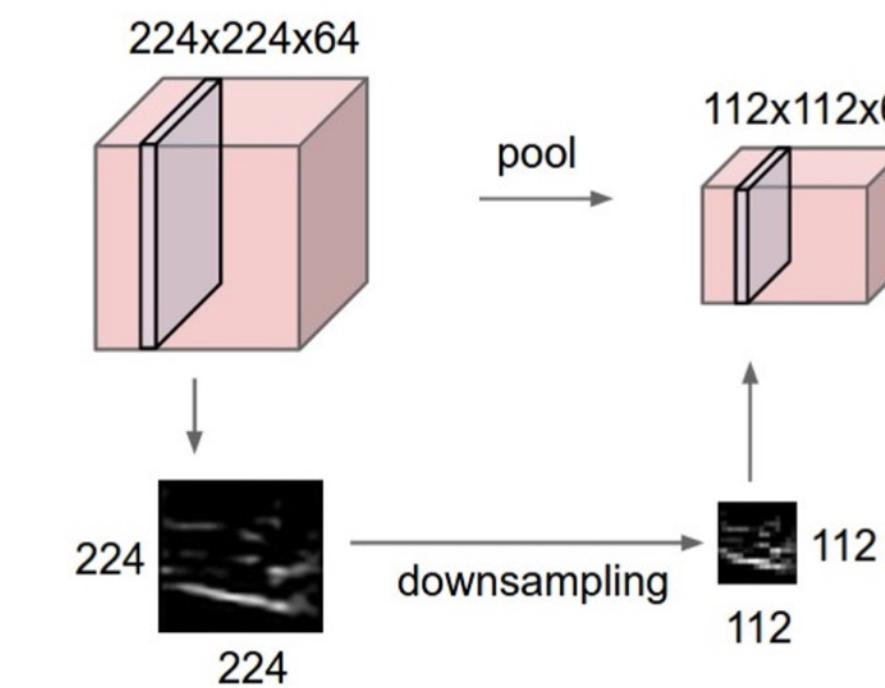


Summary: Components of Convolutional Networks

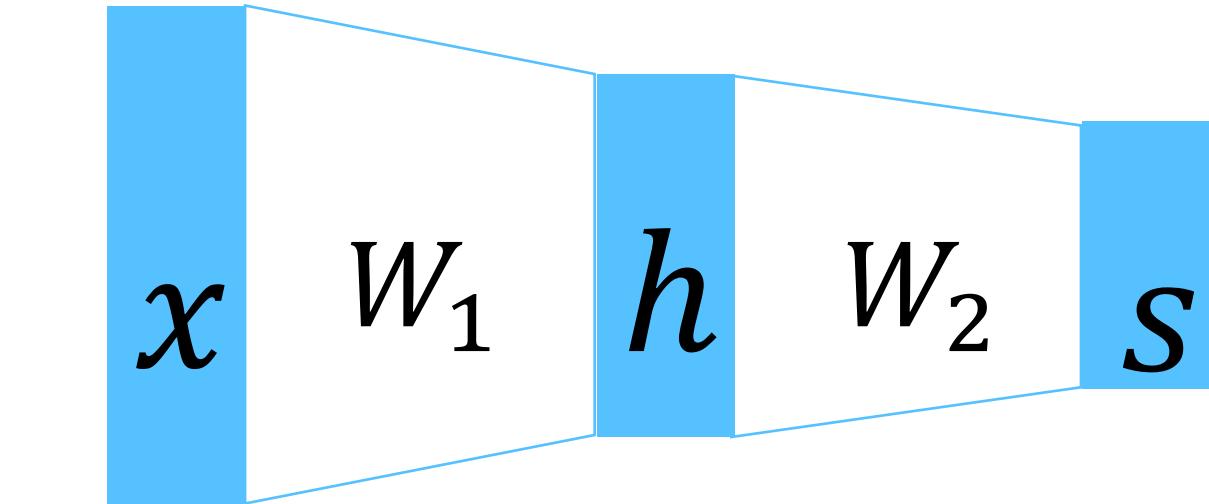
Convolution Layers



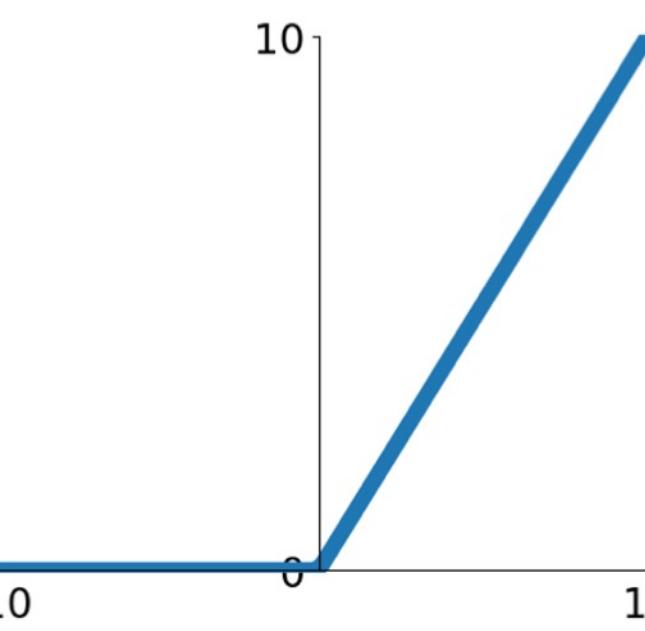
Pooling Layers



Fully-Connected Layers



Activation Function



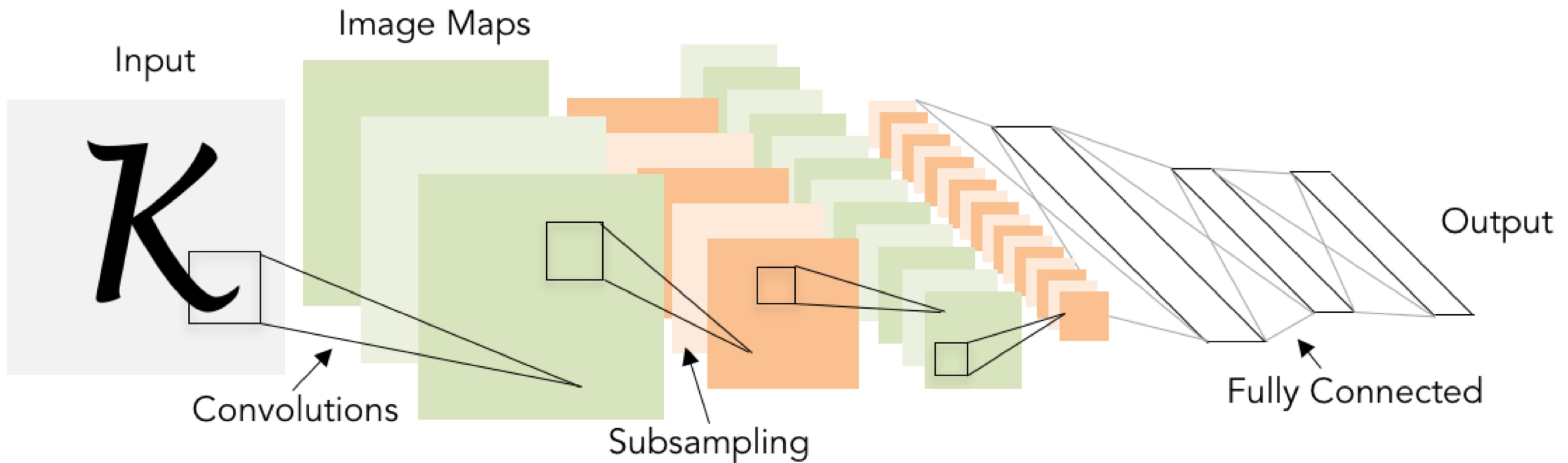
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



Summary: Components of Convolutional Network

Problem: What is the right way to combine all these components?





Project 1—Reminder

- Instructions and code available on the website
 - Here: deeprob.org/projects/project1/
- Implement KNN, linear classifier, and fully connected NN
- Due Thursday, Feb.1, 11:59 PM EST
- Discussion section: Your Thoughts?
- Late policy: 3 late tokens (24hrs each with no penalty); 25% deduction for every day the submission was late after using all three late tokens



Helpful References

- <https://cs231n.github.io/linear-classify/>
- <https://cs231n.github.io/optimization-1/>
- <https://cs231n.github.io/optimization-2/>
- https://pytorch.org/tutorials/beginner/deep_learning_60min_blitz.html



Final Project Overview

- Research-oriented final project
- Objectives
 - Gain experience reading literature
 - Reproduce published results
 - Propose a new idea and test the results!

completed in teams



Final Project Deliverables

1. A written paper review
2. In-class paper presentation
3. Reproduce published results
4. Extend results with new idea, technique or dataset
5. Document results in written report



(1) Paper Review and (2) Presentation

Final project teams will be based on overlapping interest

Students will choose from the ‘core’ list of papers on [course website](#)

Each team will be assigned one of the ‘core’ papers to review and present in-class

The 1-page paper review will be due **1-week before** the scheduled presentation

Presentation schedule will be based on paper topic as shown in [course calendar](#)

The screenshot shows a web browser window for 'dr Papers | DeepRob' at 'deeprob.org/papers/'. The page title is 'Deep Learning Research Papers for Robot Perception'. The left sidebar has links for Home, Syllabus, Calendar, Projects, PROPS Dataset, and Papers (which is selected). The main content area lists research areas with 'Core List' and 'Extended List' links: 1. RGB-D Architectures, 2. Pointcloud Processing, 3. Object Pose, Geometry, SDF, Implicit surfaces, and 4. Dense object descriptors, Category-level representations. A note at the bottom says 'This site uses Just the Docs, a documentation theme for Jekyll.'

More details on review and presentation criteria in following lectures



(3) Paper Reproduction and (4) Extension

Each team will choose a paper relating to deep learning and robot perception

Doesn't have to be same paper you presented in class

Then reimplement and reproduce at least one of the paper's published results (**not necessarily all the results**)

Then, each team will test one of their own ideas!

By extending the paper's model using new architecture or technique or dataset

Your chance to experiment with deep learning and contribute to the field!

More details on reproduction and extension
in following lectures



(5) Project Report

- The final deliverable for your final project
- A report/paper
 - What problem within robot perception or manipulation?
 - What work has been done in this area?
 - What approach did you investigate?
 - What questions and directions exist for future work?



DEEP Rob

Lecture 6
Convolutional Neural Networks
University of Michigan | Department of Robotics

