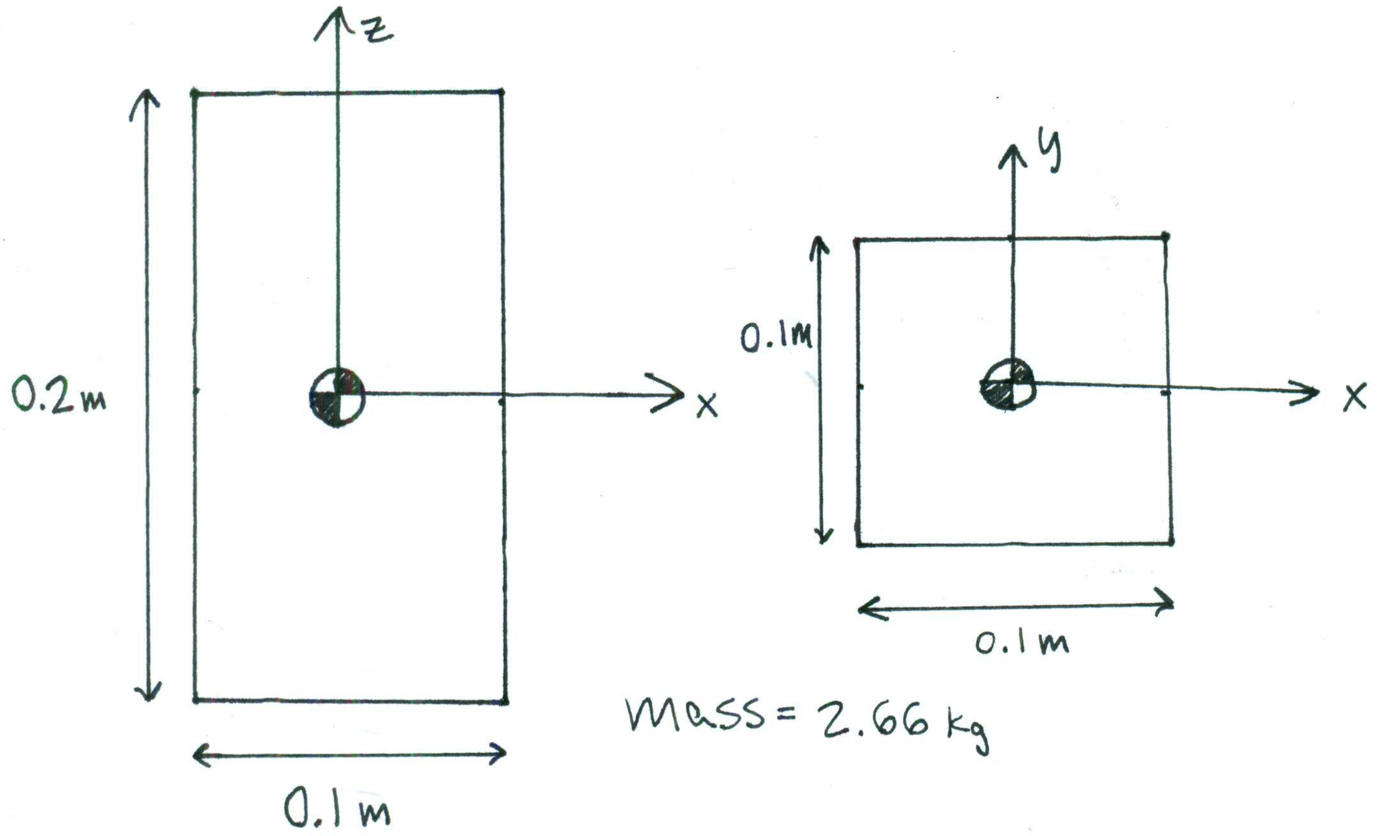
Reaction Wheel Governing Equations

(for a 2U cube sat with 4 wheels in Pyramidal Configuration)



Mass = 2.66 kg

Conservation of angular momentum:

$$I_1 W_1 + I_2 W_2 = \text{Const.}$$

(If starting with no net rotation const → Ø)

$$I_1 W_1 = -I_2 W_2$$

$$I_{S_n} W_{S_n} = -I_{RW_n} W_{RW_n}$$

, where S: satellite
RW: reaction wheel
n: axis x, y or z

$$I_{Sn} W_{Sn} = -4 \cdot \cos(\theta_n) I_{RW} W_{RW}$$

, where $\cos(\theta_n)$ is component of each RW momentum vector parallel to axis x y or z

$$\boxed{|W_{RW}| = \left| \frac{I_{Sn} W_{Sn}}{4\cos(\theta_n) I_{RW}} \right|}$$

Inertia:

$$I_{x\,\&\,y} = \frac{1}{12}(2.66\,kg)\left[(0.10\,m)^2 + (0.20\,m)^2\right]$$

$$\boxed{I_{x\,\&\,y} = 0.011083\,kg\,m^2}$$ Inertia about x & y axis

$$I_z = \frac{1}{12}(2.66\,kg)\left[(0.10\,m)^2 + (0.10\,m)^2\right]$$

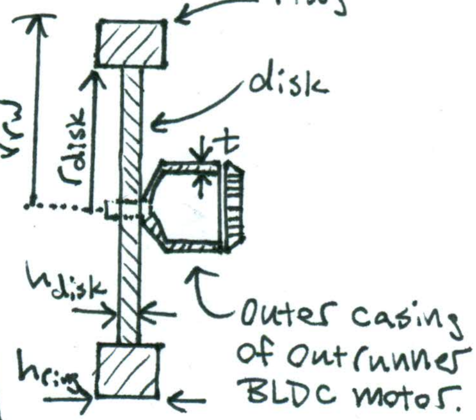
$$\boxed{I_z = 0.004433\,kg\,m^2}$$ Inertia about z axis

$$\frac{I_{x\,\&\,y}}{I_z} = \frac{0.011083\,kg\,m^2}{0.004433\,kg\,m^2}$$

$$\boxed{I_{x\,\&\,y} = 2.5 \cdot I_z}$$

$$I_{RW} = I_{disk} + I_{ring} + I_{motor}$$

$$I_{RW} = \rho\frac{\pi}{2}\left[h_{ring}\left(r_{rw}^4 - r_{disk}^4\right) + h_{disk}\,r_{disk}^4\right] + I_{motor}$$



approximate as a ring with thickness t

non-magnetic(ish)

$$\rho_{AL} \cong 2700\ kg/m^3$$

$$\rho_{\text{stainless steel (austenitic)}} \cong 7700\ kg/m^3$$

note: $(I_{disk} + I_{ring}) > I_{motor}$

Momentum Vector components:

$$\cos^2(\theta_{\hat{i}}) + \cos^2(\theta_{\hat{j}}) + \cos^2(\theta_{\hat{k}}) = 1$$

$\theta_{\hat{i}}$ and $\theta_{\hat{j}}$: Will be the same

$\theta_{\hat{k}}$: Variable

Where, $\theta_{\hat{i}}\ \theta_{\hat{j}}\ \theta_{\hat{k}}$ is the angle the RW momentum $\vec{J}$ vector makes with each component axis. For pyramidal config $\theta_{\hat{i}} = \theta_{\hat{j}}$ and $\theta_{\hat{k}}$ is chosen

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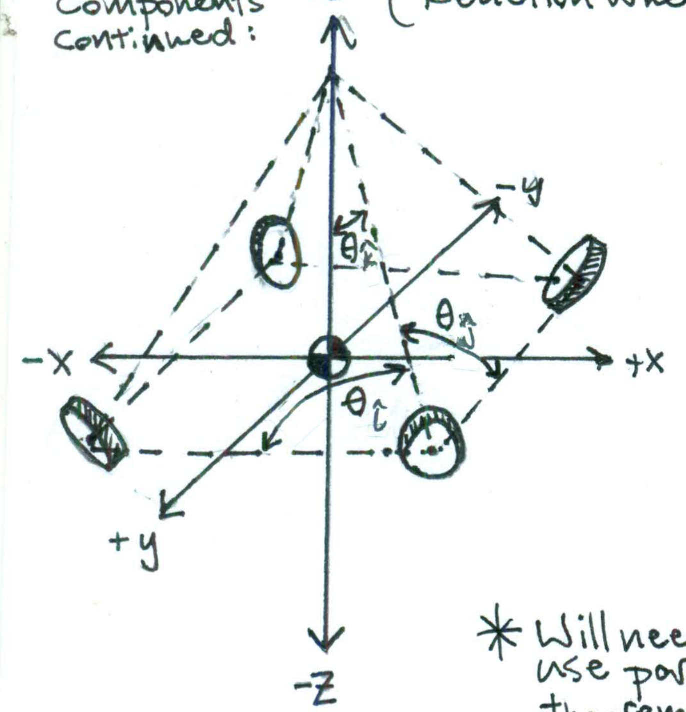
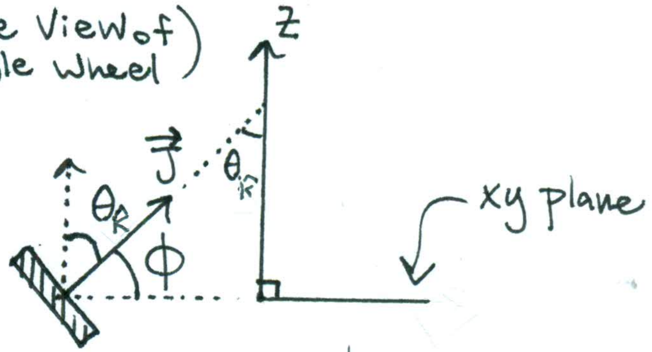
Vector components continued: (Reaction Wheels in pyramidal configuration)



* Will need to use parallel axis theorem for this datum (next page)

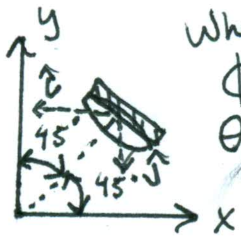
(Side View of Single Wheel)



ϕ is angle to xy plane

$\theta_z = 90^\circ - \phi$

(Top View of Single Wheel)



When $\phi = 0$, $\theta_{\hat{k}} = 90$

$\theta_{\hat{i}} = \theta_{\hat{j}} = 45^\circ$

Note on datum structure:

I chose this datum so that the calculations of I_x I_y I_z would be easier with the trade off of more trig to find angle. You could chose to have the x and y axis pass through the wheels making I_x & I_y different and $\theta_{\hat{i}}, \theta_{\hat{j}}$ easy to find either way the governing equations remain the same.

Case 1: $\theta_{\hat{i}} = \theta_{\hat{j}} = \theta_{\hat{k}} = \theta$. (All angles equal, chose this for 1U)

$$\cos^2(\theta_{\hat{i}}) + \cos^2(\theta_{\hat{j}}) + \cos^2(\theta_{\hat{k}}) = 1$$

$$3\cos^2(\theta) = 1$$

$$\cos^2(\theta) = \frac{1}{3}$$

$$\cos(\theta) = \sqrt{\frac{1}{3}}$$

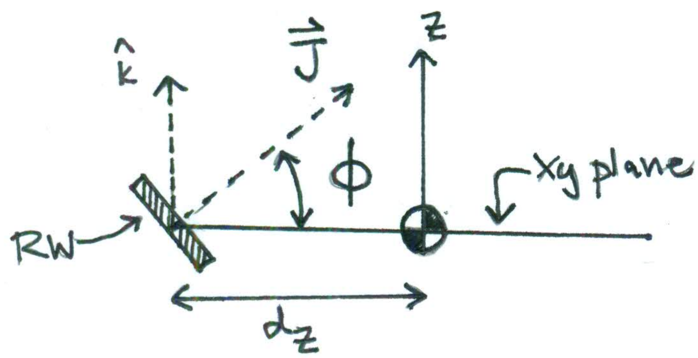
$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

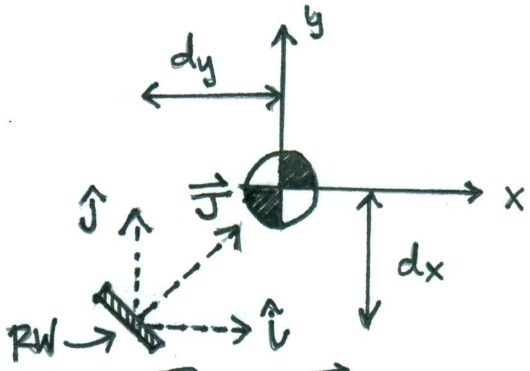
$\theta = 54.7356^\circ$ ← angle to each axis

$\phi = 35.2644^\circ$ ← angle to xy plane

Parallel axis theorem: (for this datum) ④

Since the axes x, y, z do not pass through the motors use parallel axis theorem to find $I_{s_{n'}}$ that is on axis with the component vectors of the RW momentum vector $\vec{J}$





Where, $\vec{J}$ is momentum vector
$\hat{i}, \hat{j}, \hat{k}$ are the xyz components of $\vec{J}$
$d_x d_y d_z$ is the perpendicular distance from the components to the respective axes.

$$I_{n'} = I_n + md^2$$

(d will be less than 0.05 m to allow room for the motor to be mounted)
for the motors being considered, estimate

$d_x = d_y \approx 0.035\,m$

$d_z \cong \sqrt{d_x^2 + d_y^2}$

$d_z \cong \sqrt{2(0.035\,m)^2}$

$d_z \approx 0.049497\,m$

$$\therefore I_{x'} = I_{y'} \cong (0.011083\,kgm^2) + (2.66\,kg)(0.035\,m)^2$$

$$\cong 0.014342\,kgm^2$$

$$I_{z'} \cong (0.004433\,kgm^2) + (2.66\,kg)(0.049497\,m)^2$$

$$I_{z'} \cong 0.01095\,kgm^2$$

$$\frac{I_{x'}}{I_{z'}} = 1.31$$

$I_{x'} = 1.31\, I_{z'}$ use this relation for finding ϕ

$$I_{x'} = I_{y'} = 1.31\, I_{z'}$$

$$\cos(\theta_{\hat{i}}) = \cos(\theta_{\hat{j}}) = 1.31 \cos(\theta_{\hat{k}})$$

call $\theta_{\hat{i}} = \theta_{\hat{k}} = \theta$

$$\cos(\theta) = 1.31 \cos(\theta_{\hat{k}})$$

$$\frac{\cos(\theta)}{1.31} = \cos(\theta_{\hat{k}})$$

$$\frac{\cos^2(\theta)}{1.7161} = \cos^2(\theta_{\hat{k}})$$

angle off xy plane ϕ chosen by ratio of Inertias of 2U sat with pyramidal RW config using parallel axis theorem.

$$\cos^2(\theta_{\hat{i}}) + \cos^2(\theta_{\hat{j}}) + \cos^2(\theta_{\hat{k}}) = 1$$

$$2\cos^2(\theta) + \frac{\cos^2(\theta)}{1.7161} = 1$$

$$\frac{3.4322\cos^2(\theta) + \cos^2(\theta)}{1.7161} = 1$$

$$\frac{4.4322}{1.7161}\cos^2(\theta) = 1$$

$$\cos^2(\theta) = \frac{1.7161}{4.4322}$$

$$\cos(\theta) = \sqrt{0.387189}$$

$$\theta = \cos^{-1}(0.622245)$$

$$\theta = 51.52^\circ$$

$$\cos^2(\theta_{\hat{k}}) = \frac{\cos^2(\theta)}{1.7161}$$

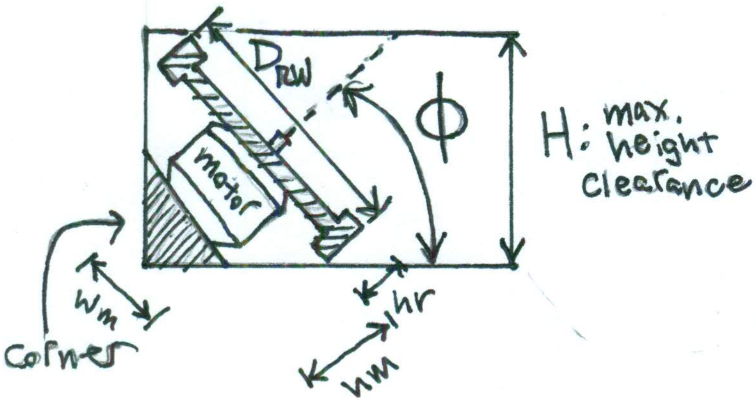
$$\cos(\theta_{\hat{k}}) = \sqrt{\frac{0.387189}{1.7161}}$$

$$\theta_{\hat{k}} = \cos^{-1}(0.474996)$$

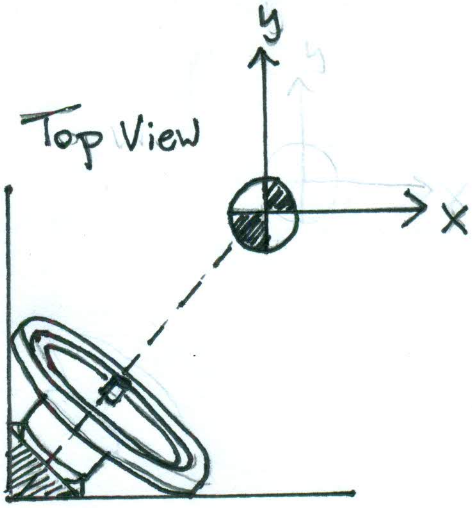
$$\theta_{\hat{k}} = 61.64^\circ \longrightarrow \phi = 28.3591^\circ \leftarrow$$ angle off xy plane

Reaction Wheel design:

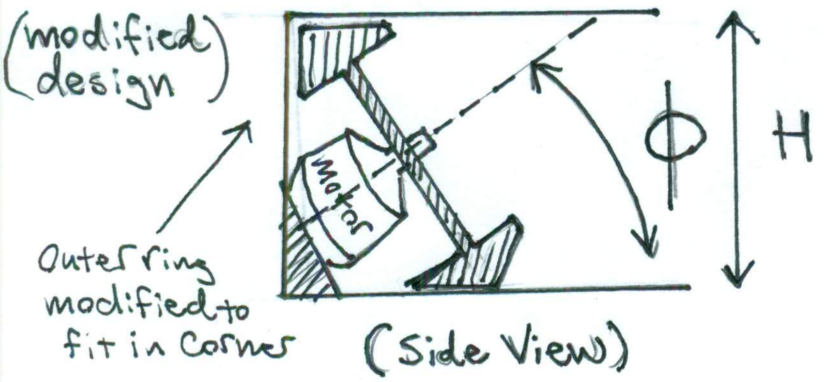
(side View)



Top View

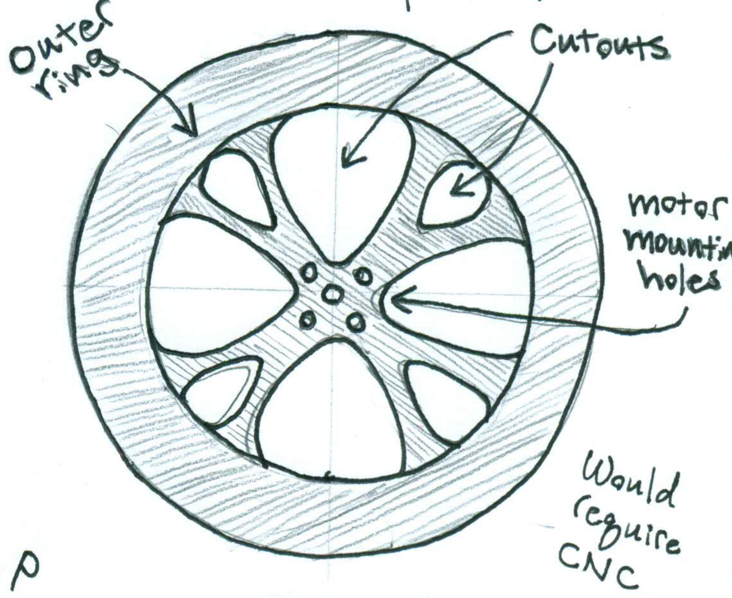


max. Diameter of RW, D_{RW}, and height of outer ring, hr, driven by maximum available height clearance, H, angle off xy plane, ϕ, as well as motor height, hm, and motor width, Wm. Distributing the wheel's mass outward increases the inertia of the wheel for a given mass. The classic ring & disk design could be modified to increase wheel inertia in a compact space:



(Side View)

Will design in Solid Works and get Inertia estimate for given ρ



(front View)

Estimate of Required Angular Velocities

$$|\omega_{RW}| = \left| \frac{I_{s_n'}}{4\cos(\theta_n) I_{RW}} \cdot \omega_{s_n} \right|$$

get estimates for I_{RW}:

assume motor is makerfire D1104 7500kV with

$r_{rw} \approx 0.020\,m$, $h_{disk} \approx 0.002\,m$, $h_{ring} \approx 0.006\,m$, $r_{disk} \approx 0.015\,m$

(I took apart the motor to measure the mass and dimensions of the rotating components to get a rough estimate of the inertia)

$$\boxed{I_{motor} \approx 9.0225\,e{-7}\,kg \cdot m^2}$$

$$I_{RW} \approx \rho \frac{\pi}{2} \left[0.006m\left((0.02m)^4 - (0.015m)^4\right) + 0.002m \cdot (0.015m)^4 \right] + 9.0225\,e{-7}\,kg \cdot m^2$$

$$I_{RW} \approx \rho \frac{\pi}{2} \left[7.575\,e{-10}\,m^5 \right] + 9.0225\,e{-7}\,kg \cdot m^2$$

$$I_{RW_{AL}} \approx \left(2700 \frac{kg}{m^3}\right) \cdot \frac{\pi}{2} \left[7.575\,e{-10}\,m^5 \right] + 9.0225\,e{-7}\,kg \cdot m^2$$

$$\boxed{I_{RW_{AL}} \approx 4.115\,e{-6}\,kg \cdot m^2}$$

$$I_{RW_{SS}} \approx \left(7700 \frac{kg}{m^3}\right) \cdot \frac{\pi}{2} \left[7.575\,e{-10}\,m^5 \right] + 9.0225\,e{-7}\,kg \cdot m^2$$

$$\boxed{I_{RW_{SS}} \approx 1.006\,e{-5}\,kg \cdot m^2}$$

estimate ω_{RW} for a range of desired $\omega_{s_{xy}}$ using $\theta_i = \theta_j = 51.52°$

$$|\omega_{RW}| = \left| \frac{(0.014342\,kg \cdot m^2)}{4 \cdot \cos(51.52)\, I_{RW}} \cdot \omega_{s_{xy}} \right|$$

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ω estimates: required angular velocities for a desired $\omega_{satellite}$ about the x or y axis using $\theta_i = \theta_j = 51.52°$

$\phi = 28.359$ off xy plane

* [all in RPM]

$\omega_{s_{x\,\&\,y}}$	$\omega_{RW_{Aluminum}}$	$\omega_{RW_{stainless\ steel}}$	$\omega_{motor\ by\ itself}$
0.125	~175	~71.6	~800
1	~1400	~572.8	~6387
2	~2801	~1145.6	~12773
5	~7002	~2864.0	~31932
10	~14003	~5727.9	~63865

[$\frac{\pi}{240}\frac{rad}{s}$ for earth tracking] → 0.125

(done in Excel)

(add ω_{s_z} as well)

Mass of Motor + Wheel System:

motor ≈ 0.006 kg ea. ×4 = 0.024 kg

AL. Wheel ≈ $(5.108e{-6}\ m^3)(2700\ kg/m^3) \times 4 = 0.055169\ kg$

S.S. wheel ≈ $(5.108e{-6}\ m^3)(7700\ kg/m^3) \times 4 = 0.157338\ kg$

(approx. Vol. of Wheel) (density of material)

Mass of Aluminum RW system ≅ 0.0792 kg
Mass of Stainless Steel RW System ≅ 0.1813 kg
} does not include mass of motor control board

– Next, estimate power required to drive the RW system at a given ω_{RW} of a given period of time based on required $\dot{\theta}_{satellite}$ (ω_{sat_n})