A Are all forces equally good?

Consider an arbitrary articulated object that has a mass matrix $\mathcal{M}(q)$. We use $q \in \mathrm{I\!R}^{\mathrm{N}}$ to represent the configuration of the object in generalized coordinates.

$$\mathcal{M}(q)\ddot{q} + C(q, \dot{q}) = Q,$$

where C is the coriolis force and Q is the external force both expressed in the generalized coordinates.

Let $J_k \in \mathrm{I\!R}^{6\times\mathrm{N}}$ be the Jacobian and $I_k \in \mathrm{I\!R}^{3\times3}$ be the inertia matrix of the rigid body k. J_k can further be split into the linear and angular constituent matrices $J_v \in \mathrm{I\!R}^{3\times\mathrm{N}}$ and $J_\omega \in \mathrm{I\!R}^{3\times\mathrm{N}}$ respectively i.e. $J_k = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$. By definition,

$$\begin{aligned} \mathcal{M}(q) &= \sum_k J_k^T(q) \begin{bmatrix} m_k I & 0 \\ 0 & m_k I_k \end{bmatrix} J_k(q) \\ &= \sum_k m_k (J_{v_k}^T J_{v_k} + J_{\omega_k}^T I_k J\omega_k). \end{aligned}$$

Assuming $\dot{q} = 0 \implies C(q, \dot{q}) = 0$, the equation of motion can be expressed as

$$\sum_k m_k A_k \ddot{q} = Q,$$

where $A_k = J_{v_k}^T J_{v_k} + J_{\omega_k}^T I_k J\omega_k$.

If $N > 6$, $A_k \in \mathrm{I\!R}^{\mathrm{N}\times\mathrm{N}}$ is not full-rank by definition. Thus there exists a null space for each A_k. If $\ddot{q}$ lies in the null-space of any A_k we cannot uniquely determine the value of m_k. If $N \leq 6$, we still cannot guarantee A_k to be full-rank because if the object is in a kinematic singularity state, $J_k(q)$ will lose rank.