Dynamic Multidimensional Scaling Algorithm for Mobile Location

Zhang-Xin Chen, Q. Wan, B. Jiang, H. Dou, W. L. Yang
Dept. of Electronic Engineering
University of Electronic Science and Technology of China
Chengdu 610054, P. R. China
zhangxinchen@uestc.edu.cn

***Abstract*-Multidimensional scaling is a set of data analysis techniques. It pictures the structure of a set of objects from data that approximate the distances between pairs of the objects, and has been proposed for wireless network based mobile location. However, classical multidimensional scaling is sensitive to the range measurements error due to non-line-of-sight propagation. This paper presents a dynamic multidimensional scaling algorithm for mobile localization. The performance is analyzed in the minimum localization system using time-of-arrival (TOA) measurements from three base stations. Simulation results show that the algorithm performs well in non-line-of-sight scenario.**

I. INTRODUCTION

Determining the position of a mobile station (MS) in wireless communication system becomes a popular issue in recent years. One of the major driving forces is due to the accuracy requirement of Enhenced-911 (E-911) safety services released by the U.S. Federal Communications Commission (FCC) [1]. With the popularization of wireless services, more and more people call for emergence services by wireless phone [2]. Besides emergency services, the mobile location information can be useful in cellular system design and management, or other location-based applications such as mobile yellow pages, route guidance, and traffic information [3]. Many methods have been proposed to estimate the source position from range measurements [3-5].

Multidimensional scaling (MDS) is a set of data analysis techniques that display the structure of a set of objects as geometrical picture from data that approximate the distances between pairs of the objects. It is commonly used in psychophysics, behavioral science, food science, etc. [6]. Since the distance measurement between the mobile station (MS) and the BS bear analogy to the requirements of MDS, the MDS technique has been proposed for mobile location using Time-of-Arrival (TOA) measurements [7-8]. Assigned a point as the origin, classical MDS can be used to determine the relative coordinates of all points by their inter-point distance. In [7], a practical mobile location algorithm is derived via modifying of the classical MDS for saving computational demand. However, it is sensitive to the range measurements error caused by non-line-of-sight (NLOS) propagation through reflected, diffracted, or scattered paths due to buildings and other obstacles. To avoid this, a complicated preprocessing for mitigating the NLOS is needed. In [8], a novel noise subspace based method is applied to estimate the source position. For dimensional knowledge of the localization problem, it performs better than several conventional approaches.

It has been shown that time filtering of the location estimates can improve the location accuracy [9-10]. In this paper, we propose a dynamic MDS algorithm for mobile location. It improves the performance of mobile location by combining the location information from measurements made at several sampling time periods together into MDS structure. The matrix constructed for employing MDS is extended, and the MS position is calculated utilizing the noise subspace method proposed in [8]. The variation of MS position between two sampling time periods is also estimated.

The rest of the paper is organized as follows. Section II gives a brief view of classical MDS positioning algorithm. Section III develops an iterative dynamic MDS algorithm for mobile location. Simulation results are shown in Section IV, followed by conclusion in Section V.

II. POSITIONING ALGORITHM BY STATIC MDS

As described in [7], when employing MDS in mobile location, we first construct a matrix of squared distances between pairs of BSs and MS, as

$$\mathbf{D} = \begin{bmatrix} \mathbf{G} & \mathbf{h} \\ \mathbf{h}^T & 0 \end{bmatrix}, \tag{1}$$

where

$$\mathbf{G} = \begin{bmatrix} 0 & d_{12}^2 & \cdots & d_{1N}^2 \\ d_{21}^2 & 0 & \cdots & d_{2N}^2 \\ \vdots & \vdots & \ddots & \vdots \\ d_{N1}^2 & d_{N2}^2 & \cdots & 0 \end{bmatrix}, \tag{2}$$

$$\mathbf{h} = \begin{bmatrix} r_1^2 & r_2^2 & \cdots & r_N^2 \end{bmatrix}^T, \tag{3}$$

where d_{ij} is the distance between the i^{th} BS and the j^{th} BS and free of error, r_i denotes the range measurement between MS and the i^{th} BS by multiplication of the TOA measurements by the speed of light, N is the number of BS, and $[\ ]^T$ denotes the transpose operation of matrix. The range measurements can be modeled as

$$r_i = l_i + n_i + \eta_i, \tag{4}$$

where l_i is the true range between the MS and the i^{th} BS, n_i is the measurement noise, η_i is the excess range due to NLOS error.

Assuming MS and BSs are the points in some space and the origin is at the point x_o contained in the space, we can obtain a matrix $\mathbf{B}$, whose element is of the form

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$$b_{ij} = \frac{1}{2}(z_{io}^2 + z_{jo}^2 - D_{ij}^2), \tag{5}$$

in which z_{io} or z_{jo} is the distance from the origin to the i^{th} point or the j^{th} point, D_{ij} is the element of **D**, $i, j = 1, \ldots, N + 1$. From cosine theorem, it is easily proved that matrix **B** is a scalar product matrix and can be expressed as

$$\mathbf{B} = \begin{bmatrix} x_{BS1} - x_o \\ x_{BS2} - x_o \\ \vdots \\ x_{BSN} - x_o \\ x_{MS} - x_o \end{bmatrix} \begin{bmatrix} x_{BS1} - x_o \\ x_{BS2} - x_o \\ \vdots \\ x_{BSN} - x_o \\ x_{MS} - x_o \end{bmatrix}^T, \tag{6}$$

where $x_{BS1}, x_{BS2}, \cdots x_{BSN}, x_{MS}$ denotes coordinates of BSs and MS with dimension 1×2. If we can obtain **B** from known matrix **D**, the MS position can be found using LS estimate based method [7] or noise subspace based method [8].

The conversion from matrix **B** to **D** is concerned with the choice of the origin. It is commonly recommended to choose the centroid of all objects as origin because a simple conversion from matrix **D** to **B** exists, as

$$\mathbf{B} = -\frac{1}{2}\left(\mathbf{I} - \frac{1}{N+1}\mathbf{E}\right)\mathbf{D}\left(\mathbf{I} - \frac{1}{N+1}\mathbf{E}\right), \tag{7}$$

where **E** is $(N + 1) \times (N + 1)$ matrix consisting entirely of 1s, **I** is identity matrix. In [7], the origin is the centroid of all BSs, and the corresponding conversion is derived. In [8], the MS' position is chosen as the origin, i.e., $x_o = x_{MS}$, then we have

$$\mathbf{b}_1 = \begin{bmatrix} x_{BS1} - x_{MS} \\ x_{BS2} - x_{MS} \\ \vdots \\ x_{BSN} - x_{MS} \end{bmatrix} \begin{bmatrix} x_{BS1} - x_{MS} \\ x_{BS2} - x_{MS} \\ \vdots \\ x_{BSN} - x_{MS} \end{bmatrix}^T = \mathbf{P}\mathbf{P}^T. \tag{8}$$

Define

$$\mathbf{Q} = \mathbf{h}\mathbf{e}_N^T + \mathbf{e}_N\mathbf{h}^T - \mathbf{G}, \tag{9}$$

where $\mathbf{e}_N$ is $N \times 1$ column vector of all ones. From (5), we have

$$\mathbf{Q} = 2\mathbf{P}\mathbf{P}^T. \tag{10}$$

Thus, the MS position can be calculated from (2), (3), (8), (9) and (10).

III. Dynamic MDS for Mobile Location

In this section, a dynamic MDS algorithm based on TOA measurements is developed. It is assumed that the measurements are made from M sampling time periods since time instance k, and the position of MS is $x_{MS}(k + j) = (a_{k+j}, b_{k+j})$, $j = 0, 1, \ldots, M$-1. The distances between MS and BSs is given by

$$\mathbf{h}_{k+j} = \left[r_1^2(k+j) \quad r_2^2(k+j) \quad \cdots \quad r_N^2(k+j)\right]^T, \tag{11}$$

and the assumed distances between $x_{MS}(k)$ and $x_{MS}(k + n)$, $n = 1, 2, \ldots, M$-1, is

$$\mathbf{g}_k = \left[g_{(k+1)k}^2 \quad g_{(k+2)k}^2 \quad \cdots \quad g_{(k+M-1)k}^2\right]^T. \tag{12}$$

In this situation, we first construct the matrix **P** in (8) and **Q** in (9). Here, the position of MS at time instance k is chosen as the origin, and we have

$$\mathbf{P} = \begin{bmatrix} x_{BS1} - x_{MS}(k) \\ x_{BS2} - x_{MS}(k) \\ \vdots \\ x_{BSN} - x_{MS}(k) \end{bmatrix}, \tag{13}$$

$$\mathbf{Q} = \mathbf{h}_{k+0}\mathbf{e}_N^T + \mathbf{e}_N\mathbf{h}_{k+0}^T - \mathbf{G}. \tag{14}$$

The next is to extend matrix **P** and **Q** as $\mathbf{P}_k$ and $\mathbf{Q}_k$, called dynamic matrix, respectively. $\mathbf{P}_k$ is constructed as

$$\mathbf{P}_k = \begin{bmatrix} \mathbf{P}^T & \begin{matrix} a_{k+1} - a_k & a_{k+2} - a_k & \cdots & a_{k+M-1} - a_k \\ b_{k+1} - b_k & b_{k+2} - b_k & \cdots & b_{k+M-1} - b_k \end{matrix} \end{bmatrix}^T, \tag{15}$$

and $\mathbf{Q}_k$ is

$$\mathbf{Q}_k = \begin{bmatrix} \mathbf{Q} & \mathbf{h}_{k+0}\mathbf{e}_{M-1}^T + \mathbf{e}_N\mathbf{g}_k^T - \mathbf{H}_k \\ (\mathbf{h}_{k+0}\mathbf{e}_{M-1}^T + \mathbf{e}_N\mathbf{g}_k^T - \mathbf{H}_k)^T & \mathbf{g}_k\mathbf{e}_{M-1}^T + \mathbf{e}_{M-1}\mathbf{g}_k^T - \mathbf{G}_k \end{bmatrix}, \tag{16}$$

where $\mathbf{e}_{M\text{-}1}$ is $(M\text{-}1) \times 1$ column vector of all ones, $\mathbf{G}_k$ is the matrix of squared distances between MSs, as

$$\mathbf{G}_k = \begin{bmatrix} 0 & g_{(k+2)(k+1)}^2 & \cdots & g_{(k+M-1)(k+1)}^2 \\ g_{(k+1)(k+2)}^2 & 0 & \cdots & g_{(k+M-1)(k+2)}^2 \\ \vdots & \vdots & \ddots & \vdots \\ g_{(k+1)(k+M-1)}^2 & g_{(k+2)(k+M-1)}^2 & \cdots & 0 \end{bmatrix}, \tag{17}$$

$\mathbf{H}_k$ is the matrix of squared distances between MS and BSs and can be expressed as

$$\mathbf{H}_k = \left[\mathbf{h}_{k+1} \quad \mathbf{h}_{k+2} \quad \cdots \quad \mathbf{h}_{k+M-1}\right]. \tag{18}$$

Similarly, we have

$$\mathbf{Q}_k = 2\mathbf{P}_k\mathbf{P}_k^T. \tag{19}$$

It is noteworthy that (16) involves two unknowns, $\mathbf{g}_k$ and $\mathbf{G}_k$, which implies that the MS position cannot be found directly as described in previous section. In the following, we will develop an iterative dynamic MDS positioning algorithm.

First of all, assuming that MS moves along a straight line with a regular velocity during M sampling time periods, i.e.,

$$(a_{k+j}, b_{k+j}) \approx (a_k, b_k) + j \times (\Delta_{ka}, \Delta_{kb}), \tag{20}$$

where $(\Delta_{ka}, \Delta_{kb})$ is the variation of MS position between two sampling time periods, $j = 0, 1, \ldots, M-1$, then the distance between $x_{MS}(k + i)$ and $x_{MS}(k + j)$ can be approximated as

$$g_{(k+i)(k+j)} \approx (j - i)\Delta_k, \tag{21}$$

where $i, j = 0, 1, \ldots, M$-1, and

$$\Delta_k = \sqrt{\Delta_{ka}^2 + \Delta_{kb}^2}. \tag{22}$$

Substituting (21) into (12) and (17), we have

$$\mathbf{g}_k \approx \Delta_k^2\left[1^2 \quad 2^2 \quad \cdots \quad (M-1)^2\right]^T, \tag{23}$$

$$\mathbf{G}_k \approx \Delta_k^2 \begin{bmatrix} 0 & 1^2 & \cdots & (M-2)^2 \\ 1^2 & 0 & \cdots & (M-3)^2 \\ \vdots & \vdots & \ddots & \vdots \\ (M-2)^2 & (M-3)^2 & \cdots & 0 \end{bmatrix}. \tag{24}$$

When Δ_k is known or can be estimated, $\mathbf{g}_k$ and $\mathbf{G}_k$ are determined, and then $\mathbf{Q}_k$ can be computed.

From (15) and (19), it is clear that the rank of matrix $\mathbf{Q}_k$ equals to 2. Because $\mathbf{Q}_k$ is symmetrical, we can let the singular value decomposition (SVD) of $\mathbf{Q}_k$ be

$$\mathbf{Q}_k = \sum_{m=1}^{M+N-1} \sigma_{km} \mathbf{u}_{km} \mathbf{u}_{km}^T, \quad (25)$$

where σ_{km} is eigenvalue of $\mathbf{Q}_k$, and $\mathbf{u}_{km}$ is corresponding eigenvector. At the same time, we have

$$\sigma_{k1} \geq \sigma_{k2} > \sigma_{k3} \approx \cdots \approx \sigma_{k(M+N-1)} \approx 0. \quad (26)$$

Define matrix

$$\mathbf{U}_k = \begin{bmatrix} \mathbf{u}_{k3} & \mathbf{u}_{k4} & \cdots & \mathbf{u}_{k(M+N-1)} \end{bmatrix}, \quad (27)$$

and from (19), (25) and (26), we can obtain

$$\mathbf{U}_k^T \mathbf{P}_k \approx \mathbf{0}_{(M+N-3)\times 2}, \quad (28)$$

where $\mathbf{0}_{(M+N-3)\times 2}$ is matrix consisting entirely of zeros. Using (20), (15) is then expressed as

$$\mathbf{P}_k \approx \begin{bmatrix} \mathbf{X} - a_k \mathbf{e}_N & \mathbf{Y} - b_k \mathbf{e}_N \\ \Delta_{ka}\mathbf{s} & \Delta_{kb}\mathbf{s} \end{bmatrix}, \quad (29)$$

where

$$\mathbf{s} = \begin{bmatrix} 1 & 2 & \cdots & M-1 \end{bmatrix}^T, \quad (30)$$

$\mathbf{X}$ and $\mathbf{Y}$ is $N\times 1$ vector consisting of BSs' abscissa and ordinate, respectively. Substituting (29) into (28), we have

$$\mathbf{U}_k^T \begin{bmatrix} \mathbf{X} - a_k \mathbf{e}_N \\ \Delta_{ka}\mathbf{s} \end{bmatrix} \approx \mathbf{0}_{(M+N-3)\times 1}, \quad (31)$$

$$\mathbf{U}_k^T \begin{bmatrix} \mathbf{Y} - b_k \mathbf{e}_N \\ \Delta_{kb}\mathbf{s} \end{bmatrix} \approx \mathbf{0}_{(M+N-3)\times 1}. \quad (32)$$

We partition $\mathbf{U}_k$ in (27) as

$$\mathbf{U}_k = \begin{bmatrix} \mathbf{U}_{k1} & \mathbf{U}_{k2} \end{bmatrix}, \quad (33)$$

where $\mathbf{U}_{k1}$ and $\mathbf{U}_{k2}$ is the first N columns and the other columns of $\mathbf{U}_k$, respectively. Sequentially, the estimates of MS position at time instance k and the variation of MS position between two sampling time periods can be given by

$$\begin{bmatrix} \hat{a}_k \\ \hat{\Delta}_{ka} \end{bmatrix} = \begin{bmatrix} \mathbf{e}_N & \mathbf{0}_N \\ \mathbf{0}_{M-1} & -\mathbf{s} \end{bmatrix}^{+} \mathbf{U}_{k1}^T \mathbf{X}, \quad (34)$$

$$\begin{bmatrix} \hat{b}_k \\ \hat{\Delta}_{kb} \end{bmatrix} = \begin{bmatrix} \mathbf{e}_N & \mathbf{0}_N \\ \mathbf{0}_{M-1} & -\mathbf{s} \end{bmatrix}^{+} \mathbf{U}_{k1}^T \mathbf{Y}, \quad (35)$$

where $[]^{+}$ denotes Moore-Penrose pseudoinverse of matrix.

Given the N BSs' position and the range measurements between MS and BSs at M sampling time periods, the procedure of our proposed iterative dynamic MDS positioning algorithm is presented as follows.

i) Initially, let $k = 1, \Delta_1 = 0$.

ii) Construct $\mathbf{Q}_k$ using (11), (14), (23) and (24).

iii) Decompose $\mathbf{Q}_k$ as $\mathbf{Q}_k = \sum_{m=1}^{M+N-1} \sigma_{km} \mathbf{u}_{km} \mathbf{u}_{km}^T$, and then construct matrix $\mathbf{U}_{k1}$.

iv) Find the estimates of MS position at time instance k, (a_k, b_k), and the variation of MS position between two sampling time periods, $(\Delta_{ka}, \Delta_{kb})$.

v) Compute $\Delta_k = \sqrt{\hat{\Delta}_{ka}^2 + \hat{\Delta}_{kb}^2}$, and let $k = k + 1$, repeat step ii) until the positioning requirement is stopped such as k equals to K.

IV. Simulation Results

In this section, we evaluate the performance of the dynamic MDS algorithm for mobile location. The evaluation is done in the minimum localization system using time-of-arrival measurements from three base stations. The BSs are presumed to be located at corners of an equilateral triangle, and their coordinate are (0, 0) m, (2500, 4330) m and (5000, 0) m. The MS user is assumed to move in the triangle.

We assume the measurement noise in (4) is modeled as zero-mean Gaussian random variable with standard deviation (SD) 10m, the NLOS error is modeled as single sided Gaussian distribution

$$p(l) = \begin{cases} \dfrac{2}{\sqrt{2\pi\sigma_G^2}} e^{-\frac{l^2}{2\sigma_G^2}}, l \geq 0 \\ 0, \ \ others \end{cases}, \quad (36)$$

where σ_G is the corresponding standard deviation.

The motion of MS is modeled as

$$(a_{k+j}, b_{k+j}) \approx (a_k, b_k) + j \times (\Delta_{ka}, \Delta_{kb}) + \xi, \quad (37)$$

where $j = 1, \ldots, M-1$, ξ is noise modeled by Gaussian random variable with mean 5m, and $(\Delta_{ka}, \Delta_{kb})$ are constants as (30, 40)m. The performance measures are the root mean square error (RMSE) of position. The RMSE is defined as

$$\mathrm{RMSE} = \sqrt{E\left[\tilde{x}^2 + \tilde{y}^2\right]}, \quad (38)$$

where E[]denotes the moment operator, $\tilde{x}$ and $\tilde{y}$ are the error in estimated x-coordinate and y-coordinate, respectively.

Fig. 1 shows the root mean square location error of the proposed method for various standard deviations of NLOS error, while the MS start from (-500, -348)m, M=25, and K is set to 50, the initial estimate of MS position's variation Δ_1 is chosen as 0. For comparison, the performance of classical MDS algorithm and Chan' method [11] are also given. The figure shows that dynamic MDS algorithm can improve the accuracy to some extent.

Let SD of NLOS error be 160m, and K is still 50. Fig. 2 shows the root mean square location error when M range from 5 to 30. From the figure, it can be seen that the dynamic MDS algorithm performs well when M>10.

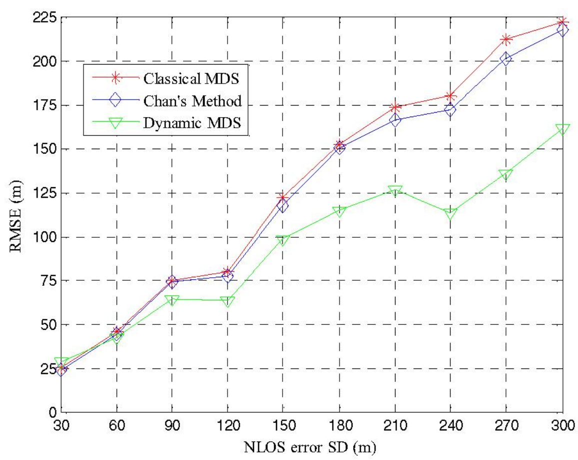

Figure 1. RMSE of dynamic MDS positioning algorithm against standard deviation of NLOS error.

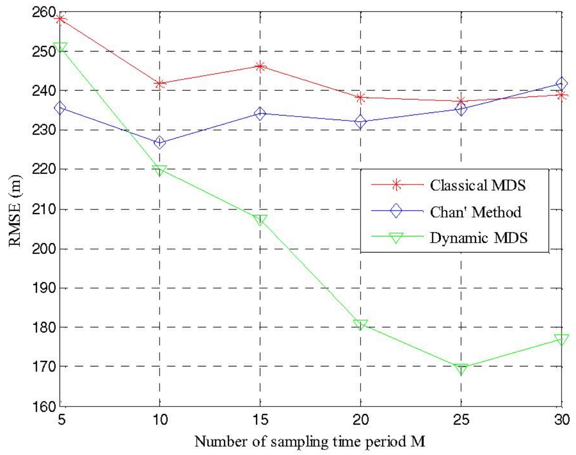

Figure 2. RMSE of dynamic MDS positioning algorithm against the number of sample time periods M.

From all the simulation results, we can find that the dynamic MDS positioning algorithm have the potential to make an improvement in mobile location accuracy. However, there are still some limitations for required conditions such as the number of sampling time periods should be more than 10.

V. CONCLUSIONS

In this paper, we develop a dynamic multidimensional scaling algorithm for mobile location. In our study, it is applied to wireless network based mobile location using range measurements. When NLOS error is modeled by single sided Gaussian model, simulation results show that the proposed algorithm has the potential to improve the mobile location performance. Future work will focus on the study for eliminating the impacts of some limitations and making further improvement of positioning accuracy.

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