Introduction to Electrical Engineering (83335)

Ori Katz

Moshe Krumbein

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https://github.com/outofink/notes

CONTENTS

INTRODUCTION TO ELECTRICAL ENGINEERING

1

LUMPED ELEMENTS (CIRCUITS)

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For example, if we attach a 1.5V with 2 wires to a light bulb, what is the current (amps) are running through the wires?

$$I = \frac{V}{R}$$

This is a major abstraction and is not always true!

We do know that Maxwell's Laws are always true:

1. $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$
2. $\vec{\nabla} \cdot \vec{B} = 0$
3. $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$
4. $\vec{\nabla} \times \vec{B} = \frac{1}{c}\left(4\pi\vec{j} + \frac{\partial \vec{E}}{\partial t}\right)$

When we can we use abstraction?

We can change an component and use its Volt-Current Relation (V-I). This can only be done if the voltage and current to be defined (injective).

Reminder. Review from electromagnetism:

1. <u>Current:</u> I which passes through area A which is defined to be the amount of charge that passes through A per time unit.

$$I \equiv \frac{dq}{dt}$$

Let's relate these equations:

$$I = \iint_A \vec{j}(\vec{r})\, ds$$

Reminder. Review from electromagnetism:

2. Voltage: Difference in electric potentials between between two points (symbolized by V, V_{ab}, V_{12})
   Voltage is a measure of the required energy/work needed in order to to move a charge from point a to point b.
   (Voltage is a amount of energy accumulated/flowing) in a component.)

$$(\text{Volt}) = \frac{(\text{Joule})}{(\text{Col})}\left(\frac{(\text{Energy})}{(\text{Charge})}\right)$$

$$W = \int_{r_1}^{r_2} \vec{F}(\vec{r})\, d\vec{r} = \int_{r_1}^{r_2} q\vec{E}(\vec{r})\, d\vec{r}$$

Only if $\vec{\boldsymbol{\nabla}} \times \vec{E} = 0$:

$$W = q(\varphi(\vec{r}_1) - \varphi(\vec{r}_2)) = q(V_2 - V_1) = qV_{12}$$

In other words, only when there is no change in the magnetic field (Maxwell's 4th law), or at the very least *quasistatic* (changes at an incredibly slow rate).

Requirements for the *lumped elements model*:

1. $\frac{\partial B}{\partial t} \approx 0$ outside of the components (required to define electric potential/voltage)

Reminder. Review from electromagnetism:

3. Power: Energy per unit of time (Watt):

$$P(t) \equiv \frac{dW}{dt} = \frac{dW}{dq} \cdot \frac{dq}{dt} = V(t) \cdot I(t)$$

   We "agree":

   - $P > 0$ - power is used in a component (resistor)
   - $P \leq 0$ - power is provided by a component

4. Direction and Signs:

$$B \to A \implies V = V_A - V_B$$

   $I > 0$ - The positive particles "move" from left to right

   The current is *positive* if the current is in the direction of the arrow.

   Note. It's easiest to define the direction of the positive current from the head of the electrical potential arrow to its tail.

Back to our requirements:

2. Current over components in series do not change:

$$
\begin{gathered}
I_A = I_B \\
I_A = I_B = 0 \\
\frac{dQ_{in}}{dt} - \frac{dQ_{out}}{dt} = 0 \\
\frac{d}{dt}(Q_{in} - Q_{out}) = 0 \\
\frac{d}{dt}\left(\sum_{\substack{\text{between} \\ A \text{ and } B}} Q\right) = 0
\end{gathered}
$$

   In other words charge is not created.

3. $V_{AB} = V_{CD} \leftarrow l \ll ct$
   In other words, we must only work with components/circuits that there size is are small enough in relation to measurement time.

$$
\begin{gathered}
2\pi f_o \Delta t \ll 2\pi \\
\Delta t \ll \frac{1}{f_0} = T
\end{gathered}
$$

$$
l \ll \frac{c}{f_0}
$$

1.1 Tricks to Solve Circuits

Ways to solve circuits:

1. By hand
2. Linear Algebra
3. “current-loop”
4. Simplification

1.1.1 Node Analysis

1. Select one node as the ground reference. The choice does not affect the element voltages (but it does affect the nodal voltages) and is just a matter of convention. Choosing the node with the most connections can simplify the analysis. For a circuit of N nodes the number of nodal equations is $N - 1$.

2. Assign a variable for each node whose voltage is unknown. If the voltage is already known, it is not necessary to assign a variable.
3. For each unknown voltage, form an equation based on Kirchhoff's Current Law (i.e. add together all currents leaving from the node and mark the sum equal to zero). The current between two nodes is equal to the voltage of the node where the current exits minus the voltage of the node where the current enters the node, both divided by the resistance between the two nodes.
4. If there are voltage sources between two unknown voltages, join the two nodes as a supernode. The currents of the two nodes are combined in a single equation, and a new equation for the voltages is formed.
5. Solve the system of simultaneous equations for each unknown voltage.

If there are only sources of current and resistors, the circuit can be plugged in directly into the matrix.