# Introduction to Signal Processing (67651)

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https://github.com/outofink/notes

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# INTRODUCTION TO SIGNAL PROCESSING

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# INTRODUCTION

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## 1.1 Background

Signal Processing is found everywhere.

In systems engineering, there are three main areas of signal processing:

- **Planning** - how to design a system to meet a given set of requirements
- **Analysis** - how to analyze a system to determine its performance
- **Control** - how to control a system to meet a given set of requirements

Why don't we just use Deep Neural Networks to process signals?

First, we don't completely know how or why DNNs work. Another reason is that it can be tricked. Third, it's a very expensive process. And fourth, they require a sample set which we often don't have.

Instead, we will use a *Fourier transform*.

## 1.2 Signal

A *signal* is a function of independent variable(s).

A signal can be with continuous or discrete time over continuous or discrete values.

Digital signals are discrete and analog signals are continuous.

Computers can only work will digital.

### 1.2.1 Converting Analog to Digital

First we *sample* the signal (measure at discrete points), do *digital signal processing*, and then *reconstruction*, which converts the digital signal back to analog.

We we loose information during sampling?

**Definition** (Nyquist–Shannon sampling theorem)**.** If a signal does not contain frequencies higher than $f_{max}$, then if we sample at $2f_{max}$, we will not lose any information.

Every signal can be split into an *even* part and an *odd* part.

### 1.2.2 Energy

$$E[x(t)] = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

It has the units of $[x]^2 T$.

*Note.* There as signals with infinite energy.

### 1.2.3 Power

$$P[x(t)] = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 \, dt$$

*Power* is the average energy of a signal.

The units are $\frac{E}{T}$.

### 1.2.4 Delta function

$$\int_{-\infty}^{\infty} f(t)\delta(t) \, dt = f(0)$$

$$\int_{-\infty}^{\infty} \delta(t) \, dt = 1$$

Shifting:

$$\int_{-\infty}^{\infty} f(t-T)\delta(t) \, dt = f(T)$$

Derivative of step function:

$$\frac{d}{dt} u(t) = \delta(t)$$

Derivative of delta:

$$\int_{-\infty}^{\infty} f(t)\delta'(t) \, dt = -f'(0)$$

It can be approximated by a rectangle, triangle, Gaussian, etc.

Discrete definition:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \qquad u[t] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

### 1.2.5 Complex Exponential

$$s = \sigma + j\omega$$
$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}(cos(\omega t) + j\sin(\omega t))$$

## 1.3 System

A *system* receives a signal as input, does some action, and returns a signal.

It is symbolized as $x(t) \implies y(f)$.

There are *single input single output* (SISO) and *multiple input multiple output* (MIMO) systems, or any combination of the two.

### 1.3.1 Connecting Systems

There are three ways to connect systems:

- *Parallel*: $y = Fu + Gu$
- *Series*: $y = G(Fu) = GFu$
- *Feedback*: $y = F(u - Gy)$

**Definition** (Linear System)**.** A system is *linear* if it has the following qualities:

- Additivity: $(x_1 \implies y_1, x_2 \implies y_2) \implies x_1 + x_2 \implies y_1 + y_2$
- Homogeneous: $(x \implies y) \implies k \cdot x \implies k \cdot y$

## 1.4 Time-invariant System

Not that the system isn't dependent on time (then nothing would happen), rather that the output given the input isn't dependent on time.

**Definition** (Time-invariant System)**.** Let $S_T$ be system shifted in time: $S_T[x(t)] = x(t - T)$. A *time-invariant system* $y$ for all $x(t)$:

$$y(S_T[x(t)]) = S_T[y(x(t))]$$

## 1.5 Linear Time-Invariant Systems

**Definition** (Linear Time-Invariant System)**.** A *linear time-invariant* system (LTI) is system that is both linear and time-invariant. There are two types of LTIs:

- LTI continuous (LTIC)
- LTI discrete (LTID)

*Claim.* Important claims:

- Chaining two LTI systems produce an LTI system
- A derivative or integral in time (from $-\infty$ to $t$) is an LTI system
- The *exponential function* is an *eigenfunction* on LTI systems

*Proof.* Let $y_1(t)$ be an LTI system. Let the input be of the form $x_1(t) = e^{\omega}t$. We define:

$$e^{\omega t} \implies y_1(t)$$

We multiply the equation by a number $e^{\omega T}$. By linearity we know that:

$$e^{\omega t}e^{\omega T} \implies y_1(t)e^{\omega T}$$

By time-invariance:

$$e^{\omega t}e^{\omega T} = e^{\omega(t+T)} \implies y_1(t+T)$$

Therefore:

$$y_1(t+T) = y_1(t)e^{\omega T}$$

If we define $t = 0$:

$$y_1(T) = y_1(0)e^{\omega T}$$

Instead of $T$ we substitute $t$:

$$y_1(t) = y_1(0)e^{\omega t} = y_1(0)x_0(t)$$

In other words:

$$e^{\omega t} \implies y_1(0)e^{\omega t}$$

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## 1.6 Causal Systems

**Definition** (Causal Systems)**.** A *causal system* as a system that dependent only on its past, and not its future.