Conflict-based Plan Merging for Multi-agent Pathfinding

Master Thesis

Aurélien SIMON

Matrikel-Nr: 806567

Supervisors

Klaus Strauch

Etienne Tignon

Torsten Schaub

Potsdam University

Master Cognitive Systems: Language, Learning and Reasoning

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Abstract

In this Master's Thesis, we aim introduce an approach for Multi-Agent Pathfinding (MAPF) problems, an artificial intelligence problem with wide-ranging applications including GPS, video games, and traffic control. MAPF involves orchestrating multiple agents to navigate from initial positions to specific destinations while avoiding collisions. This study focuses on the "Plan Merging" approach, a three-step approach based on path conflicts designed to tackle MAPF problems. The three steps are designated Individual Path Finding, Path Selection and Solving. This master thesis demonstrates that in certain cases, our proposed approaches can outperform classical MAPF methods, especially for large instances. However, it also highlights the inherent limitations of our plan merging techniques, particularly in scenarios where obtaining a complete solution is challenging or not feasible.

In dieser Masterarbeit wollen wir einen Ansatz für Multi-Agent Pathfinding (MAPF) Probleme vorstellen, ein Problem der künstlichen Intelligenz mit weitreichenden Anwendungen wie GPS, Videospiele und Verkehrssteuerung. Bei MAPF geht es darum, mehrere Agenten so zu koordinieren, dass sie von ihren Ausgangspositionen zu bestimmten Zielen navigieren und dabei Kollisionen vermeiden. Diese Studie konzentriert sich auf den "Plan Merging"-Ansatz, einen dreistufigen Ansatz, der auf Pfadkonflikten basiert und zur Lösung von MAPF-Problemen entwickelt wurde. Die drei Schritte werden als Individual Path Finding, Path Selection und Solving bezeichnet. Diese Masterarbeit zeigt, dass die von uns vorgeschlagenen Ansätze in bestimmten Fällen die klassischen MAPF-Methoden übertreffen können, insbesondere bei großen Instanzen. Sie zeigt aber auch die inhärenten Grenzen unserer Plan-Merging-Techniken auf, insbesondere in Szenarien, in denen eine vollständige Lösung schwierig oder nicht machbar ist.

Declaration of originality

I confirm that the master's thesis I have submitted is my own original work. I have not utilized any external sources or aids except for the ones explicitly mentioned. In cases where I have incorporated verbatim passages from other works, I have duly acknowledged the source through proper citations. This work has not been previously submitted for any course or examination, nor has it been presented to any other authority for approval.

Eigenständigkeitserklärung

Ich versichere, dass ich die eingereichte Masterarbeit selbstständig verfasst habe. Ich habe keine externen Quellen oder Hilfsmittel verwendet, außer denen, die ausdrücklich genannt sind. In den Fällen, in denen ich wörtliche Passagen aus anderen Arbeiten übernommen habe, habe ich die Quelle durch ordnungsgemäße Zitate ordnungsgemäß angegeben. Diese Arbeit wurde weder für einen Kurs oder eine Prüfung eingereicht, noch wurde sie einer anderen Behörde zur Genehmigung vorgelegt.

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Chapter 1

Introduction & Background

1.1 Introduction

Multi-Agent Pathfinding (MAPF) [15–17], is a fundamental artificial Intelligence problem with diverse real-world applications such as GPS, video games, routing, planning, and traffic control. In essence, MAPF involves multiple agents moving within an environment, aiming to navigate from initial positions to goal locations. The primary challenge in MAPF is to find individual paths for each agent, ensuring that they do not collide. Various techniques have been developed to tackle this problem, including search algorithms like CBS [14] (Conflict-Based Search) and reduction-solving-based methods [4].

In this Master Thesis, we focus on the "Plan Merging" approach, which aims to solve MAPF problems through a three-steps process. The first step, denoted as Individual Path Finding, involves computing paths for each agent independently, which corresponds to classic pathfinding or single-agent pathfinding [6]. The second step denoted Path Selection focuses on finding a collision-free solution using the previously computed path where selection occurs using conflict represented in two ways; potential conflict and likelihood of presence using heatmaps. The third and last step of Plan Merging is to find a solution, by reducing the size of the problem using paths selected in previous step to delimitate a subgraph. Or by using the conflict-free paths issued from previous step as a baseline for a MAPF algorithm. Each step's formalization will be described in their respective sections. The appeal of this approach lies in the lower complexity of pathfinding compared to the overall MAPF problem [13]. Plan Merging approach anticipates saving computation time at the cost of a possible loss of optimality. For instance, given some robots in a warehouse moving from a shelf to another, we, first computes some paths for them regardless of their conflict among each other. We then eliminate paths that are "bad" through a heuristic, which can be in our case, through heatmap. If a complete conflict-free solution cannot be found among the remaining paths, we use the paths that are not conflicting as a preprocessing step for a classical MAPF approach.

The approach in its definition is inspired and close to CBS's definition, however, the planning part of CBS stops if a conflict occurs and re-iter the planning part considering the conflict previously encountered, which is not the case for the Individual Path Finding; conflicts are handled after Individual Path Finding. This works falls within different MAPF topics, such as map reduction, graph-pruning, plan reuse and distributed planning.

Furthermore, our study shows that in certain cases, especially for large instances, our proposed approaches can outperform classical MAPF methods in terms of computational time. On the other hand, results will outline limits of the plan merging techniques we have developed; in certain scenarios, obtaining a complete solution might prove challenging or not feasible with the approaches we introduced, indicating the need to recognize the inherent limitations of the plan merging strategies.

1.2 Background

1.2.1 MAPF

The following definitions of Multi-Agent Path Finding (MAPF) follow the ones in [10]. MAPF is a triple (V, E, A) where V, E denotes a connected graph, V being a set of vertices and E a set of edges connecting them. Then A being a set of agents. For each agent $a = (s, g) \in A$, s is a vertex in V denoting the starting location and g is also a vertex in V denoting the goal location. We consider that every starting position and every goal position are disjoint. For each discrete time step $t \in \mathbb{N}_0$, an agent can either; wait at its current vertex or move to a neighbouring one.

The output for MAPF problems is a plan denoted as Π. A plan being a collection $(\pi_a)_{a \in A}$ of finite sequences in (V, E) where each sequence π_a is represented by a finite sequence of adjacent or identical vertices in V from s to g for agent $a = (s, g)$. We use $\pi_a(t) = v$ to denote that agent a is located at vertex v at time step t. As consequences, for each $a = (s, g) \in A$, we have $\pi_a(0) = s$ and $\pi_a(|\pi_a| - 1) = g$ (where $|\pi_a|$ gives the length of sequence π_a). Generally, for any $a = (s, g)$ and any $0 \leq t \leq |\pi_a| - 1$, we have $\pi_a(t) \in V$. In addition, we also have $(\pi_a(t), \pi_a(t + 1)) \in E$ with $0 \leq t < |\pi_a| - 1$

A plan is considered as *valid* if, taken pair wisely, sequences are collision-free. A vertex conflict occurs whenever two different agents occupy the same vertex at the same time step. Formally, we have $conflict(a, a', t)$ if given any $a, a' \in A$ and $t \in \mathbb{N}_0$, we have $\pi_a(t) = \pi_{a'}(t)$. An edge conflict (or swapping conflict) occurs whenever two agents exchange their position or are using the same vertex at the same time, which implies that edge conflict is defined on time step t and $t - 1$. We have $conflict(a, a', t)$ if given any $a, a' \in A$ and $t \in \mathbb{N}_0$, we have $\pi_a(t - 1) = \pi_{a'}(t)$ and $\pi_a(t) = \pi_{a'}(t - 1)$.

A plan $(\pi_a)_{a \in A}$ has a conflict, if a conflict (a, a', t) occurs in $(\pi_a)_{a \in A}$ for some pair $a, a' \in A$ of agents and a time step $t \in \mathbb{N}_0$.

Sum-of-costs and *makespan* of a plan $(\pi_a)_{a \in A}$ are respectively defined as

such; $\sum_{a \in A}(|\pi_a| - 1)$ and $\max_{a \in A}(|\pi_a| - 1)$.

Furthermore, we also define in which MAPF problem specification the following work has been conducted. Since we would define object that require distance and/or coordinates such as rectangle or circles, we assume that graphs have Cartesian coordinate system, which means we can represent them as a grid.

1.2.2 ASP

Answer Set Programming (ASP) [1] is an approach for declarative logic programming. In a nutshell, programmer aims to describe problems instead of solving them.

A logic program P is defined by a finite set of rules. A rule is defined with a set of atoms $\mathcal{A}$. A rule r is of the form

$$a_0 \leftarrow a_1, \ldots, a_n, not\ a_{n+1}, \ldots, not\ a_m$$

with a_i from a_0 to a_n , atoms of $\mathcal{A}$, and a_i from a_{n+1} to a_m being default negated atoms. We call literal an atom or a negated atom.

In a rule r, a_0 represent the head of the rule and $\{a_1, \ldots, a_m, not\ a_{m+1}, \ldots, not\ a_n\}$ the body of the rule. We will refer respectively to them as such; $H(r)$ the head of the rule and $B(r)$ the body of the rule. For each rule r in P, we define $B^+(r) = \{a_0 \leftarrow a_1\}$, $B^-(r) = \{a_m, not\ a_{n+1}, \ldots, not\ a_m\}$ and $H(r) = a_0$. Then, we define $B(r) = B^+(r) \cup B^-(r)$ and $B(P) = \{B(r) | r \in P\}$. A **fact** is defined as a rule with an empty body. On the other hand, with an empty head, the rule become an integrity or hard **constraint**. We define P_X as the reduct of a program P relative to a set of assigned atom X.

$$P_X = \{H(R) \leftarrow B^+(R) | r \in P, X \cap B^-(R) = \emptyset\}$$

A set X of atoms is a stable model of P if P_X is the smallest set under P_X.

For instance, lets encode the following problem. We want to pack as many object in our bag respecting the maximum authorized weight. First lets denote the possible object.

```
1 object("Trousers", 3).
2 object("T-shirt", 2).
3 object("Tooth brush", 1).
4 object("Camera", 2).
5 object("Tent", 8).
6 object("Drinks", 2).
7 object("Food", 2).
8 object("Laptop", 3).
9 object("Tablet", 2).
```

To decide which objects to pick, we utilize a choice rule that provides options for selecting objects. A choice rule provide a choices over a subset of atoms. To continue our example, we define

```
1 {pick(N,W):object(N,W)}.
```

Here, the choice rule allows for various combinations of objects to be picked. We then enforce a constraint that ensures the total weight of the selected objects does not exceed 20. This constraint is expressed using an aggregate function within an integrity constraint:

```
1 :- #sum{W : pick(N,W)} > 20.
```

Lastly, we want to maximize the weight of the bag while still keeping it under 20. To achieve this, we use an optimization statement that seeks to find the maximum weight among the available combinations:

```
1 #maximize{W : pick(N,W)}.
```

This encoding will yield the optimal combination of objects to pack into the bag, maximizing the weight without having the weight above 20.

1.2.3 Solving MAPF with ASP

Format Encoding

To define MAPF problems. The used predicates are explained in the following list 1.2.3.

1. $vertex/1$ which describe a vertex of the graph. With a tuple $(X, Y), X, Y \in \mathbb{N}^+$ expressing the position.
2. $edge/2$ which describe an edge between two vertices. With two tuple $(X, Y), X, Y \in \mathbb{N}^+$ expressing the endpoints of the edge.
3. $agent/1$ which define agents of an instance with its identifier as parameter
4. $start/2$ which describe the starting position of an agent. With an agent identifier as first parameter and a vertex as second parameter expressing the starting position.
5. $goal/2$ which describe the goal position of an agent. With an agent identifier as first parameter and a vertex as second parameter expressing the goal position.

We also have some additional predicates 1.2.3 which are optional information for the computation of individual. They are defined as such:

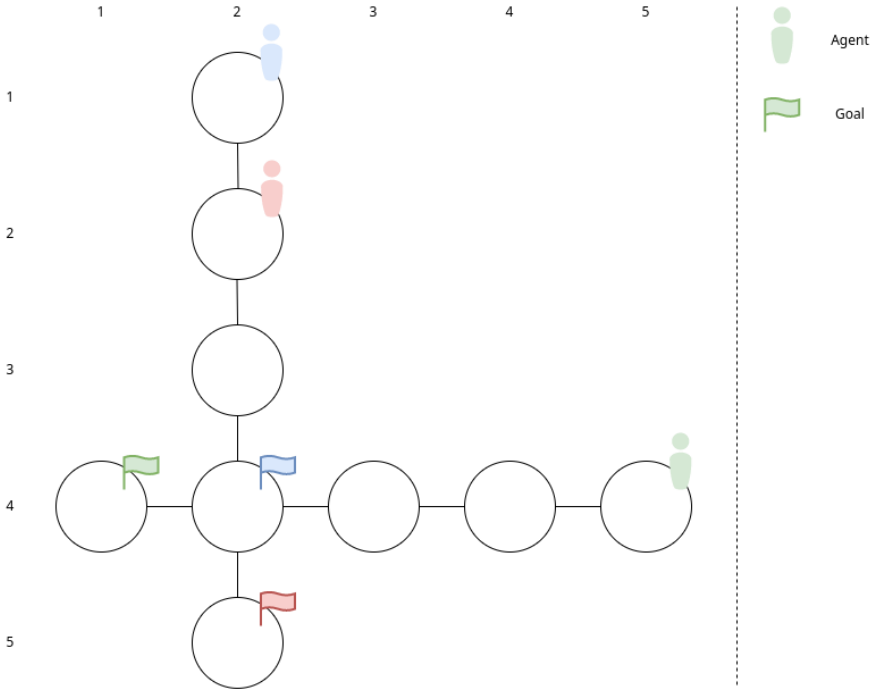
1. $shortestpath_length/2$ which represent the length of shortest path for an agent. With the identifier of an agent as first parameter and the size of a possible shortest path for the associated agent.
2. $makespan/1$ which describe the makespan of the instance with its value as parameter. It can also be defined as a *horizon* constant

For example, the following instance encoding describe the graph illustrated by figure 1.1.

Listing 1.1: Example of instance format

```
agent(1). start(1,(2,1)). goal(1,(2,4)).
agent(2). start(2,(2,3)). goal(2,(2,5)).
agent(3). start(3,(5,4)). goal(3,(1,4)).

vertex((2,3)). vertex((1,4)). vertex((2,4)).
vertex((3,4)). vertex((4,4)). vertex((2,1)).
vertex((5,4)). vertex((2,5)). vertex((2,2)).

edge((2,4),(1,4)). edge((3,4),(2,4)).
edge((4,4),(3,4)). edge((5,4),(4,4)).
edge((2,3),(2,2)). edge((2,4),(2,3)).
edge((2,5),(2,4)). edge((2,2),(2,1)).
edge((2,3),(2,4)). edge((2,4),(2,5)).
edge((2,1),(2,2)). edge((2,2),(2,3)).
edge((1,4),(2,4)). edge((2,4),(3,4)).
edge((3,4),(4,4)). edge((4,4),(5,4)).

% Optional
shortestpath_length(1,3).
shortestpath_length(2,2).
shortestpath_length(3,4).

makespan(4).
```

Figure 1.1: Illustrated graph of the instance format example 1.1



Solver encoding

As baseline for ASP MAPF solver, we use a MAPF encoding available in the Github framework *asprilo* [7], we will call it **base MAPF encoding**. We opted for this encoding due to its simplicity in comprehension and its smooth adaptability to a plan merging approach. In the encoding, we use two primary predicates outlined in the following list 1.2.3.

1. $at(R, P, T)$ representing the position P of an agent R at time step T.
2. $move(R, U, V, T)$ representing the movement from vertex U to vertex V at time step T.

Listing 1.2: Base MAPF encoding

```
1   at(R,P,0) :- start(R,P).
2   time(1..horizon).
3
4   {move(R,U,V,T) : edge(U,V)} 1 :- agent(R), time(T).
5
6   at(R,V,T) :- move(R,_,V,T).
7   :- move(R,U,_,T), not at(R,U,T-1).
8   at(R,V,T) :-
9       at(R,V,T-1),
10      not move(R,V,_,T),
11      time(T).
12
13  :- { at(R,V,T) }!=1 , agent(R), time(T).
14
15  :- {at(R,V,T) : agent(R)} > 1, vertex(V), time(T).
16  :- move(_,U,V,T), move(_,V,U,T), U < V.
17
18  :- goal(R,V), not at(R,V,horizon).
```

In the initial section of encoding 1.2, we define, respectively on lines 1 and 2, the starting positions of each agent and their allocated time using the constant *horizon*.

The rules outlined at line 4 describe how movement are performed. A possible interpretation could be; for each agent at each available time step and among all available edge going from U to V, we pick at most one to define one movement at each time.

The rule at line 6 states that if there is a move action is performed by the agent from any location (indicated by _) to a different one, the agent is positioned at this destination at this time step. Then, the rule at line 7 is a constraint that ensures consistency and coherence in the agent's locations over time. It prevents agents from moving towards a location without having been to the source location at the previous time step. Furthermore, the rule at line 8 specifies that if an agent does not make a move at a specific time step, their location remains the same. Finally, the rule 13 ensure that an agent is at, at most one defined position at a time.

To conclude the explanation of the base MAPF encoding, the last code section describe how vertex and edge (swap conflict) collisions and goal reaching satisfaction are encoded. First, line 15 outline vertex collision with a constraint rules; for each vertex, at most one agent can be positioned at a vertex. Secondly, the rule 16 ensure that, no movement from vertex U to V and no movement from V to U are issued at the same time. Lastly, the rules 18 force that, reaching the last possible time step defined by the constant *horizon*, the position of any agent have to be the same as its goal position defined by the predicate $goal/3$.

The output of the encoding is the predicate $at/3$.

1.3 Overview

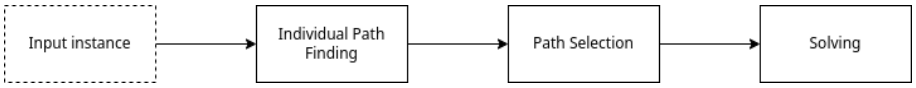
Plan merging involves the integration of multiple paths derived from various agents to construct a plan. Individual Path Finding 2 which represent the computation part of multiple paths dor agents, can be a constituent of Plan Merging but it is not necessary.

The second step of Plan Merging is Path Selection 3. Path Selection having the goal of identifying paths that closely resemble valid solutions to the problem at hand. However, it necessitates the construction of conflict-free set of path, which is computationally intensive, especially with a lot of agents and a lot of paths. This is where we incorporate Path Elimination to reduce the number of paths by eliminating potentially "problematic" paths using conflict-based strategy.

The final stage of Plan Merging involves obtaining a solution, which can be achieved by utilizing the pre-computed paths provided by Path Selection, by using the paths in their original form, or by employing the delineation of the pre-computed paths as a subgraph in order to reduce complexity for a MAPF algorithm.

The following figure 1.2 shows the different part of Plan Merging, having in order, Individual Pathfinding, Path Selection and Solving.

Figure 1.2: Overview of the thesis

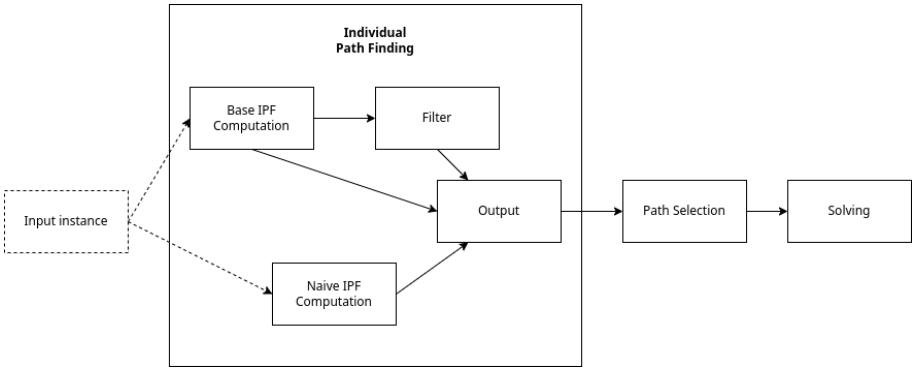


Chapter 2

Individual Path Finding

This chapter describes how Individual Path Finding is formalized and how computation can be achieved using ASP and Clingo.

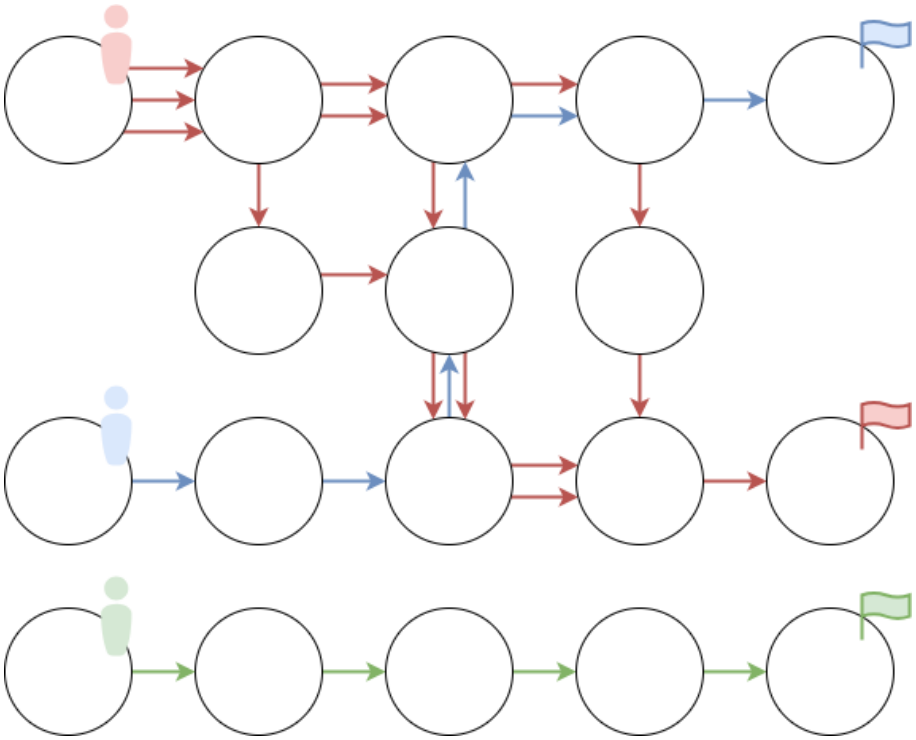
Figure 2.1: Overview of IPF



2.1 Formalization

Individual Path Finding [5, 12] (IPF) represents the computation of multiple paths for each agent without considering collision in a given MAPF problem. The idea is then to use the paths for plan merging. We formalize IPF as a triple (V, E, A) which is defined as MAPF is. The output is τ being a non-empty set of set of paths. For each agent $a \in A$, we have $\tau[a] = \{\pi_0, \ldots, \pi_n\}$. For lighter notation, we write $\gamma_a = \tau[a]$, and also γ to refer to a set of path in general. Finally, a path π is defined as MAPF background 1.2.1. Paths composing γ can be of different length. The following figure illustrate a $\tau = \{\gamma_r, \gamma_b, \gamma_g\}$ where its component being of different length; a red agents having $|\gamma_r| = 3$, a blue agent having $|\gamma_b| = 1$ and a green agent having $|\gamma_g| = 1$.

Figure 2.2: Example of a $\tau = \{\gamma_r, \gamma_b, \gamma_g\}$



Paths can have different inner properties, length, number of bends, coverage of the graph or inter-connected properties such as number of plan conflict, diversity, distant and so on. The work achieved in the thesis has been made considering only length of paths, potential conflict, diversity and distance.

Path length is a very noticeable property denoted as $|\pi|$. We can splits paths into two length category; shortest paths, and non-shortest path. (Using shortest paths only and using as makespan the longest shorter path among agents for plan merging process still guarantee optimal solution. On the other hand, using not only shortest path can break the optimality.)

2.1.1 Diversity and Distance

Diversity tends to evaluate the variety of used vertices for a given set of paths, in other words, evaluations are performed within the scope of an agent. We introduce diversity as a sum of unique vertices visited at each time step. The following function described in equation 2.1 computes diversity for a given γ.The formula is based on the Hamming distance [9].

$$distance(\gamma) = |\bigcup^{\pi \in \gamma} (\cup^{v,t \in \pi}(v,t))| \tag{2.1}$$

2.1: Diversity equation

Other diversity computation can be used, such as the Jaccard coefficient [8]. The difference between Jaccard coefficient and Hamming distance is that the first one focuses on the number of nodes that two paths have in common, while the second one focuses on the number of differences between the two paths. For our purpose, Hamming distance is more appropriate, however, it require that set are of the same length. Since paths in γ can be of different length, an adapted equation was necessary. Note that the equation 2.1 do not gives an index that can be compared to other γ's diversity. Furthermore, this equation is easy to implement with Clingo.

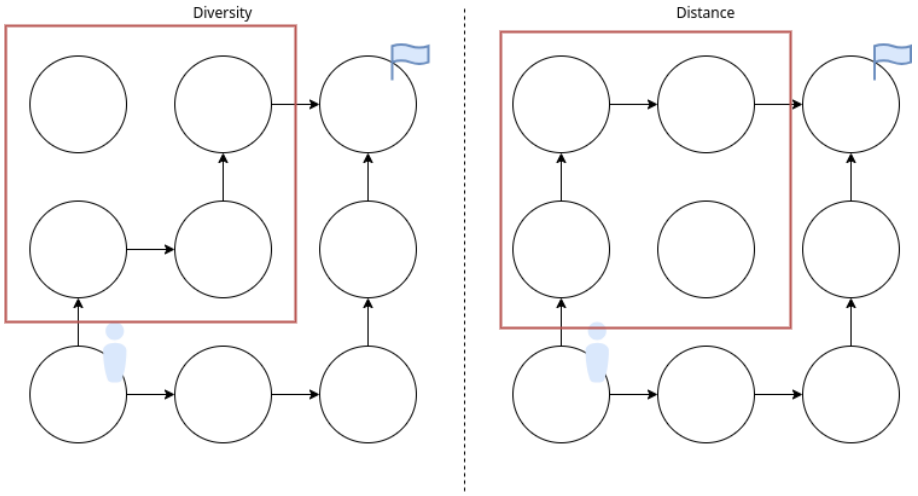
On the other hand, we can compute as property of paths the distance. It refers to how distant paths composing a γ are. Taken pair wisely, the distance between two paths is the sum of the distance between the vertices at each time step. Note that the distance between two vertices be an arbitrary function such as euclidean distance, shortest path length between these two vertices and so on. We refer to this arbitrary function as $dist(v, v')$. In our case, we consider $dist(v, v')$ as the euclidean distance between v and v'. The formalization of the distance for a given γ is defined in the following equation 2.2.

$$diversity(\gamma) = \sum_{\pi,\pi' \in \gamma, \pi \neq \pi'} (\sum_{t=0}^{t \to min(|\pi|,|\pi'|)} dist(\pi[t], \pi'[t])) \tag{2.2}$$

2.2: Diversity equation

To conclude, the figure 2.3 shows two γ of size $|\gamma| = 2$. The first one shows an example of the most diverse γ possible for this problem (multiple solutions exist, and we consider paths in γ of same length). On the other side, we have an example of the most distant γ possible for this problem. Red squares illustrate the difference between diversity and distance.

Figure 2.3: Diversity vs Distance



2.2 Computation

In this subsection, we describe two different approaches to compute multiple paths. Even if IPF is a side topic for this thesis, a concern was to be able to find x paths for each agent in a reasonable time. The following computation approaches are made with Clingo (ASP), it it likely that computing paths with programming languages such as C, C++, rust and so on, would be a better choice in terms of speed, however, sticking with Clingo has been chosen for straightforwardness and consistency of technology over the thesis. In addition, this section may give a starting point for future work on computing multiple paths using ASP and Clingo

2.2.1 Naive IPF Computation

The first approach that we describe is a direct modification of the encoding presented in the introduction. The translation adds a new constant to the encoding called *npaths* which denotes the number of paths to compute per agent. Then, we add to most of the predicate, a new argument being the identifier of the path; a unique path is now defined with an agent R and a path identifier I. As mentioned above, there is no need to consider conflicts in the encoding. With these new directives, we could obtain an encoding that we could call **Naive IPF** 2.1 which computes multiple paths for each agent in one call.

Listing 2.1: Naive IPF computation

```
1   agent(R,1..npaths) :- agent(R).
2
3   at(R,I,P,0) :- start(R,P), agent(R,I).
4
5   time(1..horizon).
6   {move(R,I,U,V,T) : edge(U,V)} 1 :- agent(R,I), time(T).
7
8   at(R,I,V,T) :- move(R,I,_,V,T).
9   :- move(R,I,U,_,T), not at(R,I,U,T-1).
10
11  at(R,I,V,T) :-
12      at(R,I,V,T-1),
13      not move(R,I,V,_,T),
14      time(T).
15
16  :- { at(R,I,V,T) }!=1 , agent(R,I), time(T).
17
18  :- goal(R,V), not at(R,I,V,horizon).
```

The encoding described in listing 2.1 has the exact same principles as the base MAPF encoding (see listing 1.2). Line 1 introduces id I for paths. Note that in this case, all agents would have the same number of paths. Of course, it is not necessary. We could use other predicates to describe how many paths are required for each agent of an instance. Then the lines 6 and 8 shows predicates *move/4* and *at/3* getting changed to predicates *move/5* and *at/4* now including the ID of paths.

The main flaw that comes with the encoding described is slowness. Requesting multiple paths per agent raises the space-search considerably. Even if we remove conflict consideration in the encoding, the fact that the grounder needs to ground the same predicate $move/5$ $|A| * npaths$ time. In addition, more variables induce a slower solving. However, this encoding gives us the opportunity to add properties to paths. For example, to implement diversity2.1, we can use *weak constraint*, for instance, having a set of diverse paths would require the addition of the following line:

```
1   #maximize{1,R,V,T : at(R,I,V,T)}.
```

More definitive additions such as constraining specific paths of specific agents to have a defined length, a defined number of bends and so on, could be used. Another flaw that comes with this encoding is that the is no guarantee that all paths for the same agent are different. However our **Naive IPF** encoding would still compute the requested number of paths for this agent a even though paths are the same. This can be fixed using inner collision; but we would tend to revert to classical MAPF solving.

2.2.2 Base IPF computation

By using multi-shot solving and assumption [11], it possible to ground both predicates *at* and *move* once for all agents. To do so, we have to tweak the naive IPF encoding described in section 2.2.1 We obtain the following encoding in listing 2.2. This encoding can be interpreted as simple pathfinding that will be executed one agent at a time. Furthermore, *start*/2 and *goal*/2 predicates are not necessary; these are handled by assumptions.

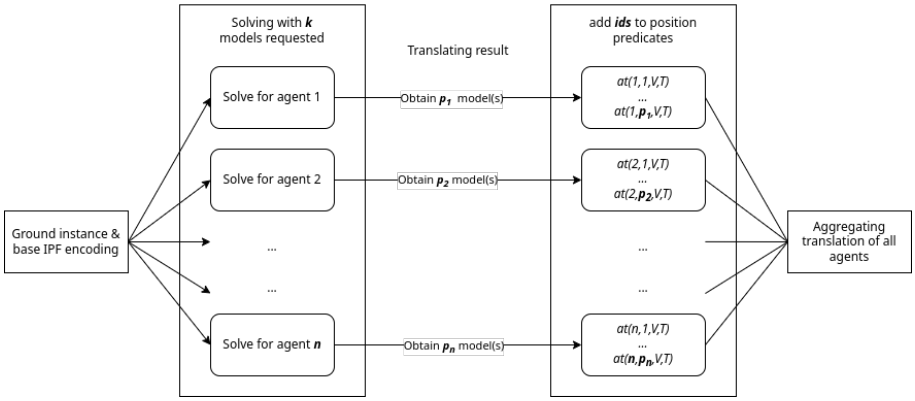
Listing 2.2: Base IPF computation

```
1   time(1..horizon).
2
3   {at(V,0) : vertex(V)} = 1.
4
5   { move(U,V,T) : edge(U,V)} 1 :- time(T).
6
7   at(V,T) :- move(_,V,T).
8           :- move(U,_,T), not at(U,T-1).
9
10  :- { at(V,T) } != 1, time(T).
11
12  at(V,T) :-
13      at(V,T-1),
14      not move(V,_,T),
15      time(T).
16
17  current_agent(R) :- at(V,0), start(R,V).
18  at(GV,T) :-
19      at(GV,T-1),
20      time(T),
21      goal(R,GV),
22      current_agent(R).
```

Lines 17 retrieve the current agent the solver is working on. By crossing the initial *at*/2 position given by assumption to fact *start*/2, we can retrieve the current agent. Line 18 is then designed to guarantee that once the path of an agent reaches its goal, the agent remains at its goal location.

The flowchart described in figure 2.4 illustrate process of base IPF computation. It first ground the instance with the encoding, then solve with the start and goal positions of an agent as assumptions. Finally, identify *at/2* with agent ID and model number. We use the model number to identify different paths of the same agent; the model enumeration represents different possible assignations for the problem, which means multiple different paths. In summary, requesting to the ASP solver n models means requesting n paths for an agent.

Figure 2.4: Flowchart of IPF computation using multi-shot solving and assumption



We define $1 \leq p \leq k$, if solving an instance with k requested models, the number of possible models p can be lower. We use a Python wrapping to control and manage assumptions. A pseudocode of the wrapper is denoted in listing 2.3

Listing 2.3: Pseudo code of base IPF encoding wrapper

```
1     result ← list()
2     ground(instance + base IPF encoding)
3
4     for each agent in instance {
5         assume(initial and goal position)
6         solve()
7
8         for each model {
9             translate ‘at/2’→‘at/4’
10            add translation to result
11        }
12    }
```

A nice property that comes with this approach is that since we use model enumeration to create different paths, if Clingo enumerates fewer models than requested, we can assume that there are no other paths possible of this length. It can be used to enumerate more paths, considering higher path length.

Base IPF computing reduces computation time significantly; given the way Clingo works, decomposing the problem into one agent at a time and reusing grounding works way better than trying to solve multiple problems at once, which naive IPF computation is doing. It does, however, compute random paths. In order to compute the shortest paths, we can use the optional predicates described in list 1.2.3. Assumptions allow us to force $goal/2$ predicate to be reached a specific time step, We can then change the goal position assumption and force it to be reached at a certain time step instead of at the horizon point.

We can use the same principle to compute paths of different lengths for one agent; this require an additional solving and translation process.

Furthermore, in this process, we can compute additional paths with a modified time step associated with the goal. This addition enables agents to reach their destination xx time steps later than the originally defined duration. This strategy proves valuable when a limited number of paths are computed, as it provides the IPF solver with extra possibilities to generate additional paths.

Given the way base IPF computation works, it seems difficult to assign different properties to the paths as the naive IPF approach does. To do so, we can request a large number of paths and create a subset out of them that satisfy chosen properties.

2.2.3 Filters

Compared to naive IPF computation, we can not manage easily path properties. We can however create subset of previously computed γ by filtering paths given some criteria. A filter f is defined as such; $f(\gamma) \subseteq \gamma$. We introduce two criteria function. The first being a function selecting k most diverse paths following 2.1 and the second one a function selecting k most distant paths following 2.2.

Listing 2.4: Encoding of diverse filter

```
1  #const npath = 5.
2
3  {dpath(R,I): at(R,I,_,_)} npath :- agent(R).
4  #maximize {1@2,R,I : dpath(R,I)}.
5
6  dpath(R,I,V,T) :- dpath(R,I), at(R,I,V,T).
7
8  #maximize {1@1R,V,T : dpath(_,_,V,T)}.
```

Diversity encoding (listing 2.4) starts by picking for each agent, at most *npath* paths among all path available denoted in line 3. On the next line 4, we use an optimization statement to have as most path possible; as written previously, it is not forced to have γ composing τ of same size. Using this optimization statement avoid unsat when no enough paths are available for an agent. Line 6 is retrieving position and time step of selected paths. From these selected path, we can, as described in equation 2.1, count the tuple (V, T) and maximize them with another optimization statement denoted line 8.

Listing 2.5: Encoding of distance filter

```
1   #const npath = 5.
2
3   {dpath(R,I): at(R,I,_,_)} npath :- agent(R).
4   #maximize {1@1,R,I : dpath(R,I)}.
5
6   dpath(R,I,V,T) :- dpath(R,I), at(R,I,V,T).
7
8   dist(R,I1,I2,T, (X1-X2)*(X1-X2) + (Y1-Y2)*(Y1-Y2)) :-
9       dpath(R,I1,(X1,Y1),T),
10      dpath(R,I2,(X2,Y2),T), I1!=I2.
11
12  dsum(R,DS) :-
13      DS=#sum{D:dist(R,I1,I2,_,D), dpath(R,I1), dpath(R,I2)},
14      agent(R).
15
16  #maximize {1@2,D : dsum(R,D)}.
```

Note that line 12 and line 16

Distance encoding (listing 2.5) starts as diversity encoding does, by picking for each agent, at most *npath* paths among all path available, maximizing the number of selected path and then retrieving their position and time step (from line 3 to 6). Line 8 introduces a new predicate $dist/4$ which denotes the root-square-less euclidean distance (in our case, computing the exact distant is not necessary) of two different paths of the same agent. We then have an intermediate rule line 12 which sums the distance of paths for each agents, which is then used in the weak constraint statement line 16 to maximize the distance issued by D.

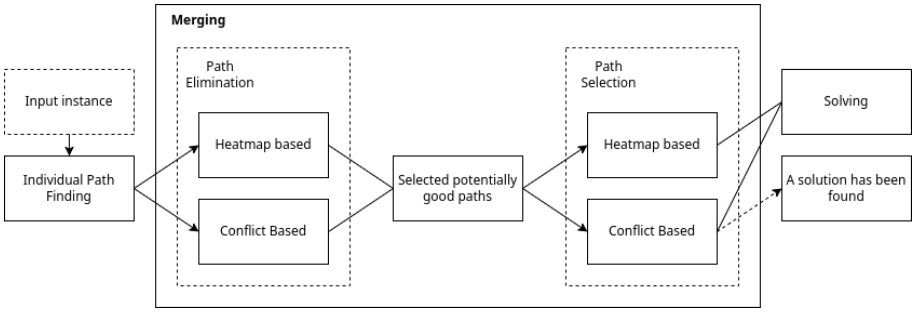
As shown in the figure 2.1 illustrating the overview of IPF, using filters is not necessary. Filters mimic that IPF could be computed considering properties for paths.

Chapter 3

Path Selection

In this chapter, we will describe what Path Selection (PS) is, how it works and which strategies we can use to take advantage of multiple computed paths per agent. The figure 3.1 shows the different steps for Path Selection used in this work.

Figure 3.1: Overview of Path Selection



Path Selection takes a τ as its input and produces a subcomponent of the same τ. We denote this extracted portion as τ'. τ' is defined such as $\forall a \in A \mid \gamma'_a \in \tau' \subseteq \gamma_a \in \tau$. Furthermore τ' can denote a plan, it requires that, for $\forall \gamma'_{a \in A' \subseteq A} \in \tau', |\gamma'_a| = 1$.

3.1 Heatmap

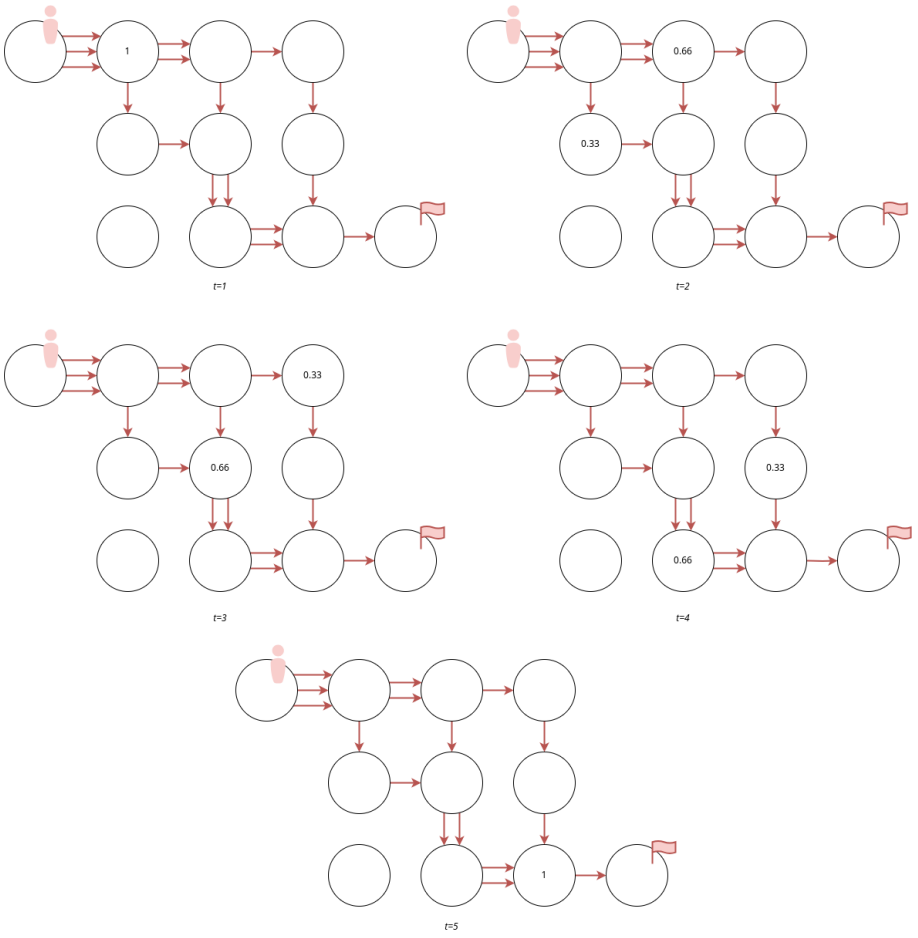
Heatmap is about projecting likelihood of presence of agents on vertices given a τ, inspired from paper [2, 3]. Likelihood refers to, in a scope of an agent, the probability of be being at a specific vertex for each time step according to a set of path γ. This heatmap interpretation stands for the scope of one agent. We will refer to agent-related heatmap as **Individual Heatmap** (IH). We formalize, in equation 3.1 an Individual Heatmap ϕ:

$$\phi(\gamma, v, t) = \frac{|\{\pi | \pi \in \gamma, \pi(t) = v\}|}{|\gamma|} \tag{3.1}$$

3.1: Individual Heatmap

ϕ is define, given a γ per vertex and time step. For example, using this definition, we can compute the Individual Heatmap of a given $|\gamma| = 3$ illustrated in figure 3.2.

Figure 3.2: Example of Individual Heatmap computed from a $|\gamma| = 3$



This representation produces critical vertices for an agent, wherein heightened values correspond to a greater likelihood of vertex utilization within a probabilistic framework. On the other hand, the mean heatmap value associated with γ serves as an indicator of path diversity, with elevated values signifying a low level of path diversity.

Through the aggregation of all Individual Heatmaps within an instance of τ, a **Global Heatmap** (GH) can be derived. It is important to note that Global Heatmaps, in this context, no longer serve as a direct indicator of the likelihood of presence but instead an indicator of "usage of vertices" .

A Global Heatmap Φ of τ is formalized in the following equation 3.2. As for Individual Heatmaps, a Global Heatmap value of Φ is computed given a vertex v and a time step t.

$$\Phi(\tau, v, t) = \frac{\sum_{\gamma \in \tau} \phi(\gamma, v, t)}{|\tau|} \tag{3.2}$$

3.2: Global Heatmap

To perform the computation of Individual Heatmaps and Global Heatmaps using Clingo, it is important to consider that Clingo inherently rounds down floating-point numbers. To obtain accurate heatmaps values in our context, we require an external program capable of performing divisions without rounding down the result. We introduce the encoding of the Individual Heatmap in listing3.1.

Listing 3.1: Individual Heatmap encoding

```
1      individual_heatmap(R,V,T,(K,N)) :-
2          K = #count{1,I : at(R,I,V,T)},
3          N = #count{1,I : at(R,I,_,_)},
4          at(R,_,V,T).
```

This short encoding introduces the *heatmap*/4 predicates, with R, V and T being respectively the agent ID, a vertex and a time point. The last argument of the predicate is a tuple (K, N) where K, computed on line 3, is the number of paths of R that go through V at time step T. And N, computed on line 3, denotes the number of paths that belong to the agent K.

Concerning the calculation of the Global Heatmap using formula 3.2, this computation is performed using **Python**[1].

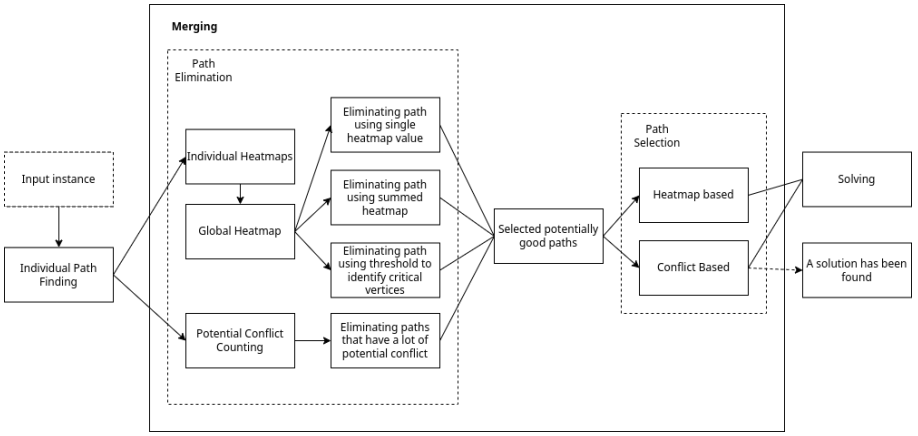
3.2 Path Elimination

Path Elimination (PE) is a process involving the removal of paths that exhibit potential issues. This step serves as a preprocessing stage for Path Selection (PS), Path Elimination, therefore, serves to chunk through the multitude of possible paths and eliminate those that are deemed "potentially problematic".

Figure 3.3 sums up the different processes in order to eliminate potentially-conflicting-paths.

[1]It is possible to provide the nominator and the denominator for the Global Heatmap at each vertex and each time step, however, compared to using a third-party programming language, it is very slow or not computationally feasible in practice.

Figure 3.3: Overview of Merging: Path Elimination



We first introduce some definition.

Definition 1. A path is classified as **killed** when it has been explicitly removed of the output of Path Elimination.

Definition 2. A path is classified as **selected** when it has been explicitly selected for the output of Path Elimination.

Definition 3. An agent is classified as **killed** when all of its paths have been explicitly killed during the Path Elimination process.

Definition 4. A vertex can be classified as **critical** at a certain time step by a Path Elimination process. By this means, PE assume that there is high chance of collision.

Definition 5. An agent is classified as **critical** when at least one of its vertex composing its plan is considered as **critical**.

Definition 6. A **potential conflict** denotes conflict among paths of different γ. It do not represent an actual conflict. Paths of agents involved in potential conflict would end-up creating conflict if they were used as they were.

3.2.1 Using Global Heatmap

Global Heatmap values often provide insights into vertices that are prone to conflicts. This information can be leveraged for path elimination.

A simplistic approach involves minimizing the cumulative Global Heatmap value by allowing the encoding to kill paths. Unfortunately, due to the interconnections among heatmaps, practical grounding of this approach becomes feasible only on small instances.

We then present three approaches for identifying potentially conflicting paths in a reasonable computation time.

Eliminating paths using unique heatmap value

One straightforward approach involves ordering all possible assignments of (v, t) within a given τ based on their global heatmap values. Subsequently, we classify the top k vertices as "critical" These critical nodes can then be used as reference points for path elimination based on various criteria. We can identify which paths pass through critical vertices using an ASP statement like this:

```
1     critical_path(R,I,V,T) :-
2         critical_vertex(V,T),
3         at(R,I,V,T),
4         not path_killed(R,I).
```

From the *critical_path*/4 predicate, we can determine that n critical paths originating from a critical node should be eliminated. For instance, we may opt to eliminate half of them as follows:

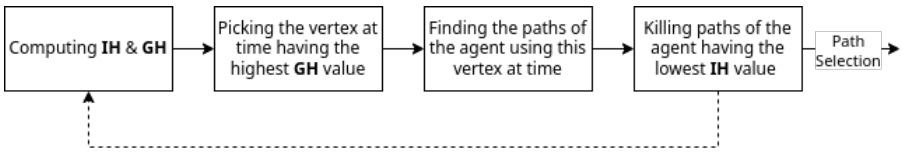
```
1     {to_kill(R,I): critical_path(R,I,V,T) } = K/2 :-
2         K = #count{1,R',I' critical_path(R',I',V,T)},
3         critical_vertex(V,T).
```

Note that we use predicate both predicates *to_kill*/2 and *path_killed*/2. They refer to the same concept. We do, however, differentiate them so the paths that are yet to be eliminated (*to_kill*/2) do not interfere with already killed paths (*path_killed*/2).

There are various parameters and variations to consider in determining the value that n can take. These may include individual heatmap values, whether an agent has already had some of its paths eliminated, and other factors in the decision-making process.

The approach we settle on is the following;

Figure 3.4: Eliminating paths using unique heatmap value process overview



As illustrated in figure 3.4, it is possible to loop over the elimination to eliminate more paths.

As mentioned earlier, the lack of float-point number handling of Clingo forces us to use a Python wrapper to sort Global and Individual Heatmap values.

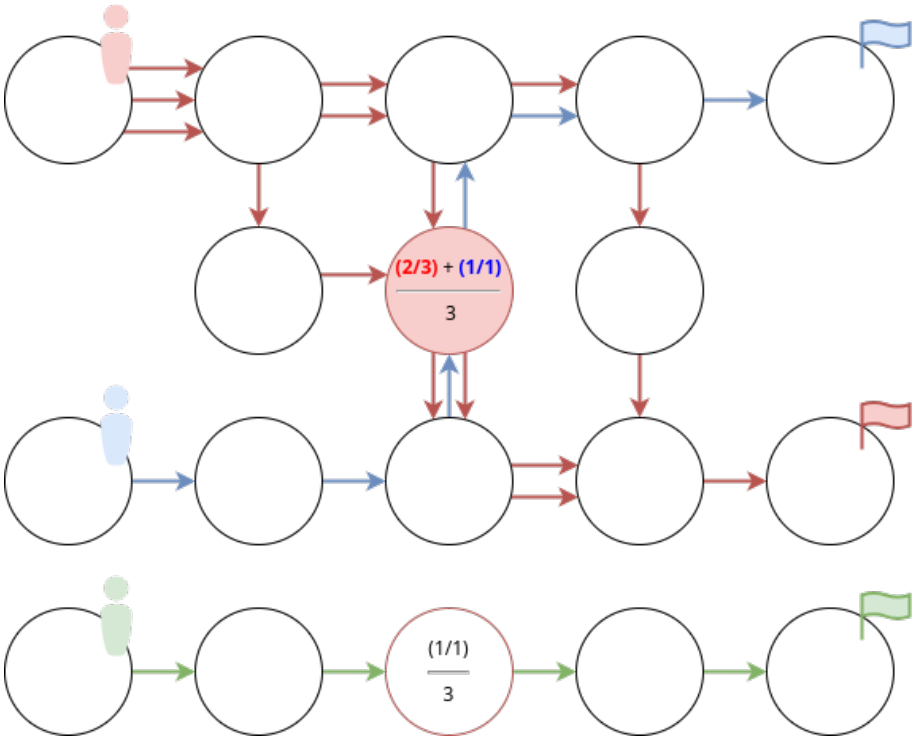
The decision to eliminate paths of agents with the lowest Individual Heatmap values can be justified on two grounds:

1. **Node Importance**: Higher Individual Heatmap values signify that a particular vertex holds more significance at this time step for the respective agent. Consequently, it is more suitable to eliminate paths of agents with lower interest in this vertex, as they are less likely to utilize it.

2. **Balanced Elimination**: Opting to eliminate paths of agents with low Individual Heatmap values helps maintain a balanced elimination process; selecting agent with the lowest heatmap means killing the least possible paths involved in the **critical vertex**. It allows us to not kill agents too swiftly.

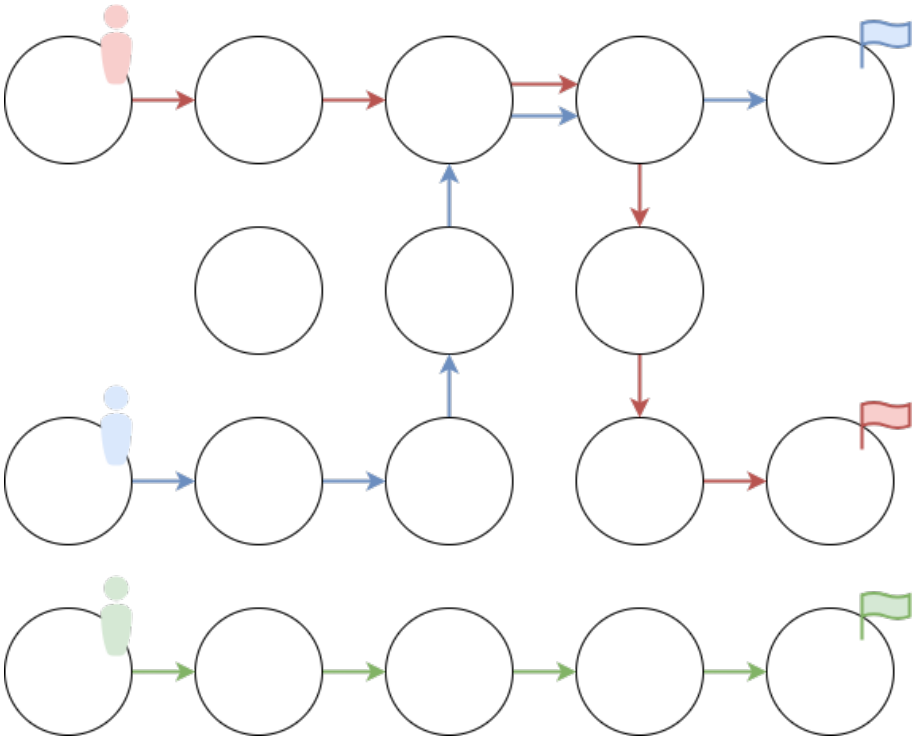
If we apply the previous approach explained above, on the example illustrated on figure 2.2. We highlighted global heatmap in red at the critical moment (time step $t = 3$), see figure 3.5.

Figure 3.5: Eliminating paths using unique Global Heatmap value process example



Between the two highlighted vertices, according to the elimination process 3.4, the vertex with the highest Global Heatmap value is the one colored in red. From this vertex, we can denote three paths going through it at time step $t = 3$. We then order the Individual Heatmap values of the two concerned agents, blue and red. from these values we can deduce that the two paths of red agents have to be killed. We obtain figure 3.5 which is in this case a valid plan:

Figure 3.6: Result of eliminating paths using unique heatmap value process



Nevertheless, despite our efforts to implement strategies aimed at balance the elimination process among agents, adopting a vertex-oriented approach tends to kill agents swiftly.

Eliminating paths using GH threshold

On flaw that comes with the previous approach is that the process is doing one vertex after the other. Thus, as described in figure 3.4, the number of loops is hard to define; it is difficult to define a value that does not kill too many agents, but on the other hand, it is required to kill enough paths. We propose a way to denotes multiple critical vertices in one call (we can still implement a loop as in the first approach described).

The principle is simple, if the Global Heatmap value of a vertex at a specific time point is higher than a defined value, the vertex is denoted as critical. We call this defined value a **threshold** denoted $\mathcal{H}$. Formally, we have;

$$critical\_vertices(\tau) = \{(v,t) | \Phi(\tau, v, t) > \mathcal{H}, (v,t) \in \mathcal{F}(\tau)\} \tag{3.3}$$

3.3: Identifying critical vertices using threshold

Where $\mathcal{F}$ is a function enumerating all possible tuple v, t in τ. It is defined as such;

$$\mathcal{F}(\tau) = \bigcup_{\gamma \in \mathcal{T}} \bigcup_{\pi \in \gamma} \cup_{t=0}^{t \to |\pi|} (\pi[t], t)$$

Threshold values can be defined through different equations, considering different properties, we defined:

$$\mathcal{H}_{sbt}(A, \Delta) = \frac{1 * \Delta}{|A|} \tag{3.4}$$

3.4: Simple (biased) threshold

The first equation 3.4, denoted as $\mathcal{H}_{sbt}(A, \Delta)$, defines a simple biased threshold. In essence, without considering the bias, this equation implies that a vertex is labeled critical if, for all n agents with k paths each, $\frac{1}{n}$ of their paths utilize this vertex at a specific time plus one additional path. The bias adjustment allows for the flexible inclusion of more or less critical vertices.

The simplicity of the basic threshold equation makes it a general tool, but it lacks vertex-specific context. To address this limitation, we introduce a more sophisticated approach with the context-dependent threshold equation, $\mathcal{H}_{bcdt}(V, T, A, \Delta)$. This threshold calculation takes into account the number of paths using a particular vertex at a specific time step (T) and the number of agents involved, providing a more precise evaluation of critical vertices. The equation incorporates the bias factor that allows a nuanced consideration.

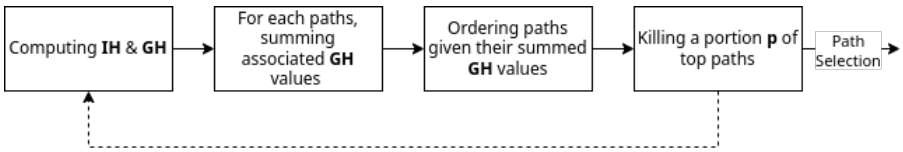
$$\mathcal{H}_{bcdt}(V, T, A, \Delta) = \frac{npath\_on(V, T) * \Delta}{nagent\_on(V, T) * |A|} \tag{3.5}$$

3.5: (Biased) Context dependent threshold

Eliminating paths using sums of heatmap value

In contrast to a vertex-oriented approach, which may label a path as "critical" even if it possesses critical attributes only on a single occurrence. An alternative strategy involves considering the scope of entire paths that may highlight conflict. The approach entails assigning a value to an entire path, facilitating comparison among paths. Specifically, we calculate this value for each path by summing the Global Heatmap values associated with its vertices. The approach is outlined in the following figure 3.7.

Figure 3.7: Eliminating paths using summed Global Heatmap values process overview



This approach tends to kill agents quite quickly when only a few paths are involved, this approach tends to be more efficient the more diverse $\gamma \in \tau$ are. Futhermore, with one call, this approach tends to chunk a huge section of the paths.

3.2.2 Using conflicts

We introduce a fourth approach that centers around **potential conflicts** to guide Path Elimination. In this method, for each agent and at every vertex along their path, we calculate the count of **potential conflicts**. This approach stands apart from the heatmap-based methods in a significant way: when an agent possesses a substantial number of paths, their individual paths do not significantly influence the Global Heatmap values, even if they have the potential to cause conflicts. However, in the Potential Conflict-based approach, every path is treated with equal weight.

The encoding for the described approach is defined in the following listing 3.2.

Listing 3.2: Conflict-based Path Elimination

```
1   #const n = 5.
2
3   to_determine(R,I) :- at(R,I,_,_), not path_killed(R,I).
4
5   potential_conflict(R,I,V,T) :-
6       to_determine(R,I), at(R,I,V,T),
7       to_determine(R',I'), at(R',I',V,T),
8       R'!=R.
9
10  path_composition(R,I,C) :-
11      C = #count{1,V,T : potential_conflict(R,I,V,T)},
12      to_determine(R,I).
13
14  {to_kill(R,I,C): path_composition(R,I,C)} = n.
15
16  #maximize {C:to_kill(_,_,C)}.
```

In order to decide which paths has to be killed, we first need to denotes which paths can be killed, line 3 labeling them with the predicate *to_determine*/2. **Po-**

tential conflicts are determined through the rule at line 5. This rule identifies situations where two or more agents could potentially conflict at a given vertex and time. The following rule, line 10 calculates the path composition of a path by counting the different **potential conflicts** composing it. Finally, lines 14 and 16 state that exactly n paths should be marked for elimination and aims to select the set of paths to eliminate that results in the highest **potential conflicts** sums. Note that n has been set arbitrarily to 5, but it can take any value.

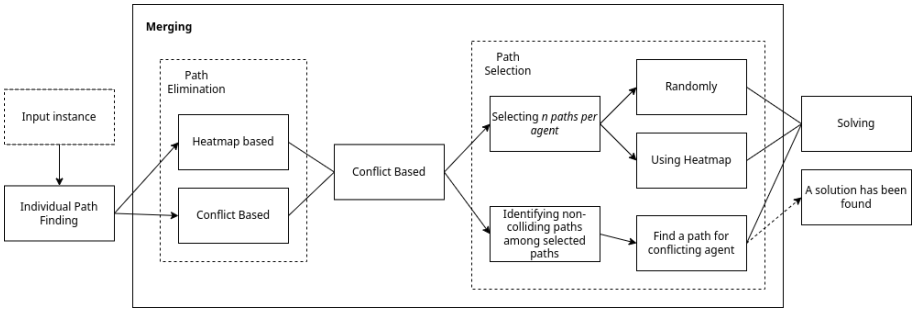
3.3 Path Selection

As mentioned above, Path Selection tends to build a τ', being a subcomponent of τ. We can describe two objectives for τ'.

1). Building τ' in order to create a plan. We can identify two kinds of plans; a valid plan Π as defined earlier in the background section 1.2, and a partial plan $\hat{\Pi}$. A partial plan possesses the same attributes as a valid plan, except for its construction of the collection of paths, which is denoted as $(\pi_a)_{a \in A' \subset A}$, while a plan Π is represented as $(\pi_a)_{a \in A}$. By this means, τ' has to be built such as each γ' composing τ' has at most one path composing them.

2). Building τ' in order to create a subgraph. We create a subgraph V', E' builded out of the paths in τ'. We have $V' = \{v \mid v \in vertices(\tau')\}$ and $V' = \{e \mid e \in edges(\tau')\}$ where $vertices(\tau)$ and $vertices(\tau)$ respectively enumerate the vertices and edges of a given set of set of path τ. By this means, τ' has to be builded such that each γ' composing τ' has to be non-empty.

Figure 3.8: Overview of Merging: Path Selection



3.3.1 Towards a (partial) plan

In this section, we explain the method for achieving the first objective outlined above (as discussed in 3.3). To summarize, the aim is to identify, for as many agents as possible, at most one conflict-free path (among paths of other agents) per agent. To achieve this, we consider the set of paths that have not been eliminated through Path Elimination. The corresponding encoding for this approach is provided in listing 3.3.

Listing 3.3: Building a conflict free τ'

```
1   % Defining collision & possible_path
2   possible_path(R,I):- at(R,I,_,_), not path_killed(R,I).
3
4   % Constructing a (partial) plan
5   {selected_path(R,I) : possible_path(R,I) }1 :- agent(R).
6
7   collision((R1,P1),(R2,P2),T) :-
8       R1 != R2,
9       at(R1,P1,V,T),
10      at(R2,P2,V,T),
11      selected_path(R1,P1),
12      selected_path(R2,P2).
13
14  collision((R1,P1),(R2,P2),T) :-
15      R1 != R2,
16      at(R1,P1,V1,T),
17      at(R1,P1,V2,T+1),
18      at(R2,P2,V2,T),
19      at(R2,P2,V1,T+1),
20      selected_path(R1,P1),
21      selected_path(R2,P2).
22
23  :- collision((R1,P1),(R2,P2),_),
24      selected_path(R1,P1),
25      selected_path(R2,P2).
26
27  selected_agent(R) :- selected_path(R,_).
28
29  #maximize {1@1,R : selected_path(R,_)}.
```

The encoding provided in listing 3.3 can be interpreted as follows: the initial step is to establish which paths are eligible for the output. Up to this step of Path Selection, paths could have been labeled as either **selected** or **killed**. For the other paths, line 2 labels them as *possible_path*/2. They become candidates for the output. Then, line 5 selects paths from the candidates, ensuring that at most one path per agent is chosen for the output. Following this, lines 7 and 14 define vertex and edge collisions, respectively, among the selected paths. The associated constraint specified in line 23 ensures that the set of **selected paths** does not contain any collisions. Lastly, the optimization statement in line 29 aims to maximize the number of **selected paths**, ensuring the selection of as many paths as possible.

3.3.2 Towards a subgraph

The second objective discussed in 3.3 can defined as a pre-processing for a MAPF algorithm. Contrary to the first objective, we require at least one path for each agent. To do so, multiple approaches can be used. By definition, the output of IPF can be used directly. We will, however, describe different approaches to select some paths per agent.

The approaches that we will described are based on the output of the encoding described in listing 3.3. The objective is to populate the set of conflict-free paths with paths of agents involved in conflicts. There are two primary methods to identify missing paths:

Utilizing Global Heatmaps: The first approach involves using Global Heatmaps and similar approaches used in Path Elimination process. This can be done either by employing the Heatmaps computed during the Path Elimination process or by recalculating them using the possible paths issued from 3.3 (or from IPF, if all paths have been eliminated)..

Path Conflict Composition: The second approach relies on path conflict composition, as outlined in listing 3.2. Similarly to the heatmap approach, the path conflict composition can be derived either from the Path Elimination process or can be recomputed based on the possible paths obtained through path selection.

We then convert the different paths into the new vertices and edges V' and E' using the following encoding 3.4.

Listing 3.4: Converting path to subgraph

```
1     nvertex(V) :- selected_path(R,I), at(R,I,V,_).
2     nedge(U,V) :-
3         edge(U,V),
4         nvertex(U),
5         nvertex(V).
```

3.3.3 Evaluating approaches

We introduced different Path Elimination approaches coupled with Path Selection, in order to evaluate them, we introduce four metrics. These metrics require a reference to compare Path Selection efficiency. We then introduce a "brute-force" Path Selection approach that computes what could be defined as the best output possible for Path Selection[2]. "Brute-force" PS approach determine which paths to kill to obtain a conflict-free set of paths. In other words, Path Selection tends to approximate of the "Brute-force" PS approach. This approach is not used in practice because of its huge computation time.

[2]The best output possible if we looking a partial plan $\hat{\Pi}$ as close possible to a complete valid plan Π

Listing 3.5: "Brute-force" PS approach

```
% Defining collision & possible_path
collision((R1,P1),(R2,P2),T) :-
    R1 != R2,
    at(R1,P1,V,T),
    at(R2,P2,V,T).

collision((R1,P1),(R2,P2),T) :-
    R1 != R2,
    at(R1,P1,V1,T),
    at(R1,P1,V2,T+1),
    at(R2,P2,V2,T),
    at(R2,P2,V1,T+1).

possible_path(R,I):- at(R,I,_,_).

% Constructing set of non conflicting paths
{usable_path(R,I) : possible_path(R,I) } :- agent(R).

:- collision((R1,P1),(R2,P2),_),
    usable_path(R1,P1),
    usable_path(R2,P2).

:- selected_path(R,I), not usable_path(R,I).

% Constructing a partial plan
{selected_path(R,I) : possible_path(R,I) }1 :- agent(R).

:- collision((R1,P1),(R2,P2),_),
    selected_path(R1,P1),
    selected_path(R2,P2).

#maximize {1@1,R : selected_path(R,I)}.
#maximize {1@2,R,I : usable_path(R,I)}.
```

The major reason for the long computation time required by the "brute-force" PS approach is the identification of vertex and edge conflicts. Contrary to the MAPF encoding presented in background section 1.2 where the identification of both edge and vertex collisions is achieved with a linear computational complexity. On the other hand, within the "brute-force" encoding, conflicts are identified in a quadratic complexity due to the manner in which paths are identified using the (R, I) tuple.

We introduce metrics to assess and compare the outcomes of various Path Selection approaches. We will compare the results of approaches to the brut-force output. To do so, we introduce two functions $brutforce$ and *approach*

which both take as argument a predicate name and return the associated set of facts. For instance, $approach(selected\_path/2)$ returns all the selected paths of the approach. Note that we could change the approach $brute force$ by another to evaluate difference between approaches results.

$$relevance = \frac{|approach(selected\_agent/1) \cap brut force(selected\_agent/1)|}{|approach(selected\_agent/1) \cup brut force(selected\_agent/1)|} \tag{3.6}$$

3.6: Relevance

Relevance computes the proportion of common selected agent designated by the approach with brutforce output.

$$absolute\_relevance = \frac{|approach(selected\_agent/1)|}{|approach(agent/1)|} \tag{3.7}$$

3.7: Absolute Relevance

Absolute Relevance evaluate proportion conflict-free agents found by the approach.

$$precision = \frac{|approach(selected\_path/2) \cap brut force(usable\_path/2)|}{|approach(selected\_path/2) \cup brut force(usable\_path/2)|} \tag{3.8}$$

3.8: Precision

Precision evaluate capacity of the approach to select path that are present in the biggest set of conflict-free paths possible.

$$precision = \frac{|approach(path\_killed/2)|}{|brut force(possible\_path/2)|} \tag{3.9}$$

3.9: Chunk Proportion

Chunk Proportion evaluate the proportion of path killed compared to the number of paths.

Chapter 4

(Partial) Solving

In this chapter, we will introduce the two solving approach that we explored. Solving can be performed using already computed-paths or a subgraph defined by paths.

Solving in both cases is basically performed using the same encoding defined in listing 4.1.

Listing 4.1: Encoding of final solver

```
1   time(1..horizon).
2
3   at(R,P,0) :- start(R,P).
4
5   { move(R,U,V,T) : nedge(U,V)} 1 :- agent(R), time(T).
6
7   at(R,V,T) :- move(R,_,V,T).
8           :- move(R,U,_,T), not at(R,U,T-1).
9
10  at(R,V,T) :-
11      at(R,V,T-1),
12      not move(R,V,_,T),
13      time(T).
14
15  :- {at(R,V,T)}!=1, agent(R), time(T).
16
17  :- { at(R,V,T) : agent(R) } > 1, nvertex(V), time(T).
18  :- move(_,U,V,T), move(_,V,U,T), U < V.
19
20  goal_reached(R) :- at(R,V,horizon), goal(R,V).
21
22  #maximize{1,R : goal_reached(R)}.
```

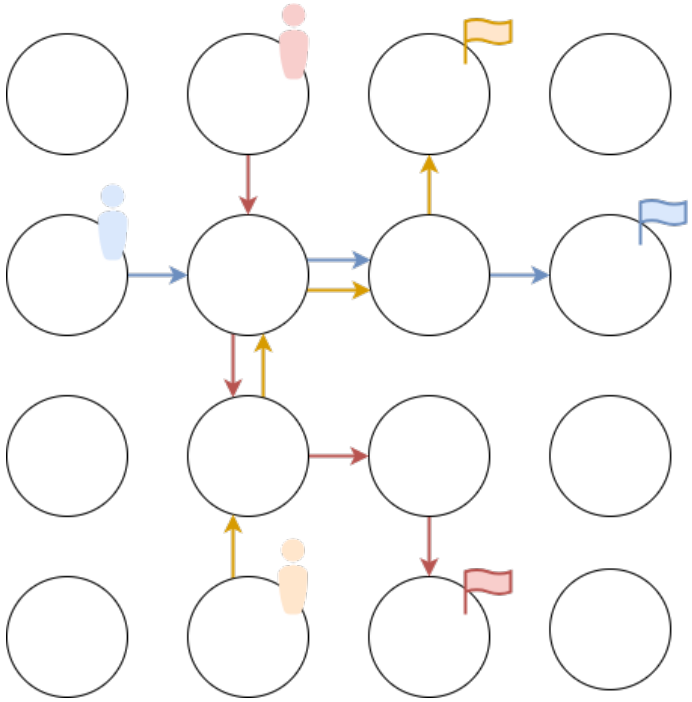
This encoding is derived from the MAPF encoding illustrated in listing 1.2

with two significant modifications. The first modification involves the redefinition of movement predicates. Instead of relying on $vertex/1$ and $edges/2$, movement is now defined on $nvertex/1$ and $nedge/2$, which are generated from the encoding detailed in listing 3.4.

The second distinction is highlighted in lines 20 and 22. Unlike classical MAPF, which either provides a complete solution or returns unsatisfiable if no solution exists, the solver in this context can produce a partial solution. This means that due to the steps taken to reduce the complexity MAPF problem, there is a possibility that the solver might not find a complete solution.

In order to highlight the differences between the two different kinds of solving approaches we will present, we introduce an example denoted in the following figure 4.1. In this example, we have on the left a τ result of IPF of a MAPF problem $\mathcal{P}$.

Figure 4.1: Example for solving approaches. Having a $|\tau| = 4$ as result of IPF



4.1 Pre-computed paths

As mentioned in the Path Selection section 3, the output can vary based on the specified objective. In the context of the primary objective, which aims to construct a (partial) plan, we utilize the set of **selected paths** to generate **pre-computed paths**. This translation is achieved through the following rules:

```
1       at(R,V,T) :-
2           selected_path(R,I),
3           at(R,I,V,T).
```

Through line 15, we ensure that the selected paths are incorporated into the solution. Conversely, for agents without a selected path, the encoding is employed to compute their paths as the MAPF encoding described here 1.2.

Figure 4.2: Possible output for Path Selection & Pre computed paths

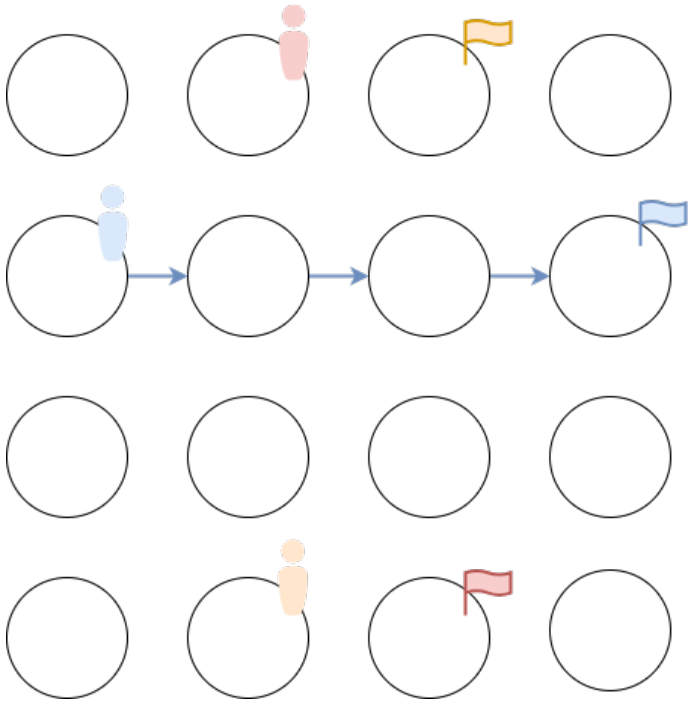
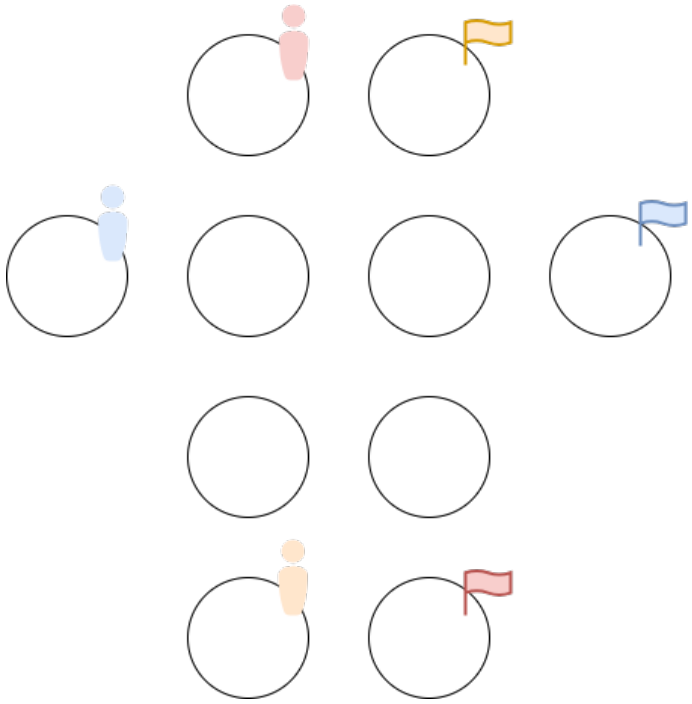


Figure 4.2 describes a partial plan $\hat{\Pi}$ where only blue agent has a path. Using pre-computed paths aims to reduce the number of paths to compute. In the example outlined in figure 4.2, the solving requires a makespan of five.

4.2 Subgraph

On the other hand, the second objective described in Section 3 aims to create a subgraph in order to reduce the size of the problem. As illustrated in Figure 4.1, we obtain the subgraph shown in Figure 4.3.

Figure 4.3: Example for solving approaches. Having a $|\tau| = 4$ as result of IPF



Contrary to pre-computing path, we aim to reduce the size of the graph the agents can move on. Applying MAPF on the problems described in figure 4.3 can find a solution with a makespan of 4. To summarize both approaches, pre-computing paths aims to reduce the number of agent for which a path has to be found at the cost of a possible lost of optimality. On the other hand, using subgraphs reduce the space search of the initial problem $\mathcal{P}$ by denoting a smaller problem $\mathcal{P}'$. Note that if an optimal solution exists for $\mathcal{P}'$ there is no guarantee that this solution is optimal for $\mathcal{P}$.

The two approaches presented can be used as one.

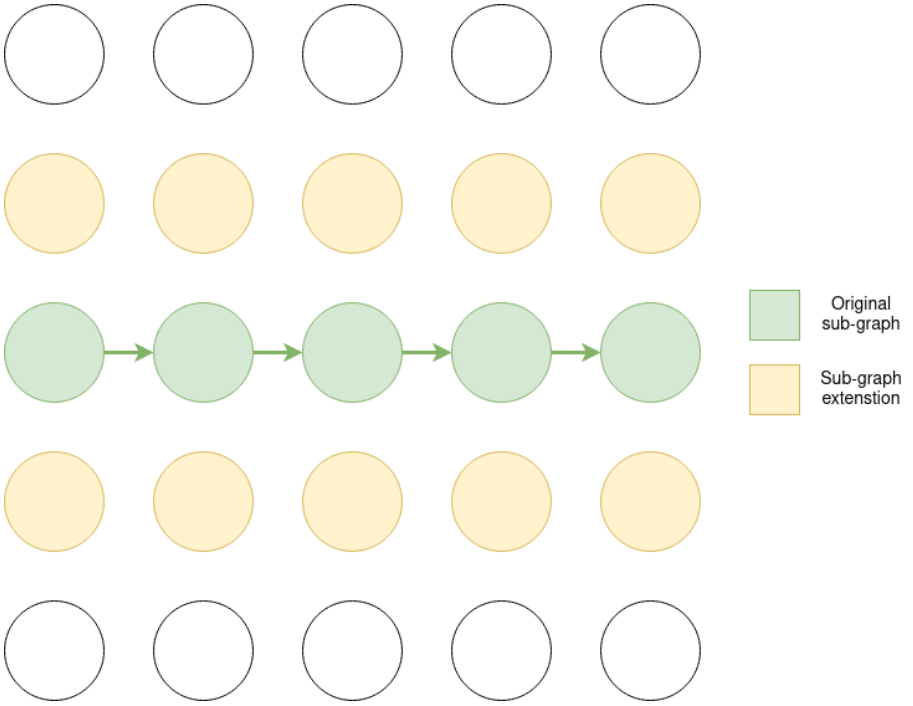
4.2.1 Subgraphs Extension Strategies

In practice, the subgraph solving approach seems to not be enough to fully solve instances; the agents seem to require too many additional time steps in order to "solve conflict". Thus, we introduce two strategies in order to extend subgraphs.

Corridor

The corridor strategy is engineered to augment the subgraph-solving process by incorporating neighboring vertices and their associated edges directly into the sub-graph. Corridors can vary in size, allowing for different levels of expansion. Figure 4.4 illustrates a corridor with a size k equal to one.

Figure 4.4: Example of corridor



The following encoding in listing 4.2 describe how corridors for paths can be computed.

Listing 4.2: Corridor extension encoding

```
1   #const corridor_level = 2.
2
3   corridor(V,0) :- selected_path_for_corridor(R,I), at(R,I,V,_).
4
5   corridor(V,K+1) :-
6       K < corridor_level,
7       corridor(U,K),
8       edge(U,V).
9
10  nvertex(V) :- corridor(V,_).
```

Predicate $selected\_path\_for\_corridor/2$ highlights a path that requires a corridor extension. In practice, we apply corridors to the conflicting paths added to the conflict-free set of paths.

With a sufficiently large k, the entire graph can be covered; using a large corridor could essentially revert the problem back to a classical MAPF scenario.

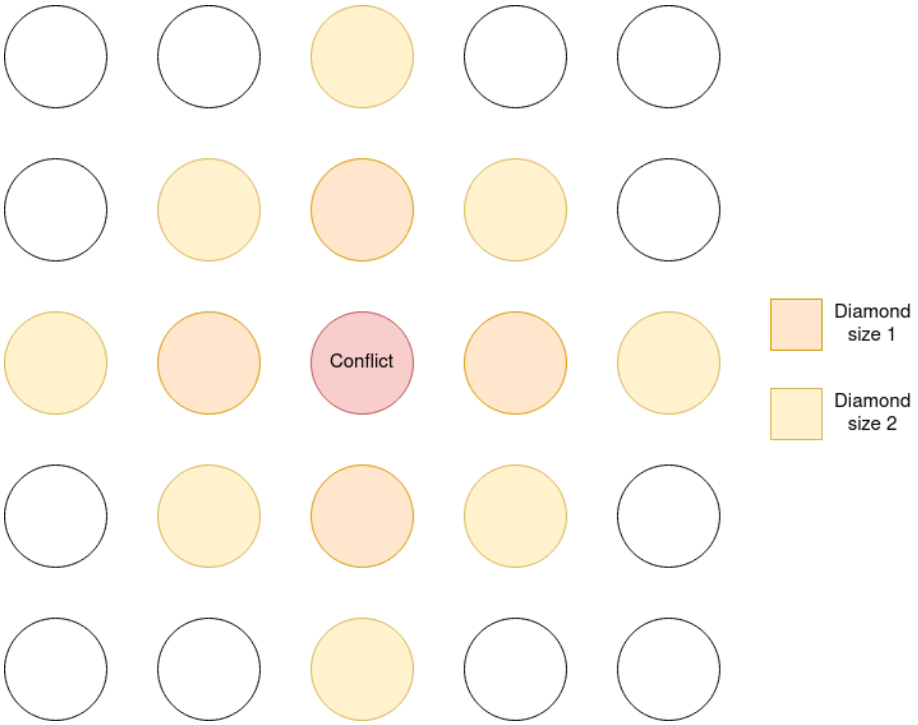
In practice, corridors are created only for paths involved in conflicts, and a choice can be made to create corridors for only one of the two agents involved

in a conflict.

Diamond

The diamond extension strategy involves expanding the sub-graph by incorporating diamond-shaped arrangements of vertices around initial vertices. As illustrated in figure 4.5, various levels of diamond extension can be applied, each increasing the size of the sub-graph and thereby expanding the possibilities for conflict resolution.

Figure 4.5: Example of diamond of size 1 and 2



The following listing 4.2 describes how diamonds are computed. As for corridors, the predicate $selected\_vertex\_for\_diamond/1$ highlight a vertex that requires a diamond extension. In practice, we apply diamond extension on every conflict induced by the set of paths composing the subgraph.

Listing 4.3: Diamond extension encoding

```
1  #const diamond_level = 2.
2  diamond(V,0) :- selected_vertex_for_diamond(V).
3
4  diamond(V,S+1) :-
5      diamond(U,S),
6      S<diamond_level,
7      edge(U,V).
8
9  nvertex(V) :- diamond(V,_).
```

Chapter 5

Benchmarks & Conclusion

5.1 Benchmarks

5.1.1 Global result

Approach	Total Time	# SAT	% Agent Completed			
			Horizon	0	1	3	5
MAPF	941.42	20		1.00	1.00	1.00	1.00
AShPc	268.18	20		0.98	0.98	0.99	>1.00
NStSgD	240.51	20		0.99	0.99	0.99	0.99
NStSgCD	240.37	20		0.99	0.99	0.99	0.99
NStSg	235.40	20		0.99	0.99	0.99	0.99
NStSgC	240.47	20		0.99	0.99	0.99	0.99
NShSg	239.73	20		0.98	0.98	0.98	0.99
AStSg	787.31	19		0.99	0.99	1.00	1.00
NShPcCD	270.90	17		0.80	0.80	0.81	0.81
NStSgPc	228.87	17		0.76	0.78	0.79	0.79
AStSgPc	781.91	16		0.78	0.78	0.79	0.80
AStSgCD	789.32	16		0.78	0.78	0.79	0.80
NShSgPc	235.05	16		0.74	0.76	0.76	0.78
NShPc	2356.84	15		0.87	0.88	0.88	0.88
NStPc	2344.96	15		0.85	0.86	0.87	0.87
AStPc	2357.29	14		0.79	0.80	0.80	0.80

5.1.2

5.2 Conclusion

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Appendix

Table 5.1: Table of the approaches reference name and their description

Name	Value
NStPc	No additional path, heatmap simple threshold elimination, pre-computed path on full graph
NStSg	No additional path, heatmap simple threshold elimination, subgraph approach
NStSgPc	no additional paths, simple threshold elimination, precomputed path on subgraph
AStPc	Additional path, heatmap simple threshold elimination, pre-computed path on full graph
AStSg	Additional path, heatmap simple threshold elimination, subgraph approach
AStSgPc	Additional paths, simple threshold elimination, precomputed path on subgraph
NStSgC	No Additional path, heatmap simple threshold elimination, subgraph approach with corridor
NStSgD	No Additional path, heatmap simple threshold elimination, subgraph approach with diamond
NStSgCD	No Additional path, heatmap simple threshold elimination, subgraph approach with diamond and corridor
AStSgCD	No Additional path, heatmap simple threshold elimination, subgraph approach with diamond and corridor
NShPc	No additional path, summed heatmap elimination, pre-computed path on full graph
NShSg	No additional path, summed heatmap elimination, subgraph approach
NShSgPc	No additional paths, summed heatmap elimination, precomputed path on subgraph
AShPc	Additional path, summed heatmap elimination, subgraph approach
NShPcCD	Additional path, summed heatmap elimination, subgraph approach with diamond and corridor
MAPF	Classical MAPF approach