

Sinsemilla hash function specification

Specifies what is needed to implement a *sinsemilla* hash function algorithm.

EXTENDS *TLC*, *Naturals*, *Integers*, *Sequences*, *Utils*, *Randomization*

--algorithm *sinsemilla*

variables

Holder for a point on the *Pallas* curve.

point = $[a \mapsto 0, b \mapsto 0]$;

Holder for a sequence of characters.

characters = $\langle \rangle$;

Holder for a sequence of bytes.

bytes = $\langle \rangle$;

Holder for a sequence of bytes when the bytes variable is already busy.

auxiliar_bytes = $\langle \rangle$;

Holder for a sequence of bits.

bits = $\langle \rangle$;

Holder for a sequence of slices.

slices = $\langle \rangle$;

Holder for a number, in particular the number of slices.

n = 0;

Holder for a number used as the current slice index in the main loop.

i = 1;

Holder for a point used as an accumulator.

accumulator = $[a \mapsto 0, b \mapsto 0]$;

Holder for the ciphertext produced by the hash function.

ciphertext = $\langle \text{"@"}, \text{"@"} \rangle$;

define

The number of bits in a chunk.

k \triangleq 10

The maximum number of chunks allowed.

c \triangleq 253

The domain separator string for the *Q* point: "z.cash.SinsemillaQ".

SinsemillaQ \triangleq

$\langle \text{"z"}, \text{"."}, \text{"c"}, \text{"a"}, \text{"s"}, \text{"h"}, \text{"."}, \text{"S"}, \text{"i"}, \text{"n"}, \text{"s"}, \text{"e"}, \text{"m"}, \text{"i"}, \text{"l"}, \text{"l"}, \text{"a"}, \text{"Q"} \rangle$

The domain separator string for the *S* point: "z.cash.SinsemillaS".

SinsemillaS \triangleq

$\langle \text{"z"}, \text{"."}, \text{"c"}, \text{"a"}, \text{"s"}, \text{"h"}, \text{"."}, \text{"S"}, \text{"i"}, \text{"n"}, \text{"s"}, \text{"e"}, \text{"m"}, \text{"i"}, \text{"l"}, \text{"l"}, \text{"a"}, \text{"S"} \rangle$

A fixed domain to be used to hash the message.

Domain \triangleq $\langle \text{"t"}, \text{"e"}, \text{"s"}, \text{"t"}, \text{" "}, \text{"S"}, \text{"i"}, \text{"n"}, \text{"s"}, \text{"e"}, \text{"m"}, \text{"i"}, \text{"l"}, \text{"l"}, \text{"a"} \rangle$

A fixed message to be hashed.

Message \triangleq $\langle \text{"m"}, \text{"e"}, \text{"s"}, \text{"s"}, \text{"a"}, \text{"g"}, \text{"e"} \rangle$

The incomplete addition operator. Sums the *x* and *y* coordinates of two points on the *Pallas* curve.

$IncompleteAddition(x, y) \triangleq [a \mapsto x.a + y.a, b \mapsto x.b + y.b]$

Type invariants.

$TypeInvariantPoint \triangleq point \in [a : Nat, b : Nat]$

$TypeInvariantCharacters \triangleq characters \in Seq(STRING)$

$TypeInvariantBytes \triangleq bytes \in Seq(Nat)$

$TypeInvariantAuxiliarBytes \triangleq bytes \in Seq(Nat)$

$TypeInvariantBits \triangleq bits \in Seq(\{0, 1\})$

$TypeInvariantSlices \triangleq slices \in Seq(Seq(\{0, 1\}))$

Check all type invariants.

$InvType \triangleq TypeInvariantPoint \wedge TypeInvariantCharacters \wedge TypeInvariantBytes$
 $\wedge TypeInvariantBytes \wedge TypeInvariantBits \wedge TypeInvariantSlices$

Point holder will eventually end up with a point different than the starting one.

$LivenessPoint \triangleq \Diamond(point \neq [a \mapsto 0, b \mapsto 0])$

Accumulator accumulates.

$LivenessAccumulator \triangleq \Diamond(accumulator \neq [a \mapsto 0, b \mapsto 0])$

Index should always be incremented.

$LivenessIndex \triangleq \Diamond(i > 1)$

Slices should always be produced.

$LivenessSlices \triangleq \Diamond(Len(slices) > 0)$

Ciphertext should be produced.

$LivenessCipherValue \triangleq \Diamond(ciphertext \neq \langle "0", "0" \rangle)$

Check all liveness properties.

$Liveness \triangleq LivenessPoint \wedge LivenessAccumulator \wedge LivenessIndex \wedge LivenessSlices \wedge LivenessCipher$

Bytes should always be a sequence of integers representing bytes.

$SafetyBytesSequence \triangleq \wedge bytes = \langle \rangle \vee (\forall index \in 1 \dots Len(bytes) : bytes[index] \in 0 \dots 255)$

Slices should always be a sequence of sequences of bits and each slice should have no length greater than k .

We only can have a slice with length $<$ than k when we are building the slices in the "PadLastSlice" label of the pad procedure.

$SafetySlicesSequence \triangleq$

$\wedge slices = \langle \rangle \vee (\forall index \in 1 \dots Len(slices) : slices[index] \in Seq(\{0, 1\}) \wedge Len(slices[index]) \leq k)$

The number of slices should be less than or equal to the maximum number of chunks allowed.

$SafetyMaxChunks \triangleq n \leq c$

Check that the ciphertext has the correct fixed size.

$SafetyCipherSize \triangleq Len(ciphertext) = 2$

Check all safety properties.

$Safety \triangleq SafetyBytesSequence \wedge SafetySlicesSequence \wedge SafetyMaxChunks \wedge SafetyCipherSize$

end define ;

Convert a sequence of characters to a sequence of bytes.

macro *characters_to_bytes*()

begin

bytes := [*char* $\in 1 \dots Len(characters)$ \mapsto *Ord(characters[*char*])*];

end macro ;

Convert a sequence of bytes to a flat sequence of bits.

```
macro bytes_to_bits()
begin
  bits := FlattenSeq([byte ∈ 1 .. Len(bytes) ↦ ByteToBitSequence(bytes[byte])]);
end macro ;
```

Convert a sequence of bytes to a sequence of characters.

```
macro bytes_to_characters()
begin
  characters := [b ∈ 1 .. Len(bytes) ↦ Chr(bytes[b])];
end macro ;
```

Convert a *Pallas* point to a sequence of fixed bytes. Here we just use the point coordinates as bytes.

```
macro point_to_bytes()
begin
  bytes := ⟨point.a, point.b⟩;
end macro ;
```

The starting procedure that do all the conversion needed with the domain and message constants, call the main procedure to hash the message and decodes the resulting point coordinates to characters.

```
procedure sinsemilla_hash()
begin
```

Encode the domain characters as bytes and store them in *auxiliar_bytes* for later use.

EncodeDomain:

```
  characters := Domain;
  characters_to_bytes();
  auxiliar_bytes := bytes;
```

Encode the message characters as bits and store them in bits for later use.

EncodeMessage:

```
  characters := Message;
  characters_to_bytes();
  bytes_to_bits();
```

With the domain bytes in bytes and the message bits in bits, call the main procedure to hash the message.

SinsemillaHashToPoint:

```
  bytes := auxiliar_bytes;
  call sinsemilla_hash_to_point();
```

Decode the point coordinates to characters.

DecodeCipherText:

```
  point_to_bytes();
  bytes_to_characters();
```

Ciphertext:

```
  ciphertext := characters;
  return;
```

```
end procedure ;
```

the main procedure convert the message bits into a *Pallas* point, using the domain bytes stored in bytes as the

domain separator and the message bits stored in bits as the message.

procedure *sinsemilla_hash_to_point*()

begin

CalculateN:

Calculate the number of slices needed to hash the message.

$n := \text{Len}(\text{bits}) \div k$;

CallPad:

Use the global bits as input and get slices in slices.

call *pad*() ;

CallQ:

Produce a *Pallas* point with the bytes stored, these bytes are set in the caller as domain bytes.

call *q*() ;

InitializeAcc:

With the point we got from calling *q*, initialize the accumulator.

accumulator := *point* ;

MainLoop:

Loop over the slices.

while $i \leq n$ **do**

CallS:

Produce a *Pallas* point calling *s* given the padded bits (10 bits).

bits := *slices*[*i*] ;

call *s*() ;

Accumulate:

Incomplete addition of the accumulator and the point.

accumulator :=

IncompleteAddition(*IncompleteAddition*(*accumulator*, *point*), *accumulator*) ;

IncrementIndex:

$i := i + 1$;

end while ;

AssignAccumulatorToPoint:

point := *accumulator* ;

return ;

end procedure ;

Pad the message bits with zeros until the length is a multiple of *k*. Create chunks of *k* bits.

procedure *pad*()

begin

GetSlices:

slices := [$index \in 1 \dots n \mapsto \text{IF } (index * k + k) \geq \text{Len}(\text{bits}) \text{ THEN}$

SubSeq(*bits*, $index * k$, $\text{Len}(\text{bits})$)

ELSE *SubSeq*(*bits*, $index * k$, $index * k + k - 1$)] ;

PadLastSlice:

slices[$\text{Len}(\text{slices})$] := [$index \in 1 \dots k \mapsto \text{IF } index \leq \text{Len}(\text{slices}[\text{Len}(\text{slices})]) \text{ THEN}$

slices[$\text{Len}(\text{slices})$][*index*]

ELSE 0] ;

```

    return ;
end procedure ;

```

Produce a *Pallas* point with the bytes stored in *bytes*, these bytes are set in the caller as domain bytes.

```

procedure q()
begin
    Q:
        call hash_to_pallas(SinsemillaQ, bytes) ;
    return ;
end procedure ;

```

Produce a *Pallas* point given the padded bits (10 bits). First we call *IntToLEOSP* on the bits and then we call *hash_to_pallas* with the result.

```

procedure s()
begin
    CallI2LEOSP:
        call IntToLEOSP32() ;
    S:
        call hash_to_pallas(SinsemillaS, bytes) ;
    return ;
end procedure ;

```

Produce a *Pallas* point with the separator and message bytes stored in *separator* and *message_bytes*.

```

procedure hash_to_pallas(separator, message_bytes)
begin
    HashToPallas:
        Here we decouple the input message and separator from the outputs by choosing random coordinates.
        From now on, in this model, we can't relate the original message with the ciphertext anymore.
        point := [
            a  $\mapsto$  CHOOSE r  $\in$  RandomSubset(1, 1 .. 3) : TRUE,
            b  $\mapsto$  CHOOSE r  $\in$  RandomSubset(1, 1 .. 3) : TRUE
        ] ;
    return ;
end procedure ;

```

Integer to Little-Endian Octet String Pairing.

This procedure assumes $k = 10$, so we have 8 bits to build the first byte and 2 bits for the second. The second byte is formed by the first two bits of the second byte of the input and 6 zeros. We reach the 32 bytes by adding two zero bytes at the end.

This algorithm is the one implemented in Zebra.

```

procedure IntToLEOSP32()
begin
    IntToLEOSP:
        bytes := (
            BitSequenceToByte(SubSeq(bits, 1, 8)),
            BitSequenceToByte(SubSeq(bits, 9, 10)[1], SubSeq(bits, 9, 10)[2], 0, 0, 0, 0, 0, 0),

```

```

        0,
        0
    } ;
    return ;
end procedure ;

```

Single process that calls the starting procedure.

```

fair process main = "MAIN"
begin

```

```

    SinSemillaHashCall:
        call sinsemilla_hash();

```

```

end process ;
end algorithm ;

```

```

    BEGIN TRANSLATION (chksum(pcal) = "e42dec70" ∧ chksum(tla) = "77d018cd")
    CONSTANT defaultInitValue
    VARIABLES point, characters, bytes, auxiliar_bytes, bits, slices, n, i,
               accumulator, ciphertext, pc, stack

```

```

    define statement
    k ≜ 10

```

```

    c ≜ 253

```

```

    SinsemillaQ ≜
    ⟨ "z", ".", "c", "a", "s", "h", ".", "S", "I", "n", "s", "e", "m", "I", "l", "l", "a", "Q" ⟩

```

```

    SinsemillaS ≜
    ⟨ "z", ".", "c", "a", "s", "h", ".", "S", "I", "n", "s", "e", "m", "I", "l", "l", "a", "S" ⟩

```

```

    Domain ≜ ⟨ "t", "e", "s", "t", " ", "S", "I", "n", "s", "e", "m", "I", "l", "l", "a" ⟩

```

```

    Message ≜ ⟨ "m", "e", "s", "s", "a", "g", "e" ⟩

```

```

    IncompleteAddition(x, y) ≜ [a ↦ x.a + y.a, b ↦ x.b + y.b]

```

```

    TypeInvariantPoint ≜ point ∈ [a : Nat, b : Nat]
    TypeInvariantCharacters ≜ characters ∈ Seq(STRING)
    TypeInvariantBytes ≜ bytes ∈ Seq(Nat)
    TypeInvariantAuxiliarBytes ≜ bytes ∈ Seq(Nat)
    TypeInvariantBits ≜ bits ∈ Seq({0, 1})
    TypeInvariantSlices ≜ slices ∈ Seq(Seq({0, 1}))

```

```

    InvType ≜ TypeInvariantPoint ∧ TypeInvariantCharacters ∧ TypeInvariantBytes
    ∧ TypeInvariantBytes ∧ TypeInvariantBits ∧ TypeInvariantSlices

```

```

    LivenessPoint ≜ ◇(point ≠ [a ↦ 0, b ↦ 0])

```

$$LivenessAccumulator \triangleq \Diamond(accumulator \neq [a \mapsto 0, b \mapsto 0])$$

$$LivenessIndex \triangleq \Diamond(i > 1)$$

$$LivenessSlices \triangleq \Diamond(Len(slices) > 0)$$

$$LivenessCipherValue \triangleq \Diamond(ciphertext \neq \langle \text{"@"}, \text{"@"} \rangle)$$

$$Liveness \triangleq LivenessPoint \wedge LivenessAccumulator \wedge LivenessIndex \wedge LivenessSlices \wedge LivenessCipherValue$$

$$SafetyBytesSequence \triangleq \wedge bytes = \langle \rangle \vee (\forall index \in 1 \dots Len(bytes) : bytes[index] \in 0 \dots 255)$$

$$SafetySlicesSequence \triangleq \wedge slices = \langle \rangle \vee (\forall index \in 1 \dots Len(slices) : slices[index] \in Seq(\{0, 1\}) \wedge Len(slices[index]) \leq k)$$

$$SafetyMaxChunks \triangleq n \leq c$$

$$SafetyCipherSize \triangleq Len(ciphertext) = 2$$

$$Safety \triangleq SafetyBytesSequence \wedge SafetySlicesSequence \wedge SafetyMaxChunks \wedge SafetyCipherSize$$

VARIABLES *separator, message_bytes*

$$vars \triangleq \langle point, characters, bytes, auxiliar_bytes, bits, slices, n, i, accumulator, ciphertext, pc, stack, separator, message_bytes \rangle$$

$$ProcSet \triangleq \{ \text{"MAIN"} \}$$

$$\begin{aligned} Init &\triangleq \text{Global variables} \\ &\wedge point = [a \mapsto 0, b \mapsto 0] \\ &\wedge characters = \langle \rangle \\ &\wedge bytes = \langle \rangle \\ &\wedge auxiliar_bytes = \langle \rangle \\ &\wedge bits = \langle \rangle \\ &\wedge slices = \langle \rangle \\ &\wedge n = 0 \\ &\wedge i = 1 \\ &\wedge accumulator = [a \mapsto 0, b \mapsto 0] \\ &\wedge ciphertext = \langle \text{"@"}, \text{"@"} \rangle \\ &\text{Procedure } hash_to_pallas \\ &\wedge separator = [self \in ProcSet \mapsto defaultInitValue] \\ &\wedge message_bytes = [self \in ProcSet \mapsto defaultInitValue] \\ &\wedge stack = [self \in ProcSet \mapsto \langle \rangle] \\ &\wedge pc = [self \in ProcSet \mapsto \text{"SinSemillaHashCall"}] \end{aligned}$$

$$\begin{aligned} EncodeDomain(self) &\triangleq \wedge pc[self] = \text{"EncodeDomain"} \\ &\wedge characters' = Domain \end{aligned}$$

$$\begin{aligned}
& \wedge \text{bytes}' = [\text{char} \in 1 \dots \text{Len}(\text{characters}') \mapsto \text{Ord}(\text{characters}'[\text{char}])] \\
& \wedge \text{auxiliar_bytes}' = \text{bytes}' \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![\text{self}] = \text{"EncodeMessage"}] \\
& \wedge \text{UNCHANGED } \langle \text{point}, \text{bits}, \text{slices}, n, i, \text{accumulator}, \\
& \quad \text{ciphertext}, \text{stack}, \text{separator}, \\
& \quad \text{message_bytes} \rangle \\
\text{EncodeMessage}(\text{self}) & \triangleq \wedge \text{pc}[\text{self}] = \text{"EncodeMessage"} \\
& \wedge \text{characters}' = \text{Message} \\
& \wedge \text{bytes}' = [\text{char} \in 1 \dots \text{Len}(\text{characters}') \mapsto \text{Ord}(\text{characters}'[\text{char}])] \\
& \wedge \text{bits}' = \text{FlattenSeq}([\text{byte} \in 1 \dots \text{Len}(\text{bytes}') \mapsto \text{ByteToBitSequence}(\text{bytes}'[\text{byte}])] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![\text{self}] = \text{"SinsemillaHashToPoint"}] \\
& \wedge \text{UNCHANGED } \langle \text{point}, \text{auxiliar_bytes}, \text{slices}, n, i, \\
& \quad \text{accumulator}, \text{ciphertext}, \text{stack}, \\
& \quad \text{separator}, \text{message_bytes} \rangle \\
\text{SinsemillaHashToPoint}(\text{self}) & \triangleq \wedge \text{pc}[\text{self}] = \text{"SinsemillaHashToPoint"} \\
& \wedge \text{bytes}' = \text{auxiliar_bytes} \\
& \wedge \text{stack}' = [\text{stack} \text{ EXCEPT } ![\text{self}] = \langle [\text{procedure} \mapsto \text{"sinsemilla_hash_to_p"} \\
& \quad \text{pc} \mapsto \text{"DecodeCipherText"}] \rangle \\
& \quad \circ \text{stack}[\text{self}]] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![\text{self}] = \text{"CalculateN"}] \\
& \wedge \text{UNCHANGED } \langle \text{point}, \text{characters}, \\
& \quad \text{auxiliar_bytes}, \text{bits}, \text{slices}, n, \\
& \quad i, \text{accumulator}, \text{ciphertext}, \\
& \quad \text{separator}, \text{message_bytes} \rangle \\
\text{DecodeCipherText}(\text{self}) & \triangleq \wedge \text{pc}[\text{self}] = \text{"DecodeCipherText"} \\
& \wedge \text{bytes}' = \langle \text{point}.a, \text{point}.b \rangle \\
& \wedge \text{characters}' = [b \in 1 \dots \text{Len}(\text{bytes}') \mapsto \text{Chr}(\text{bytes}'[b])] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![\text{self}] = \text{"Ciphertext"}] \\
& \wedge \text{UNCHANGED } \langle \text{point}, \text{auxiliar_bytes}, \text{bits}, \text{slices}, \\
& \quad n, i, \text{accumulator}, \text{ciphertext}, \text{stack}, \\
& \quad \text{separator}, \text{message_bytes} \rangle \\
\text{Ciphertext}(\text{self}) & \triangleq \wedge \text{pc}[\text{self}] = \text{"Ciphertext"} \\
& \wedge \text{ciphertext}' = \text{characters} \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![\text{self}] = \text{Head}(\text{stack}[\text{self}]).\text{pc}] \\
& \wedge \text{stack}' = [\text{stack} \text{ EXCEPT } ![\text{self}] = \text{Tail}(\text{stack}[\text{self}])] \\
& \wedge \text{UNCHANGED } \langle \text{point}, \text{characters}, \text{bytes}, \text{auxiliar_bytes}, \\
& \quad \text{bits}, \text{slices}, n, i, \text{accumulator}, \text{separator}, \\
& \quad \text{message_bytes} \rangle \\
\text{sinsemilla_hash}(\text{self}) & \triangleq \text{EncodeDomain}(\text{self}) \vee \text{EncodeMessage}(\text{self}) \\
& \vee \text{SinsemillaHashToPoint}(\text{self}) \\
& \vee \text{DecodeCipherText}(\text{self}) \vee \text{Ciphertext}(\text{self})
\end{aligned}$$

$$\begin{aligned}
\text{CalculateN}(\text{self}) &\triangleq \wedge pc[\text{self}] = \text{"CalculateN"} \\
&\wedge n' = (\text{Len}(\text{bits}) \div k) \\
&\wedge pc' = [pc \text{ EXCEPT } ![\text{self}] = \text{"CallPad"}] \\
&\wedge \text{UNCHANGED } \langle \text{point}, \text{characters}, \text{bytes}, \text{auxiliar_bytes}, \\
&\quad \text{bits}, \text{slices}, i, \text{accumulator}, \text{ciphertext}, \\
&\quad \text{stack}, \text{separator}, \text{message_bytes} \rangle \\
\\
\text{CallPad}(\text{self}) &\triangleq \wedge pc[\text{self}] = \text{"CallPad"} \\
&\wedge \text{stack}' = [\text{stack} \text{ EXCEPT } ![\text{self}] = \langle [\text{procedure} \mapsto \text{"pad"}, \\
&\quad \quad \quad pc \quad \quad \mapsto \text{"CallQ"}] \rangle \\
&\quad \quad \quad \circ \text{stack}[\text{self}]] \\
&\wedge pc' = [pc \text{ EXCEPT } ![\text{self}] = \text{"GetSlices"}] \\
&\wedge \text{UNCHANGED } \langle \text{point}, \text{characters}, \text{bytes}, \text{auxiliar_bytes}, \\
&\quad \text{bits}, \text{slices}, n, i, \text{accumulator}, \text{ciphertext}, \\
&\quad \text{separator}, \text{message_bytes} \rangle \\
\\
\text{CallQ}(\text{self}) &\triangleq \wedge pc[\text{self}] = \text{"CallQ"} \\
&\wedge \text{stack}' = [\text{stack} \text{ EXCEPT } ![\text{self}] = \langle [\text{procedure} \mapsto \text{"q"}, \\
&\quad \quad \quad pc \quad \quad \mapsto \text{"InitializeAcc"}] \rangle \\
&\quad \quad \quad \circ \text{stack}[\text{self}]] \\
&\wedge pc' = [pc \text{ EXCEPT } ![\text{self}] = \text{"Q"}] \\
&\wedge \text{UNCHANGED } \langle \text{point}, \text{characters}, \text{bytes}, \text{auxiliar_bytes}, \text{bits}, \\
&\quad \text{slices}, n, i, \text{accumulator}, \text{ciphertext}, \\
&\quad \text{separator}, \text{message_bytes} \rangle \\
\\
\text{InitializeAcc}(\text{self}) &\triangleq \wedge pc[\text{self}] = \text{"InitializeAcc"} \\
&\wedge \text{accumulator}' = \text{point} \\
&\wedge pc' = [pc \text{ EXCEPT } ![\text{self}] = \text{"MainLoop"}] \\
&\wedge \text{UNCHANGED } \langle \text{point}, \text{characters}, \text{bytes}, \\
&\quad \text{auxiliar_bytes}, \text{bits}, \text{slices}, n, i, \\
&\quad \text{ciphertext}, \text{stack}, \text{separator}, \\
&\quad \text{message_bytes} \rangle \\
\\
\text{MainLoop}(\text{self}) &\triangleq \wedge pc[\text{self}] = \text{"MainLoop"} \\
&\wedge \text{IF } i \leq n \\
&\quad \text{THEN } \wedge pc' = [pc \text{ EXCEPT } ![\text{self}] = \text{"CallS"}] \\
&\quad \text{ELSE } \wedge pc' = [pc \text{ EXCEPT } ![\text{self}] = \text{"AssignAccumulatorToPoint"}] \\
&\wedge \text{UNCHANGED } \langle \text{point}, \text{characters}, \text{bytes}, \text{auxiliar_bytes}, \\
&\quad \text{bits}, \text{slices}, n, i, \text{accumulator}, \text{ciphertext}, \\
&\quad \text{stack}, \text{separator}, \text{message_bytes} \rangle \\
\\
\text{CallS}(\text{self}) &\triangleq \wedge pc[\text{self}] = \text{"CallS"} \\
&\wedge \text{bits}' = \text{slices}[i] \\
&\wedge \text{stack}' = [\text{stack} \text{ EXCEPT } ![\text{self}] = \langle [\text{procedure} \mapsto \text{"s"}, \\
&\quad \quad \quad pc \quad \quad \mapsto \text{"Accumulate"}] \rangle \\
&\quad \quad \quad \circ \text{stack}[\text{self}]]
\end{aligned}$$

$$\begin{aligned}
& \wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"CallI2LEOSP"}] \\
& \wedge \text{UNCHANGED } \langle point, characters, bytes, auxiliar_bytes, \\
& \quad slices, n, i, accumulator, ciphertext, \\
& \quad separator, message_bytes \rangle \\
\\
Accumulate(self) & \triangleq \wedge pc[self] = \text{"Accumulate"} \\
& \wedge accumulator' = IncompleteAddition(IncompleteAddition(accumulator, point), acc \\
& \wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"IncrementIndex"}] \\
& \wedge \text{UNCHANGED } \langle point, characters, bytes, auxiliar_bytes, \\
& \quad bits, slices, n, i, ciphertext, stack, \\
& \quad separator, message_bytes \rangle \\
\\
IncrementIndex(self) & \triangleq \wedge pc[self] = \text{"IncrementIndex"} \\
& \wedge i' = i + 1 \\
& \wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"MainLoop"}] \\
& \wedge \text{UNCHANGED } \langle point, characters, bytes, \\
& \quad auxiliar_bytes, bits, slices, n, \\
& \quad accumulator, ciphertext, stack, \\
& \quad separator, message_bytes \rangle \\
\\
AssignAccumulatorToPoint(self) & \triangleq \wedge pc[self] = \text{"AssignAccumulatorToPoint"} \\
& \wedge point' = accumulator \\
& \wedge pc' = [pc \text{ EXCEPT } ![self] = Head(stack[self]).pc] \\
& \wedge stack' = [stack \text{ EXCEPT } ![self] = Tail(stack[self])] \\
& \wedge \text{UNCHANGED } \langle characters, bytes, \\
& \quad auxiliar_bytes, bits, slices, \\
& \quad n, i, accumulator, \\
& \quad ciphertext, separator, \\
& \quad message_bytes \rangle \\
\\
sinsemilla_hash_to_point(self) & \triangleq CalculateN(self) \vee CallPad(self) \\
& \vee CallQ(self) \vee InitializeAcc(self) \\
& \vee MainLoop(self) \vee CallS(self) \\
& \vee Accumulate(self) \\
& \vee IncrementIndex(self) \\
& \vee AssignAccumulatorToPoint(self) \\
\\
GetSlices(self) & \triangleq \wedge pc[self] = \text{"GetSlices"} \\
& \wedge slices' = [index \in 1 \dots n \mapsto \text{IF } (index * k + k) \geq Len(bits) \text{ THEN} \\
& \quad SubSeq(bits, index * k, Len(bits)) \\
& \quad \text{ELSE } SubSeq(bits, index * k, index * k + k - 1)] \\
& \wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"PadLastSlice"}] \\
& \wedge \text{UNCHANGED } \langle point, characters, bytes, auxiliar_bytes, \\
& \quad bits, n, i, accumulator, ciphertext, stack, \\
& \quad separator, message_bytes \rangle \\
\\
PadLastSlice(self) & \triangleq \wedge pc[self] = \text{"PadLastSlice"}
\end{aligned}$$

$$\begin{aligned}
& \wedge \text{ slices}' = [\text{ slices } \text{ EXCEPT } ![\text{ Len}(\text{ slices})] = \text{ slices}[\text{ Len}(\text{ slices})][\text{ index}] \quad [\text{ index } \in 1 \dots k \mapsto \text{ ELSE } 0]] \\
& \wedge \text{ pc}' = [\text{ pc } \text{ EXCEPT } ![\text{ self}] = \text{ Head}(\text{ stack}[\text{ self}]).\text{ pc}] \\
& \wedge \text{ stack}' = [\text{ stack } \text{ EXCEPT } ![\text{ self}] = \text{ Tail}(\text{ stack}[\text{ self}])] \\
& \wedge \text{ UNCHANGED } \langle \text{ point, characters, bytes, auxiliar_bytes,} \\
& \quad \text{ bits, n, i, accumulator, ciphertext,} \\
& \quad \text{ separator, message_bytes} \rangle \\
\text{ pad}(\text{ self}) & \triangleq \text{ GetSlices}(\text{ self}) \vee \text{ PadLastSlice}(\text{ self}) \\
\text{ Q}(\text{ self}) & \triangleq \wedge \text{ pc}[\text{ self}] = \text{ "Q"} \\
& \wedge \text{ message_bytes}' = [\text{ message_bytes } \text{ EXCEPT } ![\text{ self}] = \text{ bytes}] \\
& \wedge \text{ separator}' = [\text{ separator } \text{ EXCEPT } ![\text{ self}] = \text{ SinsemillaQ}] \\
& \wedge \text{ stack}' = [\text{ stack } \text{ EXCEPT } ![\text{ self}] = \langle [\text{ procedure } \mapsto \text{ "hash_to_pallas",} \\
& \quad \text{ pc} \mapsto \text{ Head}(\text{ stack}[\text{ self}]).\text{ pc}, \\
& \quad \text{ separator} \mapsto \text{ separator}[\text{ self}], \\
& \quad \text{ message_bytes} \mapsto \text{ message_bytes}[\text{ self}]] \\
& \quad \circ \text{ Tail}(\text{ stack}[\text{ self}])] \\
& \wedge \text{ pc}' = [\text{ pc } \text{ EXCEPT } ![\text{ self}] = \text{ "HashToPallas"}] \\
& \wedge \text{ UNCHANGED } \langle \text{ point, characters, bytes, auxiliar_bytes, bits,} \\
& \quad \text{ slices, n, i, accumulator, ciphertext} \rangle \\
\text{ q}(\text{ self}) & \triangleq \text{ Q}(\text{ self}) \\
\text{ CallI2LEOSP}(\text{ self}) & \triangleq \wedge \text{ pc}[\text{ self}] = \text{ "CallI2LEOSP"} \\
& \wedge \text{ stack}' = [\text{ stack } \text{ EXCEPT } ![\text{ self}] = \langle [\text{ procedure } \mapsto \text{ "IntToLEOSP32",} \\
& \quad \text{ pc} \mapsto \text{ "S"}] \\
& \quad \circ \text{ stack}[\text{ self}]] \\
& \wedge \text{ pc}' = [\text{ pc } \text{ EXCEPT } ![\text{ self}] = \text{ "IntToLEOSP"}] \\
& \wedge \text{ UNCHANGED } \langle \text{ point, characters, bytes, auxiliar_bytes,} \\
& \quad \text{ bits, slices, n, i, accumulator,} \\
& \quad \text{ ciphertext, separator, message_bytes} \rangle \\
\text{ S}(\text{ self}) & \triangleq \wedge \text{ pc}[\text{ self}] = \text{ "S"} \\
& \wedge \text{ message_bytes}' = [\text{ message_bytes } \text{ EXCEPT } ![\text{ self}] = \text{ bytes}] \\
& \wedge \text{ separator}' = [\text{ separator } \text{ EXCEPT } ![\text{ self}] = \text{ SinsemillaS}] \\
& \wedge \text{ stack}' = [\text{ stack } \text{ EXCEPT } ![\text{ self}] = \langle [\text{ procedure } \mapsto \text{ "hash_to_pallas",} \\
& \quad \text{ pc} \mapsto \text{ Head}(\text{ stack}[\text{ self}]).\text{ pc}, \\
& \quad \text{ separator} \mapsto \text{ separator}[\text{ self}], \\
& \quad \text{ message_bytes} \mapsto \text{ message_bytes}[\text{ self}]] \\
& \quad \circ \text{ Tail}(\text{ stack}[\text{ self}])] \\
& \wedge \text{ pc}' = [\text{ pc } \text{ EXCEPT } ![\text{ self}] = \text{ "HashToPallas"}] \\
& \wedge \text{ UNCHANGED } \langle \text{ point, characters, bytes, auxiliar_bytes, bits,} \\
& \quad \text{ slices, n, i, accumulator, ciphertext} \rangle \\
\text{ s}(\text{ self}) & \triangleq \text{ CallI2LEOSP}(\text{ self}) \vee \text{ S}(\text{ self})
\end{aligned}$$

$$\begin{aligned}
HashToPallas(self) &\triangleq \wedge pc[self] = \text{"HashToPallas"} \\
&\wedge point' = [\\
&\quad a \mapsto \text{CHOOSE } r \in \text{RandomSubset}(1, 1 \dots 3) : \text{TRUE}, \\
&\quad b \mapsto \text{CHOOSE } r \in \text{RandomSubset}(1, 1 \dots 3) : \text{TRUE} \\
&] \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).pc] \\
&\wedge separator' = [separator \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).separator] \\
&\wedge message_bytes' = [message_bytes \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).message_bytes] \\
&\wedge stack' = [stack \text{ EXCEPT } ![self] = \text{Tail}(\text{stack}[self])] \\
&\wedge \text{UNCHANGED } \langle characters, bytes, auxiliar_bytes, bits, \\
&\quad slices, n, i, accumulator, ciphertext \rangle \\
\\
hash_to_pallas(self) &\triangleq HashToPallas(self) \\
\\
IntToLEOSP(self) &\triangleq \wedge pc[self] = \text{"IntToLEOSP"} \\
&\wedge bytes' = \langle \\
&\quad \text{BitSequenceToByte}(\text{SubSeq}(bits, 1, 8)), \\
&\quad \text{BitSequenceToByte}(\langle \text{SubSeq}(bits, 9, 10)[1], \text{SubSeq}(bits, 9, 10)[2], 0, \\
&\quad 0, \\
&\quad 0 \\
&\rangle) \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).pc] \\
&\wedge stack' = [stack \text{ EXCEPT } ![self] = \text{Tail}(\text{stack}[self])] \\
&\wedge \text{UNCHANGED } \langle point, characters, auxiliar_bytes, bits, \\
&\quad slices, n, i, accumulator, ciphertext, \\
&\quad separator, message_bytes \rangle \\
\\
IntToLEOSP32(self) &\triangleq IntToLEOSP(self) \\
\\
SinSemillaHashCall &\triangleq \wedge pc[\text{"MAIN"}] = \text{"SinSemillaHashCall"} \\
&\wedge stack' = [stack \text{ EXCEPT } ![\text{"MAIN"}] = \langle [procedure \mapsto \text{"sinsemilla_hash"}, \\
&\quad pc \mapsto \text{"Done"}] \rangle \\
&\quad \circ stack[\text{"MAIN"}]] \\
&\wedge pc' = [pc \text{ EXCEPT } ![\text{"MAIN"}] = \text{"EncodeDomain"}] \\
&\wedge \text{UNCHANGED } \langle point, characters, bytes, auxiliar_bytes, \\
&\quad bits, slices, n, i, accumulator, \\
&\quad ciphertext, separator, message_bytes \rangle \\
\\
main &\triangleq SinSemillaHashCall \\
\\
\text{Allow infinite stuttering to prevent deadlock on termination.} \\
Terminating &\triangleq \wedge \forall self \in ProcSet : pc[self] = \text{"Done"} \\
&\wedge \text{UNCHANGED } vars \\
\\
Next &\triangleq main \\
&\quad \vee (\exists self \in ProcSet : \vee sinsemilla_hash(self) \\
&\quad \vee sinsemilla_hash_to_point(self))
\end{aligned}$$

$$\begin{aligned}
& \vee \text{pad}(\text{self}) \vee q(\text{self}) \quad \vee s(\text{self}) \\
& \vee \text{hash_to_pallas}(\text{self}) \vee \text{IntToLEOSP32}(\text{self}) \\
& \vee \text{Terminating} \\
\text{Spec} & \triangleq \wedge \text{Init} \wedge \square[\text{Next}]_{\text{vars}} \\
& \wedge \wedge \text{WF}_{\text{vars}}(\text{main}) \\
& \wedge \text{WF}_{\text{vars}}(\text{sinsemilla_hash}(\text{"MAIN"})) \\
& \wedge \text{WF}_{\text{vars}}(\text{sinsemilla_hash_to_point}(\text{"MAIN"})) \\
& \wedge \text{WF}_{\text{vars}}(\text{pad}(\text{"MAIN"})) \\
& \wedge \text{WF}_{\text{vars}}(q(\text{"MAIN"})) \\
& \wedge \text{WF}_{\text{vars}}(s(\text{"MAIN"})) \\
& \wedge \text{WF}_{\text{vars}}(\text{hash_to_pallas}(\text{"MAIN"})) \\
& \wedge \text{WF}_{\text{vars}}(\text{IntToLEOSP32}(\text{"MAIN"})) \\
\text{Termination} & \triangleq \diamond(\forall \text{self} \in \text{ProcSet} : \text{pc}[\text{self}] = \text{"Done"})
\end{aligned}$$

END TRANSLATION