COMP4141 Homework 9

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Exercise 1 (5 marks) Identify the error in the following incorrect proof that $\mathbf{P} \neq \mathbf{NP}$.

> We assume that $\mathbf{P} = \mathbf{NP}$ and prove a contradiction. If $\mathbf{P} = \mathbf{NP}$ then $SAT \in \mathbf{P}$, hence for some k, we have $SAT \in \mathbf{TIME}(n^k)$. Because every language in $\mathbf{NP}$ is polynomial time reducible to SAT, we have $\mathbf{NP} \subseteq \mathbf{TIME}(n^k)$. Therefore $\mathbf{P} \subseteq \mathbf{TIME}(n^k)$. But by the time hierarchy theorem, $\mathbf{TIME}(n^{k+1})$ contains a language that is not in $\mathbf{TIME}(n^k)$, which contradicts that $\mathbf{P} \subseteq \mathbf{TIME}(n^k)$. Therefore $\mathbf{P} \neq \mathbf{NP}$.

Exercise 2 (10 marks) As the Rolling Bones say, you can't always get what you want. But at least you can try to get as many of the things you want as possible! We can formulate this more precisely as the language of all $\langle S, k\rangle$ such that S is a finite set of formulas of propositional logic, and k is the largest number such that there exists a subset $\{\phi_1, \ldots, \phi_k\}$ of S of size k, such that $\phi_1 \wedge \ldots \wedge \phi_k$ is satisfiable. With proof, give a complexity class (of those discussed in this course) that contains this language. The smaller your complexity class, the higher your grade! (However, you do not need to prove that the problem is complete for your class.)

Exercise 3 (15 marks) Prove that $\mathbf{BPP}$ is closed under concatentation. That is, if A, B are languages in $\mathbf{BPP}$, then so is $AB = \{xy \mid x \in A,\ y \in B\}$.