COMP4141 Tutorial 1

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Exercise 1 (Functions over strings, induction) In the lecture it was mentioned that many functions are defined recursively on the structure of strings.

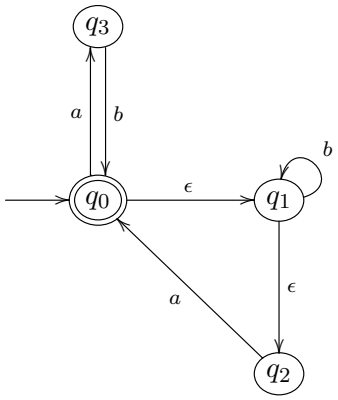
1. Give the recursive definition of a function $\texttt{rev} : \Sigma^* \to \Sigma^*$, which takes an arbitrary string as input and returns the reversed string. For example, $\texttt{rev}(string) = gnirts$ and $\texttt{rev}(racecar) = racecar$.
2. Prove by induction that, for all $u \in \Sigma^*$, $|\texttt{rev}(u)| = |u|$.
   Hint: You may find it useful to first prove some simpler properties (also using induction) about concatenation and/or the length function.

Exercise 2 (Deterministic Finite Automata) Assume the alphabet $\Sigma = \{a, b\}$. Define a DFA that accepts the language

$$\{w \in \Sigma^* \ : \ w \text{ has odd length and contains } \textit{both} \text{ an } a \text{ and a } b\} \ .$$

Present your answer in two ways: as a diagram and as a 5-tuple, i.e., you have to define Q, Σ, q_0, δ, and F. Explain the intuition for what each of the states is doing. Draw the diagram with as few edge crossings as possible, for clarity.

Exercise 3 (Nondeterministic Finite Automata) Assume the alphabet $\Sigma = \{a, b\}$. Consider the following ϵ-NFA in graphical representation.



List all the strings of length ≤ 3 that this automaton accepts.

Exercise 4 (Determinisation) Using the subset construction, build a DFA that accepts the same language as the NFA from the previous question. Give the graphical representation of this DFA, indicating clearly the set of NFA states associated to each DFA state. Show only the reachable states.

Exercise 5 (Regular Expressions) Write a regular expression for each of the following languages over alphabet $\Sigma = \{a, b\}$:

1. the set E of words of even length,
2. the set O of words of odd length,
3. the set of words of even length that contain both an a and a b. (Hint: first argue that a word in this language must contain an *adjacent* a and b.)