COMP4141 Tutorial 8

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Exercise 1 Let $\#$ be a symbol in an alphabet Σ^*. A $\#$-*chain* in a language L from $\alpha \in \Sigma^*$ to $\beta \in \Sigma^*$ is a sequence of words

$$\begin{array}{c} \gamma_0\#\gamma_1, \\ \gamma_1\#\gamma_2, \\ \vdots \\ \gamma_{n-1}\#\gamma_n \end{array}$$

such that $\alpha = \gamma_0$ and $\beta \in \gamma_n$, and for all $i = 0 \ldots n-1$ we have $\gamma_i\#\gamma_{i+1} \in L$ and $|\gamma_i| = |\gamma_{i+1}|$. Show that the following problem is in **PSPACE**: Given a context-free grammar G and words α, β, does there exist a $\#$-chain (of any length) in $L(G)$ from α to β?

Exercise 2 Show that the problem of the previous question is **PSPACE**-hard (and hence **PSPACE**-complete, by the previous exercise.)

Exercise 3 Show that the balanced-parenthesis language over $\Sigma = \{(,)\}$ defined by the context free grammar

$$S ::= \epsilon \mid (S) \mid SS$$

is in **L**.

Exercise 4 (Sipser 8.29) Show that the language

$$A_{\text{NFA}} = \{\langle N, w\rangle \mid N \text{ is an NFA that accepts word } w\}$$

is **NL**-complete. You may use the fact that PATH is NL-complete.