COMP4141 Tutorial 9
Alternation, Turing Reduction, Probabilistic Complexity

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Exercise 1 Show that $\mathbf{NP}^{\mathbf{NP}} = \mathbf{NP}^{\mathbf{co\text{-}NP}}$.

Exercise 2 Two undirected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there exists a bijective function $f : V_1 \to V_2$ such that for all $u, v \in V_1$, we have $\{u, v\} \in E_1$ iff $\{f(u), f(v)\} \in E_2$. A subgraph of a graph $G = (V_1, E_1)$ is a graph $H = (V_2, E_2)$ with $V_2 \subseteq V_1$ and $E_2 = \{\{u, v\} \in E_1 \mid u \in V_2 \text{ and } v \in V_2\}$.

Show that the following problem is in the polynomial hierarchy: given graphs G_1 and G_2 and a number k, determine if for every subgraph H_1 of G_1 with exactly k vertices, there exists a subgraph H_2 of G_2, such that H_1 and H_2 are isomorphic. What is the best that you can say about what *level* of the hierarchy it lives?

Exercise 3 In the lecture, we showed that $SAT \in \mathbf{PP}$ and concluded that $\mathbf{NP} \subseteq \mathbf{PP}$. There is actually a missing step in the argument, relating to how $\mathbf{PP}$ interacts with polynomial time reductions. Formulate and prove a result about this, and use it to complete the argument.

Exercise 4 Prove or disprove: $\mathbf{BPP}$ is closed under intersection. That is, if $A \in \mathbf{BPP}$ and $B \in \mathbf{BPP}$ then $A \cap B \in \mathbf{BPP}$.