COMP4141 Homework 6

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Due date: Wed April 10, 2019, 2019, 14:05

Exercise 1 Define a state q of a TM M to be *reachable* if there exists an input string w, such that q is reached at some point during the computation of M on input w . If q is not reachable, we say it is *unreachable*. Consider the problem of determining whether a Turing machine has any unreachable states. Formulate this as a language and use the reduction proof technique to show that it is undecidable. (Note that this problem is a lot like a problem of interest in program analysis: does a given program have any unreachable code location? A similar result applies in that case, once the programming language is sufficiently expressive, e.g. like C, Java, ...!)

Exercise 2 Show that the following language is **NP**-complete:

$$\begin{aligned} L = \{\langle \mathcal{C}, k\rangle \mid\ & \mathcal{C} = \{C_1, \ldots, C_n\} \text{ is a finite collection of finite sets, } k \in \mathbb{N}, \\ & \mathcal{C} \text{ contains } k \text{ distinct sets } S_1, \ldots, S_k \text{ such that } S_i \cap S_j \neq \emptyset \text{ for all } i \neq j\ \} \end{aligned}$$

You may use a reduction from any problem mentioned in the lecture notes.

Exercise 3 Consider the problem of solving systems of linear inequalities $\mathbf{Ax} \leq \mathbf{b}$, where $\mathbf{A}$ is an $n \times m$ matrix with integer entries, $\mathbf{x}$ is a column vector of m distinct variables, and $\mathbf{b}$ is a is a column vector of n integer constants. That is, if

$$A = \begin{pmatrix} a_{1,1} & \ldots & a_{1,m} \\ \vdots & \vdots & \vdots \\ a_{n,1} & \ldots & a_{n,m} \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

then a solution is a set of values for the x_i such that

$$\sum_{j=1\ldots m} a_{i,j} x_j \leq b_i \text{ for all } i = 1 \ldots n$$

An *integer* solution of such a problem has all the x_i as *integers*. Consider the language Ł $= \{\langle \mathbf{A}, \mathbf{x}, \mathbf{b}\rangle \mid \mathbf{Ax} \leq \mathbf{b}$ has an integer solution$\}$. Note the constants in $\mathbf{A}$ and $\mathbf{b}$ are encoded in binary. Show using a reduction from 3SAT that L is **NP**-hard. (Note: you are not required to show that L is in **NP**. All the numbers in this question may be positive or negative integers.)