COMP4141 Homework 8
Logspace, Turing Reduction

Ron van der Meyden
meyden@cse.unsw.edu.au

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Exercise 1 Let USAT be the set of all boolean formulas that have *exactly one* satisfying assignment. Show that USAT$\in \mathbf{P}^{SAT}$.

Exercise 2 A 2-CNF formula is a formula of propositional logic in which each clause has *exactly* two literals. That is, the formula is a conjunction of clauses, where each clause is of the form ℓ_1 or $\ell_1 \vee \ell_2$ where each ℓ_i is either p or $\neg p$ for some atomic proposition p. (Note that we may treat a single literal ℓ as equivalent to $\ell \vee \ell$, so these formulas can also express clauses containing just one literal.)

Use the fact that PATH is complete for **NL** under logspace reductions to show that the following language is **NL**-hard under logspace reductions:

$$L = \{\phi \mid \phi \text{ is a satisfiable 2-CNF formula}\}$$

Exercise 3 Given a 2-CNF formula ϕ with variables $x_1 \ldots, x_n$, define the directed graph $G_\phi = \langle V, E\rangle$ as follows. The set of vertices V consists of all the literals over the variables of ϕ, i.e., $V = \{x_i \mid i = 1 \ldots n\} \cup \{\neg x_i \mid i = 1 \ldots n\}$. The set of edges E contains two edges for each clause $\ell_1 \vee \ell_2$ of ϕ: the edge $(\overline{\ell_1}, \ell_2)$ and the edge $(\overline{\ell_2}, \ell_1)$. Here $\overline{\ell}$ is the complementary literal, defined by $\overline{x} = \neg x$ and $\overline{\neg x} = x$. (By way of intuition, note that $\ell_1 \vee \ell_2$ is equivalent to $\overline{\ell_1} \Rightarrow \ell_2$.)

1. Show that ϕ is satisfiable iff G_ϕ does not contain any cycles (paths from a vertex to itself) that contain both a variable x_i and its complement $\neg x_i$.
2. Use this to show that the language L of the previous question is in **NL**.

(Combining exercise 2 and 3, we get that L is complete for **NL**.)