COMP4141 Homework 7

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Exercise 1 Define a language L to be **co-NP**-complete if it is in **co-NP** and all languages in **co-NP** can be polynomial-time reduced to L. Say that a formula of quantified boolean logic is a *universal sentence* if it is a sentence (i.e., has no free variables) of the form $\forall x_1 \ldots \forall x_n(\psi)$ where ψ is a propositional logic formula (contains no quantifiers). Show that the language

$$\{\phi \mid \phi \text{ is a true universal sentence}\}$$

is **co-NP**-complete. (Hint: use the fact that SAT is **NP**-complete.)

Exercise 2 A *boolean program* is a program in which all variables have type Boolean, and which may use the following constructions:

- Boolean expressions e may be formed from boolean variables using the operators $\wedge, \vee, \neg$ and constants $0, 1$
- if x is a variable and e is a Boolean expression, then the statement $x := e$ is a program. It runs by calculating the value of e and updating x to have this value.
- If P and Q are programs, then $P; Q$ is a program that runs by first executing P and then executing Q.
- If P and Q are programs and e is a boolean expression, then

  $$\textbf{if } e \textbf{ then } \{P\} \textbf{ else } \{Q\}$$

  is a program that runs by first calculating the value of e and executing P if e is true and executing Q otherwise.
- If P is a program and e is a boolean expression, then

  $$\textbf{while } e \textbf{ do } \{P\}$$

  is a program that runs by repeatedly testing if e is true and executing P if so. The loop exits as soon as e evaluates to false.

Boolean programs run starting from some given initial assignment π to their boolean variables, and update this assignment during their execution. They may terminate or not, possibly depending on the initial assignment. For example

$$\textbf{while } x \textbf{ do } \{\textbf{if } y \textbf{ then } \{y := 0\} \textbf{ else } \{y := 1\}\}$$

when run from initial assignment π, halts immediately if $\pi(x) = 0$ and is non-terminating if $\pi(x) = 1$.

Show that the computational complexity of the following problem is in PSPACE: given a boolean program P, does there exists an assignment π such that P terminates when run from initial assignment π?

Exercise 3 Show that the problem from Exercise 2 is PSPACE-hard.