

# COMP4141 Tutorial 7

## P, NP, PSPACE

**Exercise 1** Fill in some of the details of the Tseitin reduction discussed in lectures, by converting the following formulas to equivalent 3cnf formulas using boolean equivalences.

- $p \Leftrightarrow \neg q$
- $p \Leftrightarrow (q \vee r)$
- $p \Leftrightarrow (q \wedge r)$

**Exercise 2 (Sipser Theorem 7.44)** Work through the proof of NP-completeness of VERTEX-COVER.

**Exercise 3** Show that if every **NP**-hard problem is also **PSPACE**-hard, then **NP=PSPACE**.

**Exercise 4 (Sipser 8.9)** A ladder in a language  $L$  is a sequence of strings in  $L$ , all of the same length, such that each differs from the preceding one by at most one character. For example, the following is a ladder in English, starting with “head” and ending in “free”.

head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free.

Let  $LADDER_{DFA} = \{\langle M, s, t \rangle \mid M \text{ is a DFA and } L(M) \text{ contains a ladder from } s \text{ to } t\}$ . Show that  $LADDER_{DFA} \in \mathbf{PSPACE}$ .