COMP4141 Tutorial 6
Reductions, Asymptotics, P, NP

Exercise 1 (Sisper 5.23) Show that a language A is decidable iff $A \leq_m 0^*1^*$.

Exercise 2 Using the definitions of O and o, determine which of the following assertions are true, carefully justifying your answer.

- $10^{100} \cdot n = O(\frac{1}{10^{100}} \cdot n)$
- $\frac{1}{10^{100}} \cdot n = o(10^{100} \cdot n)$
- $257n^3 + 3n^2 + 28 = O(n^3)$
- $257n^3 + 3n^2 + 28 = o(10^{100} \cdot n^3)$
- $n\,log\;n = o(n^2)$
- $n^{100} = o(2^n)$

Exercise 3 Prove using the definitions that $f = o(g)$ implies $f = O(g)$.

Exercise 4 Suppose f and g are strictly positive functions, with $g = O(f)$. Is it possible that $f = o(g)$? (Prove or disprove.)

Exercise 5 Show that **NP** is closed under Kleene star, i.e., if the language $L \in$ **NP**, then $L^* \in$ **NP**.

Exercise 6 Show that if **P** = **NP**, then every language $A \in$ **P** except $A = \emptyset$ and $A = \Sigma^*$, is **NP**-complete.

Exercise 7 An undirected graph G with vertices V and edges E is *3-colourable* if there exists a mapping $c : V \to \{r, b, g\}$ such that for all edges $\{u, v\}$ in E, we have $c(u) \neq c(v)$. Define the language

$$3COL = \{\langle V, E\rangle \mid G = (V, E) \text{ is 3-colourable}\}\ .$$

Suppose we knew that $3COL$ is **NP**-complete. Using this fact, give a new proof by reduction that SAT is **NP**-complete.