COMP4141 Homework 3

Due date: Wed Mar 13, 2019, 14:05

Exercise 1 For any string $w = w_1w_2\cdots w_n$ the *reverse* of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^{\mathcal{R}} = \{\ w^{\mathcal{R}} \mid w \in A\ \}$. Show that the set of context free languages is closed under $^{\mathcal{R}}$, i.e., if L is a context-free language, then so is $L^{\mathcal{R}}$.

Exercise 2 (Sipser's Problem 2.44) If A and B are languages, define

$$A \diamond B = \{\ xy \mid x \in A, y \in B, |x| = |y|\ \}\ \ .$$

Show that if A and B are regular languages, then $A \diamond B$ is a context-free language. (Hint: use a push-down automaton.)

Exercise 3 (Chomsky hierarchy) Consider the language

$$L = \{a^n b^m c^n d^m \mid n, m > 0\}.$$

What is the *largest* i such that this language is of type i in the Chomsky hierarchy? Prove your answer. (Note that you need to show that L is of type i but not of type $i + 1$, for some i.)