COMP4141 Homework 1

Due date: Wed Feb 27, 2019, 14:05

Homework may be submitted solo or in groups of at most two. Submit homework *legibly handwritten in ink*. Use block letters if your cursive writing is not sufficiently legible.

Exercise 1 For an NFA $A = (Q, \Sigma, q_0, \delta, F)$, define $L^{\forall}(A)$ to be the set of words $w \in \Sigma^*$ such that *for all* runs $q_0, \ldots, q$ of A on w, the last state q is in F. (Compare with the definition of $L(A)$, which is the same, except that it uses "for some" in place of "for all".) Intuitively, this definition amounts to a change of the acceptance condition for NFA's. Show that it does not actually make a difference to the set of languages that can be accepted. That is, show that for all languages L, there exists an NFA A such that $L = L(A)$, if and only if there exists an NFA B such that $L = L^{\forall}(B)$.

Exercise 2 For a language $L \subseteq \Sigma^*$ and a word $w \in \Sigma^*$, define the left quotient

$$L/w = \{x \in \Sigma^* \mid xw \in L\}$$

and the right quotient

$$L\backslash w = \{x \in \Sigma^* \mid wx \in L\}$$

Show that if L is regular, then so are L/w and $L\backslash w$.

Exercise 3 Let $w \in \Sigma^*$. Construct a *DFA* A_w such that

$$L(A_w) = \{x \cdot w \cdot y \mid x, y \in \Sigma^*\} \ .$$

That is, $L(A_w)$ is the set of all words that contain w. For this question, half marks for a correct answer, full marks if A_w has at most $|w| + 1$ states. (Hint: consider $w = ababb$ and input word *babababba*.)