Histograms

A histogram is a graph. A graph that shows frequency of anything. Usually histogram have bars that represent frequency of occurring of data in the whole data set.

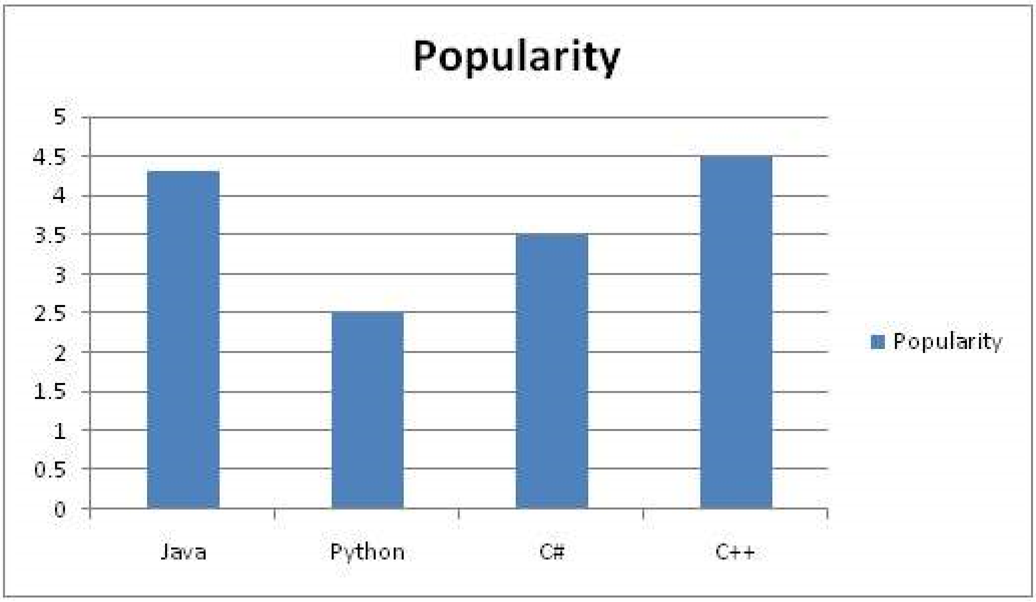
A Histogram has two axis the x axis and the y axis.

The x axis contains event whose frequency you have to count.

The y axis contains frequency.

The different heights of bar shows different frequency of occurrence of data.

Usually a histogram looks like this.



Now we will see an example of this histogram is build

Example

Consider a class of programming students and you are teaching python to them.

At the end of the semester, you got this result that is shown in table. But it is very messy and does not show your overall result of class. So you have to make a histogram of your result, showing the overall frequency of occurrence of grades in your class. Here how you are going to do it.

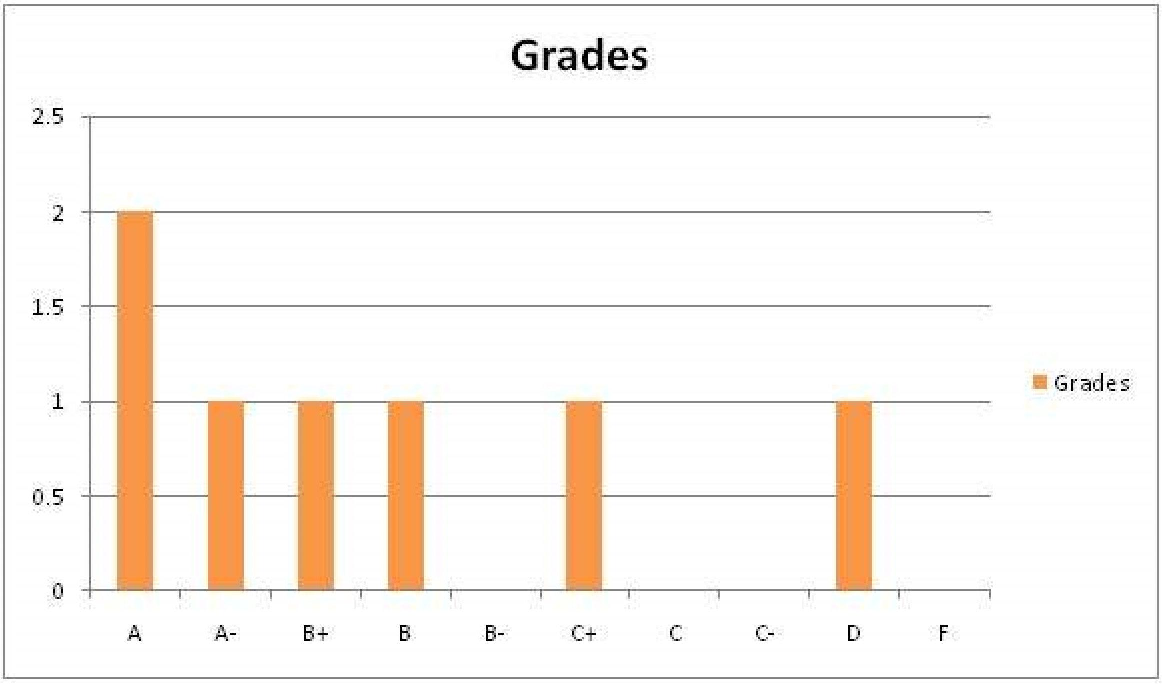
Result sheet

Name	Grade
John	A
Jack	D
Carter	B
Tommy	A
Lisa	C+
Derek	A-
Tom	B+

Histogram of result sheet

Now what you are going to do is, that you have to find what comes on the x and the y axis.

There is one thing to be sure, that y axis contains the frequency, so what comes on the x axis. X axis contains the event whose frequency has to be calculated. In this case x axis contains grades.

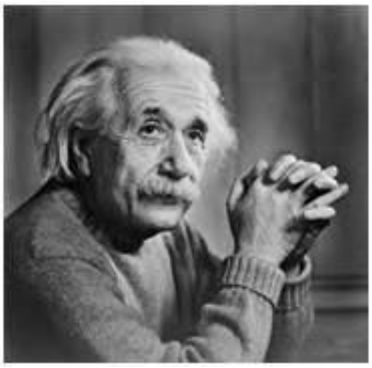


Now we will how do we use a histogram in an image.

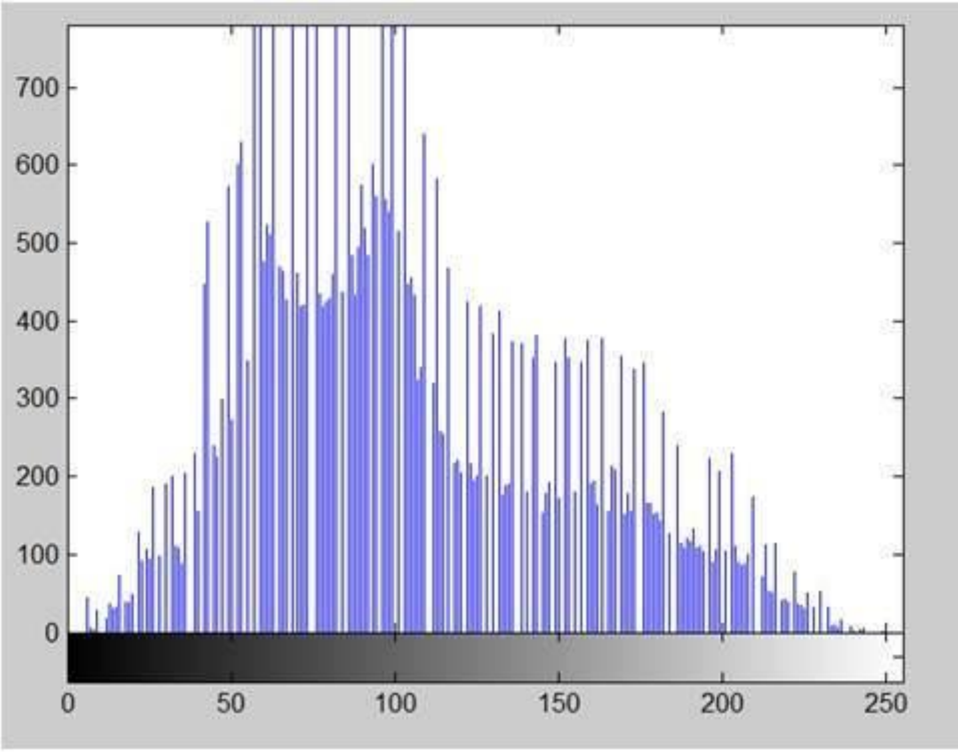
Histogram of an image

Histogram of an image, like other histograms also shows frequency. But an image histogram, shows frequency of pixels intensity values. In an image histogram, the x axis shows the gray level intensities and the y axis shows the frequency of these intensities.

For example



The histogram of the above picture of the Einstein would be something like this



The x axis of the histogram shows the range of pixel values. Since its an 8 bpp image, that means it has 256 levels of gray or shades of gray in it. Thats why the range of x axis starts from 0 and end at 255 with a gap of 50. Whereas on the y axis, is the count of these intensities.

As you can see from the graph, that most of the bars that have high frequency lies in the first half portion which is the darker portion. That means that the image we have got is darker. And this can be proved from the image too.

Applications of Histograms

Histograms has many uses in image processing. The first use as it has also been discussed above is the analysis of the image. We can predict about an image by just looking at its histogram. Its like looking an x ray of a bone of a body.

The second use of histogram is for brightness purposes. The histograms has wide application in image brightness. Not only in brightness, but histograms are also used in adjusting contrast of an image.

Another important use of histogram is to equalize an image.

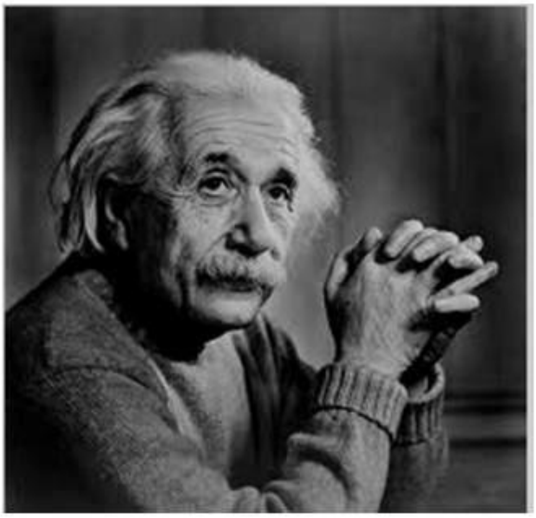
And last but not the least, histogram has wide use in thresholding. This is mostly used in computer vision.

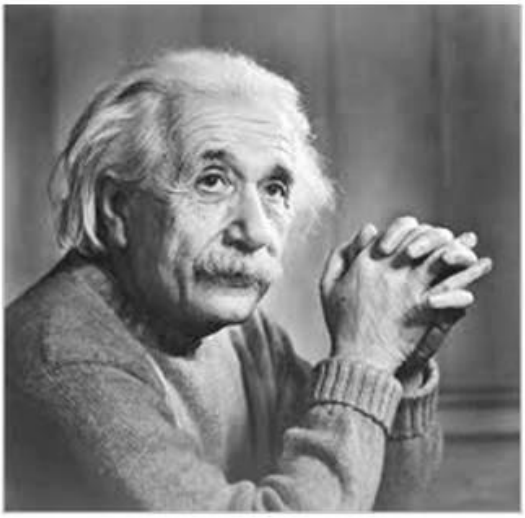
Brightness

Brightness is a relative term. It depends on your visual perception. Since brightness is a relative term, so brightness can be defined as the amount of energy output by a source of light relative to the source we are comparing it to. In some cases we can easily say that the image is bright, and in some cases, its not easy to perceive.

For example

Just have a look at both of these images, and compare which one is brighter.





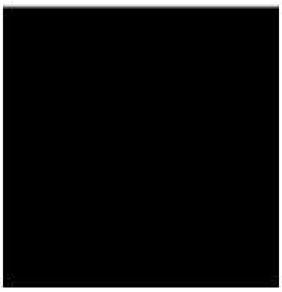
We can easily see, that the image on the right side is brighter as compared to the image on the left.

But if the image on the right is made more darker then the first one, then we can say that the image on the left is more brighter then the left.

How to make an image brighter.

Brightness can be simply increased or decreased by simple addition or subtraction, to the image matrix.

Consider this black image of 5 rows and 5 columns



Since we already know, that each image has a matrix at its behind that contains the pixel values. This image matrix is given below.

0	0	0	0	0
0	0	0	0	0

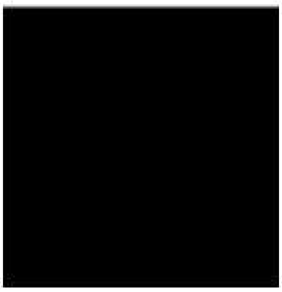
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

Since the whole matrix is filled with zero, and the image is very much darker.

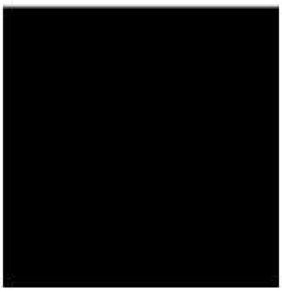
Now we will compare it with another same black image to see this image got brighter or not.

Still, both images look the same. To make the second one brighter we just need to add the value 1 to each element in the matrix representing it.

What we will do is, that we will simply add a value of 1 to each of the matrix value of image 1. After adding the image 1 would something like this.



Now we will again compare it with image 2, and see any difference.



We see, that still we cannot tell which image is brighter as both images looks the same.

Now what we will do, is that we will add 50 to each of the matrix value of the image 1 and see what the image has become.

The output is given below.

Now again, we will compare it with image 2.

Now you can see that the image 1 is slightly brighter then the image 2. We go on, and add another 45 value to its matrix of image 1, and this time we compare again both images.

Now when you compare it, you can see that this image1 is clearly brighter then the image 2.

Even it is brighter then the old image1. At this point the matrix of the image1 contains 100 at each index as first add 5, then 50, then 45. So 5 + 50 + 45 = 100.

Contrast

Contrast can be simply explained as the difference between maximum and minimum pixel intensity in an image.

For example.

Consider the final image1 in brightness.

The matrix of this image is:

100	100	100	100	100
100	100	100	100	100
100	100	100	100	100
100	100	100	100	100
100	100	100	100	100

The maximum value in this matrix is 100.

The minimum value in this matrix is 100.

Contrast = maximum pixel intensity(subtracted by) minimum pixel intensity

= 100 (subtracted by) 100

= 0

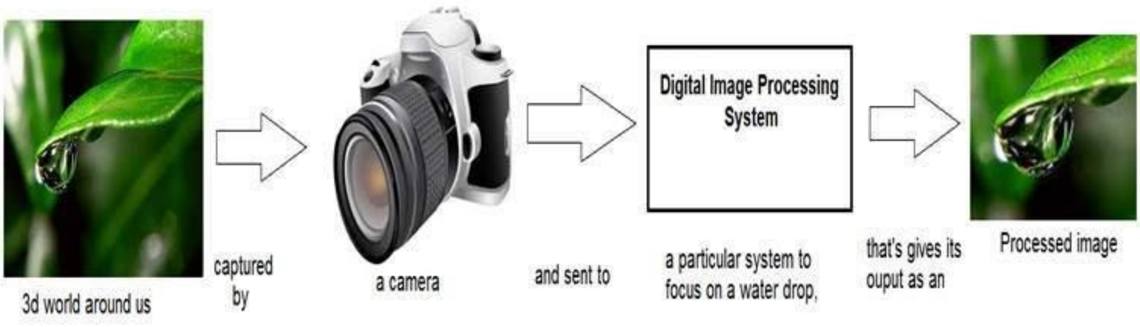
0 means that this image has 0 contrast.

Transformation

Transformation is a function. A function that maps one set to another set after performing some operations.

Digital Image Processing system

We have already seen in the introductory tutorials that in digital image processing, we will develop a system that whose input would be an image and output would be an image too. And the system would perform some processing on the input image and gives its output as an processed image. It is shown below.



Now function applied inside this digital system that process an image and convert it into output can be called as transformation function.

As it shows transformation or relation, that how an image1 is converted to image2.

Image transformation.

Consider this equation

G(x,y) = T{ f(x,y) }

In this equation,

F(x,y) = input image on which transformation function has to be applied.

G(x,y) = the output image or processed image.

T is the transformation function.

This relation between input image and the processed output image can also be represented as.

s = T (r)

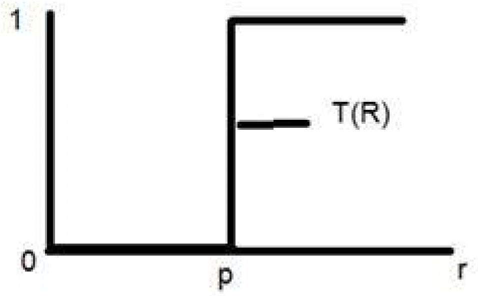
where r is actually the pixel value or gray level intensity of f(x,y) at any point. And s is the pixel value or gray level intensity of g(x,y) at any point.

The basic gray level transformation has been discussed in our tutorial of basic gray level transformations.

Now we are going to discuss some of the very basic transformation functions.

Examples

Consider this transformation function.



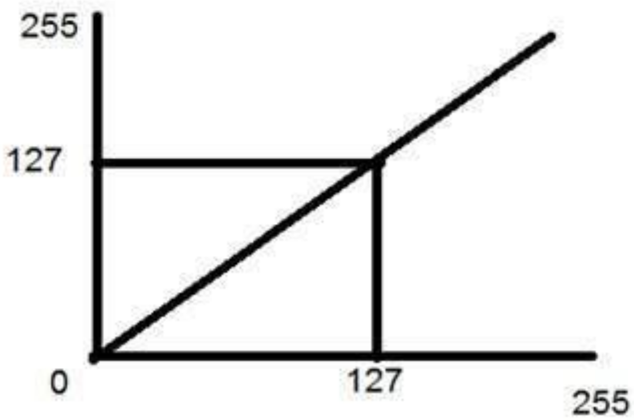
Lets take the point r to be 256, and the point p to be 127. Consider this image to be a one bpp image. That means we have only two levels of intensities that are 0 and 1. So in this case the transformation shown by the graph can be explained as.

All the pixel intensity values that are below 127 (point p) are 0, means black. And all the pixel intensity values that are greater then 127, are 1, that means white. But at the exact point of 127, there is a sudden change in transmission, so we cannot tell that at that exact point, the value would be 0 or 1.

Mathematically this transformation function can be denoted as:

$$g(x,y) = \begin{cases} 0 & f(x,y) < 127 \\ 1 & f(x,y) > 127 \end{cases}$$

Consider another transformation like this



Now if you will look at this particular graph, you will see a straight transition line between input image and output image.

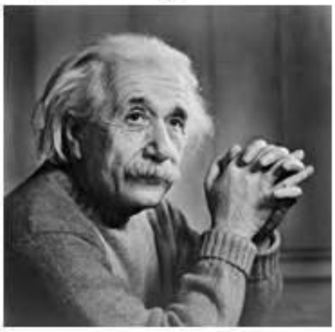
It shows that for each pixel or intensity value of input image, there is a same intensity value of output image. That means the output image is exact replica of the input image.

It can be mathematically represented as:

$g(x,y) = f(x,y)$

the input and output image would be in this case are shown below.

Input image



Output image

Histogram

Histogram is nothing but a graph that shows frequency of occurrence of data. Histograms has many use in image processing, out of which we are going to discuss one user here which is called histogram sliding.

Histogram sliding

In histogram sliding, we just simply shift a complete histogram rightwards or leftwards. Due to shifting or sliding of histogram towards right or left, a clear change can be seen in the image.In this tutorial we are going to use histogram sliding for manipulating brightness.

The term i-e: Brightness has been discussed in our tutorial of introduction to brightness and contrast. But we are going to briefly define here.

Brightness

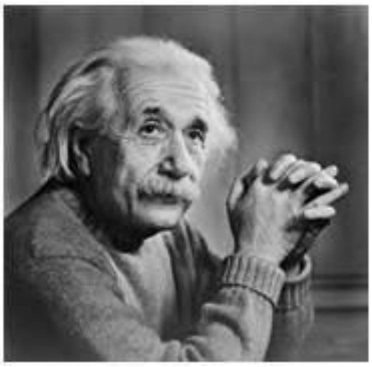
Brightness is a relative term. Brightness can be defined as intensity of light emit by a particular light source.

Contrast

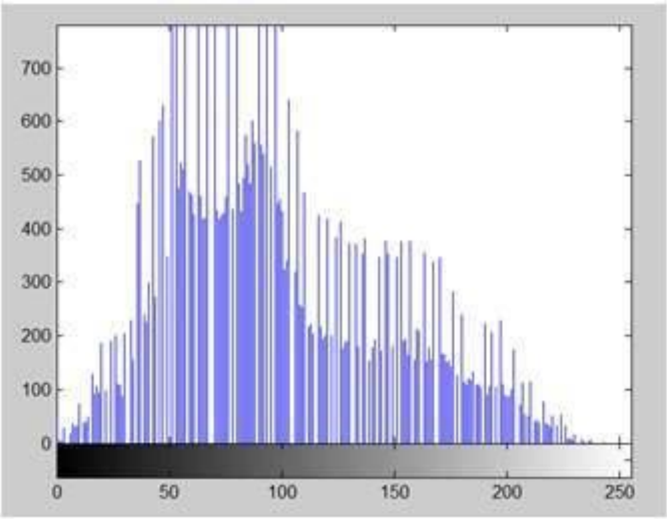
Contrast can be defined as the difference between maximum and minimum pixel intensity in an image.

Sliding Histograms

Increasing brightness using histogram sliding



Histogram of this image has been shown below.



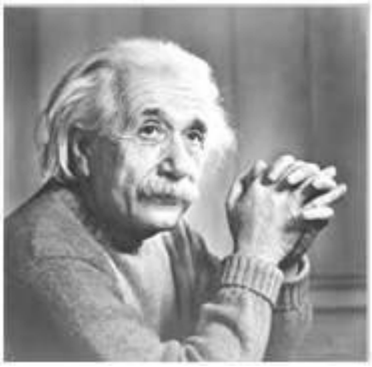
On the y axis of this histogram are the frequency or count. And on the x axis, we have gray level values. As you can see from the above histogram, that those gray level intensities whose count is more then 700, lies in the first half portion, means towards blacker portion. Thats why we got an image that is a bit darker.

In order to bright it, we will slide its histogram towards right, or towards whiter portion. In order to do we need to add atleast a value of 50 to this image. Because we can see from the histogram above, that this image also has 0 pixel intensities, that are pure black. So if we add 0 to 50, we will shift all the values lies at 0 intensity to 50 intensity and all the rest of the values will be shifted accordingly.

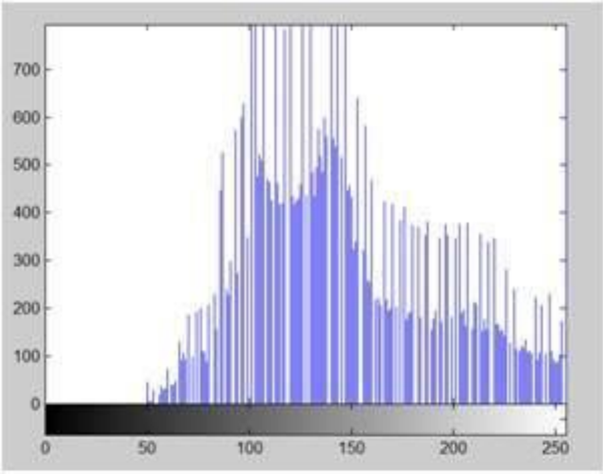
Lets do it.

Here what we got after adding 50 to each pixel intensity.

The image has been shown below.



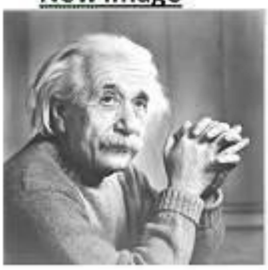
And its histogram has been shown below.



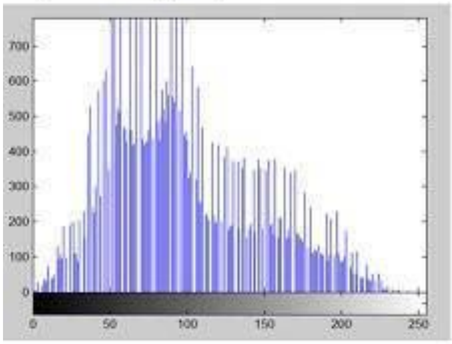
Lets compare these two images and their histograms to see that what change have to got.

<u>Old image</u>

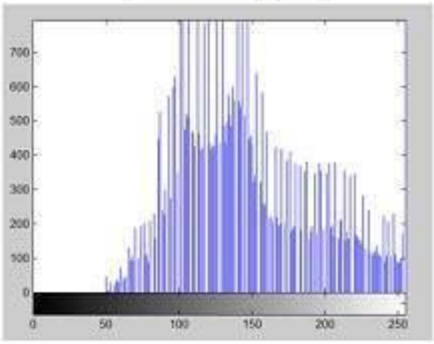
<u>New image</u>



<u>Old histogram</u>



<u>New Histogram</u>



Conclusion

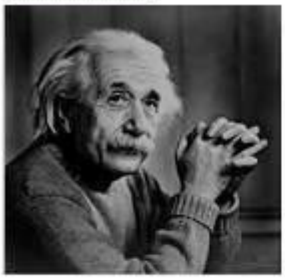
As we can clearly see from the new histogram that all the pixels values has been shifted towards right and its effect can be seen in the new image.

Decreasing brightness using histogram sliding

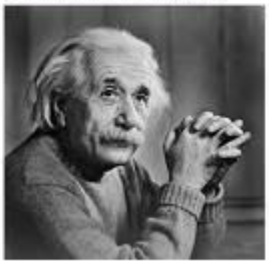
Now if we were to decrease brightness of this new image to such an extent that the old image look brighter, we got to subtract some value from all the matrix of the new image. The value which we are going to subtract is 80. Because we already add 50 to the original image and we got a new brighter image, now if we want to make it darker, we have to subtract at least more than 50 from it.

And this what we got after subtracting 80 from the new image.

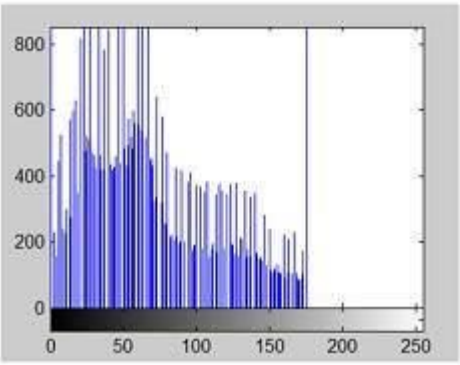
New image.



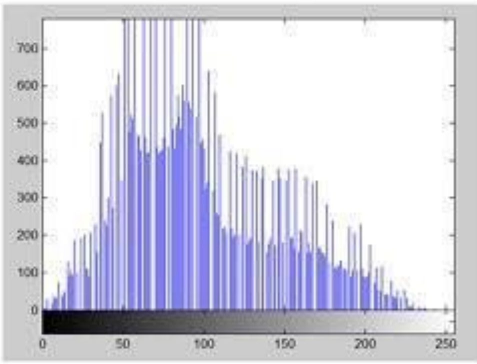
Original image.



New Histogram.



Original Histogram.



Conclusion

It is clear from the histogram of the new image, that all the pixel values has been shifted towards right and thus, it can be validated from the image that new image is darker and now the original image look brighter as compare to this new image.

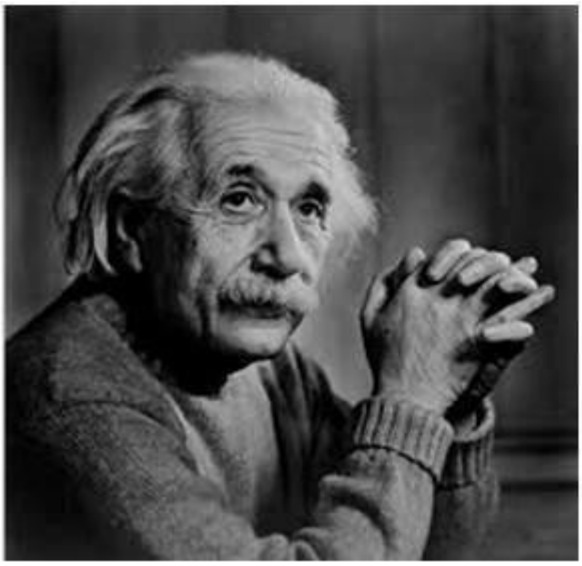
There are two methods of enhancing contrast. The first one is called Histogram stretching that increase contrast. The second one is called Histogram equalization that enhance contrast and it has been discussed in our tutorial of histogram equalization.

Before we will discuss the histogram stretching to increase contrast, we will briefly define contrast.

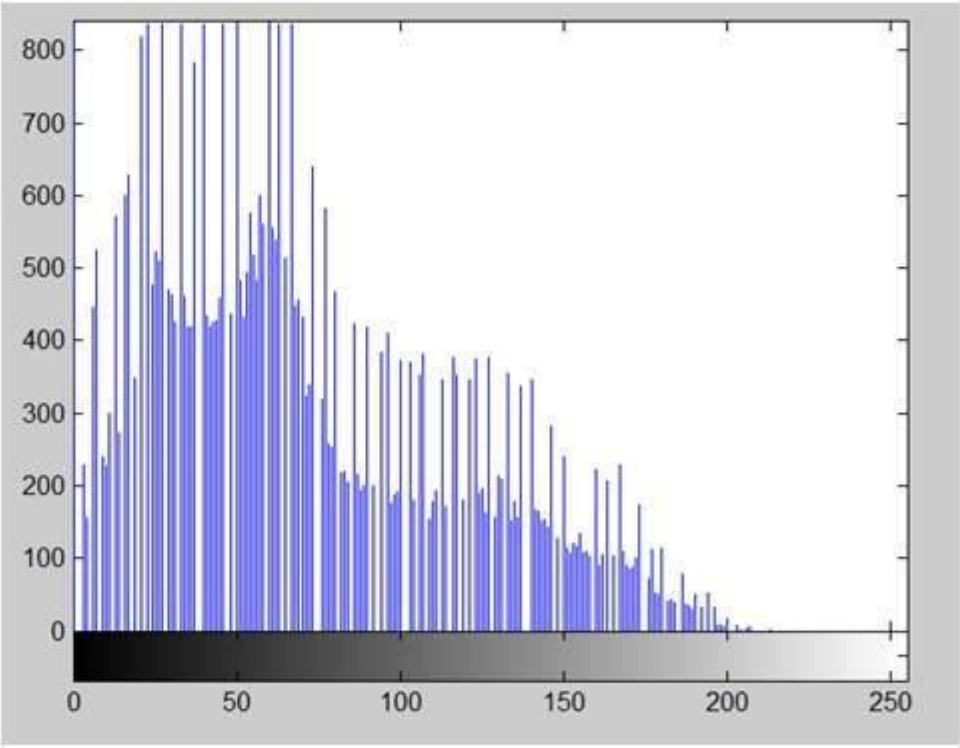
Contrast

Contrast is the difference between maximum and minimum pixel intensity.

Consider this image.



The histogram of this image is shown below.



Now we calculate contrast from this image.

Contrast = 225.

Now we will increase the contrast of the image.

Increasing the contrast of the image

The formula for stretching the histogram of the image to increase the contrast is

$$g(x,y) = \frac{f(x,y) - fmin}{fmax - fmin} * 2^{bpp}$$

The formula requires finding the minimum and maximum pixel intensity multiply by levels of gray. In our case the image is 8bpp, so levels of gray are 256.

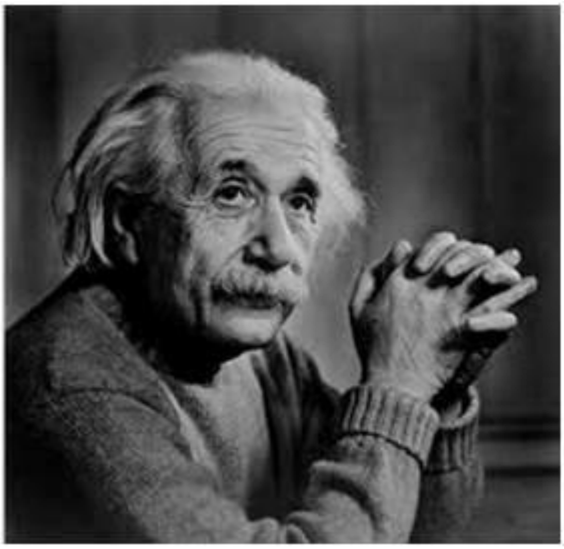
The minimum value is 0 and the maximum value is 225. So the formula in our case is

$$g(x,y) = \frac{f(x,y) - 0}{225 - 0} * 255$$

where f(x,y) denotes the value of each pixel intensity. For each f(x,y) in an image , we will calculate this formula.

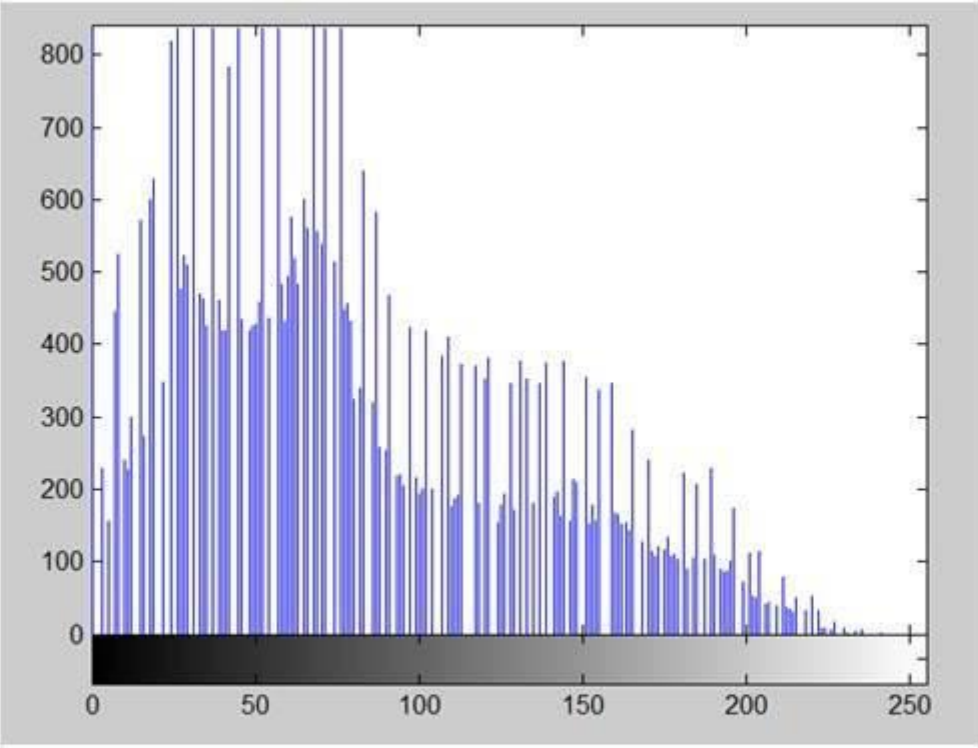
After doing this, we will be able to enhance our contrast.

The following image appear after applying histogram stretching.



The stretched histogram of this image has been shown below.

Note the shape and symmetry of histogram. The histogram is now stretched or in other means expand. Have a look at it.



In this case the contrast of the image can be calculated as

Contrast = 240

Hence we can say that the contrast of the image is increased.

Note : this method of increasing contrast doesnot work always, but it fails on some cases.

Failing of histogram stretching

As we have discussed , that the algorithm fails on some cases. Those cases include images with when there is pixel intensity 0 and 255 are present in the image

Because when pixel intensities 0 and 255 are present in an image, then in that case they become the minimum and maximum pixel intensity which ruins the formula like this.

Original Formula

$$g(x,y) = \frac{f(x,y) - fmin}{fmax - fmin} * 2^{bpp}$$

Putting fail case values in the formula:

$$g(x,y) = \frac{f(x,y) - 0}{255 - 0} * 255$$

Simplify that expression gives

$$g(x,y) = \frac{f(x,y)}{255} * 255$$

$$g(x,y) = f(x,y)$$

That means the output image is equal to the processed image. That means there is no effect of histogram stretching has been done at this image.

What is Probability Mass Funcion (PMF) ?

PMF stands for probability mass function. As it name suggest, it gives the probability of each number in the data set or you can say that it basically gives the count or frequency of each element.

How PMF is calculated

We will calculate PMF from two different ways. First from a matrix, because in the next tutorial, we have to calculate the PMF from a matrix, and an image is nothing more then a two dimensional matrix.

Then we will take another example in which we will calculate PMF from the histogram.

Consider this matrix.

1	2	7	5	6
7	2	3	4	5
0	1	5	7	3
1	2	5	6	7
6	1	0	3	4

Now if we were to calculate the PMF of this matrix, here how we are going to do it.

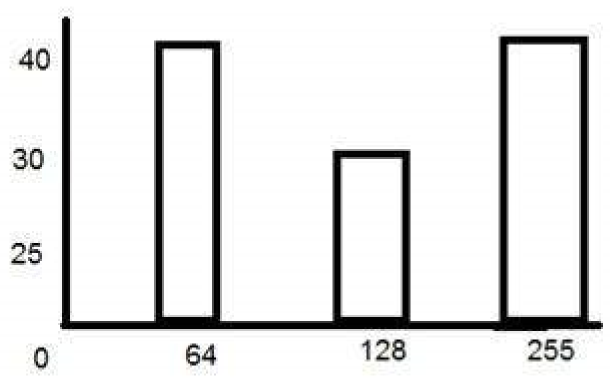
At first, we will take the first value in the matrix , and then we will count, how much time this value appears in the whole matrix. After count they can either be represented in a histogram, or in a table like this below.

PMF

0	2	2/25
1	4	4/25
2	3	3/25
3	3	3/25
4	2	2/25
5	4	4/25
6	3	3/25
7	4	4/25

Note that the sum of the count must be equal to total number of values.

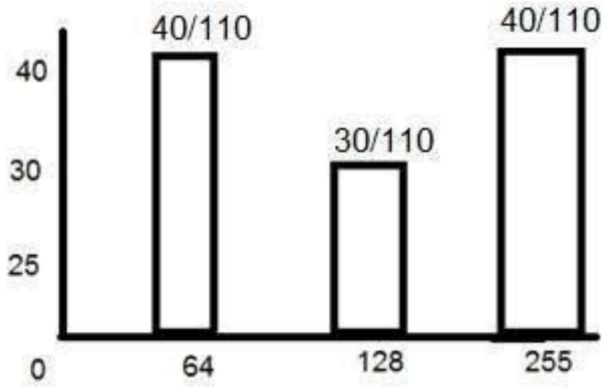
Calculating PMF from histogram



The above histogram shows frequency of gray level values for an 8 bits per pixel image.

Now if we have to calculate its PMF, we will simple look at the count of each bar from vertical axis and then divide it by total count.

So the PMF of the above histogram is this.



Another important thing to note in the above histogram is that it is not monotonically increasing. So in order to increase it monotonically, we will calculate its CDF.

What is (Cumulative Distribution Function) CDF?

CDF stands for cumulative distributive function. It is a function that calculates the cumulative sum of all the values that are calculated by PMF. It basically sums the previous one.

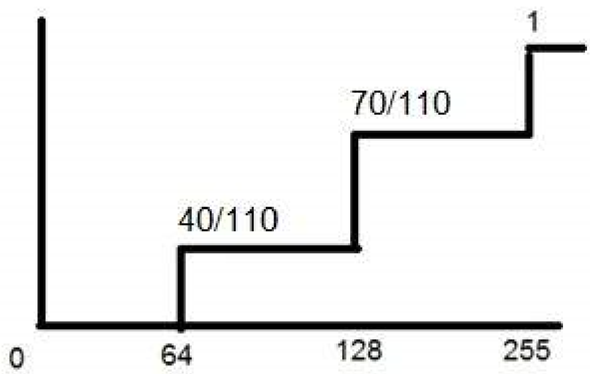
How it is calculated?

We will calculate CDF using a histogram. Here how it is done. Consider the histogram shown above which shows PMF.

Since this histogram is not increasing monotonically, so will make it grow monotonically.

We will simply keep the first value as it is, and then in the 2nd value , we will add the first one and so on.

Here is the CDF of the above PMF function.



Now as you can see from the graph above, that the first value of PMF remain as it is. The second value of PMF is added in the first value and placed over 128. The third value of PMF is added in the second value of CDF , that gives 110/110 which is equal to 1.

And also now, the function is growing monotonically which is necessary condition for histogram equalization.

PMF and CDF usage in histogram equalization

Histogram equalization

Histogram equalization is discussed in the next tutorial but a brief introduction of histogram equalization is given below.

Histogram equalization is used for enhancing the contrast of the images.

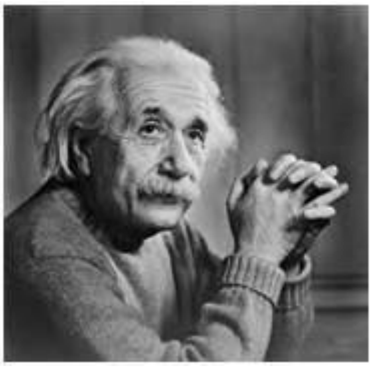
PMF and CDF are both use in histogram equalization as it is described in the beginning of this tutorial. In the histogram equalization, the first and the second step are PMF and CDF. Since in histogram equalization, we have to equalize all the pixel values of an image. So PMF helps us calculating the probability of each pixel value in an image. And CDF gives us the cumulative sum of these values. Further on, this CDF is multiplied by levels, to find the new pixel intensities, which are mapped into old values, and your histogram is equalized.

Histogram Equalization

Histogram equalization is used to enhance contrast. It is not necessary that contrast will always be increase in this. There may be some cases were histogram equalization can be worse. In that cases the contrast is decreased.

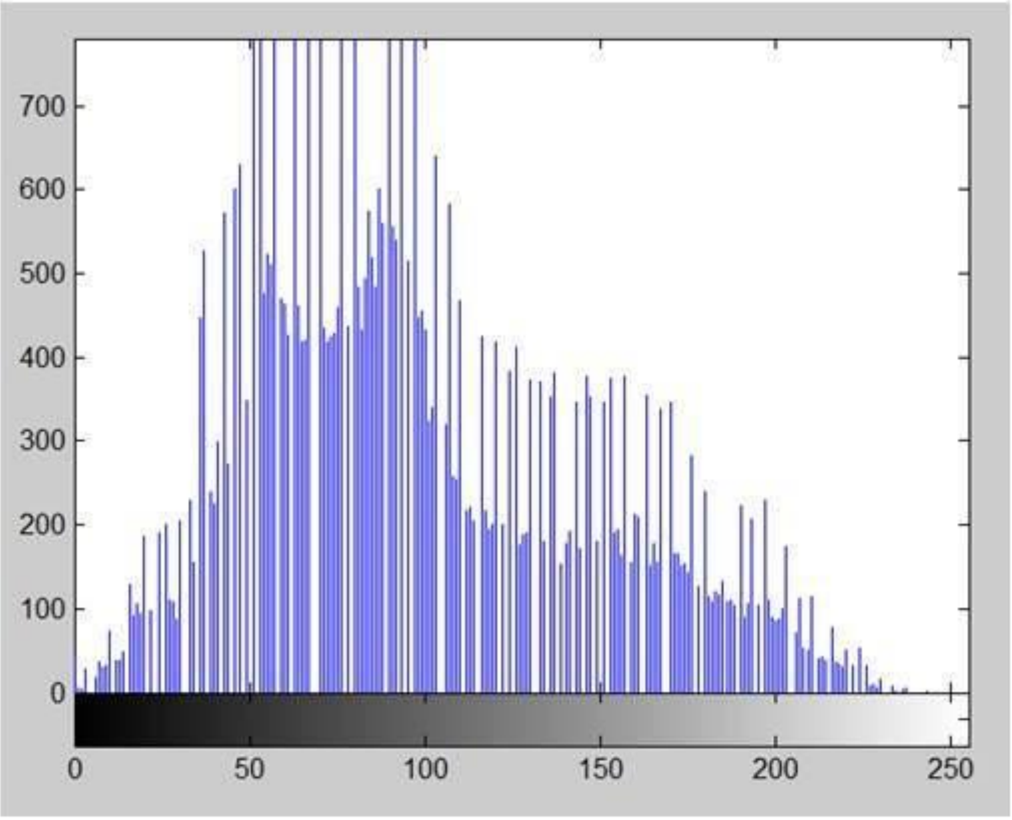
Lets start histogram equalization by taking this image below as a simple image.

Image



Histogram of this image

The histogram of this image has been shown below.



Now we will perform histogram equalization to it.

PMF

First we have to calculate the PMF (probability mass function) of all the pixels in this image. If you donot know how to calculate PMF, please visit our tutorial of PMF calculation.

CDF

Our next step involves calculation of CDF (cumulative distributive function). Again if you donot know how to calculate CDF , please visit our tutorial of CDF calculation.

Calculate CDF according to gray levels

Lets for instance consider this , that the CDF calculated in the second step looks like this.

Gray Level Value	CDF
0	0.11
1	0.22

2	0.55
3	0.66
4	0.77
5	0.88
6	0.99
7	1

Then in this step you will multiply the CDF value with (Gray levels (minus) 1) .

Considering we have an 3 bpp image. Then number of levels we have are 8. And 1 subtracts 8 is 7. So we multiply CDF by 7. Here what we got after multiplying.

Gray Level Value	CDF	CDF * (Levels-1)
0	0.11	0
1	0.22	1
2	0.55	3
3	0.66	4
4	0.77	5
5	0.88	6
6	0.99	6
7	1	7

Now we have is the last step, in which we have to map the new gray level values into number of pixels.

Lets assume our old gray levels values has these number of pixels.

Gray Level Value	Frequency
0	2
1	4
2	6
3	8
4	10
5	12
6	14
7	16

Now if we map our new values to , then this is what we got.

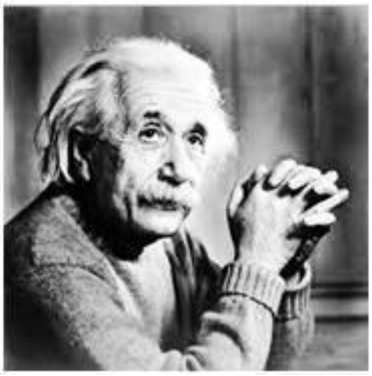
Gray Level Value	New Gray Level Value	Frequency
0	0	2
1	1	4
2	3	6
3	4	8

4	5	10
5	6	12
6	6	14
7	7	16

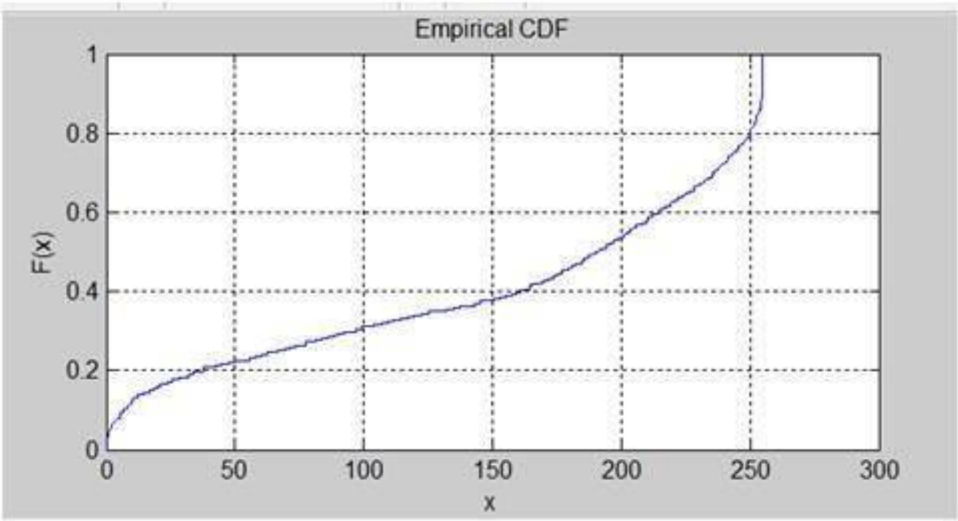
Now map these new values you are onto histogram, and you are done.

Lets apply this technique to our original image. After applying we got the following image and its following histogram.

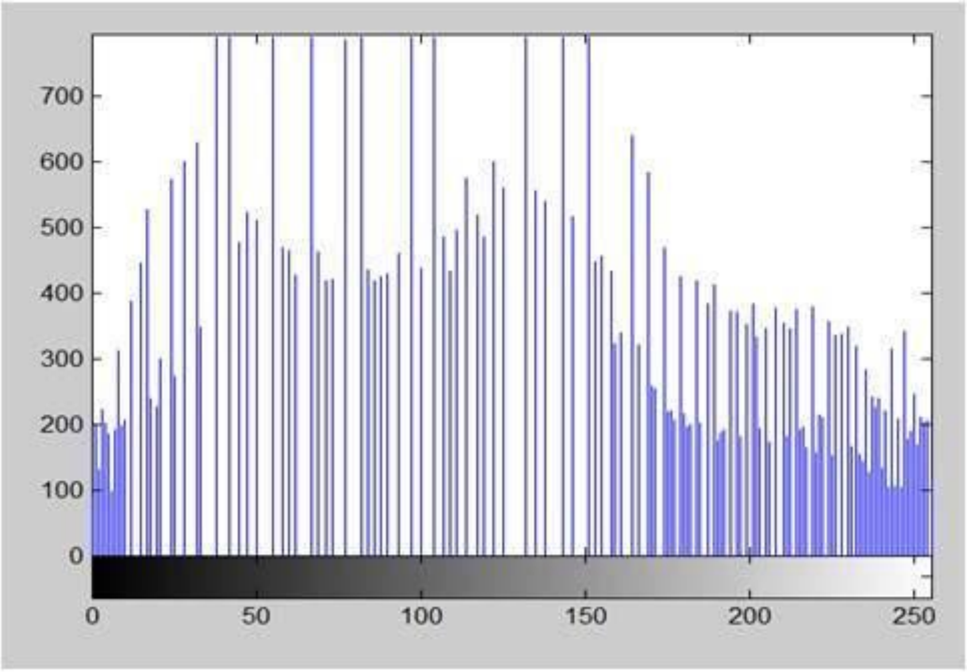
Histogram Equalization Image



Cumulative Distributive function of this image

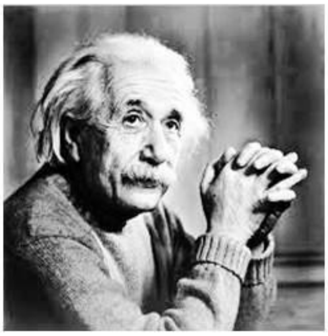


Histogram Equalization histogram

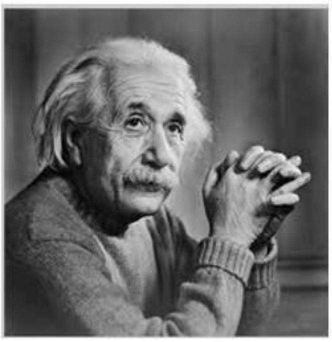


Comparing both the histograms and images

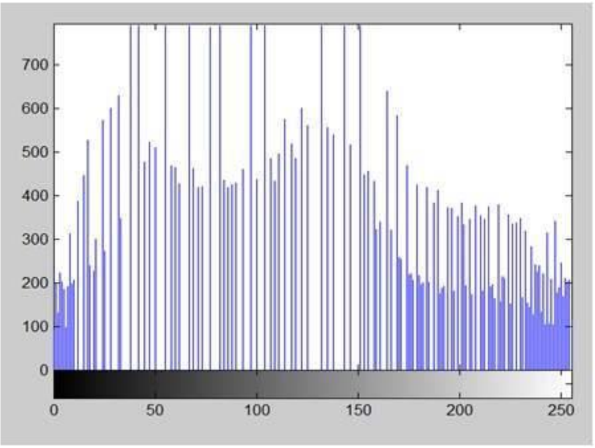
New Image



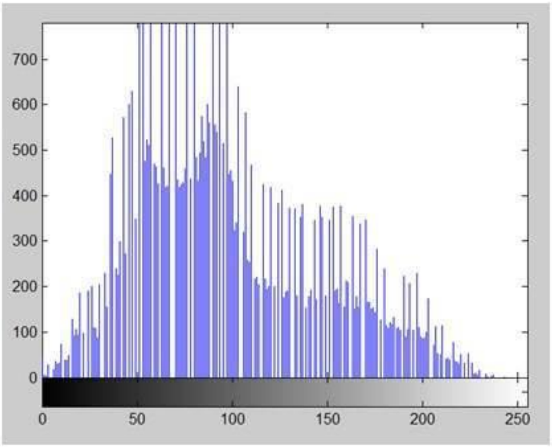
Old image



New Histogram



Old Histogram



Conclusion

As you can clearly see from the images that the new image contrast has been enhanced and its histogram has also been equalized. There is also one important thing to be note here that during histogram equalization the overall shape of the histogram changes, where as in histogram stretching the overall shape of histogram remains same.