Estimation and Control Library

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***Abstract*— A library for optimal estimation and control, as well as classical/modern control theories. Algorithms are applied on simulated robots using Ignition Gazebo.**

I. INTRODUCTION

A library for optimal estimation and control, as well as classical/modern control theories. Algorithms are applied on simulated robots using ROS [1] and Ignition Gazebo [2].

II. CARTPOLE DYNAMICS

This section derives the equation of motion of the cartpole system in the following two ways: (1) standard vector analysis; (2) using Lagrangian.

A. *Standard vector analysis*

The position of the cart is given as

$$\mathbf{r}_1 = x\mathbf{E}_x \tag{1}$$

Computing the 1st and 2nd order rate of change of $\mathbf{r}_1$ in reference frame $\mathcal{F}_i$, we obtain the velocity and acceleration as

$$\mathbf{v}_1 =^{\mathcal{F}_i} \frac{d}{dt}(\mathbf{r}_1) = \dot{x}\mathbf{E}_x \tag{2}$$

$$\mathbf{a}_1 =^{\mathcal{F}_i} \frac{d}{dt}(\mathbf{v}_1) = \ddot{x}\mathbf{E}_x \tag{3}$$

The position of the pole is given as

$$\mathbf{r}_2 = x\mathbf{E}_x + l\mathbf{e}_r \tag{4}$$

Similarly, the velocity and acceleration of the pole can be computed as

$$\mathbf{v}_2 =^{\mathcal{F}_i} \frac{d}{dt}(\mathbf{r}_2) = \dot{x}\mathbf{E}_x + l\dot{\theta}\mathbf{e}_\theta \times \mathbf{e}_r = \dot{x}\mathbf{E}_x - l\dot{\theta}\mathbf{e}_z \tag{5}$$

$$\mathbf{a}_2 =^{\mathcal{F}_i} \frac{d}{dt}(\mathbf{v}_2) = \ddot{x}\mathbf{E}_x - l\ddot{\theta}\mathbf{e}_z - l\dot{\theta}^2\mathbf{e}_r \tag{6}$$

Rewriten $\mathbf{a}_2$ in $\mathbf{E}_x$ and $\mathbf{E}_z$ as

$$\mathbf{a}_2 = (\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)\mathbf{E}_x - (l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta)\mathbf{E}_z \tag{7}$$

Applying Netwon 2nd law on the $\mathbf{E}_x$-direciton of the cart and pole, we have

$$F - T_x = m_1\ddot{x} \tag{8}$$

$$T_x = m_2(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) \tag{9}$$

Adding the above two euqations we get the first equation,

$$F = (m_1 + m_2)\ddot{x} + m_2(l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) \tag{10}$$

The cartpole system is consisted of two systems, i.e. the cart and the pole. The internal force can be canceled out by analyzing the two systems as a whole. A simplier way is to apply the balance of angular momemtun relative to the cart.

$$\frac{d}{dt}\mathbf{H}_c = \mathbf{M}_c - (\mathbf{r}_2 - \mathbf{r}_1) \times m_2\mathbf{a}_1 \tag{11}$$

where $\mathbf{H}_c$ is the angular momemtun of the cartpole system relative to the cart.

$$\mathbf{H}_c = (\mathbf{r}_2 - \mathbf{r}_1) \times m_2(\mathbf{v}_2 - \mathbf{v}_1) = m_2 l^2\dot{\theta}\mathbf{E}_y \tag{12}$$

And $\mathbf{M}_c$ is the moment relative to the cart.

$$\mathbf{M}_c = (\mathbf{r}_2 - \mathbf{r}_1) \times (-m_2 g)\mathbf{E}_z = m_2 g l \sin\theta\mathbf{E}_y \tag{13}$$

$$(\mathbf{r}_2 - \mathbf{r}_1) \times m_2\mathbf{a}_1 = l\mathbf{e}_r \times m_2\ddot{x}\mathbf{E}_x \tag{14}$$

Thus, we get the second equation

$$\cos\theta\ddot{x} + l\ddot{\theta} = -g\sin\theta \tag{15}$$

In summary, the equation of motion is

$$F = (m_1 + m_2)\ddot{x} + m_2(l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) \tag{16}$$

$$\cos\theta\ddot{x} + l\ddot{\theta} = -g\sin\theta \tag{17}$$

REFERENCES

[1] M. Quigley, K. Conley, B. Gerkey, J. Faust, T. Foote, J. Leibs, R. Wheeler, and A. Y. Ng, "Ros: an open-source robot operating system," in *ICRA workshop on open source software*, vol. 3, no. 3.2. Kobe, Japan, 2009, p. 5.

[2] I. Gazebo, "https://ignitionrobotics.org."

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