

## Who invented ray tracing?

### A historical remark

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We provide some remarks on the very early developments of the visualization techniques conducted during the European Renaissance. It is shown that the basic principle of ray tracing was already presented by Albrecht Dürer (1471–1528) in 1525. This article is intended to be of common interest; it is not a scientific report.

**Key words:** Ray tracing – Perspective projection – Historical remarks

**R**eading this special issue of the *Visual Computer*, one may ask: who invented ray tracing? Ray tracing is not a technique that necessarily depends on electronic computers. We know that the first computing machines were not electronic but mechanical instruments.

In 1624 Wilhelm Schickard developed the first calculation machine for adding, subtracting, multiplying, and dividing (Freytag Löringhoff 1958); the machine was reconstructed according to his drawings in 1958. Another early mechanical adder was built by Blaise Pascal in 1642. The first machine for adding, subtracting, dividing, and extracting square roots was built by the philosopher and mathematician Gottfried Wilhelm Leibnitz in 1673 (Encyclopaedia Britannica 1985).

In order to identify the first ray tracer, we have to ask what is essential for ray tracing. We think that this is the "tracing of the rays," even if this is done mechanically and even if it is not done automatically.

Since ray tracing is impossible without perspective projection, it could not have been invented before the calculus of perspective projection was developed. The mathematical method of perspective projection was developed in Italy during the European Renaissance in the years between 1480 and 1515 (the precise data cannot be established).

The pioneers of perspective projection were architects and painters, namely Leon B. Alberti, Leonardo da Vinci, Bramante and Raphael. They were able to draw and paint true perspective projections. This is clearly demonstrated in Raphael's fresco *School of Athens* (Krömker and Hofmann 1986; Mazzola et al. 1987); the fresco shows not only true perspective projection but also very detailed figures and subtle lighting (see Fig. 1).

The great Renaissance artist, mathematician and painter, Albrecht Dürer, was responsible for two important books published in Nuremberg, Germany. Both books have the same main title *Underweysung der messung* (Dürer 1525, 1538).

— The first book was printed in the year 1525. The full title inscription of the book is *Underweysung der messung mit dem zirckel un richtscheyt / in Linien ebenen unnd gantzen corporen / durch Albrecht Dürer zusammen getzoge / und zu nutz alle kunstlieb habenden mit zu gehörigen figuren / in truck gebracht / im jar. M.D.XXv..*

This inscription in old German translates roughly as: "Instructions for measuring with the compass and the ruler / in lines, planes and whole solids / compiled by Albrecht Dürer / which is useful for all those who like the fine



Fig. 1

arts — with pictures and figures / printed / in the year. 1525."

At the time when Albrecht Dürer died, in 1528, there was obviously a lot of posthumous material. Therefore ...

- A second book was printed in the year 1538. This second volume contains most of the material of the 1525 print.

Here the title inscription is *Underweysung der Messung / mit dem Zirckel un richtscheyt / in Linien Ebnen unnd gantzen Corporen / durch Albrecht Dürer zusammen getzogen / un durch jn selbs (als er noch auff erden war) an vil orten gebessert / in sonderheyt mit xxii figure gemert / die selbigen auch mit eygner handt auffgerissen / wie es dann eyn yder werckman erkennen wirdt / Nun aber zu nutz allen kunst liebhabenden in truck geben. 1538.*

An approximate translation would be: "Instructions for measuring with the compass and the ruler / in lines, planes and whole solids / compiled by Albrecht Dürer / improved by him personally at many places (while he was still on earth) / especially expanded with 22 figures / all of which were drawn by his own hand / as would be recognized by every craftsman / for the use of all those who like the fine arts printed. 1538." (see Fig. 2).

Figures 2–6 were obtained from the second book (Dürer 1538). The figures, as shown in Figs. 3–5, are also contained in the first book (Dürer 1528).

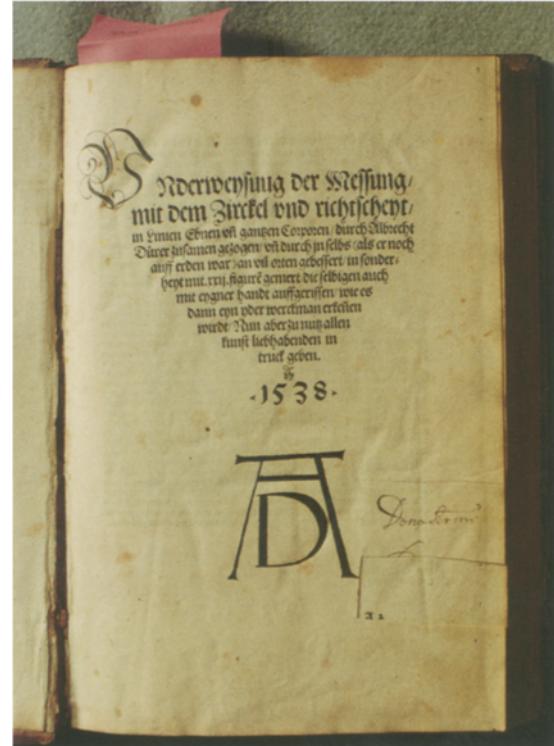


Fig. 2

In his books, A. Dürer deals with general geometry, the design of monuments, the design of text fonts, and perspective projection. In chapters of his books he provides instructions for the construction of perspective projections (also with lights and shadows). The methods, as they were described in the *Underweysung der messung*, can be clearly identified as "object scanning" and "ray tracing."

Let us look at some figures from Dürer's books. The first method deals with the drawing of shadows in a picture: see Figs. 3 and 4.

Dürer realized that the shadow is on those parts of a scene which are hidden from a light source. The shadow parts can be found by a perspective projection, in which the light source is seen as the view point. The projection of the light rays onto the cube and the ground is depicted in Fig. 3. Note that the projection of the silhouette of the cube is identified as the silhouette of the cube's shadow. The finished drawing of the cube with the shadows can be seen in Fig. 4.

The next figure shows a method we would now call "ray tracing": see Fig. 5, left page, and Fig. 6, left page top.

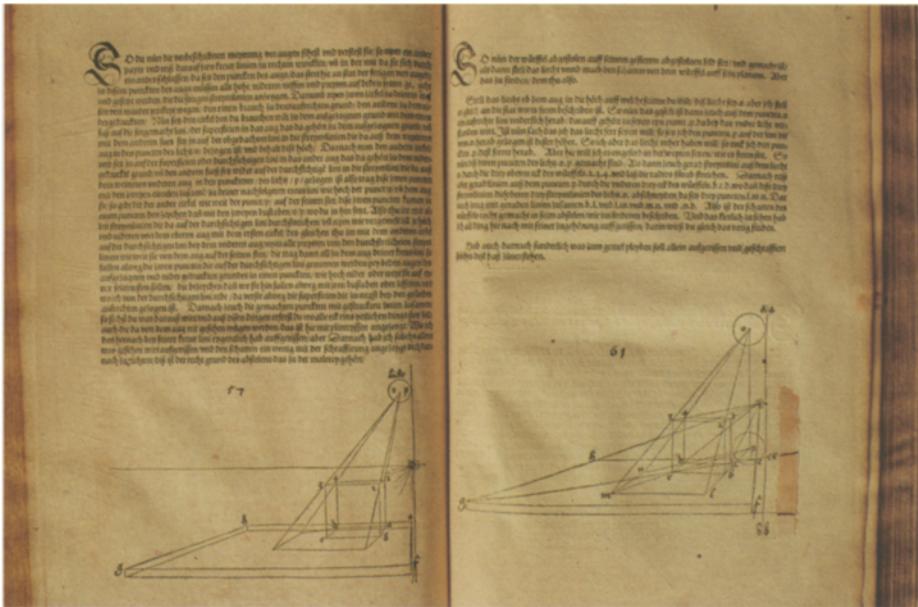


Fig. 3

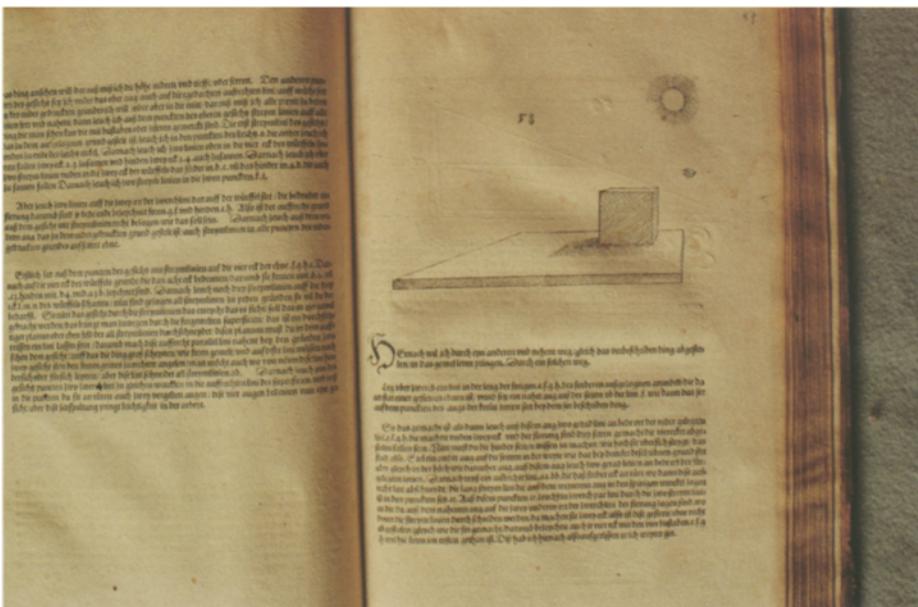


Fig. 4

In Fig. 5 the artist is painting the portrait of a man; in Fig. 6 he draws the picture of a tankard. The artist has fixed the eye on the top of a stick and is looking through the picture plane and painting onto the picture plane what is seen. Therefore, the artist has to paint onto a transparent material, e.g. glass. If one layer of paint has been applied, the artist is no longer able to see the object (the tankard, the man) which he has to paint.

To deal with this disadvantage, Dürer provides another method; see Fig. 6, left page bottom. Here, the artist has again fixed the eye on the top of a stick. But, in contrast to the previous methods, he is not painting directly onto the projection plane. The projection plane is represented by a grid of lines, while the same grid is drawn on the artist's canvas. The artist draws what he sees according to the grid.



Fig. 5



Fig. 6

Please note that the artist is “sampling” the objects behind the picture plane. The rays are traced from the artist’s eye through the grid of the picture plane (which is nowadays the pixel plane) until they hit an object. That is exactly the principle of the ray tracing algorithms and programs of today!

Another figure deals with “object scanning”; see Fig. 5, right page. Here the artist is drawing the picture of a lute. The eye point is marked by the

eyebolt in the wall. A thread runs through the eyebolt: one end of the thread is loaded, the other end is fixed on a stick. The artist’s assistant holds the stick and points to the part of the lute (the object point) which the artist wants to paint. The artist observes the point in the projection plane through which the thread runs. This point is marked later on the picture plane. Having marked a large number of these points, the artist is able

to draw a wire-frame picture of the lute; the wire-frame may be completed as a full-colored painting.

We call Dürer's method in Fig. 5, left page, and in Fig. 6 ray tracing because a ray is traced from the eye through the projection plane until it hits an object. The main loop runs over all points in the projection plane. In contrast to this, we call Dürer's method in Fig. 5, right page, object scanning because the object is scanned point by point, and the point in the projection plane is determined as a consequence of perspective projection. The main loop runs over all points of the object.

The longer one looks at Dürer's figures, the more one realizes how little visualization technique has changed.

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## References

1. Baron von Freytag Löringhoff B (1958) Über die erste Rechenmaschine. *Physikalische Blätter*, 14th year, pp 361–365
2. Encyclopaedia Britannica (1985) 15th edn, vol 21, p 564
3. Dürer A (1525) *Underweysung der messung mit dem zirckel un richtscheyd...* Nuremberg. Reprints (1972) Unterschneidheim and (1983) Nördlingen
4. Dürer A (1538) *Underweysung der Messung mit dem Zirckel un richtscheyd...* Nuremberg
5. Krömker D, Hofmann GR (1986) Reconstructing and modeling Raphael's school of Athens. *Computer Graphics* 10:381–385
6. Mazzola G, Krömker D, Hofmann GR (1987) *Rasterbild-Bildraster, Anwendung der Graphischen Datenverarbeitung zur geometrischen Analyse eines Meisterwerks der Renaissance: Raphaels 'Schule von Athen.'* Springer, Berlin Heidelberg New York Paris London Tokyo



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