

1 Chapter 1

Exercise 1.1(a)

Let X be the sum of the two number obtained.

Let X_1 be the number obtained on Die 1.

Let X_2 be the number obtained on Die 2.

Thus, $X = X_1 + X_2$, and

$$E_x = \{X = x\} = \{X_1 + X_2 = x\} = \{X_1 = x_1, X_2 = x - x_1 \mid 1 \leq x_1, x - x_1 \leq 6\}$$

Now,

$$1 \leq x - x_1 \leq 6 \iff -1 \geq x_1 - x \geq -6 \iff x - 1 \geq x_1 \geq x - 6 \iff x - 6 \leq x_1 \leq x - 1$$

Hence,

$$E_x = \{X = x\} = \{X_1 + X_2 = x\} = \{X_1 = x_1, X_2 = x - x_1 \mid \max\{1, x - 6\} \leq x_1 \leq \min\{6, x - 1\}\}$$

$$\begin{aligned} P(E_x) &= \sum_{x_1=\max\{1, x-6\}}^{\min\{6, x-1\}} P(X_1 = x_1, X_2 = x - x_1) = \sum_{x_1=\max\{1, x-6\}}^{\min\{6, x-1\}} \frac{1}{6^2} \\ &= \frac{1}{6^2} (\min\{6, x - 1\} - \max\{1, x - 6\} + 1) \end{aligned}$$

Next, note that

$$\min\{6, x - 1\} = \begin{cases} x - 1, & \text{if } x = 2, 3, \dots, 6 \\ 6, & \text{if } x = 7, 8, \dots, 12 \end{cases} \quad \text{and} \quad \max\{1, x - 6\} = \begin{cases} 1, & \text{if } x = 2, 3, \dots, 6 \\ x - 6, & \text{if } x = 7, 8, \dots, 12 \end{cases}$$

Hence,

$$\begin{aligned} P(E_x) &= \frac{1}{6^2} (\min\{6, x - 1\} - \max\{1, x - 6\} + 1) = \frac{1}{36} \begin{cases} (x - 1) - 1 + 1, & \text{if } x = 2, 3, \dots, 6 \\ 6 - (x - 6) + 1, & \text{if } x = 7, 8, \dots, 12 \end{cases} \\ &= \frac{1}{36} \begin{cases} x - 1, & \text{if } x = 2, 3, \dots, 6 \\ 13 - x, & \text{if } x = 7, 8, \dots, 12 \end{cases} \end{aligned}$$

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References