02576 Physically Based Rendering

Scattering by Particles

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The colours of materials

- What causes materials to look the way they do?
- The appearance of a material is largely determined by its particle composition.
- We therefore turn our attention to the general theory of light scattering by spherical particles: the Lorenz-Mie theory.
- This is a general case of appearance modelling.
  - Lorenz showed in 1890 that the rainbow and the sky (Rayleigh scattering) are simply special cases.
  - Mie derived the theory again in 1908 to predict the appearance of colloidal gold solutions.
- In general, we can use the theory to obtain the optical properties of turbid materials consisting of approximately spherical particles.
- First: How do we use the optical properties to compute an image?

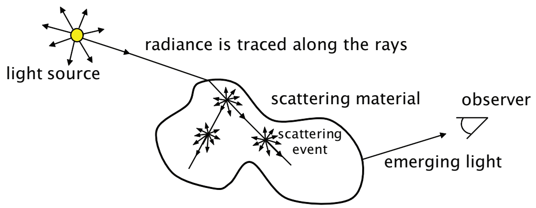
Computing appearance from optical properties

- Prediction requires solving the radiative transfer equation:

$$(\vec{\omega}\cdot\nabla)L(\mathbf{x},\vec{\omega}) = -\sigma_t(\mathbf{x})L(\mathbf{x},\vec{\omega})+\sigma_s(\mathbf{x})\int_{4\pi} p(\mathbf{x},\vec{\omega}',\vec{\omega})L(\mathbf{x},\vec{\omega}')\,\mathrm{d}\omega'+L_e(\mathbf{x},\vec{\omega})\,.$$

- The solution method of choice today:

  Stochastic ray tracing (Monte Carlo integration).



- Required input: The scattering coefficient σ_s, the extinction coefficient σ_t, and the phase function p.
- How do we compute these input from the particle composition of a material?

Scattering of a plane wave by a spherical particle

- A plane wave scattered by a spherical particle gives rise to a spherical wave.
- The components of a spherical wave are spherical functions.
- To evaluate these spherical functions, we use spherical harmonic expansions.
- Coefficients in these spherical harmonic expansions are referred to as Lorenz-Mie coefficients a_n and b_n.
- Lorenz [1890] and Mie [1908] derived formal expressions for a_n and b_n using the spherical Bessel functions j_n and y_n.
- These expressions are written more compactly if we use the Riccati-Bessel functions:

$$\begin{aligned}\psi_n(z) &= z\,j_n(z)\\ \zeta_n(z) &= z(j_n(z) - i\,y_n(z))\ ,\end{aligned}$$

  where z is (in general) a complex number.

The Lorenz-Mie coefficients (a_n and b_n)

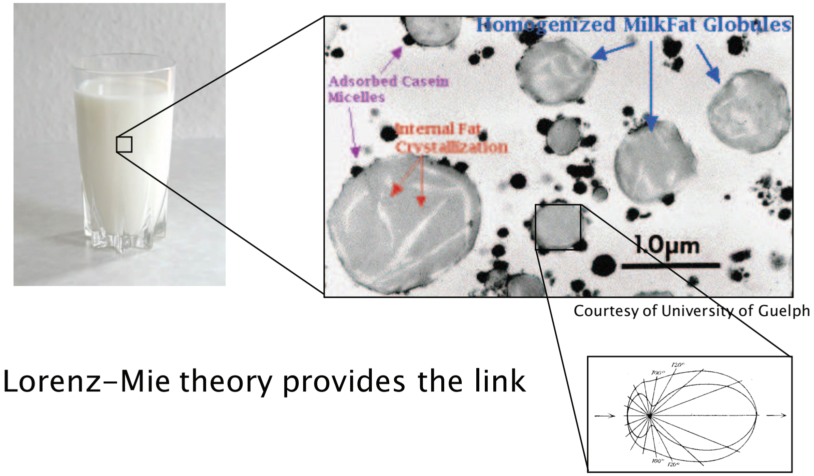
- Using the Riccati-Bessel functions ψ_n and ζ_n, the expressions for the Lorenz-Mie coefficients are

$$\begin{aligned} a_n &= \frac{n_{\text{med}}\psi_n'(y)\psi_n(x) - n_p\psi_n(y)\psi_n'(x)}{n_{\text{med}}\psi_n'(y)\zeta_n(x) - n_p\psi_n(y)\zeta_n'(x)} \\ b_n &= \frac{n_p\psi_n'(y)\psi_n(x) - n_{\text{med}}\psi_n(y)\psi_n'(x)}{n_p\psi_n'(y)\zeta_n(x) - n_{\text{med}}\psi_n(y)\zeta_n'(x)} \ . \end{aligned}$$

  - Primes $'$ denote derivative.
  - n_{med} and n_p are the refractive indices of the host medium and the particle respectively.
  - x and y are called size parameters.
- If r is the particle radius and λ is the wavelength *in vacuo*, then x and y are defined by

$$x = \frac{2\pi r n_{\text{med}}}{\lambda} \quad , \quad y = \frac{2\pi r n_p}{\lambda} \ .$$

From particles to appearance



Courtesy of University of Guelph

Lorenz–Mie theory provides the link

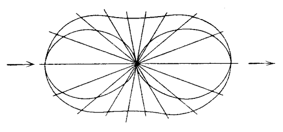
Scattering by spherical particles

- The Lorenz-Mie theory:

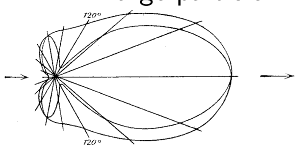
$$\begin{aligned} p(\theta) &= \frac{|S_1(\theta)|^2 + |S_2(\theta)|^2}{2|k|^2 C_s} \\ S_1(\theta) &= \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)}\left(a_n\pi_n(\cos\theta) + b_n\tau_n(\cos\theta)\right) \\ S_2(\theta) &= \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)}\left(a_n\tau_n(\cos\theta) + b_n\pi_n(\cos\theta)\right) \ . \end{aligned}$$

- a_n and b_n are the Lorenz-Mie coefficients.
- π_n and τ_n are spherical functions associated with the Legendre polynomials.

small particle



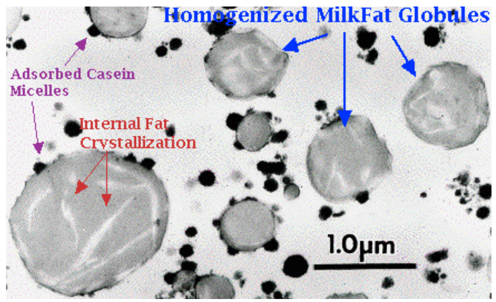
large particle



Quantity of scattering

- Lorenz-Mie theory continued:
  The scattering and extinction cross sections of a particle:

$$\begin{aligned} C_s &= \frac{\lambda^2}{2\pi|n_{\text{med}}|^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2 + |b_n|^2\right) \\ C_t &= \frac{\lambda^2}{2\pi}\sum_{n=1}^{\infty}(2n+1)\mathrm{Re}\left(\frac{a_n + b_n}{n_{\text{med}}^2}\right) \ . \end{aligned}$$



Bulk optical properties of a material

- Start with volume fraction of a component v

$$v = \frac{4\pi}{3}\int_{r_{\text{min}}}^{r_{\text{max}}} r^3 N(r)\,\mathrm{d}r$$

- Use it to scale the number density distribution N.
- Use that to find the bulk properties σ_s (and σ_t likewise)

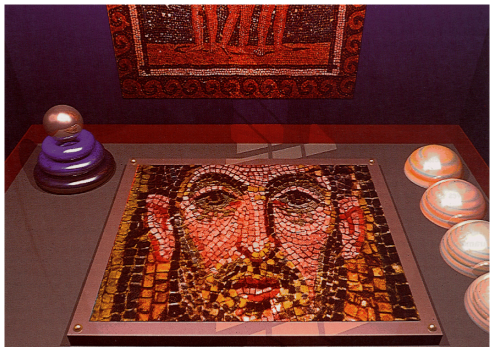
$$\sigma_s = \int_{r_{\text{min}}}^{r_{\text{max}}} C_s(r)N(r)\,\mathrm{d}r$$

Computing scattering properties

- Input needed for computing scattering properties:
  - Particle composition (volume fractions, particle shapes).
  - Refractive index for host medium n_{med}.
  - Refractive index for each particle type n_p.
  - Size distribution for each particle size (r).
- Lorenz-Mie theory uses a series expansion. How many terms should we include?
- Number of terms to sum $N = \left\lceil |x| + p|x|^{1/3} + 1 \right\rceil$.
  - Empirically justified [Wiscombe 1980, Mackowski et al. 1990].
  - Theoretically justified [Cachorro and Salcedo 1991].
  - For a maximum error of 10^{-8}, use $p = 4.3$.
- Code for evaluating the expansions in the Lorenz-Mie theory is available online [Frisvad et al. 2007]:
  http://www.imm.dtu.dk/pubdb/p.php?5288

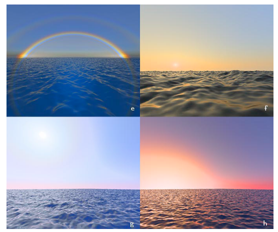
Graphics and Lorenz-Mie theory (early encounters)

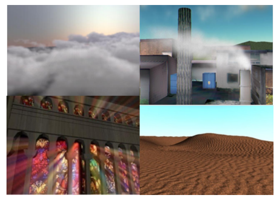
- Nishita et al. [1987]: Mie theory is very complicated, let us use the empirical approximations by Gibbons [1958] instead.
- Gibbons' empirical expressions become known as "hazy and murky Mie" ☺
- Rushmeier [1995]: Mie theory is too restrictive.
- Callet [1996]: Mie theory is useful for pigmented materials.

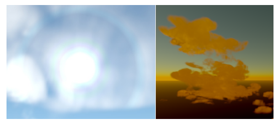


Graphics and Lorenz-Mie theory

- Jackèl and Walter [1997]: Atmospheric phenomena.
- Nishita and Dobashi [2001]: Natural phenomena consisting of particles in air.
- Riley et al. [2004]: Real-time simulation of atmospheric phenomena.
- Frisvad et al. [2007]: Computing scattering properties (particles in an absorbing host).







Particle contents (examples)

- Natural water
  - Refractive index of host: saline water.
  - Mineral and alga contents: user input in volume fractions.
  - Refractive indices of mineral and algae: empirical formulae.
  - Shape of mineral and algal particles: spheres.
  - Size distributions: power laws.
- Icebergs
  - Refractive index of host: pure ice.
  - Brine and air contents: depend on temperature, salinity, and density.
  - Refractive index of brine and air: empirical formula, measured absorption spectrum, and $n_{\mathrm{air}} = 1.00$.
  - Shape of brine pores and air pockets: closed cylinders and ellipsoids.
  - Size distributions: power laws.
- Milk
  - Refractive index of host: water + dissolved vitamin B2.
  - Fat and protein contents: user input in wt.-%.
  - Refractive index of milk fat and casein: measured spectra.
  - Shape of fat globules and casein micelles: spheres and a volume to surface area ratio.
  - Size distributions: log-normal with mean depending on fat content and homogenization pressure.

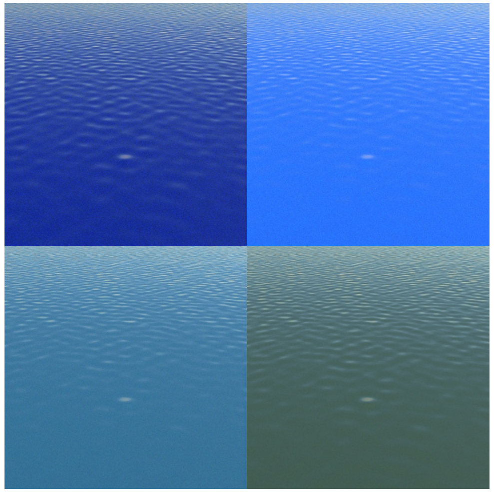
Case study: natural waters

- Glacial melt water with rock flour mixing with purer water from melted snow to give Lake Pukaki in New Zealand its beautiful bright blue colour.

Oceanic and coastal waters

Cold Atlantic

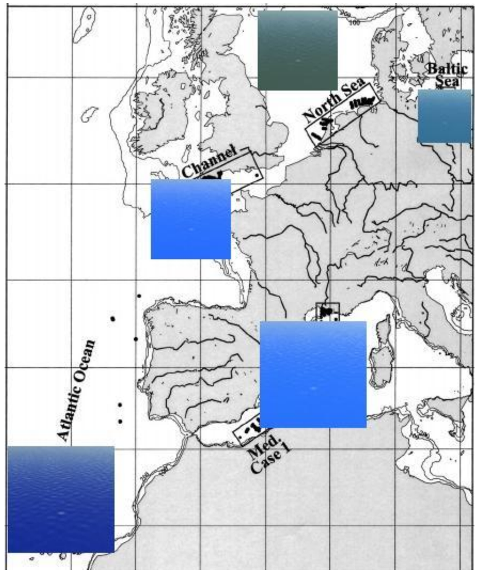
Mediterranean



Baltic

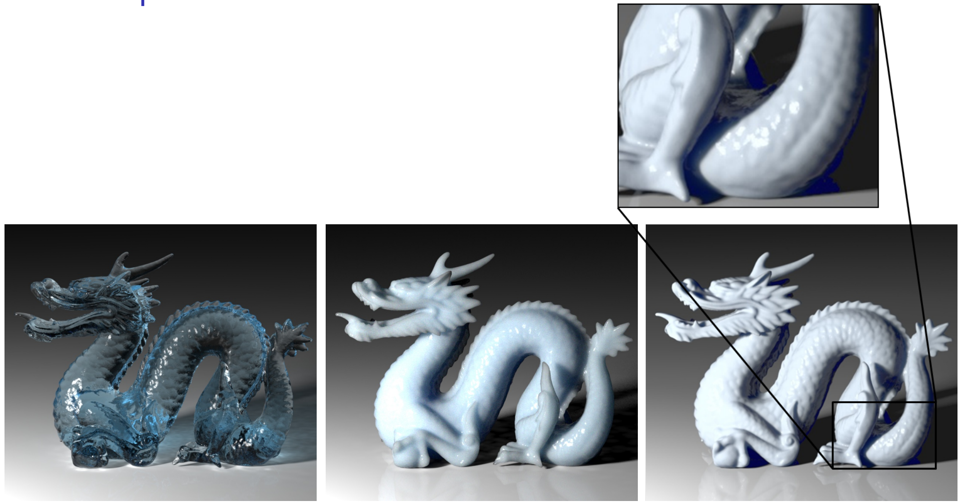
North Sea

Oceanic and coastal waters



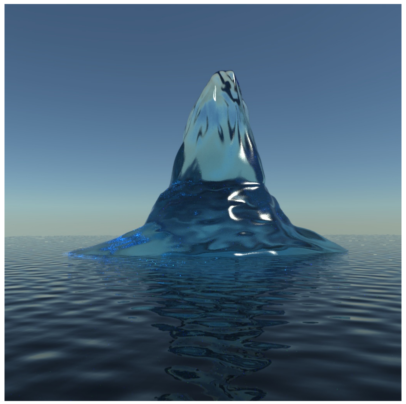
Case study: icebergs

Ice sculptures

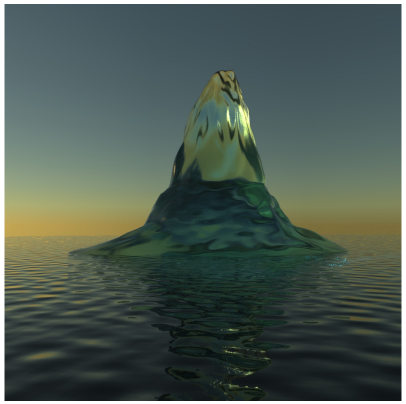


pure ice compacted snow white ice

Bottle green icebergs

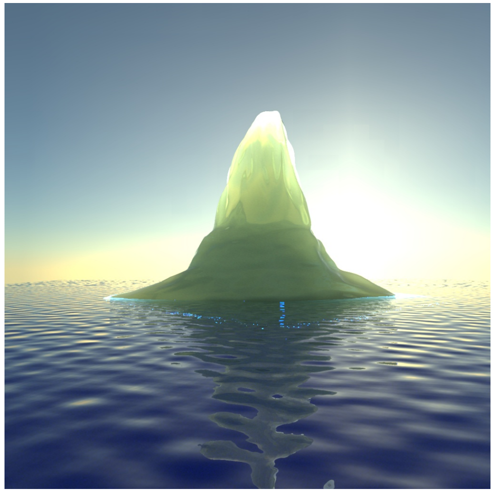


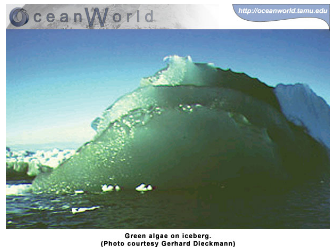
10 am



7 pm

Algal ice



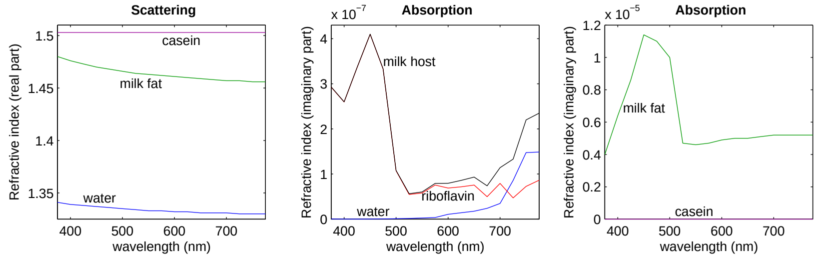


Case study: milk

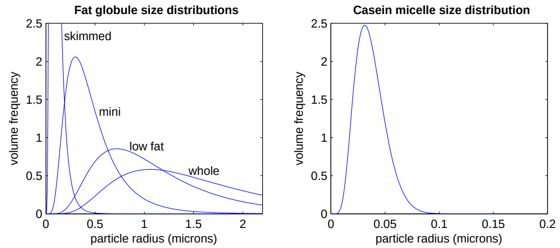

- Reddish on forward scattering, subtle bluish on side scattering, white on back scattering.

Measurements used for the milk model

- Refractive indices:



- Particle size distributions:



Predicting appearance based on a content declaration

water vitamin B2 protein fat skimmed low fat whole

- Vitamin B2 content: 0.17 mg / 100 g [Fox and McSweeney 1998]
- Protein content: 3 g / 100 g
- Fat content: 0.1 g (skimmed), 1.5 g (low fat), 3.5 g (whole) / 100 g
- Homogenization pressure: 0 MPa (model: [0, 50] MPa)

Exercises

- Choose a material or visible phenomenon that you would like to model and take/find a picture of it.
- Find out what rendering method and optical properties you would need to render it.
- Search the literature (give references).
- Describe how you would do the rendering using the methods covered in this course.
- **Project proposal:**

  Implement your appearance model (do the rendering).

Optics references (in chronological order)

- Lorenz, L. Lysbevægelser i og uden for en af plane Lysbølger belyst Kugle. *Det kongelig danske Videnskabernes Selskabs Skrifter*, pp. 2–62. 6. Række, Naturvidenskabelig og Mathematisk Afdeling VI, 1, 1890.
- Mie, G. Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen. *Annalen der Physik 25*(3), pp. 377–445. IV. Folge, 1908.
- Infeld, L. The influence of the width of the gap upon the theory of antennas. *Quarterly of Applied Mathematics 5*(2), pp. 113–132, 1947.
- Gibbons, M. G. Radiation received by an uncollimated receiver from a 4π source. *Journal of the Optical Society of America 48*(8), pp. 550–555, August 1958.
- Kattawar, G. W., and Plass, G. N. Electromagnetic scattering from absorbing spheres. *Applied Optics 6*(8), pp. 1377–1382, August 1967.
- Dave, J. V. Scattering of electromagnetic radiation by a large, absorbing sphere. *IBM Journal of Research and Development 13*(3), pp. 302–313, May 1969.
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- Yang, W. Improved recursive algorithm for light scattering by a multilayered sphere. *Applied Optics 42*(9), pp. 1710–1720, March 2003.

Graphics references (in chronological order)

- Nishita, T., Miyawaki, Y., and Nakamae, E. A shading model for atmospheric scattering considering luminous intensity distribution of light sources. Computer Graphics (Proceedings of ACM SIGGRAPH 87) 21(4), pp. 303–310, July 1987.
- Rushmeier, H. Input for participating media. In *Realistic Input for Realistic Images*, ACM SIGGRAPH 95 Course Notes, ACM Press, 1995. Also appeared in the ACM SIGGRAPH 98 Course Notes - A Basic Guide to Global Illumination.
- Callet, P. Pertinent data for modelling pigmented materials in realistic rendering. *Computer Graphics Forum 15*(2), pp. 119–127, 1996.
- Jackèl, D., and Walter, B. Modeling and rendering of the atmosphere using Mie-scattering. *Computer Graphics Forum 16*(4), pp. 201–210, 1997.
- Nishita, T., and Dobashi, Y. Modeling and rendering of various natural phenomena consisting of particles. In *Proceedings of Computer Graphics International 2001*, pp. 149–156, 2001.
- Riley, K., Ebert, D. S., Kraus, M., Tessendorf, J., and Hansen, C. Efficient rendering of atmospheric phenomena. In *Proceedings of the 15th Eurographics Symposium on Rendering*, pp. 375–386, 2004.
- Frisvad, J. R., Christensen, N. J., and Jensen, H. W. Computing the scattering properties of participating media using Lorenz-Mie theory. *ACM Transactions on Graphics 26*(3), Article 60, July 2007.

02576 Physically Based Rendering

High Dynamic Range Imaging

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High dynamic range imaging

- Why doesn't the camera see what I see?
  - The camera has a much smaller dynamic range (several orders of magnitude measured in cd/m^2).
  - The part of the visible dynamic range captured by the camera is determined by the exposure time.
- Exposure time is usually changed in stops.
  - A stop is a power-of-two exposure step (half the exposure time if aperture is kept constant).
- High dynamic range imaging:
  - Keep the camera still and take images at multiple exposures.
  - Combine several low dynamic range images into one high dynamic range image (HDR image capture).
  - Map the high dynamic range image to a low dynamic range display (tone reproduction).
- HDRI was once Hollywood's best kept secret [Bloch 2007].

References

- Bloch, C. *The HDRI Handbook: High Dynamic Range Imaging for Photographers and CG Artists*. Rocky Nook, 2007.
- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. *High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting*, second edition, Morgan Kaufmann/Elsevier, 2010.

HDR image capture

- Exposure time from 30 s to 1 ms in 1-stop increments.

- Combining to get high dynamic range:

$$L_{ij} = \sum_{k=1}^{N} \frac{f^{-1}(Z_{ij}) w(Z_{ij})}{\Delta t_k} \Bigg/ \sum_{k=1}^{N} w(Z_{ij}) \ ,$$

where Z_{ij} are pixel values, Δt_k is exposure time, w is a weighting function to tone down extreme pixel values, and f is the camera response function.

References

- Debevec, P. E., and Malik, J. Recovering high dynamic range radiance maps from photographs. In *Proceedings of ACM SIGGRAPH 97*, pp. 369–378, August 1997.
- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. *High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting*, second edition, Morgan Kaufmann/Elsevier, 2010.

Tone reproduction

left Linear mapping of all dynamic range.

middle Linear mapping of lower 0.1% of dynamic range.

right Histogram adjustment [Ward et al. 1997].

References

- Debevec, P. E., and Malik, J. Recovering high dynamic range radiance maps from photographs. In *Proceedings of ACM SIGGRAPH 97*, pp. 369–378, August 1997.
- Ward, G., Rushmeier, H., and Piatko, C. A visibility matching tone reproduction operator for high dynamic range scenes. *IEEE Transactions on Visualization and Computer Graphics 3*(4), pp. 291–306, 1997.

RGBE encoding (the .hdr format)

- RGBE $\rightarrow$ RGBA

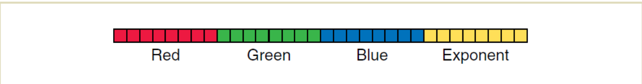


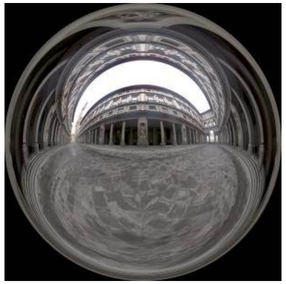
FIGURE 3.2 Bit breakdown for the 32-bit/pixel RGBE (and XYZE) encodings.

$$\begin{aligned} R_W &= \frac{R_M + 0.5}{256} 2^{E-128} \\ G_W &= \frac{G_M + 0.5}{256} 2^{E-128} \\ B_W &= \frac{B_M + 0.5}{256} 2^{E-128} \end{aligned}$$

References

- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. *High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting*, second edition, Morgan Kaufmann/Elsevier, 2010.

Light probes



- The angular map

$$\begin{aligned} r &= \frac{\arccos(-D_z)}{2\pi\sqrt{D_x^2 + D_y^2}} \\ (u, v) &= \left(\frac{1}{2} + rD_x, \frac{1}{2} + rD_y\right) \ , \end{aligned}$$

where (D_x, D_y, D_z) is the look-up direction into the environment map.

References

- Debevec, P. Image-based lighting. *IEEE Computer Graphics and Applications 22*(2), pp. 2634, 2002.

Exercises

- Load an HDR image to be used as a texture (loading functionality in the framework).
- Convert from RGBE to floating point RGBA.
- Use an HDR light probe as environment map.