2 (1) Part 1 discussed in class

minimize $J(A_r) = \|A - A_F\|_F^2$

given $A_{m \times n}$ $\rho(A) > r$

A_r is a rank r matrix

$r < \min(m,n)$

Find SVD of A

$$A = U \Sigma V^T$$

$A_{m \times n}$

$V_{n \times n}$, $U_{m \times m}$: Orthogonal Matrix

Σ : $m \times n$ diagonal entries whose entries are non-negative. (Singular Values)

Columns of U & V are called left singular and right singular ~~values~~ vectors of M.

Consider the r largest singular values & corresponding singular vectors

$A_r = U_r S_r V_r^T$ solⁿ $\boxed{}$

A rank-k approximation to the covariance matrix is used in PCA

(2) $\min_{\substack{R \\ \text{st } RR^T = I}} J(R) = \|RB - A\|_F^2$

$A_{n \times m}$ $B_{n \times m}$ $R_{n \times n}$

Rigid Transform Estimation (Orthogonal Procrustes Problem)

Sol^n $AB^T = USV^T$ (SVD)

$R^T = UV^T$

Application:

Learning the Bases: Method 3- Union of Ortho-Normal Bases (Dictionary Learning)

Algo

for $m = 1:M$

{

$X_m = X - \sum_{j \neq m} A_j S_j$;

$S_m X_m^T = U \Lambda V^T$;

$A_m = VU^T$ → Orthogonal Procrustes

}

Updating S_m: $S_m = \text{argmin}_{S^*} \|X_m - A_m S^*\|^2 + \lambda_t \|S^*\|_1$

Because we are solving

$\min_A \|X_m - A S_m\|^2$ s.t. $AA^T = I$

$S = \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{pmatrix}$ Coefficient Matrix

$\vec{S}_m$

A: dictionary.

$(X_m)_{p \times k}$