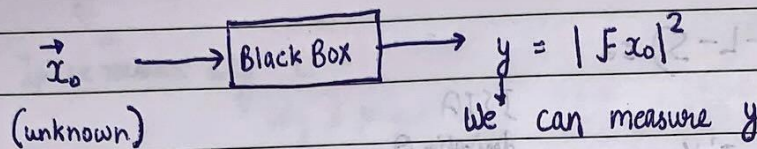


## Recent Advances in Phase Retrieval

### 1 Phase Retrieval



If  $x_0$  is modelled as a vector in  $\mathbb{C}^n$ , without additional information, this inverse problem (finding  $x_0$  given  $y$ ) is ill-posed. More structure can be added by assuming sparsity and compact support.

In a general phase retrieval setting, in which we receive  $y = |Ax_0|^2$  for some known matrix  $A \in \mathbb{C}^{m \times n}$ . We seek to solve

→ find  $x$  subject to  $|Ax|^2 = y, x \in S, (1)$

where  $S \subseteq \mathbb{C}^n$  corresponds to the imposed structure. Focus will be on  $S = \mathbb{C}^n$  or set of  $k$ -sparse vectors

### Uniqueness

$$x \in S \Leftrightarrow e^{i\phi} x \in S \quad \forall \phi \in [0, 2\pi)$$

Unique solution upto a global phase factor.

$$[x]_0 := \{e^{i\phi} x_0 : \phi \in [0, 2\pi)\}$$

Let  $a_i^*$  denote  $i^{\text{th}}$  row of  $A$

(here  $a_i \in \mathbb{C}^n$  is column vector, and  $a_i^*$  denote its conjugate-transpose)



b) D: DFT matrix (1D)

for real-valued

$$S \mathbf{a} = \mathbf{R}^n$$

$\vec{d}_i$ :  $i^{\text{th}}$  column of D

$\vec{d}_i^T$ :  $i^{\text{th}}$  row of D

$$\therefore y[i] = \vec{d}_i^T X \vec{d}_i$$

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$$X = \vec{x} \vec{x}^T$$

Special case when  $\vec{x}$  is real

For an  $n$ -dimensional (non-zero) vector its null space has dimension  $n-1$ .

Hence by Rank Nullity Theorem  $X$  has rank 1.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 x_2 & \dots & x_1 x_n \\ x_2 x_1 & x_2^2 & \dots & x_2 x_n \\ \vdots & \vdots & \ddots & \vdots \\ x_n x_1 & x_n x_2 & \dots & x_n^2 \end{bmatrix}$$

Any two columns  $i$  &  $j$  are  $\frac{x_i}{x_j}$  multiple apart.

hence linearly independent.

d) For Hermitian Matrices

Singular values are same as Eigenvalues

hence Trace = Nuclear Norm

(Sum of Eigenvalues) (Sum of singular values)



(e)

$$m = \Omega(n \text{ poly } \log n)$$

as ~~more~~ mentioned on 4<sup>th</sup> last line of pg 3 in the paper.