Assignment 5

Q1 a)

b)

2 (i) Part 1 discussed in class

minimize $J(A_r) = \|A - A_F\|_F^2$

given $A_{m \times n}$ $\rho(A) > r$

A_r is a rank r matrix

$r < \min(m, n)$

Find SVD of A

$$A = U \Sigma V^T$$

$A_{m\times n}$

$V_{n\times n}$, $U_{m\times m}$: Orthogonal Matrix

Σ : $m \times n$ diagonal entries whose entries are non-negative. (Singular Values)

Columns of U & V are called left singular and right singular ~~values~~ vectors of M.

Consider the r largest singular values & corresponding singular vectors

$$A_r = U_r S_r V_r^T \quad \text{sol}^n \ \blacksquare$$

A rank-k approximation to the covariance matrix is used in PCA

Proof: (for completeness, adapted from wikipedia)

Σ $m \times n$ diagonal matrix with entries $(\sigma_1, \sigma_2, \ldots, \sigma_m)$ such that

$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \geq \sigma_m \geq 0$

Claim: Best k rank approximation given by.

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

If $\vec{v}_{d\times 1}$ is vectorised image and d is large, then dimensionality reduction is performed to reduce number of dimensions to k ($\ll d$) with minimal loss in image properties. For this best k-rank approx. is required. With this matrix operations can be carried out in computationally efficient manner.

(2) $\min_{\text{st } RR^T = I} J(R) = \|RB - A\|_F^2$

$A_{n \times m} \quad B_{n \times m} \quad R_{n \times n}$

Rigid Transform Estimation (Orthogonal Procrustes Problem)

~~E(R)~~

Sol^n $AB^T = USV^T$ (SVD)

$R^T = UV^T$

Application:

Learning the Bases: Method 3 - Union of Ortho-Normal Bases (Dictionary Learning)

Algo

for $m = 1:M$

{

$X_m = X - \sum_{j \neq m} A_j S_j$;

$S_m X_m^T = U \Lambda V^T$;

$A_m = VU^T$ → Orthogonal Procrustes

}

Updating S_m: $S_m = \arg\min_{S^*} \|X_m - A_m S^*\|^2 + \lambda_t \|S^*\|_1$

Because we are solving

$\min_A \|X_m - A S_m\|^2$ s.t. $AA^T = I$

$S = \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_M \end{pmatrix}$ Coefficient Matrix

$\vec{S}_m$

A: dictionary

$(X_m)_{p \times k}$