# Recent Advances in Phase Retrieval

## 1 Phase Retrieval

$$\vec{x}_0 \longrightarrow \boxed{\text{Black Box}} \longrightarrow y = |Fx_0|^2$$

(unknown) We can measure $y$

If $x_0$ is modelled as a Vector in $\mathbb{C}^n$, without additional information, this inverse problem (finding $x_0$ given $y$) is ill-posed.

More structure can be added by assuming sparsity and compact support.

In a general phase retreival setting, in which we receive $y = |Ax_0|^2$ for some known matrix $A \in \mathbb{C}^{m \times n}$

We seek to solve

→ find $x$ subject to $|Ax|^2 = y$, $x \in S$, (1)

where $S \subseteq \mathbb{C}^n$ corresponds to the imposed structure.

Focus will be on $S = \mathbb{C}^n$ or set of k-sparse vectors

**Uniqueness**

$x \in S \Leftrightarrow e^{i\phi} x \in S \quad \forall\, \phi \in [0, 2\pi)$

Unique solution upto a global phase factor.

$[x]_0 := \{ e^{i\phi} x_0 : \phi \in [0, 2\pi) \}$

Let $a_i^*$ denote $i^{th}$ row of $A$

(here $a_i \in \mathbb{C}^n$ is column vector, and $a_i^*$ denote its conjugate-transpose)

b) D: DFT matrix (1D)

for real-valued

$S = R^n$

$\vec{d}_i$ : $i^{th}$ column of D

$\therefore \vec{d}_i^{\,T}$ : $i^{th}$ row of D

$\therefore y[i] = \vec{d}_i^{\,T} X \vec{d}_i$

c) $X = \vec{x}\vec{x}^T$

Special case when $\vec{x}$ is real

For an n-dimensional (non-zero) vector its null space has dimension n-1.

Hence by Rank Nullity Theorem X has rank 1.

$$X = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} \begin{bmatrix} r_1 & r_2 & \cdots & r_n \end{bmatrix} = \begin{bmatrix} r_1^2 & r_1 r_2 & \cdots & r_1 r_n \\ r_2 r_1 & r_2^2 & \cdots & r_2 r_n \\ \vdots & & & \\ r_n r_1 & r_n r_2 & \cdots & r_n^2 \end{bmatrix}$$

Any two columns i & j are $\frac{r_i}{r_j}$ multiple apart

hence Linearly independent.

d) For Hermitian Matrices

Singular values are same as EigenValues

hence Trace = Nuclear Norm

(Sum of EigenValues) (Sum of singular values)

(e) $m = \Omega(n \text{ poly} \log n)$

as ~~me~~ mentioned on $4^{th}$ last line of pg 3 in the paper.