3

Let $\boldsymbol{\mathcal{S}}$ be the set of 10^5 subsets that will be used to implement the algorithm in Q2.

Let $\hat{\lambda}_{\max}$ and $\hat{\lambda}_{\min}$ be the maximum (minimum) of the maximal (minimal) eigenvalue of any matrix $\boldsymbol{A}_{\mathcal{S}}^T\boldsymbol{A}_{\mathcal{S}}$ where $\mathcal{S} \in \boldsymbol{\mathcal{S}}$.

$$\begin{aligned}
\text{Now, } \boldsymbol{\mathcal{S}} \in n^{|S|} &\Rightarrow \hat{\lambda}_{\max} \leq \lambda_{\max} \text{ and } \hat{\lambda}_{\min} \geq \lambda_{\min} && (1)\\
&\Rightarrow \hat{\lambda}_{\max} - 1 \leq \lambda_{\max} - 1 \text{ and } 1 - \hat{\lambda}_{\min} \leq 1 - \lambda_{\min} && (2)\\
&\Rightarrow \max(\hat{\lambda}_{\max} - 1, 1 - \hat{\lambda}_{\min}) \leq \max(\lambda_{\max} - 1, 1 - \lambda_{\min}) && (3)\\
&\Rightarrow \hat{\delta}_s \leq \delta_s && (4)
\end{aligned}$$

$$\begin{aligned}
3 \text{ follows as } \max(\hat{\lambda}_{\max} - 1, 1 - \hat{\lambda}_{\min}) &= \hat{\lambda}_{\max} - 1 \leq \lambda_{\max} - 1 \leq \max(\lambda_{\max} - 1, 1 - \lambda_{\min}) && (5)\\
\text{or } &= 1 - \hat{\lambda}_{\min} \leq 1 - \lambda_{\min} \leq \max(\lambda_{\max} - 1, 1 - \lambda_{\min}) && (6)
\end{aligned}$$

Hence, the correct answer is option (1) $\hat{\delta}_s \leq \delta_s$.