1 **RIP**

$(1-\delta_s)\|\theta\|^2 \le \|A\theta\|^2 \le (1+\delta_s)\|\theta\|^2 \quad \forall\, \theta$ s-sparse

$$\mu(A) = \max_{i,j,\, i\neq j} \frac{|A_i^t A_j|}{\|A_i\|_2 \|A_j\|_2}$$

→ WLOG consider $\|A_i\|_2 = 1 \quad \forall i$

Let $A_{m\times n} = \left[\vec{A}_1 \;\; \vec{A}_2 \;\cdots\; \vec{A}_n\right] \qquad (\vec{A}_i)_{m\times 1}$

Slide 101:

$\delta_s = \max\{1-\lambda_{min}, \lambda_{max}-1\}$ 0

$$\lambda_{max} = \max_{\theta_T \in \mathbb{R}^s, |T|\le s} \frac{\|A_T\theta_T\|^2}{\|\theta_T\|^2} \;(1), \quad \lambda_{min} = \min_{\theta_T \in \mathbb{R}^s, |T|\le s} \frac{\|A_T\theta_T\|^2}{\|\theta_T\|^2}$$

∴ $\lambda_{max}, \lambda_{min}$ are maximum and minimum Eigen values of $(A_T)^T A_T$ over all sets T.

Consider $A^T A = \begin{bmatrix} \vec{A}_1^T \\ \vec{A}_2^T \\ \vdots \\ \vec{A}_n^T \end{bmatrix}_{n\times m} \begin{bmatrix} \vec{A}_1 & \vec{A}_2 & \vec{A}_3 & \cdots & \vec{A}_n \end{bmatrix}_{m\times n}$

$$= \begin{bmatrix} 1 & \vec{A}_1^T\vec{A}_2 & \vec{A}_1^T\vec{A}_3 & \cdots & \vec{A}_1^T\vec{A}_n \\ \vec{A}_2^T\vec{A}_1 & 1 & \vec{A}_2^T\vec{A}_3 & \cdots & \vec{A}_2^T\vec{A}_n \\ \vdots & & & & \\ \vec{A}_n^T A_1 & \vec{A}_n^T\vec{A}_2 & \vec{A}_n^T\vec{A}_3 & \cdots & \cancel{\vec{A}_n^T A_n}\, 1 \end{bmatrix}$$

each off-diagonal entry atmost μ

As per (1) consider $s\times s$ submatrix indexed by $|T|$

wlog consider ~~λmax~~ (A) λ_{min} is an eigen value of these matrix if $1-\lambda_{min} > \lambda_{max}-1$

else (B) λ_{max} is an eigen value.

Now applying Gershgorin Theorem (Slide 104)

we get Eigen value is contained in

$$|x-1| \le \mu(s-1) \qquad \text{(C)}$$

~~x-1 can be is max of (1~~

~~x / is~~

λ_{min} satisfies (C) if (A) holds else λ_{max}

we get using (0) $\quad \delta_s \le \mu(s-1)$