$P(f|y,R)$

4 $y_i \sim \text{Poisson}(I_0 \exp(-R^i \vec{f}))$ denote $\lambda_i := I_0 \exp(-R^i \vec{f})$

Given y_i probability of its generation is $\dfrac{e^{-\lambda_i} (\lambda_i)^{y_i}}{y!}$

Considering all elements in $\vec{y}$, probability of its generation is.

$$p = P(f|y,R) = \prod_{i=1}^{m} \frac{e^{-\lambda_i}(\lambda_i)^{y_i}}{y!}$$

R^i : i^{th} row

max p is same as min $-\log p$

$\therefore$ which is

$$-\sum_{i=1}^{m} \left[-\lambda_i + y_i \log \lambda_i - y_i! \right]$$

Since y_i is known we can ignore $y_i!$ (constant)

$$\Rightarrow \sum_{i=1}^{m} \left[\lambda_i - y_i \log \lambda_i \right]$$

$$\Rightarrow \sum_{i=1}^{m} \left[I_0 \exp(-R^i \vec{f}) - y_i \log \left(I_0 e^{-R^i \vec{f}} \right) \right]$$

$$\Rightarrow \sum_{i=1}^{m} \left[I_0 \exp(-R^i \vec{f}) + y_i (R^i \vec{f}) \right]$$

$-y_i \log I_0$ constant

~~$\min_f$~~

Good estimate is $\vec{f}' = \underset{\vec{f}}{\text{argmin}} \left(\sum_{i=1}^{m} \left(I_0 e^{-R^i \vec{f}} + y_i (R^i \vec{f}) \right) \right)$

b) $y = y_{original} + \eta$ $\lambda_i = I_0 \exp(-R^i \vec{f})$

$y_{original} \sim \text{Poisson}(I_0 \exp(-Rf))$

$\eta \sim N(0, \sigma^2)$

$\cancel{P(f \mid y, R) = \sum}$

$$P(y_i \mid f, R) = \sum_{n=0}^{\infty} \frac{e^{-\lambda_i} (\lambda_i)^n}{n!} \frac{e^{-\frac{(y_i - n)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}$$

$$\max_f P(y \mid f, R)$$

$$\equiv \min_f -\log P(y \mid f, R)$$

$$\equiv \min_f -\log \prod_{i=1}^{m} P(y_i \mid f, R)$$ By independence

$$= \min_f -\sum_{i=1}^{m} \log P(y_i \mid f, R)$$

$$= \min_f -\sum_i \log \left(\sum_{n=0}^{\infty} \frac{e^{-I_0 \exp(-R^i \vec{f})} \left(I_0 e^{-R^i \vec{f}}\right)^n}{n!} \frac{e^{-\frac{(y_i - n)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma} \right)$$

argmin of above would give estimate of f