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1 Paper Details

Title Group-Based Sparse Representation for Image Restoration

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Problem Description is to improve the traditional patch-based sparse representation of natural images used in image restoration which considers each patch independently, with an *efficient* (low complexity) method exploiting both the *local sparsity and the nonlocal similarity of patches.* This novel is concept called group-based sparse representation (GSR). An *algorithm* is also needed to be developed to solve the resultant GSR-driven ℓ_0 minimization problem for image restoration.

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

Here, $\boldsymbol{x}$ is the original image, $\boldsymbol{y}$ is the measurement vector, $\boldsymbol{H}$ is an $m \times n$ measurement matrix such that m is much smaller than n and $\boldsymbol{n}$ is additive Gaussian noise.

Sensing matrix is a Gaussian random projection matrix applied to the image at block level (block-based CS). It is constructed by designing a measurement matrix (32×32) for each block independently (Gaussian IID) and then concatenating all these matrices.

Sparsifying basis is a novel sparse representation called group-based sparse representation (GSR) which first involves group construction where each group $\boldsymbol{G_k}$ contains all the patches with similar structures as measured my euclidean distance. Then each group is represented accurately by a self-adaptive learning dictionary $\boldsymbol{D_{G_k}}$. Concatenating all such dictionaries gives us $\boldsymbol{D_G}$, the sparsifying basis.

CS-based Estimator This results in the formulation of the following minimization problem for the GSR image restoration scheme.

$$\hat{\boldsymbol{\alpha}}_{\boldsymbol{G}} = \underset{\boldsymbol{\alpha_G}}{\operatorname{argmin}} \frac{1}{2}\|\boldsymbol{H}\boldsymbol{D_G} \circ \boldsymbol{\alpha_G} - \boldsymbol{y}\|_2^2 + \lambda\|\boldsymbol{\alpha_G}\|_0 \tag{2}$$

where the original image is $\boldsymbol{x} = \boldsymbol{D_G} \circ \boldsymbol{\alpha_G}$ and the reconstructed image is $\hat{\boldsymbol{x}} = \boldsymbol{D_G} \circ \hat{\boldsymbol{\alpha_G}}$. $\boldsymbol{D_G}$ is the sparsifying basis and $\boldsymbol{\alpha_G}$ is the concatenation of all sparse codes $\boldsymbol{\alpha_{G_k}}$ of the corresponding group $\boldsymbol{G_k}$ over dictionary

$\boldsymbol{D}_{\boldsymbol{G}_{\boldsymbol{k}}}$. The $\circ$ operator essentially reconstructs the image vector by using the information $\boldsymbol{D}_{\boldsymbol{G}}$ and $\boldsymbol{\alpha}_{\boldsymbol{G}}$.

To solve this problem, the estimator used is Split Bregman Iteration (SBI) which splits the minimization problem into two sub-problems $(\boldsymbol{u}, \boldsymbol{\alpha}_{\boldsymbol{G}})$, where after $\boldsymbol{u}$ is intended to be $\boldsymbol{D}_{\boldsymbol{G}} \circ \boldsymbol{\alpha}_{\boldsymbol{G}}$.

We start by initialising $\boldsymbol{b}^0$, $\boldsymbol{u}^0$ and $\boldsymbol{\alpha}_{\boldsymbol{G}}^0$ to $\mathbf{0}$ and then iteratively compute the following until convergence

$$\boldsymbol{u}^{t+1} = \underset{\boldsymbol{u}}{\operatorname{argmin}} \frac{1}{2}\|\boldsymbol{H}\boldsymbol{u} - \boldsymbol{y}\|_2^2 + \frac{\mu}{2}\|\boldsymbol{u} - \boldsymbol{D}_{\boldsymbol{G}} \circ \boldsymbol{\alpha}_{\boldsymbol{G}}^t - \boldsymbol{b}^t\|_2^2 \tag{3}$$

$$\boldsymbol{\alpha}_{\boldsymbol{G}}^{t+1} = \underset{\boldsymbol{\alpha}_{\boldsymbol{G}}}{\operatorname{argmin}} \lambda\|\boldsymbol{\alpha}_{\boldsymbol{G}}\|_0 + \frac{\mu}{2}\|\boldsymbol{u}^{t+1} - \boldsymbol{D}_{\boldsymbol{G}} \circ \boldsymbol{\alpha}_{\boldsymbol{G}} - \boldsymbol{b}^t\|_2^2 \tag{4}$$

$$\boldsymbol{b}^{t+1} = \boldsymbol{b}^t - (\boldsymbol{u}^{t+1} - \boldsymbol{D}_{\boldsymbol{G}} \circ \boldsymbol{\alpha}_{\boldsymbol{G}}^{t+1}) \tag{5}$$

where μ is a positive hyperparameter.