Polynomial Roots

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Here are formulas of roots of polynomial functions:

1. Solution of ax + b is:

 $-\frac{b}{a}$

(1)

(2)

(3)

2. Solution of $ax^2 + bx + c$ is:

 $r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \equiv \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$

3. Solution of $ax^3 + bx^2 + cx + d$ is:

$$r_{1} = -\frac{1}{3a} \left[b + \sqrt[3]{\frac{2b^{3} - 9abc + 27a^{2}d \pm \sqrt{(2b^{3} - 9abc + 27a^{2}d)^{2} - 4(b^{2} - 3ac)^{3}}}{2}} + \frac{b^{2} - 3ac}{\sqrt[3]{\frac{2b^{3} - 9abc + 27a^{2}d \pm \sqrt{(2b^{3} - 9abc + 27a^{2}d)^{2} - 4(b^{2} - 3ac)^{3}}}{2}} \right]$$

$$r_{2} = -\frac{1}{3a} \left[b + \left(\frac{-1 + \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^{3} - 9abc + 27a^{2}d \pm \sqrt{(2b^{3} - 9abc + 27a^{2}d)^{2} - 4(b^{2} - 3ac)^{3}}}{2}} + \frac{b^{2} - 3ac}{\left(\frac{-1 + \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^{3} - 9abc + 27a^{2}d \pm \sqrt{(2b^{3} - 9abc + 27a^{2}d)^{2} - 4(b^{2} - 3ac)^{3}}}{2}} \right]$$

$$r_{3} = -\frac{1}{3a} \left[b + \left(\frac{-1 - \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^{3} - 9abc + 27a^{2}d \pm \sqrt{(2b^{3} - 9abc + 27a^{2}d)^{2} - 4(b^{2} - 3ac)^{3}}}{2}} + \frac{b^{2} - 3ac}{\left(\frac{-1 + \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^{3} - 9abc + 27a^{2}d \pm \sqrt{(2b^{3} - 9abc + 27a^{2}d)^{2} - 4(b^{2} - 3ac)^{3}}}} \right]$$

4. Solution of $x^4 + ax^3 + bx^2 + cx + d$ is:

$$r_1 = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3}} + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)^{\frac{1}{3}} \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4\left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4\left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} \\ - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3}} + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4\left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} - \frac{-a^3 + 4ab - 8c}{4\sqrt{\frac{a^2}{4} - \frac{2b}{3}} + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} - \frac{-a^3 + 4ab - 8c}{4\sqrt{\frac{a^2}{4} - \frac{2b}{3}} + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}} - \frac{-a^3 + 4ab - 8c}{4\sqrt{\frac{a^2}{4} - \frac{2b}{3}} + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}}} - \frac{-a^3 + 4ab - 8c}{4\sqrt{\frac{a^2}{4} - \frac{2b}{3}} + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)^{\frac{1}{3}}}} + \frac{a^3 + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3}} + \frac{a^3 + \frac{2^{\frac{1}{2}} \left(b^2 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd}}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3}}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3}} + \frac{a^3 + \frac{2^{\frac{1}{2}} \left(b^3 - 3ac + 12d\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^3 + \frac{a^3 + a^3 + a^$$

$$r_{2} = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^{2}}{4} - \frac{2b}{3}} + \frac{2\frac{1}{3}(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}} + \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{a^{2}}{2} - \frac{4b}{3}} + \frac{2\frac{1}{3}(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{1}(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{1}(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{1}(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{2} - 2b}}}} + \sqrt{\frac{a^{3} + 4ab - 8c}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{3} - 2ab + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{3} - 2ab + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{3} - 2ab + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{3}}}}}} - \sqrt{\frac{a^{3} + 4ab - 8c}}{4\sqrt{\frac{a^{3} - 2ab + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{3$$

$$r_{3} = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}} + \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}} - \sqrt{\frac{a^{2}}{2} - \frac{4b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{$$

$$r_{4} = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}})^{\frac{1}{3}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}{54}\right)^{\frac{1}{3}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}\right)^{\frac{1}{3}} + \frac{-a^{3} + 4ab - 8c}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2b(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2b(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}^{\frac{1}{3}}} + \frac{(ab^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2b(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}{4(b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}^{\frac{1}{3}}}}}}$$

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