Polynomial Roots

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September 8, 2020

Here are formulas of roots of polynomial functions:

1. Solution of ax + b is:

(1)

(2)

(3)

2. Solution of $ax^2 + bx + c$ is:

 $r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \equiv \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$

3. Solution of $ax^3 + bx^2 + cx + d$ is:

$$c_1 = -\frac{1}{3a} \left[b + \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d \pm \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} + \frac{b^2 - 3ac}{\sqrt[3]{\frac{2b^3 - 9abc + 27a^2d \pm \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}}} \right]$$

$$c_2 = -\frac{1}{3a} \left[b + \left(\frac{-1 + \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d \pm \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} + \frac{b^2 - 3ac}{\left(\frac{-1 + \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d \pm \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \right]$$

$$c_3 = -\frac{1}{3a} \left[b + \left(\frac{-1 - \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d \pm \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} + \frac{b^2 - 3ac}{\left(\frac{-1 + \sqrt{-3}}{2} \right) \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d \pm \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \right]$$

4. Solution of $x^4 + ax^3 + bx^2 + cx + d$ is:

$$r_1 = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3}} + \frac{2^{\frac{1}{4}}(b^2 - 3ac + 12d)}{3(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}})^{\frac{1}{4}} \\ - \frac{1}{2} \sqrt{\frac{a^2}{2} - \frac{4b}{3}} + \frac{2^{\frac{1}{4}}(b^2 - 3ac + 12d)}{3(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}} \\ - \frac{a^3}{4} - \frac{a^3}{3} + \frac{2^{\frac{1}{4}}(b^2 - 3ac + 12d)}{3(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}} \\ - \frac{a^3}{4} - \frac{a^3}{3} - \frac{2^{\frac{1}{4}}(b^2 - 3ac + 12d)}{3(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}} \\ - \frac{a^3}{4} - \frac{a^3}{3} - \frac{2^{\frac{1}{4}}(b^2 - 3ac + 12d)}{3(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}} \\ - \frac{a^3}{4} - \frac{a^3}{3} - \frac{2^{\frac{1}{4}}(b^2 - 3ac + 12d)}{3(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^{\frac{3}{4}}}} \\ - \frac{a^3}{4} - \frac{a^3}{3} - \frac{a^3}{4} - \frac{a^3}{3} - \frac{a^3}{4} - \frac{a^3}{4} - \frac{a^3}{3} - \frac{a^3}{4} - \frac{a^3}{4} - \frac{a^3}{3} - \frac{a^3}{4} - \frac{a^$$

$$r_{2} = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}\right)^{\frac{1}{3}}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}\right)^{\frac{1}{3}}} - \frac{1}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}\right)^{\frac{1}{3}}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}\right)^{\frac{1}{3}}}}} + \frac{(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}\right)^{\frac{1}{3}}}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}\right)^{\frac{1}{3}}}}}}$$

$$r_{3} = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}(b^{2} - 3ac + 12d)}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}} + \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{a^{2}}{2} - \frac{4b}{3} + \frac{2^{\frac{1}{3}(b^{2} - 3ac + 12d)}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}})^{\frac{1}{3}}}} - \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}} + \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3} + (2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}{4\sqrt{\frac{a^{2} - \frac{2b}{3} + \frac{2a^{2} - 2b^{2}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}} + \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd}}{4\sqrt{\frac{a^{2} - 2b^{2}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}} + \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd}}{4\sqrt{\frac{a^{2} - 2b^{2}}{3(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd)^{2}}}}}} + \sqrt{\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4(b^{2} - 3ac + 12d)^{3}}}}}{4\sqrt{\frac{a^{2} - 2b^{2}}{3(2b^{3}$$

$$r_{4} = -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^{2} - 3ac + 12d)}{3\left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}\right)^{\frac{1}{3}}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}\right)^{\frac{1}{3}}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}\right)^{\frac{1}{3}}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}\right)^{\frac{1}{3}}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(2^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(2^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}}} + \left(\frac{2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}}}{4\sqrt{\frac{a^{2}}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(2^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd + \sqrt{-4\left(b^{2} - 3ac + 12d\right)^{3} + \left(2b^{3} - 9abc + 27c^{2} + 27a^{2}d - 72bd\right)^{2}}}}}} \right)^{\frac{1}{3}}}}$$