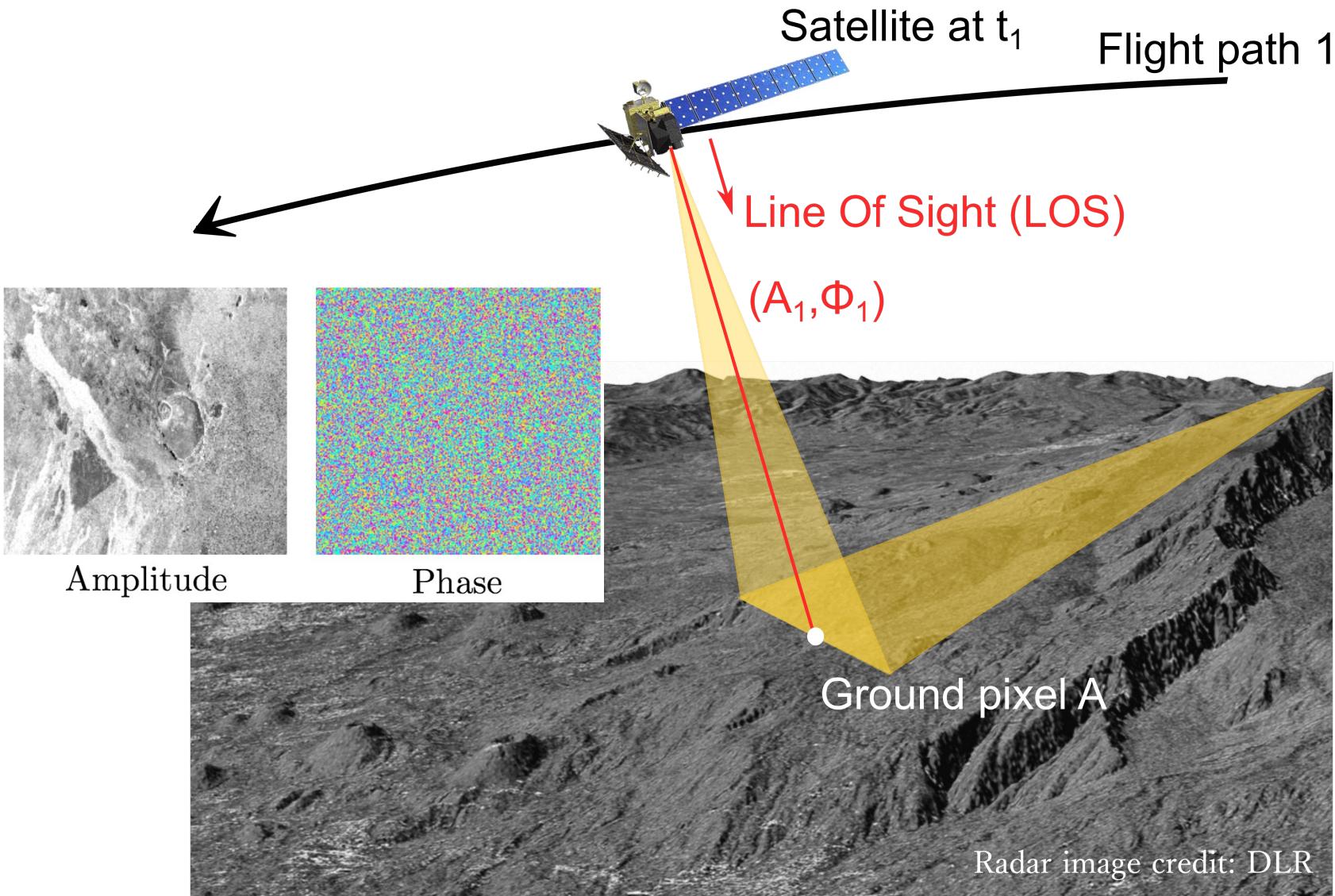


Synthetic Aperture Radar Interferometry

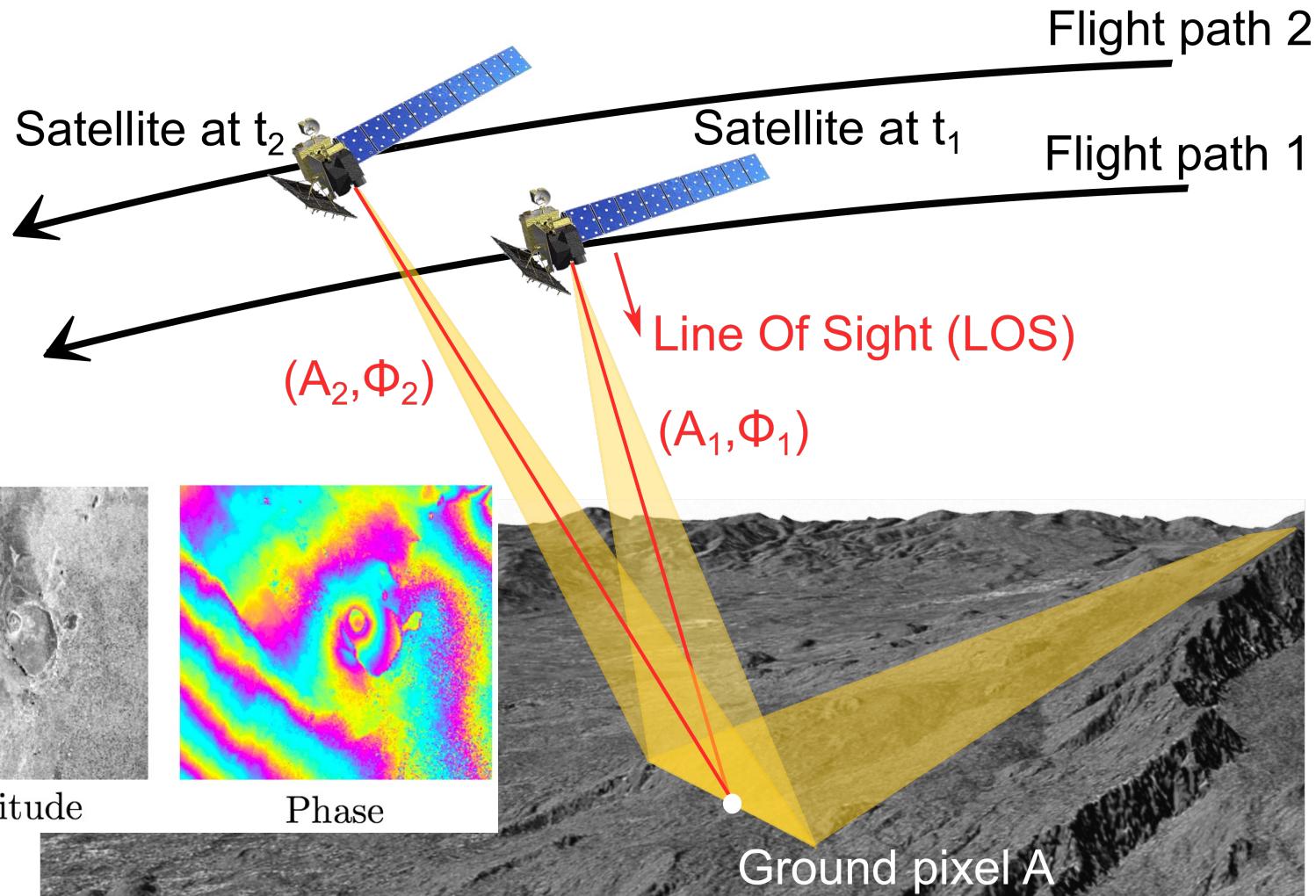
Ann Chen
The University of Texas at Austin

* This lecture was originally developed by Paul A Rosen and Scott Hensley.

Synthetic Aperture Radar (SAR)



Interferometric Synthetic Aperture Radar (InSAR)



Radar image credit: DLR

SAR interferometry

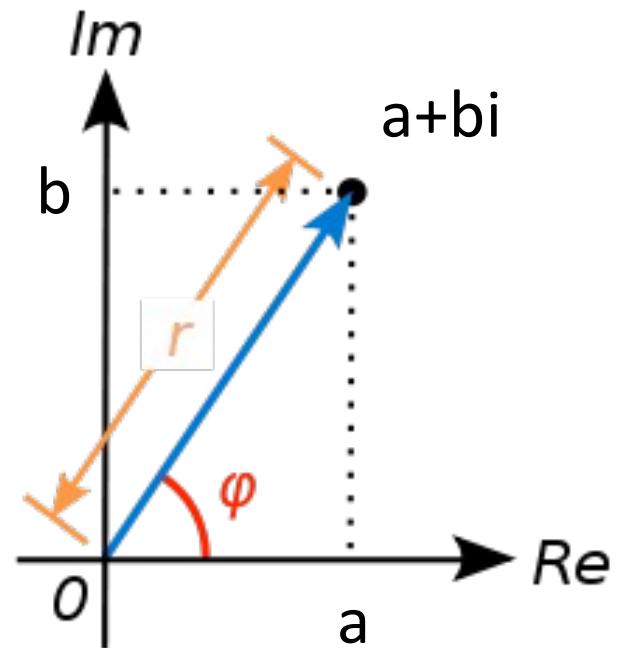
- Complex numbers

$$z = r e^{i\varphi}$$

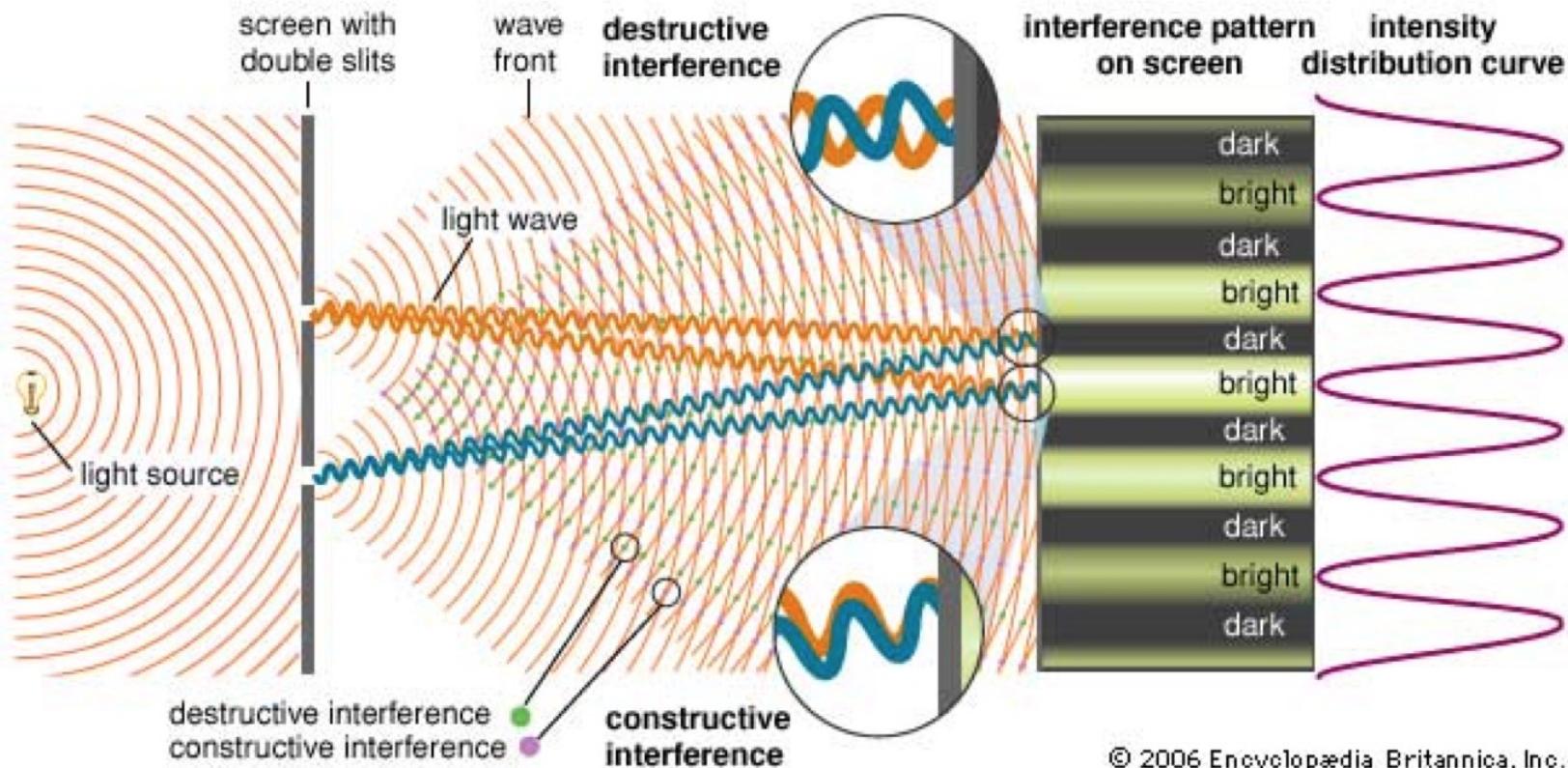
$$= r(\cos \varphi + i \sin \varphi)$$

- Interferogram formation

$$\begin{aligned} z_1 z_2^* &= r_1 e^{i\varphi_1} r_2 e^{-i\varphi_2} \\ &= r_1 r_2 e^{i(\varphi_1 - \varphi_2)} \end{aligned}$$



Wave Interference

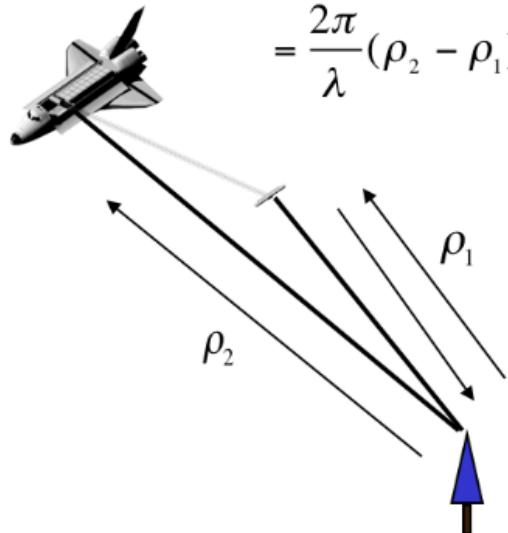


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The relationship of phase to propagation distance

Classic

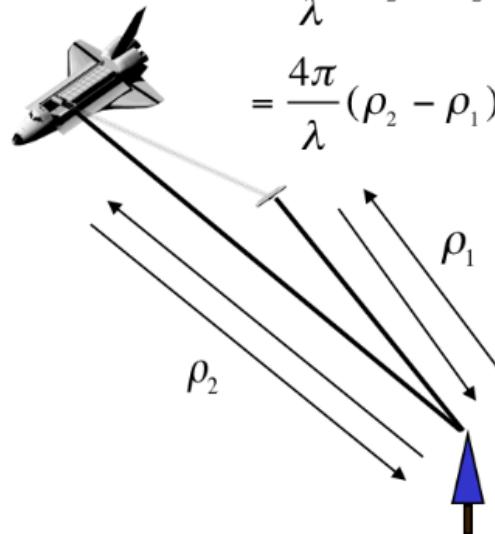
$$\Delta\phi = \frac{2\pi}{\lambda}(\rho_2 + \rho_1) - \frac{2\pi}{\lambda}(\rho_1 + \rho_1)$$
$$= \frac{2\pi}{\lambda}(\rho_2 - \rho_1)$$



$$\Delta\phi = \frac{2\pi p}{\lambda} \delta\rho,$$
$$p = 1$$

Ping-Pong

$$\Delta\phi = \frac{2\pi}{\lambda}(\rho_2 + \rho_2) - \frac{2\pi}{\lambda}(\rho_1 + \rho_1)$$
$$= \frac{4\pi}{\lambda}(\rho_2 - \rho_1)$$

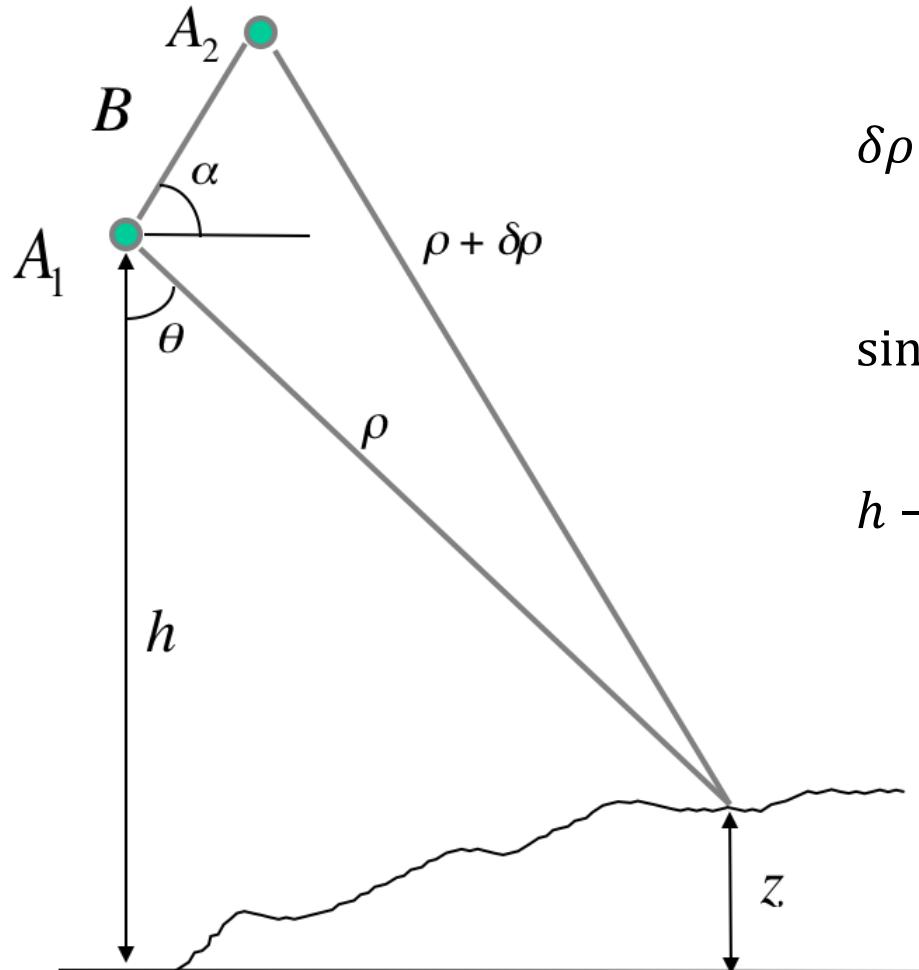


$$\Delta\phi = \frac{2\pi p}{\lambda} \delta\rho,$$
$$p = 2$$

Classic: one antenna transmits, and both receives.

Ping pong: each antenna transmits and receives its own echo.

Interferometry for topography



$$\delta\rho = -\frac{\lambda}{2\pi p}(\phi_2 - \phi_1) = -\frac{\lambda}{2\pi p}\Delta\phi$$

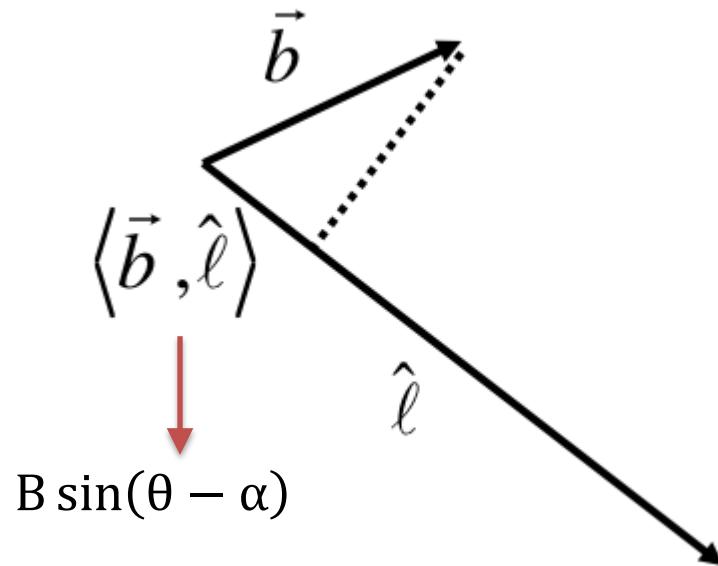
$$\sin(\theta - \alpha) = \frac{\rho^2 + B^2 - (\rho + \delta\rho)^2}{2\rho B}$$

$$h - z = \rho \cos \theta$$

We can measure $\Delta\phi$ (unwrapped) from radar. Given h, B, α and ρ , we can solve for θ , which gives us z at every pixel location.

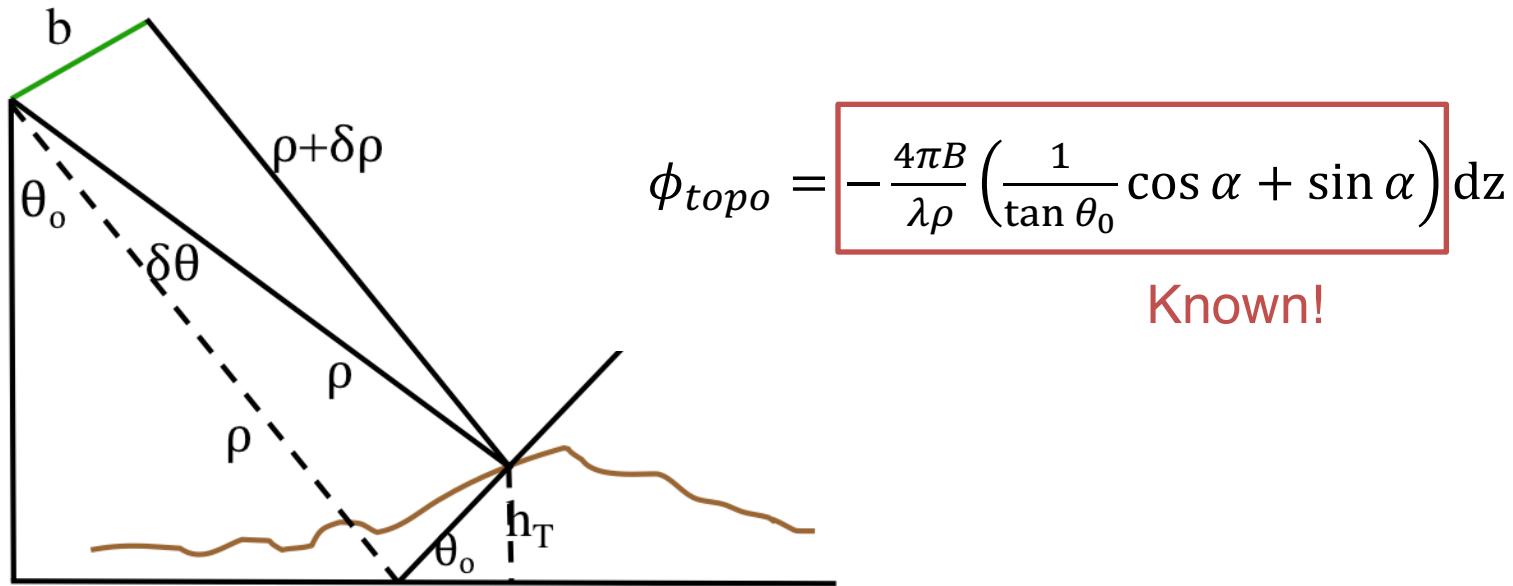
Interferometry for topography

- The so-called parallel-ray approximation:



Solution for topography from interferometric phase

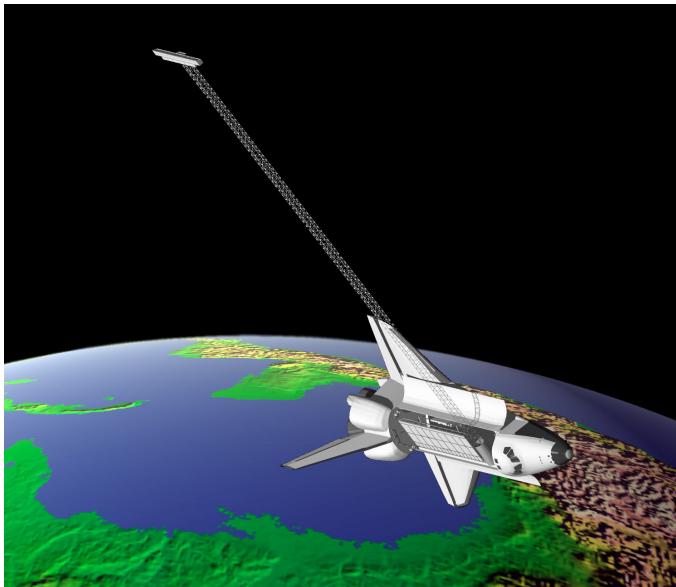
- After subtracting a flat earth fringe:



The constant-phase contours will appear at the constant height on the imaged topography. The phase will repeat every 2π radians, so therefore each “fringe” (one phase cycle) corresponds to a constant height difference (the ambiguity height).

Implementation options

- A single spacecraft with two displaced antennas
- Two spacecraft, each with a synthetic aperture radar, flying in formation to form the interferometer baseline.



The Shuttle Radar
Topography Mission

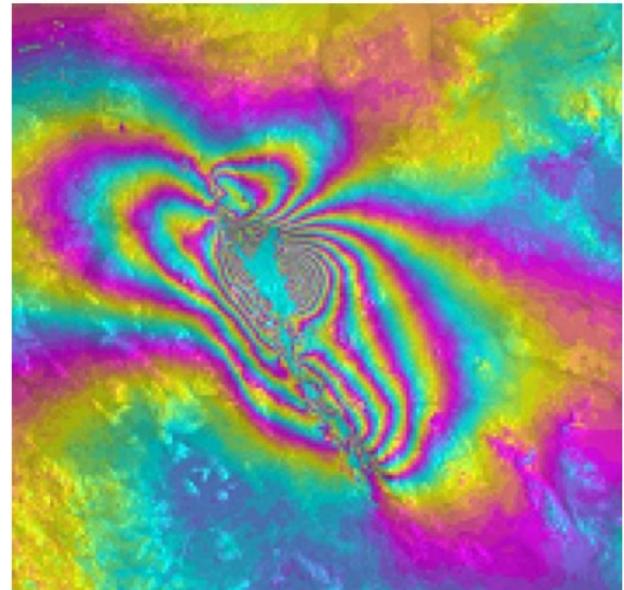
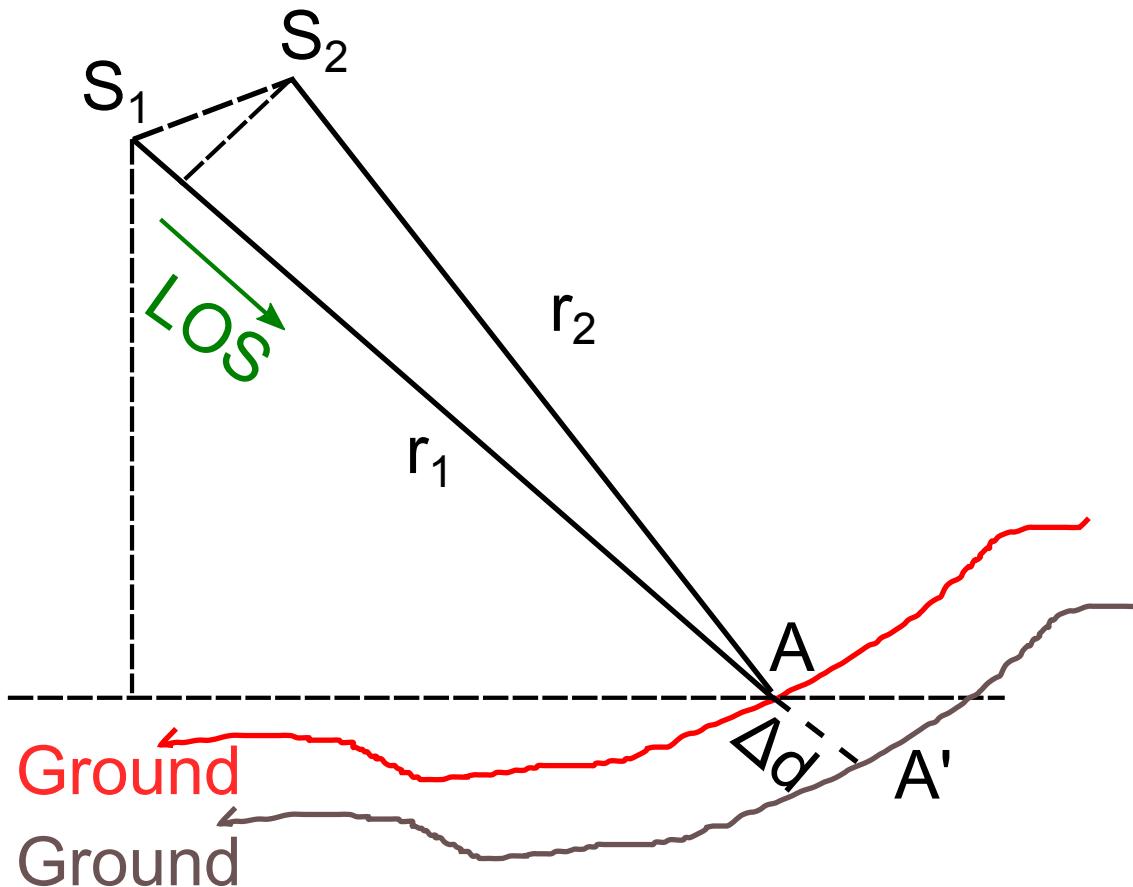


**TanDEM-X (TerraSAR-X add-on for
Digital Elevation Measurement)**

Technology for generation of DEM

- Optical-stereo instrumentation
- Laser profiling instruments
- Radar interferometry

Interferometry for surface deformation



$$\phi_2 - \phi_1 = \phi_{topo} + \frac{4\pi}{\lambda} \Delta d$$

λ : radar wavelength
 Δd : LOS deformation

Interferometry Sensitivities

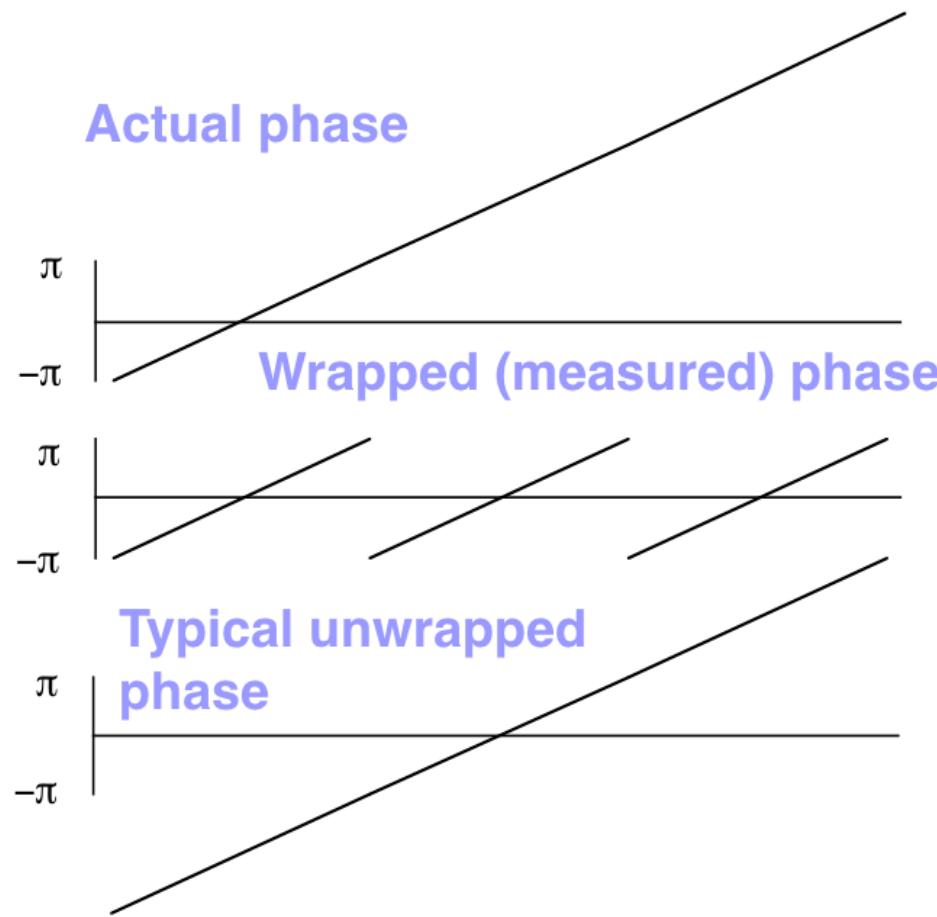
- Topographic Sensitivity (meter level topography)

$$\frac{\partial \Delta\phi}{\partial h} = \frac{2\pi pb \cos(\theta - \alpha)}{\lambda \rho \sin \theta}$$

- Displacement Sensitivity (cm-mm level deformation)

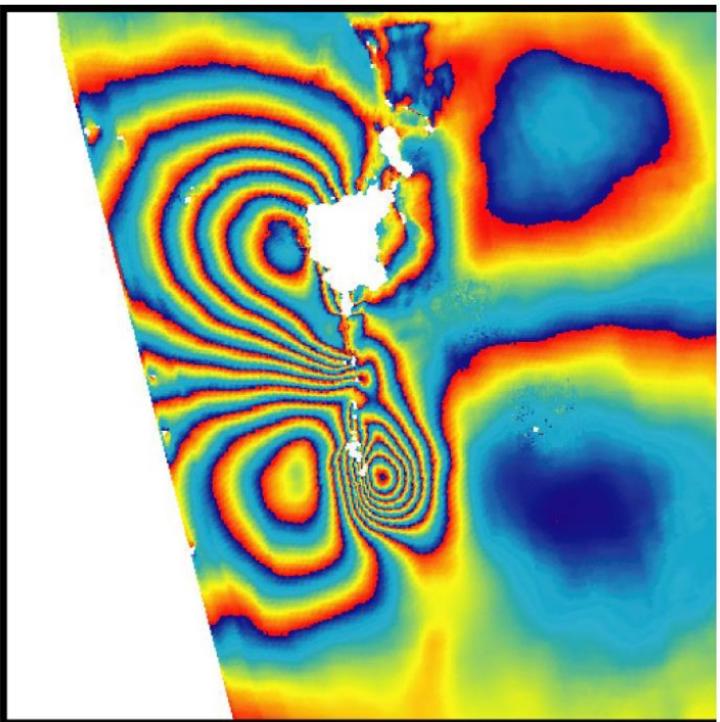
$$\frac{\partial \Delta\phi}{\partial \Delta\rho} = \frac{4\pi}{\lambda}$$

Phase unwrapping

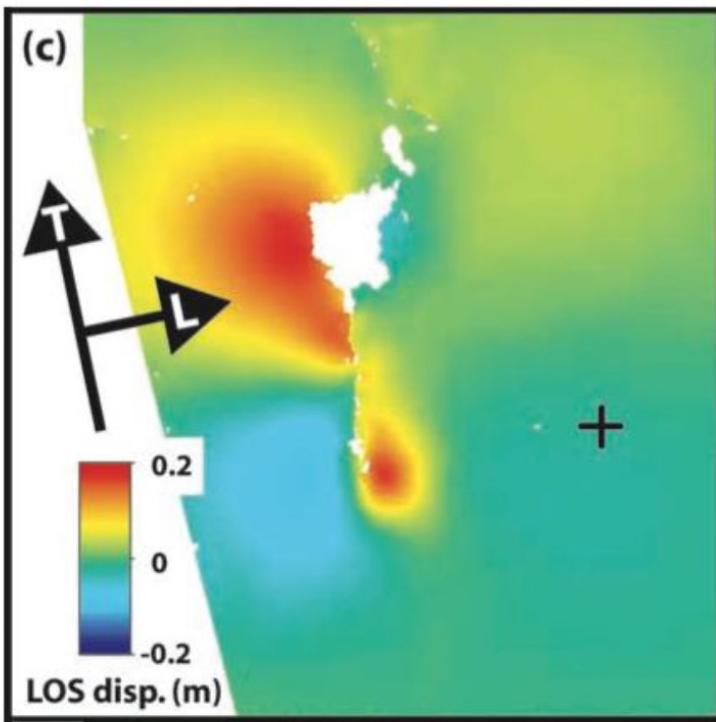


Phase unwrapping

Wrapped



Unwrapped

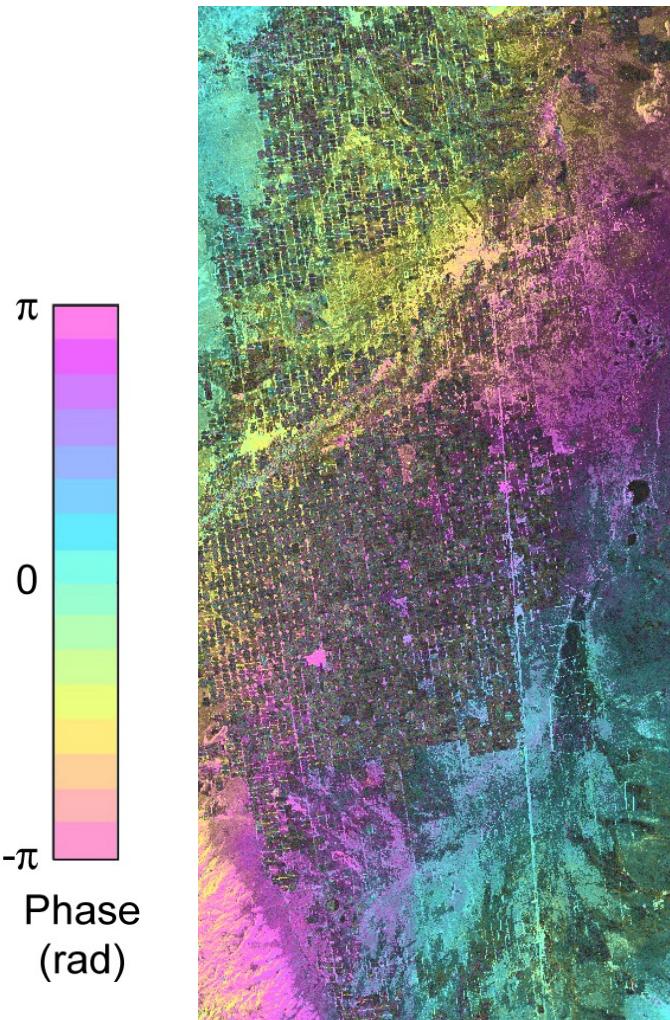


Error Sources in LOS measurements

$$\Delta\varphi = \frac{4\pi}{\lambda} \Delta d_{\text{LOS}} + \Delta\varphi_{\text{dem}} + \Delta\varphi_{\text{atm}} + \Delta\varphi_{\text{iono}} + \Delta\varphi_{\text{orb}} + \Delta\varphi_{\text{decor}} + \Delta\varphi_{\text{unwarp}} + \Delta\varphi_n$$

- Orbital errors
- DEM errors
- Ionospheric and tropospheric errors
- Decorrelation and phase unwrapping errors

Orbital errors



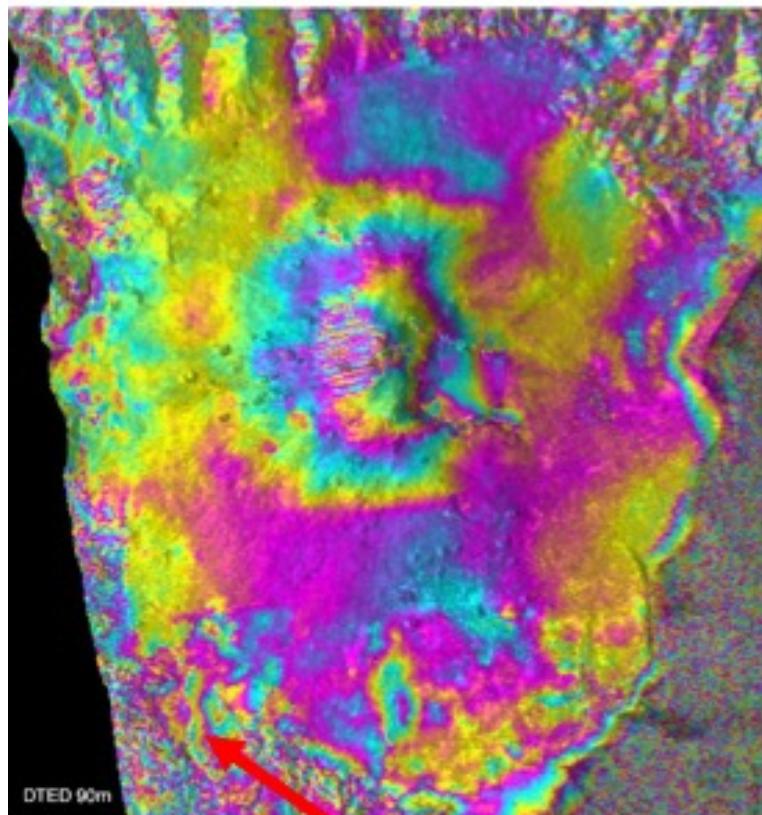
An L-band interferogram before (left) and after (right) the orbital ramp removal.

The negative LOS motion (in light yellow) is associated with surface uplift due to aquifer recharge.

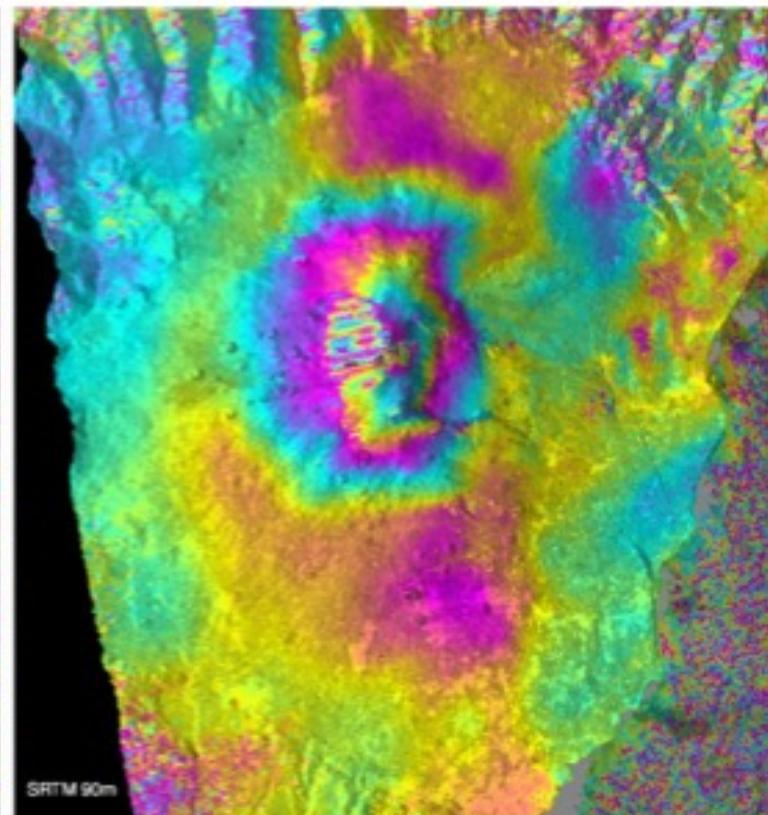
The spatial variation of ionospheric and tropospheric delays may also cause a phase ramp.

DEM errors

Poor DEM

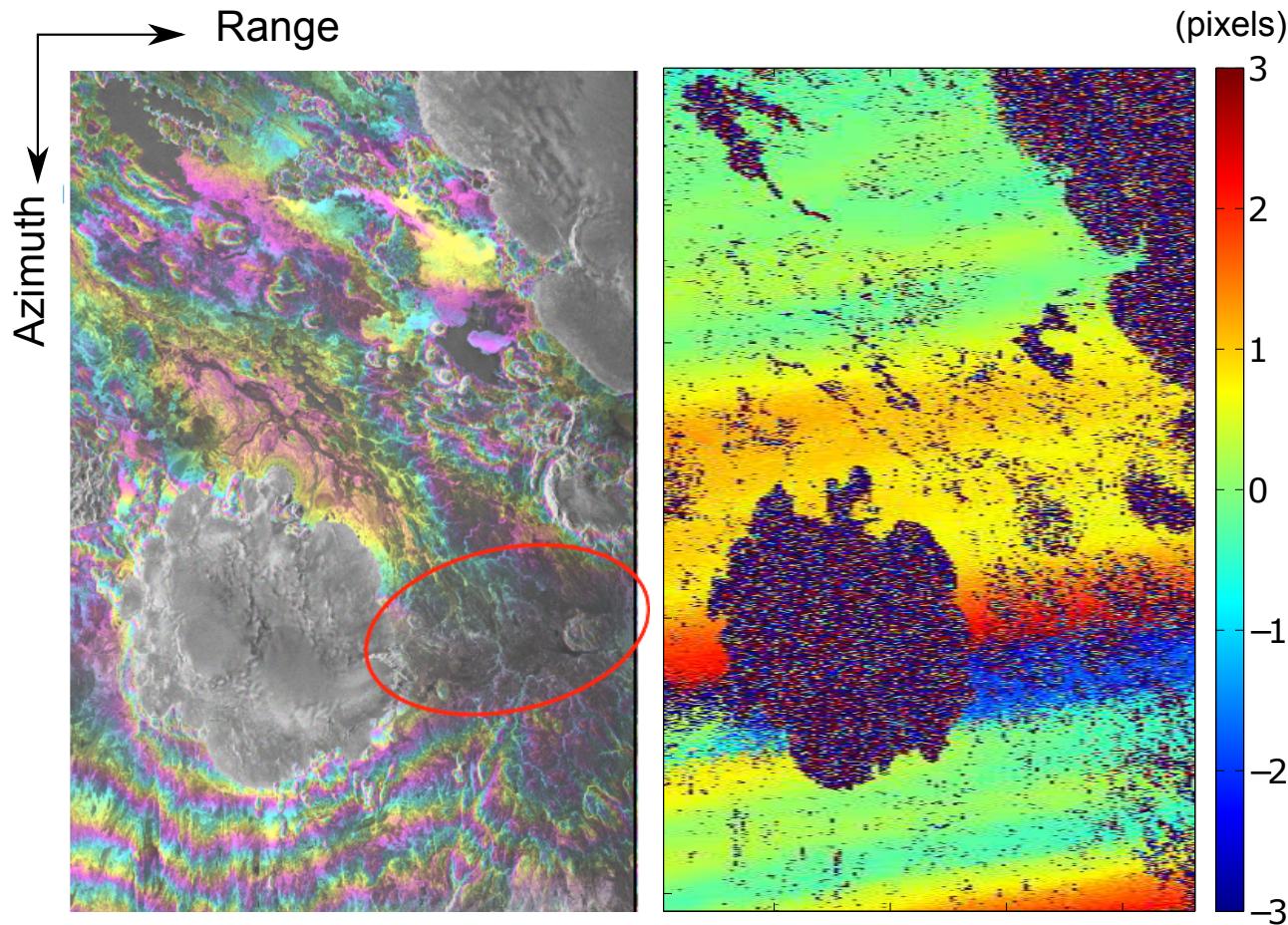


SRTM DEM



Result of DEM Error

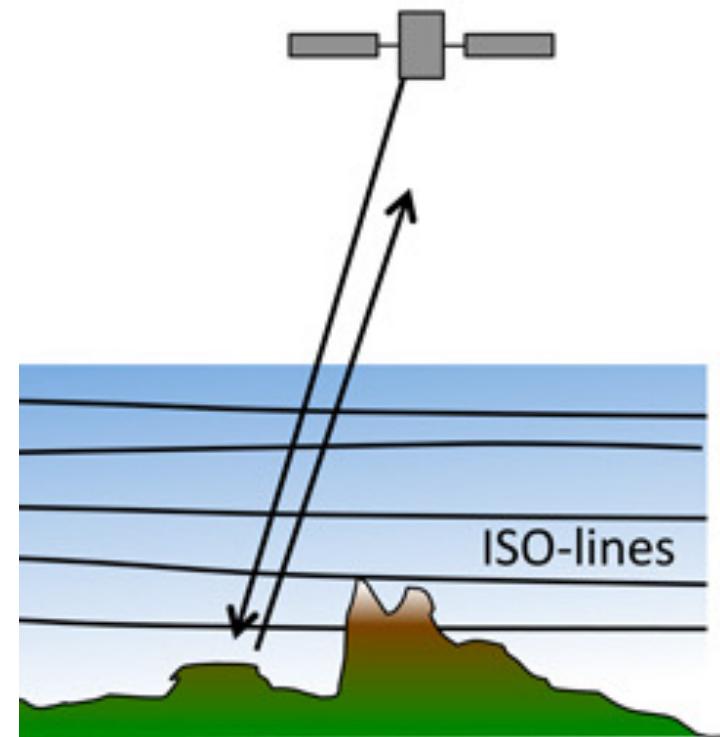
Ionospheric artifacts



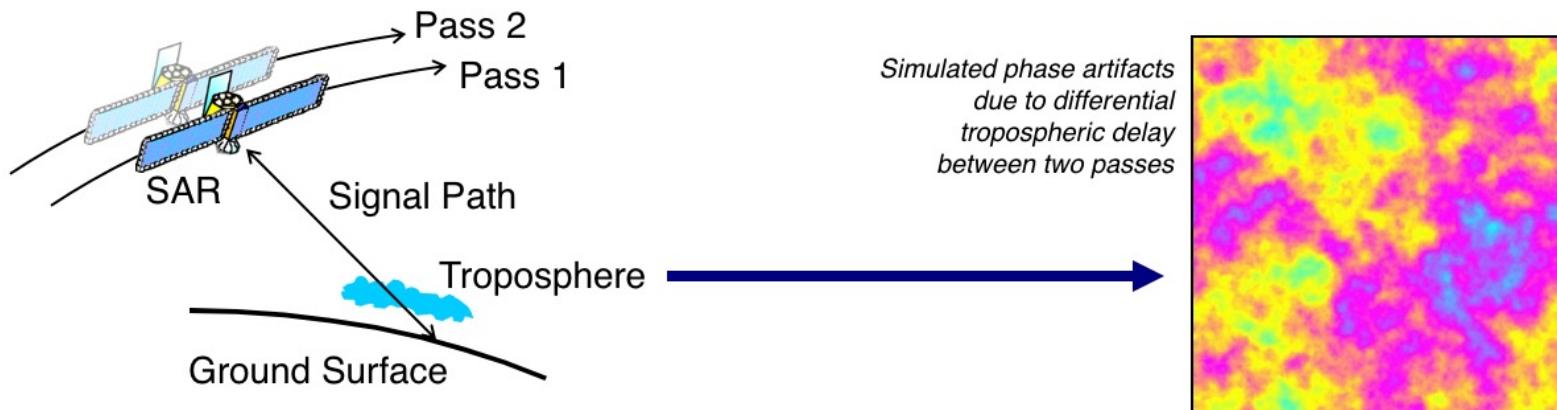
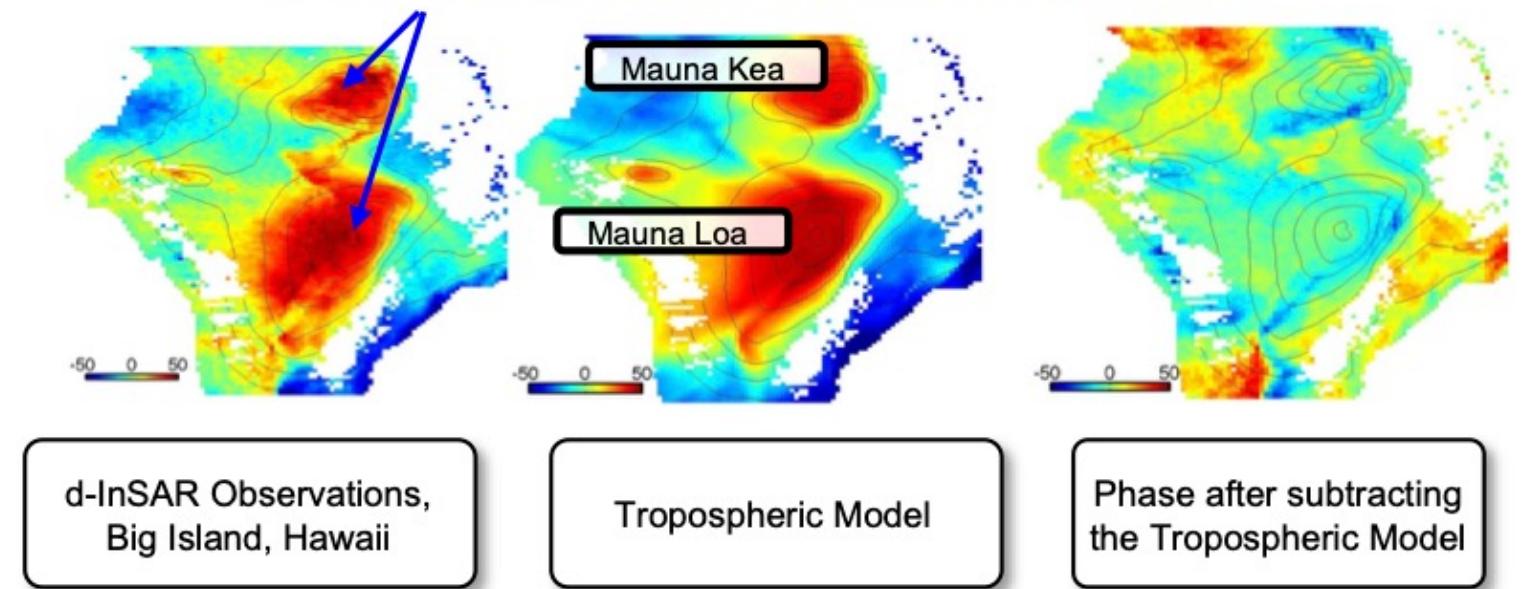
An L-band Iceland interferogram (Left) and the azimuth misregistration measured in pixels due to a local ionospheric TEC gradient.

Tropospheric noise

- Introduced by variation in pressure, temperature, and water vapor in troposphere.
- Affect all radar wavelengths in the same way
- Up to 15 cm
- May contain a component that is correlated with topography
- Often contain a component that is random in time.

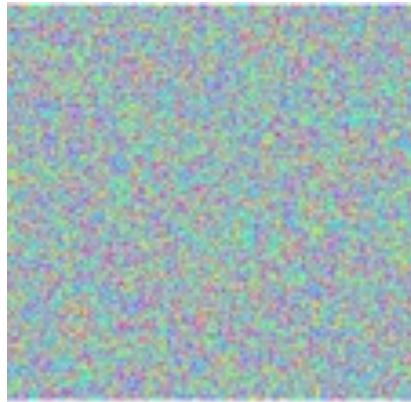


Atmospheric phase correlated with Topography

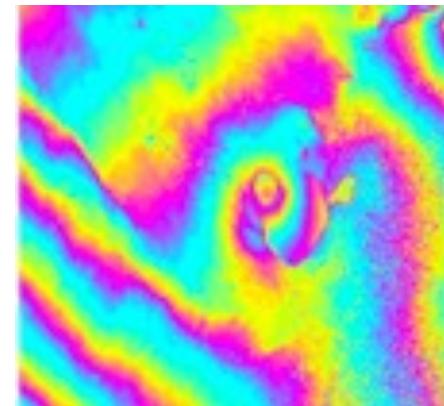


Correlation

- An important measure of interferogram quality is the correlation of the two SAR images. This quantity varies from 0 to 1.



Correlation equals 0
(decorrelated), phase
measurements are
completely random.



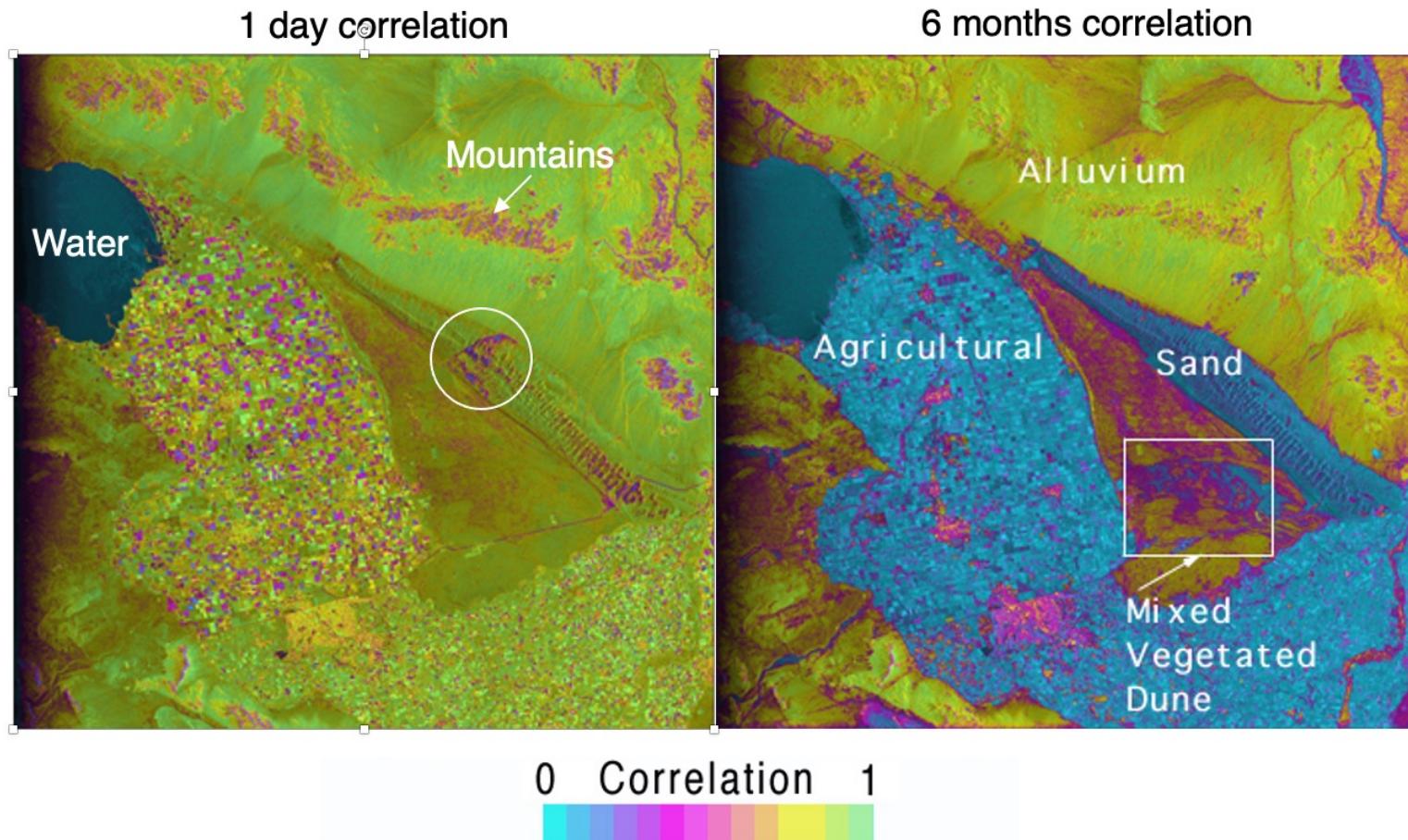
High correlation, phase
measurements contains high
quality topographic and
deformation information

Correlation

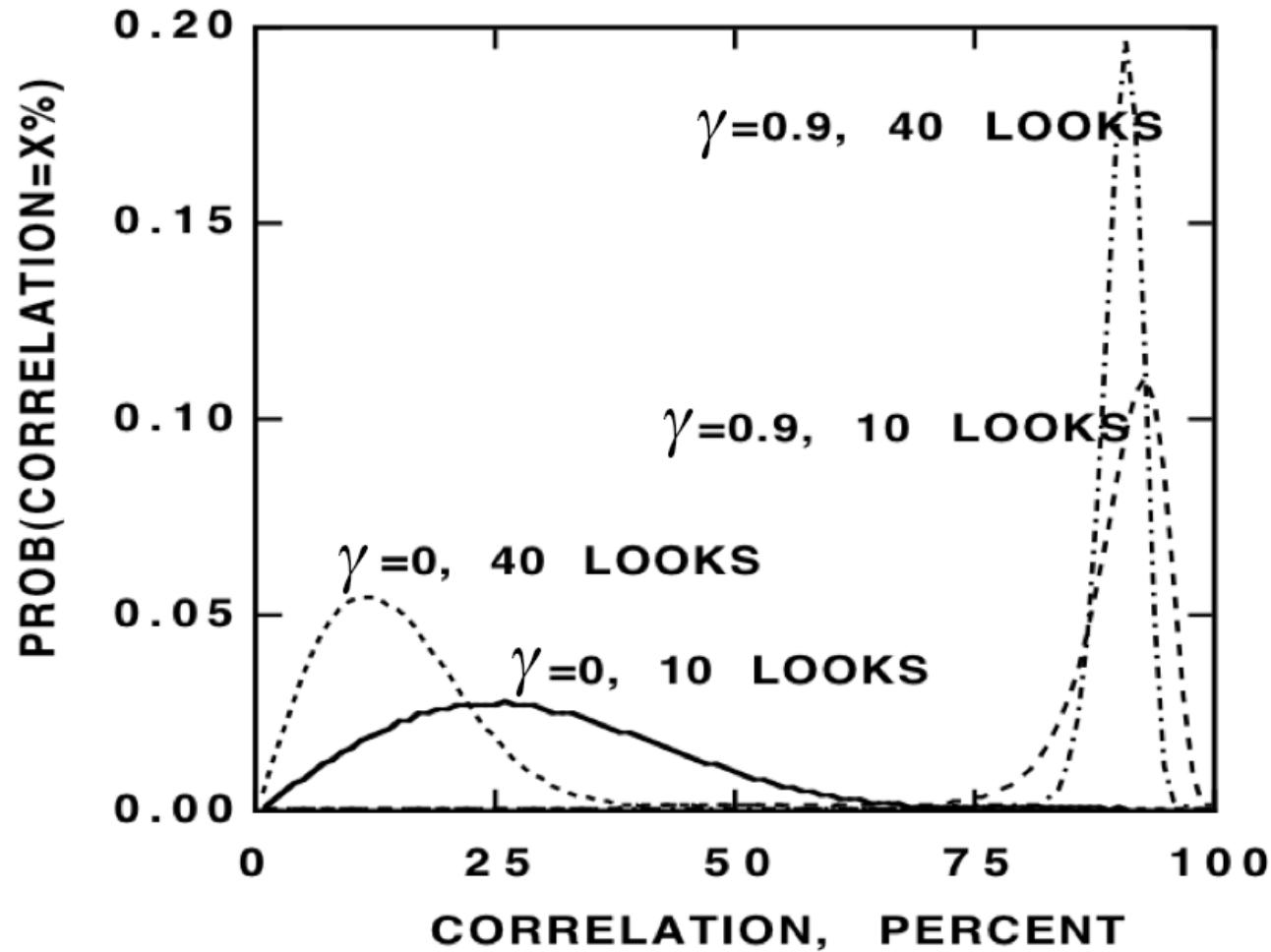
- We can calculate the correlation when we generate the multi-looked interferogram, according to the following definition:

$$\hat{\gamma} = \frac{|\langle s_1 s_2^* \rangle|}{\sqrt{\langle s_1 s_1^* \rangle \langle s_2 s_2^* \rangle}}$$
$$\rightarrow \frac{|\sum_{mn} s_1(\rho_{mn}, x_{mn}) s_2^*(\rho_{mn}, x_{mn})|}{\sqrt{\sum_{mn} s_1(\rho_{mn}, x_{mn}) s_1^*(\rho_{mn}, x_{mn}) \sum_{mn} s_2(\rho_{mn}, x_{mn}) s_2^*(\rho_{mn}, x_{mn})}}$$

An example



Correlation estimator statistics



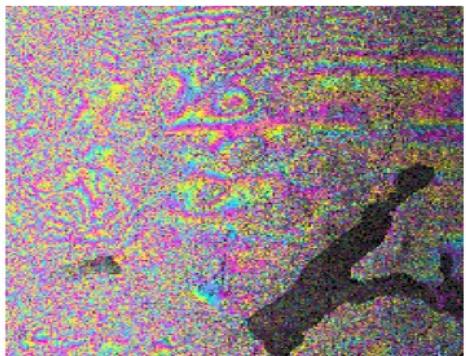
Total correlation

- The total correlation is the product of the individual correlation components:

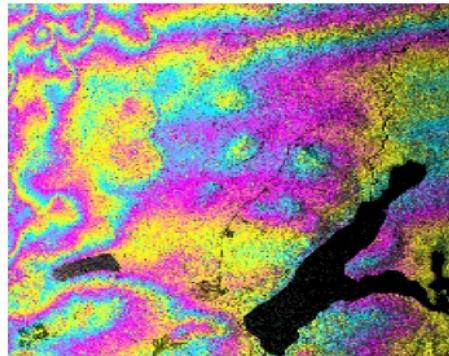
$$\rho_{total} = \rho_{thermal} \cdot \rho_{spatial} \cdot \rho_{temporal}$$

where $\rho_{thermal}$ represents electronic changes inside the radar system, $\rho_{spatial}$ referred to a change in sensor location (including rotation), $\rho_{temporal}$ represents decorrelation from a change in the surface between observations.

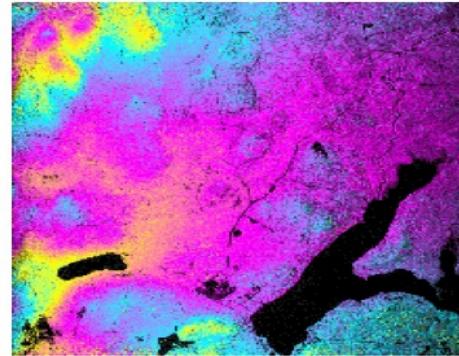
Interferograms



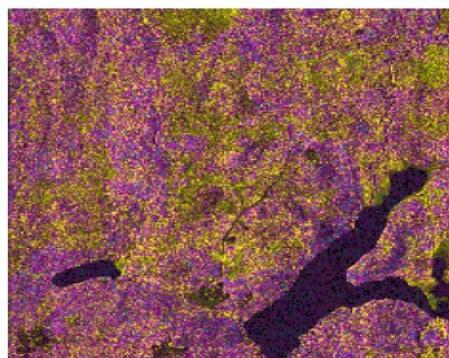
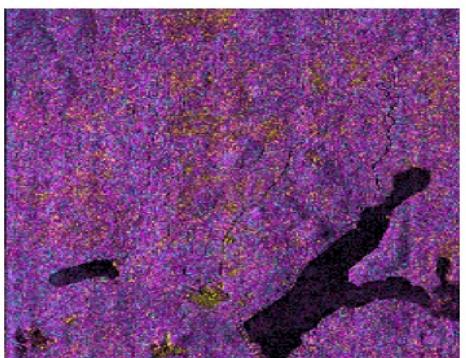
C-band



L-band



P-band



Correlation