Advanced Image Processing

Part V:

Image Sensor Models.
Noise Models.

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Recommended books

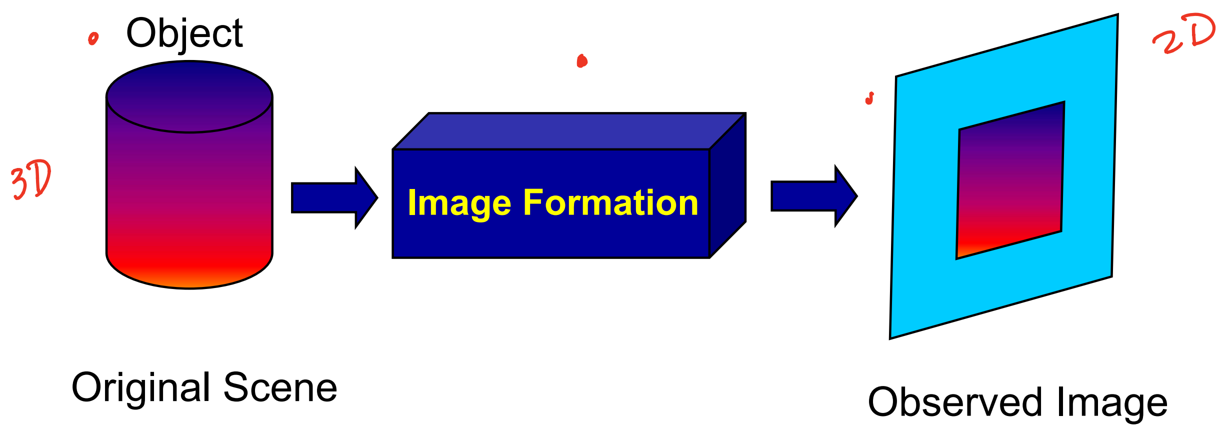
- A. K. Jain, Fundamentals of Digital Image Processing, Prentice-Hall, 1989.
- R. Lagendijk and J. Biemond, Iterative Identification and restoration of Images, Kluwer Academic Publishers, 1991.
- M. Bertero and P. Boccacci, Introduction to Inverse Problems in Imaging, IOP Publishing LTD, 1998.
- A.N. Tikhonov and V.Y. Arsenin, Solutions of ill-posed problems, Washington: Winston/Willey, 1977.
- V.A. Morozov, Methods for Solving Incorrectly Posed Problems, Springer, 1984.

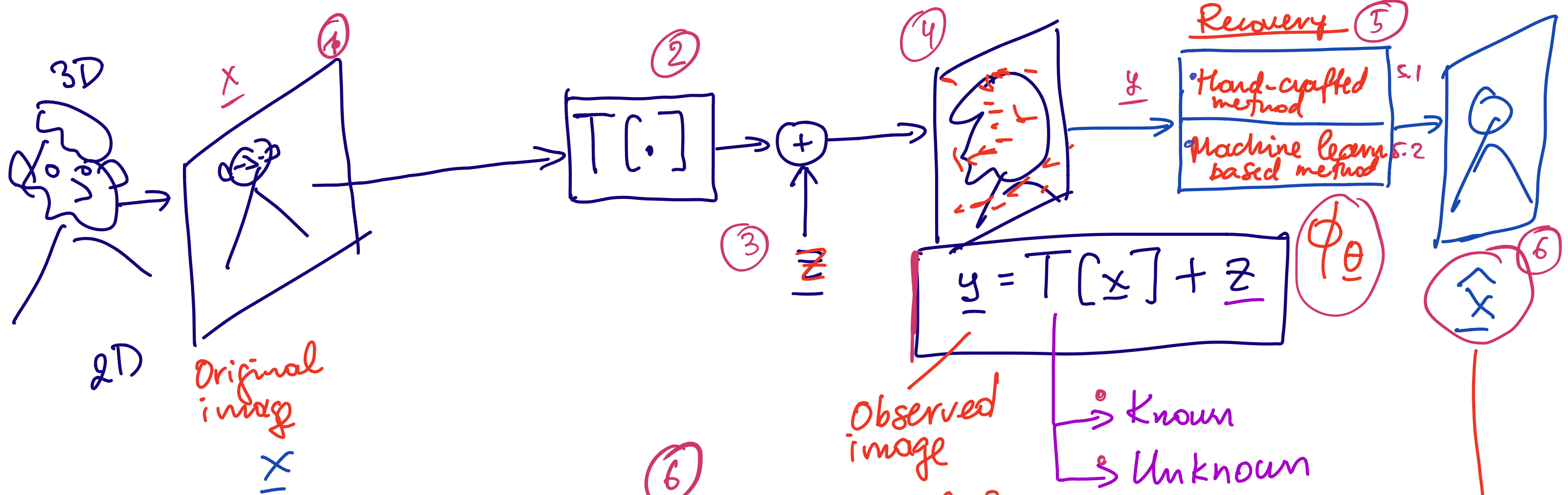
Roadmap:

1. Introduction
2. Generalized Model of Imaging Systems
3. Image Sensor Models
   - Linear motion blur
   - Defocusing
   - Diffraction Limited Imaging
   - Sparse Imaging Devices
   - Phase Errors in Focusing
   - Atmospheric Turbulence
4. Noise Models
   - Additive Noise
   - Discrete Noise
   - Multiplicative Noise

1. Introduction

- Images are produced to record or to display useful information.
- Due to imperfections in the electronic or photographic medium, communication channel and digitizing equipment, however the resulted image often represents a degraded version of the original scene.





6

$$\min D = \|\underline{x} - \hat{\underline{x}}\|_2^2$$

5.1

5.2

7

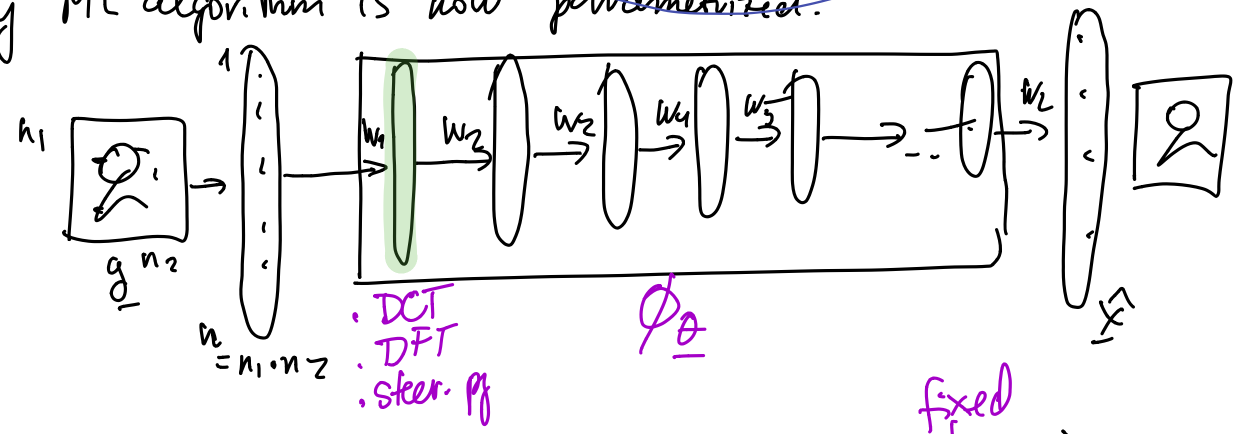
$$\hat{\underline{x}} = \phi_\theta(\underline{y})$$

$$\boxed{\hat{\underline{\theta}} \leftarrow \min_{\underline{\theta}} \sum_{i=1}^{N} \|\underline{x}_i - \underbrace{\phi_\theta(\underline{y}_i)}_{\hat{x}}\|_2^2}$$

ADAM

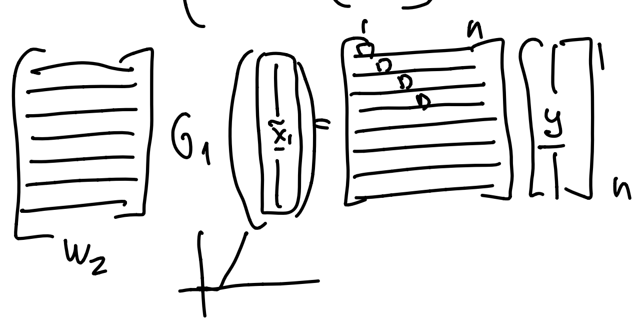
$$\underline{\theta} = (W_1, W_2, \ldots, W_L)$$

Any ML algorithm is now parametrized.



fixed

$$\phi_\theta = \sigma_L(W_L \;\ldots\;\ldots\; \sigma_2(W_2\, \sigma_1(W_1 y)))$$

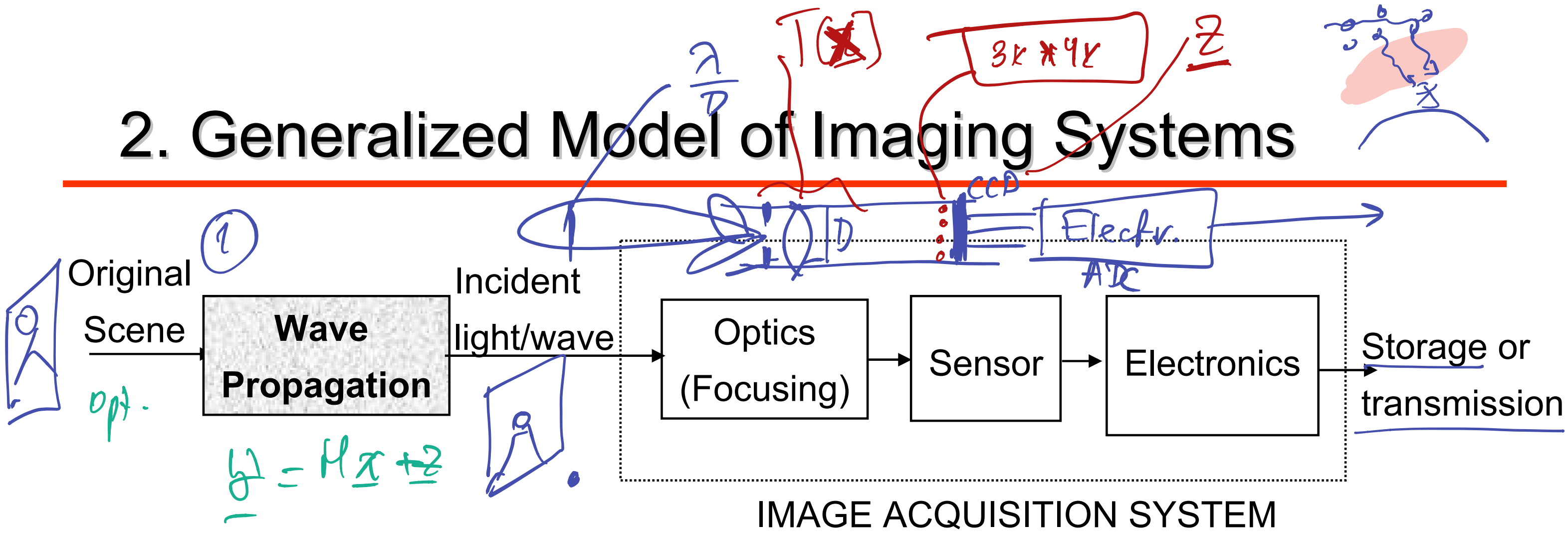


where $\underline{\theta} = (W_1, W_2, \ldots W_L)$

2. Generalized Model of Imaging Systems

- The degradations may have many causes, but two types of degradations are often dominant:
  - blurring $T[\cdot]$
  - noise $\underline{z}$
- Blurring is a form of bandwith reduction of the image due to the imperfect image formation process or due to the physical constraints of the imaging system design.
- Noise is a form of random image corruption that may be introduced by:
  - ① the transmission medium (both imaging and communication);
  - ② the recording medium (film grain noise, failures in CCDs);
  - ③ measurement and quantization errors.

2. Generalized Model of Imaging Systems



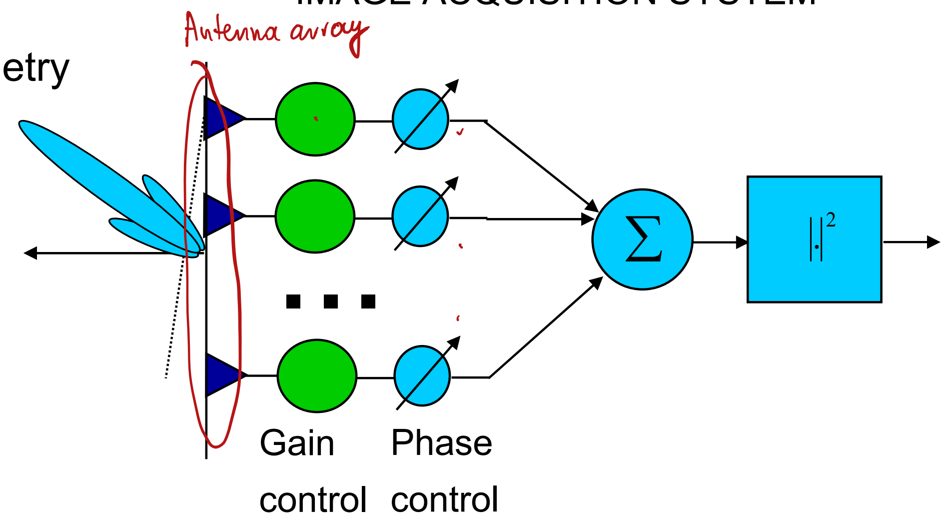
1

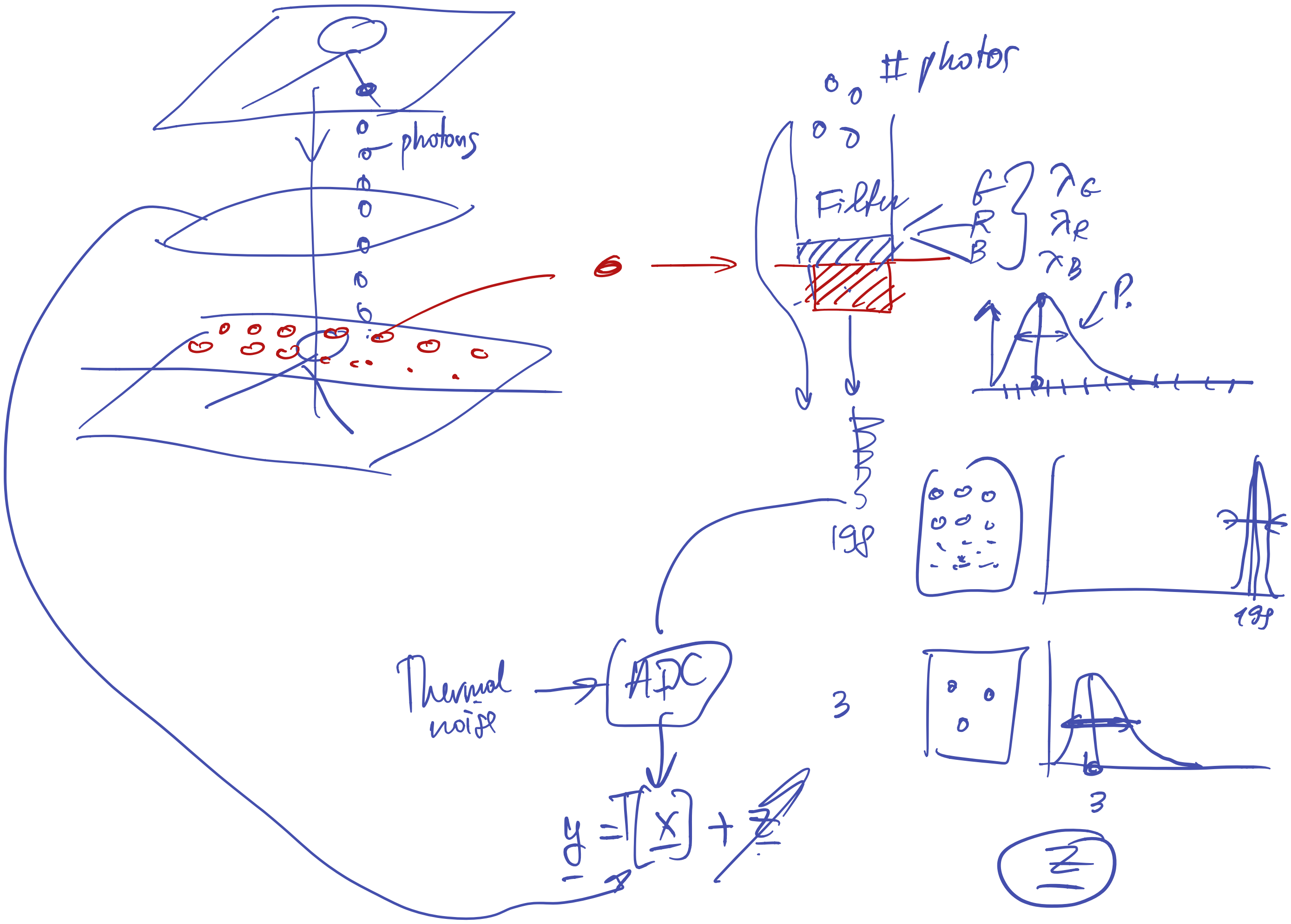
$y = Mx + z$

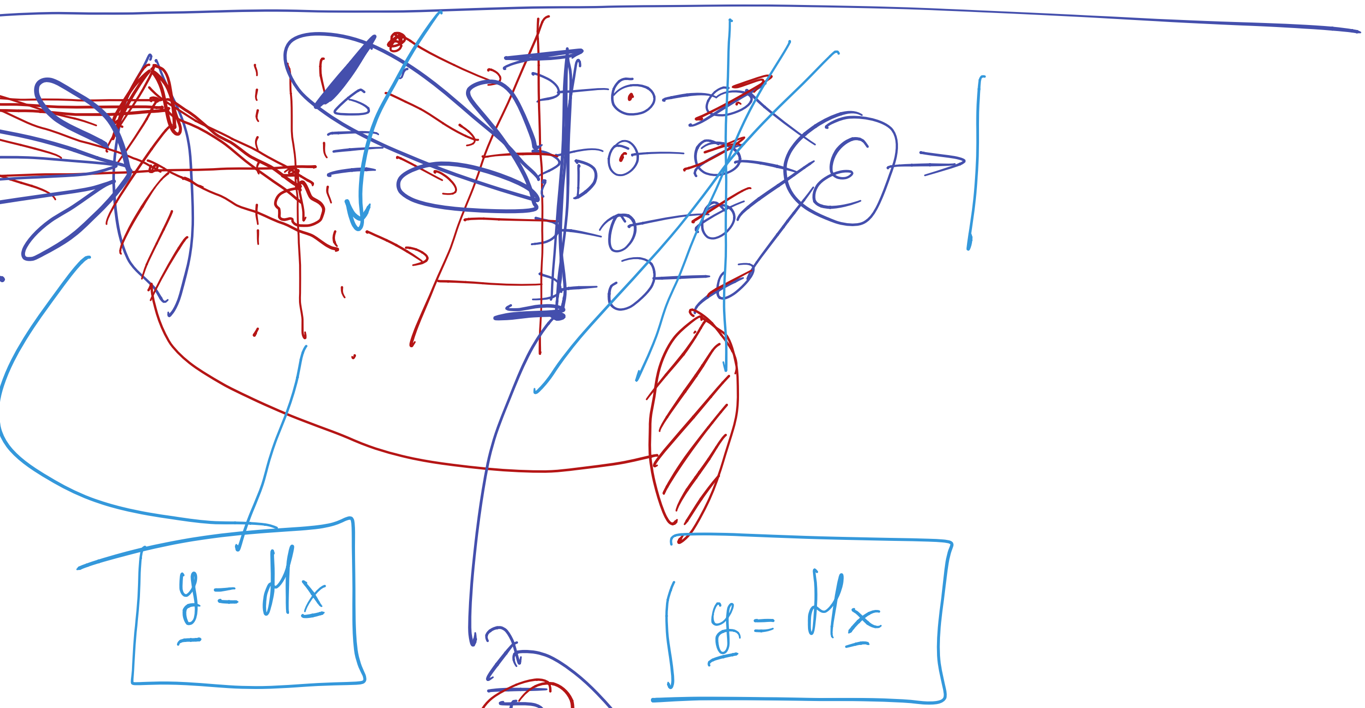
2 Radar and Radiometry Imaging Systems

radar.

$y = Mx + z$







$\frac{\lambda}{D}$ $\frac{380\cdot10^{-9}}{0.02.}$

$2\cdot 10^{-2}$

$\frac{\cancel{3}.\ 10^{-7}}{\cancel{3}\ 10^{-2}}$

10^{-5}

$\frac{\lambda}{D}$

$\lambda_{opt} = 380 \div 720\ nm$

$380\cdot10^{-9} \div 720\cdot10^{-9}$

$\lambda_{radar} = 0.01 - 1\ m$

$10^{-2} - 10^{1}\ m.$

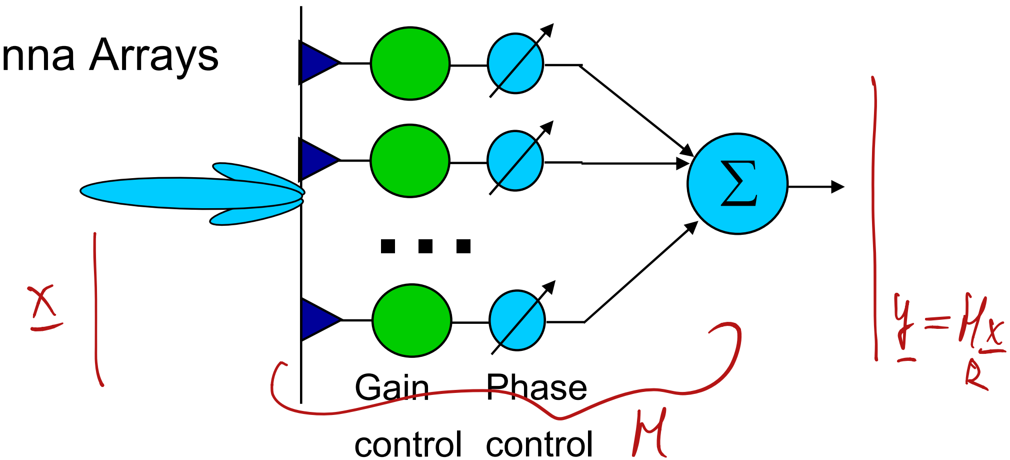
10m

$\frac{0.01}{10} = 10^{-3}$

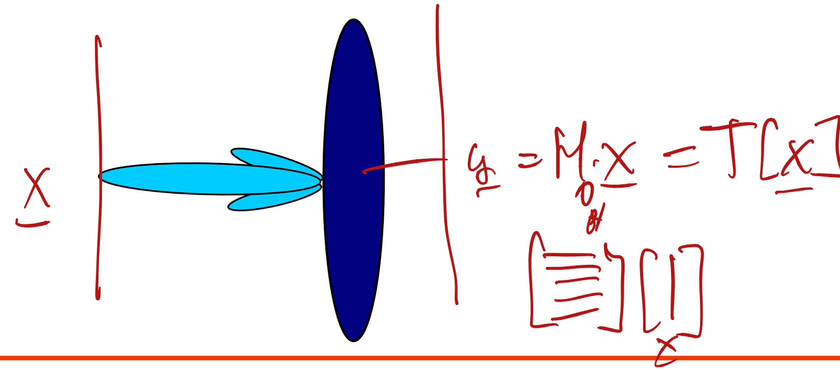
10^{-5}

2. Antenna Arrays and Lens: Analogy

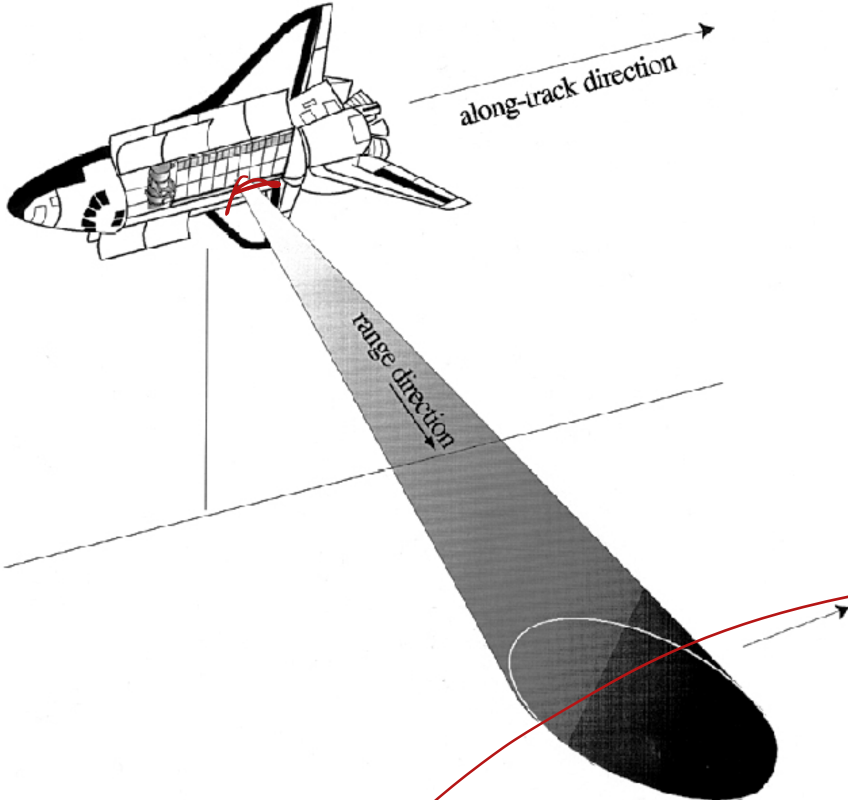
Antenna Arrays



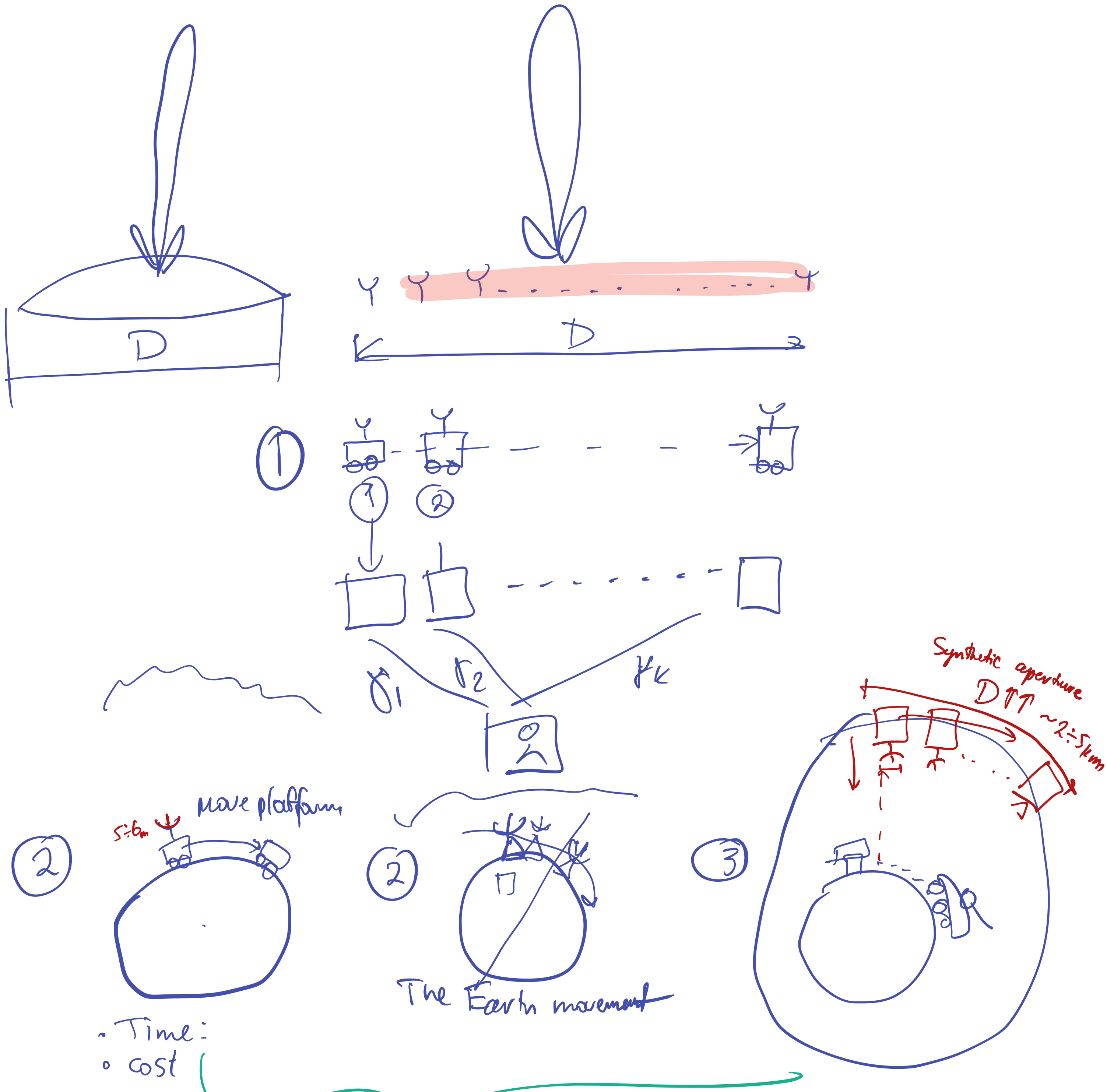
Lens



2. Synthetic Aperture Radar (SAR)

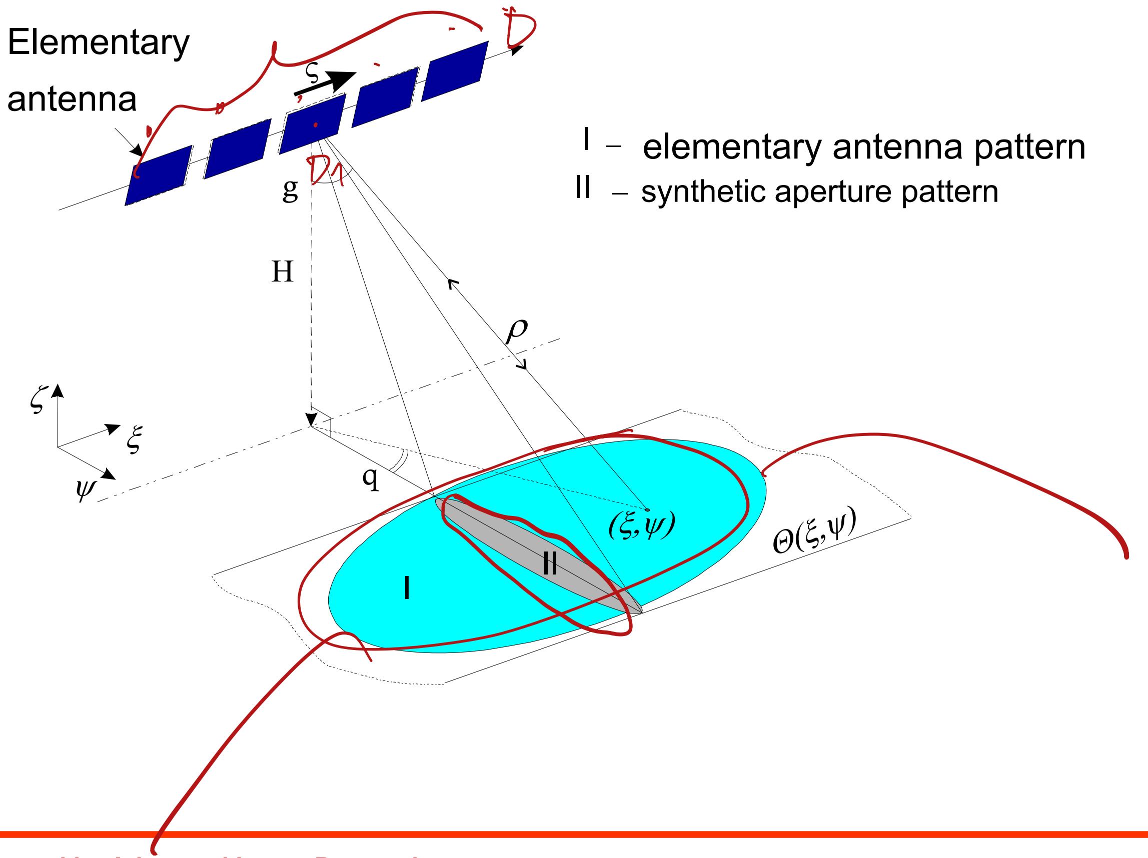


Proc. IEEE, March, 2000

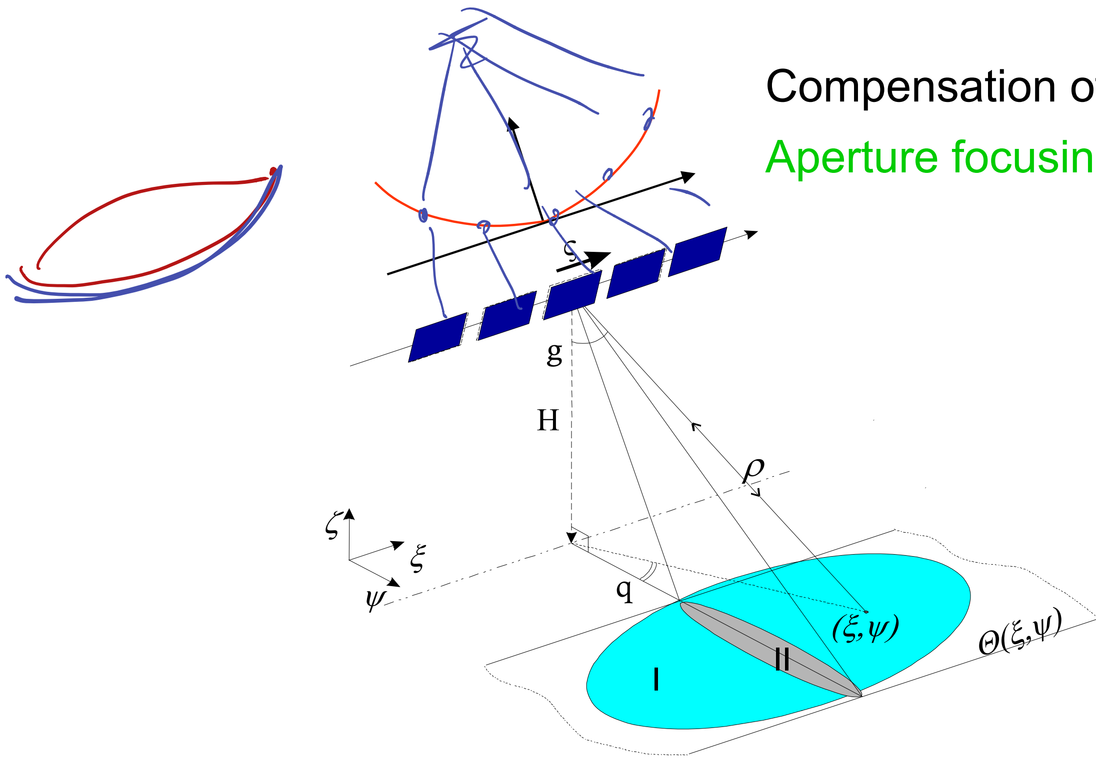


$$\underline{y} = H\underline{x}$$

2. Synthetic Aperture Radar (SAR)



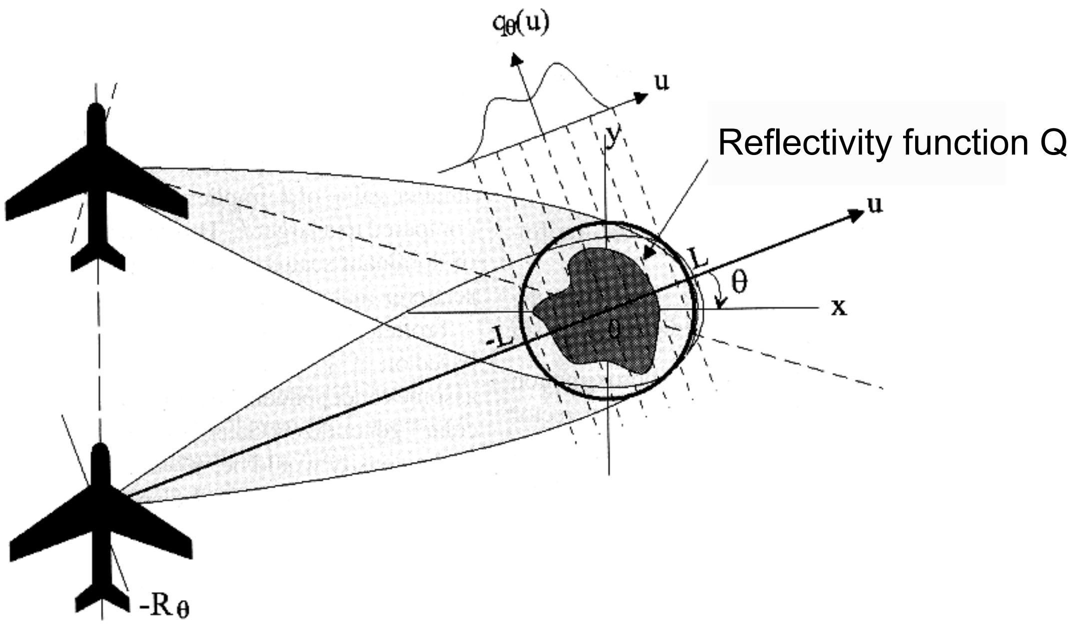
2. Synthetic Aperture Radar (SAR): Focusing



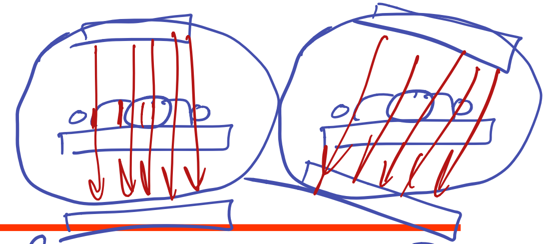
Compensation of phase delay.

Aperture focusing.

2. Spotlight-mode SAR



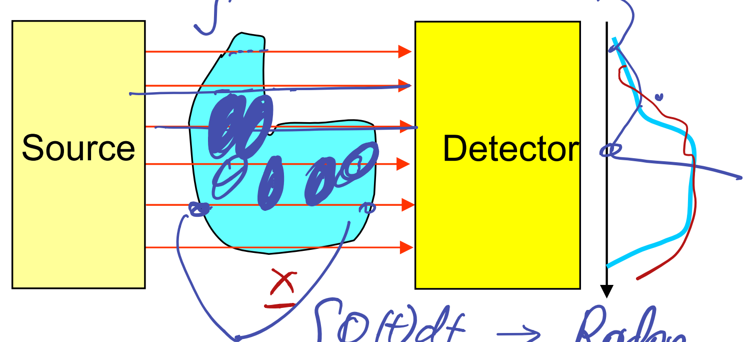
$y = Hx$



2. Tomography and Geophysics

Applications: $y = Hx$

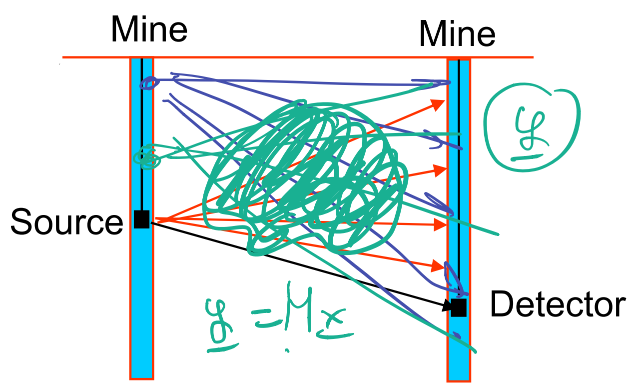
- Medical Imaging Systems;
- Non-destructive Testing;
- Geophysics.



$\int \rho(t)\,dt \to$ Radon transform

- Image Reconstruction: Radon or modified Radon transform.
- Limiting factors:
  - finite time of observation
  - limited number of projections
- Noise:
  - Poisson
  - Mixture noise

$y = Hx$



2. Generalized Model of Imaging Systems

Linear model

$y = Hx$

$[y] = [H][x]$

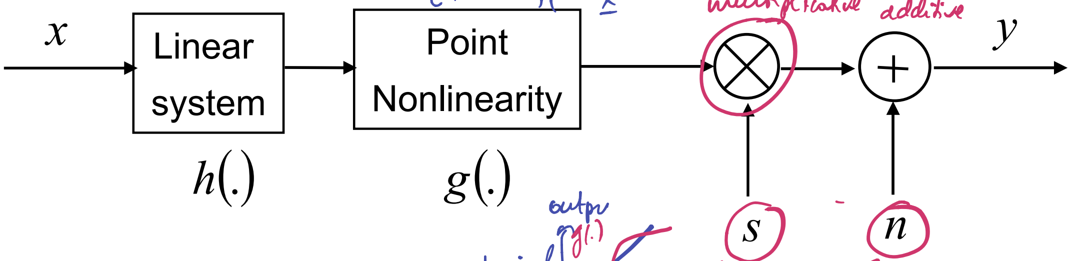
Defocusing / optical

Radar imaging

SAR

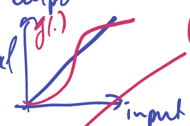
Medical (Radon transform)

multiplicative additive



output $g(.)$

physical

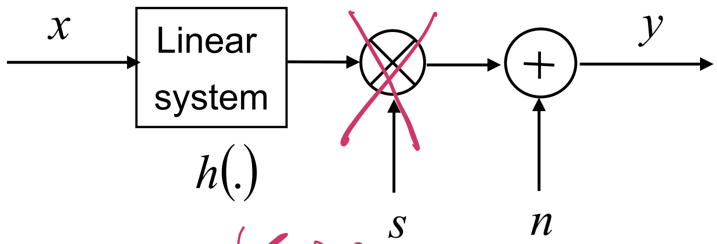


$$\boxed{y = g(Hx)s + n}$$

$\sigma(Wx)$

ReLU

2. Linear Model of Imaging Systems



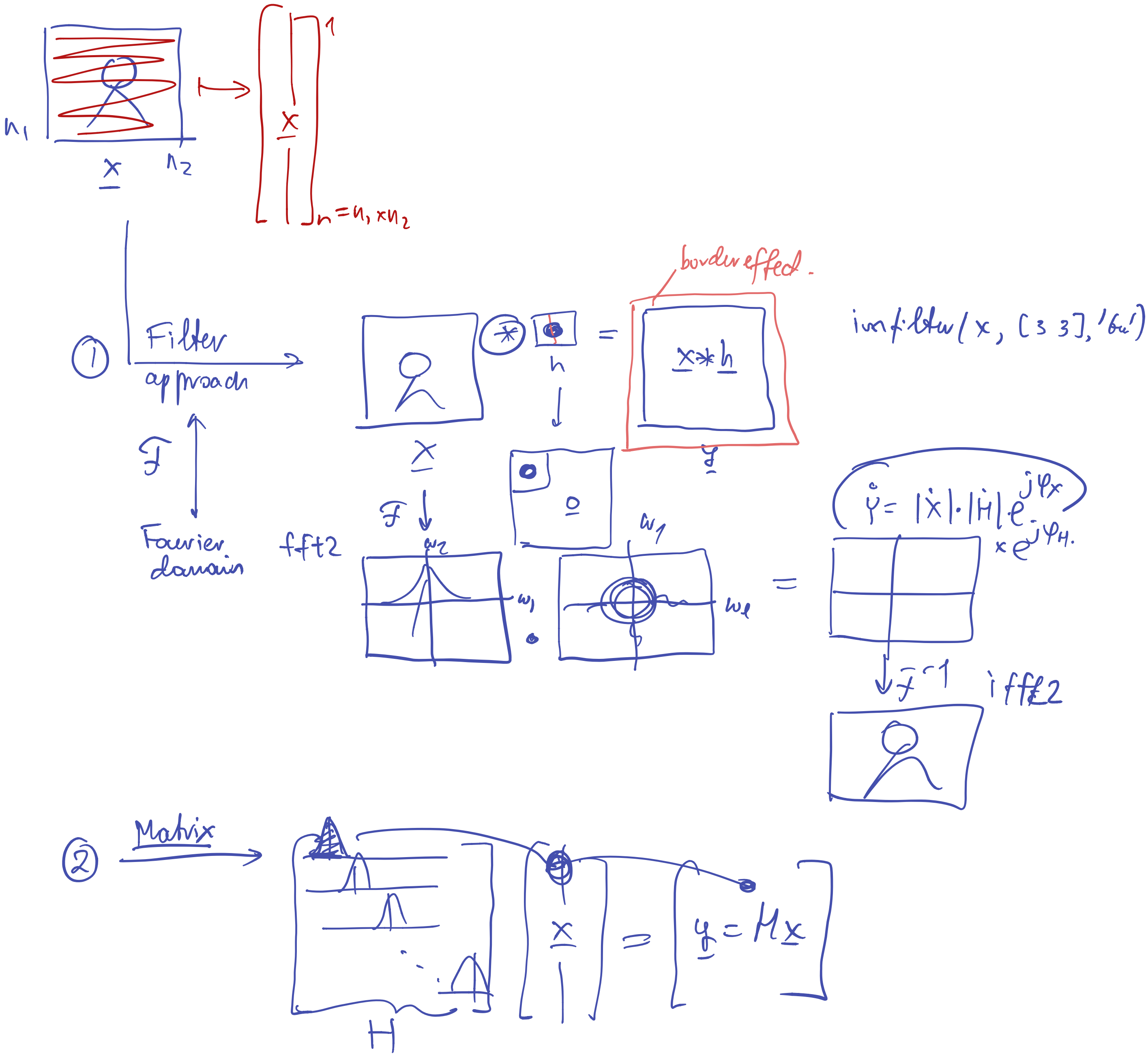
$$\boxed{y=(Hx)\cancel{s}+n}$$

$$\boxed{\underline{y} = H\underline{x} + \underline{n}}$$

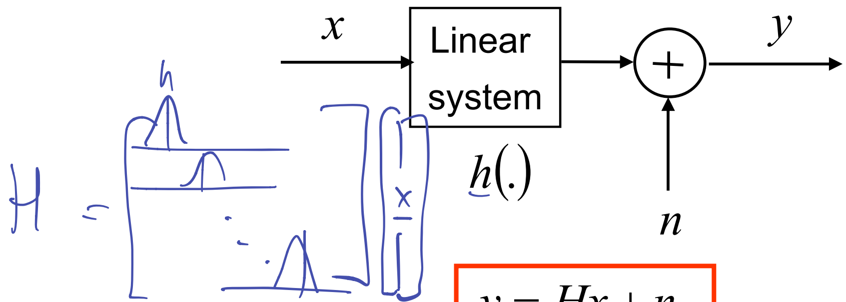
<u>2-D convolution</u>: $y(t_1,t_2)=\iint\limits_{\infty} h(t_1-t_1',t_2-t_2')x(t_1',t_2')dt_1'dt_2' + n(t_1',t_2') =$

$$= h(t_1,t_2) * x(t_1,t_2) + n(t_1,t_2)$$

$\sum\sum$


n_1
$\underline{x}$
n_2
$\underline{x}$
$n = n_1 \times n_2$
border effect.
① Filter approach
$\mathcal{F}$
Fourier domain
$\circledast$
$=$
$\underline{x} * \underline{h}$
imfilter(x, [3 3], 'fu')
h
$\underline{x}$
$\underline{y}$
$\mathcal{F}$
fft2
ω_2
ω_1
ω_1
ω_2
$=$
$\hat{y} = |\hat{x}| \cdot |\hat{H}| e^{j\varphi_x} \times e^{j\varphi_H}$.
$\mathcal{F}^{-1}$
ifft2
② Matrix
$\underline{x}$
$=$
$\underline{y} = H\underline{x}$
H

2. Imaging Systems: Discrete Formulation



$$y = Hx + n$$

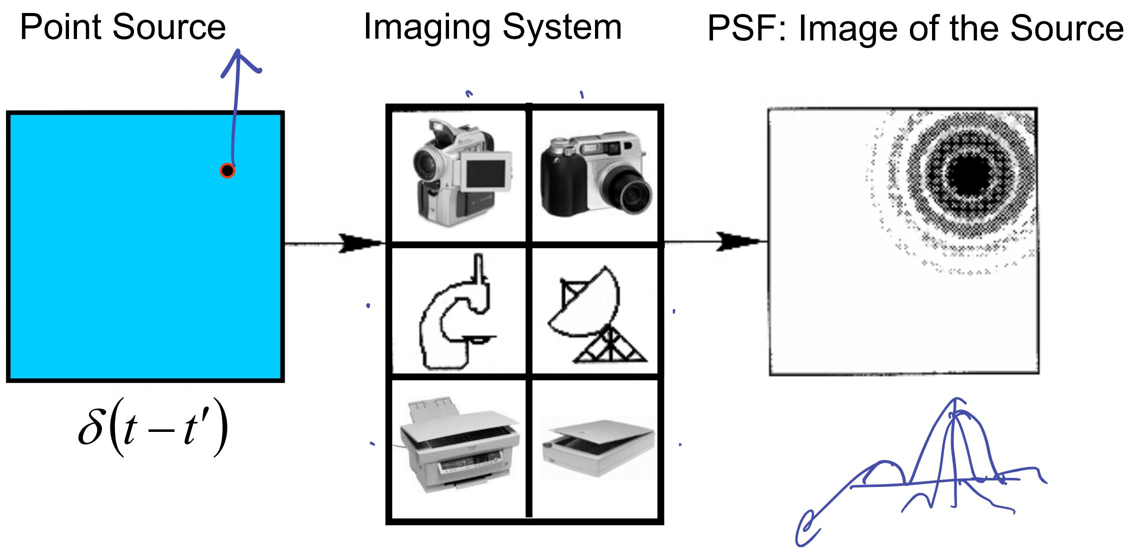
$y \quad | \; 2x$

$h = h_1 \circ h_2$

2-D convolution: $$\underline{y(n_1,n_2)} = \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{M-1} h(n_1-k_1, n_2-k_2)\,x(k_1,k_2)$$

Frequency Domain: $$\underline{Y(m_1,m_2)} = H(m_1,m_2)\,X(m_1,m_2)$$

2. Imaging Systems



$\delta(t-t')$

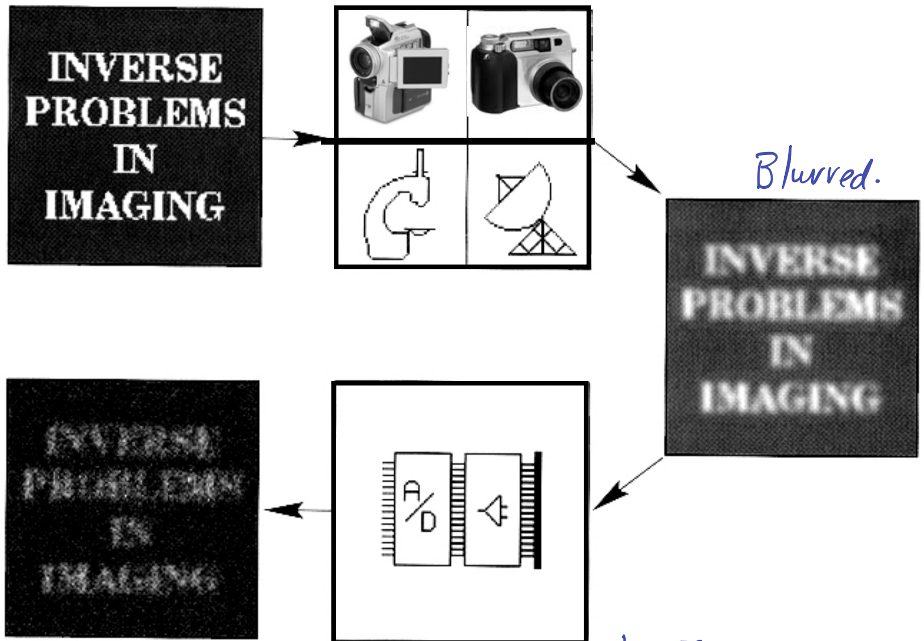
$\underline{y} = H\underline{x}$

2. Imaging Systems

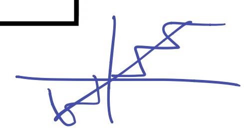
Blurred.

$\underline{y} = H\underline{x}$

$\underline{y} = H\underline{x} + \underline{z}$

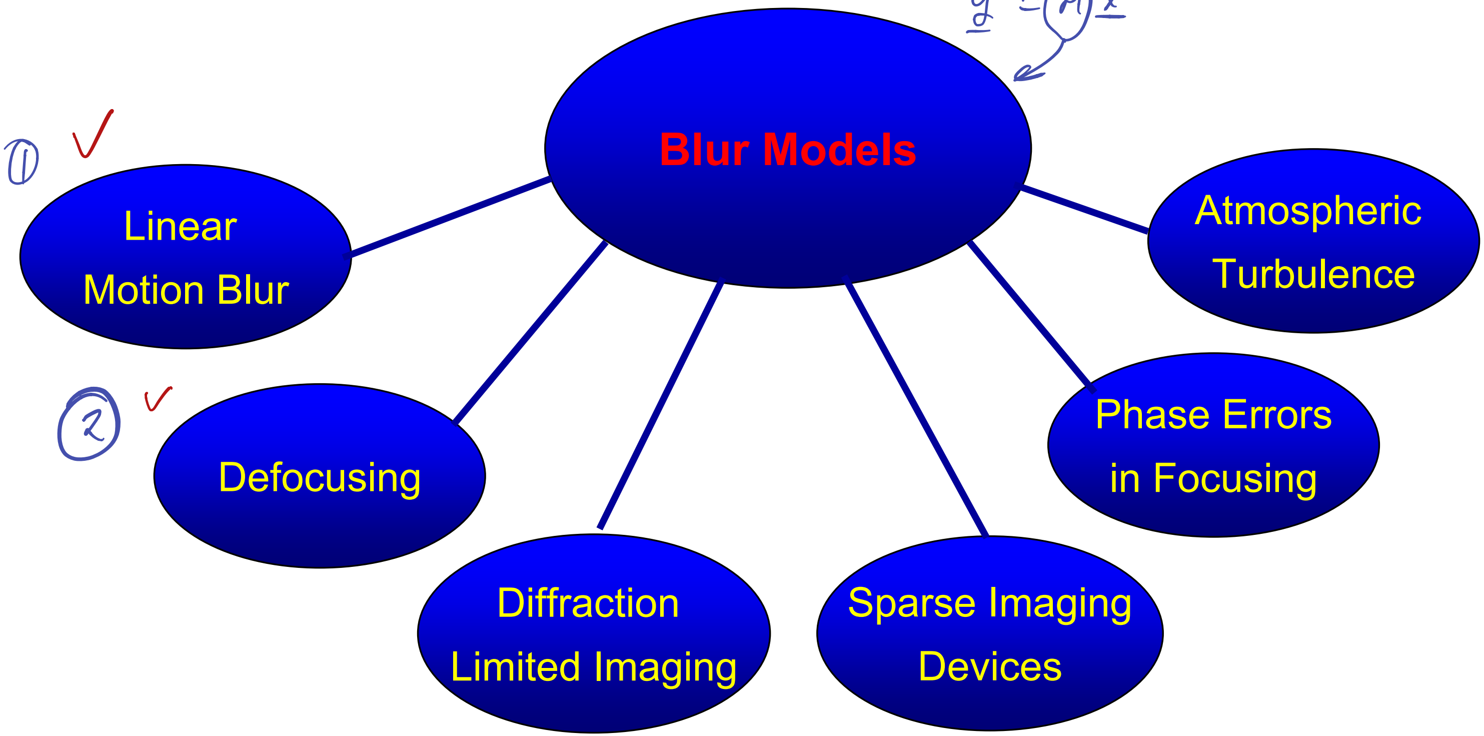


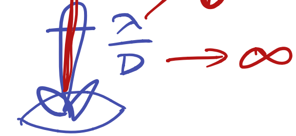
$+\ \underline{n}$



3. Classification of Blurring

$\underline{y} = H \underline{x}$

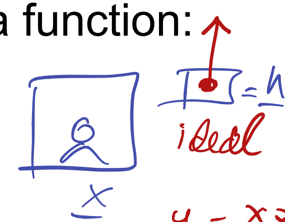




3. No Blur: Ideal Imaging System

- The PSF of ideal imaging system is a Dirac delta function:

$$h(t_1,t_2)=\delta(t_1,t_2)$$

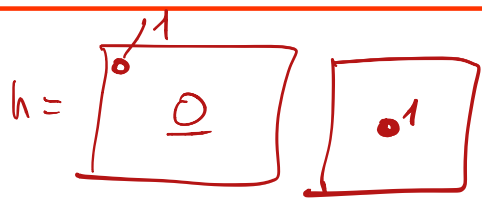


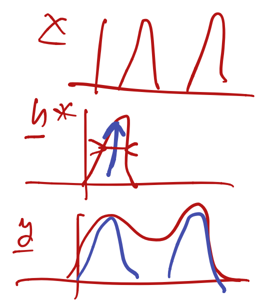
$\underline{y} = \underline{x} * \underline{h}$

$\underline{y} = \underline{x}$?

- The spatially discrete PSF is a unit pulse:

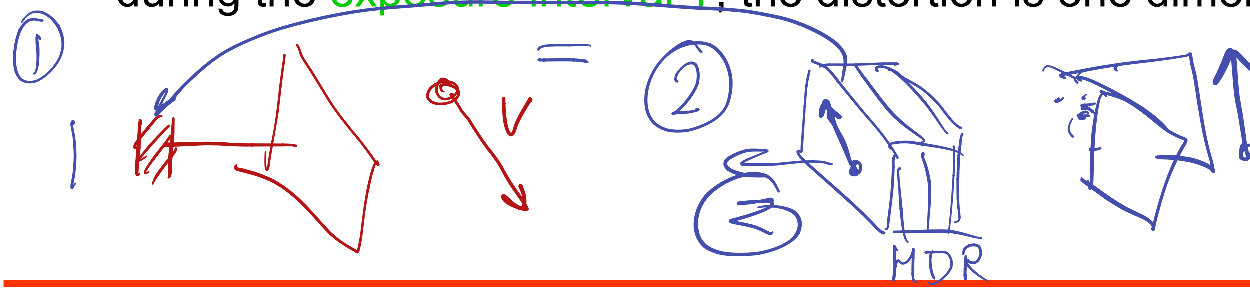
$$h(n_1,n_2)=\delta(n_1,n_2)=\begin{cases}1, \ if \ n_1 = n_2 = 0,\\ 0, \ elsewhere.\end{cases}$$





3. Linear Motion Blur

- Linear motion blur is due to relative motion of the object and the imaging device during exposure.
- This can be in the form of a translation, a rotation, a sudden change of scale, or some combinations of these (affine transforms).
- When the scene to be recorded translates relative to the camera at a constant velocity V under some angle ϕ with the horizontal axis during the exposure interval T, the distortion is one dimensional.

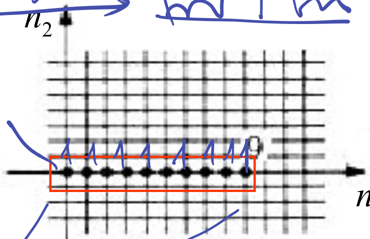


3. Linear Motion Blur

- The length of motion is L=TV. The PSF is given by:

$$h(t_1,t_2;L,\phi)=\begin{cases}\frac{1}{L},\ if\ \sqrt{t_1^2+t_2^2}\le\frac{L}{2},\ \frac{t_1}{t_2}=-\tan\phi, \\ 0,\ elsewhere.\end{cases}$$

The PSF of Motion Blur



$\frac{1}{L}$

L

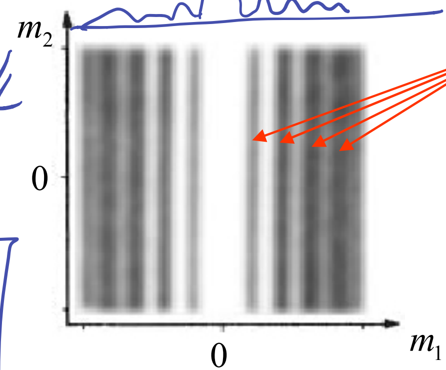
1D

$h=$

$1/L$

The Magnitude Spectrum of Motion Blur

1.1

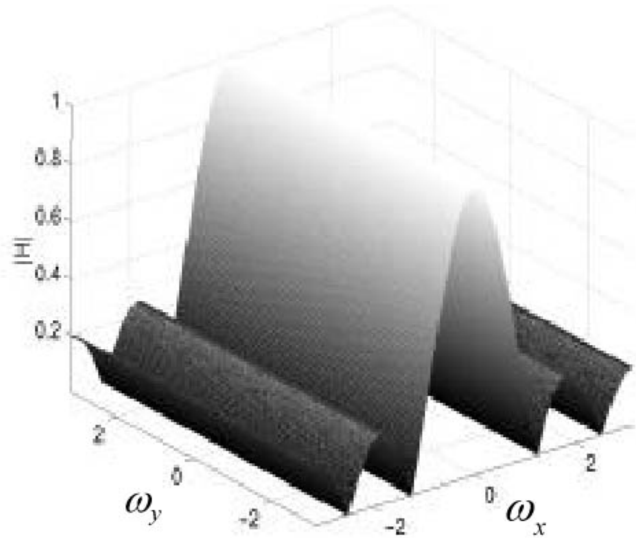


3. Linear Motion Blur: Horizontal

Cameraman blurred horizontally

Filter impulse response

$$\frac{1}{5}\begin{pmatrix}1 & 1 & [1] & 1 & 1\end{pmatrix}$$

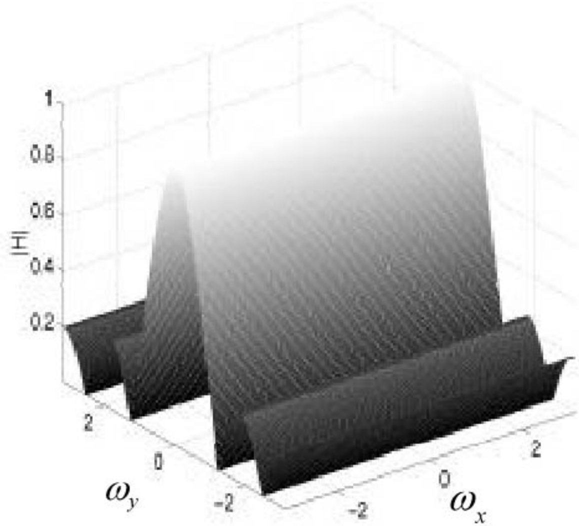


3. Linear Motion Blur: Vertical

Cameraman blurred vertically

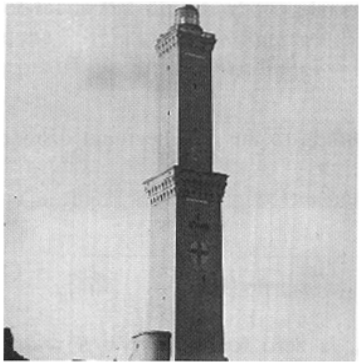
Filter impulse response

$$\frac{1}{5}\begin{pmatrix}1\\ 1\\ {[1]}\\ 1\\ 1\end{pmatrix}$$

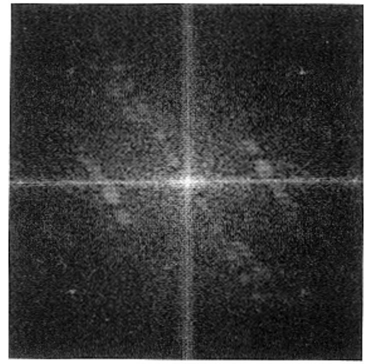


3. Linear Motion Blur

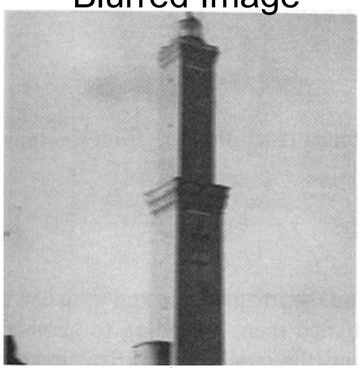
Original Image



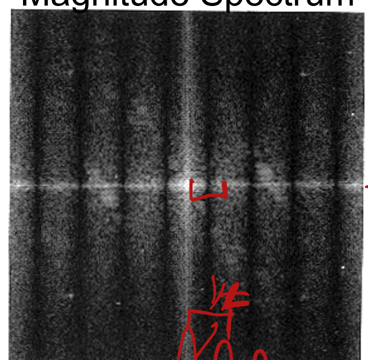
Magnitude Spectrum



Blurred Image



Magnitude Spectrum



log(|x|+1)

TV

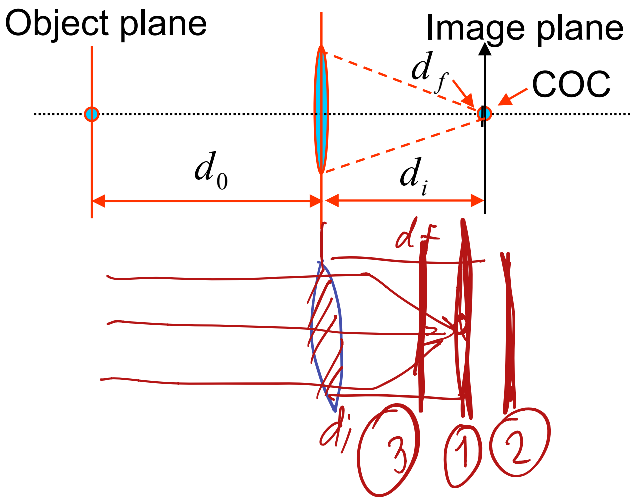
3. Out-of-Focus Blur: Defocusing

- When a camera images a 3-D scene onto a 2-D image plane, some parts of the scene are in focus while other parts are not.
- If the aperture of the of the camera is circular, the image of any point source is a small disc, known a circle of confusion (COC).
- The degree of defocus (diameter of the COC) depends on:
  - the focal length d_f
  - the aperture size
  - the distance between camera and object d_0.

3. Out-of-Focus Blur: Defocusing

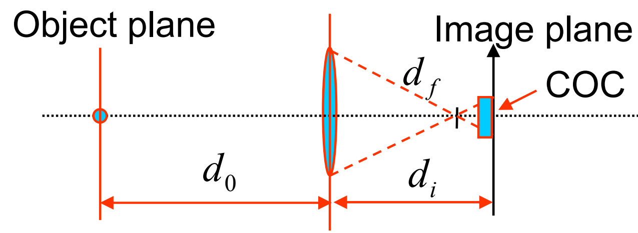
No defocusing: COC is point

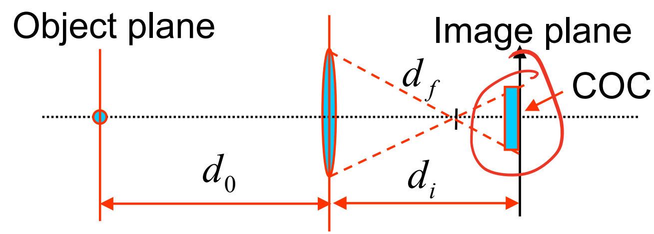
$$\boxed{\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{d_f}}$$



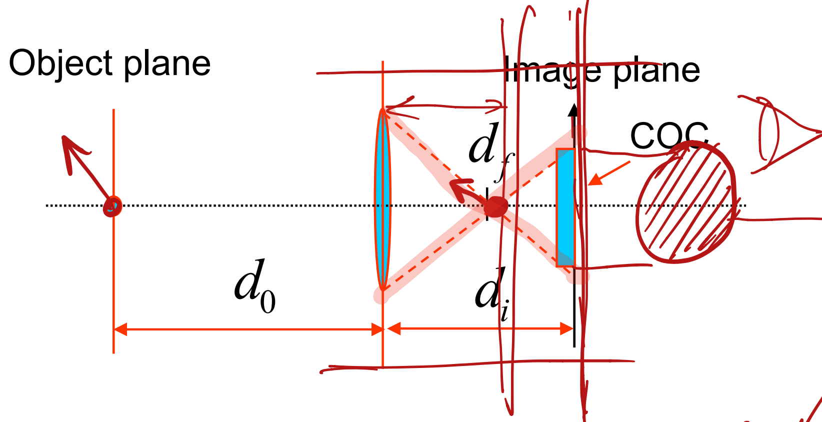
Defocusing: COC is disc

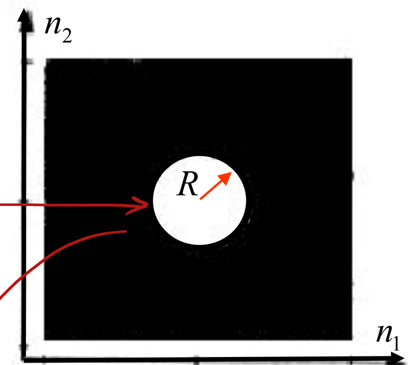
$$\boxed{\frac{1}{d_0}+\frac{1}{d_i}\neq\frac{1}{d_f}}$$





3. Out-of-Focus Blur: 2-D





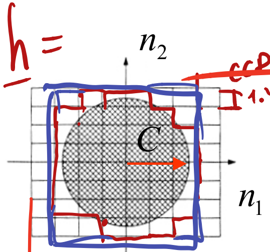
Cont

$$h(t_1,t_2;R)=\begin{cases}\dfrac{1}{\pi R^2}=1, & \text{if } \sqrt{t_1^2+t_2^2}\le R^2,\\ 0, & \text{elsewhere.}\end{cases}$$

R^2

Discrete

Discrete version: $h =$

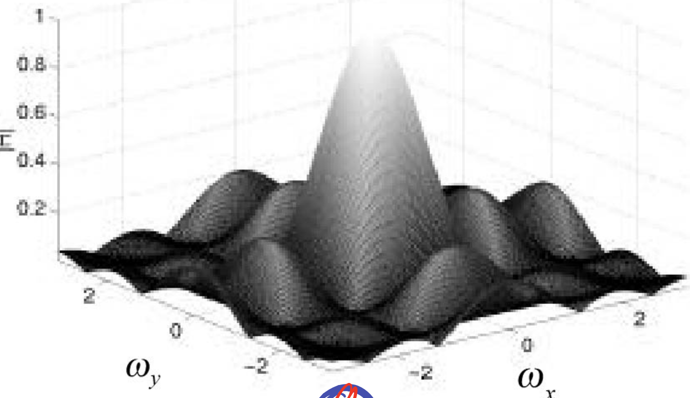


$$h(n_1,n_2;R)=\begin{cases}\dfrac{1}{C}, & \text{if } \sqrt{n_1^2+n_2^2}\le R^2,\\ 0, & \text{elsewhere.}\end{cases}$$

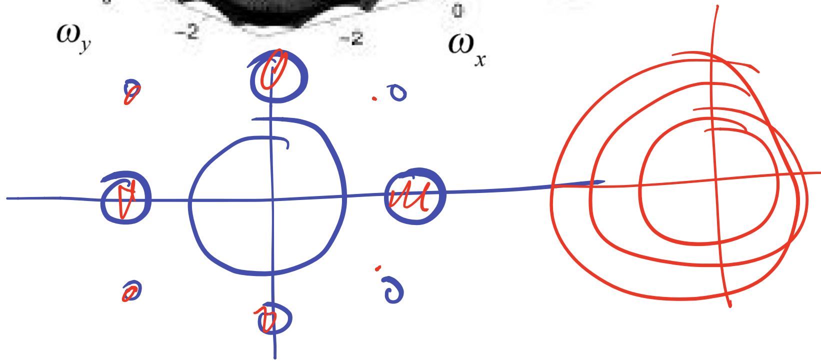
0

3. Out-of-Focus Blur: Approximation

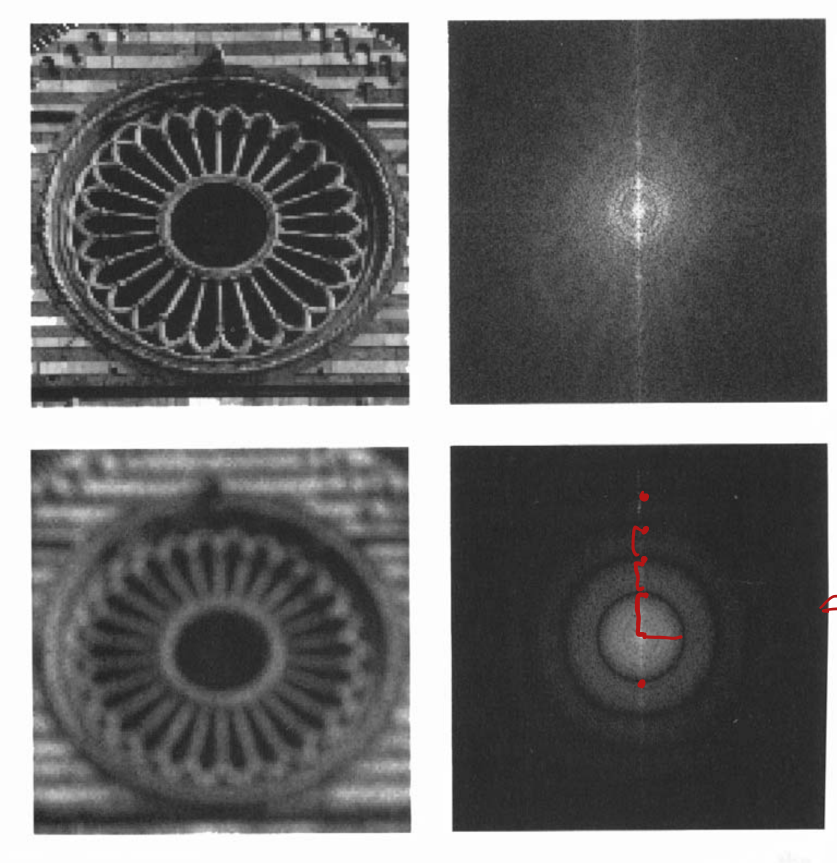
$$\frac{1}{25}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & [1] & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

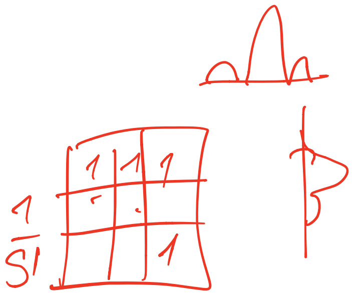


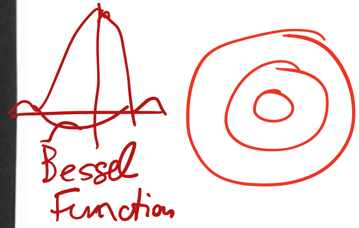
"approximation"!



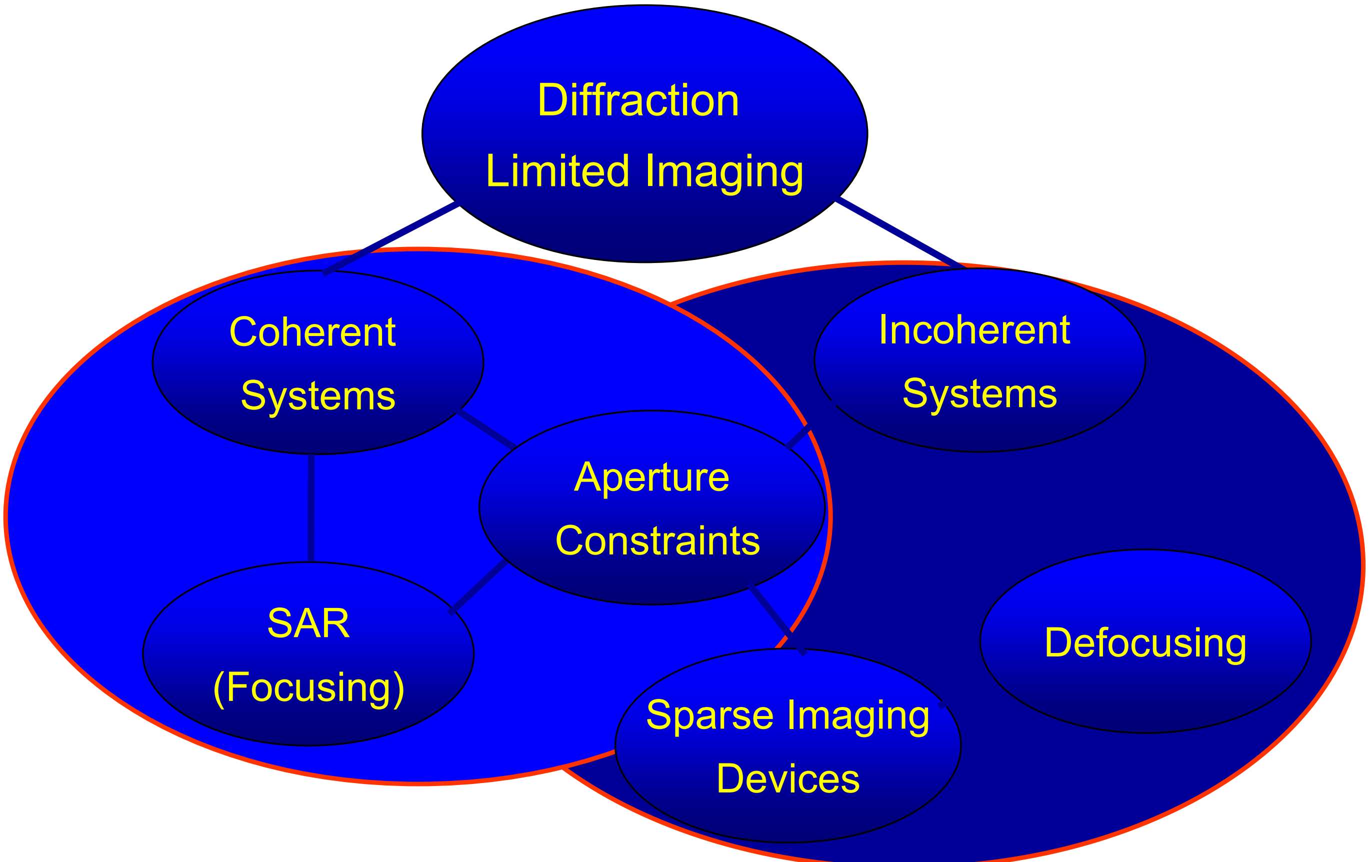
3. Out-of-Focus Blur: Circular Aperture







3. Diffraction Limited Imaging Systems



3.Imaging Systems: Fundamental Connections

General Case (Coherent Imaging Systems)

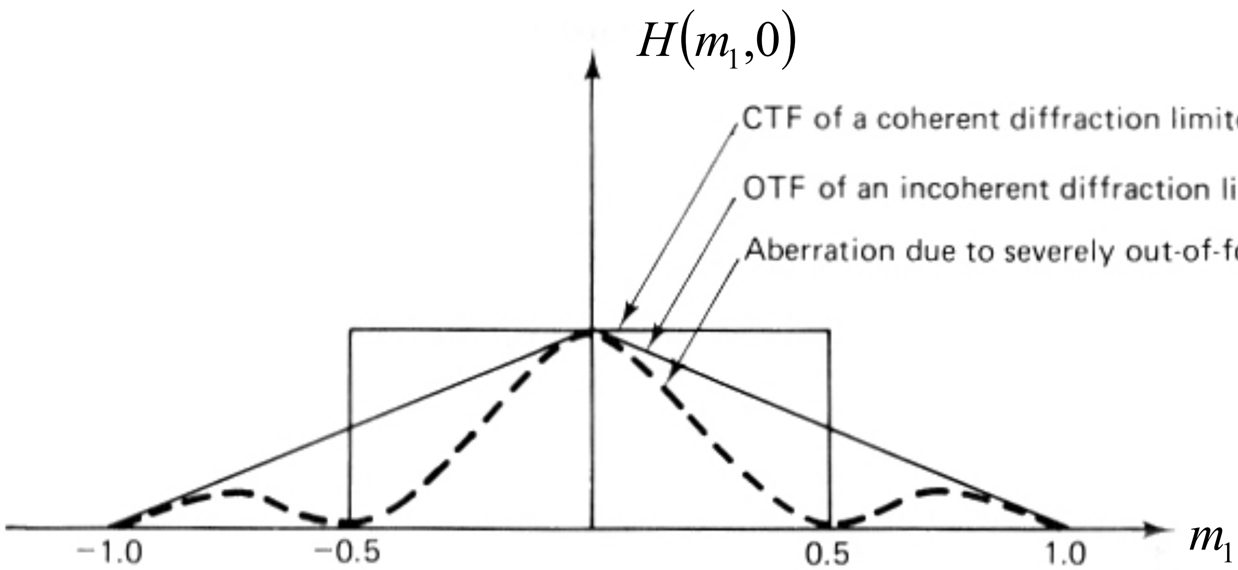
Complex filed distribution in the imaging aperture $\dot{\chi}(\vartheta_1, \vartheta_2)$ $\xrightarrow{\Im\{.\}}$ Directional Antenna Pattern (DAP) Point Spread Function (PSF) $h(t_1, t_2)$ $\xrightarrow{\Im\{.\}}$ Spatial System Spectrum $H(m_1, m_2)$

Incoherent Imaging Systems

Complex filed distribution in the imaging aperture $\dot{\chi}(\vartheta_1, \vartheta_2)$ $\xrightarrow{\Im\{.\}}$ Directional Antenna Pattern (DAP) Point Spread Function (PSF) $|.|^2$ $h(t_1, t_2)$ $\xrightarrow{\Im\{.\}}$ Spatial System Spectrum $H(m_1, m_2)$

Autocovariance $\otimes$

3. Generalized Model of Imaging Systems

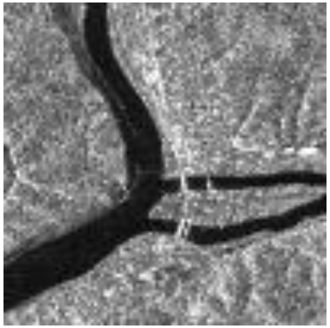


CTF - the coherent transfer function (spatial spectrum of coherent imaging systems)

OTF - the optical transfer function (spatial spectrum of coherent imaging systems)

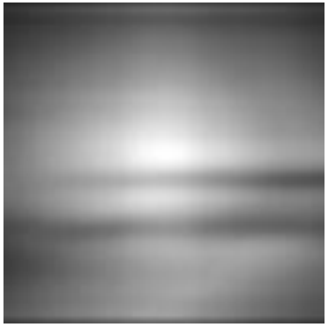
3. Generalized Model of Imaging Systems

Synthetic Aperture Radar (SAR)

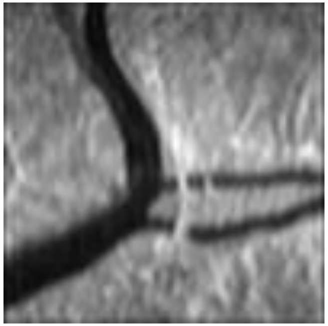


Scattering surface

SAR image with 5 elements.



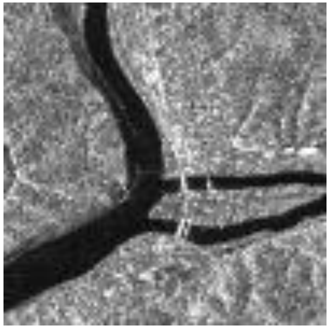
SAR image with 100 elements without quadratic delay compensation.



SAR image with 100 elements with quadratic delay compensation.

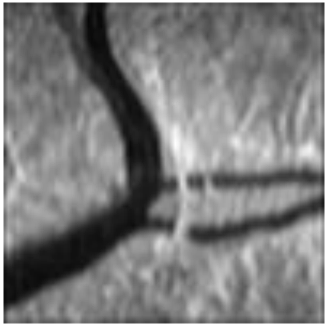
3. Generalized Model of Imaging Systems

Incoherent Imaging (Radiometry Imaging)



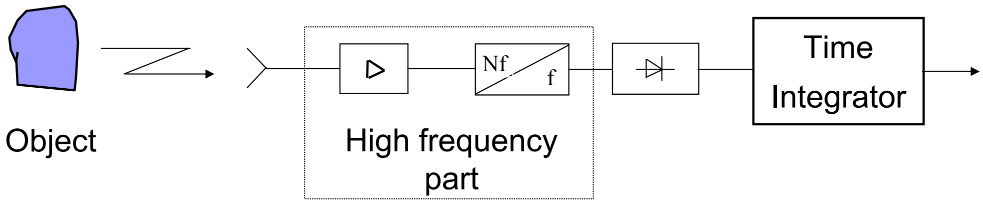
Scattering surface

Radiometry image with 100 antenna elements and zero-phase CFD.



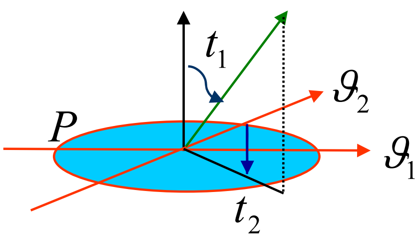
SAR image with 100 elements with quadratic delay compensation.

3. Incoherent Imaging Systems



Directional Antenna Pattern

$$h(t_1 - t_1', t_2 - t_2') = \left| \iint_P \dot{\chi}(\vartheta_1, \vartheta_2) \cdot \exp\left(\frac{2\pi \cdot \left((t_1 - t_1')\vartheta_1 + (t_2 - t_2')\vartheta_2 \right)}{\lambda} \right) d\vartheta_1 d\vartheta_2 \right|^2$$



P - antenna aperture

$\dot{\chi}(\vartheta_1, \vartheta_2)$ - complex field (current) distribution (CFD) in the antenna aperture

λ - wavelength

3. Incoherent Imaging Systems

Complex field distribution (CFD) in the antenna aperture (lens)

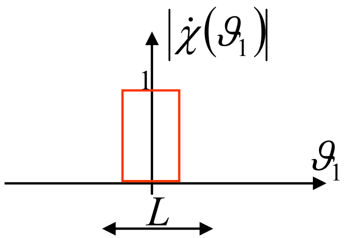
$$\dot{\chi}(\vartheta_1,\vartheta_2)=\left|\dot{\chi}(\vartheta_1,\vartheta_2)\right|e^{-j\varphi_I(\vartheta_1,\vartheta_2)}$$

- Role of the CFD magnitude:
  - physical size of antenna determines the width of the directional antenna pattern (PSF);
  - the shape of the magnitude (CFD) determines the level of side lobes and their relationships.
- Role of the CFD phase:
  - the phase distribution determines the focusing properties of the imaging system (focused system has the min width of the PSF);
  - beam scanning of the antennas;
  - phase aberrations in the imaging systems.

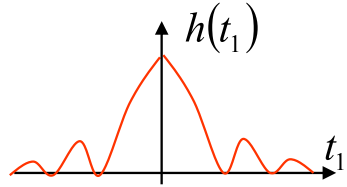
3. Incoherent Imaging Systems: CFD Magnitude

Assume zero-phased or focussed imaging system $\dot{\chi}(\vartheta_1, \vartheta_2) = \left|\dot{\chi}(\vartheta_1, \vartheta_2)\right|$

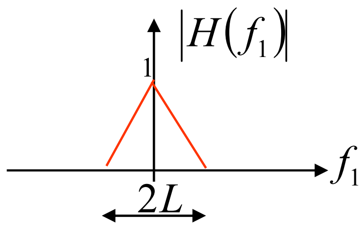
Field distribution



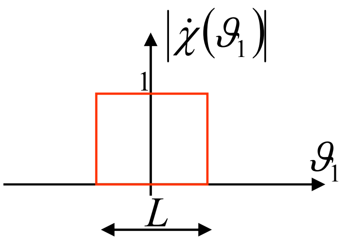
Impulse response



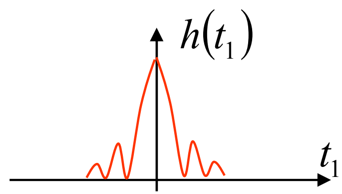
Spatial Spectrum



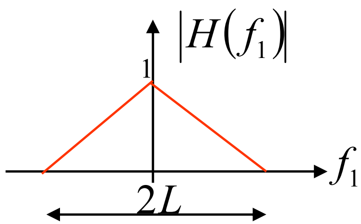
Field distribution



Impulse response

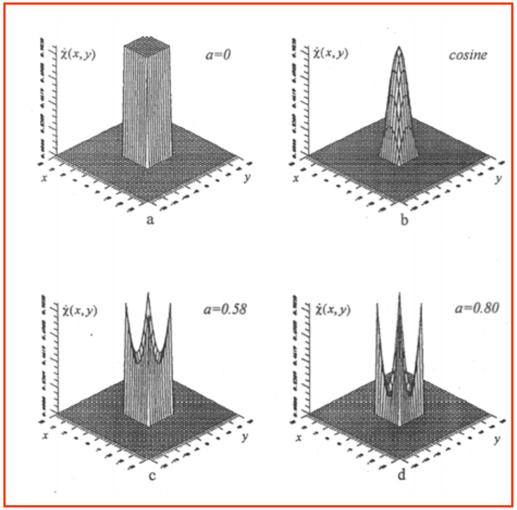


Spatial Spectrum

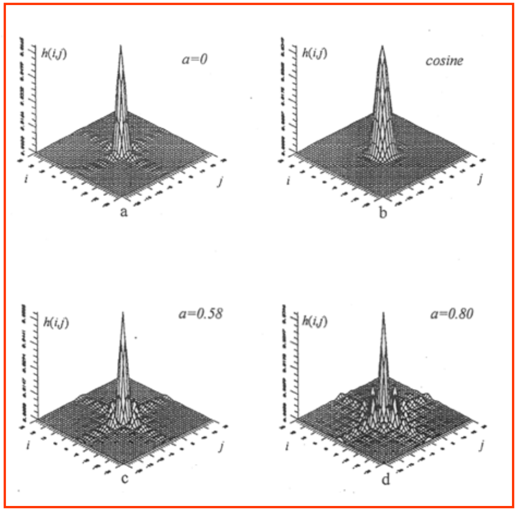


3. Incoherent Imaging Systems: CFD Magnitude

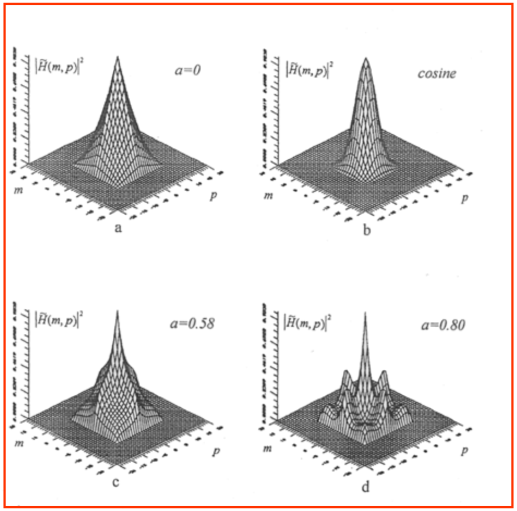
Magnitudes of CFD



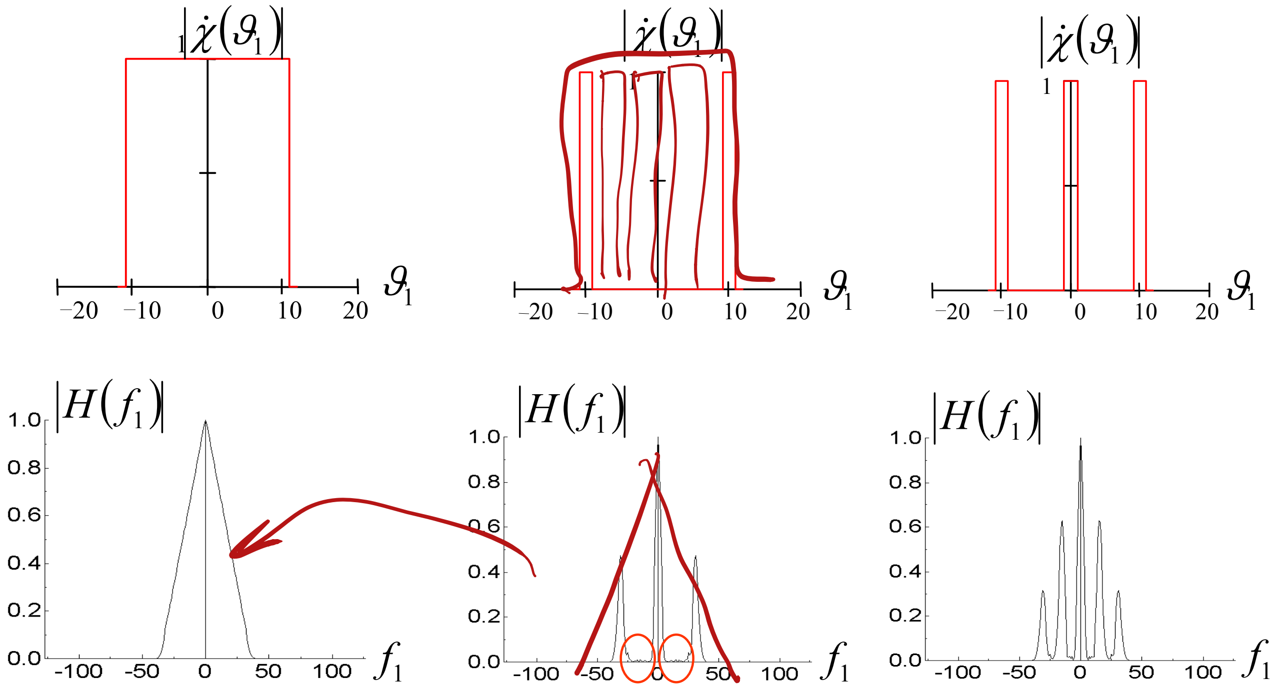
Directional Antenna Patterns



Spatial Spectra

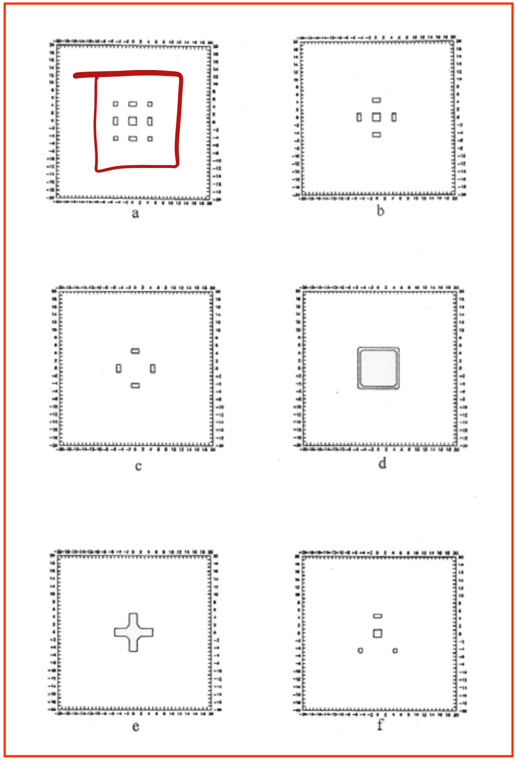


3. Sparse Incoherent Imaging Systems

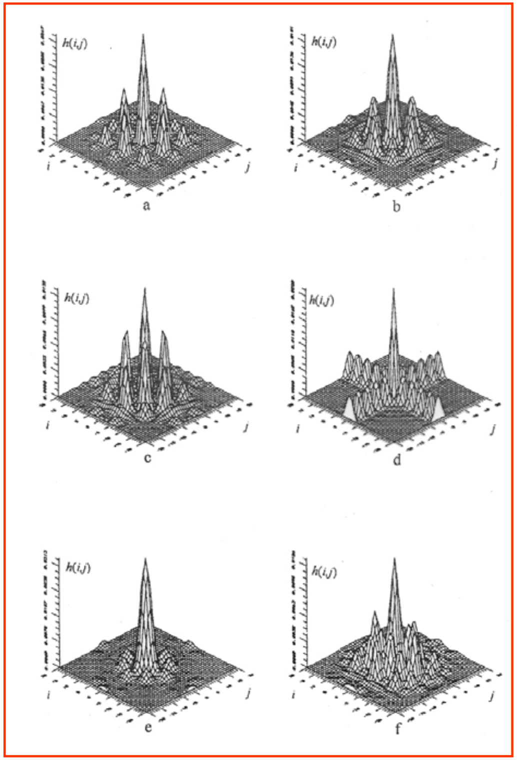


3. Sparse Incoherent Imaging Systems: CFD Magnitude

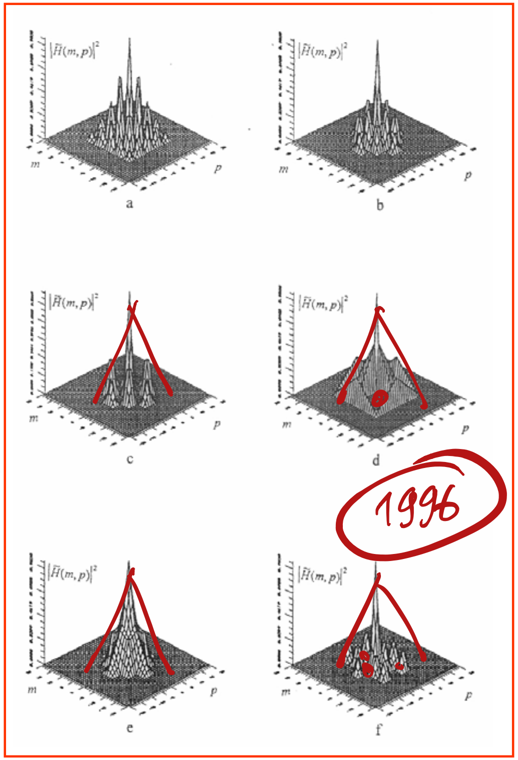
Magnitudes of CFD



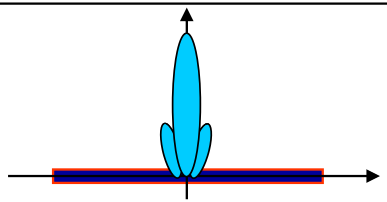
Directional Antenna Patterns

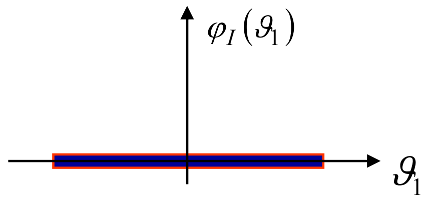


Spatial Spectra

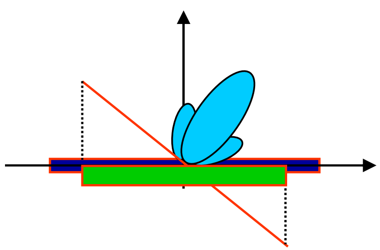


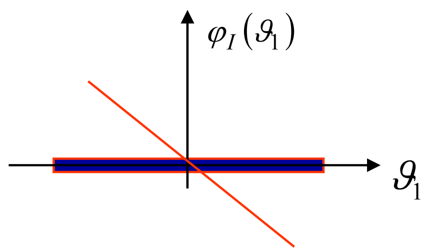
3. Incoherent Imaging Systems: CFD Phase





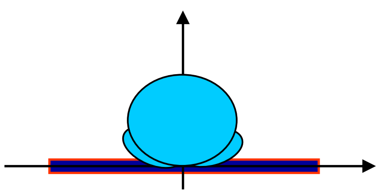
Zero-phased or focused system:
the PSF has the min width and oriented in perpendicular direction to antenna aperture.

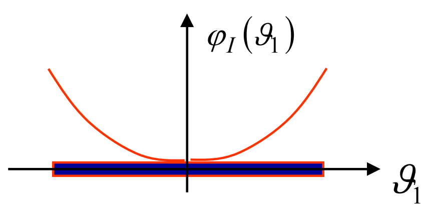




Linear phase delay along the aperture:

- beam scanning (radar mode);
- equivalent antenna size is decreased;
- the beam becomes broader.





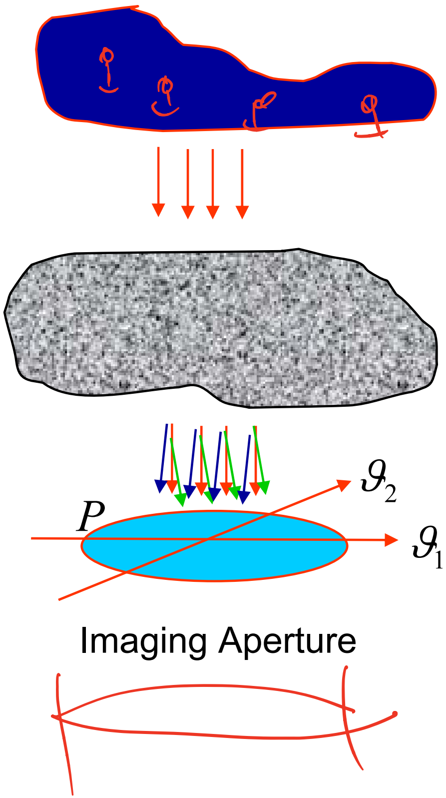
Quadratic phase delay along the aperture (model of defocusing):

- the SAR mode;
- the system becomes unfocused;
- the focusing is done on very far zone.

3. Atmospheric Turbulence Blur

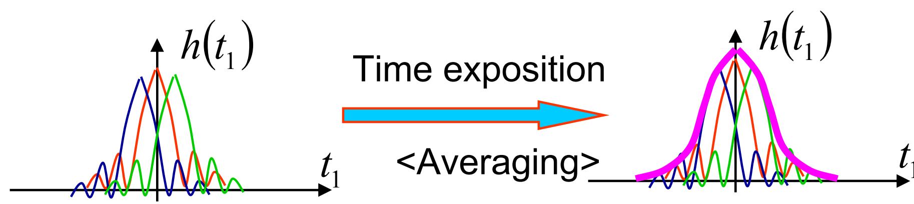
<u>Wave propagation media</u>:

- atmosphere (index of refraction)
- ground
- object subjected to non-destructive testing

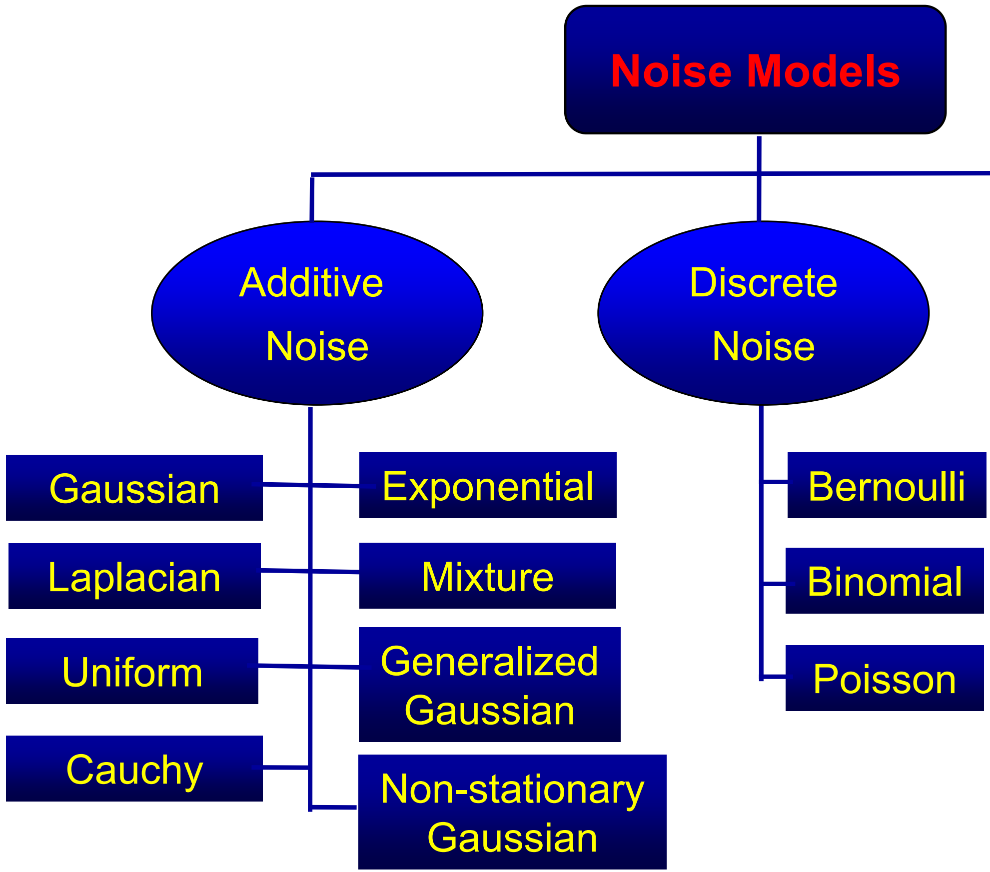


Object

Wave propagation media with time (spatial) varying parameters



4. Classification of Noise in Imaging Systems



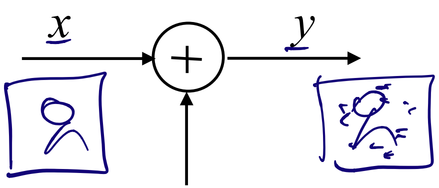
$\underline{y} = H\underline{x} + \underline{z}$

Blurs

Coherent
- radars
- Ultrasound

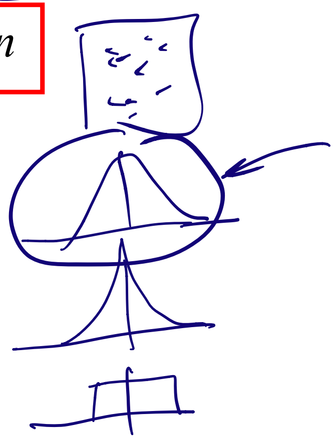
$\underline{y} = H\underline{x} + \underbrace{\underline{n}_1 + \underline{n}_2 + \underline{n}_3 \ldots}_{\underline{n} \sim \text{Gaussian}}$

4. Additive Gaussian Noise



$n \sim$ **i.i.d.** $N(0, \sigma_n^2)$

$$\boxed{y = x + n}$$



- Thermal noise
- This model reflects the limiting behavior of other models.

Properties:

- linear operations (transforms and filtering) on Gaussian R.V. yield Gaussian R.V.
- Central Limit Theorem (CLT)

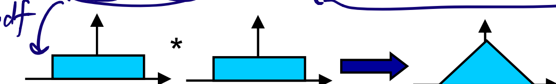
The distribution of sum of a large number of independent, small random variables has a Gaussian distribution.

$$z = n_1 + n_2 \longrightarrow \boxed{p_z = p_{n_1} * p_{n_2}}$$

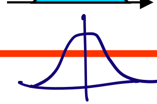
$\mathcal{F}: * \leftrightarrow \odot$

$\odot \leftrightarrow *$

pdf

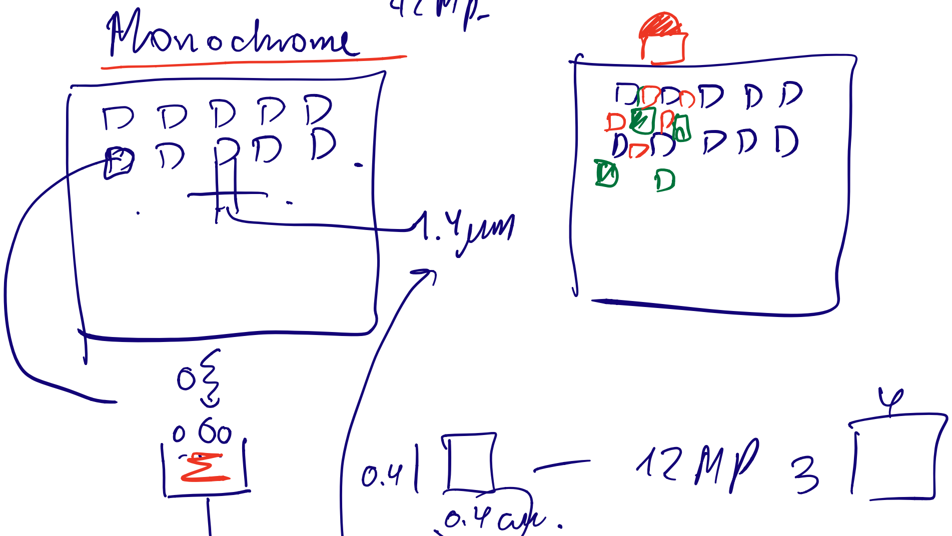


p.d.f.



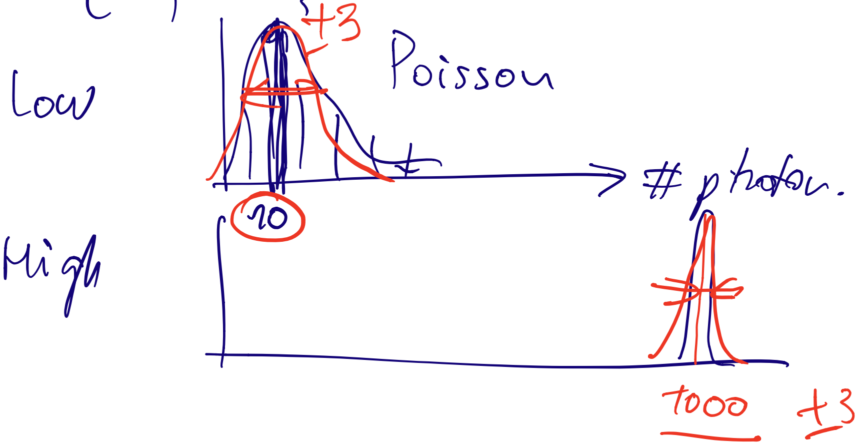
12 Mp

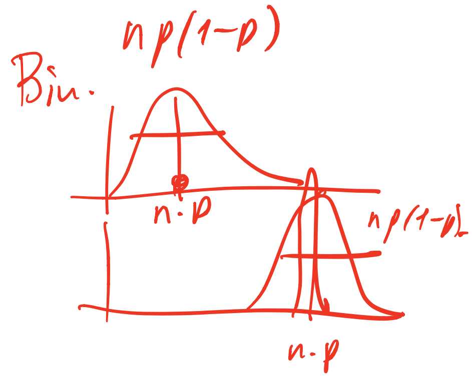
Monochrome

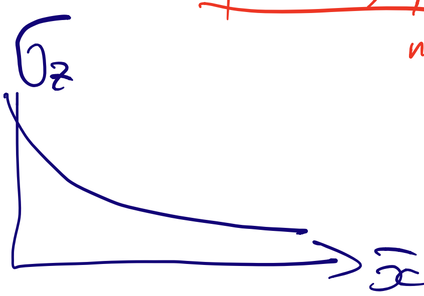


$$\frac{0.4\,\text{cm.}}{3 \cdot 10^3} =$$

(a)

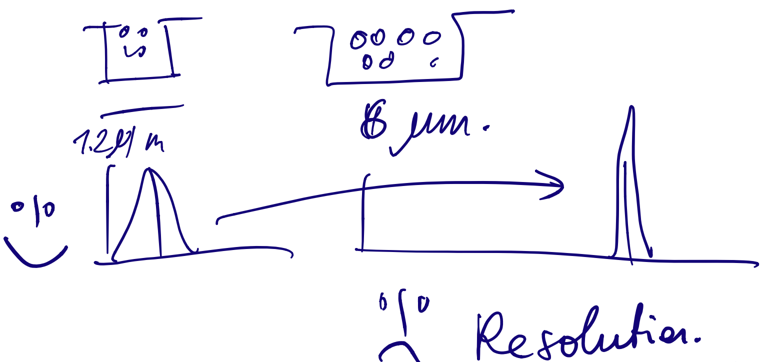






(b)

$\underline{y} = H\underline{x} + \underline{z}$



Resolution.

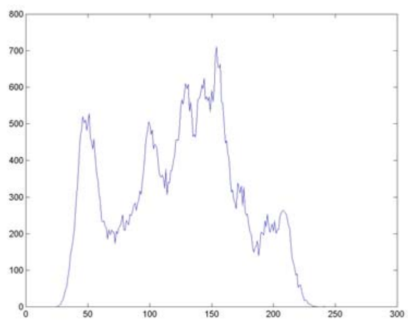
4. Additive Gaussian Noise

<u>CLT further requirements</u>:

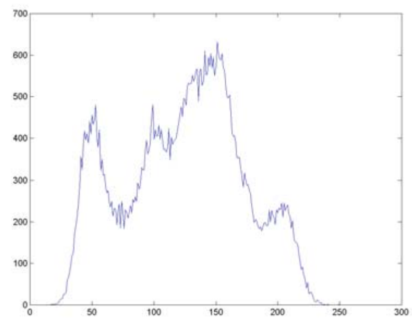
- The number of of R.V. that contribute to the sum should be large enough.
- The individual R.V. in the sum must be independent.
- Each term in the sum must be small, negligible compared to the sum.

4. Additive Gaussian Noise

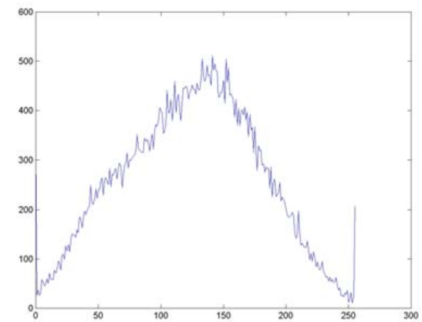
Original Lena



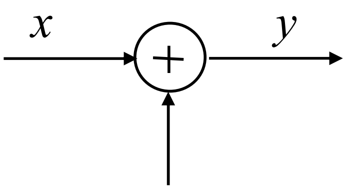
Gaussian noise: var 25



Gaussian noise: var 625



4. Additive Heavy-Tailed Noises



$$\boxed{y = x + n}$$

$n \sim$ **i.i.d.**

- Laplacian
- Exponential
- Mixture
- Cauchy
- Generalized Gaussian

- In many situations, the conditions of the CLT are almost, but not quite, true.

<u>Reasons</u>:

- There may not be a large enough number of terms in the sum.
- The terms may not be sufficiently independent.

<u>Conclusions</u>:

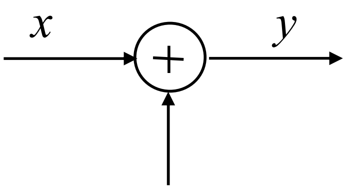
- The Gaussian approximation may not be very accurate.
- Even when the center of the pdf is approximately Gaussian, the tails may not be.

4. Additive Heavy-Tailed Noises

<u>Practical models of</u>:

- outliers in measurement;
- broadcasting noise;
- wave propagation;
- failures in registration equipment;
- failures in networks or modems;
- industrial noise and Electro-Magnetic Compatibility (EMC);
- random processes with partially known statistics (robust modeling);
- failures in scanning equipment (both electronic and mechanic): scanners, CCDs, antennas and so on.

4. Additive Laplacian (Double Exponential) Noise



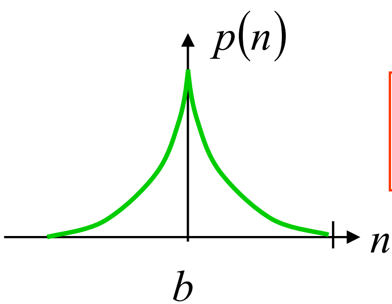
$$\boxed{y = x + n}$$

$n \sim$ **i.i.d.** Laplacian

Features:

- pdf has heavy-tails;
- log-likelihood function of Laplacian pdf is abs function.

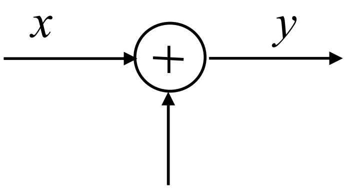
$$\boxed{p(n) = \frac{a}{2} e^{-a|n-b|}} \qquad a > 0, \quad -\infty < b < \infty$$



$$\boxed{E[n] = b, \quad Var[n] = 2/a^2}$$

$$b = 0$$

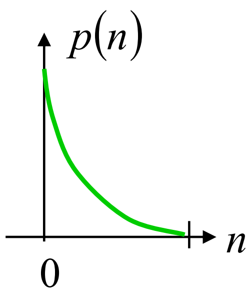
4. Additive Exponential Noise



$$y = x + n$$

$n \sim$ **i.i.d.** Exponential

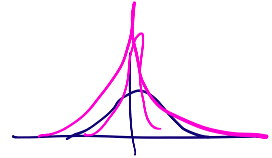
$$p(n) = \begin{cases} a e^{-an}, & x \geq 0, \\ 0, & otherwise. \end{cases} \qquad a > 0$$



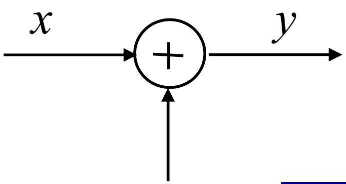
$$E[n] = 1/a, \quad Var[n] = 1/a^2$$

Features:

- pdf has heavy-tails;
- single side (asymmetric) pdf
- the model is used to model non-negative R.V. (radiometry, astronomy noise or false objects like stars with random intensity).



4. Additive Mixture Noise



$$\boxed{y = x + n}$$

$n \sim$ **i.i.d.**

- Mixture
  - Gaussian&Gaussian
  - Gaussian&Laplacian

$$\boxed{p(n) = (1-q)p_0(n) + qp_1(n)}$$

- Mixture Gaussian and Gaussian

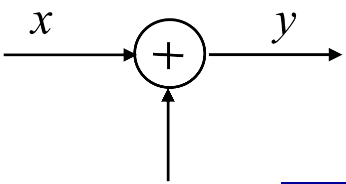
$$p_0(n) = N\left(0, \sigma_0^2\right)$$
$$p_1(n) = N\left(0, \sigma_1^2\right)$$
$$\sigma_1^2 >> \sigma_0^2$$

q is small positive constant that controls the heavy-tails (or outliers).

It is so-called q-contaminated model.

- Mixture Gaussian and Laplacian (so-called Huber model).

4. Additive Cauchy Noise



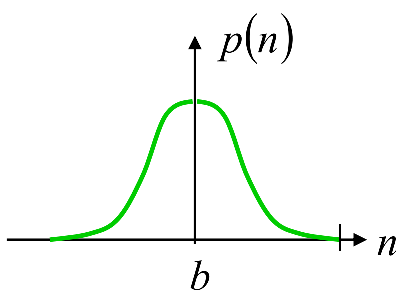
$$\boxed{y = x + n}$$

$n \sim$ **i.i.d.** Cauchy

$$\boxed{p(n) = \frac{1}{\pi}\frac{a}{a^2 + (n-b)^2}}$$

$a > 0, \quad -\infty < b < \infty$

Scale parameter
(but not variance)

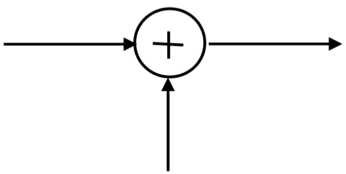


$b = 0$

Features:

- pdf has heavy-tails;
- algebraic pdf (not from exponential family);
- log-likelihood function of Cauchy pdf is close to Gaussian near the origin.

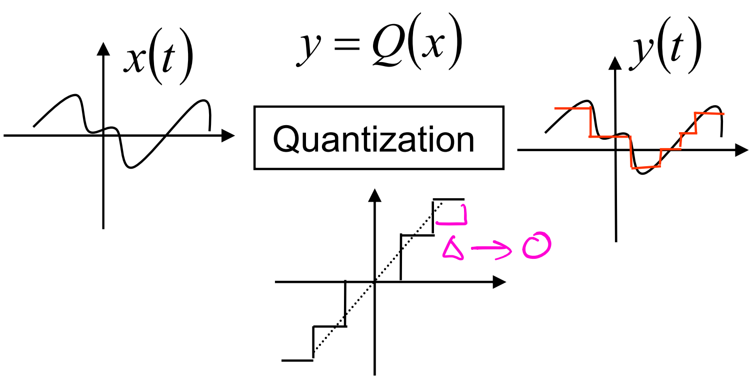
4. Additive Uniform Noise: Model of Quantization



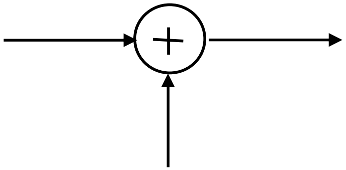
$$\boxed{y = x + n}$$

$n \sim$ **i.i.d.** Uniform

- Quantization noise results when a continuous R.V. is converted to a discrete one.
- Or when a discrete R.V. is converted to one with fewer levels.
- Another source of uniform noise is dithering.



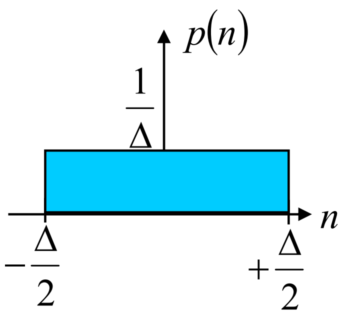
4. Additive Uniform Noise: Model of Quantization



$$y = Q(x) = x + n$$

$n \sim$ **i.i.d.** Uniform

$$p(n) = \begin{cases} 1/\Delta, & -\frac{\Delta}{2} \le n < \frac{\Delta}{2}, \\ 0, & otherwise. \end{cases}$$



- Assume *L* levels are used for quantization.
- If the number of levels is large (so Δ is small), quantization noise is usually assumed to be uniform.

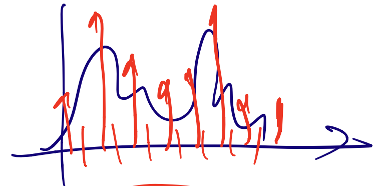
$$E[n] = 0, \quad Var[n] = (b-a)^2/12 = \frac{\Delta^2}{12}$$

- Since $\Delta \sim 2^{-L}, Var[n] \sim \frac{2^{-2L}}{12}$
- Note: this model is only valid for large L. Then noise is signal independent.

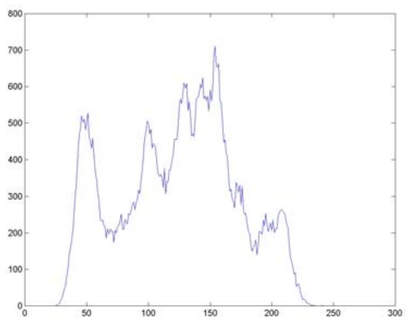
4. Additive Uniform Noise: Example

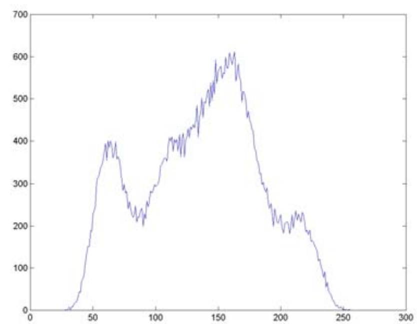
Original Lena

Uniform noise: std 25

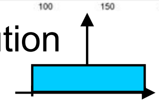


$x_Q + n = x$

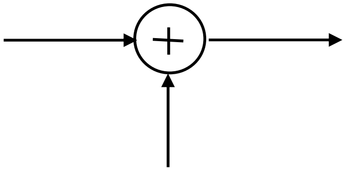




Convolution



4. Generalized Model: Generalized Gaussian Noise



$$y = x + n$$

$n \sim$ **i.i.d.** GG

$$p(n) = \left(\frac{\gamma \eta(\gamma)}{2\Gamma\left(\frac{1}{\gamma}\right)} \right)^{\frac{1}{2}} \cdot \frac{1}{\sqrt{\sigma_n}} \cdot \exp\left\{ -\eta(\gamma) \left| \frac{n}{\sigma_n} \right|^{\gamma} \right\}$$

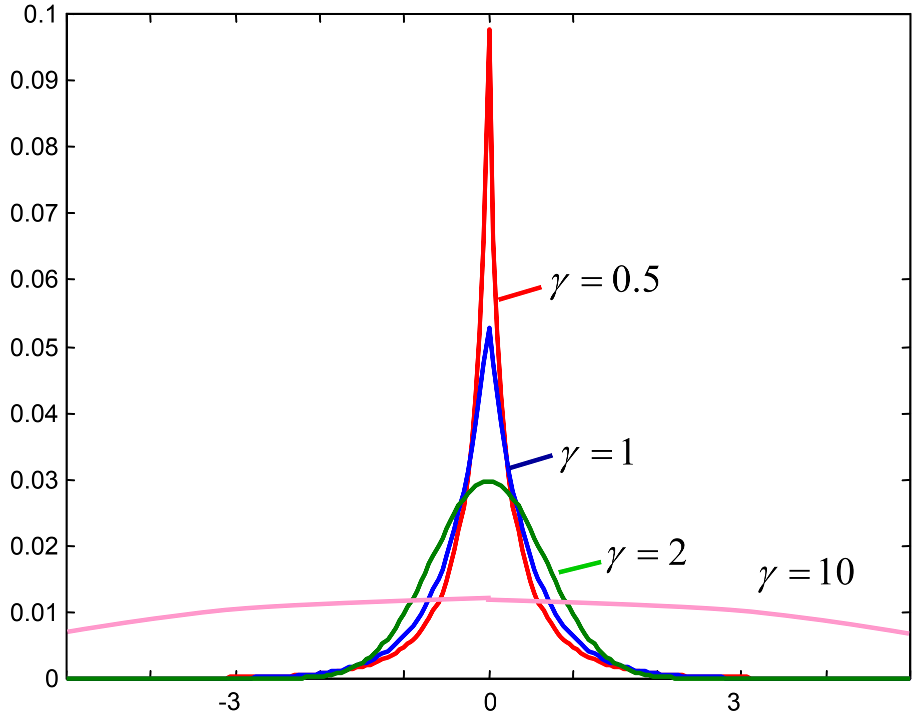
$$\eta(\gamma) = \sqrt{\frac{\Gamma(3/\gamma)}{\Gamma(1/\gamma)}}$$

$$\Gamma(t) = \int_0^{\infty} e^{-u} u^{t-1} du$$

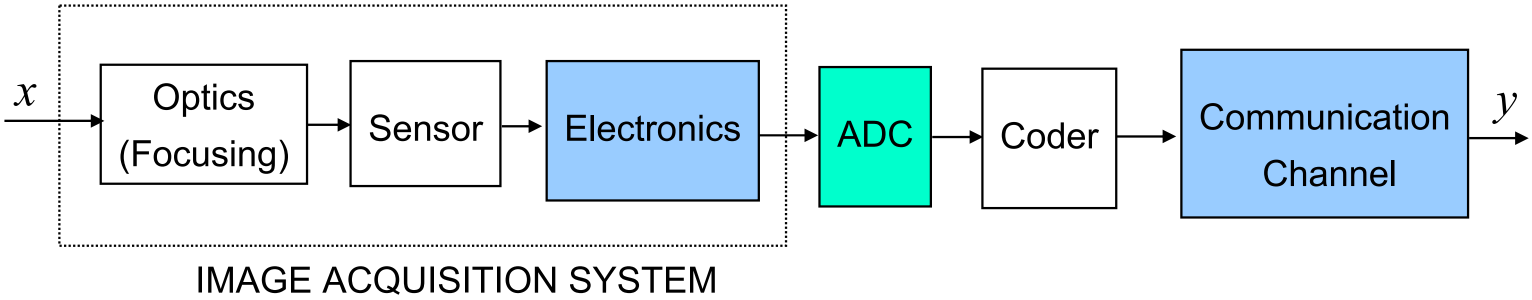
Features:

- Generalized model for many distributions from the exponential family:
  - $\gamma = 2$ Gaussian
  - $\gamma = 1$ Laplacian
  - $\gamma \to \infty$ Uniform

4. Generalized Model: Generalized Gaussian Noise



4. Discrete Noise

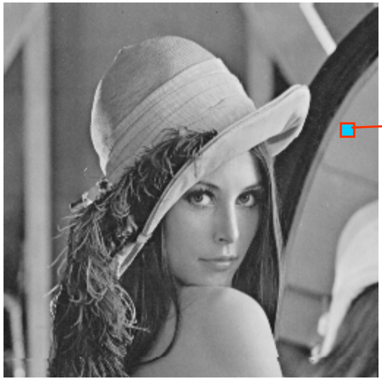


- Failures in the control electronics.
- Failures in Analog-to-Digital Converter (ADC) (mostly on the pixel-wise level).
- Failures in Communication Channel (losses of on the pixel-wise and on package levels).

4. Discrete Noise: General Reasons

- MPEG and DCT-based JPEG: failures and block loosing during transmission.
- Irregular sampling:
  - Dispersion and anisotropy of imaging environment (small particles).
  - Failures in scanners, non-uniform motion of scanning head.
  - Failures in radar imaging systems
    - electronic failures in phase control
    - mechanic failures in the rotation step control.
- The influence of hard-limiters in the receiver entire block of EMC (results in min and max values exceeding the threshold).

4. Discrete Noise



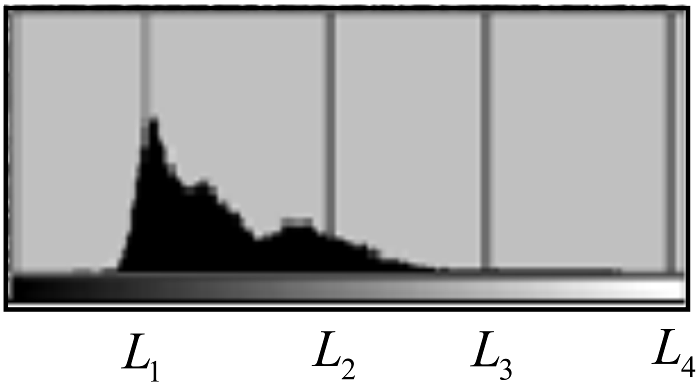
One pixel corresponds to 8 bits

8							1

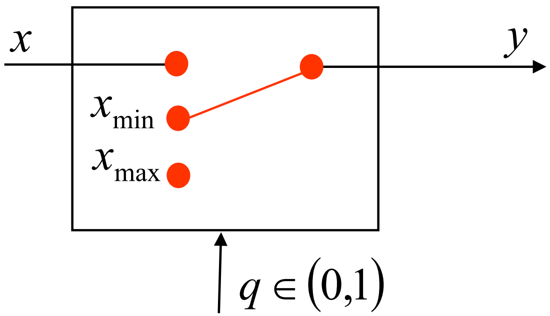
Type of distortion	Structure of the byte
Salt and pepper noise (bimodal) (extreme possible distortions)	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1
Unimodal noise (fixed error byte structure)	1 0 1 1 0 0 0 1
Multimodal noise (more than 2)	1 0 1 1 0 0 0 1 1 1 1 1 1 1 1 1 (more than 2)

4. Discrete Noise: Binomial and Bernoulli Noises

$$y = \begin{cases} x, & \text{Prob}(y = x) = 1 - q, \\ L_1, & \text{Prob}(y = L_1) = q_1, \\ L_2, & \text{Prob}(y = L_2) = q_2, \\ \vdots & \\ L_m, & \text{Prob}(y = L_m) = q_m. \end{cases} \qquad \sum_{i=1}^{m} q_i = 1$$



4. Bernoulli Noise: Salt and Pepper Noise

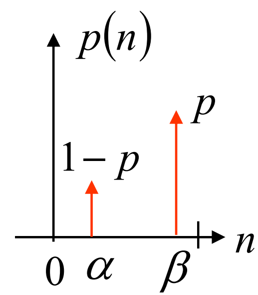


$$n_{S\&P} = \begin{cases} \alpha, \ \mathbf{Prob}(n = \alpha) = 1 - p, \\ \beta, \ \mathbf{Prob}(n = \beta) = p. \end{cases}$$

<u>Salt and Pepper</u>

$$\alpha = x_{\min}$$
$$\beta = x_{\max}$$

$$\boxed{y_{S\&P} = \begin{cases} x, & \mathrm{Prob}(y = x) = 1 - q, \\ x_{\min}, & \mathrm{Prob}(y = x_{\min}) = q/2, \\ x_{\max}, & \mathrm{Prob}(y = x_{\max}) = q/2. \end{cases}}$$



$$p(n) = (1 - p)\delta(n = \alpha) + p\delta(n = \beta)$$

4. Binomial Noise - Unimodal Example

Original image

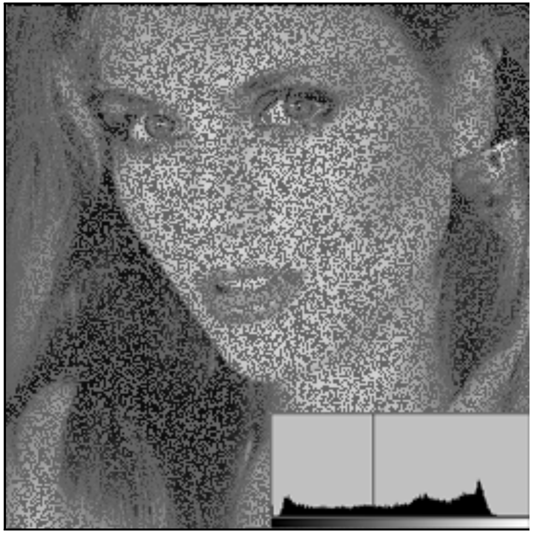
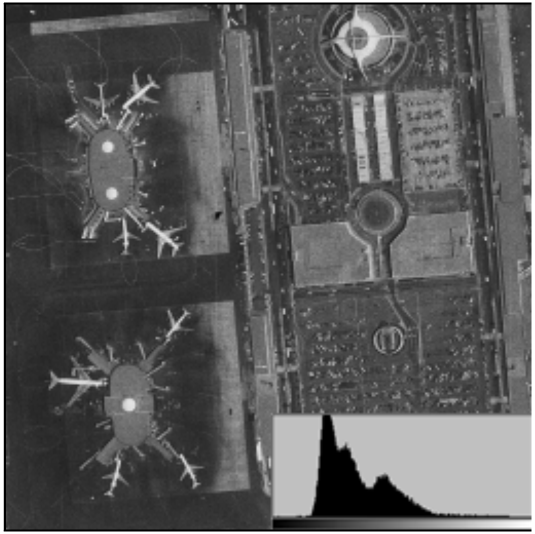


Image corrupted by q=50% unimodal noise (L=100)

4. Binomial Noise - Multimodal Example



Original image

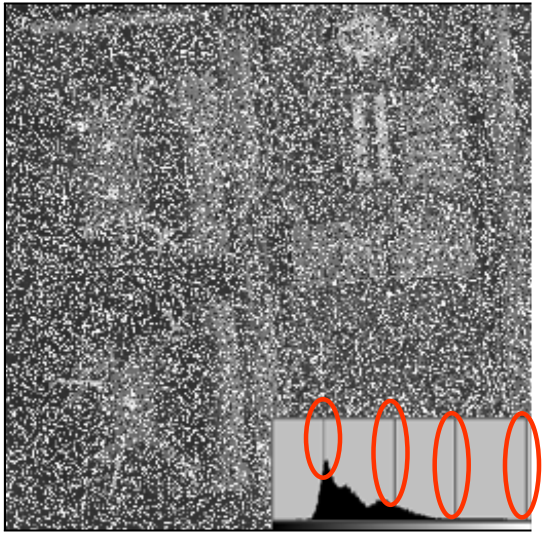


Image corrupted by four-component noise

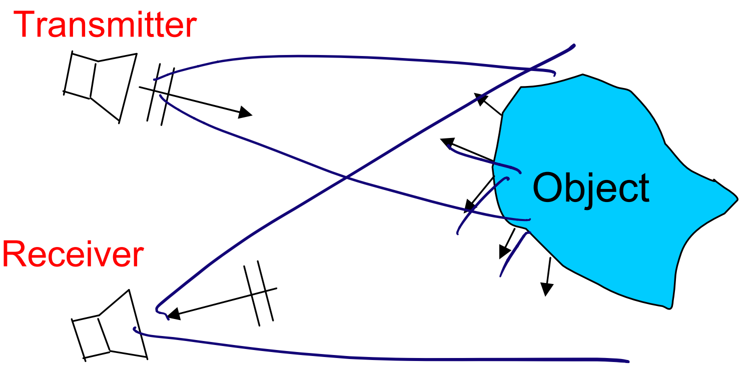
$q = 50\%$

$L_1 = 50$, $L_2 = 120$, $L_3 = 180$, $L_4 = 250$,

$q_1 = q_2 = q_3 = q_4 = 0.25$

4. Speckle noise

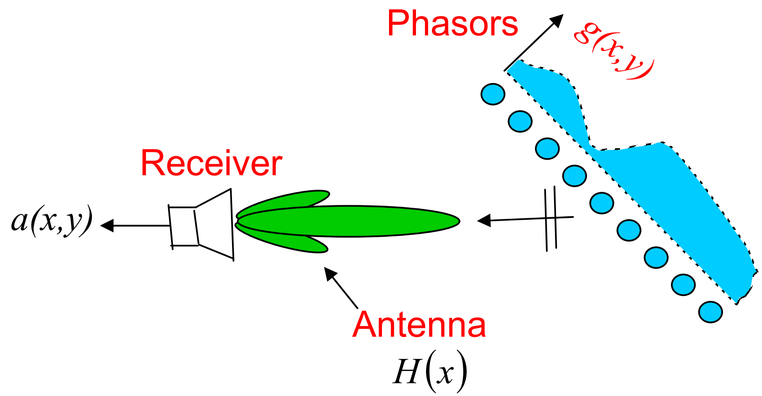
- Radar, sonar, laser, SAR: monochromatic coherent radiation is scattered from a surface whose roughness is of the order of wavelength, interference of waves produces a noise called *speckle*.
- The presence of speckle noise reduces the resolution of imaging system, particularly for low-contrast images.
- Speckle noise is typical for all coherent systems independently on wavelength: microwave (10e-3m), infrared (10e-6m), optical (10e-9m).
- Speckle noise is not additive.



4. Speckle noise

Speckle Representation

In free space, speckle can be considered as an infinite sum of independent, identical phasors with random amplitude and phase $g(x,y)$.

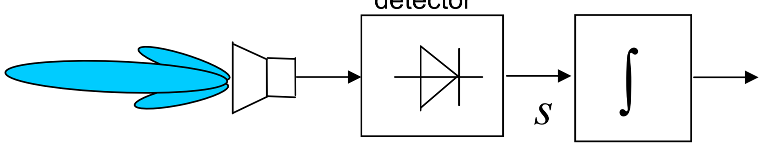


Complex amplitude in receiver: $a(x,y) = a_R(x,y) + ja_I(x,y)$

where a_R and a_I are zero mean, independent Gaussian random variables (for each x,y) with variance σ_a^2

4. Speckle noise

Detector

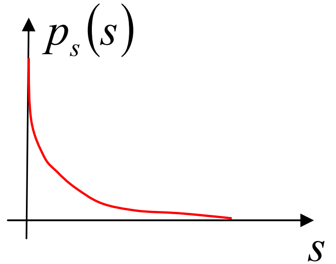


$$|\cdot|^2$$

<u>Intensity field</u>: $s = s(x,y) = |a(x,y)|^2 = a^2{}_R + a^2{}_I$

Has the exponential single sided distribution with variance $\sigma^2 = 2\sigma_a^2$ and mean $\mu_a = E[s] = \sigma^2$

$$p_s(s) = \left\{ \frac{1}{\sigma^2} \exp\left(-\frac{s}{\sigma^2} \right), s \geq 0 \right.$$



4. Speckle noise

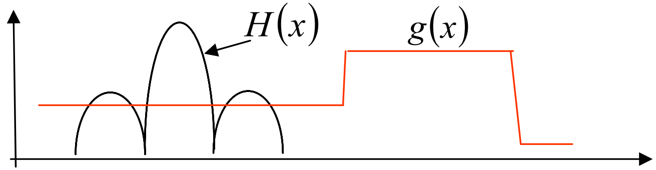
<u>Given</u>: coherent linear imaging system with impulse response $H\left(x, y; x', y'\right)$

$\phi\left(x', y'\right)$ is the phase distribution on the object

$\eta\left(x, y\right)$ is the additive detector noise

$$\boxed{v\left(x, y\right) = \left| \iint H\left(x, y; x', y'\right) g\left(x', y'\right) e^{j\phi\left(x', y'\right)} dx' dy' \right|^2 + \eta\left(x, y\right)}$$

4. Speckle noise



If the impulse response decays rapidly outside a region $R_{cell}(x,y)$ called the resolution cell, and $g(x,y)$ is nearly constant in this region:

$$\boxed{v(x,y) \cong |g(x,y)|^2 |h(x,y)|^2 + \eta(x,y) = u(x,y)s(x,y) + \eta(x,y)}$$

Homomorph.

Multiplicative noise

$$u(x,y) \equiv |g(x,y)|^2$$

$$h(x,y) = \iint H(x,y;x',y')e^{j\phi(x',y')}dx'dy'$$

Image intensity

$\log(a \cdot b) = \log a + \log b$

<u>Conclusions</u>:

- The appearance of of the speckle is due to low resolution of imaging systems.
- There is not speckle in an ideal imaging system.