OPTIMAL PUBLIC EXPENDITURE WITH INEFFICIENT UNEMPLOYMENT

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MAIN IDEA

The paper proposes a theory of optimal public expenditure when unemployment is inefficient.
Embedding Samuelson’s theory into a matching model

- sufficient statistics
  - elasticity of substitution between public and private consumption
  - the unemployment gap
  - the unemployment multiplier

MOTIVATION

Public expenditure is one of main tools used by government to tackle high unemployment

- with efficient unemployment: Samuelson's rule is valid
- with inefficient unemployment: Optimal public expenditure deviates (stimulus spending) from the Samuelson's rule to bring unemployment closer to its efficient level

MODEL

$$Y(t) = C(t) + G(t), Y(t) < k$$

$$u(t) = \frac{[k - Y(t)]}{K}$$

*idle aggregate capacity becomes unemployment rate

Matching function:

$$h(t) = wv(t)^{1-\eta}[k - Y(t)]^{\eta}$$

$$x(t) = \frac{v(t)}{k - Y(t)}$$

$$v_0 = \frac{sY_0}{q(x)}, \rho v_0 = \frac{\rho s Y_0}{q(x)}, Y_0 = [1 + \tau(x)]y_0$$

$$\tau(x) = \frac{\rho s}{q(x) - \rho s}$$

SUFFICIENT STATISTICS

- Efficient Tightness

$$c = \frac{C}{(1 - \tau(x))}, g = \frac{G}{(1 - \tau(x))}, y(x,t) = \frac{1 - u(x)}{1 + \tau(x)} \cdot k$$

the efficient tightness x^* is implicitly defined by:

$$(1 - \eta)u(x^*) - \eta(x^*) = 0$$

SUFFICIENT STATISTICS

- Unemployment Gap

$$u - u^*$$

- Unemployment Multiplier

$$m = -y \cdot \frac{du}{dg}$$

- The elasticity of substitution between public and private consumption

$$\frac{1}{\epsilon} = -\frac{dln(MRS_{gc})}{dln(\frac{g}{c})}$$

DERIVATION OF OPTIMAL PUBLIC EXPENDITURE

- Maximize welfare $U(y(x(g), k) - g, g)$, FOC:

$$0 = \frac{\partial U}{\partial g} - \frac{\partial U}{\partial c} + \frac{\partial U}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}$$

  divide both side by $\frac{\partial U}{\partial c}$

- Lemma 2: Optimal Public Expenditure Satisfies

$$1 = MRS_{gc} + \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}$$

- Lemma 3: $1 - MRS_{gc}$ can be approximated as:

$$1 - MRS_{gc} \approx \frac{1}{\epsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

DERIVATION OF OPTIMAL PUBLIC EXPENDITURE

- $$\frac{x}{y} \cdot \frac{\partial y}{\partial x} \approx \frac{u - u^*}{1 - u^*}$$

$$\frac{y}{x} \cdot \frac{dx}{dg} \approx \frac{m}{(1 - \eta)(1 - u)u}$$

- Lemma 4: Optimal Stimulus Spending satisfies:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \frac{u - u^*}{u^*}$$

$$z_0 = \frac{1}{(1 - \eta)(1 - u^*)^2}$$

this is useful to assess whether current stimulus spending is optimal or not but cannot be used to compute the optimal stimulus spending

MAIN FORMULA

A first order Taylor expansion of u at u_0 yields:

$$u \approx u_0 - constant \cdot m \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{z_0 \epsilon m}{1 + z_1 z_0 \epsilon m^2} \cdot \frac{u_0 - u^*}{u^*}$$

$$z_1 = \frac{g/y - (c/y)^*}{u^*}$$

under the optimal policy, the unemployment rate is:

$$u \approx u^* + \frac{u_0 - u^*}{1 + z_1 z_0 \epsilon m^2}$$

THREE IMPLICATIONS

- Sign of optimal stimulus spending
- Role of the unemployment multiplier

$$\frac{g/c-(g/c)^*}{(g/c)^*}=\frac{1}{2}\cdot\sqrt{\frac{z_0\epsilon}{z_1}}\cdot\frac{u_0-u^*}{u^*}$$

  increase in m for $m\in\ [0,\frac{1}{\sqrt{z_0\epsilon z_1}}]$, decrease in m for $m\in\ [\frac{1}{\sqrt{z_0\epsilon z_1}},+\infty]$

- Role of the elasticity of substitution between public and private consumption

$$\frac{g/c-(g/c)^*}{(g/c)^*}=\frac{1}{z_1 m}\cdot\frac{u_0-u^*}{u^*}$$