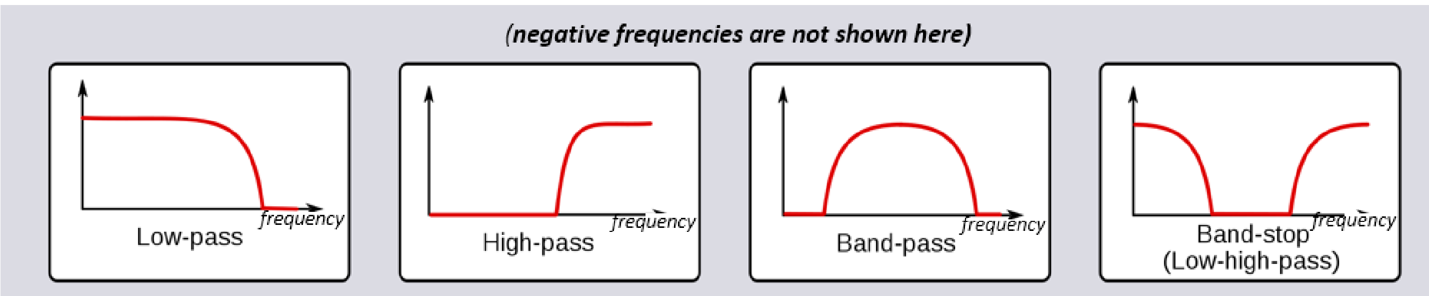
DSP Lab 3 Solutions

Introduction

Digital filters are essential in the many forms of communication we enjoy today. The frequency response of these filters hinges on the number of "taps" or coefficients for these filters. Using the GNURadio companion GUI developed, students were able to get practical experience visualizing the effects that different parameters like cutoff frequency, transition bandwidth, and window type may or may not have on the number of taps a filter system expresses. Students performed this lab using 4 filters, the low pass, high pass, band pass and band reject filters. The window types used in this lab were Rectangular, Hanning, Hamming, Blackman, and Kaiser with value of $\beta = 6.76$.



Results and Discussion

No. of Taps	Transition band width (kHz)	Cut-off Frequency (kHz)	Window Type
77	1	9	Hamming
51	1.5	9	Hamming
39	2	9	Hamming
25	3	9	Hamming
19	4	9	Hamming
11	7	9	Hamming
09	8	9	Hamming
05	15	9	Hamming

Figure 1: Low pass filter

No. of Taps	Transition band width (kHz)	Cut-off Frequency (kHz)	Window Type
77	1.4	7	Blackman
51	2.1	7	Blackman
39	2.7	7	Blackman
25	4.3	7	Blackman
19	5.5	7	Blackman
11	9	7	Blackman
09	13	7	Blackman
05	--	--	Blackman

Figure 2: High Pass filter

No. of Taps	Transition band width (kHz)	Cut-off Frequency (kHz)	Window Type
77	--	5-11	Rectangular
51	0.6	5-11	Rectangular
39	0.8	5-11	Rectangular
25	1.2	5-11	Rectangular
19	1.6	5-11	Rectangular
11	3	5-11	Rectangular
09	3.8	5-11	Rectangular
05	6	5-11	Rectangular

Figure 3: Band Pass filter

No. of Taps	Transition band width (kHz)	Cut-off Frequency (kHz)	Window Type
77	0.84	5-11	Hanning
51	1.25	5-11	Hanning
39	1.67	5-11	Hanning
25	2.5	5-11	Hanning
19	3.3	5-11	Hanning
11	6	5-11	Hanning
09	7	5-11	Hanning
05	13	5-11	Hanning

Figure 4: Band Reject filter

Remarks:

In the ideal condition, the **number of taps** is not **dependent** on the **center frequency** only but precisely depend on the **transition width** (or more accurately, on the **ratio** of the **sampling rate** to **the transition width**). For the demonstration purposes, Let's take constant cut off of 14 kHz and then vary the transition BW, that will change your No. of Taps.

No. of Taps	Transition BW	Cut off
77	1000	14000
51	1500	14000
39	2000	14000
25	3000	14000
19	4000	14000
11	6500	14000
09	8000	14000
05	1300	14000

This is expected since M is the window length in the time domain and the transition bandwidth is inversely proportional to the window length. M + 1 is the number of coefficients or taps, therefore it follows that the transition band width would be inversely proportional to the number of taps.

Using a fixed filter (a low pass filter was chosen for ease of measurement), the stopband attenuation was measured for each window type using a transition bandwidth of 2kHz and a cutoff frequency of 9kHz.

As seen in figure 5, using a rectangular window, the stopband attenuation was measured to be -20dB, which would amount to the ability to handle 1% error. The number of taps seen in the rectangular at transition bandwidth of 2kHz was 39, and this value did not change by varying the windowing type.

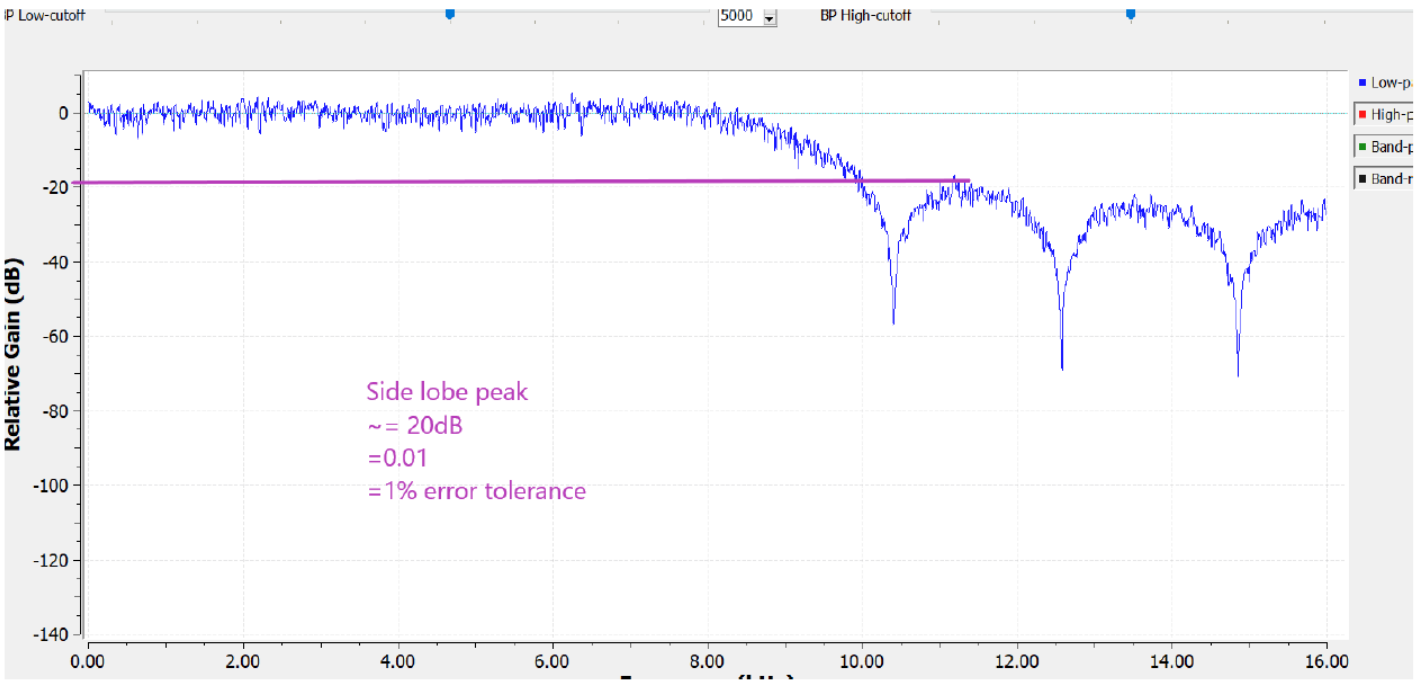


Figure 5: Rectangle window

As seen in Figure 6, the stopband attenuation of the Hanning window is -45dB, which allows for 0.003% error tolerance. The ripple effect for the Hanning window is less than with the Rectangular window.

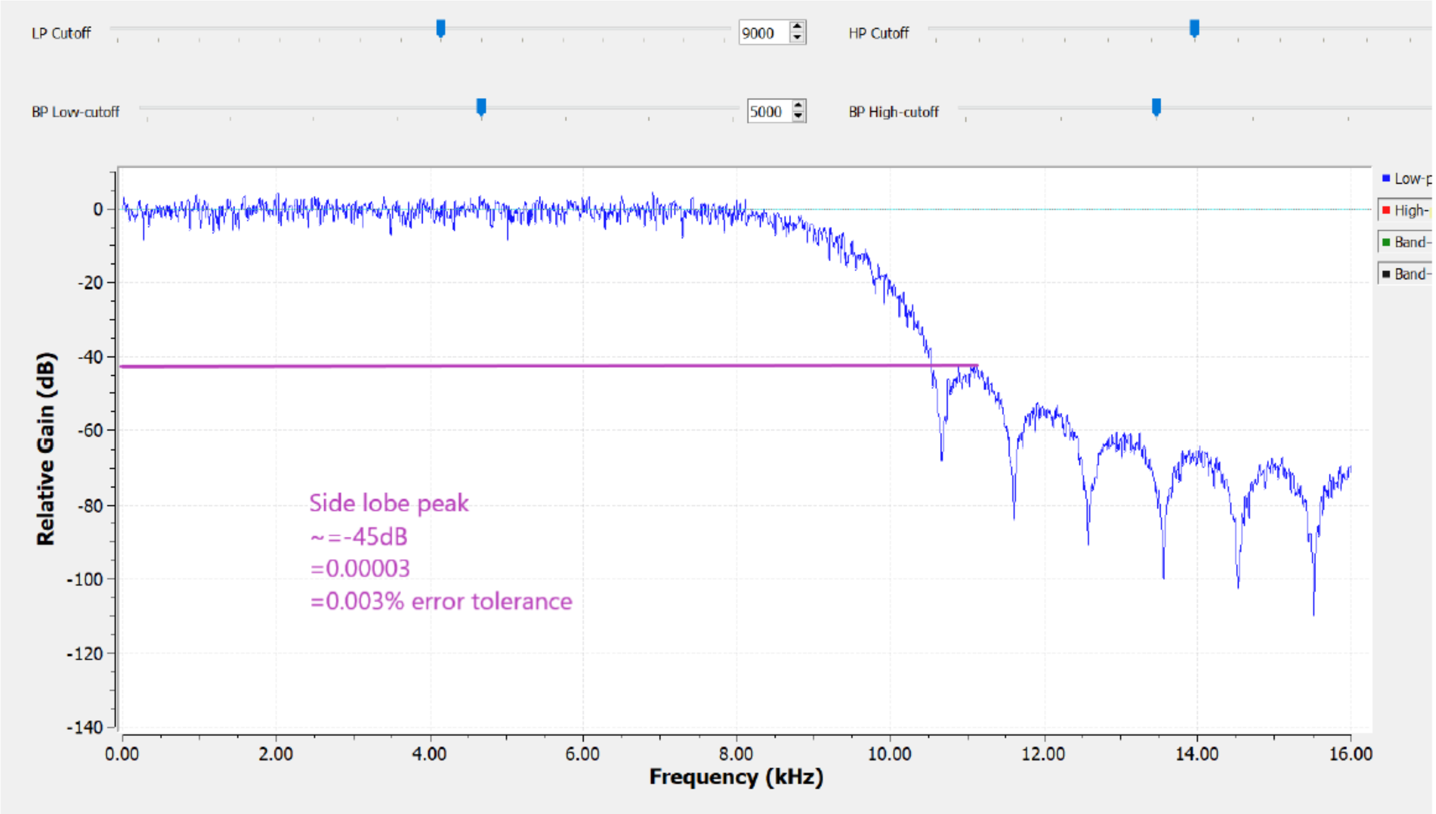


Figure 6: Hanning window

In Figure 7 it can be seen that the stopband attenuation of the Hamming window is -50dB, which allows for 0.001% error tolerance, a little better than the Hanning window. The ripple effect for the Hamming window is less than with the Hanning window.

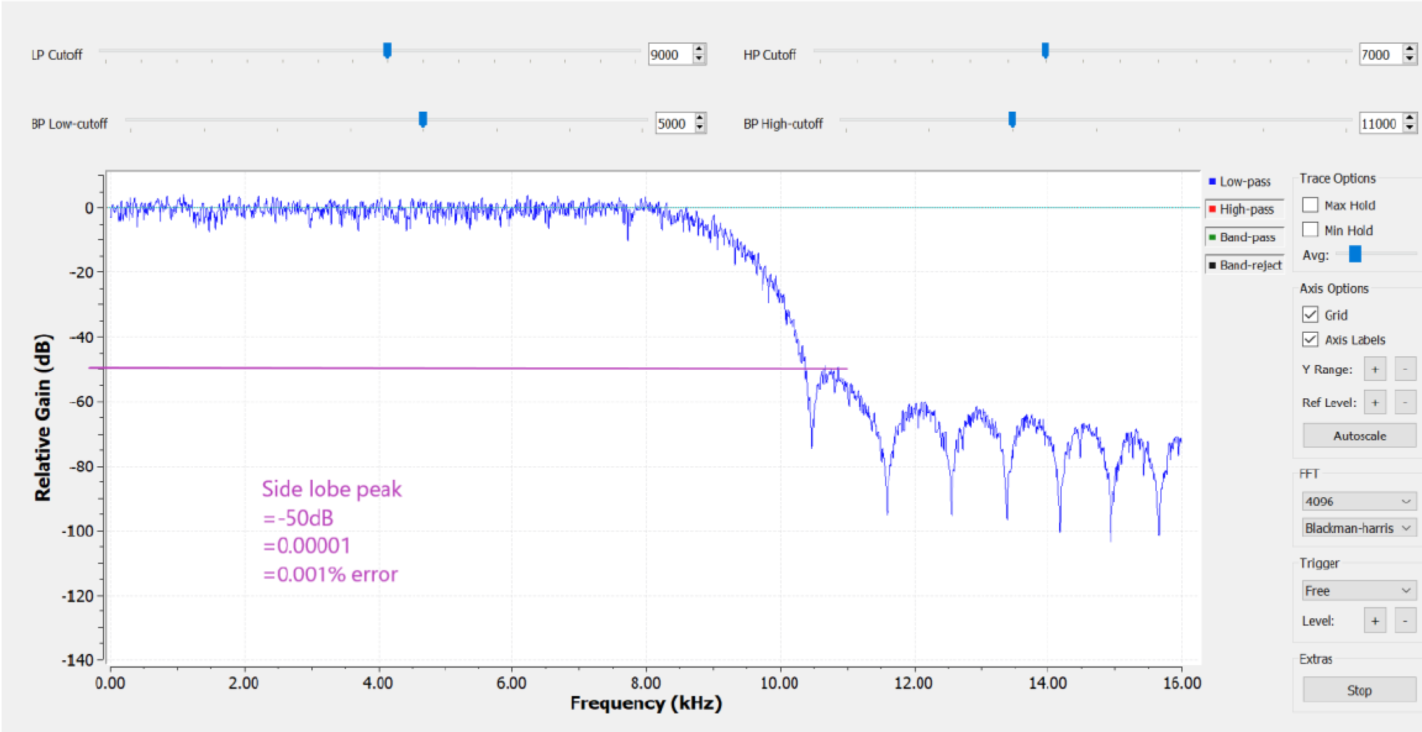


Figure 7: Hamming window

The Blackman window seen in Figure 8 has a stopband attenuation of -70dB with an error tolerance of 1e-5 %. The ripple effect in the Blackman window is even less than with the Hamming window.

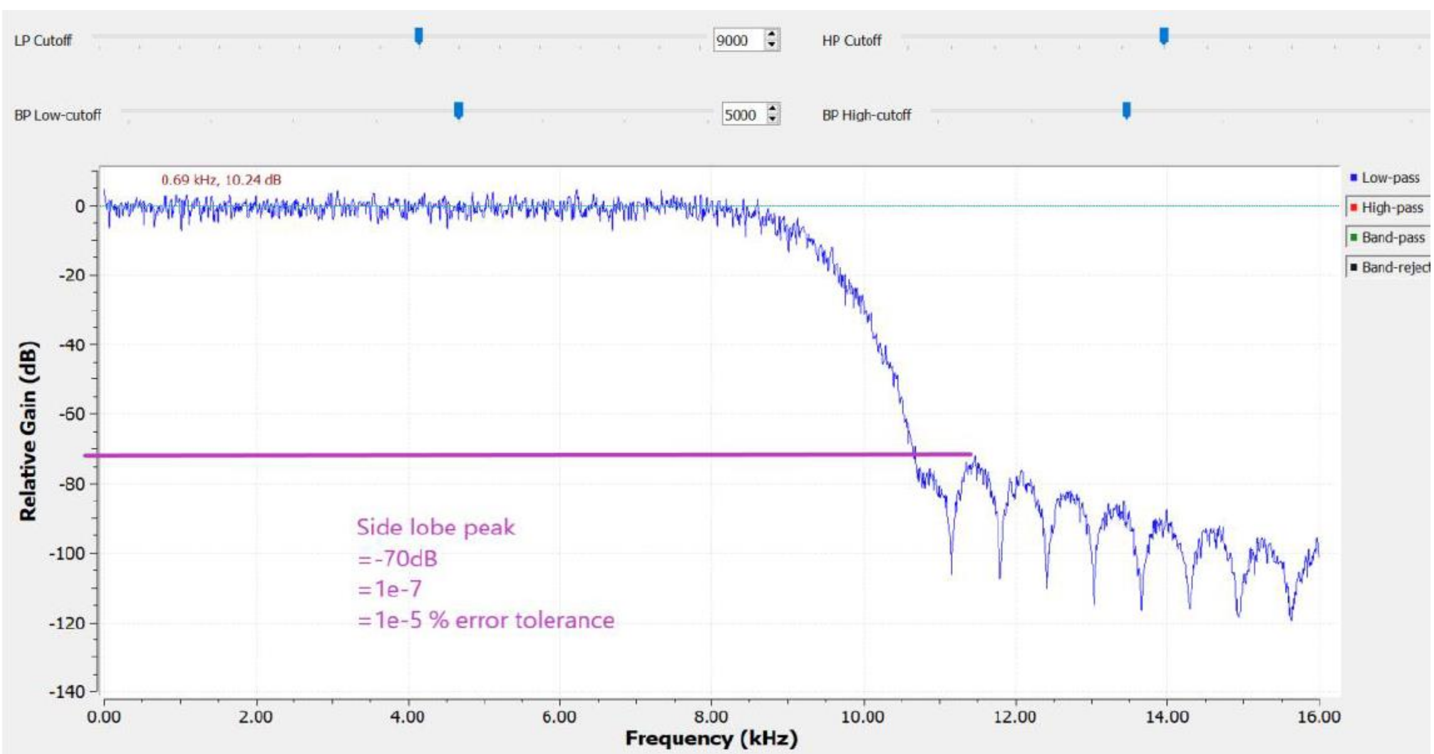


Figure 8: Blackman window

Optional:

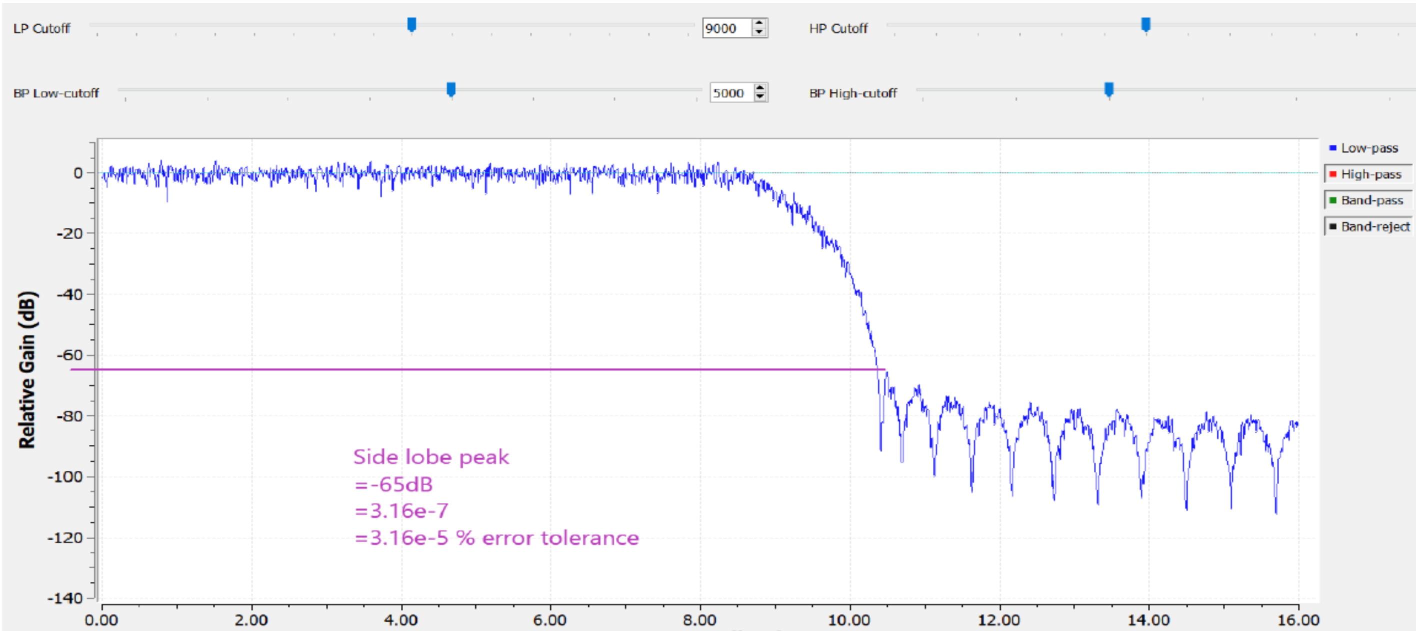
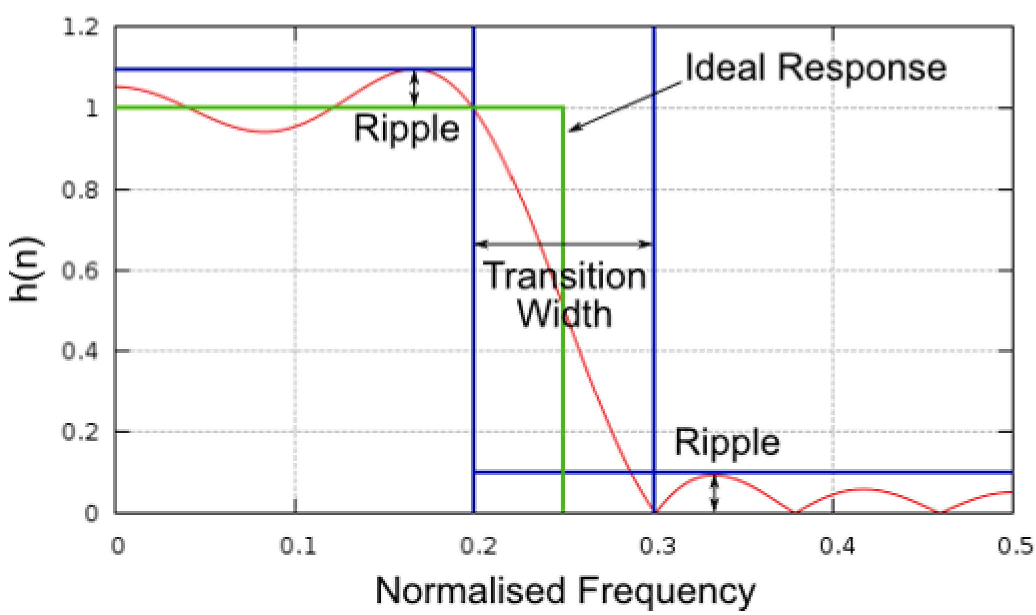


Figure 9: Kaiser with beta = 6.76

Lastly the frequency response of the Kaiser window with β= 6.76 on the Low pass filter is seen in figure 9. The stopband attenuation here is -65dB, which is slightly worse than the Blackman window. This is to be expected since the Blackman window performance lies between the β= 7 and β= 8 Kaiser window with regard to stopband attenuation.

Remarks:

Let's visualize transiton width. In the diagram below, the green line represents the ideal response for transitioning between a passband and stopband, which essentially has a transition width of zero. The Red line demonstrates the result of a realistic filter, which has some ripple and a certain transition width.



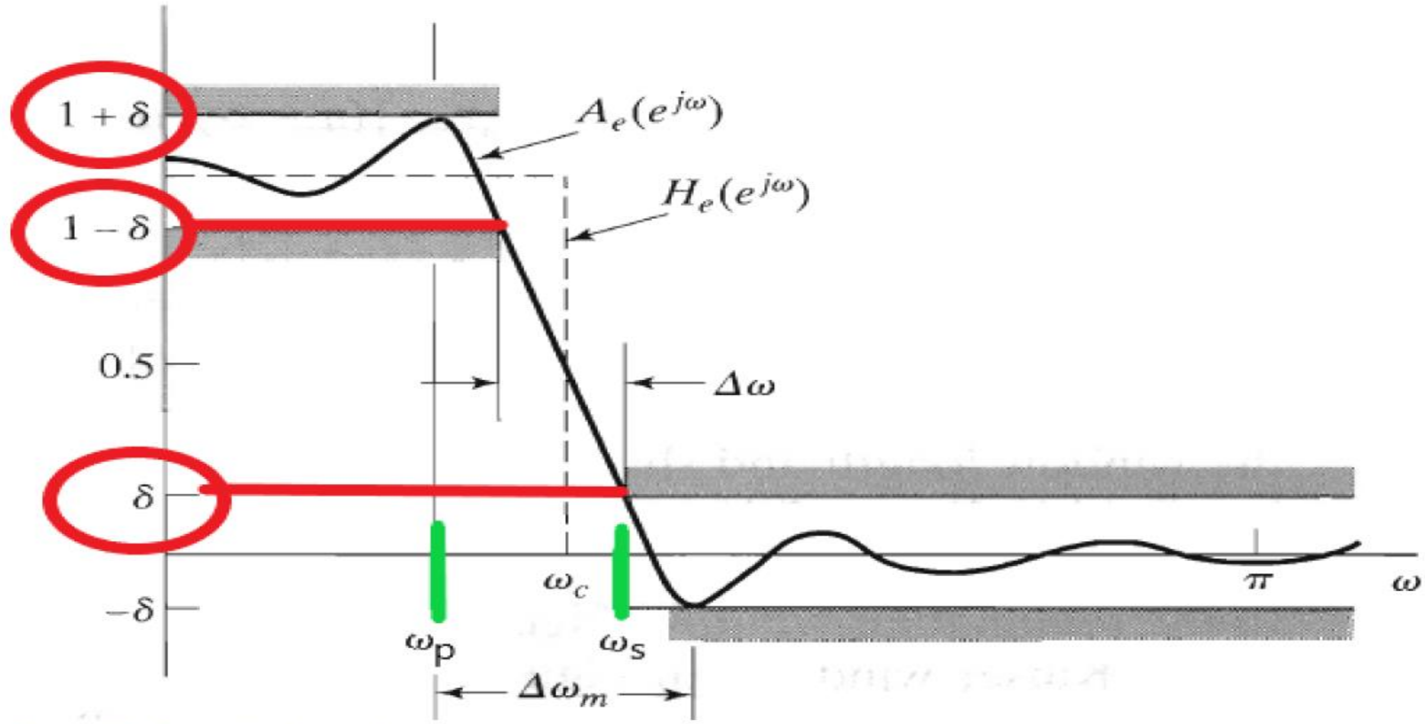


Figure 10: Transition band limits

The name of the separation between the passband and stopband in FIR filter designs is called the transition band. This band extends from (1 – δ) to δ on the vertical axis and from ω_p to ω_s on the horizontal axis, as seen in figure 10 below. The ripple or oscillations around the $H(e^{j\omega})$ ideal filter dotted line seen in the figure are determined by the window type (or value of β for Kaiser windowing method), with the ripple being no greater than the value of δ.

Table 7.2 COMPARISON OF COMMONLY USED WINDOWS

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Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20 \log_{10} \delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hann	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

Discrete-Time Signal Processing, Third Edition
Alan V. Oppenheim • Ronald W. Schafer

Lesson Learned:

The lesson is that the FIR filter requires vastly more computational resources than the IIR to perform roughly the same filtering operation.

Here are some real-world examples of FIR and IIR filters that you may have used before.

If you perform a "moving average" across a list of numbers, that's just an FIR filter with taps of 1's: - h = [1 1 1 1 1 1 1 1 1 1] for a moving average filter with a window size of 10. It also happens to be a low-pass type filter; why is that? What's the difference between using 1's and using taps that decay to zero?

Answer:

A moving average filter is a low-pass filter because it smooths out "high frequency" changes, which is usually why people will use one. The reason to use taps that decay to zero on both ends is to avoid a sudden change in the output, like if the signal being filtered was zero for a while and then suddenly jumped up.

Now for an IIR example. Have any of you ever done this:

x = x*0.99 + new_value*0.01

where the 0.99 and 0.01 represent the speed the value updates (or rate of decay, same thing). It's a convenient way to slowly update some variable without having to remember the last several values. This is actually a form of low-pass IIR Filter. Hopefully you can see why IIR filters have less stability than FIR. Input values never fully go away!