

## 1 One Pole

$$\begin{aligned}H(s) &= \frac{1}{s/p - 1} = \frac{p}{s - p} \\|H(j\omega)|^2 &= \frac{p^2}{\omega^2 + p^2} \\ \int_0^\infty |H(j\omega)|^2 d\omega &= \frac{p}{4}\end{aligned}$$

## 2 Two Poles

$$\begin{aligned}H(s) &= \frac{1}{(s/p_1 - 1)(s/p_2 - 2)} \\ &= p_1 p_2 \frac{1}{s^2 - s(p_1 + p_2) + p_1 p_2} \\ |H(j\omega)|^2 &= (p_1 p_2)^2 \frac{1}{(p_1 p_2 - \omega^2)^2 + \omega^2 (p_1 + p_2)^2} \\ &= (p_1 p_2)^2 \frac{1}{\omega^4 + \omega^2 (p_1^2 + p_2^2) + (p_1 p_2)^2} \\ &= (p_1 p_2)^2 \frac{1}{p_2^2 - p_1^2} \left( \frac{1}{\omega^2 + p_1^2} - \frac{1}{\omega^2 + p_2^2} \right) \\ \int_0^\infty |H(j\omega)|^2 d\omega &= \frac{(p_1 p_2)^2}{p_2^2 - p_1^2} \left( \frac{1}{4p_1} - \frac{1}{4p_2} \right) \\ &= \frac{p_1 p_2}{4(p_2^2 - p_1^2)} (p_2 - p_1) = \frac{p_1 p_2^2 - p_1^2 p_2}{4(p_2^2 - p_1^2)}\end{aligned}$$

### 3 Two Poles, One Zero

$$\begin{aligned}
 H(s) &= \frac{s/z - 1}{(s/p_1 - 1)(s/p_2 - 2)} \\
 &= \frac{p_1 p_2}{z} \frac{s - z}{s^2 - s(p_1 + p_2) + p_1 p_2} \\
 |H(j\omega)|^2 &= \left( \frac{p_1 p_2}{z} \right)^2 \frac{\omega^2 + z^2}{(p_1 p_2 - \omega^2)^2 + \omega^2 (p_1 + p_2)^2} \\
 &= \left( \frac{p_1 p_2}{z} \right)^2 \frac{\omega^2 + z^2}{\omega^4 + \omega^2 (p_1^2 + p_2^2) + (p_1 p_2)^2} \\
 &= \left( \frac{p_1 p_2}{z} \right)^2 \frac{1}{p_2^2 - p_1^2} \left( \frac{z^2 - p_1^2}{\omega^2 + p_1^2} - \frac{z^2 - p_2^2}{\omega^2 + p_2^2} \right) \\
 \int_0^\infty |H(j\omega)|^2 d\omega &= \frac{(p_1 p_2)^2}{z^2 (p_2^2 - p_1^2)} \left( \frac{z^2 - p_1^2}{4p_1} - \frac{z^2 - p_2^2}{4p_2} \right) \\
 &= \frac{p_1 p_2}{4z^2 (p_2^2 - p_1^2)} \left( z^2 (p_2 - p_1) + p_1 p_2^2 - p_2 p_1^2 \right)
 \end{aligned}$$