Definitions[1]

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February 8, 2020

[1] 24.979: Topics in semantics

Getting high: Scope, projection, and evaluation order

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1 *Tower notation*

(1) Tower values (def.)

$$\frac{f\ []}{x} \coloneqq \lambda k\ .\ f\ (k\ x)$$

(2) Tower types (def.)

$$\frac{\mathsf{b}}{\mathsf{a}} \coloneqq (\mathsf{a} \to \mathsf{b}) \to \mathsf{b}$$

N.b. we also use $\mathsf{K_b\ a}$ as an abbreviation for types of the form $(\mathsf{a} \to \mathsf{b}) \to \mathsf{b}$.

2 *Composition rules*

2.1 *Bidirectional Function Application (FA)*

(3) Function Application (FA) (def.)

a. $f\ \mathsf{A}\ x \coloneqq f\ x$ $\qquad \mathsf{A} : (\mathsf{a} \to \mathsf{b}) \to \mathsf{a} \to \mathsf{b}$

b. $x\ \mathsf{A}\ f \coloneqq f\ x$ $\qquad \mathsf{A} : \mathsf{a} \to (\mathsf{a} \to \mathsf{b}) \to \mathsf{b}$

2.2 *LIFT*

LIFT (a generalization of *Montague lift*) lifts a value into a trivially continuized value.

(4) LIFT (def.)

$a^{\uparrow} \coloneqq \lambda k\ .\ k\ a \qquad (\uparrow) : \mathsf{a} \to \mathsf{K_t\ a}$

(5) LIFT (tower ver.)

$$a^{\uparrow} \coloneqq \frac{[]}{a}$$

Since LIFT is polymorphic, we can use it to lift continuized values – we call this *external lift* (although it's really just LIFT).

2.3 *Scopal Function Application (SFA)*

(6) Scopal Function Application (SFA) (def.)
$m \mathsf{S} n \coloneqq \lambda k \,.\, m\ (\lambda a \,.\, n\ (\lambda b \,.\, k\ (a \mathsf{A} b)))$

(7) Scopal Function Application (SFA) (tower ver.)
$$\frac{f\ []}{x} \mathsf{S} \frac{g\ []}{y} \coloneqq \frac{f\ (g\ [])}{x \mathsf{A} y}$$

2.4 LOWER

(8) LOWER (def.)
$m^{\downarrow} \coloneqq m\ id$
$\mathsf{LOWER} : ((\mathtt{a} \to \mathtt{a}) \to \mathtt{a}) \to \mathtt{a}$

(9) LOWER (tower ver.)
$$\left(\frac{f\ []}{p}\right)^{\downarrow} = f\ p \qquad (\downarrow) : \mathtt{K_t\ t} \to \mathtt{t}$$