Stochastic random process HW1 陳劭珩 110064533

1. $X \sim Exp(\lambda)$, $Y = X^2$, find $E[Y] = ?$

由定義 $E[Y] = \int_{-\infty}^{\infty} Y \cdot f_Y(y)\,dy$

令 $Y = g(x) = x^2$

$E[Y] = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$

$= \int_0^{\infty} x^2 \cdot \lambda e^{-\lambda x}\,dx$

$= \lambda \cdot \int_0^{\infty} x^2 e^{-\lambda x}\,dx$

+	x^2	$e^{-\lambda x}dx$
−	$2x$	$-\frac{1}{\lambda}e^{-\lambda x}$
+	2	$\frac{1}{\lambda^2}e^{-\lambda x}$
−	0	$\frac{1}{-\lambda^3}e^{-\lambda x}$

$\xrightarrow{\text{分部積分}} = \lambda \cdot \left[x^2 \cdot \frac{1}{-\lambda} e^{-\lambda x}, -2x \cdot \frac{1}{\lambda^2} e^{-\lambda x} + 2 \cdot \frac{1}{-\lambda^3} e^{-\lambda x} - 0\right]\Big|_0^{\infty}$

$= \lambda \cdot \left[(0-0-0-0) - (0 - 0 - \frac{2}{\lambda^3} - 0)\right]$

$= \frac{2}{\lambda^2}$ #

2. X是負非 continuous r.v., pdf為 $f_X(x)$, cdf為 $F_X(x)$.

請由期望值定義試證 $E[X] = \int_0^{\infty} (1 - F_X(x))\,dx$ = cdf曲線上面積 (到Y軸, $Y=1$)

由 expectation 定义.

$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx \xrightarrow[\text{by parts}]{\text{integration}}$ $u = x$ $du = 1 \cdot dx$, $dv = f_X(x)dx$ $v = F_X(x)$

$= u \cdot v - \int v\,du = x\,F_X(x)\Big|_0^{\infty} - \int_0^{\infty} F_X(x)\,dx$

$= x\Big|_0^{\infty} \cdot F_X(x)\Big|_0^{\infty} - \int_0^{\infty} F_X(x)\,dx$

$= x\Big|_0^{\infty} \cdot 1 - \int_0^{\infty} F_X(x)\,dx = \int_0^{\infty} 1\,dx - \int_0^{\infty} F_X(x)\,dx$

$= \int_0^{\infty} (1 - F_X(x))\,dx$ 得證 #