CS 5291: Stochastic Processes for Networking

HW4

1. Suppose that X_1 and X_2 are two independent nonnegative continuous random variables with probability density functions $f_1(x) = \lambda_1 e^{-\lambda_1 x}$ and $f_2(x) = \lambda_2 e^{-\lambda_2 x}$
   (a) Derive the expected value $E[\min(X_1, X_2)]$.
   (b) Derive the expected value $E[\max(X_1, X_2)]$.

2. A certain scientific theory supposes that mistakes in cell division occur according to a Poisson process with rate 2.5 per year, and that an individual dies when 196 such mistakes have occurred. Assuming this theory, find
   (a) the mean lifetime of an individual
   (b) the variance of the lifetime of an individual
   (c) an approximate of the probability that an individual dies before age 67.2
   Hint: Use the central limit theorem. Suppose the value of the complementary cumulative distribution function of the normal distribution, $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-0.5z^2} dz$, can be known by table lookup.

3. Consider a Poisson process $N(t)$ is decomposed into two counting processes $N_1(t)$ and $N_2(t)$ with probability p and $1 - p$, respectively.
   (a) Find the joint probability mass function $\Pr\{N_1(t) = n, N_2(t) = m\}$
   (b) Prove that $N_1(t)$ and $N_2(t)$ are independent of each other. That is, $\Pr\{N_1(t) = n, N_2(t) = m\} = \Pr\{N_1(t) = n\} \cdot \Pr\{N_2(t) = m\}$.

4. On Friday night, the number of customers arriving at a lounge bar can be modeled by a Poisson process with intensity that twelve customers per hour.
   (a) Find the probability that there are 2 customers between 21:00 and 21:40.
   (b) Find the probability that there are 4 customers between 21:00 and 21:40 and 6 customers between 21:40 and 22:00.