Stochastic random process for networking 110064533

1. show that $P\{X_1 > s, X_2 > t\} = e^{-\lambda_1 s - \lambda_2 t - \lambda_3 \cdot \max(s,t)}$

X_1, X_2 denote 1^{st}, 2^{nd} component's survival time

type 1 shock ~ Poisson(λ_1)		1^{st} fail
2	λ_2	2^{nd} fail
3	λ_3	both fail

survival time 大於 s or t 代表 1^{st} fail 發生在 s or t 之後

let $S_{1,i}$ 代表 1^{st} type i shock 的 arrival time, i=1,2,3

$$P\{X_1 > s, X_2 > t\} = P\{\underbrace{S_{1,1} > s}_{\text{type 1 shock 比 s 晚到}}, \underbrace{S_{1,2} > t}_{\text{type 2 shock 比 t 晚到}}, \underbrace{S_{1,3} > s, S_{1,3} > t}_{\text{type 3 shock 比 s 跟 t 都晚到}}\}$$

$\boxed{S_{1,i} \sim \text{Erlang}(1, \lambda_i)}$

$$= P\{S_{1,1} > s\} \cdot P\{S_{1,2} > t\} \cdot P\{S_{1,3} > s, S_{1,3} > t\}$$

$$= e^{-\lambda_1 s} \cdot e^{-\lambda_2 t} \cdot \underbrace{P\{S_{1,3} > \max(s,t)\}}_{= e^{-\lambda_3 \cdot \max(s,t)}}$$

$$\therefore P\{X_1 > s, X_2 > t\} = e^{-\lambda_1 s - \lambda_2 t - \lambda_3 \cdot \max(s,t)}$$

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4. $\boxed{\lim_{t\to\infty} \frac{N(t)}{t} \to \frac{1}{E(T)}} = \text{rate}$

$F_S(t) = P(X \le t)$ = shock arrival time 小於 t の机率

$$E(T) = \int_0^\infty \underline{f(t)} \cdot (1 - F_S(t))\, \underline{dt} = \int_0^\infty f(t) \cdot e^{-\lambda t}\, dt$$

$P(\text{job length} = t) = f(t)dt$

$$= \int_0^\infty F'(t) \cdot e^{-\lambda t}\, dt \qquad \therefore \text{rate} = \frac{1}{E(T)} = \frac{1}{\int_0^\infty F'(t) e^{-\lambda t}\, dt}$$

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2. $N(t)$, $M(t)$ indep. non-homo. Poisson with $\lambda(t)$, $\mu(t)$

let $N^*(t) = N(t) + M(t) \rightarrow N^*(t)$ compound Poisson

$P\{$ the event is from $N(t)$ | an event occurs in $(t, t+h)\}$

$$= \frac{P\{\text{the event is from } N(t), \text{ not from } M(t) \text{ in } (t, t+h)\}}{P\{\text{either } N(t) \text{ or } M(t) \text{ in } (t, t+h)\}}$$

$$= \frac{P\{N(t+h) - N(t) = 1\} \cdot (1 - \underbrace{P\{M(t+h) - M(t) = 1\}}_{\to o(h)})}{[\lambda(t) \cdot h + o(h)] + [\mu(t) \cdot h + o(h)] - [\lambda(t)h + o(h)] \cdot [\mu(t)h + o(h)]}$$

$$= \frac{[\lambda(t)h + o(h)] \cdot [1 - \mu(t)h - o(h)]}{[\lambda(t) + \mu(t)]h + 2 \cdot o(h) + \underbrace{o(h)}_{\to o(h)}}$$

$$= \frac{\lambda(t)h + o(h)}{[\lambda(t) + \mu(t)]h + o(h)} = \frac{\lambda(t) + \frac{o(h)}{h} \to 0}{\lambda(t) + \mu(t) + \frac{o(h)}{h} \to 0} = \frac{\lambda(t)}{\lambda(t) + \mu(t)}$$

$\therefore \{N^*(t)\}$ 從 $\{N(t)\}$ 分流來の机率為 $\frac{\lambda(t)}{\lambda(t) + \mu(t)}$

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3. let $N(t) \triangleq \{\#$ of failed batteries by time $t\}$

$$\boxed{\lim_{t \to \infty} \frac{N(t)}{t} \to \frac{1}{E(X)}} = \frac{1}{\frac{30+60}{2}} = \frac{1}{45} = \text{rate}$$

$\therefore$ in the long run, the changing rate is every 45 hours

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