Quiz 1

Student ID: Name:

(a). The continuous random variable X is exponentially distributed with parameter λ. Its cumulative distribution function is

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

Another random variable Y is defined as $Y = 8X$.

Please find $\mathrm{E}[Y]$.

$$E[Y] = E[8X] = \int_0^\infty 8x\lambda e^{-\lambda}\, dx = 8\int_0^\infty x\lambda e^{-\lambda x} dx$$

$$= 8[x(-e^{-\lambda}\,)\big|_0^\infty - \int_0^\infty (-e^{-\lambda x})dx] = 8\int_0^\infty e^{-\lambda x} dx = \frac{8}{\lambda}$$

(b). Suppose X has the following probability mass function:

$$\mathrm{p}(0) = 0.1, \quad \mathrm{p}(1) = 0.6, \quad \mathrm{p}(2) = 0.3$$

Please Calculate $\mathrm{E}[X^2]$.

Letting $\mathrm{Y} = \mathrm{X}^2$, we have that Y is a random variable that can take on one of the values $0^2, 1^2, 2^2$ with respective probabilities

$$p_Y(0) = P\{Y = 0^2\} = 0.1$$

$$p_Y(1) = P\{Y = 1^2\} = 0.6$$

$$p_Y(4) = P\{Y = 2^2\} = 0.3$$

Hence,

$$\mathrm{E}[Y] = \mathrm{E}[X^2] = 0 \cdot (0.1) + 1 \cdot (0.6) + 4 \cdot (0.3) = 1.8$$