Quiz 4

Student ID: Name:

1. Let $\{N(t), t \geq 0\}$ be a Poisson process with $P\{N(1) = 0\} = e^{-3}$. Let S_n denote the time of the n-th event. Find $E[N(4) - N(2)|N(1) = 3]$.

$$\mathrm{e}^{-\lambda\times 1}\frac{(\lambda\times 1)^0}{0!} = e^{-3} \Longrightarrow \lambda = 3$$

$$\mathrm{E}[\mathrm{N}(4) - \mathrm{N}(2)|\mathrm{N}(1) = 3] = E[N(2)] = \lambda \times t = 3 \times 2 = 6$$

2. On Friday night, the number of customers arriving at a lounge bar can be modeled by a Poisson process with intensity that twelve customers per hour.
   (a) Find the probability that there are 2 customers between 21:00 and 21:40.
   (b) Find the probability that there are 4 customers between 21:00 and 21:40 and 6 customers between 21:40 and 22:00.

(a) We know that $\lambda = 12$ and the interval between 21:00 and 21:40 has the length $\tau = \frac{2}{3}$ hours. If X is the number of arrivals in that interval, we can write $\mathrm{X} \sim \mathrm{Poisson}(8)$. Therefore,

$$\mathrm{P}(\mathrm{X} = 2) = \frac{e^{-8}(8)^2}{2!}$$

(b) Here, we have two non-overlapping intervals $I_1 = (21{:}00, 21{:}40]$ and $I_2 = (21{:}40, 22{:}00]$. Thus, we can write

$$\mathrm{P}(\text{ 4 arrivals in } I_1 \textit{ and } \text{6 arrivals in } I_2 \text{ })$$

$$= \mathrm{P}(\text{ 4 arrivals in } I_1\text{ }) \cdot P(\text{ 6 arrivals in } I_2\text{ })$$

$$= \frac{e^{-8}(8)^4}{4!} \cdot \frac{e^{-4}(4)^6}{6!}$$