CS 5291: Stochastic Processes for Networking

HW1

1. X is an exponentially distributed random variable with parameter λ. $Y = X^2$. Find the mathematical expectation of Y.

$$E[Y] = E[X^2] = \int_0^\infty x^2 \lambda e^{-\lambda x} dx$$

$$= x^2(-e^{-\lambda x})\Big|_0^\infty - \int_0^\infty 2x(-e^{-\lambda x})dx = \frac{2}{\lambda}\int_0^\infty x\lambda e^{-\lambda x}dx = \frac{2}{\lambda^2}$$

2. Suppose X is a non-negative and continuous random variable whose pdf is $f_X(x)$ and whose cdf is $F_X(x)$. Starting from the definition of the mathematical expectation, prove that $E[X] = \int_0^\infty (1 - F_X(x))dx$.

$$E[X] = \int_0^\infty xf(x)dx$$

$$\int_0^\infty [1 - F_X(x)]dx = x(1 - F_X(x))\Big|_0^\infty + \int_0^\infty xf(x)dx = \int_0^\infty xf(x)dx = E[X]$$