CS 5291: Stochastic Processes for Networking

HW5

1. Some components of a two-component system fail after receiving a shock. Shocks of three types arrive independently and in accordance with Poisson processes. Shocks of the first type arrive at a Poisson rate λ_1 and cause the first component to fail. Those of the second type arrive at a Poisson rate λ_2 and cause the second component to fail. The third type of shock arrives at a Poisson rate λ_3 and causes both components to fail. Let X_1 and X_2 denote the survival times for the two components. Show that the joint distribution of X_1 and X_1 is given by

$$\mathrm{P}\{X_1 > \mathrm{s}, X_2 > \mathrm{t}\} = \exp\{-\lambda_1 \mathrm{s} - \lambda_2 \mathrm{t} - \lambda_3 \max(\mathrm{s}, \mathrm{t})\}$$

2. Let $N(t)$ and $M(t)$ are two independent non-homogeneous Poisson process, with respective intensity function $\lambda(t)$ and $\mu(t)$. Let $N^*(t) = N(t) + M(t)$. Please use $\boldsymbol{o}(\cdot)$ to explain that why an event of $\{N^*(t)\}$ occurs at time t then the event at time t is from $\{\, N(t)\}$ process with the probability $\frac{\lambda(t)}{\lambda(t)+\mu(t)}$.

   (Hint: P{the event is from $\boldsymbol{N(t)}$ given an event occurs in $(\boldsymbol{t}, \boldsymbol{t}+\boldsymbol{h})$})

3. Beverly has a radio that works on a single battery. As soon as the battery in use fails, Beverly immediately replaces it with a new battery. If the lifetime of a battery (in hours) is distributed uniformly over the interval (30, 60), then at what rate does Beverly have to change batteries?

4. A worker sequentially works on jobs. Each time a job is completed, a new one is begun. Each job, independently, takes a random amount of time having distribution F to complete. However, independently of this, shocks occur according to a Poisson process with rate λ. Whenever a shock occurs, the worker discontinuous working on the present job and starts a new one. In the long run, at what rate are jobs completed?