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CS 5291 Stochastic Process for Networking HW6

1. $X_1, X_2, \ldots$ indep. (like flipping coin?) $P(X_i=1)=p$, $P(X_i=0)=1-p$

$N_1 \triangleq \min\{n : X_1+\cdots+X_n=5\}$, $N_2=\begin{cases}3, & X_1=0\\5, & X_1=1\end{cases}$, $N_3=\begin{cases}3, & X_4=0\\2, & X_4=1\end{cases}$

(a) N_1, N_2, N_3 是不是 stopping time?

N_i is stopping time $\Longleftrightarrow$ N_i depends only on $X_1, \ldots, X_i$

1° N_1 (擲到5个1(正面)最少要几次) 只跟 $X_1, X_2, \ldots, X_i$ 有关 $\Rightarrow$ Yes ※

2° N_2 只跟 X_1 有关 $\Rightarrow$ Yes ※ 3° N_3 跟 X_4 有关 $\Rightarrow$ No ※

$\therefore N_1, N_2$ are stopping time, N_3 is not ※

(b) derive $E(N)$, $E(X)$, or $E(\sum_{i=1}^{N} X_i)$ by using Wald's equation

$E(\sum_{i=1}^{N} X_i) = E(\underbrace{X_1+\cdots+X_{N_1}}_{5}) = E(X)\cdot E(N_1)$ $\because$ $\boxed{E(\sum_{n=1}^{T} X_n) = E(X)\cdot E(T)}$

$\because X_1+\cdots+X_{N_1}=5$, $E(X_i=1)=p$ $\therefore E(N_1)=\dfrac{E(5)}{p}=\dfrac{5}{p}$

$E(X_1+\cdots+X_{N_2}) = E(X)\cdot E(N_2) = p\cdot(3(1-p)+5\cdot p) = p(3+2p)$

$\therefore E(N_1)=\dfrac{5}{p}$, $E(N_2)=3+2p$, $E(X)=p$

$E(X_1+\cdots+X_{N_1})=5$, $E(X_1+\cdots+X_{N_2})=p(3+2p)$

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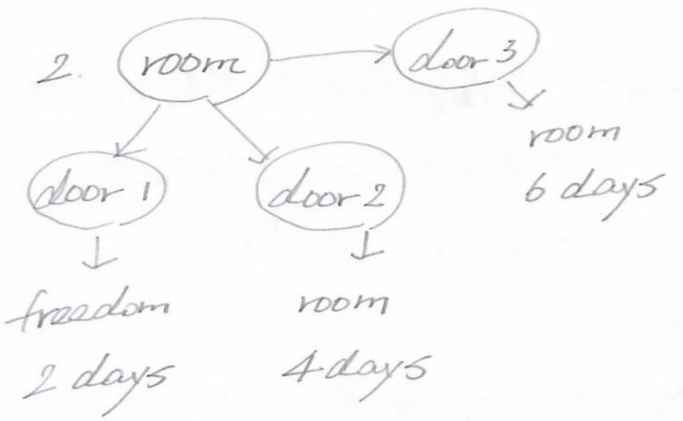
2.



assume equal likely choice,

$T \triangleq$ time it takes to be free

也就是滿足 $T=\sum_{i=1}^{N} X_i$

(a) 定義合適的 iid r.v. X_i 及 stopping time N 來 model 該問題

$X_i \triangleq$ traveling time after i^{th} choice #

$N \triangleq$ # of choice that has been made until freedom #

(b) find $E(T) = \boxed{E(\sum_{i=1}^{N} X_i) = E(X)\cdot E(N)}$ ← Wald's Eq.

$\Rightarrow N \sim Geo(\frac{1}{3})$, $E(N) = \frac{1}{p} = 3$, $var(N) = \frac{1-p}{p^2} = 6$

$E(X) = \frac{2+4+6}{3} = 4$ $\quad \therefore E(T) = 3 \cdot 4 = 12$ #

(c) find $E(\sum_{i=1}^{N} X_i \mid N=n)$

↳ 第n次才會選 door 1, 前 n-1 次都選 2 or 3

$\underbrace{X_1, X_2, \dots, X_{n-1}}, \underline{X_n}$ ↳ 2, p(door 1) = 1) $\quad E(X_n) = 2$

$X_i = \begin{cases} 4, & p(door2) = \frac{1}{2} \\ 6, & p(door3) = \frac{1}{2} \end{cases}$ $\quad E(X_1 + \dots + X_{n-1}) = (n-1)\cdot\left[4\cdot\frac{1}{2} + 6\cdot\frac{1}{2}\right] = 5n-5$

$\therefore E(\sum_{i=1}^{N} X_i \mid N=n) = 5n-3$ # $\quad 5n-5+2 = 5n-3$

(d) 用 (c) 的結果去推 $E(T) = E(E(\sum_{i=1}^{N} X_i \mid N=n))$ ∵ law of total expectation

$\Rightarrow E(T) = E(5N-3) = 5\cdot E(N) - 3 = 5\cdot 3 - 3 = 12$

$\therefore E(T) = 12$ 跟(b)一樣 #