Quiz 2

Student ID: Name:

For an exponential random variable X, $X \sim Exp(\lambda)$, we know its moment generating function is $\phi(t) = \frac{\lambda}{\lambda - t}$ for $t < \lambda$. What is the third moment of X? (That is, what is $E[X^3]$?)

$$\phi(t) = E[e^{tx}] = \int_0^\infty e^{tx} \cdot \lambda e^{-\lambda x} dx = \int_0^\infty \lambda \cdot e^{-(\lambda - t)x} dx$$

$$= -\left(\frac{\lambda}{\lambda - t}\right) e^{-(\lambda - t)x} \Big|_0^\infty = \frac{\lambda}{\lambda - t}, \qquad t < \lambda$$

$$\phi'(t) = \frac{\lambda}{(\lambda - t)^2}$$

$$\phi''(t) = \frac{2\lambda}{(\lambda - t)^3}$$

$$\phi'''(t) = \frac{6\lambda}{(\lambda - t)^4}$$

$$E[X^3] = \phi'''(0) = \frac{6}{\lambda^3}$$