

Simple Energy

Paul Kim

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When tangent-point energy was used to evolve the curve, it turned out that in a discrete setting, it is minimized when the points are colinear.

Instead, it might be worth investigating the simplest of curve energies.

1 Simple Energy

For a closed curve $C = \gamma(t)$ over $t \in M$, define **simple energy** as:

$$\mathcal{E}^\alpha(\gamma) := \iint_{M^2} \frac{1}{|\gamma(x) - \gamma(y)|^\alpha} d\gamma_x d\gamma_y \quad (1)$$

where $\alpha > 0$.

For $\alpha \geq 1$, this energy is ill-defined in an analytic framework. However, in a discrete scheme inspired by this, there is a natural way to make this well-defined.

2 Discrete Simple Energy

Definition 1. Given a closed, non-intersection polygonal curve $\Gamma := (x_0, x_1, \dots, x_{J-1})$ ($x_0 = x_J$) define **discrete simple energy of first kind** as:

$$E_1^\alpha(\Gamma) = \sum_{i=0}^{J-1} \sum_{j \neq i} \frac{1}{|x_i - x_j|^\alpha} |x_i - x_{i+1}| |x_j - x_{j+1}| \quad (2)$$

where $\alpha > 0$.

This is the simplest energy to compute, but it may not be symmetric.

To address the issue of non-symmetricty, a simple modification can be done:

Definition 2. Given a closed, non-intersection polygonal curve $\Gamma := (x_0, x_1, \dots, x_{J-1})$ ($x_0 = x_J$) define **discrete simple energy of second kind** as:

$$E_2^\alpha(\Gamma) = \sum_{i=0}^{J-1} \sum_{|i-j|>1} k_{i,j}^\alpha |x_i - x_{i+1}| |x_j - x_{j+1}| \quad (3)$$

where

$$k_{i,j}^\alpha = \frac{1}{4} \left(\frac{1}{|x_i - x_j|^\alpha} + \frac{1}{|x_i - x_{j+1}|^\alpha} + \frac{1}{|x_{i+1} - x_j|^\alpha} + \frac{1}{|x_{i+1} - x_{j+1}|^\alpha} \right) \quad (4)$$

and $\alpha > 0$. Note that the indexing must be done cyclically, so $|i - j| > 1$ is modulo J .