Curve Repulsion - 1

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1 Theory behind Discretization

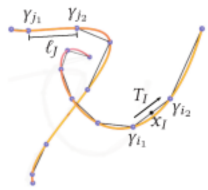

Figure 1: Discretization process

We now consider a discretization of a curve. Index the points on the curve by $\mathcal{I} = \{1, 2, \cdots, M\}$, such that points are given by $\{\gamma_1, \gamma_2, \cdots, \gamma_M\}$

Using the similar notation to the paper by Yu, Schumacher, and Crane, for edge $I = \{\gamma_i, \gamma_j\} \in E$ and for function $u : \mathbb{R}^3 \to \mathbb{R}$

- $l_I \coloneqq |\gamma_i - \gamma_j|$
- $T_I \coloneqq \frac{\gamma_j - \gamma_i}{l_I}$
- $\mathbf{x}_I \coloneqq \frac{\gamma_i + \gamma_j}{2}$
- $u_I \coloneqq \frac{u_i + u_j}{2}$
  - Syntactic sugar: $u_i \equiv u(\gamma_i)$
- $u[I] \coloneqq \begin{pmatrix} u_i \\ u_j \end{pmatrix}$

1.1 Discrete Energy

The naïve discretization of $\mathcal{E}^\alpha_\beta \coloneqq \iint_{M^2} k^\alpha_\beta (\gamma(x), \gamma(y), T(x))\, \mathrm{d}x_\gamma\, \mathrm{d}y_\gamma$ where $k^\alpha_\beta(p, q, T) \coloneqq \frac{|T \times (p-q)|^\alpha}{|p-q|^\beta}$ is given by

$$\sum_{I \in E} \sum_{J \in E} \int_{\bar{I}} \int_{\bar{J}} k^\alpha_\beta (\gamma(x), \gamma(y), T_I)\, \mathrm{d}x_\gamma\, \mathrm{d}y_\gamma \tag{1}$$

However, in a polygonal curve (hence the discretized curve), (1) is ill-defined.

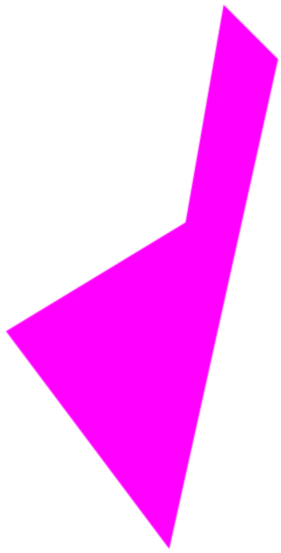

Figure 2: Near each vertex, the integrand is unbounded.

So resolve this by removing the two neighboring edges.[1] Also approximate the kernel by the average of the kernel evaluated at each pair of appropriate edges (total: 4)

$$\hat{\mathcal{E}}_\beta^\alpha := \sum_{I,J \in E, I \cap J = \emptyset} \left(\hat{k}_\beta^\alpha\right)_{I,J} l_I l_J \tag{2}$$

$$\left(\hat{k}_\beta^\alpha\right)_{I,J} := \frac{1}{4} \sum_{i \in J, j \in J} k_\beta^\alpha \left(\gamma_i, \gamma_j, T_I\right) \tag{3}$$

2 Discrete Gradient Flow in L^2

[1]In the limit, the contribution from this removed edge goes to zero.