

Given $\gamma : M \rightarrow \mathbb{R}^3$, $\gamma = \gamma(s)$ (I suppose arc-length parameterised for simplicity), and functional $\mathcal{E} = \mathcal{E}(\gamma)$, characterize first derivative operator $\tilde{\nabla}$ as:

$$\left(\tilde{\nabla}\mathcal{E}(\gamma)\right)(s) = \underbrace{\frac{d\mathcal{E}(\gamma(s))}{ds}}_{\text{Scalar}} \underbrace{\frac{d\gamma(s)}{ds}}_{\text{Tangent Vector}} \quad (1)$$

Discretized versions are:

$$\tilde{\nabla}_{\mathbf{\Gamma}}^+ E(\mathbf{\Gamma})[i] = \frac{E(\mathbf{\Gamma}[i+1]) - E(\mathbf{\Gamma}[i])}{|e_i|} (\mathbf{\Gamma}[i+1] - \mathbf{\Gamma}[i]) / |e_i| \quad (2)$$

$$\tilde{\nabla}_{\mathbf{\Gamma}}^- E(\mathbf{\Gamma})[i] = \frac{E(\mathbf{\Gamma}[i]) - E(\mathbf{\Gamma}[i-1])}{|e_{i-1}|} (\mathbf{\Gamma}[i] - \mathbf{\Gamma}[i-1]) / |e_{i-1}| \quad (3)$$

where $e_i := \mathbf{\Gamma}[i+1] - \mathbf{\Gamma}[i]$.

Characterize second derivative operator $\tilde{\Delta}$ similarly by second derivative with respect to s ? Discretization by applying the discretized version of first derivative operator in forward and backward form.