Constrained Model Predictive Control using Feedforward Steady-State Target Optimization tracking approach for an Isolated Power System

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***Abstract*— A constrained Linear Quadratic Model Predictive Model Control (LQ-MPC) with offset-free tracking and disturbance rejection using the Feedforward Steady-State Target Optimization (SSTO) approach is presented in this paper. The MPC control law uses Finite-Receding-Horizon method in dual-mode. Moreover, the stability and recursive feasibility are guaranteed by off-line dual-mode and on-line optimization. The results are presented by simulation.**

I. STATE-SPACE MODEL OF THE SYSTEM

The isolated power system [1] to be studied is a linear Single-Input Single-Output (SISO) model with the addition of the disturbance in the output, Fig. 1.

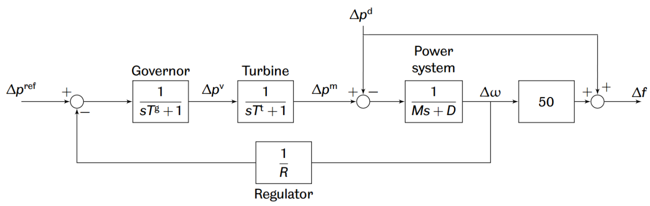

Fig. 1: Isolated power system

The linear continuous-time state-space model is as follows,

$$\begin{bmatrix}\Delta\dot{\omega}\\ \Delta\dot{p}^m\\ \Delta\dot{p}^v\end{bmatrix} = \begin{bmatrix}-D/M & 1/M & 0\\ 0 & -1/T^t & -1/T^t\\ -1/(RT^g) & 0 & -1/T^g\end{bmatrix}\begin{bmatrix}\Delta\omega\\ \Delta p\\ \Delta p^v\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1/T^g\end{bmatrix}\Delta p^{ref} + \begin{bmatrix}-1/M\\ 0\\ 0\end{bmatrix}\Delta p^d \quad (1)$$

$$\Delta f = \begin{bmatrix}50 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\omega\\ \Delta p\\ \Delta p^v\end{bmatrix} + \Delta p^d$$

where Δf is the output values, Δp^{ref} is the input, and Δp^d is the disturbance, the rest of the parameters can be consulted at [1].

Thus, the Discrete Linear Time-Invariant (LTI) state-space system is

$$\begin{aligned} x(k+1) &= A\ x(k) + B\ u(k) + B_d\ d(k)\\ y(k) &= C\ x(k) + D_d\ d(k), \quad k = 0,1,2,\dots \end{aligned} \quad (2)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and $d \in \mathbb{R}^q$ are the states, input, output, and disturbance vectors respectively. A, B, C, are the state, input, and output matrices, and B_d, D_d are the disturbance matrices in the input, and in the output.

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II. FORMULATION OF THE LQ-MPC TRACKING PROBLEM

A. *Feedforward tracking with SSTO*

The Feedfoward tracking approach by SSTO requires a target optimizer that calculates the steady-state values for the states x_{ss} and the input u_{ss} given a reference r in the presence of a disturbance d, Fig. 2.

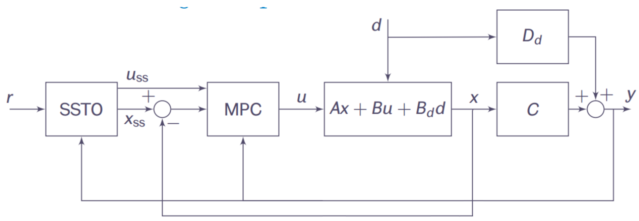


Fig. 2: MPC and SSTO model

B. *Cost function*

Because it is desired that $x \to x_{ss}$ and $u \to u_{ss}$, new variables can be defined as $\xi \equiv x - x_{ss}$ and $v \equiv u - u_{ss}$, where $\xi \to 0$ and $v \to 0$.

Therefore, the constrained LQ-MPC problem is to minimize the cost function

$$\begin{aligned} \mathbb{P}_N(v(k)) : \min_{v(k)} \sum_{j=0}^{N-1} &(\xi^\intercal(k+j|k)\ Q\ \xi(k+j|k)\\ &+ v^\intercal(k+j|k)\ R\ v(k+j|k))\\ &+ \xi^\intercal(k+N|k)\ P\ \xi(k+N|k) \end{aligned} \quad (3)$$

subject to, for $j = 0,1,2,\dots,N-1$

$$\begin{aligned} \xi(k+1+j|k) &= A\ \xi(k+j|k) + B\ v(k+j|k)\\ \xi(k|k) &= \xi(k)\\ P_x\ \xi(k+j|k) &\le q_x - P_x\ x_{ss} \quad (4)\\ P_u\ v(k+j|k) &\le q_u - P_u\ u_{ss} \quad (5)\\ P_{x_N}\ \xi(k+N|k) &\le \tilde{q}_{x_N} \quad (6) \end{aligned}$$

where Q and R are weight matrices, P is the terminal cost matrix, (4), (5), (6) are the linear inequality (polyhedra) constraints, and N is the finite-horizon value. Note that B_d, D_d and d are not present because the SSTO will manage the disturbance rejection.

C. Defining the inequality constraints

Given $|\Delta p^{ref}| = |u| \leq u_{min,max}$, and $|\Delta f| = |y| \leq y_{min,max}$, for $y = C\ x$,

$$\begin{bmatrix} C \\ -C \end{bmatrix} x(k+j|k) \leq \begin{bmatrix} y_{max} \\ -y_{min} \end{bmatrix}$$
$$\begin{bmatrix} I \\ -I \end{bmatrix} u(k+j|k) \leq \begin{bmatrix} u_{max} \\ -u_{min} \end{bmatrix}$$

can be written in terms of ξ and v as follows,

$$\begin{aligned} \underbrace{\begin{bmatrix} C \\ -C \end{bmatrix}}_{P_x} \xi(k+j|k) &\leq \underbrace{\begin{bmatrix} y_{max} \\ -y_{min} \end{bmatrix}}_{q_x} - \underbrace{\begin{bmatrix} C \\ -C \end{bmatrix}}_{P_x} x_{ss} \\ \underbrace{\begin{bmatrix} I \\ -I \end{bmatrix}}_{P_u} v(k+j|k) &\leq \underbrace{\begin{bmatrix} u_{max} \\ -u_{min} \end{bmatrix}}_{q_u} - \underbrace{\begin{bmatrix} I \\ -I \end{bmatrix}}_{P_u} u_{ss} \end{aligned} \tag{7}$$

and using a proper deadbeat mode-2 terminal inequality constraint to ensure a fast convergence to zero,

$$\begin{aligned} &\underbrace{\mathbf{I}_{n\times n} \otimes \begin{bmatrix} P_x \\ P_u\ K \end{bmatrix} \begin{bmatrix} (A+BK_\infty)^0 \\ \vdots \\ (A+BK_\infty)^{N-1} \end{bmatrix}}_{P_{x_N}} \xi(k+N|k) \\ &\leq \underbrace{\mathbf{1}_n \otimes \begin{bmatrix} q_x - Px\ x_{ss} \\ q_u - P_u\ u_{ss} \end{bmatrix}}_{\tilde{q}_{x_N}} \end{aligned} \tag{8}$$

where K_∞ is the deadbeat mode-2 gain calculated using Linear Quadratic Regulator (LQR) method.

D. Target equilibrium optimization under constraints (SSTO)

The offset-free equilibrium points (x_{ss}, u_{ss}) that satisfies the constraints exits if only if $x_{ss} = Ax_{ss} + Bu_{ss} + B_d d$ and $y_{ss} = Cx_{ss} + D_d d = r$.

Thus, the SSTO problem is

$$\min_{\{x_{ss},u_{ss}\}} \|C\ x_{ss} + D_d\ d - r\| + \rho\|u_{ss}\| \tag{9}$$

subject to

$$\begin{aligned} \underbrace{\begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix}}_{A_{eq}} \underbrace{\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix}}_{x_{eq}} &= \underbrace{\begin{bmatrix} Bd\ d \\ r - D_d\ d \end{bmatrix}}_{b_{eq}} \\ \underbrace{\begin{bmatrix} P_x & 0 \\ 0 & P_u \end{bmatrix}}_{A_{neq}} \underbrace{\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix}}_{x_{neq}} &\leq \underbrace{\begin{bmatrix} q_x \\ q_u \end{bmatrix}}_{b_{neq}} \end{aligned} \tag{10}$$

III. SOLVING THE LQ-MPC OPTIMIZATION PROBLEM

The solution of the LQ-MPC problem (3) in the compress form can written as,

$$\min_{v(k)} \frac{1}{2}\mathbf{v}^\intercal(k)\ H\ \mathbf{v}(k) + \xi^\intercal(k)\ L^\intercal\ \mathbf{v}(k) + \xi^\intercal(k)\ M\ \xi(k) \tag{11}$$

subject to

$$P_c\mathbf{v}(k) \leq q_c + S_c\ \xi(k) \tag{12}$$

where H, L, M are the cost matrices given Q and R. P_c, q_c, S_c are the stacked inequality constraints for $P_x, P_u, P_{x_N}, q_u, q_x, \tilde{q}_{x_N}$ given the predictions matrices $\mathbf{x} = Fx(k) + G\mathbf{u}(k)$, [1]. Note that the prediction matrices are constructed using the states and input without the tracking model.

The solution of (11) is the optimal control input sequence,

$$\mathbf{v}^* = \{v^*(k|k), v^*(k+1|k), \ldots, v^*(k+N-1|k)\}$$

which can be solved numerically.

Because MPC uses the receding-horizon principle, only the first optimized control input is needed

$$v(k) = v^*(k|k) = K_N(\xi(k))$$

but the real control input applied to the system is

$$u(k) = u_{ss} + v^*(k|k) \tag{13}$$

A. Stability

In order to ensure stability it is necessary to compute a terminal cost matrix P that satisfies the Lyapunov equation

$$(A + B\ K)^\intercal P(A + B\ K) - P + (Q + K^\intercal R\ K) = 0 \tag{14}$$

which solution can be computed numerically. A good approach is to use a deadbeat mode-2 control gain K that converges the states to the origin $x = 0$ as long as $(A+BK)$ is stable, which means that the eigenvalues λ are inside the unit circle $|\lambda| < 1$.

IV. ALGORITHM FOR THE IMPLEMENTATION

Now that the solution of the LQ-MPC is established and the stability can be guaranteed, there are basic conditions to meet such as,

- (A, B) is stabilizable which can be proved by the controllability, in Matlab `rank(ctrb(A,B))`.
- $Q \succeq 0$, can be $C^\intercal C$
- $R \succ 0$, changed later for tuning.

The following steps reflect the Matlab implementation algorithm

Off-line mode:

1. Discretization of the model (1) using zero holder (zoh), in Matlab will be `sys=c2d(idss(Ac,Bc,C,D,Bd,x0,0),Ts)`, where T_s is the sampling time.
2. Construct the target equilibrium constraints (10)
3. Define the optimization equation as `z=[xss;uss]` and `fun=@(z) C(1)*z(1)+Dd*d-r+rho*z(4)`
4. Solve the optimization problem for SSTO using `f=fmincon(fun,z0,Aneq,bneq,Aeq,beq)` in Matlab, where `z0=[r,0,0,0]`, `fun` is (9), and `f=[xss,uss]` is the optimized vector solution.
5. Construct the LQ-MPC inequality constraints (11), and compute the deadbeat mode-2 terminal constraint using `K=-dlqr(A,B,Q,R)`.

6. Construct the prediction matrices F, G using `[F,G]=predict_mats(A,B,N)` and the constraint matrices (12) using `[Pc,qc,Sc]=...` `constraint_mats(F,G,Pu,qu,Px,qx,PxN,qxN);`
7. Compute the deadbeat mode-2 control Kl using `Kl=-acker(A,B,[0,0,0])`
8. Calculate the Lyapunov solution of (14) with `P=dlyap((A+B*K)',Q+K'*R*K)`, which gives a terminal cost matrix P that guaranties stability.
9. Finally before the on-line mode is initiated, set the initial conditions as `x=[0;0;0;]`

On-line mode: iteration for k steps

1. Recall steps 3-4 of the off-line mode for the SSTO optimization but at each iteration `z0=[x;us(1,k)]`, where `us(1,k)` is the first optimized control input.
2. Calculate $\xi(k)$ and $v(k)$ using `xi(:,k)=x-xss` `v(:,k)=us(:,k)-uss`.
3. Compute the constraints (10) with the new x_{ss} and u_{ss}
4. Compute (12) like it was done in the off-line mode step 6.
5. Store the states `xs(:,k)=x` for the iteration process.
6. Solve the LQ-MPC optimization problem (11) without using the matrix M as follows, `[v,fval,flag]=quadprog(H,L*xi(:,k),...` `Pc,qc+Sc*xi(:,k))` where `v` is the optimal control input $v(k)$
7. Check feasibility using `flag`, if it is < 1 the solution is infeasible.
8. Use (13) as the real control input applied to the system, `us(:,k)=v(1)+uss` using only the first optimized value (receding principle).
9. Close the loop (2), `x=A*x+B*us(1,k)+Bd*d` `y=C*xs+Dd*d`
10. Calculate the cost value of (11) using the matrix M.
11. Return to step 1.

V. Design Requirements

For the simulation the following specifications are demanded,

- Reference $r = 0.3$
- Frequency settles to $|\Delta f| = |y| \leq 0.01$ within 2 [s] of the disturbance initiaion.
- $|\Delta p^{ref}| = |u| \leq 0.5$, and $|\Delta f| = |y| \leq 0.5$.
- Zero steady-state error in the output.

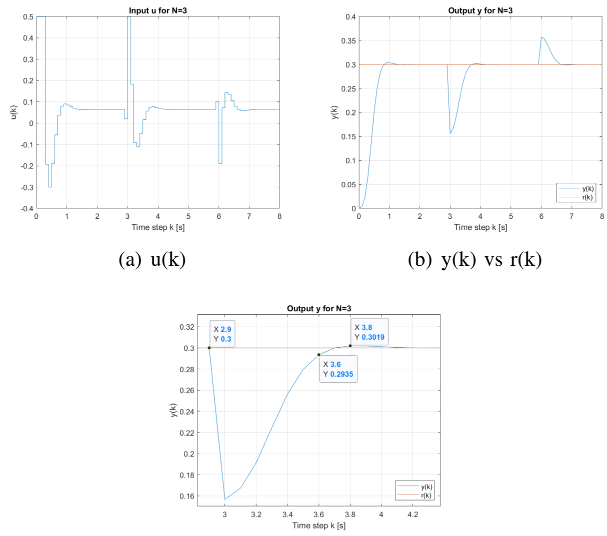
VI. Simulation and Results

The tuning and design parameters, as well as the simulation value results are shown in TableI

Fig. 3a shows that the input constraint is satisfied, and in Fig. 3b it can be seen that the zero offset-free tracking is also satisfied rejecting disturbances at different times. The low value of R is to minimize the overshoot as much as possible which is 0.6%, the best Q is $C^{\intercal}C$ where the settling time t_s is around 0.7. Also, the selection of $N = 3$ was lowest possible to increase performance, Fig. 3c.

Parameter	**Value**
Q	$C^{\intercal}C$
R	0.08
N	3
d_1	0.5 at $t = 30[s]$
d_2	-0.2 at $t = 60[s]$
r	0.3
x_0	$[0; 0; 0]^{\intercal}$
t_s	$0.7[s]$
overshoot	0.6%

TABLE I: Tuning and design parameters

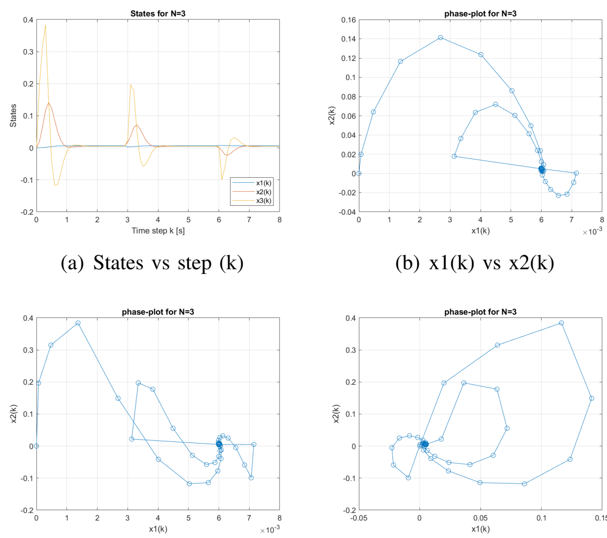


(a) u(k) (b) y(k) vs r(k)

(c) y(k) vs r(k)

Fig. 3: Input and output

Fig. 4 shows that the constraints are satisfied for the three states. And the cost value shown in Fig. 5 is relatively low due the low value of R.



(a) States vs step (k) (b) x1(k) vs x2(k)

(c) x1(k) vs x3(k) (d) x2 (k) vs x3(k)

Fig. 4: States and phase plots

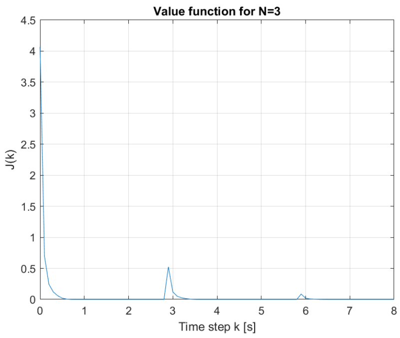


Fig. 5: Cost value $J(k)$ function

One of the drawbacks of the SSTO approach is that it does not have robustness because it needs a perfect model of the plant. Also, when $d > 0.5$, the solution is infeasible, in order to handle bigger disturbances it is necessary to change the cost function, [2] shows an alternative solution.

REFERENCES

[1] P. Trodden, Lecture Notes ACS616 2018/19

[2] S. Dughman, J.A. Rossiter, A survey of guaranteeing feasibility and stability in MPC during target changes 9th International Symposium on Advanced Control of Chemical Processes, June 2015.