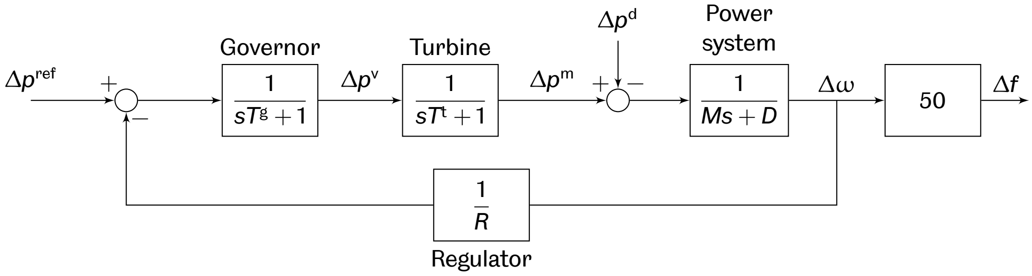
The operation of an isolated power system under primary frequency control is modelled by the following block diagram.




In this system, a steam turbine produces mechanical power, which is subsequently converted to electrical power via a synchronous generator connected to the grid. A change in power demand, $\Delta p^{\text{d}}$, causes the system frequency $f$ (Hz) to change. The control objective is to drive frequency deviations, $\Delta f$, to zero following a demand change, $\Delta p^{\text{d}}$. To aid this, a governor controls the steam flow input to the turbine in response to the error between the reference power $\Delta p^{\text{ref}}$ and the regulated frequency $\Delta\omega/R$, where $R > 0$ is the regulation factor.

The primary frequency control loop present in the system is, unfortunately, unable to regulate the frequency error to zero following demand changes (why?). Therefore, the aim is to design a *secondary* frequency control loop that will adjust the reference power $\Delta p^{\text{ref}}$ in response to frequency deviations $\Delta\omega$, in order to eliminate error and improve transient performance. To this end, a continuous-time state-space model of the system is given as:

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{p}^{\text{m}} \\ \Delta\dot{p}^{\text{v}} \end{bmatrix} = \begin{bmatrix} -D/M & 1/M & 0 \\ 0 & -1/T^{\text{t}} & 1/T^{\text{t}} \\ -1/(RT^{\text{g}}) & 0 & -1/T^{\text{g}} \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta p^{\text{m}} \\ \Delta p^{\text{v}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/T^{\text{g}} \end{bmatrix} \Delta p^{\text{ref}} + \begin{bmatrix} -1/M \\ 0 \\ 0 \end{bmatrix} \Delta p^{\text{d}}$$

$$\Delta f = \begin{bmatrix} 50 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta p^{\text{m}} \\ \Delta p^{\text{v}} \end{bmatrix}$$

In this model, the input, $u$, is the change in reference power to the turbine governor, i.e., $\Delta p^{\text{ref}}$ (in *per unit* (p.u.) – that is, normalized with respect to a base value), and the output, $y$, is the frequency of the power system, i.e., $\Delta f$ (Hz). The states are the (deviations from operating points in) angular frequency, $\Delta\omega$, mechanical output power of the steam turbine, $\Delta p^{\text{m}}$, and output power reference from the turbine governor, $\Delta p^{\text{v}}$. The demand change $\Delta p^{\text{d}}$ is a disturbance, $d$. For the particular power system under consideration, the model parameters are

$$M = 10, D = 0.8, R = 0.1, T^{\text{t}} = 0.5, T^{\text{g}} = 0.2$$

Your task is to design, implement and tune an MPC controller for this system in order to meet the specification on the following page. Your design should specify, as a minimum,

- Constraints
- Cost weighting matrices $Q$ and $R$
- Horizon length $N$
- Any terminal conditions employed — the cost matrix $P$ and any constraints on $x(k+N|k)$
- Any modifications to the formulation for tracking and disturbance rejection

Designs with smaller $N$ are typically (but not necessarily) preferable. A strong design will achieve *guarantees* of stability and recursive feasibility. You may assume that the state $x$ and disturbance $d$ are fully measurable. To obtain the discrete-time prediction model for controller, use a sampling time of 0.1 seconds and zero-order hold sampling (*i.e.* `sysd = c2d(sysc,0.1)` in MATLAB).

**Specification**
In response to a large step-change in demand—up to 0.3 p.u. in magnitude—the closed-loop system

- maintains stability
- frequency settles to $|\Delta f| \leqslant 0.01$ Hz within 2 seconds of the disturbance initiation
- satisfies the constraints

$$\begin{aligned} |\Delta p^{\text{ref}}| &\leqslant 0.5 \\ |\Delta f| &\leqslant 0.5 \end{aligned}$$

- exhibits zero steady-state error in the output

**(40 marks)**