

Two step nitrification model study

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$$s_1 \xrightarrow{\mu_1(s,x)} s_2 + y_1 x_1 \quad (\text{R G1})$$

$$s_2 \xrightarrow{\mu_2(s,x)} s_3 + y_2 x_2 \quad (\text{R G2})$$

$$\dot{x}_1 = (\mu_1(s, x) - D) x_1 \quad (1)$$

$$\dot{x}_2 = (\mu_2(s, x) - D) x_2 \quad (2)$$

$$\dot{s}_1 = (s_{in} - s_1)D - k_1 \mu_1(s, x) x_1 \quad (3)$$

$$\dot{s}_2 = -s_2 D + k_1 \mu_1(s, x) x_1 - k_2 \mu_2(s, x) x_2 \quad (4)$$

$$\dot{s}_3 = -s_3 D + k_2 \mu_2(s, x) x_2 \quad (5)$$

Case 1:

$$\mu_1(s, x) = \bar{\mu}_1 \frac{s_1}{K_1 + s_1} \quad (6)$$

$$\mu_2(s, x) = \bar{\mu}_2 \frac{s_2}{K_1 + s_2} \quad (7)$$

Equilibrium points It is assumed that the points here presented are non-negative. Three cases can take place:

Coexistence: From (1) one gets:

$$s_1^{eq} = \frac{K_1 D}{\bar{\mu}_1 - D} \quad (8)$$

$$x_1^{eq} = \frac{s_{in} - s_1^{eq}}{k_1} \quad (9)$$

$$s_2^{eq} = \frac{K_2 D}{\bar{\mu}_2 - D} \quad (10)$$

$$x_2^{eq} = \frac{s_{in} - s_1^{eq} - s_2^{eq}}{k_2} \quad (11)$$

$$s_3^{eq} = s_{in} - s_1^{eq} - s_2^{eq} \quad (12)$$

Washout of x_2

$$s_1^{eq} = \frac{K_1 D}{\bar{\mu}_1 - D} \quad (13)$$

$$x_1^{eq} = \frac{s_{in} - \frac{K_1 D}{\bar{\mu}_1 - D}}{k_1} \quad (14)$$

$$s_2^{eq} = s_{in} - s_1^{eq} \quad (15)$$

$$x_2^{eq} = 0 \quad (16)$$

$$s_3^{eq} = 0 \quad (17)$$

Washout

$$s_1^{eq} = s_{in} \tag{18}$$

$$x_1^{eq} = 0 \tag{19}$$

$$s_2^{eq} = 0 \tag{20}$$

$$x_2^{eq} = 0 \tag{21}$$

$$s_3^{eq} = 0 \tag{22}$$

Jacobian of the system:

$$J_{11} = \bar{\mu}_1 \frac{s_1}{K_1 + s_1} - D \quad (23)$$

$$J_{12} = 0 \quad (24)$$

$$J_{13} = \bar{\mu}_1 \frac{K_1}{(K_1 + s_1)^2} x_1 \quad (25)$$

$$J_{14} = 0 \quad (26)$$

$$J_{15} = 0 \quad (27)$$

$$J_{21} = 0 \quad (28)$$

$$J_{22} = \bar{\mu}_1 \frac{s_2}{K_2 + s_2} - D \quad (29)$$

$$J_{23} = 0 \quad (30)$$

$$J_{24} = \bar{\mu}_2 \frac{K_1}{(K_1 + s_2)^2} x_2 \quad (31)$$

$$J_{25} = 0 \quad (32)$$

$$J_{31} = -k_1 \bar{\mu}_1 \frac{s_1}{K_1 + s_1} \quad (33)$$

$$J_{32} = 0 \quad (34)$$

$$J_{33} = -D - k_1 \bar{\mu}_1 \frac{K_1}{(K_1 + s_1)^2} x_1 \quad (35)$$

$$J_{34} = 0 \quad (36)$$

$$J_{35} = 0 \quad (37)$$

$$J_{41} = k_1 \bar{\mu}_1 \frac{s_1}{K_1 + s_1} \quad (38)$$

$$J_{42} = -k_2 \bar{\mu}_2 \frac{s_2}{K_2 + s_2} \quad (39)$$

$$J_{43} = k_1 \bar{\mu}_1 \frac{K_1}{(K_1 + s_1)^2} x_1 \quad (40)$$

$$J_{44} = -D - k_2 \bar{\mu}_2 \frac{K_2}{(K_2 + s_2)^2} x_2 \quad (41)$$

$$J_{45} = 0 \quad (42)$$

$$J_{51} = 0 \quad (43)$$

$$J_{52} = 0 \quad (44)$$

$$J_{53} = 0 \quad (45)$$

$$J_{54} = 0 \quad (46)$$

$$J_{55} = -D \quad (47)$$

$$(48)$$