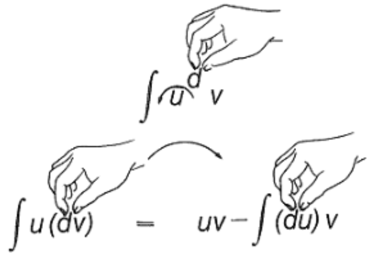
Integration by parts - notes

Saturday, December 27, 2014
11:16 AM

Q. When to use this method?
R. Use this method when given integration is of the form $\int F(x)G'(x)\,dx$

$$= F(x)G(x) - \int F'(x)G(x)\,dx$$



This method is particularly very interesting because it gives us some very cool results:-

$$\int f(x)\,dx = \int 1 . f(x)\,dx \quad \begin{bmatrix} F(x) = f(x) \\ G'(x) = 1 \end{bmatrix}$$

$$= f(x)\int 1 .\,dx - \int x f'(x)\,dx$$

$$= x f(x) - \int x f'(x)\,dx$$

Let $y = f(x)$ [common sense]

$\Rightarrow dy = f'(x)\,dx$

-and-

$x = f^{-1}(y) = g(y)$ [Assume]

$$\therefore \int f(x)\,dx = x f(x) - \int x\,dy$$

$$= x f(x) - \int g(y)\,dy$$

- or simply -

$$\boxed{\int f(x)\,dx = xy - \int x\,dy}$$

Example

Integrate : $\int \operatorname{cosec}^{-1}\sqrt{x}\,dx$

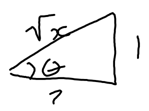
— Looks tough, but is not !

Here, $f(x) = y = \operatorname{cosec}^{-1}\sqrt{x}$

or $\operatorname{cosec}(y) = \sqrt{x}$ [comm sense]

$$\left\{ \operatorname{cosec}(\theta) = \frac{\text{Hypo.}}{\text{Perp.}} \right\}$$

$$\therefore \frac{\sqrt{x}}{1} = \frac{H}{P}$$



$(?)^2 = x - 1$

$? = \sqrt{x-1}$

Let's use the formula

$$\int \underbrace{1}_{g'(x)} . \underbrace{\operatorname{cosec}^{-1}\sqrt{x}}_{f(x)}\,dx = x . \operatorname{cosec}^{-1}\sqrt{x} - \int x\,dy$$

$$= x \operatorname{cosec}^{-1}\sqrt{x} - \int \operatorname{cosec}^2 y\,dx \quad \text{[from common sense step]}$$

$$= x \operatorname{cosec}^{-1}\sqrt{x} + \cot y + C$$

[1. Learn identities
2. Always add coff. of integration]

Q:? why I made the $\triangle$ diagram ?

A: Because $\theta = y$ in this question !!!

$$\therefore \Rightarrow x \operatorname{cosec}^{-1}\sqrt{x} + \cot(\operatorname{cosec}^{-1}\sqrt{x}) + C$$

$\therefore \Rightarrow x \operatorname{cosec}^{-1}\sqrt{x} + \cot\left(\operatorname{cosec}^{-1}\sqrt{x}\right) + C$

$$\cot\theta = \frac{\text{Base}}{\text{Perp.}} = \frac{\sqrt{x-1}}{1}$$

$$\Rightarrow \boxed{x \operatorname{cosec}^{-1}\sqrt{x} + \sqrt{x-1} + C}$$