第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e_n}$$

极化体电荷:

$$\rho_{\mathbf{P}} = -\nabla \cdot \mathbf{P}$$

极化电荷代数和应为0:

$$0 = \oint_S \sigma_{\mathbf{P}} dS + \int_V \rho_{\mathbf{P}} dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D_{n2}} - \mathbf{D_{n1}} = \sigma$$

电位连续性、E的切向连续性:

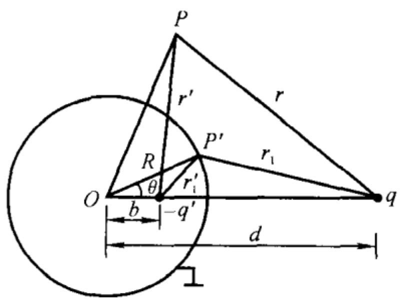
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b=\frac{R^2}{d}$$
$$q'=\frac{R}{d}q$$

两种理想介质

在$r_1=r_2$处

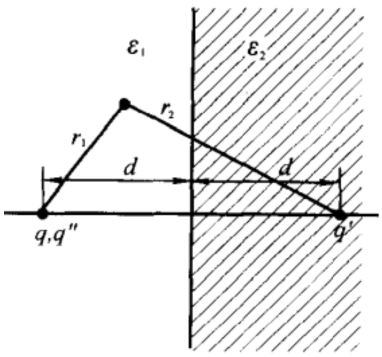


图 1 - 30　点电荷对无限大介质分界平面的镜像

$$\begin{cases}\frac{q}{\varepsilon_1}+\frac{q'}{\varepsilon_1}=\frac{q''}{\varepsilon_2}\\ q-q'=q''\end{cases}$$

$$\therefore\begin{cases}q'=\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}q\\ q''=\frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}q\end{cases}$$

几个电容公式

同轴夹层线缆

$$C=\frac{2\pi\varepsilon}{\ln(b/a)}$$

同心夹层球

$$C=\frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}$$

孤立球

$$C=4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电，容易想到

$$
\begin{aligned}
W_e &= \frac{1}{2}\int_V \rho\varphi dV \\
&= \frac{1}{2}\int_S \sigma\varphi dS
\end{aligned}
$$

静电场能量密度

$$w_e' = \frac{1}{2}\mathbf{D}\cdot\mathbf{E}$$

虚功原理

$$dW = dW_e + fdg$$

不与电源相连

$$0 = dW_e + fdg$$

$$\therefore f = -\frac{\partial W_e}{\partial g}$$

与电源相连,各带电体电位不变

$$dW_e = \frac{1}{2}\sum_k \varphi_k dq_k = \frac{1}{2}dW$$

$$\therefore f = \frac{\partial W_e}{\partial g}$$

第二章

电流密度

$$\mathbf{J} = \rho\mathbf{v}$$

作用于垂直于v的dS

$$dI = \mathbf{J}\cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma\mathbf{v}$$

作用于垂直于v的dl

$$dI = \mathbf{K}\cdot\mathbf{e_n}dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v}dq = \mathbf{J}dV = \mathbf{K}dS = Id\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma\mathbf{E}$$

γ为电导率

焦耳定律微分形式

$$p = \frac{dP}{dV} = \mathbf{J} \cdot \mathbf{E}$$

含源欧姆定律

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{E_e})$$

电流连续性方程

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$\begin{gathered}\mathbf{J_{1n}} = \mathbf{J_{2n}}, \text{体现一个电荷守恒} \\ \mathbf{E_{1t}} = \mathbf{E_{2t}}, \text{体现一个保守场} \\ \frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\gamma_1}{\gamma_2}, \text{谓之折射定律}\end{gathered}$$

第三章

BS定律

$$\begin{aligned}\mathbf{B} &= \frac{\mu}{4\pi}\oint_L \frac{Id\mathbf{l}\times\mathbf{e_R}}{R^2} \\ &= \frac{\mu}{4\pi}\oint_V \frac{\mathbf{J}dV\times\mathbf{e_R}}{R^2} \\ &= \frac{\mu}{4\pi}\oint_S \frac{\mathbf{K}dS\times\mathbf{e_R}}{R^2}\end{aligned}$$
$$B\text{在场点, 其余均在源点}$$

安培力

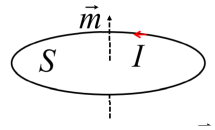
$$d\mathbf{F} = Id\mathbf{l}\times\mathbf{B}$$

安培环路定律

$$\nabla\times\mathbf{H} = \mathbf{J}$$
$$\oint_L \mathbf{H}\cdot d\mathbf{l} = \sum_k I_k$$

有介质

分子磁矩



$$\mathbf{m} = I\mathbf{S}$$

磁力矩

$$\mathbf{T} = \mathbf{m}\times\mathbf{B}$$

磁化强度

$$\mathbf{M} = \lim_{\Delta V \to 0} \frac{\sum_i \mathbf{m_i}}{\Delta V}$$

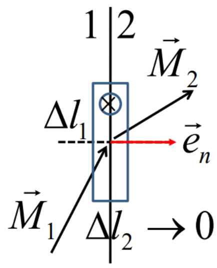
$$\text{理想介质中}, \mathbf{M} = \chi_m \mathbf{H}, \chi_m\text{是磁化率}$$

磁化电流

$$I_m = \oint_L \mathbf{J_m} \cdot d\mathbf{l}$$

$$\mathbf{J_m} = \nabla \times \mathbf{M}$$

磁化强度衔接条件



$$(\mathbf{M_1} - \mathbf{M_2}) \times \mathbf{e_n} = \mathbf{K_m}$$

磁场强度

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$\mathbf{B} = \mu \mathbf{H}(\text{理想介质})$$

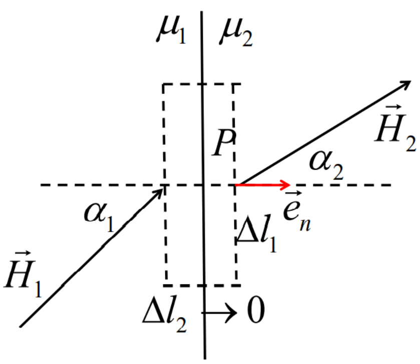
$$\nabla \times \mathbf{H} = \mathbf{J}, \text{自由电流密度, 安培环路定律2.0}$$

不存在磁单极子,磁场是无源的

$$\nabla \cdot \mathbf{B} = 0(\text{微分形式})$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0(\text{积分形式})$$

衔接条件、折射定律



考虑安培环路定律$\nabla \times \mathbf{H} = \mathbf{J}$:

$$(\mathbf{H_1} - \mathbf{H_2}) \times \mathbf{e_n} = \mathbf{K}$$

$$H_{1t} - H_{2t} = K, \text{分量形式}$$

磁场无源:

$$(\mathbf{B_1} - \mathbf{B_2}) \cdot \mathbf{e_n} = 0$$

$$\mathbf{B_{1n}} = \mathbf{B_{2n}}, \text{分量形式}$$

折射定律

$$\text{理想介质, 无面电流}$$
$$\frac{\tan\alpha_1}{\tan\alpha_2}=\frac{\mu_1}{\mu_2}$$

磁矢位

磁场无源,所以将$\mathbf{B}$看做一个场的旋度场(旋度无散)

$$\mathbf{B}=\nabla\times\mathbf{A}$$
$$\text{称}\mathbf{A}\text{为磁矢位}$$

代入安培环路定律和磁场构造方程

$$\nabla\times\nabla\times\mathbf{A}=\mu\mathbf{J}$$

利用矢量恒等式,并令$\nabla\cdot\mathbf{A}=\mathbf{0}$(库伦规范条件),得到一个三维泊松方程

$$\nabla^2\mathbf{A}=-\mu\mathbf{J}$$

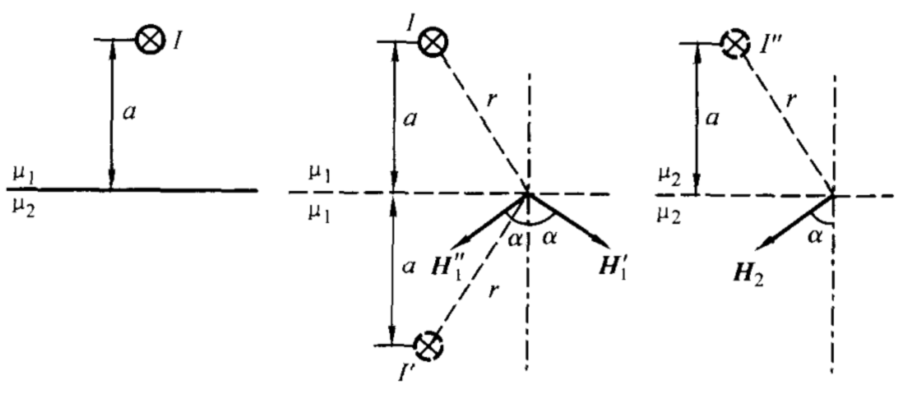
类比静电场的泊松方程的解

$$\begin{aligned}\mathbf{A}&=\frac{\mu}{4\pi}\int_{V'}\frac{\mathbf{J}dV'}{R}\\&=\frac{\mu}{4\pi}\int_{S'}\frac{\mathbf{K}dS'}{R}\\&=\frac{\mu}{4\pi}\oint_{L'}\frac{Id\mathbf{l}'}{R}\end{aligned}$$

衔接方程

$$\mathbf{A_1}=\mathbf{A_2}$$
$$\frac{1}{\mu_1}\frac{\partial A_1}{\partial n}-\frac{1}{\mu_2}\frac{\partial A_2}{\partial n}=K,\text{平行平面场}$$

镜像法



(a) (b) (c)

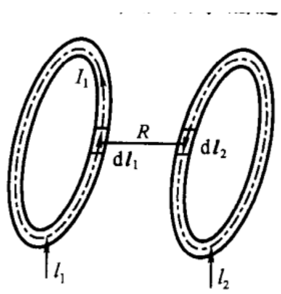
由衔接条件

$$\frac{I}{2\pi r}\sin\alpha-\frac{I'}{2\pi r}\sin\alpha=\frac{I''}{2\pi r}\sin\alpha$$
$$\mu_1(\frac{I}{2\pi r}\cos\alpha+\frac{I'}{2\pi r}\cos\alpha)=\mu_2\frac{I''}{2\pi r}\cos\alpha$$

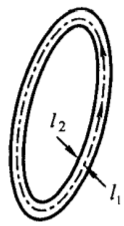
解得

$$
I' = \frac{\mu_2 - \mu_1}{\mu_1 + \mu 2} I
$$
$$
I'' = \frac{2\mu_1}{\mu_1 + \mu_2} I
$$

Neumann公式



$$
M_{12} = M_{21} = \frac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}
$$



$$
L_o = \frac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}
$$
$$
L = L_i + L_o \approx L_o
$$
$$
L_i \approx \frac{\mu l}{8\pi}
$$

第四章

含时Maxwell's Equation

涡旋电场

$$
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B})
$$

在静止媒质中$\mathbf{v} = \mathbf{0}$

$$
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
$$

全电流定律

位移电流

$$
\mathbf{J_d} = \frac{\partial \mathbf{D}}{\partial t}
$$

$$
\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J_d} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}
$$

理想导体边界条件

导体内部(静电屏蔽)

$$
\begin{gathered}
\mathbf{E_1}=\mathbf{0}\\
\mathbf{B_1}=\mathbf{0}
\end{gathered}
$$

导体外部(衔接方程)

$$
\begin{gathered}
(\mathbf{E_1}-\mathbf{E_2})\times\mathbf{e_n}=\mathbf{0}\Rightarrow E_{2t}=0\\
(\mathbf{H_1}-\mathbf{H_2})\times\mathbf{e_n}=\mathbf{K}\Rightarrow B_{2t}=K\\
(\mathbf{D_1}-\mathbf{D_2})\cdot\mathbf{e_n}=\sigma\Rightarrow D_{2n}=\sigma\\
(\mathbf{B_1}-\mathbf{B_2})\cdot\mathbf{e_n}=0\Rightarrow B_{2n}=0
\end{gathered}
$$

导体附近$\mathbf{E}$线垂直于表面,$\mathbf{B}$线平行于表面

动态位

$$
\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t}=-\nabla\varphi
$$

于是

$$
\begin{gathered}
\mathbf{E}=-\nabla\varphi-\frac{\partial\mathbf{A}}{\partial t}\\
\mathbf{B}=\nabla\times\mathbf{A}
\end{gathered}
$$

达朗贝尔方程

$$
\begin{gathered}
\square=\nabla^2-\mu\varepsilon\frac{\partial^2}{\partial t^2}=\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\\
\square\mathbf{A}=-\mu\mathbf{J}\\
\square\varphi=-\frac{\rho}{\varepsilon}
\end{gathered}
$$

洛伦兹规范条件

$$
\nabla\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0
$$

达朗贝尔方程的解

$$
\begin{gathered}
R=|\mathbf{r}-\mathbf{r}'|\\
\varphi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon}\int_{V'}\frac{\rho(\mathbf{r}',t-\frac{R}{c})dV'}{R}\\
\mathbf{A}(\mathbf{r},t)=\frac{\mu}{4\pi}\int_{V'}\frac{\mathbf{J}(\mathbf{r}',t-\frac{R}{c})dV'}{R}
\end{gathered}
$$

电磁场能量

波印廷定理(能量守恒)

$$
\frac{\partial W}{\partial t}=-\oint_{\partial V}\mathbf{E}\times\mathbf{H}\cdot\mathbf{dS}+\int_V\mathbf{E_e}\cdot\mathbf{J}dV-\int_V\frac{\mathbf{J}^2}{\gamma}dV
$$

> 电磁场能量变化=-电磁波向外辐射+电源提供-焦耳热

波印廷矢量

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

正弦电磁场

$$\mathbf{E}(x,y,z,t) = \mathrm{Re}[\sqrt{2}\dot{\mathbf{E}}(x,y,z)e^{j\omega t}]$$

$$\dot{\mathbf{E}}(x,y,z) = \sum_{i\in\{x,y,z\}} \frac{1}{\sqrt{2}} E_{mi} e^{j\phi_i} \mathbf{e_i}$$

波印廷矢量均值

$$\mathbf{S_{av}} = (\mathbf{E} \times \mathbf{H}) \cos{(\phi_E - \phi_B)}$$

复波印廷矢量

$$\dot{\mathbf{S}} = \dot{\mathbf{E}} \times \dot{\mathbf{H}}^*$$

$$\mathbf{S_{av}} = \mathrm{Re}[\dot{\mathbf{S}}]$$

达朗贝尔方程得复数形式

$$\nabla^2 \dot{\mathbf{A}} + \beta^2 \dot{\mathbf{A}} = -\mu \dot{\mathbf{J}}$$

$$\nabla^2 \dot{\varphi} + \beta^2 \dot{\varphi} = -\frac{\dot{\rho}}{\varepsilon}$$

$$\text{其中} \beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$