第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e_n}$$

极化体电荷:

$$\rho_{\mathbf{P}} = -\nabla \cdot \mathbf{P}$$

极化电荷代数和应为0:

$$0 = \oint_S \sigma_{\mathbf{P}} dS + \int_V \rho_{\mathbf{P}} dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D_{n2}} - \mathbf{D_{n1}} = \sigma$$

电位连续性、E的切向连续性:

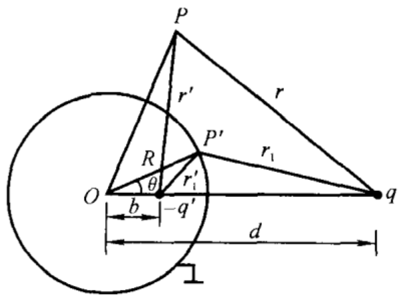
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b = \frac{R^2}{d}$$

$$q' = \frac{R}{d}q$$

两种理想介质

在$r_1 = r_2$处

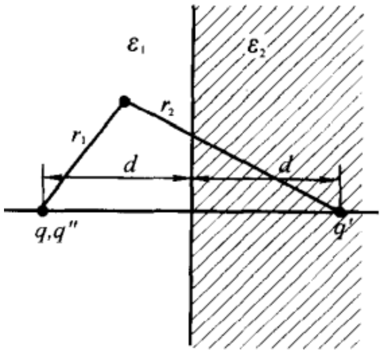


图 1 − 30　点电荷对无限大介质分界平面的镜像

$$\begin{cases} \frac{q}{\varepsilon_1} + \frac{q'}{\varepsilon_1} = \frac{q''}{\varepsilon_2} \\ q - q' = q'' \end{cases}$$

$$\therefore \begin{cases} q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \\ q'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

同心夹层球

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

孤立球

$$C = 4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电，容易想到

$$\begin{aligned} W_e &= \frac{1}{2}\int_V \rho\varphi dV \\ &= \frac{1}{2}\int_S \sigma\varphi dS \end{aligned}$$

静电场能量密度

$$w_e' = \frac{1}{2}\mathbf{D} \cdot \mathbf{E}$$

虚功原理

$$dW = dW_e + fdg$$

不与电源相连

$$0 = dW_e + fdg$$

$$\therefore f = -\frac{\partial W_e}{\partial g}$$

与电源相连,各带电体电位不变

$$dW_e = \frac{1}{2}\sum_k \varphi_k dq_k = \frac{1}{2}dW$$

$$\therefore f = \frac{\partial W_e}{\partial g}$$

第二章

电流密度

$$\mathbf{J} = \rho\mathbf{v}$$

作用于垂直于v的dS

$$dI = \mathbf{J}\cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma\mathbf{v}$$

作用于垂直于v的dl

$$dI = \mathbf{K}\cdot\mathbf{e_n}dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v}dq = \mathbf{J}dV = \mathbf{K}dS = Id\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma\mathbf{E}$$

$$\gamma\text{为电导率}$$

焦耳定律微分形式

$$p = \frac{dP}{dV} = \mathbf{J}\cdot\mathbf{E}$$

含源欧姆定律

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{E_e})$$

电流连续性方程

$$\oint_S \mathbf{J}\cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$
\begin{gathered}
\mathbf{J_{1n}} = \mathbf{J_{2n}}, \text{体现一个电荷守恒} \\
\mathbf{E_{1t}} = \mathbf{E_{2t}}, \text{体现一个保守场} \\
\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\gamma_1}{\gamma_2}, \text{谓之折射定律}
\end{gathered}
$$

第三章

BS定律

$$
\begin{aligned}
\mathbf{B} &= \frac{\mu}{4\pi}\oint_L \frac{I d\mathbf{l} \times \mathbf{e_R}}{R^2} \\
&= \frac{\mu}{4\pi}\oint_V \frac{\mathbf{J} dV \times \mathbf{e_R}}{R^2} \\
&= \frac{\mu}{4\pi}\oint_S \frac{\mathbf{K} dS \times \mathbf{e_R}}{R^2}
\end{aligned}
$$

$$B\text{在场点,其余均在源点}$$

安培力

$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$$

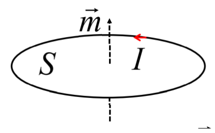
安培环路定律

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \sum_k I_k$$

有介质

分子磁矩



$$\mathbf{m} = I\mathbf{S}$$

磁力矩

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

磁化强度

$$\mathbf{M} = \lim_{\Delta V \to 0} \frac{\sum_i \mathbf{m_i}}{\Delta V}$$

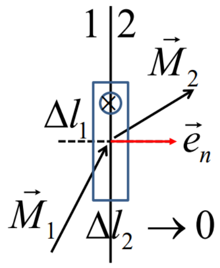
$$\text{理想介质中}, \mathbf{M} = \chi_m \mathbf{H}, \chi_m\text{是磁化率}$$

磁化电流

$$I_m = \oint_L \mathbf{J_m} \cdot d\mathbf{l}$$

$$\mathbf{J_m} = \nabla \times \mathbf{M}$$

磁化强度衔接条件



$$(\mathbf{M_1}-\mathbf{M_2})\times\mathbf{e_n}=\mathbf{K_m}$$

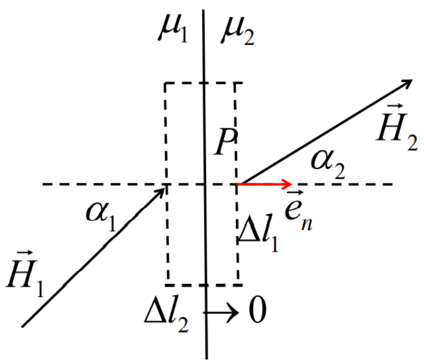
磁场强度

$$\mathbf{H}=\frac{\mathbf{B}}{\mu_0}-\mathbf{M}$$
$$\mathbf{B}=\mu\mathbf{H}(\text{理想介质})$$
$$\nabla\times\mathbf{H}=\mathbf{J},\text{自由电流密度},\text{安培环路定律}2.0$$

不存在磁单极子,磁场是无源的

$$\nabla\cdot\mathbf{B}=0(\text{微分形式})$$
$$\oint_S\mathbf{B}\cdot d\mathbf{S}=0(\text{积分形式})$$

衔接条件、折射定律



考虑安培环路定律$\nabla\times\mathbf{H}=\mathbf{J}$:

$$(\mathbf{H_1}-\mathbf{H_2})\times\mathbf{e_n}=\mathbf{K}$$
$$H_{1t}-H_{2t}=K,\text{分量形式}$$

磁场无源:

$$(\mathbf{B_1}-\mathbf{B_2})\cdot\mathbf{e_n}=0$$
$$\mathbf{B_{1n}}=\mathbf{B_{2n}},\text{分量形式}$$

折射定律

$$\text{理想介质},\text{无面电流}$$
$$\frac{\tan\alpha_1}{\tan\alpha_2}=\frac{\mu_1}{\mu_2}$$

磁矢位

磁场无源,所以将$\mathbf{B}$看做一个场的旋度场(旋度无散)

$$\mathbf{B}=\nabla\times\mathbf{A}$$
称$\mathbf{A}$为磁矢位

代入安培环路定律和磁场构造方程

$$\nabla\times\nabla\times\mathbf{A}=\mu\mathbf{J}$$

利用矢量恒等式,并令$\nabla\cdot\mathbf{A}=\mathbf{0}$(库伦规范条件),得到一个三维泊松方程

$$\nabla^2\mathbf{A}=-\mu\mathbf{J}$$

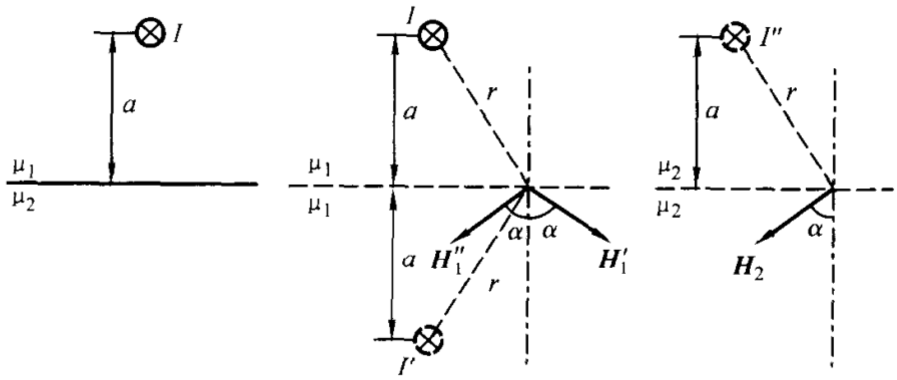
类比静电场的泊松方程的解

$$\begin{aligned}\mathbf{A}&=\frac{\mu}{4\pi}\int_{V'}\frac{\mathbf{J}dV'}{R}\\&=\frac{\mu}{4\pi}\int_{S'}\frac{\mathbf{K}dS'}{R}\\&=\frac{\mu}{4\pi}\oint_{L'}\frac{Id\mathbf{l}'}{R}\end{aligned}$$

衔接方程

$$\mathbf{A_1}=\mathbf{A_2}$$
$$\frac{1}{\mu_1}\frac{\partial A_1}{\partial n}-\frac{1}{\mu_2}\frac{\partial A_2}{\partial n}=K,\text{平行平面场}$$

镜像法



(a) (b) (c)

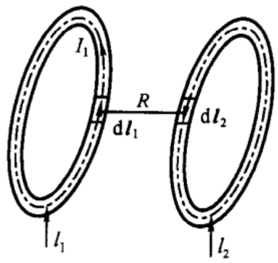
由衔接条件

$$\frac{I}{2\pi r}\sin\alpha-\frac{I'}{2\pi r}\sin\alpha=\frac{I''}{2\pi r}\sin\alpha$$
$$\mu_1(\frac{I}{2\pi r}\cos\alpha+\frac{I'}{2\pi r}\cos\alpha)=\mu_2\frac{I''}{2\pi r}\cos\alpha$$

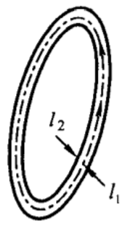
解得

$$I'=\frac{\mu_2-\mu_1}{\mu_1+\mu 2}I$$
$$I''=\frac{2\mu_1}{\mu_1+\mu_2}I$$

Neumann公式



$$M_{12} = M_{21} = \frac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}$$



$$L_o = \frac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}$$

$$L = L_i + L_o \approx L_o$$

$$L_i \approx \frac{\mu l}{8\pi}$$