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有介质 磁矢位

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含时Maxwell's Equation

动态位

电磁场能量

正弦电磁场

第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e_n}$$

极化体电荷:

$$ho_{\mathbf{P}} = -
abla \cdot \mathbf{P}$$

极化电荷代数和应为0:

$$0 = \oint_S \sigma_{f P} dS + \int_V
ho_{f P} dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D_{n2}} - \mathbf{D_{n1}} = \sigma$$

电位连续性、E的切向连续性:

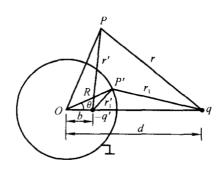
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b=rac{R^2}{d}$$
 $q'=rac{R}{d}q$

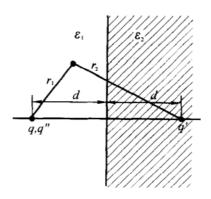


图 1-30 点电荷对无限大介质分界平面的镜像

$$egin{cases} rac{q}{arepsilon_1} + rac{q'}{arepsilon_1} = rac{q''}{arepsilon_2} \ q - q' = q'' \end{cases}$$

$$\therefore egin{cases} q' = rac{arepsilon_1 - arepsilon_2}{arepsilon_1 + arepsilon_2} q \ q'' = rac{2arepsilon_2}{arepsilon_1 + arepsilon_2} q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\varepsilon}{\ln\left(b/a\right)}$$

同心夹层球

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

孤立球

$$C = 4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电, 容易想到

$$egin{aligned} W_e &= rac{1}{2} \int_V
ho arphi dV \ &= rac{1}{2} \int_S \sigma arphi dS \end{aligned}$$

静电场能量密度

$$w_e' = rac{1}{2} {f D} \cdot {f E}$$

虚功原理

$$dW = dW_e + fdg$$

不与电源相连

$$0 = dW_e + fdg$$

$$\therefore f = -\frac{\partial W_e}{\partial g}$$

与电源相连,各带电体电位不变

$$egin{aligned} dW_e &= rac{1}{2} \sum_k arphi_k dq_k = rac{1}{2} dW \ &\therefore f = rac{\partial W_e}{\partial g} \end{aligned}$$

第二章

电流密度

$$\mathbf{J} = \rho \mathbf{v}$$

作用于垂直于v的dS

$$dI = \mathbf{J} \cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma \mathbf{v}$$

作用于垂直于v的dl

$$dI = \mathbf{K} \cdot \mathbf{e_n} dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v}dq = \mathbf{J}dV = \mathbf{K}dS = Id\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma \mathbf{E}$$
 γ 为电导率

焦耳定律微分形式

$$p = rac{dP}{dV} = {f J} \cdot {f E}$$

含源欧姆定律

$$\mathbf{J} = \gamma (\mathbf{E} + \mathbf{E_e})$$

电流连续性方程

$$\oint_{S} \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$\mathbf{J_{1n}} = \mathbf{J_{2n}}$$
,体现一个电荷守恒 $\mathbf{E_{1t}} = \mathbf{E_{2t}}$,体现一个保守场 $\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}$,谓之折射定律

第三章

BS定律

$$\mathbf{B} = rac{\mu}{4\pi} \oint_L rac{Id\mathbf{l} imes \mathbf{e_R}}{R^2}$$

$$= rac{\mu}{4\pi} \oint_V rac{\mathbf{J} dV imes \mathbf{e_R}}{R^2}$$

$$= rac{\mu}{4\pi} \oint_S rac{\mathbf{K} dS imes \mathbf{e_R}}{R^2}$$
 B 在场点,其余均在源点

安培力

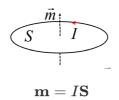
$$d\mathbf{F} = Id\mathbf{l} \times \mathbf{B}$$

安培环路定律

$$abla extbf{ iny H} = extbf{J} \ \oint_L extbf{H} \cdot d extbf{l} = \sum_k I_k$$

有介质

分子磁矩



磁力矩

$$\mathbf{T} = \mathbf{m} {\times} \mathbf{B}$$

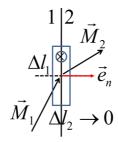
磁化强度

$$\mathbf{M}=\lim_{\Delta V o 0}rac{\sum_{i}\mathbf{m_{i}}}{\Delta V}$$
理想介质中, $\mathbf{M}=\chi_{m}\mathbf{H},\chi_{m}$ 是磁化率

磁化电流

$$I_m = \oint_L \mathbf{J_m} \cdot d\mathbf{l} \ \mathbf{J_m} =
abla imes \mathbf{M}$$

磁化强度衔接条件



$$(\mathbf{M_1} - \mathbf{M_2}) {\times} \mathbf{e_n} = \mathbf{K_m}$$

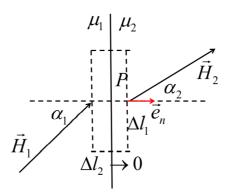
磁场强度

$$\mathbf{H}=rac{\mathbf{B}}{\mu_{\mathbf{0}}}-\mathbf{M}$$
 $\mathbf{B}=\mu\mathbf{H}($ 理想介质 $)$ $abla imes\mathbf{H}=\mathbf{J},$ 自由电流密度 $,$ 安培环路定律 $\mathbf{2}.0$

不存在磁单极子,磁场是无源的

$$abla \cdot \mathbf{B} = 0 ($$
微分形式 $)$ $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 ($ 积分形式 $)$

衔接条件、折射定律



考虑安培环路定律 $\nabla \times \mathbf{H} = \mathbf{J}$:

$$(\mathbf{H_1} - \mathbf{H_2}) imes \mathbf{e_n} = \mathbf{K}$$
 $H_{1t} - H_{2t} = K,$ 分量形式

磁场无源:

$$(\mathbf{B_1} - \mathbf{B_2}) \cdot \mathbf{e_n} = 0$$
 $\mathbf{B_{1n}} = \mathbf{B_{2n}},$ 分量形式

理想介质,无面电流

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2}$$

磁矢位

磁场无源,所以将图看做一个场的旋度场(旋度无散)

$$\mathbf{B} =
abla imes \mathbf{A}$$

称A为磁矢位

代入安培环路定律和磁场构造方程

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

利用矢量恒等式,并令 $\nabla \cdot \mathbf{A} = \mathbf{0}$ (库伦规范条件),得到一个三维泊松方程

$$abla^2 \mathbf{A} = -\mu \mathbf{J}$$

类比静电场的泊松方程的解

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}dV'}{R}$$

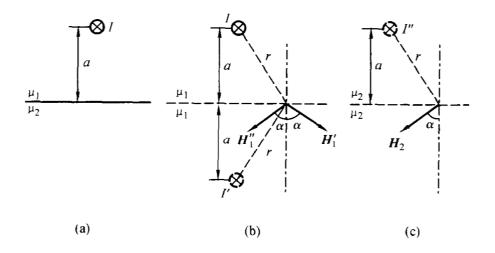
$$= \frac{\mu}{4\pi} \int_{S'} \frac{\mathbf{K}dS'}{R}$$

$$= \frac{\mu}{4\pi} \oint_{L'} \frac{Id\mathbf{l}'}{R}$$

衔接方程

$$egin{aligned} \mathbf{A_1} &= \mathbf{A_2} \ &rac{1}{\mu_1} rac{\partial A_1}{\partial n} - rac{1}{\mu_2} rac{\partial A_2}{\partial n} = K,$$
平行平面场

镜像法



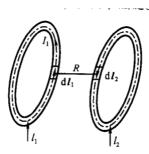
由衔接条件

$$\frac{I}{2\pi r}\sin\alpha - \frac{I'}{2\pi r}\sin\alpha = \frac{I''}{2\pi r}\sin\alpha$$
$$\mu_1(\frac{I}{2\pi r}\cos\alpha + \frac{I'}{2\pi r}\cos\alpha) = \mu_2\frac{I''}{2\pi r}\cos\alpha$$

$$I' = rac{\mu_2 - \mu_1}{\mu_1 + \mu_2} I$$

$$I'' = rac{2\mu_1}{\mu_1 + \mu_2} I$$

Neumann公式



$$M_{12} = M_{21} = rac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} rac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}$$



$$egin{align} L_o &= rac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} rac{\mathbf{dl_1 \cdot dl_2}}{R} \ L &= L_i + L_o pprox L_o \ L_i &pprox rac{\mu l}{8\pi} \ \end{pmatrix}$$

第四章

含时Maxwell's Equation

涡旋电场

$$abla{ imes}\mathbf{E} = -rac{\partial \mathbf{B}}{\partial t} +
abla{ imes}(\mathbf{v}{ imes}\mathbf{B})$$

在静止媒质中v = 0

$$abla imes \mathbf{E} = -rac{\partial \mathbf{B}}{\partial t}$$

全电流定律

位移电流

$$\mathbf{J_d} = rac{\partial \mathbf{D}}{\partial t}$$
 $abla imes \mathbf{H} = \mathbf{J} + \mathbf{J_d} = \mathbf{J} + rac{\partial \mathbf{D}}{\partial t}$

理想导体边界条件

导体内部(静电屏蔽)

$$\mathbf{E_1} = \mathbf{0}$$
 $\mathbf{B_1} = \mathbf{0}$

导体外部(衔接方程)

$$(\mathbf{E_1} - \mathbf{E_2}) \times \mathbf{e_n} = \mathbf{0} \Rightarrow E_{2t} = 0$$

$$(\mathbf{H_1} - \mathbf{H_2}) \times \mathbf{e_n} = \mathbf{K} \Rightarrow B_{2t} = K$$

$$(\mathbf{D_1} - \mathbf{D_2}) \cdot \mathbf{e_n} = \sigma \Rightarrow D_{2n} = \sigma$$

$$(\mathbf{B_1} - \mathbf{B_2}) \cdot \mathbf{e_n} = 0 \Rightarrow B_{2n} = 0$$

导体附近E线垂直于表面,B线平行于表面

动态位

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi$$

于是

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}$$

 $\mathbf{B} = \nabla \times \mathbf{A}$

达朗贝尔方程

$$\Box = \nabla^2 - \mu \varepsilon \frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$
$$\Box \mathbf{A} = -\mu \mathbf{J}$$
$$\Box \varphi = -\frac{\rho}{\varepsilon}$$

洛伦兹规范条件

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

达朗贝尔方程的解

$$R = |\mathbf{r} - \mathbf{r}'|$$
 $arphi(\mathbf{r},t) = rac{1}{4\piarepsilon} \int_{V'} rac{
ho(\mathbf{r}',t-rac{R}{c})dV'}{R}$ $\mathbf{A}(\mathbf{r},t) = rac{\mu}{4\pi} \int_{V'} rac{\mathbf{J}(\mathbf{r}',t-rac{R}{c})dV'}{R}$

电磁场能量

波印廷定理(能量守恒)

$$rac{\partial W}{\partial t} = -\oint_{\partial V} \mathbf{E} imes \mathbf{H} \cdot \mathbf{dS} + \int_{V} \mathbf{E_e} \cdot \mathbf{J} dV - \int_{V} \frac{\mathbf{J}^2}{\gamma} dV$$

电磁场能量变化=-电磁波向外辐射+电源提供-焦耳热

波印廷矢量

正弦电磁场

$$\mathbf{E}(x,y,z,t) = \mathrm{Re}[\sqrt{2}\dot{\mathbf{E}}(x,y,z)e^{j\omega t}] \ \dot{\mathbf{E}}(x,y,z) = \sum_{i\in\{x,y,z\}} rac{1}{\sqrt{2}} E_{mi}e^{j\phi_i}\mathbf{e_i}$$

波印廷矢量均值

$$\mathbf{S_{av}} = (\mathbf{E} { imes} \mathbf{H}) \cos \left(\phi_E - \phi_B
ight)$$

复波印廷矢量

$$\dot{\mathbf{S}} = \dot{\mathbf{E}} \times \dot{\mathbf{H}}^*$$
 $\mathbf{S}_{\mathbf{a}\mathbf{v}} = \mathrm{Re}[\dot{\mathbf{S}}]$

达朗贝尔方程的复数形式

$$abla^2 \dot{\mathbf{A}} + eta^2 \dot{\mathbf{A}} = -\mu \dot{\mathbf{J}}$$

$$abla^2 \dot{\varphi} + eta^2 \dot{\varphi} = -\frac{\dot{\rho}}{\varepsilon}$$

$$\sharp \oplus \beta = \omega \sqrt{\mu \varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

洛伦兹规范条件复数形式

$$\nabla \cdot \mathbf{\dot{A}} + j\omega\mu\varepsilon\dot{\varphi} = 0$$

动态位复数形式

$$egin{aligned} \dot{\mathbf{E}} &= -
abla \dot{arphi} - j\omega\muarepsilon\dot{\mathbf{A}} \ &= rac{
abla (
abla \cdot \dot{\mathbf{A}})}{j\omega\muarepsilon} - j\omega\muarepsilon\dot{\mathbf{A}} \end{aligned}$$

$$\mathbf{\dot{B}} =
abla imes \mathbf{\dot{A}}$$

达朗贝尔方程的复数解

$$R = |\mathbf{r} - \mathbf{r}'| \ \dot{arphi} = rac{1}{4\piarepsilon} \int_{V'} rac{\dot{
ho}e^{-jrac{2\pi R}{\lambda}}dV'}{R} \ \dot{\mathbf{A}} = rac{\mu}{4\pi} \int_{V'} rac{\dot{\mathbf{J}}e^{-jrac{2\pi R}{\lambda}}dV'}{R}$$

似稳条件

$$\frac{2\pi R}{\lambda} << 1$$