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第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e}_n$$

极化体电荷:

$$\rho_{\mathbf{P}} = -\nabla \cdot \mathbf{P}$$

极化电荷代数数和应为0:

$$0 = \oint_S \sigma_{\mathbf{P}} dS + \int_V \rho_{\mathbf{P}} dV$$

在理想电介质中:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D}_{n2} - \mathbf{D}_{n1} = \sigma$$

电位连续性、E的切向连续性:

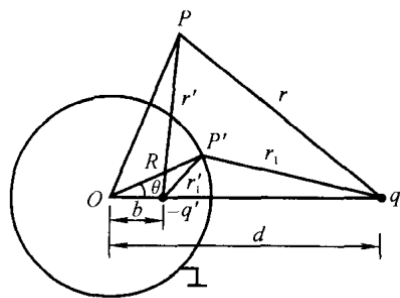
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b = \frac{R^2}{d}$$
$$q' = \frac{R}{d}q$$

两种理想介质
在 $r_1 = r_2$ 处

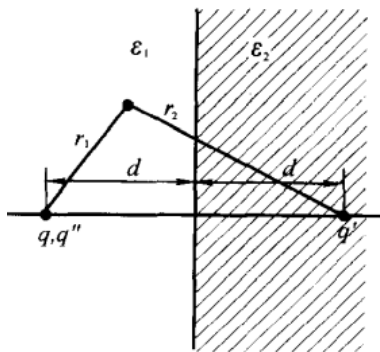


图 1 - 30 点电荷对无限大
介质分界平面的镜像

$$\begin{cases} \frac{q}{\varepsilon_1} + \frac{q'}{\varepsilon_1} = \frac{q''}{\varepsilon_2} \\ q - q' = q'' \end{cases}$$
$$\therefore \begin{cases} q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \\ q'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

同心夹层球

$$C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$$

孤立球

$$C = 4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电，容易想到

$$\begin{aligned} W_e &= \frac{1}{2} \int_V \rho\varphi dV \\ &= \frac{1}{2} \int_S \sigma\varphi dS \end{aligned}$$

静电场能量密度

$$w'_e = \frac{1}{2} \mathbf{D} \cdot \mathbf{E}$$

虚功原理

$$dW = dW_e + fdg$$

不与电源相连

$$\begin{aligned} 0 &= dW_e + fdg \\ \therefore f &= -\frac{\partial W_e}{\partial g} \end{aligned}$$

与电源相连,各带电体电位不变

$$\begin{aligned} dW_e &= \frac{1}{2} \sum_k \varphi_k dq_k = \frac{1}{2} dW \\ \therefore f &= \frac{\partial W_e}{\partial g} \end{aligned}$$

第二章

电流密度

$$\mathbf{J} = \rho \mathbf{v}$$

作用于垂直于 \mathbf{v} 的 dS

$$dI = \mathbf{J} \cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma \mathbf{v}$$

作用于垂直于 \mathbf{v} 的 dl

$$dI = \mathbf{K} \cdot \mathbf{e}_n dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v}dq = \mathbf{J}dV = \mathbf{K}dS = Id\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma \mathbf{E}$$

γ 为电导率

焦耳定律微分形式

$$p = \frac{dP}{dV} = \mathbf{J} \cdot \mathbf{E}$$

含源欧姆定律

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{E}_e)$$

电流连续性方程

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$\mathbf{J}_{1n} = \mathbf{J}_{2n}$, 体现一个电荷守恒

$\mathbf{E}_{1t} = \mathbf{E}_{2t}$, 体现一个保守场

$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}$, 谓之折射定律

第三章

BS定律

$$\begin{aligned} \mathbf{B} &= \frac{\mu}{4\pi} \oint_L \frac{Id\mathbf{l} \times \mathbf{e}_R}{R^2} \\ &= \frac{\mu}{4\pi} \oint_V \frac{\mathbf{J}dV \times \mathbf{e}_R}{R^2} \\ &= \frac{\mu}{4\pi} \oint_S \frac{\mathbf{K}dS \times \mathbf{e}_R}{R^2} \end{aligned}$$

B 在场点, 其余均在源点

安培力

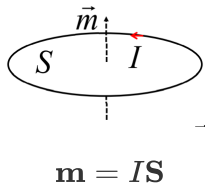
$$d\mathbf{F} = Id\mathbf{l} \times \mathbf{B}$$

安培环路定律

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} \\ \oint_L \mathbf{H} \cdot d\mathbf{l} &= \sum_k I_k \end{aligned}$$

有介质

分子磁矩



磁力矩

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

磁化强度

$$\mathbf{M} = \lim_{\Delta V \rightarrow 0} \frac{\sum_i \mathbf{m}_i}{\Delta V}$$

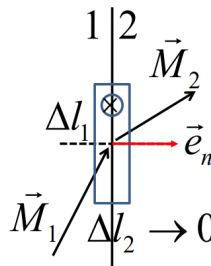
理想介质中, $\mathbf{M} = \chi_m \mathbf{H}$, χ_m 是磁化率

磁化电流

$$I_m = \oint_S \mathbf{J}_m \cdot d\mathbf{S}$$

$$\mathbf{J}_m = \nabla \times \mathbf{M}$$

磁化强度衔接条件



$$(\mathbf{M}_1 - \mathbf{M}_2) \times \mathbf{e}_n = \mathbf{K}_m$$

磁场强度

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$\mathbf{B} = \mu \mathbf{H} \text{ (理想介质)}$$

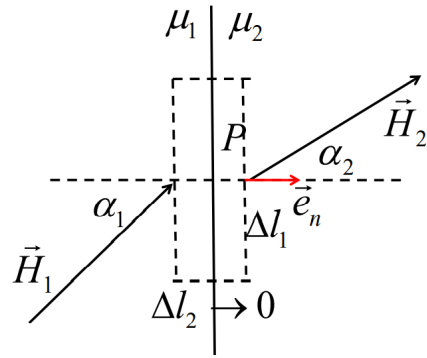
$$\nabla \times \mathbf{H} = \mathbf{J}, \text{ 自由电流密度, 安培环路定律2.0}$$

不存在磁单极子, 磁场是无源的

$$\nabla \cdot \mathbf{B} = 0 \text{ (微分形式)}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \text{ (积分形式)}$$

衔接条件、折射定律



考虑安培环路定律 $\nabla \times \mathbf{H} = \mathbf{J}$:

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{e}_n = \mathbf{K}$$

$$H_{1t} - H_{2t} = K, \text{分量形式}$$

磁场无源:

$$(\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{e}_n = 0$$

$$\mathbf{B}_{1n} = \mathbf{B}_{2n}, \text{分量形式}$$

折射定律

理想介质, 无面电流

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2}$$

磁矢位

磁场无源, 所以将 \mathbf{B} 看做一个场的旋度场(旋度无散)

$$\mathbf{B} = \nabla \times \mathbf{A}$$

称 \mathbf{A} 为磁矢位

代入安培环路定律和磁场构造方程

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

利用矢量恒等式, 并令 $\nabla \cdot \mathbf{A} = 0$ (库伦规范条件), 得到一个三维泊松方程

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

类比静电场的泊松方程的解

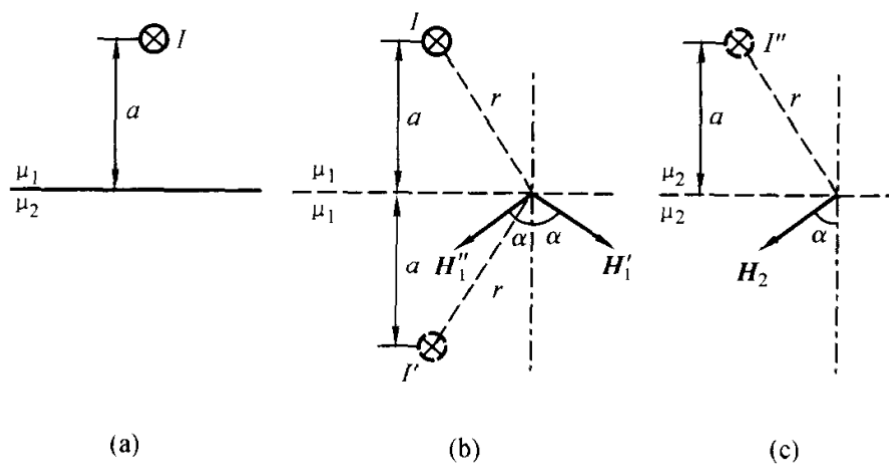
$$\begin{aligned} \mathbf{A} &= \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J} dV'}{R} \\ &= \frac{\mu}{4\pi} \int_{S'} \frac{\mathbf{K} dS'}{R} \\ &= \frac{\mu}{4\pi} \oint_{L'} \frac{I dl'}{R} \end{aligned}$$

衔接方程

$$\mathbf{A}_1 = \mathbf{A}_2$$

$$\frac{1}{\mu_1} \frac{\partial A_1}{\partial n} - \frac{1}{\mu_2} \frac{\partial A_2}{\partial n} = K, \text{平行平面场}$$

镜像法



由衔接条件

$$\frac{I}{2\pi r} \sin \alpha - \frac{I'}{2\pi r} \sin \alpha = \frac{I''}{2\pi r} \sin \alpha$$

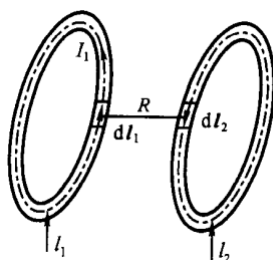
$$\mu_1 \left(\frac{I}{2\pi r} \cos \alpha + \frac{I'}{2\pi r} \cos \alpha \right) = \mu_2 \frac{I''}{2\pi r} \cos \alpha$$

解得

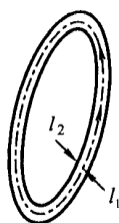
$$I' = \frac{\mu_2 - \mu_1}{\mu_1 + \mu_2} I$$

$$I'' = \frac{2\mu_1}{\mu_1 + \mu_2} I$$

Neumann公式



$$M_{12} = M_{21} = \frac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl}_1 \cdot \mathbf{dl}_2}{R}$$



$$L_o = \frac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl}_1 \cdot \mathbf{dl}_2}{R}$$

$$L = L_i + L_o \approx L_o$$

$$L_i \approx \frac{\mu l}{8\pi}$$

第四章

含时Maxwell's Equation

涡旋电场

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B})$$

在静止媒质中 $\mathbf{v} = \mathbf{0}$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

全电流定律

位移电流

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

理想导体边界条件

导体内部(静电屏蔽)

$$\mathbf{E}_1 = \mathbf{0}$$

$$\mathbf{B}_1 = \mathbf{0}$$

导体外部(衔接方程)

$$(\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{e}_n = \mathbf{0} \Rightarrow E_{2t} = 0$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{e}_n = \mathbf{K} \Rightarrow B_{2t} = K$$

$$(\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{e}_n = \sigma \Rightarrow D_{2n} = \sigma$$

$$(\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{e}_n = 0 \Rightarrow B_{2n} = 0$$

导体附近 \mathbf{E} 线垂直于表面, \mathbf{B} 线平行于表面

动态位

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi$$

于是

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

达朗贝尔方程

$$\square = \nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$

$$\square \mathbf{A} = -\mu \mathbf{J}$$

$$\square \varphi = -\frac{\rho}{\epsilon}$$

洛伦兹规范条件

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

达朗贝尔方程的解

$$R = |\mathbf{r} - \mathbf{r}'|$$
$$\varphi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon} \int_{V'} \frac{\rho(\mathbf{r}', t - \frac{R}{c}) dV'}{R}$$
$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}(\mathbf{r}', t - \frac{R}{c}) dV'}{R}$$

电磁场能量

波印廷定理(能量守恒)

$$\frac{\partial W}{\partial t} = - \oint_{\partial V} \mathbf{E} \times \mathbf{H} \cdot d\mathbf{S} + \int_V \mathbf{E}_e \cdot \mathbf{J} dV - \int_V \frac{\mathbf{J}^2}{\gamma} dV$$

电磁场能量变化=-电磁波向外辐射+电源提供-焦耳热

波印廷矢量

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

正弦电磁场

$$\mathbf{E}(x, y, z, t) = \text{Re}[\sqrt{2}\dot{\mathbf{E}}(x, y, z)e^{j\omega t}]$$
$$\dot{\mathbf{E}}(x, y, z) = \sum_{i \in \{x, y, z\}} \frac{1}{\sqrt{2}} E_{mi} e^{j\phi_i} \mathbf{e}_i$$

波印廷矢量均值

$$\mathbf{S}_{av} = (\mathbf{E} \times \mathbf{H}) \cos(\phi_E - \phi_B)$$

复波印廷矢量

$$\dot{\mathbf{S}} = \dot{\mathbf{E}} \times \dot{\mathbf{H}}^*$$
$$\mathbf{S}_{av} = \text{Re}[\dot{\mathbf{S}}]$$

波印廷定理复数形式

$$j\omega \int_V \mu |\dot{\mathbf{H}}|^2 - \epsilon |\dot{\mathbf{E}}|^2 dV = - \oint_{\partial V} \dot{\mathbf{E}} \times \dot{\mathbf{H}} \cdot d\mathbf{S} - \int_V \frac{|\dot{\mathbf{J}}|^2}{\gamma} dV + \int_V \dot{\mathbf{E}}_e \cdot \dot{\mathbf{J}}^* dV$$

达朗贝尔方程的复数形式

$$\nabla^2 \dot{\mathbf{A}} + \beta^2 \dot{\mathbf{A}} = -\mu \dot{\mathbf{J}}$$
$$\nabla^2 \dot{\varphi} + \beta^2 \dot{\varphi} = -\frac{\dot{\rho}}{\epsilon}$$

其中 $\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

洛伦兹规范条件复数形式

$$\nabla \cdot \dot{\mathbf{A}} + j\omega\mu\epsilon\dot{\varphi} = 0$$

动态位复数形式

$$\begin{aligned}\dot{\mathbf{E}} &= -\nabla\dot{\varphi} - j\omega\mu\varepsilon\dot{\mathbf{A}} \\ &= \frac{\nabla(\nabla\cdot\dot{\mathbf{A}})}{j\omega\mu\varepsilon} - j\omega\mu\varepsilon\dot{\mathbf{A}}\end{aligned}$$

$$\dot{\mathbf{B}} = \nabla\times\dot{\mathbf{A}}$$

达朗贝尔方程的复数解

$$\begin{aligned}R &= |\mathbf{r} - \mathbf{r}'| \\ \dot{\varphi} &= \frac{1}{4\pi\varepsilon} \int_{V'} \frac{\dot{\rho}e^{-j\frac{2\pi R}{\lambda}}dV'}{R} \\ \dot{\mathbf{A}} &= \frac{\mu}{4\pi} \int_{V'} \frac{\dot{\mathbf{J}}e^{-j\frac{2\pi R}{\lambda}}dV'}{R}\end{aligned}$$

似稳条件

$$\frac{2\pi R}{\lambda} \ll 1$$

单元偶极子的辐射

$$P = \frac{2\pi\mu c}{3}I^2\left(\frac{\Delta l}{\lambda}\right)^2 \approx 80\pi^2I^2\left(\frac{\Delta l}{\lambda}\right)^2$$

方向性因子

$$f(\theta, \phi) = \sin\theta$$

准静态电磁场

电准静态场

磁场的变化率约等于0

$$\frac{\partial\mathbf{B}}{\partial t} \approx \mathbf{0}$$

磁准静态场

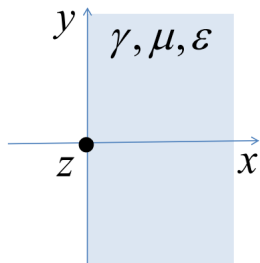
电场的变化率约等于0

$$\frac{\partial\mathbf{E}}{\partial t} \approx \mathbf{0}$$

以下关系成立

$$\begin{aligned}\nabla^2\mathbf{H} - \mu\gamma\frac{\partial\mathbf{H}}{\partial t} &= \mathbf{0} \\ \nabla^2\mathbf{E} - \mu\gamma\frac{\partial\mathbf{E}}{\partial t} &= \mathbf{0} \\ \nabla^2\mathbf{J} - \mu\gamma\frac{\partial\mathbf{J}}{\partial t} &= \mathbf{0}\end{aligned}$$

电流的集肤效应



$x > 0$ 半无限大空间为导体，正弦电流 i 沿 y 方向流过， \vec{J} 只有 y 分量并在 yoz 平面上处处相等。求 i 在半无限大导体中的分布。

$$\frac{\partial^2 \dot{J}_y}{\partial x^2} = j\omega\mu\gamma\dot{J}_y$$

$$\text{定义 } k = \sqrt{j\omega\mu\gamma} = \alpha + j\beta$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\gamma}{2}}$$

$$\dot{J}_y = \dot{J}_0 e^{-\alpha x} e^{-j\beta x}$$

透入深度

$$|e^{-kd}| = |e^{-1}|$$

$$\alpha d = 1$$

$$d = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\gamma}}$$

TEM波

理想介质中在 x 方向上传播的TEM波

$$\frac{\partial^2 E_y}{\partial x^2} - \mu\epsilon \frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} - \mu\epsilon \frac{\partial^2 H_z}{\partial t^2} = 0$$

E 和 H 的关系

$$\frac{E_y^+(x, t)}{H_z^+(x, t)} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\frac{E_y^-(x, t)}{H_z^-(x, t)} = -\sqrt{\frac{\mu}{\epsilon}}$$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}$$

正弦波

$$\dot{\mathbf{E}}(x, t) = \dot{E}_{y0} e^{-kx} \mathbf{e}_y$$

$$k = j\beta = j\frac{\omega}{c}$$

导电介质

$$\frac{\partial^2 E_y}{\partial x^2} - \mu\gamma \frac{\partial E_y}{\partial t} - \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} - \mu\gamma \frac{\partial H_z}{\partial t} - \mu\varepsilon \frac{\partial^2 H_z}{\partial t^2} = 0$$

等效介电常数

$$\varepsilon' = \varepsilon + \frac{\gamma}{j\omega}$$

$$k = j\omega\sqrt{\mu\varepsilon'}$$

波印廷矢量的均值

$$\dot{\mathbf{S}}_{\text{av}} = \text{Re}[\dot{\mathbf{E}} \times \dot{\mathbf{H}}^*] = \frac{1}{|Z_0|} E_0^{+2} e^{-2\alpha x} \cos \phi \mathbf{e}_x$$

$$\phi = \arg \sqrt{\frac{\mu}{\varepsilon'}}$$

极化

$$\phi_z - \phi_y = \frac{\pi}{2}, \text{右旋}$$

$$\phi_z - \phi_y = -\frac{\pi}{2}, \text{左旋}$$

反射与折射

平行极化波

反射系数

$$\Gamma_{\parallel} = \frac{E_{\parallel}^{-}}{E_{\parallel}^{+}} = \frac{Z_{02} \cos \theta_2 - Z_{01} \cos \theta_1}{Z_{02} \cos \theta_2 + Z_{01} \cos \theta_1} = \frac{\tan (\theta_1 - \theta_2)}{\tan (\theta_1 + \theta_2)}$$

折射系数

$$T_{\parallel} = \frac{E'_{\parallel}}{E_{\parallel}^{+}} = \frac{2 Z_{02} \cos \theta_1}{Z_{02} \cos \theta_2 + Z_{01} \cos \theta_1} = \frac{2 \sin \theta_2 \cos \theta_1}{\sin (\theta_1 + \theta_2) \cos (\theta_1 - \theta_2)}$$

垂直极化波

反射系数

$$\Gamma_{\perp} = \frac{E_{\perp}^{-}}{E_{\perp}^{+}} = \frac{Z_{02} \cos \theta_1 - Z_{01} \cos \theta_2}{Z_{02} \cos \theta_1 + Z_{01} \cos \theta_2}$$

折射系数

$$T_{\parallel} = \frac{E'_{\parallel}}{E_{\parallel}^{+}} = \frac{2 Z_{02} \cos \theta_1}{Z_{02} \cos \theta_1 + Z_{01} \cos \theta_2}$$

理想导体处反射

$$E^- = -E^+$$

$$H^- = H^+$$

$$e.g. E_y^+ = E_m \cos(\omega t - \beta x)$$

$$H_z^+ = \frac{E_m}{Z_0} \cos(\omega t - \beta x)$$

$$E_y^- = -E_m \cos(\omega t + \beta x)$$

$$H_z^- = \frac{E_m}{Z_0} \cos(\omega t + \beta x)$$

理想介质反射、折射

$$\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

$$T = \frac{2Z_{02}}{Z_{02} + Z_{01}}$$

$$\text{若 } \dot{E}^+(x) = \dot{E}^+ e^{-j\beta_1 x}$$

$$\text{则 } \dot{H}^+(x) = \frac{\dot{E}^+}{Z_{01}} e^{-j\beta_1 x}$$

$$\dot{E}^-(x) = \Gamma \dot{E}^+ e^{+j\beta_1 x}$$

$$\dot{H}^-(x) = -\frac{\Gamma \dot{E}^+}{Z_{01}} e^{j\beta_1 x}$$

$$\dot{E}'(x) = T \dot{E}^+ e^{-j\beta_2 x}$$

$$\dot{H}'(x) = \frac{T \dot{E}^+}{Z_{02}} e^{-j\beta_2 x}$$

合成电场

$$\dot{E}(x) = \dot{E}^+ + \dot{E}^-$$

为行驻波, 极值为 $|\dot{E}^+|(1 \pm |\Gamma|)$

$$\text{定义驻波比 } S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

传输线

L_0 是单位长度的电感

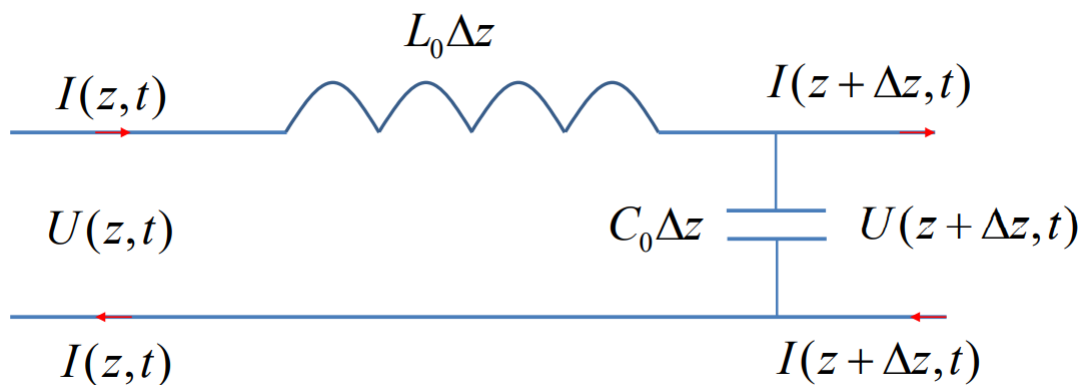
C_0 是单位长度的电容

$$L_0 C_0 = \mu \varepsilon$$

电报方程

$$\frac{\partial U}{\partial z} + L_0 \frac{\partial I}{\partial t} = 0$$

$$\frac{\partial I}{\partial z} + C_0 \frac{\partial U}{\partial t} = 0$$



传输线波动方程

$$\frac{\partial^2 U}{\partial z^2} - L_0 C_0 \frac{\partial^2 U}{\partial t^2} = 0$$

$$\frac{\partial^2 I}{\partial z^2} - L_0 C_0 \frac{\partial^2 I}{\partial t^2} = 0$$

传播速度

$$v = \frac{1}{\sqrt{L_0 C_0}}$$

传输线特性阻抗

$$Z = \sqrt{\frac{L_0}{C_0}}$$

电压波与电流波

$$U(z, t) = U^+(t - \frac{z}{v}) + U^-(t + \frac{z}{v})$$

$$I(z, t) = I^+(t - \frac{z}{v}) + I^-(t + \frac{z}{v})$$

$$I(z, t) = \frac{1}{Z_0} [U^+(t - \frac{z}{v}) - U^-(t + \frac{z}{v})]$$

正弦稳态解

$$\text{令 } k^2 = (j\omega)^2 L_0 C_0$$

$$\text{解得 } \begin{cases} \dot{U}(x, t) = \dot{U}^+ e^{-kt} + \dot{U}^- e^{kt} \\ \dot{I}(x, t) = \frac{\dot{U}^+}{Z_0} e^{-kt} - \frac{\dot{U}^-}{Z_0} e^{kt} \end{cases}$$

若已知端始电压电流 $\dot{U}(-l), \dot{I}(-l)$

$$\dot{U}^+ = \frac{1}{2} (\dot{U}_1 + Z_0 \dot{I}_1) e^{-j\beta l}$$

$$\dot{U}^- = \frac{1}{2} (\dot{U}_1 - Z_0 \dot{I}_1) e^{j\beta l}$$

$$\begin{cases} \dot{U}(z) = \dot{U}_1 \cos \beta(l + z) - j Z_0 \dot{I}_1 \sin \beta(l + z) \\ \dot{I}(z) = \dot{I}_1 \cos \beta(l + z) - j \frac{\dot{U}_1}{Z_0} \sin \beta(l + z) \end{cases}$$

平行板传输线

$$C_0 = \frac{\varepsilon S}{d\Delta z} = \frac{\varepsilon W}{d}$$

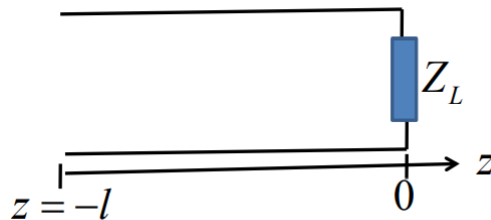
d 是板间距离

W 是板的宽度

反射与透射

反射原因:传输线阻抗 \neq 负载阻抗,传输线1阻抗 \neq 传输线1阻抗

带负载



负载阻抗

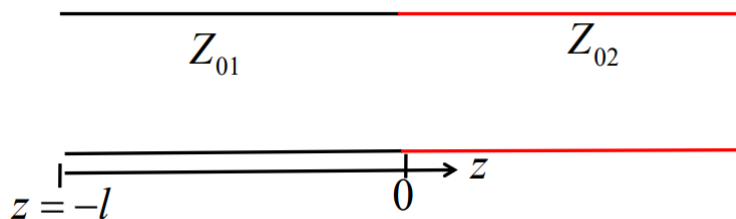
$$Z_L = \frac{\dot{U}(0)}{\dot{I}(0)} = Z_0 \frac{\dot{U}^+ + \dot{U}^-}{\dot{U}^+ - \dot{U}^-}$$

反射系数

$$\text{负载处 } \Gamma_L = \frac{\dot{U}^-}{\dot{U}^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\text{任意 } z \text{ 处 } \Gamma_z = \frac{\dot{U}^- e^{j\beta z}}{\dot{U}^+ e^{-j\beta z}} = \Gamma_L e^{2j\beta z}$$

串联的两传输线



$$\begin{cases} \dot{U}^+ + \dot{U}^- = \dot{U}', & \text{电位连续} \\ \dot{I}^+ + \dot{I}^- = \dot{I}', & \text{电流连续} \end{cases}$$

反射系数与透射系数

$$\Gamma = \frac{\dot{U}^-}{\dot{U}^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

$$T = \frac{\dot{U}'}{\dot{U}^+} = \frac{2Z_{02}}{Z_{02} + Z_{01}}$$

行波状态

$$\Gamma = 0$$

振幅不变、能量效率最高、电压电流同相位

驻波状态

$$|\Gamma| = 1$$

负载处短路、开路、纯电抗

行驻波

$$0 < |\Gamma| < 1$$

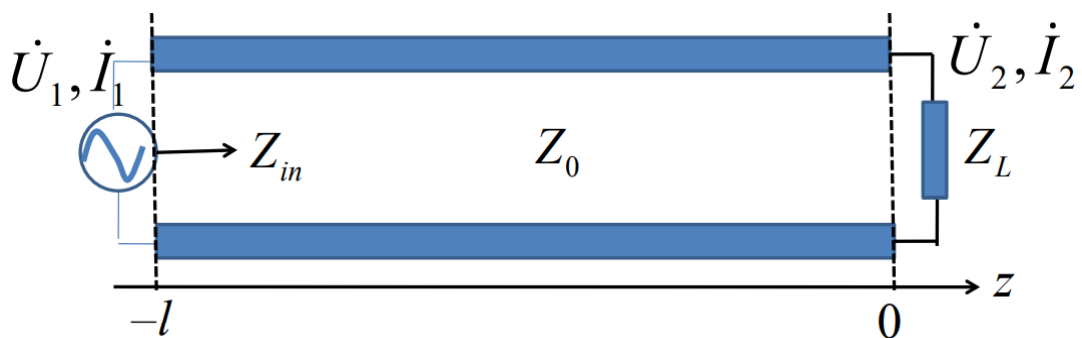
驻波比

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\text{第一个最大值位置: } |z|_{\max 1} = \frac{\phi_L}{4\pi} \lambda + \frac{1}{2} \lambda$$

$$\text{第一个最小值位置: } |z|_{\min 1} = \frac{\phi_L}{4\pi} \lambda + \frac{\lambda}{4}$$

入端阻抗



$$Z_{in} = \frac{\dot{U}_1}{\dot{I}_1} = \frac{\dot{U}(-l)}{\dot{I}(-l)} = Z_0 \frac{Z_L + jZ_0 \tan \frac{2\pi}{\lambda} l}{Z_0 + jZ_L \tan \frac{2\pi}{\lambda} l}$$

阻抗匹配

当 $Z_0 = Z_L$ 时, Z_{in} 始终等于 Z_0

终端短路

$$Z_L = 0$$

$$Z_{in} = jZ_0 \tan \beta l = jX_i$$

$X_i > 0$ 呈感性

$X_i < 0$ 呈容性

$X_i = 0$ 电感电容串联

$X_i = \pm\infty$ 电感电容并联

终端开路

$$Z_L = \infty$$

$$Z_{in} = -jZ_0 \cot \beta l = jX_i$$

终端纯电抗

$$Z_{in} = jX$$

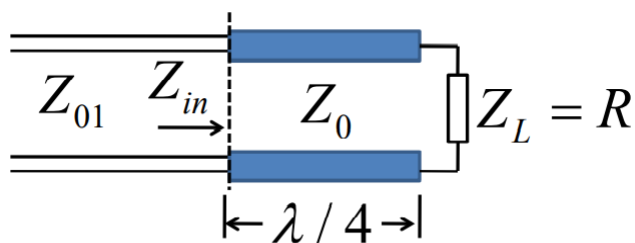
终端纯电阻

$$R_L > Z_0 : R_L = SZ_0$$

$$R_L < Z_0 : R_L = \frac{Z_0}{S}$$

阻抗匹配

$\frac{\lambda}{4}$ 阻抗变换器



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$l = \frac{\lambda}{4}$$

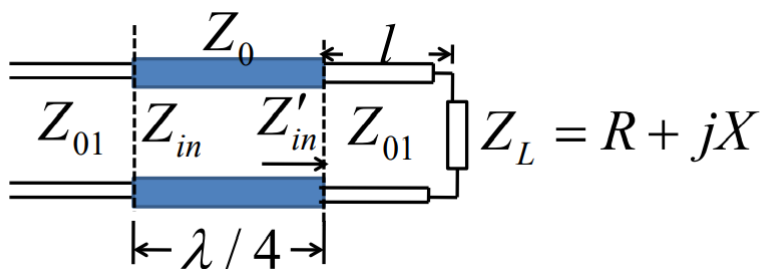
$$Z_{in} = \frac{Z_0^2}{R}$$

欲使阻抗匹配

$$Z_{in} = Z_{01}$$

$$\therefore Z_0 = \sqrt{RZ_{01}}$$

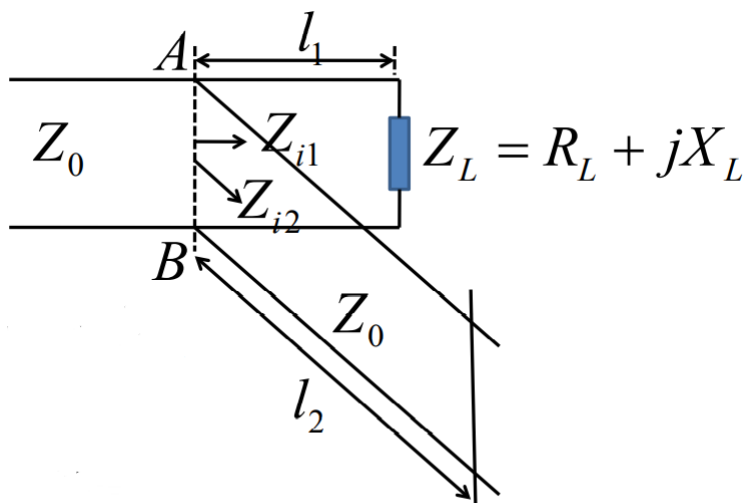
若负载不为纯电阻



则需求解 l ,使得 Z'_{in} 为纯电阻

$$Z'_{in} = Z_{01} \frac{(R + jX) + jZ_{01} \tan \beta l}{Z_{01} + j(R + jX) \tan \beta l}$$

单短截线变换器



$$\frac{1}{Z_0} = \frac{1}{Z_{i1}} + \frac{1}{Z_{i2}}$$

需满足

$$\begin{aligned} \frac{1}{Z_{i1}} &= \frac{1}{Z_0} + jB_{i1} \\ \frac{1}{Z_{i2}} &= -jB_{i1} \end{aligned}$$

其中

$$\begin{aligned} Z_{i2} &= jZ_0 \tan \beta l_2 \\ Z_{i1} &= Z_0 \frac{Z_L + jZ_0 \tan \beta l_1}{Z_0 + jZ_L \tan \beta l_1} \end{aligned}$$