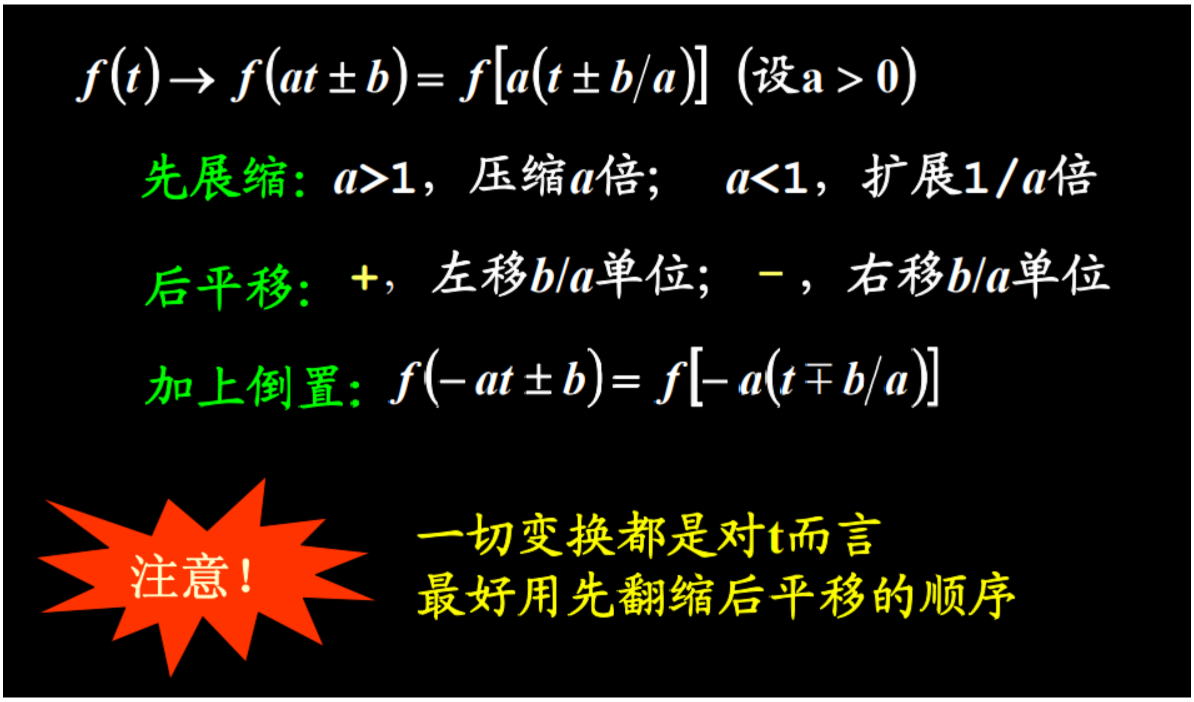
$$\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$$

$$sinc(t) = \frac{\sin \pi t}{\pi t}$$

钟形脉冲函数

$$f(t) = Ee^{-(\frac{t}{\tau})^2}$$



门函数

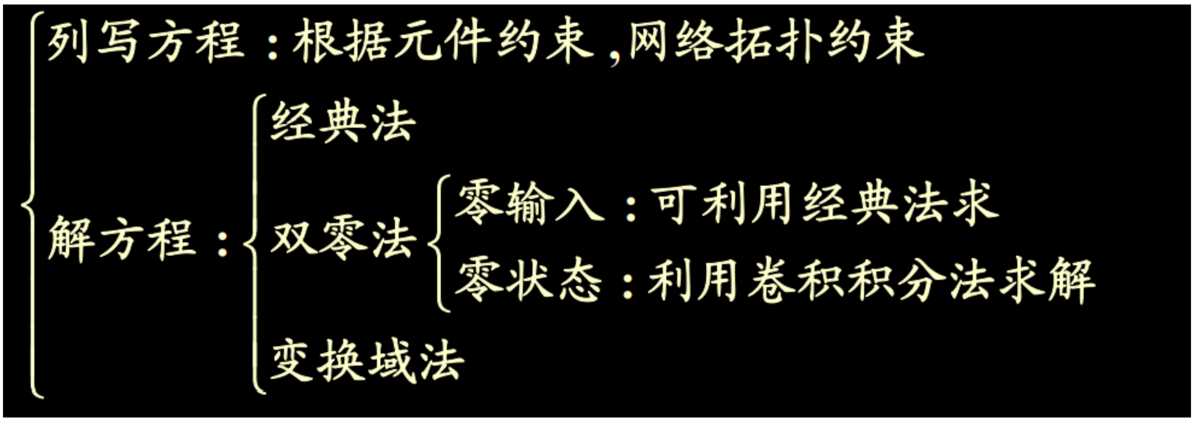
$$f(t) = u(t + \frac{\tau}{2}) - u(t - \frac{\tau}{2})$$

$$\int_{-\infty}^{+\infty} \delta^{(k)}(t) f(t) dt = (-1)^k f^{(k)}(0)$$

$$\int_{-\infty}^{+\infty} \delta^{(k)}(t - t_0) f(t) dt = (-1)^k f^{(k)}(t_0)$$

$$f(t)\delta'(t) = f(0)\delta'(t) - f'(0)\delta(t)$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$



齐次解法求冲击响应

1.求齐次通解,保留待定系数

2.令非齐次项为$\delta(t)$,对$(0^-,0^+)$积分，求出$\hat{h}^{(n-1)}(0^+)$

3.根据零状态特性，得出$\hat{h}^{(i)}(0^+)=0,(i=n-2,n-3,\ldots,0)$

4.带入$\hat{h}$及各阶导数，确定系数

5.$h(t)$是$\hat{h}(t)$及其各阶导数的线性组合

信号的脉冲分解

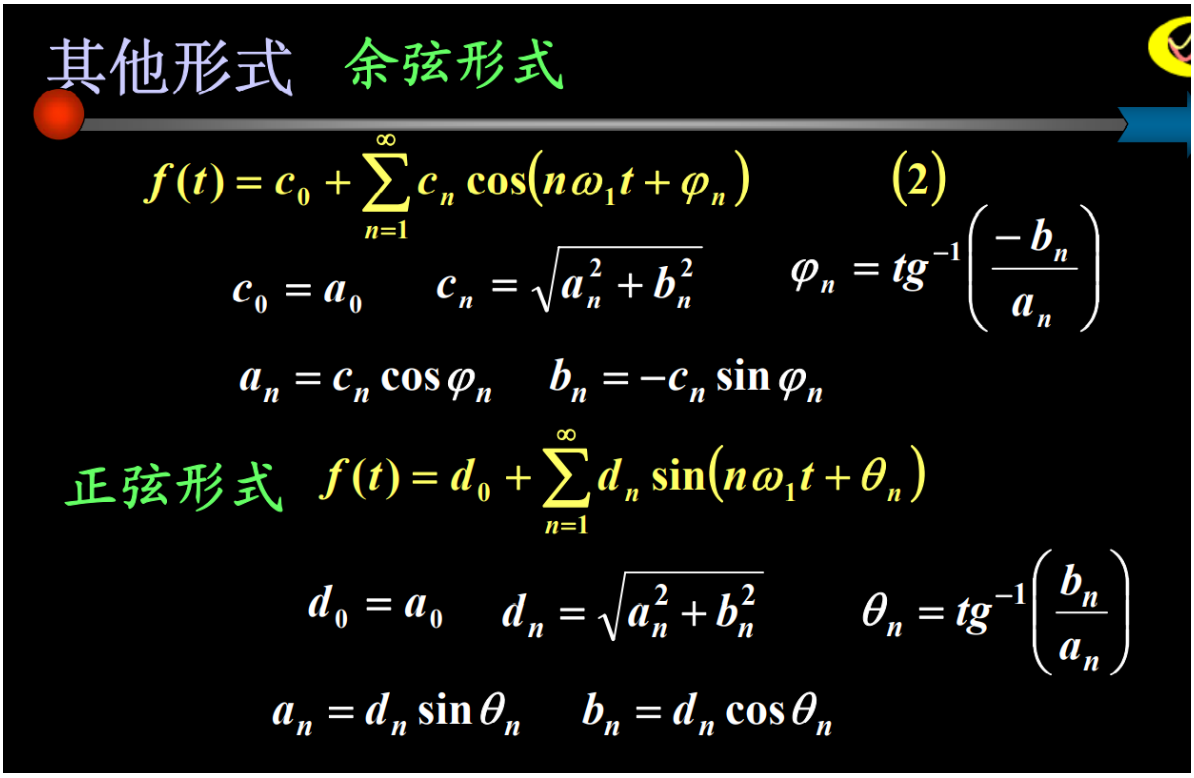
$$f(t)=\int_{-\infty}^{+\infty}f(\tau)\delta(t-\tau)d\tau$$

卷积的分析性质

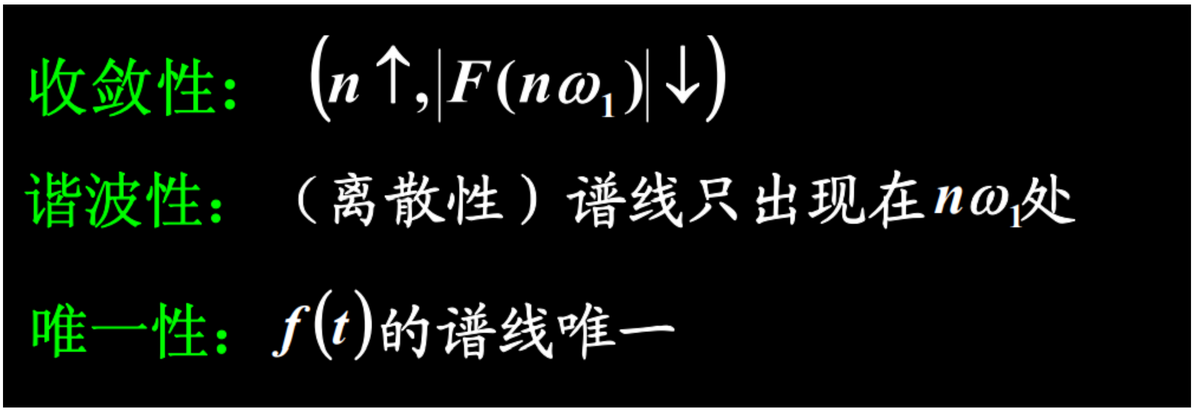
$$\begin{gathered}g(t)=f(t)\otimes h(t)\\ g^{(n-m)}(t)=f^{(n)}(t)\otimes h^{(-m)}(t)=f^{(-m)}(t)\otimes h^{(n)}(t)\\ g(t)=f^{(n)}(t)\otimes h^{(-n)}(t)=f^{(-n)}(t)\otimes h^{(n)}(t)\\ \text{需满足松弛条件}\end{gathered}$$

常用卷积公式

$$f(t)\otimes\delta^{(n)}(t-t_0)=f^{(n)}(t-t_0)$$



周期信号频谱具有离散性，谐波性，收敛性



帕塞瓦尔定理

能量形式

功率形式(傅里叶级数):

$$f(t)=a_0+\sum_{n=1}^{\infty}(a_n\cos n\omega_1 t+b_n\sin n\omega_1 t)=\sum_{n=-\infty}^{+\infty}F_n e^{jn\omega_1 t}$$

$$P=\frac{1}{T}\int_0^T[f(t)]^2=a_0^2+\sum_{n=1}^{\infty}\frac{a_n^2+b_n^2}{2}=\sum_{n=-\infty}^{+\infty}|F_n|^2$$

傅里叶变换

矩形脉冲

$$\mathscr{F}[E*(u(t-\frac{\tau}{2})-u(t+\frac{\tau}{2}))](\omega)=E\tau Sa(\frac{\omega\tau}{2})$$

指数信号

$$\mathscr{F}[e^{-\alpha t}u(t)](\omega)=\frac{1}{\alpha+j\omega}$$

直流信号

$$\mathscr{F}[1](\omega)=2\pi\delta(\omega)$$

符号函数

$$\mathscr{F}[sgn(t)](\omega)=\frac{2}{j\omega}$$

微分性质

$$\mathscr{F}[f^{(n)}(t)](\omega)=(j\omega)^n F(\omega)$$

$$\mathscr{F}^{-1}[\frac{\partial^n F(\omega)}{\partial\omega^n}](t)=(-jt)^n f(t)$$

尺缩变换

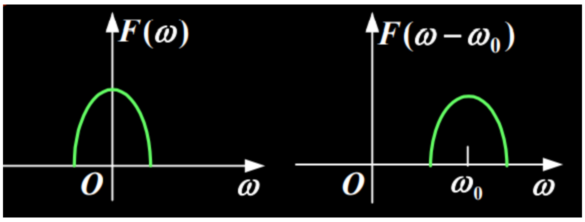
$$\mathscr{F}[f(at)](\omega)=\frac{1}{|a|}F(\frac{\omega}{a})$$

尺度变换+时移变换

$$\mathscr{F}[f(at+b)](\omega)=\frac{1}{|a|}F(\frac{\omega}{a})e^{jb\frac{\omega}{a}}$$

频移特性

$$\mathscr{F}[f(t)e^{j\omega_0 t}](\omega) = F(\omega - \omega_0)$$

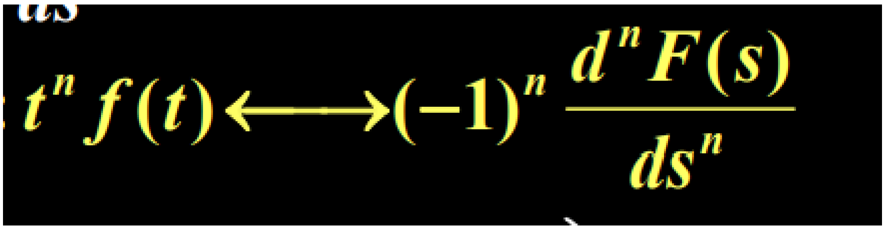


冲击列得傅里叶变换

$$\mathscr{F}[\delta_{T_s}(t)](\omega) = \frac{2\pi}{T_s} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_s)$$

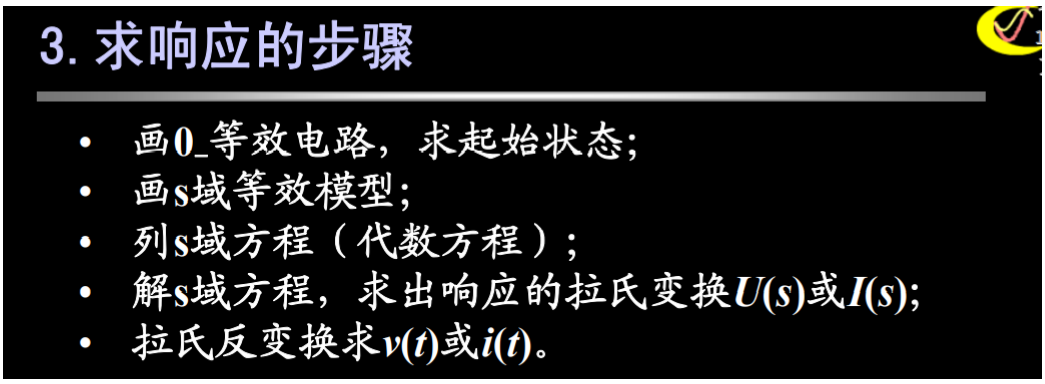
拉普拉斯变换

$$\mathscr{L}[t^n](s) = \frac{n!}{s^{n+1}}$$
$$\mathscr{L}[\cos\omega_0 t](s) = \frac{s}{s^2 + \omega_0^2}$$
$$\mathscr{L}[\sin\omega_0 t](s) = \frac{\omega_0}{s^2 + \omega_0^2}$$
$$\mathscr{L}[f(t)e^{-s_0 t}](s) = F(s + s_0)b$$
$$\mathscr{L}[f(t - t_0)u(t - t_0)](s) = F(s)e^{-st_0}$$
$$\mathscr{L}[\frac{d^n f(t)}{dt^n}](s) = s^n F(s) - \sum_{r=0}^{n-1} s^{n-r-1} \frac{d^r f(t)}{dt^r}\Big|_{t=0_-}$$



周期化定理

$$\mathscr{L}[f_T(t)](s) = \mathscr{L}[f_0(t)] * \mathscr{L}[\delta_T(t)] = F_0(s)\frac{1}{1 - e^{-sT}}$$

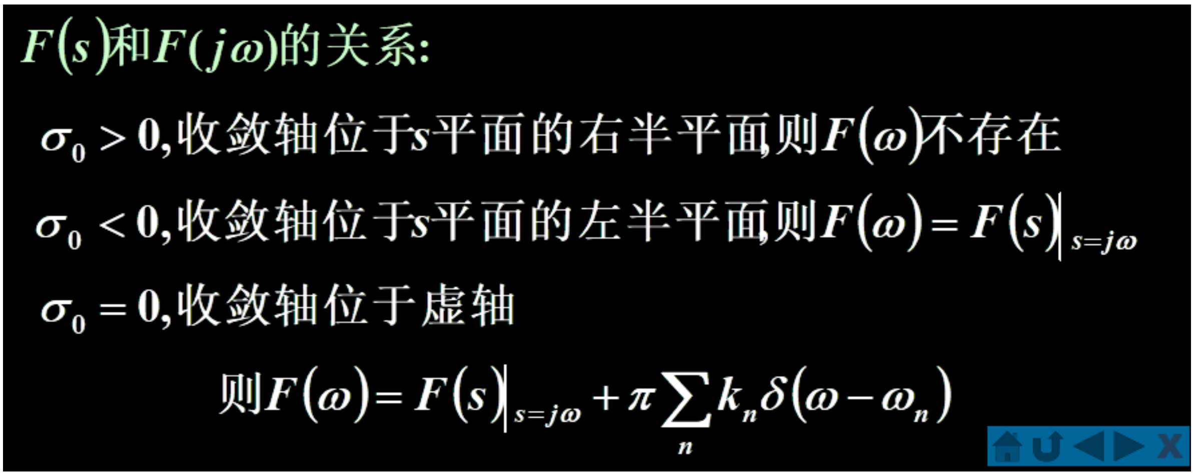


无失真传输条件

$$h(t) = K\delta(t - t_0)$$
$$H(j\omega) = Ke^{-j\omega t_0}$$
$$|H(j\omega)| = K$$
$$\phi(j\omega) = -\omega t_0$$

理想低通滤波器

$$H(j\omega) = \begin{cases} 1 * e^{-j\omega t_0} & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$
$$h(t) = \frac{\omega_c}{\pi} Sa[\omega_c(t - t_0)]$$



相关

$$R_{12}(t) = \int_{-\infty}^{+\infty} f_1(\tau + t) f_2^*(\tau) d\tau$$
$$R_{12}(t) = R_{21}^*(-t)$$
$$\mathscr{F}[R_{12}(t)](\omega) = F_1(\omega) F_2^*(\omega)$$

z变换

$$\mathscr{Z}[n^m x[n]](z) = (z^{-1} \frac{d}{d(z^{-1})})^m X(z)$$
$$\mathscr{Z}[n^m x[n]](z) = (-z\frac{d}{dz})^m X(z)$$
$$\mathscr{Z}[a^n u[n]](z) = \frac{z}{z-a}, |z| > |a|$$
$$\mathscr{Z}[-a^n u[-n-1]](z) = \frac{z}{z-a}, |z| < |a|$$
$$\mathscr{Z}[a^n x[n]](z) = X(\frac{z}{a}), R_{x1} < |\frac{z}{a}| < R_{x2}$$

初值定理

$$x[0] = \lim_{z->\infty} X(z)$$
$$x[1] = \lim_{z->\infty} z(X(z) - x[0])$$

终值定理

$$\text{若} x[\infty] \text{存在：}$$
$$x[\infty] = \lim_{z->1} (z-1)X(z)$$

时域卷积定理

$$\mathscr{Z}[x[n] \otimes h[n]](z) = X(z)H(z)$$