第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e_n}$$

极化体电荷:

$$\rho_{\mathbf{P}} = -\nabla \cdot \mathbf{P}$$

极化电荷代数和应为0:

$$0 = \oint_S \sigma_{\mathbf{P}} dS + \int_V \rho_{\mathbf{P}} dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D_{n2}} - \mathbf{D_{n1}} = \sigma$$

电位连续性、E的切向连续性:

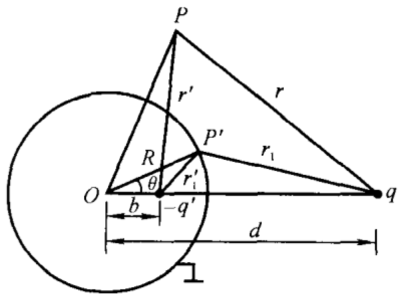
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b = \frac{R^2}{d}$$
$$q' = \frac{R}{d} q$$

两种理想介质
在$r_1 = r_2$处

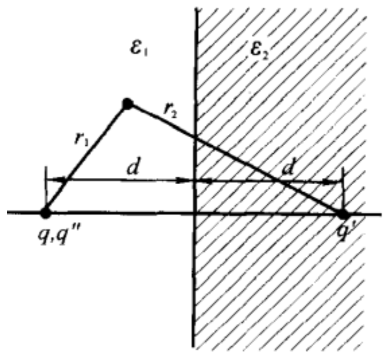


图 1－30　点电荷对无限大介质分界平面的镜像

$$\begin{cases} \frac{q}{\varepsilon_1} + \frac{q'}{\varepsilon_1} = \frac{q''}{\varepsilon_2} \\ q - q' = q'' \end{cases}$$

$$\therefore \begin{cases} q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \\ q'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

同心夹层球

$$C=\frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}$$

孤立球

$$C=4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电，容易想到

$$\begin{aligned}W_e&=\frac{1}{2}\int_V\rho\varphi dV\\&=\frac{1}{2}\int_S\sigma\varphi dS\end{aligned}$$

静电场能量密度

$$w_e'=\frac{1}{2}\mathbf{D}\cdot\mathbf{E}$$

虚功原理

$$dW=dW_e+fdg$$

不与电源相连

$$\begin{gathered}0=dW_e+fdg\\ \therefore f=-\frac{\partial W_e}{\partial g}\end{gathered}$$

与电源相连,各带电体电位不变

$$\begin{gathered}dW_e=\frac{1}{2}\sum_k\varphi_k dq_k=\frac{1}{2}dW\\ \therefore f=\frac{\partial W_e}{\partial g}\end{gathered}$$

第二章

电流密度

$$\mathbf{J}=\rho\mathbf{v}$$

作用于垂直于v的dS

$$dI=\mathbf{J}\cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K}=\sigma\mathbf{v}$$

作用于垂直于v的dl

$$dI=\mathbf{K}\cdot\mathbf{e_n}dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v}dq = \mathbf{J}dV = \mathbf{K}dS = Id\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma\mathbf{E}$$
$$\gamma\text{为电导率}$$

焦耳定律微分形式

$$p = \frac{dP}{dV} = \mathbf{J}\cdot\mathbf{E}$$

含源欧姆定律

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{E_e})$$

电流连续性方程

$$\oint_S \mathbf{J}\cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$\mathbf{J_{1n}} = \mathbf{J_{2n}}, \text{体现一个电荷守恒}$$
$$\mathbf{E_{1t}} = \mathbf{E_{2t}}, \text{体现一个保守场}$$
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\gamma_1}{\gamma_2}, \text{谓之折射定律}$$

第三章

BS定律

$$\begin{aligned}\mathbf{B} &= \frac{\mu}{4\pi}\oint_L \frac{Id\mathbf{l}\times\mathbf{e_R}}{R^2} \\ &= \frac{\mu}{4\pi}\oint_V \frac{\mathbf{J}dV\times\mathbf{e_R}}{R^2} \\ &= \frac{\mu}{4\pi}\oint_S \frac{\mathbf{K}dS\times\mathbf{e_R}}{R^2}\end{aligned}$$
$$B\text{在场点, 其余均在源点}$$

安培力

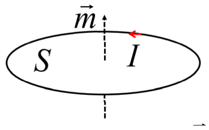
$$d\mathbf{F} = Id\mathbf{l}\times\mathbf{B}$$

安培环路定律

$$\nabla\times\mathbf{H} = \mathbf{J}$$
$$\oint_L \mathbf{H}\cdot d\mathbf{l} = \sum_k I_k$$

有介质

分子磁矩



$$\mathbf{m} = I\mathbf{S}$$

磁力矩

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

磁化强度

$$\mathbf{M} = \lim_{\Delta V \to 0} \frac{\sum_i \mathbf{m_i}}{\Delta V}$$

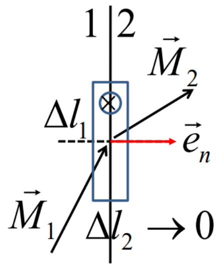
$$\text{理想介质中}, \mathbf{M} = \chi_m \mathbf{H}, \chi_m\text{是磁化率}$$

磁化电流

$$I_m = \oint_L \mathbf{J_m} \cdot d\mathbf{l}$$

$$\mathbf{J_m} = \nabla \times \mathbf{M}$$

磁化强度衔接条件



$$(\mathbf{M_1} - \mathbf{M_2}) \times \mathbf{e_n} = \mathbf{K_m}$$

磁场强度

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$\mathbf{B} = \mu \mathbf{H}(\text{理想介质})$$

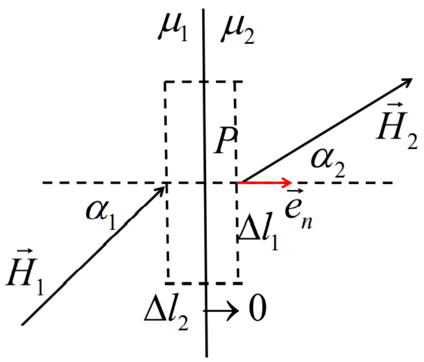
$$\nabla \times \mathbf{H} = \mathbf{J}, \text{自由电流密度}, \text{安培环路定律}2.0$$

不存在磁单极子,磁场是无源的

$$\nabla \cdot \mathbf{B} = 0(\text{微分形式})$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0(\text{积分形式})$$

衔接条件、折射定律



考虑安培环路定律$\nabla\times\mathbf{H}=\mathbf{J}$:

$$(\mathbf{H_1}-\mathbf{H_2})\times\mathbf{e_n}=\mathbf{K}$$
$$H_{1t}-H_{2t}=K,\text{分量形式}$$

磁场无源:

$$(\mathbf{B_1}-\mathbf{B_2})\cdot\mathbf{e_n}=0$$
$$\mathbf{B_{1n}}=\mathbf{B_{2n}},\text{分量形式}$$

折射定律

$$\text{理想介质,无面电流}$$
$$\frac{\tan\alpha_1}{\tan\alpha_2}=\frac{\mu_1}{\mu_2}$$

磁矢位

磁场无源,所以将$\mathbf{B}$看做一个场的旋度场(旋度无散)

$$\mathbf{B}=\nabla\times\mathbf{A}$$
$$\text{称}\mathbf{A}\text{为磁矢位}$$

代入安培环路定律和磁场构造方程

$$\nabla\times\nabla\times\mathbf{A}=\mu\mathbf{J}$$

利用矢量恒等式,并令$\nabla\cdot\mathbf{A}=\mathbf{0}$(库伦规范条件),得到一个三维泊松方程

$$\nabla^2\mathbf{A}=-\mu\mathbf{J}$$

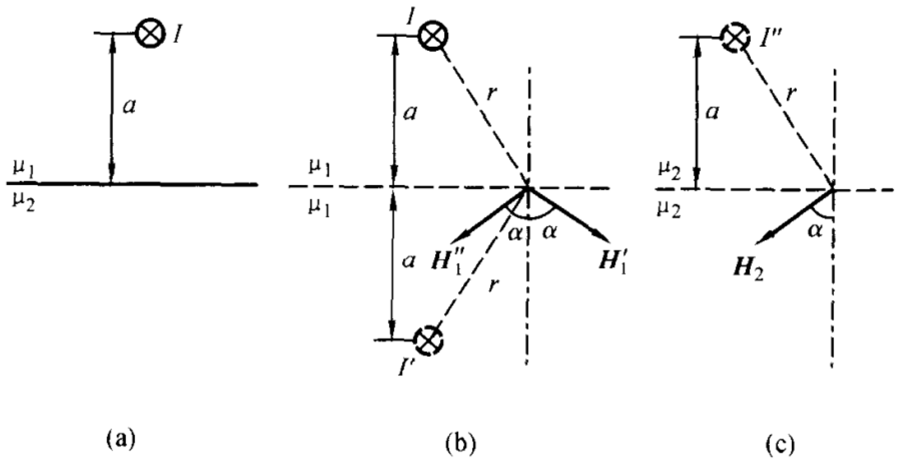
类比静电场的泊松方程的解

$$\begin{aligned}\mathbf{A}&=\frac{\mu}{4\pi}\int_{V'}\frac{\mathbf{J}dV'}{R}\\&=\frac{\mu}{4\pi}\int_{S'}\frac{\mathbf{K}dS'}{R}\\&=\frac{\mu}{4\pi}\oint_{L'}\frac{Id\mathbf{l}'}{R}\end{aligned}$$

衔接方程

$$\mathbf{A_1}=\mathbf{A_2}$$
$$\frac{1}{\mu_1}\frac{\partial A_1}{\partial n}-\frac{1}{\mu_2}\frac{\partial A_2}{\partial n}=K,\text{平行平面场}$$

镜像法



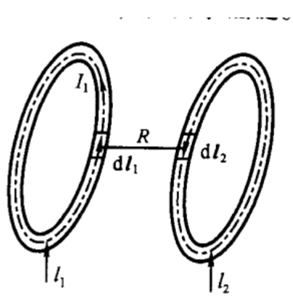
由衔接条件

$$\frac{I}{2\pi r}\sin\alpha-\frac{I'}{2\pi r}\sin\alpha=\frac{I''}{2\pi r}\sin\alpha$$
$$\mu_1(\frac{I}{2\pi r}\cos\alpha+\frac{I'}{2\pi r}\cos\alpha)=\mu_2\frac{I''}{2\pi r}\cos\alpha$$

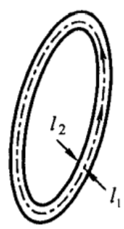
解得

$$I'=\frac{\mu_2-\mu_1}{\mu_1+\mu 2}I$$
$$I''=\frac{2\mu_1}{\mu_1+\mu_2}I$$

Neumann公式



$$M_{12}=M_{21}=\frac{N_1N_2\mu}{4\pi}\oint_{L_1}\oint_{L_2}\frac{\mathbf{dl_1}\cdot\mathbf{dl_2}}{R}$$



$$L_o=\frac{N^2\mu}{4\pi}\oint_{L_1}\oint_{L_2}\frac{\mathbf{dl_1}\cdot\mathbf{dl_2}}{R}$$
$$L=L_i+L_o\approx L_o$$
$$L_i\approx\frac{\mu l}{8\pi}$$

第四章

含时Maxwell's Equation

涡旋电场

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}+\nabla\times(\mathbf{v}\times\mathbf{B})$$

在静止媒质中$\mathbf{v}=\mathbf{0}$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

全电流定律

位移电流

$$\mathbf{J_d}=\frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\times\mathbf{H}=\mathbf{J}+\mathbf{J_d}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$$

理想导体边界条件

导体内部(静电屏蔽)

$$\begin{aligned}\mathbf{E_1}&=\mathbf{0}\\ \mathbf{B_1}&=\mathbf{0}\end{aligned}$$

导体外部(衔接方程)

$$\begin{aligned}(\mathbf{E_1}-\mathbf{E_2})\times\mathbf{e_n}=\mathbf{0}&\Rightarrow E_{2t}=0\\ (\mathbf{H_1}-\mathbf{H_2})\times\mathbf{e_n}=\mathbf{K}&\Rightarrow B_{2t}=K\\ (\mathbf{D_1}-\mathbf{D_2})\cdot\mathbf{e_n}=\sigma&\Rightarrow D_{2n}=\sigma\\ (\mathbf{B_1}-\mathbf{B_2})\cdot\mathbf{e_n}=0&\Rightarrow B_{2n}=0\end{aligned}$$

导体附近$\mathbf{E}$线垂直于表面,$\mathbf{B}$线平行于表面

动态位

$$\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t}=-\nabla\varphi$$

于是

$$\begin{aligned}\mathbf{E}&=-\nabla\varphi-\frac{\partial\mathbf{A}}{\partial t}\\ \mathbf{B}&=\nabla\times\mathbf{A}\end{aligned}$$

达朗贝尔方程

$$\begin{aligned}\Box=\nabla^2-\mu\varepsilon\frac{\partial^2}{\partial t^2}&=\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\\ \Box\mathbf{A}&=-\mu\mathbf{J}\\ \Box\varphi&=-\frac{\rho}{\varepsilon}\end{aligned}$$

洛伦兹规范条件

$$\nabla\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0$$

达朗贝尔方程的解

$$R=|\mathbf{r}-\mathbf{r}'|$$
$$\varphi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon}\int_{V'}\frac{\rho(\mathbf{r}',t-\frac{R}{c})dV'}{R}$$
$$\mathbf{A}(\mathbf{r},t)=\frac{\mu}{4\pi}\int_{V'}\frac{\mathbf{J}(\mathbf{r}',t-\frac{R}{c})dV'}{R}$$

电磁场能量

波印廷定理(能量守恒)

$$\frac{\partial W}{\partial t}=-\oint_{\partial V}\mathbf{E}\times\mathbf{H}\cdot\mathbf{dS}+\int_V\mathbf{E_e}\cdot\mathbf{J}dV-\int_V\frac{\mathbf{J}^2}{\gamma}dV$$

> 电磁场能量变化=-电磁波向外辐射+电源提供-焦耳热

波印廷矢量

$$\mathbf{S}=\mathbf{E}\times\mathbf{H}$$

正弦电磁场

$$\mathbf{E}(x,y,z,t)=\mathrm{Re}[\sqrt{2}\dot{\mathbf{E}}(x,y,z)e^{j\omega t}]$$
$$\dot{\mathbf{E}}(x,y,z)=\sum_{i\in\{x,y,z\}}\frac{1}{\sqrt{2}}E_{mi}e^{j\phi_i}\mathbf{e_i}$$

波印廷矢量均值

$$\mathbf{S_{av}}=(\mathbf{E}\times\mathbf{H})\cos{(\phi_E-\phi_B)}$$

复波印廷矢量

$$\dot{\mathbf{S}}=\dot{\mathbf{E}}\times\dot{\mathbf{H}}^*$$
$$\mathbf{S_{av}}=\mathrm{Re}[\dot{\mathbf{S}}]$$

波印廷定理复数形式

$$j\omega\int_V\mu|\dot{\mathbf{H}}|^2-\varepsilon|\dot{\mathbf{E}}^2|dV=-\oint_{\partial V}\dot{\mathbf{E}}\times\dot{\mathbf{H}}\cdot\mathbf{dS}-\int_V\frac{|\dot{\mathbf{J}}|^2}{\gamma}dV+\int_V\dot{\mathbf{E}}_\mathbf{e}\cdot\dot{\mathbf{J}}^*dV$$

达朗贝尔方程的复数形式

$$\nabla^2\dot{\mathbf{A}}+\beta^2\dot{\mathbf{A}}=-\mu\dot{\mathbf{J}}$$
$$\nabla^2\dot{\varphi}+\beta^2\dot{\varphi}=-\frac{\dot{\rho}}{\varepsilon}$$
$$\text{其中}\beta=\omega\sqrt{\mu\varepsilon}=\frac{\omega}{c}=\frac{2\pi}{\lambda}$$

洛伦兹规范条件复数形式

$$\nabla\cdot\dot{\mathbf{A}}+j\omega\mu\varepsilon\dot{\varphi}=0$$

动态位复数形式

$$
\begin{aligned}
\dot{\mathbf{E}} &= -\nabla\dot{\varphi} - j\omega\mu\varepsilon\dot{\mathbf{A}} \\
&= \frac{\nabla(\nabla\cdot\dot{\mathbf{A}})}{j\omega\mu\varepsilon} - j\omega\mu\varepsilon\dot{\mathbf{A}}
\end{aligned}
$$

$$
\dot{\mathbf{B}} = \nabla\times\dot{\mathbf{A}}
$$

达朗贝尔方程的复数解

$$
\begin{aligned}
R &= |\mathbf{r} - \mathbf{r}'| \\
\dot{\varphi} &= \frac{1}{4\pi\varepsilon}\int_{V'}\frac{\dot{\rho}e^{-j\frac{2\pi R}{\lambda}}dV'}{R} \\
\dot{\mathbf{A}} &= \frac{\mu}{4\pi}\int_{V'}\frac{\dot{\mathbf{J}}e^{-j\frac{2\pi R}{\lambda}}dV'}{R}
\end{aligned}
$$

似稳条件

$$
\frac{2\pi R}{\lambda} << 1
$$

单元偶极子的辐射

$$
P = \frac{2\pi\mu c}{3}I^2(\frac{\Delta l}{\lambda})^2 \approx 80\pi^2 I^2(\frac{\Delta l}{\lambda})^2
$$

方向性因子

$$
f(\theta,\phi) = \sin\theta
$$

准静态电磁场

电准静态场
磁场的变化率约等于0

$$
\frac{\partial\mathbf{B}}{\partial t} \approx \mathbf{0}
$$

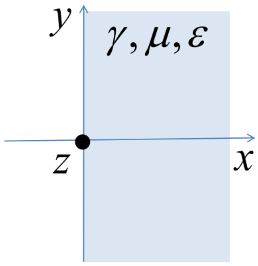
磁准静态场
电场的变化率约等于0

$$
\frac{\partial\mathbf{B}}{\partial t} \approx \mathbf{0}
$$

以下关系成立

$$
\begin{aligned}
\nabla^2\mathbf{H} - \mu\gamma\frac{\partial\mathbf{H}}{\partial t} &= \mathbf{0} \\
\nabla^2\mathbf{E} - \mu\gamma\frac{\partial\mathbf{E}}{\partial t} &= \mathbf{0} \\
\nabla^2\mathbf{J} - \mu\gamma\frac{\partial\mathbf{J}}{\partial t} &= \mathbf{0}
\end{aligned}
$$

电流的集肤效应



$x>0$半无限大空间为导体，正弦电流i沿y方向流过，$\vec{j}$ 只有y分量并在yoz平面上处处相等。求i在半无限大导体中的分布。

$$\frac{\partial^2 \dot{j}_y}{\partial x^2} = \dot{j}\omega\mu\gamma J_y$$

$$\text{定义}k = \sqrt{j\omega\mu\gamma} = \alpha + j\beta$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\gamma}{2}}$$

$$\dot{j}_y = \dot{j}_0 e^{-\alpha x} e^{-j\beta x}$$

透入深度

$$|e^{-kd}| = |e^{-1}|$$

$$\alpha d = 1$$

$$d = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\gamma}}$$

TEM波

理想介质中在x方向上传播的TEM波

$$\frac{\partial^2 E_y}{\partial x^2} - \mu\varepsilon\frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} - \mu\varepsilon\frac{\partial^2 H_z}{\partial t^2} = 0$$

E和H的关系

$$\frac{E_y^+(x,t)}{H_z^+(x,t)} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\frac{E_y^-(x,t)}{H_z^-(x,t)} = -\sqrt{\frac{\mu}{\varepsilon}}$$

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

正弦波

$$\dot{\mathbf{E}}(x,t) = \dot{E}_{y0} e^{-kx} \mathbf{e_x}$$

$$k = j\beta = j\frac{\omega}{c}$$

导电介质

$$\frac{\partial^2 E_y}{\partial x^2} - \mu\gamma\frac{\partial E_y}{\partial t} - \mu\varepsilon\frac{\partial^2 E_y}{\partial t^2} = 0$$
$$\frac{\partial^2 H_z}{\partial x^2} - \mu\gamma\frac{\partial H_z}{\partial t} - \mu\varepsilon\frac{\partial^2 H_z}{\partial t^2} = 0$$

等效介电常数

$$\varepsilon' = \varepsilon + \frac{\gamma}{j\omega}$$

$$k = j\omega\sqrt{\mu\varepsilon'}$$

波印廷矢量的均值

$$\dot{\mathbf{S}}_{\mathbf{av}} = \mathrm{Re}[\dot{\mathbf{E}}\times\dot{\mathbf{H}}^*] = \frac{1}{|Z_0|}E_0^{+2}e^{-2\alpha x}\cos\phi\mathbf{e_x}$$
$$\phi = \arg\sqrt{\frac{\mu}{\varepsilon'}}$$

极化

$$\phi_z - \phi_y = \frac{\pi}{2}, \text{右旋}$$
$$\phi_z - \phi_y = -\frac{\pi}{2}, \text{左旋}$$

反射与折射

平行极化波

反射系数

$$\Gamma_{\parallel} = \frac{E_{\parallel}^-}{E_{\parallel}^+} = \frac{Z_{02}\cos\theta_2 - Z_{01}\cos\theta_1}{Z_{02}\cos\theta_2 + Z_{01}\cos\theta_1} = \frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}$$

折射系数

$$T_{\parallel} = \frac{E_{\parallel}'}{E_{\parallel}^+} = \frac{2Z_{02}\cos\theta_1}{Z_{02}\cos\theta_2 + Z_{01}\cos\theta_1} = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)}$$

垂直极化波

反射系数

$$\Gamma_{\perp} = \frac{E_{\perp}^-}{E_{\perp}^+} = \frac{Z_{02}\cos\theta_1 - Z_{01}\cos\theta_2}{Z_{02}\cos\theta_1 + Z_{01}\cos\theta_2}$$

折射系数

$$T_{\parallel} = \frac{E_{\parallel}'}{E_{\parallel}^+} = \frac{2Z_{02}\cos\theta_1}{Z_{02}\cos\theta_1 + Z_{01}\cos\theta_2}$$

理想导体处反射

$$E^- = -E^+$$
$$H^- = H^+$$
$$e.g. E_y^+ = E_m \cos(\omega t - \beta x)$$
$$H_z^+ = \frac{E_m}{Z_0}\cos(\omega - \beta x)$$
$$E_y^- = -E_m \cos(\omega t + \beta x)$$
$$H_z^- = \frac{E_m}{Z_0}\cos(\omega t + \beta x)$$

理想介质反射、折射

$$\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$
$$T = \frac{2Z_{02}}{Z_{02} + Z_{01}}$$
$$\text{若}\dot{E}^+(x) = \dot{E}^+ e^{-j\beta_1 x}$$
$$\text{则}\dot{H}^+(x) = \frac{\dot{E}^+}{Z_{01}} e^{-j\beta_1 x}$$
$$\dot{E}^-(x) = \Gamma \dot{E}^+ e^{+j\beta_1 x}$$
$$\dot{H}^-(x) = -\frac{\Gamma \dot{E}^+}{Z_{01}} e^{j\beta_1 x}$$
$$\dot{E}'(x) = T\dot{E}^+ e^{-j\beta_2 x}$$
$$\dot{H}'(x) = \frac{T\dot{E}^+}{Z_{02}} e^{-j\beta_2 x}$$

合成电场

$$\dot{E}(x) = \dot{E}^+ + \dot{E}^-$$
$$\text{为行驻波, 极值为}|\dot{E}^+|(1 \pm |\Gamma|)$$
$$\text{定义驻波比}S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

传输线

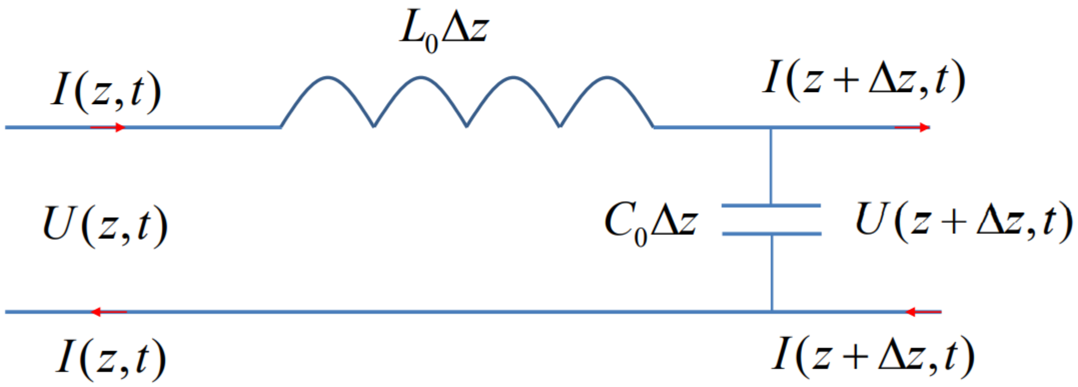
L_0是单位长度的电感

C_0是单位长度的电容

$$L_0 C_0 = \mu\varepsilon$$

电报方程

$$\frac{\partial U}{\partial z} + L_0 \frac{\partial I}{\partial t} = 0$$
$$\frac{\partial I}{\partial z} + C_0 \frac{\partial U}{\partial t} = 0$$



传输线波动方程

$$\frac{\partial^2 U}{\partial z^2} - L_0 C_0 \frac{\partial^2 U}{\partial t^2} = 0$$
$$\frac{\partial^2 I}{\partial z^2} - L_0 C_0 \frac{\partial^2 I}{\partial t^2} = 0$$

传播速度

$$v = \frac{1}{\sqrt{L_0 C_0}}$$

传输线特性阻抗

$$Z = \sqrt{\frac{L_0}{C_0}}$$

电压波与电流波

$$U(z,t) = U^+(t-\frac{z}{v}) + U^-(t+\frac{z}{v})$$
$$I(z,t) = I^+(t-\frac{z}{v}) + I^-(t+\frac{z}{v})$$
$$I(z,t) = \frac{1}{Z_0}[U^+(t-\frac{z}{v}) - U^-(t+\frac{z}{v})]$$

正弦稳态解

$$令k^2 = (j\omega)^2 L_0 C_0$$
$$解得\begin{cases}\dot{U}(x,t) = \dot{U}^+ e^{-kt} + \dot{U}^- e^{kt} \\ \dot{I}(x,t) = \frac{\dot{U}^+}{Z_0} e^{-kt} - \frac{\dot{U}^-}{Z_0} e^{kt}\end{cases}$$

若已知端始电压电流$\dot{U}(-l), \dot{I}(-l)$

$$\dot{U}^+ = \frac{1}{2}(\dot{U}_1 + Z_0 \dot{I}_1) e^{-j\beta l}$$
$$\dot{U}^- = \frac{1}{2}(\dot{U}_1 - Z_0 \dot{I}_1) e^{j\beta l}$$
$$\begin{cases}\dot{U}(z) = \dot{U}_1 \cos\beta(l+z) - jZ_0 \dot{I}_1 \sin\beta(l+z) \\ \dot{I}(z) = \dot{I}_1 \cos\beta(l+z) - j\frac{\dot{U}_1}{Z_0} \sin\beta(l+z)\end{cases}$$

平行板传输线

$$C_0 = \frac{\varepsilon S}{d\Delta z} = \frac{\varepsilon W}{d}$$
$$d\text{是板间距离}$$
$$W\text{是板的宽度}$$

反射与透射

反射原因:传输线阻抗≠负载阻抗,传输线1阻抗≠传输线1阻抗

带负载

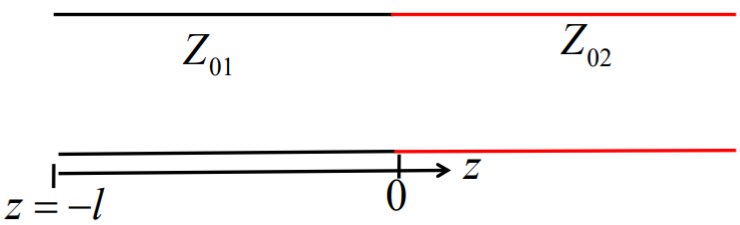
负载阻抗

$$Z_L = \frac{\dot{U}(0)}{\dot{I}(0)} = Z_0 \frac{\dot{U}^+ + \dot{U}^-}{\dot{U}^+ - \dot{U}^-}$$

反射系数

$$\text{负载处}\Gamma_L = \frac{\dot{U}^-}{\dot{U}^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$\text{任意}z\text{处}\Gamma_z = \frac{\dot{U}^- e^{j\beta z}}{\dot{U}^+ e^{-j\beta z}} = \Gamma_L e^{2j\beta z}$$

串联的两传输线



$$\begin{cases} \dot{U}^+ + \dot{U}^- = \dot{U}', \text{电位连续} \\ \dot{I}^+ + \dot{I}^- = \dot{I}', \text{电流连续} \end{cases}$$

反射系数与透射系数

$$\Gamma = \frac{\dot{U}^-}{\dot{U}^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$
$$T = \frac{\dot{U}'}{\dot{U}^+} = \frac{2Z_{02}}{Z_{02} + Z_{01}}$$

行波状态

$$\Gamma = 0$$
$$\text{振幅不变、能量效率最高、电压电流同相位}$$

驻波状态

$$|\Gamma| = 1$$

负载处短路、开路、纯电抗

行驻波

$$0 < |\Gamma| < 1$$

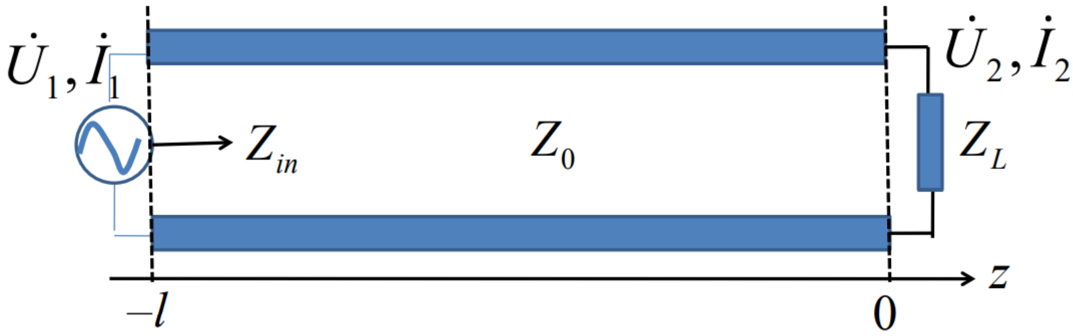
驻波比

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\text{第一个最大值位置：} |z|_{\max 1} = \frac{\phi_L}{4\pi}\lambda + \frac{1}{2}\lambda$$

$$\text{第一个最小值位置：} |z|_{\min 1} = \frac{\phi_L}{4\pi}\lambda + \frac{\lambda}{4}$$

入端阻抗



$$Z_{in} = \frac{\dot{U}_1}{\dot{I}_1} = \frac{\dot{U}(-l)}{\dot{I}(-l)} = Z_0\frac{Z_L + jZ_0\tan\frac{2\pi}{\lambda}l}{Z_0 + jZ_L\tan\frac{2\pi}{\lambda}l}$$

阻抗匹配

当$Z_0 = Z_L$时,Z_{in}始终等于Z_0

终端短路

$Z_L = 0$

$$Z_{in} = jZ_0\tan\beta l = jX_i$$

$X_i > 0$ 呈感性

$X_i < 0$ 呈容性

$X_i = 0$ 电感电容串联

$X_i = \pm\infty$ 电感电容并联

终端开路

$Z_L = \infty$

$$Z_{in} = -jZ_0\cot\beta l = jX_i$$

终端纯电抗

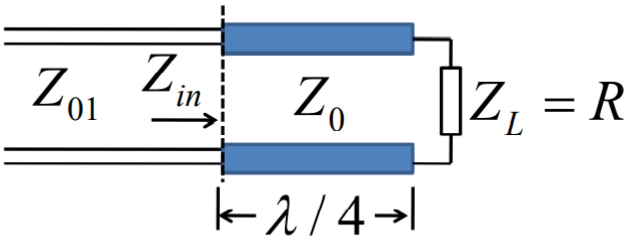
$$Z_{in} = jX$$

终端纯电阻

$$R_L > Z_0 : R_L = SZ_0$$
$$R_L < Z_0 : R_L = \frac{Z_0}{S}$$

阻抗匹配

$\frac{\lambda}{4}$阻抗变换器

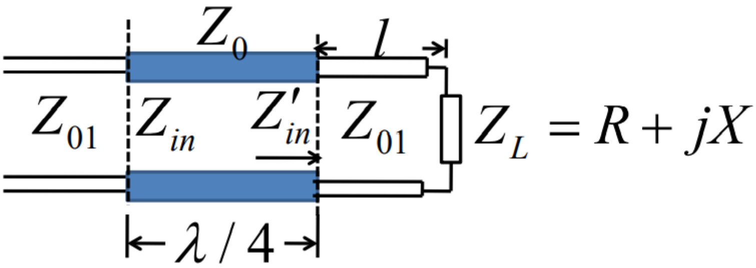


$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$
$$l = \frac{\lambda}{4}$$
$$Z_{in} = \frac{Z_0^2}{R}$$

欲使阻抗匹配

$$Z_{in} = Z_{01}$$
$$\therefore Z_0 = \sqrt{RZ_{01}}$$

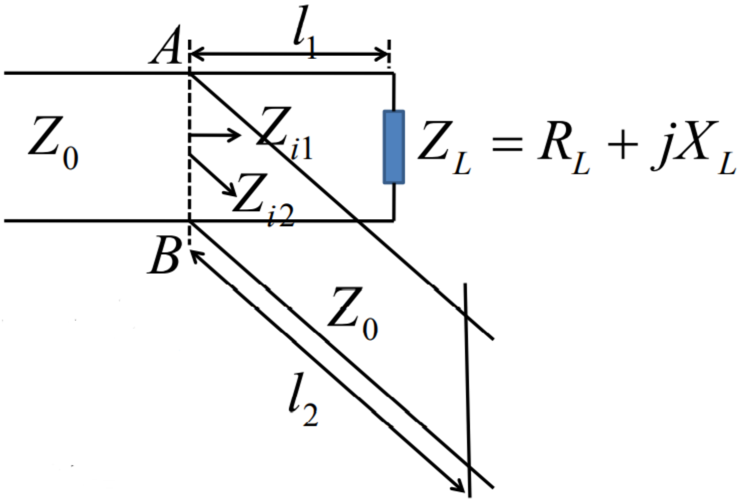
若负载不为纯电阻



则需求解l,使得Z'_{in}为纯电阻

$$Z'_{in} = Z_{01} \frac{(R + jX) + jZ_{01} \tan \beta l}{Z_{01} + j(R + jX) \tan \beta l}$$

单短截线变换器



$$\frac{1}{Z_0} = \frac{1}{Z_{i1}} + \frac{1}{Z_{i2}}$$

需满足

$$\frac{1}{Z_{i1}} = \frac{1}{Z_0} + jB_{i1}$$
$$\frac{1}{Z_{i2}} = -jB_{i1}$$

其中

$$Z_{i2} = jZ_0 \tan \beta l_2$$
$$Z_{i1} = Z_0 \frac{Z_L + jZ_0 \tan \beta l_1}{Z_0 + jZ_L \tan \beta l_1}$$