

第一章

第二章

第三章

有介质

磁矢位

第四章

含时Maxwell's Equation

动态位

电磁场能量

正弦电磁场

单元偶极子的辐射

准静态电磁场

TEM波

理想介质中在*x*方向上传播的TEM波

导电介质

反射与折射

传输线

# 第一章

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极化面电荷:

$$\sigma_P = \mathbf{P} \cdot \mathbf{e}_n$$

极化体电荷:

$$\rho_P = -\nabla \cdot \mathbf{P}$$

极化电荷代数数和应为0:

$$0 = \oint_S \sigma_P dS + \int_V \rho_P dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D}_{n2} - \mathbf{D}_{n1} = \sigma$$

电位连续性、E的切向连续性:

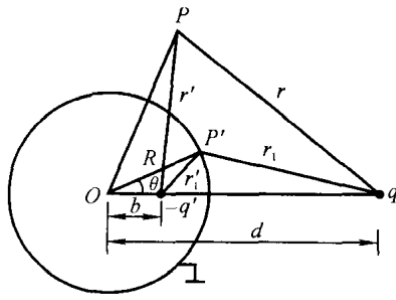
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b = \frac{R^2}{d}$$

$$q' = \frac{R}{d}q$$

两种理想介质  
在 $r_1 = r_2$ 处

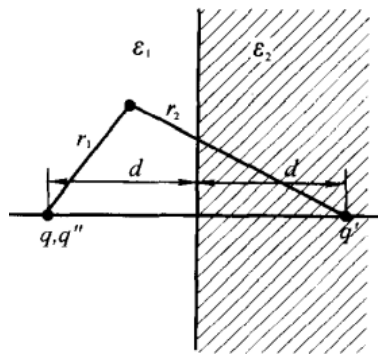


图 1 - 30 点电荷对无限大  
介质分界平面的镜像

$$\begin{cases} \frac{q}{\epsilon_1} + \frac{q'}{\epsilon_1} = \frac{q''}{\epsilon_2} \\ q - q' = q'' \end{cases}$$

$$\therefore \begin{cases} q' = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} q \\ q'' = \frac{2\epsilon_2}{\epsilon_1 + \epsilon_2} q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\epsilon}{\ln(b/a)}$$

同心夹层球

$$C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$$

孤立球

$$C = 4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电，容易想到

$$W_e = \frac{1}{2} \int_V \rho \varphi dV$$

$$= \frac{1}{2} \int_S \sigma \varphi dS$$

静电场能量密度

$$w'_e = \frac{1}{2} \mathbf{D} \cdot \mathbf{E}$$

虚功原理

$$dW = dW_e + f dg$$

不与电源相连

$$0 = dW_e + f dg$$

$$\therefore f = - \frac{\partial W_e}{\partial g}$$

与电源相连,各带电体电位不变

$$dW_e = \frac{1}{2} \sum_k \varphi_k dq_k = \frac{1}{2} dW$$

$$\therefore f = \frac{\partial W_e}{\partial g}$$

## 第二章

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电流密度

$$\mathbf{J} = \rho \mathbf{v}$$

作用于垂直于 $v$ 的 $dS$

$$dI = \mathbf{J} \cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma \mathbf{v}$$

作用于垂直于 $v$ 的 $dl$

$$dI = \mathbf{K} \cdot \mathbf{e}_n dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v} dq = \mathbf{J} dV = \mathbf{K} dS = I dl$$

欧姆定律微分形式

$$\mathbf{J} = \gamma \mathbf{E}$$

$\gamma$ 为电导率

焦耳定律微分形式

$$p = \frac{dP}{dV} = \mathbf{J} \cdot \mathbf{E}$$

含源欧姆定律

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{E}_e)$$

电流连续性方程

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$\mathbf{J}_{1n} = \mathbf{J}_{2n}, \text{体现一个电荷守恒}$$

$$\mathbf{E}_{1t} = \mathbf{E}_{2t}, \text{体现一个保守场}$$

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}, \text{谓之折射定律}$$

## 第三章

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BS定律

$$\begin{aligned} \mathbf{B} &= \frac{\mu}{4\pi} \oint_L \frac{Id\mathbf{l} \times \mathbf{e}_R}{R^2} \\ &= \frac{\mu}{4\pi} \oint_V \frac{\mathbf{J}dV \times \mathbf{e}_R}{R^2} \\ &= \frac{\mu}{4\pi} \oint_S \frac{\mathbf{K}dS \times \mathbf{e}_R}{R^2} \end{aligned}$$

$B$ 在场点, 其余均在源点

安培力

$$d\mathbf{F} = Id\mathbf{l} \times \mathbf{B}$$

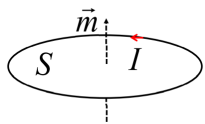
安培环路定律

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} \\ \oint_L \mathbf{H} \cdot d\mathbf{l} &= \sum_k I_k \end{aligned}$$

## 有介质

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分子磁矩



$$\mathbf{m} = I\mathbf{S}$$

磁力矩

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

磁化强度

理想介质中,  $\mathbf{M} = \chi_m \mathbf{H}$ ,  $\chi_m$  是磁化率

$$I_m = \oint_L \mathbf{J}_m \cdot d\mathbf{l}$$
$$\mathbf{J}_m = \nabla \times \mathbf{M}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 (\text{积分形式})$$

$$\mathbf{B}_{1n} = \mathbf{B}_{2n}, \text{分量形式}$$

## 折射定律

理想介质, 无面电流

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2}$$

## 磁矢位

磁场无源, 所以将 $\mathbf{B}$ 看做一个场的旋度场(旋度无散)

$$\mathbf{B} = \nabla \times \mathbf{A}$$

称 $\mathbf{A}$ 为磁矢位

代入安培环路定律和磁场构造方程

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

利用矢量恒等式, 并令 $\nabla \cdot \mathbf{A} = 0$ (库伦规范条件), 得到一个三维泊松方程

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

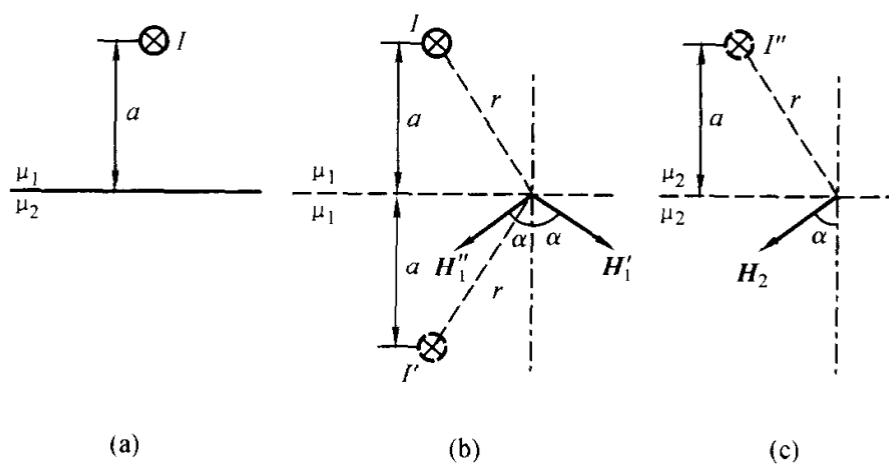
类比静电场的泊松方程的解

$$\begin{aligned} \mathbf{A} &= \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J} dV'}{R} \\ &= \frac{\mu}{4\pi} \int_{S'} \frac{\mathbf{K} dS'}{R} \\ &= \frac{\mu}{4\pi} \oint_{L'} \frac{I d\mathbf{l}'}{R} \end{aligned}$$

衔接方程

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{A}_2 \\ \frac{1}{\mu_1} \frac{\partial A_1}{\partial n} - \frac{1}{\mu_2} \frac{\partial A_2}{\partial n} &= K, \text{ 平行平面场} \end{aligned}$$

镜像法



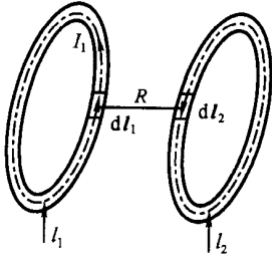
由衔接条件

$$\begin{aligned} \frac{I}{2\pi r} \sin \alpha - \frac{I'}{2\pi r} \sin \alpha &= \frac{I''}{2\pi r} \sin \alpha \\ \mu_1 \left( \frac{I}{2\pi r} \cos \alpha + \frac{I'}{2\pi r} \cos \alpha \right) &= \mu_2 \frac{I''}{2\pi r} \cos \alpha \end{aligned}$$

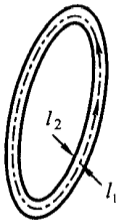
解得

$$I' = \frac{\mu_2 - \mu_1}{\mu_1 + \mu_2} I$$
$$I'' = \frac{2\mu_1}{\mu_1 + \mu_2} I$$

Neumann公式



$$M_{12} = M_{21} = \frac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl}_1 \cdot \mathbf{dl}_2}{R}$$



$$L_o = \frac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl}_1 \cdot \mathbf{dl}_2}{R}$$
$$L = L_i + L_o \approx L_o$$
$$L_i \approx \frac{\mu l}{8\pi}$$

## 第四章

### 含时Maxwell's Equation

涡旋电场

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B})$$

在静止媒质中 $\mathbf{v} = \mathbf{0}$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

全电流定律

位移电流

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

理想导体边界条件

导体内部(静电屏蔽)

$$\mathbf{E}_1 = \mathbf{0}$$

$$\mathbf{B}_1 = \mathbf{0}$$

导体外部(衔接方程)

$$(\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{e}_n = \mathbf{0} \Rightarrow E_{2t} = 0$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{e}_n = \mathbf{K} \Rightarrow B_{2t} = K$$

$$(\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{e}_n = \sigma \Rightarrow D_{2n} = \sigma$$

$$(\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{e}_n = 0 \Rightarrow B_{2n} = 0$$

导体附近 $\mathbf{E}$ 线垂直于表面, $\mathbf{B}$ 线平行于表面

## 动态位

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi$$

于是

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

达朗贝尔方程

$$\square = \nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$

$$\square \mathbf{A} = -\mu \mathbf{J}$$

$$\square \varphi = -\frac{\rho}{\epsilon}$$

洛伦兹规范条件

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

达朗贝尔方程的解

$$R = |\mathbf{r} - \mathbf{r}'|$$

$$\varphi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon} \int_{V'} \frac{\rho(\mathbf{r}', t - \frac{R}{c}) dV'}{R}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}(\mathbf{r}', t - \frac{R}{c}) dV'}{R}$$

## 电磁场能量

波印廷定理(能量守恒)

$$\frac{\partial W}{\partial t} = - \oint_{\partial V} \mathbf{E} \times \mathbf{H} \cdot d\mathbf{S} + \int_V \mathbf{E}_e \cdot \mathbf{J} dV - \int_V \frac{\mathbf{J}^2}{\gamma} dV$$

电磁场能量变化=-电磁波向外辐射+电源提供-焦耳热

波印廷矢量



$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

## 正弦电磁场

$$\mathbf{E}(x, y, z, t) = \text{Re}[\sqrt{2}\dot{\mathbf{E}}(x, y, z)e^{j\omega t}]$$

$$\dot{\mathbf{E}}(x, y, z) = \sum_{i \in \{x, y, z\}} \frac{1}{\sqrt{2}} E_{mi} e^{j\phi_i} \mathbf{e}_i$$

波印廷矢量均值

$$\mathbf{S}_{\text{av}} = (\mathbf{E} \times \mathbf{H}) \cos(\phi_E - \phi_B)$$

复波印廷矢量

$$\dot{\mathbf{S}} = \dot{\mathbf{E}} \times \dot{\mathbf{H}}^*$$

$$\mathbf{S}_{\text{av}} = \text{Re}[\dot{\mathbf{S}}]$$

波印廷定理复数形式

$$j\omega \int_V \mu |\dot{\mathbf{H}}|^2 + \varepsilon |\dot{\mathbf{E}}|^2 dV = - \oint_{\partial V} \dot{\mathbf{E}} \times \dot{\mathbf{H}} \cdot d\mathbf{S} - \int_V \frac{|\dot{\mathbf{J}}|^2}{\gamma} dV + \int_V \dot{\mathbf{E}}_e \cdot \dot{\mathbf{J}}^* dV$$

达朗贝尔方程的复数形式

$$\nabla^2 \dot{\mathbf{A}} + \beta^2 \dot{\mathbf{A}} = -\mu \dot{\mathbf{J}}$$

$$\nabla^2 \dot{\varphi} + \beta^2 \dot{\varphi} = -\frac{\dot{\rho}}{\varepsilon}$$

其中  $\beta = \omega \sqrt{\mu \varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

洛伦兹规范条件复数形式

$$\nabla \cdot \dot{\mathbf{A}} + j\omega \mu \varepsilon \dot{\varphi} = 0$$

动态位复数形式

$$\dot{\mathbf{E}} = -\nabla \dot{\varphi} - j\omega \mu \varepsilon \dot{\mathbf{A}}$$

$$= \frac{\nabla(\nabla \cdot \dot{\mathbf{A}})}{j\omega \mu \varepsilon} - j\omega \mu \varepsilon \dot{\mathbf{A}}$$

$$\dot{\mathbf{B}} = \nabla \times \dot{\mathbf{A}}$$

达朗贝尔方程的复数解

$$R = |\mathbf{r} - \mathbf{r}'|$$

$$\dot{\varphi} = \frac{1}{4\pi\varepsilon} \int_{V'} \frac{\dot{\rho} e^{-j\frac{2\pi R}{\lambda}}}{R} dV'$$

$$\dot{\mathbf{A}} = \frac{\mu}{4\pi} \int_{V'} \frac{\dot{\mathbf{J}} e^{-j\frac{2\pi R}{\lambda}}}{R} dV'$$

似稳条件

$$\frac{2\pi R}{\lambda} \ll 1$$

## 单元偶极子的辐射

$$P = \frac{2\pi\mu c}{3} I^2 \left(\frac{\Delta l}{\lambda}\right)^2 \approx 80\pi^2 I^2 \left(\frac{\Delta l}{\lambda}\right)^2$$

方向性因子

$$f(\theta, \phi) = \sin \theta$$

## 准静态电磁场

电准静态场

磁场的变化率约等于0

$$\frac{\partial \mathbf{B}}{\partial t} \approx 0$$

磁准静态场

电场的变化率约等于0

$$\frac{\partial \mathbf{E}}{\partial t} \approx 0$$

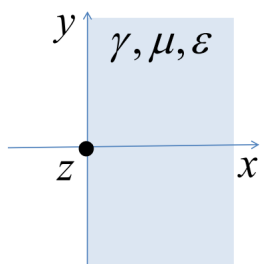
以下关系成立

$$\nabla^2 \mathbf{H} - \mu\gamma \frac{\partial \mathbf{H}}{\partial t} = 0$$

$$\nabla^2 \mathbf{E} - \mu\gamma \frac{\partial \mathbf{E}}{\partial t} = 0$$

$$\nabla^2 \mathbf{J} - \mu\gamma \frac{\partial \mathbf{J}}{\partial t} = 0$$

电流的集肤效应



$x > 0$  半无限大空间为导体，正弦电流  $i$  沿  $y$  方向流过， $\vec{J}$  只有  $y$  分量并在  $yoz$  平面上处处相等。求  $i$  在半无限大导体中的分布。

$$\frac{\partial^2 \dot{J}_y}{\partial x^2} = j\omega\mu\gamma \dot{J}_y$$

$$\text{定义 } k = \sqrt{j\omega\mu\gamma} = \alpha + j\beta$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\gamma}{2}}$$

$$\dot{J}_y = \dot{J}_0 e^{-\alpha x} e^{-j\beta x}$$

透入深度

$$|e^{-kd}| = |e^{-1}|$$

$$\alpha d = 1$$

$$d = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\gamma}}$$

## TEM波

### 理想介质中在 $x$ 方向上传播的TEM波

$$\frac{\partial^2 E_y}{\partial x^2} - \mu\epsilon \frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} - \mu\epsilon \frac{\partial^2 H_z}{\partial t^2} = 0$$

$E$ 和 $H$ 的关系

$$\frac{E_y^+(x, t)}{H_z^+(x, t)} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\frac{E_y^-(x, t)}{H_z^-(x, t)} = -\sqrt{\frac{\mu}{\epsilon}}$$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}$$

正弦波

$$\dot{\mathbf{E}}(x, t) = \dot{E}_{y0} e^{-kx} \mathbf{e}_x$$

$$k = j\beta = j\frac{\omega}{c}$$

## 导电介质

$$\frac{\partial^2 E_y}{\partial x^2} - \mu\gamma \frac{\partial E_y}{\partial t} - \mu\epsilon \frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} - \mu\gamma \frac{\partial H_z}{\partial t} - \mu\epsilon \frac{\partial^2 H_z}{\partial t^2} = 0$$

等效介电常数

$$\epsilon' = \epsilon + \frac{\gamma}{j\omega}$$

$$k = j\omega\sqrt{\mu\epsilon'}$$

波印廷矢量的均值

$$\dot{\mathbf{S}}_{\text{av}} = \text{Re}[\dot{\mathbf{E}} \times \dot{\mathbf{H}}^*] = \frac{1}{|Z_0|} E_0^{+2} e^{-2\alpha x} \cos \phi \mathbf{e}_x$$

$$\phi = \arg \sqrt{\frac{\mu}{\epsilon'}}$$

极化

$$\phi_z - \phi_y = \frac{\pi}{2}, \text{右旋}$$

$$\phi_z - \phi_y = -\frac{\pi}{2}, \text{左旋}$$

## 反射与折射

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平行极化波

反射系数

$$\Gamma_{\parallel} = \frac{E_{\parallel}^{-}}{E_{\parallel}^{+}} = \frac{Z_{02} \cos \theta_2 - Z_{01} \cos \theta_1}{Z_{02} \cos \theta_2 + Z_{01} \cos \theta_1} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

折射系数

$$T_{\parallel} = \frac{E'_{\parallel}}{E_{\parallel}^{+}} = \frac{2Z_{02} \cos \theta_1}{Z_{02} \cos \theta_2 + Z_{01} \cos \theta_1} = \frac{2 \sin \theta_2 \cos \theta_1}{\sin(\theta_1 + \theta_2) \cos(\theta_1 - \theta_2)}$$

垂直极化波

反射系数

$$\Gamma_{\perp} = \frac{E_{\perp}^{-}}{E_{\perp}^{+}} = \frac{Z_{02} \cos \theta_1 - Z_{01} \cos \theta_2}{Z_{02} \cos \theta_1 + Z_{01} \cos \theta_2}$$

折射系数

$$T_{\parallel} = \frac{E'_{\parallel}}{E_{\parallel}^{+}} = \frac{2Z_{02} \cos \theta_1}{Z_{02} \cos \theta_1 + Z_{01} \cos \theta_2}$$

理想导体处反射

$$E^{-} = -E^{+}$$

$$H^{-} = H^{+}$$

$$e.g. E_y^{+} = E_m \cos(\omega t - \beta x)$$

$$H_z^{+} = \frac{E_m}{Z_0} \cos(\omega t - \beta x)$$

$$E_y^{-} = -E_m \cos(\omega t + \beta x)$$

$$H_z^{-} = \frac{E_m}{Z_0} \cos(\omega t + \beta x)$$

理想介质反射、折射

$$\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

$$T = \frac{2Z_{02}}{Z_{02} + Z_{01}}$$

$$\text{若 } \dot{E}^+(x) = \dot{E}^+ e^{-j\beta_1 x}$$

$$\text{则 } \dot{H}^+(x) = \frac{\dot{E}^+}{Z_{01}} e^{-j\beta_1 x}$$

$$\dot{E}^-(x) = \Gamma \dot{E}^+ e^{+j\beta_1 x}$$

$$\dot{H}^-(x) = -\frac{\Gamma \dot{E}^+}{Z_{01}} e^{j\beta_1 x}$$

$$\dot{E}'(x) = T \dot{E}^+ e^{-j\beta_2 x}$$

$$\dot{H}'(x) = \frac{T \dot{E}^+}{Z_{02}} e^{-j\beta_2 x}$$

合成电场

$$\dot{E}(x) = \dot{E}^+ + \dot{E}^-$$

为行驻波, 极值为  $|\dot{E}^+|(1 \pm |\Gamma|)$

$$\text{定义驻波比 } S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

## 传输线

$L_0$ 是单位长度的电感

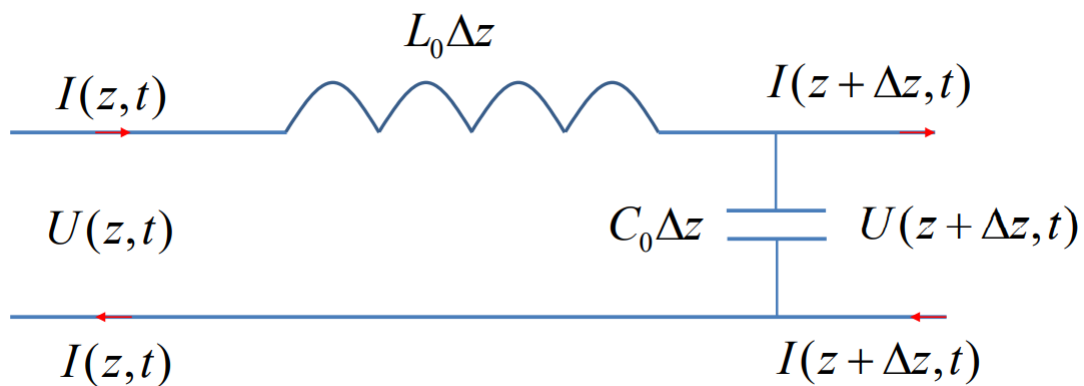
$C_0$ 是单位长度的电容

$$L_0 C_0 = \mu \varepsilon$$

电报方程

$$\frac{\partial U}{\partial z} + L_0 \frac{\partial I}{\partial t} = 0$$

$$\frac{\partial I}{\partial z} + C_0 \frac{\partial U}{\partial t} = 0$$



传输线波动方程

$$\frac{\partial^2 U}{\partial z^2} - L_0 C_0 \frac{\partial^2 U}{\partial t^2} = 0$$

$$\frac{\partial^2 I}{\partial z^2} - L_0 C_0 \frac{\partial^2 I}{\partial t^2} = 0$$

传播速度

$$v = \frac{1}{\sqrt{L_0 C_0}}$$

传输线特性阻抗

$$Z = \sqrt{\frac{L_0}{C_0}}$$