第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e_n}$$

极化体电荷:

$$\rho_{\mathbf{P}} = -\nabla \cdot \mathbf{P}$$

极化电荷代数和应为0:

$$0 = \oint_S \sigma_{\mathbf{P}} dS + \int_V \rho_{\mathbf{P}} dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D_{n2}} - \mathbf{D_{n1}} = \sigma$$

电位连续性、E的切向连续性:

$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球

$$b = \frac{R^2}{d}$$

$$q' = \frac{R}{d}q$$

两种理想介质

在$r_1 = r_2$处

图 1 – 30　点电荷对无限大
介质分界平面的镜像

$$\begin{cases} \frac{q}{\varepsilon_1} + \frac{q'}{\varepsilon_1} = \frac{q''}{\varepsilon_2} \\ q - q' = q'' \end{cases}$$

$$\therefore \begin{cases} q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}q \\ q'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2}q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\varepsilon}{\ln{(b/a)}}$$

同心夹层球

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

孤立球

$$C = 4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电，容易想到

$$W_e = \frac{1}{2} \int_V \rho\varphi dV$$

$$= \frac{1}{2} \int_S \sigma\varphi dS$$

静电场能量密度

$$w'_e = \frac{1}{2}\mathbf{D} \cdot \mathbf{E}$$

虚功原理

$$dW = dW_e + fdg$$

不与电源相连

$$0 = dW_e + fdg$$

$$\therefore f = -\frac{\partial W_e}{\partial g}$$

与电源相连,各带电体电位不变

$$dW_e = \frac{1}{2} \sum_k \varphi_k dq_k = \frac{1}{2} dW$$

$$\therefore f = \frac{\partial W_e}{\partial g}$$

第二章

电流密度

$$\mathbf{J} = \rho\mathbf{v}$$

作用于垂直于v的dS

$$dI = \mathbf{J} \cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma\mathbf{v}$$

作用于垂直于v的dl

$$dI = \mathbf{K} \cdot \mathbf{e_n}dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v}dq = \mathbf{J}dV = \mathbf{K}dS = Id\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma\mathbf{E}$$
γ为电导率

焦耳定律微分形式

$$p = \frac{dP}{dV} = \mathbf{J} \cdot \mathbf{E}$$

含源欧姆定律

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{E_e})$$

电流连续性方程

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$\mathbf{J_{1n}} = \mathbf{J_{2n}}, \text{体现一个电荷守恒}$$
$$\mathbf{E_{1t}} = \mathbf{E_{2t}}, \text{体现一个保守场}$$
$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}, \text{谓之折射定律}$$

第三章

BS定律

$$\mathbf{B} = \frac{\mu}{4\pi} \oint_L \frac{Id\mathbf{l} \times \mathbf{e_R}}{R^2}$$
$$= \frac{\mu}{4\pi} \oint_V \frac{\mathbf{J}dV \times \mathbf{e_R}}{R^2}$$
$$= \frac{\mu}{4\pi} \oint_S \frac{\mathbf{K}dS \times \mathbf{e_R}}{R^2}$$

B在场点，其余均在源点

安培力
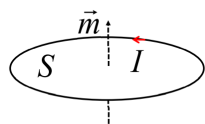
$$d\mathbf{F} = Id\mathbf{l} \times \mathbf{B}$$

安培环路定律

$$\nabla \times \mathbf{H} = \mathbf{J}$$
$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \sum_k I_k$$

有介质

分子磁矩

$$\mathbf{m} = I\mathbf{S}$$

磁力矩

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

磁化强度

$$\mathbf{M} = \lim_{\Delta V \to 0} \frac{\sum_i \mathbf{m_i}}{\Delta V}$$

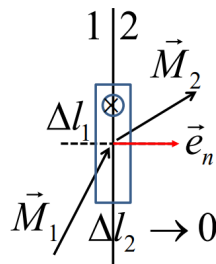理想介质中, $\mathbf{M} = \chi_m \mathbf{H}$, χ_m是磁化率

磁化电流

$$I_m = \oint_L \mathbf{J_m} \cdot d\mathbf{l}$$

$$\mathbf{J_m} = \nabla \times \mathbf{M}$$

磁化强度衔接条件

$$(\mathbf{M_1} - \mathbf{M_2}) \times \mathbf{e_n} = \mathbf{K_m}$$

磁场强度

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$
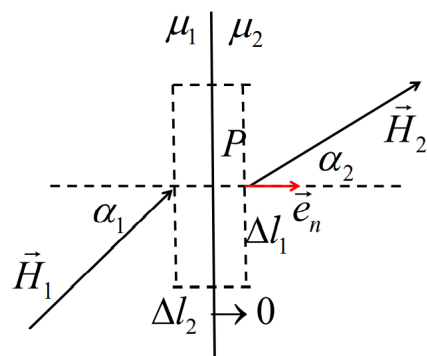
$$\mathbf{B} = \mu \mathbf{H}(\text{理想介质})$$

$$\nabla \times \mathbf{H} = \mathbf{J}, \text{自由电流密度}, \text{安培环路定律2.0}$$

不存在磁单极子,磁场是无源的

$$\nabla \cdot \mathbf{B} = 0 (\text{微分形式})$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 (\text{积分形式})$$

衔接条件、折射定律

考虑安培环路定律$\nabla \times \mathbf{H} = \mathbf{J}$:

$$(\mathbf{H_1} - \mathbf{H_2}) \times \mathbf{e_n} = \mathbf{K}$$

$$H_{1t} - H_{2t} = K, \text{分量形式}$$

磁场无源:

$$(\mathbf{B_1} - \mathbf{B_2}) \cdot \mathbf{e_n} = 0$$

$$\mathbf{B_{1n}} = \mathbf{B_{2n}}, \text{分量形式}$$

折射定律

$$理想介质, 无面电流$$
$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2}$$

磁矢位

磁场无源,所以将$\mathbf{B}$看做一个场的旋度场(旋度无散)

$$\mathbf{B} = \nabla \times \mathbf{A}$$

称$\mathbf{A}$为磁矢位

代入安培环路定律和磁场构造方程

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

利用矢量恒等式,并令$\nabla \cdot \mathbf{A} = \mathbf{0}$(库伦规范条件),得到一个三维泊松方程
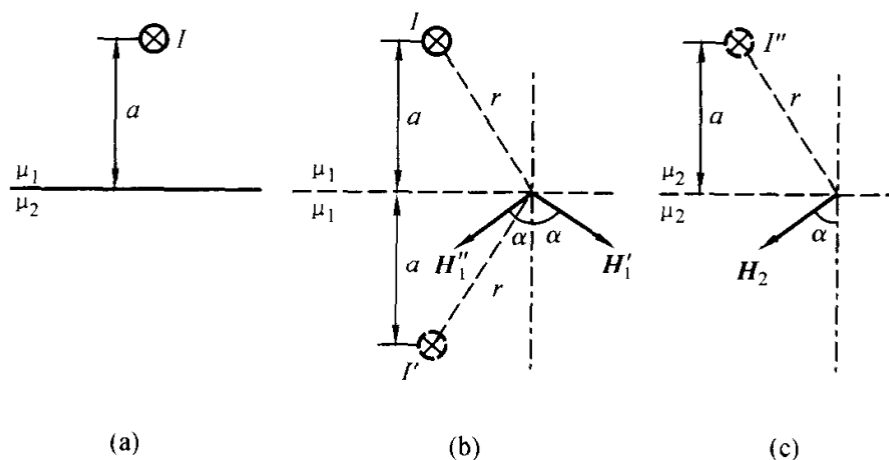
$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

类比静电场的泊松方程的解

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J} dV'}{R}$$
$$= \frac{\mu}{4\pi} \int_{S'} \frac{\mathbf{K} dS'}{R}$$
$$= \frac{\mu}{4\pi} \oint_{L'} \frac{I d\mathbf{l}'}{R}$$

衔接方程

$$\mathbf{A_1} = \mathbf{A_2}$$
$$\frac{1}{\mu_1} \frac{\partial A_1}{\partial n} - \frac{1}{\mu_2} \frac{\partial A_2}{\partial n} = K, {}_{平行平面场}$$

镜像法

(a)   (b)   (c)
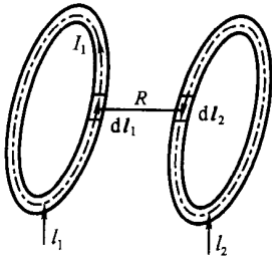
由衔接条件

$$\frac{I}{2\pi r} \sin \alpha - \frac{I'}{2\pi r} \sin \alpha = \frac{I''}{2\pi r} \sin \alpha$$
$$\mu_1 \left(\frac{I}{2\pi r} \cos \alpha + \frac{I'}{2\pi r} \cos \alpha \right) = \mu_2 \frac{I''}{2\pi r} \cos \alpha$$
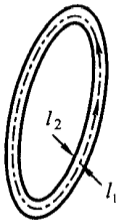
解得

$$I' = \frac{\mu_2 - \mu_1}{\mu_1 + \mu2} I$$

$$I'' = \frac{2\mu_1}{\mu_1 + \mu_2} I$$

Neumann公式

$$M_{12} = M_{21} = \frac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}$$

$$L_o = \frac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}$$

$$L = L_i + L_o \approx L_o$$

$$L_i \approx \frac{\mu l}{8\pi}$$

第四章

含时Maxwell's Equation

涡旋电场

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B})$$

在静止媒质中$\mathbf{v} = \mathbf{0}$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

全电流定律

位移电流

$$\mathbf{J_d} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J_d} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

理想导体边界条件

导体内部(静电屏蔽)

$$\mathbf{E_1} = \mathbf{0}$$
$$\mathbf{B_1} = \mathbf{0}$$

导体外部(衔接方程)

$$(\mathbf{E_1} - \mathbf{E_2}) \times \mathbf{e_n} = \mathbf{0} \Rightarrow E_{2t} = 0$$
$$(\mathbf{H_1} - \mathbf{H_2}) \times \mathbf{e_n} = \mathbf{K} \Rightarrow B_{2t} = K$$
$$(\mathbf{D_1} - \mathbf{D_2}) \cdot \mathbf{e_n} = \sigma \Rightarrow D_{2n} = \sigma$$
$$(\mathbf{B_1} - \mathbf{B_2}) \cdot \mathbf{e_n} = 0 \Rightarrow B_{2n} = 0$$

导体附近$\mathbf{E}$线垂直于表面,$\mathbf{B}$线平行于表面

动态位

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi$$

于是

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}$$
$$\mathbf{B} = \nabla \times \mathbf{A}$$

达朗贝尔方程

$$\square = \nabla^2 - \mu\varepsilon \frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$
$$\square \mathbf{A} = -\mu \mathbf{J}$$
$$\square \varphi = -\frac{\rho}{\varepsilon}$$

洛伦兹规范条件

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

达朗贝尔方程的解

$$R = |\mathbf{r} - \mathbf{r'}|$$
$$\varphi(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon} \int_{V'} \frac{\rho(\mathbf{r'}, t - \frac{R}{c}) dV'}{R}$$
$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}(\mathbf{r'}, t - \frac{R}{c}) dV'}{R}$$

电磁场能量

波印廷定理(能量守恒)

$$\frac{\partial W}{\partial t} = -\oint_{\partial V} \mathbf{E} \times \mathbf{H} \cdot d\mathbf{S} + \int_V \mathbf{E_e} \cdot \mathbf{J} dV - \int_V \frac{\mathbf{J}^2}{\gamma} dV$$

电磁场能量变化=-电磁波向外辐射+电源提供-焦耳热

波印廷矢量

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

正弦电磁场

$$\mathbf{E}(x, y, z, t) = \text{Re}[\sqrt{2}\dot{\mathbf{E}}(x, y, z)e^{j\omega t}]$$

$$\dot{\mathbf{E}}(x, y, z) = \sum_{i \in \{x,y,z\}} \frac{1}{\sqrt{2}} E_{mi} e^{j\phi_i} \mathbf{e_i}$$

波印廷矢量均值

$$\mathbf{S_{av}} = (\mathbf{E} \times \mathbf{H}) \cos(\phi_E - \phi_B)$$

复波印廷矢量

$$\dot{\mathbf{S}} = \dot{\mathbf{E}} \times \dot{\mathbf{H}}^*$$

$$\mathbf{S_{av}} = \text{Re}[\dot{\mathbf{S}}]$$

达朗贝尔方程的复数形式

$$\nabla^2 \dot{\mathbf{A}} + \beta^2 \dot{\mathbf{A}} = -\mu \dot{\mathbf{J}}$$

$$\nabla^2 \dot{\varphi} + \beta^2 \dot{\varphi} = -\frac{\dot{\rho}}{\varepsilon}$$

$$其中 \beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

洛伦兹规范条件复数形式

$$\nabla \cdot \dot{\mathbf{A}} + j\omega\mu\varepsilon\dot{\varphi} = 0$$

动态位复数形式

$$\dot{\mathbf{E}} = -\nabla\dot{\varphi} - j\omega\mu\varepsilon\dot{\mathbf{A}}$$

$$= \frac{\nabla(\nabla \cdot \dot{\mathbf{A}})}{j\omega\mu\varepsilon} - j\omega\mu\varepsilon\dot{\mathbf{A}}$$

$$\dot{\mathbf{B}} = \nabla \times \dot{\mathbf{A}}$$

达朗贝尔方程的复数解

$$R = |\mathbf{r} - \mathbf{r}'|$$

$$\dot{\varphi} = \frac{1}{4\pi\varepsilon} \int_{V'} \frac{\dot{\rho}e^{-j\frac{2\pi R}{\lambda}}dV'}{R}$$

$$\dot{\mathbf{A}} = \frac{\mu}{4\pi} \int_{V'} \frac{\dot{\mathbf{J}}e^{-j\frac{2\pi R}{\lambda}}dV'}{R}$$

似稳条件

$$\frac{2\pi R}{\lambda} << 1$$