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## 第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e_n}$$

极化体电荷:

$$\rho_{\mathbf{P}} = -\nabla \cdot \mathbf{P}$$

极化电荷代数和应为0:

$$0 = \oint_S \sigma_{f P} dS + \int_V 
ho_{f P} dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D_{n2}} - \mathbf{D_{n1}} = \sigma$$

电位连续性、E的切向连续性:

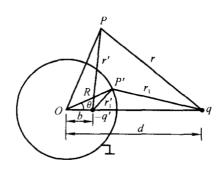
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b=rac{R^2}{d}$$
  $q'=rac{R}{d}q$ 

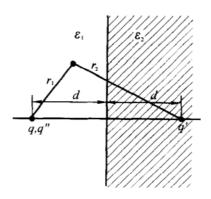


图 1-30 点电荷对无限大介质分界平面的镜像

$$egin{cases} rac{q}{arepsilon_1}+rac{q'}{arepsilon_1}=rac{q''}{arepsilon_2}\ q-q'=q'' \end{cases}$$

$$\therefore egin{cases} q' = rac{arepsilon_1 - arepsilon_2}{arepsilon_1 + arepsilon_2} q \ q'' = rac{2arepsilon_2}{arepsilon_1 + arepsilon_2} q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\varepsilon}{\ln\left(b/a\right)}$$

同心夹层球

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

孤立球

$$C = 4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电, 容易想到

$$egin{aligned} W_e &= rac{1}{2} \int_V 
ho arphi dV \ &= rac{1}{2} \int_S \sigma arphi dS \end{aligned}$$

静电场能量密度

$$w_e' = rac{1}{2} {f D} \cdot {f E}$$

虚功原理

$$dW = dW_e + fdg$$

不与电源相连

$$0 = dW_e + fdg$$
 
$$\therefore f = -\frac{\partial W_e}{\partial g}$$

与电源相连,各带电体电位不变

$$egin{aligned} dW_e &= rac{1}{2} \sum_k arphi_k dq_k = rac{1}{2} dW \ &\therefore f = rac{\partial W_e}{\partial g} \end{aligned}$$

# 第二章

电流密度

$$\mathbf{J} = \rho \mathbf{v}$$

作用于垂直于v的dS

$$dI = \mathbf{J} \cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma \mathbf{v}$$

作用于垂直于v的dl

$$dI = \mathbf{K} \cdot \mathbf{e_n} dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v}dq = \mathbf{J}dV = \mathbf{K}dS = Id\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma \mathbf{E}$$
  $\gamma$ 为电导率

焦耳定律微分形式

$$p = rac{dP}{dV} = {f J} \cdot {f E}$$

含源欧姆定律

$$\mathbf{J} = \gamma (\mathbf{E} + \mathbf{E_e})$$

电流连续性方程

$$\oint_{S} \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$\mathbf{J_{1n}} = \mathbf{J_{2n}}$$
,体现一个电荷守恒  $\mathbf{E_{1t}} = \mathbf{E_{2t}}$ ,体现一个保守场  $\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}$ ,谓之折射定律

# 第三章

BS定律

$$\mathbf{B} = rac{\mu}{4\pi} \oint_L rac{Id\mathbf{l} imes \mathbf{e_R}}{R^2}$$

$$= rac{\mu}{4\pi} \oint_V rac{\mathbf{J} dV imes \mathbf{e_R}}{R^2}$$

$$= rac{\mu}{4\pi} \oint_S rac{\mathbf{K} dS imes \mathbf{e_R}}{R^2}$$
 $B$ 在场点,其余均在源点

安培力

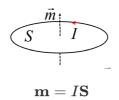
$$d\mathbf{F} = Id\mathbf{l} \times \mathbf{B}$$

安培环路定律

$$abla extbf{ iny H} = extbf{J} \ \oint_L extbf{H} \cdot d extbf{l} = \sum_k I_k$$

## 有介质

分子磁矩



磁力矩

$$\mathbf{T} = \mathbf{m} {\times} \mathbf{B}$$

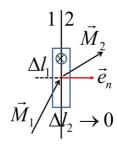
磁化强度

$$\mathbf{M}=\lim_{\Delta V o 0}rac{\sum_{i}\mathbf{m_{i}}}{\Delta V}$$
理想介质中, $\mathbf{M}=\chi_{m}\mathbf{H},\chi_{m}$ 是磁化率

磁化电流

$$I_m = \oint_L \mathbf{J_m} \cdot d\mathbf{l} \ \mathbf{J_m} = 
abla imes \mathbf{M}$$

磁化强度衔接条件



$$(\mathbf{M_1} - \mathbf{M_2}) \times \mathbf{e_n} = \mathbf{K_m}$$

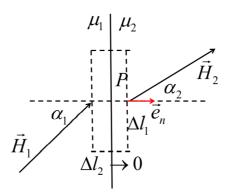
磁场强度

$$\mathbf{H}=rac{\mathbf{B}}{\mu_{\mathbf{0}}}-\mathbf{M}$$
  $\mathbf{B}=\mu\mathbf{H}($ 理想介质 $)$   $abla imes\mathbf{H}=\mathbf{J},$  自由电流密度 $,$  安培环路定律 $\mathbf{2}.0$ 

不存在磁单极子,磁场是无源的

$$abla \cdot \mathbf{B} = 0 ($$
微分形式 $)$   $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 ($ 积分形式 $)$ 

衔接条件、折射定律



考虑安培环路定律 $\nabla \times \mathbf{H} = \mathbf{J}$ :

$$(\mathbf{H_1} - \mathbf{H_2}) imes \mathbf{e_n} = \mathbf{K}$$
  $H_{1t} - H_{2t} = K,$  分量形式

磁场无源:

$$(\mathbf{B_1} - \mathbf{B_2}) \cdot \mathbf{e_n} = 0$$
  $\mathbf{B_{1n}} = \mathbf{B_{2n}},$  分量形式

理想介质,无面电流

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2}$$

## 磁矢位

磁场无源,所以将图看做一个场的旋度场(旋度无散)

$$\mathbf{B} = 
abla imes \mathbf{A}$$

称A为磁矢位

代入安培环路定律和磁场构造方程

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

利用矢量恒等式,并令 $\nabla \cdot \mathbf{A} = \mathbf{0}$ (库伦规范条件),得到一个三维泊松方程

$$abla^2 \mathbf{A} = -\mu \mathbf{J}$$

类比静电场的泊松方程的解

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}dV'}{R}$$

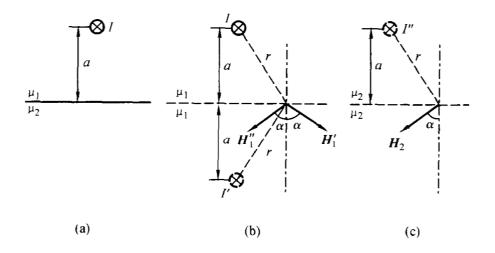
$$= \frac{\mu}{4\pi} \int_{S'} \frac{\mathbf{K}dS'}{R}$$

$$= \frac{\mu}{4\pi} \oint_{L'} \frac{Id\mathbf{l}'}{R}$$

衔接方程

$$egin{aligned} \mathbf{A_1} &= \mathbf{A_2} \ &rac{1}{\mu_1} rac{\partial A_1}{\partial n} - rac{1}{\mu_2} rac{\partial A_2}{\partial n} = K,$$
平行平面场

镜像法

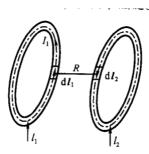


由衔接条件

$$\frac{I}{2\pi r}\sin\alpha - \frac{I'}{2\pi r}\sin\alpha = \frac{I''}{2\pi r}\sin\alpha$$
$$\mu_1(\frac{I}{2\pi r}\cos\alpha + \frac{I'}{2\pi r}\cos\alpha) = \mu_2\frac{I''}{2\pi r}\cos\alpha$$

$$I' = rac{\mu_2 - \mu_1}{\mu_1 + \mu_2} I$$
 $I'' = rac{2\mu_1}{\mu_1 + \mu_2} I$ 

Neumann公式



$$M_{12} = M_{21} = rac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} rac{\mathbf{dl_1} \cdot \mathbf{dl_2}}{R}$$



$$egin{align} L_o &= rac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} rac{\mathbf{dl_1 \cdot dl_2}}{R} \ L &= L_i + L_o pprox L_o \ L_i &pprox rac{\mu l}{8\pi} \ \end{pmatrix}$$

# 第四章

## 含时Maxwell's Equation

涡旋电场

$$abla{ imes}\mathbf{E} = -rac{\partial \mathbf{B}}{\partial t} + 
abla{ imes}(\mathbf{v}{ imes}\mathbf{B})$$

在静止媒质中v = 0

$$abla imes \mathbf{E} = -rac{\partial \mathbf{B}}{\partial t}$$

全电流定律

位移电流

$$\mathbf{J_d} = rac{\partial \mathbf{D}}{\partial t}$$
  $abla imes \mathbf{H} = \mathbf{J} + \mathbf{J_d} = \mathbf{J} + rac{\partial \mathbf{D}}{\partial t}$ 

理想导体边界条件

导体内部(静电屏蔽)

$$\mathbf{E_1} = \mathbf{0}$$
 $\mathbf{B_1} = \mathbf{0}$ 

导体外部(衔接方程)

$$(\mathbf{E_1} - \mathbf{E_2}) \times \mathbf{e_n} = \mathbf{0} \Rightarrow E_{2t} = 0$$

$$(\mathbf{H_1} - \mathbf{H_2}) \times \mathbf{e_n} = \mathbf{K} \Rightarrow B_{2t} = K$$

$$(\mathbf{D_1} - \mathbf{D_2}) \cdot \mathbf{e_n} = \sigma \Rightarrow D_{2n} = \sigma$$

$$(\mathbf{B_1} - \mathbf{B_2}) \cdot \mathbf{e_n} = 0 \Rightarrow B_{2n} = 0$$

导体附近E线垂直于表面,B线平行于表面

### 动态位

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi$$

于是

$$\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}$$
  
 $\mathbf{B} = \nabla \times \mathbf{A}$ 

达朗贝尔方程

$$\Box = \nabla^2 - \mu \varepsilon \frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$
$$\Box \mathbf{A} = -\mu \mathbf{J}$$
$$\Box \varphi = -\frac{\rho}{\varepsilon}$$

洛伦兹规范条件

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

达朗贝尔方程的解

$$R = |\mathbf{r} - \mathbf{r}'|$$
  $arphi(\mathbf{r},t) = rac{1}{4\piarepsilon} \int_{V'} rac{
ho(\mathbf{r}',t-rac{R}{c})dV'}{R}$   $\mathbf{A}(\mathbf{r},t) = rac{\mu}{4\pi} \int_{V'} rac{\mathbf{J}(\mathbf{r}',t-rac{R}{c})dV'}{R}$ 

### 电磁场能量

波印廷定理(能量守恒)

$$rac{\partial W}{\partial t} = -\oint_{\partial V} \mathbf{E} imes \mathbf{H} \cdot \mathbf{dS} + \int_{V} \mathbf{E_e} \cdot \mathbf{J} dV - \int_{V} \frac{\mathbf{J}^2}{\gamma} dV$$

电磁场能量变化=-电磁波向外辐射+电源提供-焦耳热

波印廷矢量

### 正弦电磁场

$$\mathbf{E}(x,y,z,t) = \mathrm{Re}[\sqrt{2}\mathbf{\dot{E}}(x,y,z)e^{j\omega t}] \ \mathbf{\dot{E}}(x,y,z) = \sum_{i\in\{x,y,z\}} rac{1}{\sqrt{2}} E_{mi}e^{j\phi_i}\mathbf{e_i}$$

波印廷矢量均值

$$\mathbf{S_{av}} = (\mathbf{E}{ imes}\mathbf{H})\cos{(\phi_E - \phi_B)}$$

复波印廷矢量

$$\dot{\mathbf{S}} = \dot{\mathbf{E}} \times \dot{\mathbf{H}}^*$$
  
 $\mathbf{S}_{av} = \mathrm{Re}[\dot{\mathbf{S}}]$ 

波印廷定理复数形式

$$\int_V \mu |\dot{\mathbf{H}}|^2 + arepsilon |\dot{\mathbf{E}}|^2 dV = - \oint_{\partial V} \dot{\mathbf{E}} imes \dot{\mathbf{H}} \cdot \mathbf{dS} - \int_V rac{|\dot{\mathbf{J}}|^2}{\gamma} dV + \int_V \dot{\mathbf{E}}_\mathbf{e} \cdot \dot{\mathbf{J}}^* dV$$

达朗贝尔方程的复数形式

$$abla^2 \dot{\mathbf{A}} + eta^2 \dot{\mathbf{A}} = -\mu \dot{\mathbf{J}}$$

$$abla^2 \dot{\varphi} + eta^2 \dot{\varphi} = -\frac{\dot{\rho}}{\varepsilon}$$

$$\sharp \oplus \beta = \omega \sqrt{\mu \varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

洛伦兹规范条件复数形式

$$abla \cdot \dot{\mathbf{A}} + j\omega\muarepsilon\dot{arphi} = 0$$

动态位复数形式

$$egin{aligned} \dot{\mathbf{E}} &= -
abla \dot{ec{\phi}} - j\omega\muarepsilon\dot{\mathbf{A}} \ &= rac{
abla (
abla \cdot \dot{\mathbf{A}})}{j\omega\muarepsilon} - j\omega\muarepsilon\dot{\mathbf{A}} \end{aligned}$$

$$\mathbf{\dot{B}} = 
abla imes \mathbf{\dot{A}}$$

达朗贝尔方程的复数解

$$R = |\mathbf{r} - \mathbf{r}'| \ \dot{arphi} = rac{1}{4\piarepsilon} \int_{V'} rac{\dot{
ho}e^{-jrac{2\pi R}{\lambda}}dV'}{R} \ \dot{\mathbf{A}} = rac{\mu}{4\pi} \int_{V'} rac{\dot{\mathbf{J}}e^{-jrac{2\pi R}{\lambda}}dV'}{R}$$

似稳条件

$$\frac{2\pi R}{\lambda} << 1$$

### 单元偶极子的辐射

$$P = rac{2\pi\mu c}{3}I^2(rac{\Delta l}{\lambda})^2 pprox 80\pi^2I^2(rac{\Delta l}{\lambda})^2$$

方向性因子

$$f(\theta, \phi) = \sin \theta$$

# 准静态电磁场

电准静态场

磁场的变化率约等于0

$$\frac{\partial \mathbf{B}}{\partial t} pprox \mathbf{0}$$

磁准静态场

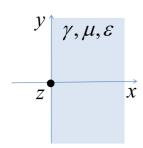
电场的变化率约等于0

$$\frac{\partial \mathbf{B}}{\partial t} \approx \mathbf{0}$$

以下关系成立

$$egin{aligned} 
abla^2 \mathbf{H} - \mu \gamma rac{\partial \mathbf{H}}{\partial t} &= \mathbf{0} \ 
abla^2 \mathbf{E} - \mu \gamma rac{\partial \mathbf{E}}{\partial t} &= \mathbf{0} \ 
abla^2 \mathbf{J} - \mu \gamma rac{\partial \mathbf{J}}{\partial t} &= \mathbf{0} \end{aligned}$$

电流的集肤效应



x > 0半无限大空间为导体,正弦电流i沿y方向流过, $\vec{j}$  只有y分量 并在yoz平面上处处相等。求i在半无限大导体中的分布。

$$egin{aligned} rac{\partial^2 \dot{J}_y}{\partial x^2} &= \dot{j}\omega\mu\gamma J_y \ rac{\dot{z}}{\partial x^2} &= \dot{z}\omega\mu\gamma J_y \end{aligned}$$
定义 $k = \sqrt{j\omega\mu\gamma} = lpha + jeta \ lpha &= eta = \sqrt{rac{\omega\mu\gamma}{2}} \ \dot{J}_y &= \dot{J}_0 e^{-lpha x} e^{-jeta x} \end{aligned}$ 

透入深度

$$|e^{-kd}| = |e^{-1}|$$
  $\alpha d = 1$   $d = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\gamma}}$ 

# TEM波

### 理想介质中在x方向上传播的TEM波

$$\frac{\partial^2 E_y}{\partial x^2} - \mu \varepsilon \frac{\partial^2 E_y}{\partial t^2} = 0$$
$$\frac{\partial^2 H_z}{\partial x^2} - \mu \varepsilon \frac{\partial^2 H_z}{\partial t^2} = 0$$

E和H的关系

$$egin{aligned} rac{E_y^+(x,t)}{H_z^+(x,t)} &= \sqrt{rac{\mu}{arepsilon}} \ rac{E_y^-(x,t)}{H_z^-(x,t)} &= -\sqrt{rac{\mu}{arepsilon}} \ Z_0 &= \sqrt{rac{\mu}{arepsilon}} \end{aligned}$$

正弦波

$$\dot{\mathbf{E}}(x,t)=\dot{E}_{y0}e^{-kx}\mathbf{e_{x}} 
onumber \ k=jeta=jrac{\omega}{c}$$

### 导电介质

$$\begin{split} \frac{\partial^2 E_y}{\partial x^2} - \mu \gamma \frac{\partial E_y}{\partial t} - \mu \varepsilon \frac{\partial^2 E_y}{\partial t^2} &= 0\\ \frac{\partial^2 H_z}{\partial x^2} - \mu \gamma \frac{\partial H_z}{\partial t} - \mu \varepsilon \frac{\partial^2 H_z}{\partial t^2} &= 0 \end{split}$$

等效介电常数

$$\varepsilon' = \varepsilon + \frac{\gamma}{j\omega}$$

$$k = j\omega\sqrt{\mu\varepsilon'}$$

波印廷矢量的均值

$$egin{align} \dot{\mathbf{S}}_{\mathbf{av}} &= \mathrm{Re}[\dot{\mathbf{E}} imes \dot{\mathbf{H}}^*] = rac{1}{|Z_0|} E_0^{+2} e^{-2lpha x} \cos\phi \mathbf{e_x} \ \phi &= \arg\sqrt{rac{\mu}{arepsilon'}} \ \end{aligned}$$

极化

$$\phi_z - \phi_y = rac{\pi}{2}$$
,右旋 $\phi_z - \phi_y = -rac{\pi}{2}$ ,左旋

### 反射与折射

平行极化波

反射系数

$$\Gamma_{\parallel} = rac{E_{\parallel}^{-}}{E_{\parallel}^{+}} = rac{Z_{02}\cos heta_{2} - Z_{01}\cos heta_{1}}{Z_{02}\cos heta_{2} + Z_{01}\cos heta_{1}} = rac{ an\left( heta_{1} - heta_{2}
ight)}{ an\left( heta_{1} + heta_{2}
ight)}$$

折射系数

$$T_{\parallel} = rac{E_{\parallel}'}{E_{\parallel}^+} = rac{2Z_{02}\cos heta_1}{Z_{02}\cos heta_2 + Z_{01}\cos heta_1} = rac{2\sin heta_2\cos heta_1}{\sin{( heta_1+ heta_2)}\cos{( heta_1- heta_2)}}$$

垂直极化波

反射系数

$$\Gamma_{\perp} = rac{E_{\perp}^{-}}{E_{\perp}^{+}} = rac{Z_{02}\cos heta_{1} - Z_{01}\cos heta_{2}}{Z_{02}\cos heta_{1} + Z_{01}\cos heta_{2}}$$

折射系数

$$T_{\parallel} = rac{E_{\parallel}'}{E_{\parallel}^+} = rac{2 Z_{02} \cos heta_1}{Z_{02} \cos heta_1 + Z_{01} \cos heta_2}$$

理想导体处反射

$$E^{-} = -E^{+} \ H^{-} = H^{+} \ e.\,g.\,E_{y}^{+} = E_{m}\cos{(\omega t - eta x)} \ H_{z}^{+} = rac{E_{m}}{Z_{0}}\cos{(\omega - eta x)} \ E_{y}^{-} = -E_{m}\cos{(\omega t + eta x)} \ H_{z}^{-} = rac{E_{m}}{Z_{0}}\cos{(\omega t + eta x)}$$

理想介质反射、折射

$$egin{aligned} \Gamma &= rac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \ T &= rac{2Z_{02}}{Z_{02} + Z_{01}} \ \ddot{E}\dot{E}^+(x) &= \dot{E}^+e^{-jeta_1x} \ \ddot{B}\dot{H}^+(x) &= rac{\dot{E}^+}{Z_{01}}e^{-jeta_1x} \ \dot{E}^-(x) &= \Gamma\dot{E}^+e^{+jeta_1x} \ \dot{H}^-(x) &= -rac{\Gamma\dot{E}^+}{Z_{01}}e^{jeta_1x} \ \dot{E}'(x) &= T\dot{E}^+e^{-jeta_2x} \ \dot{H}'(x) &= rac{T\dot{E}^+}{Z_{02}}e^{-jeta_2x} \end{aligned}$$

合成电场

$$\dot{E}(x)=\dot{E}^++\dot{E}^-$$
  
为行驻波, 极值为 $|\dot{E}^+|(1\pm|\Gamma|)$   
定义驻波比 $S=rac{1+|\Gamma|}{1-|\Gamma|}$ 

# 传输线

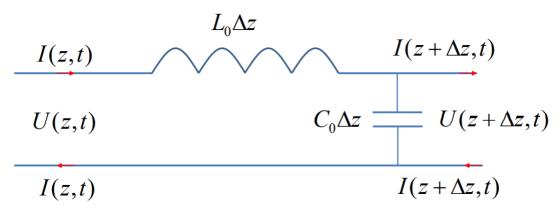
 $L_0$ 是单位长度的电感

 $C_0$ 是单位长度的电容

$$L_0C_0=\mu\varepsilon$$

电报方程

$$\frac{\partial U}{\partial z} + L_0 \frac{\partial I}{\partial t} = 0$$
$$\frac{\partial I}{\partial z} + C_0 \frac{\partial U}{\partial t} = 0$$



传输线波动方程

$$\begin{aligned} \frac{\partial^2 U}{\partial z^2} - L_0 C_0 \frac{\partial^2 U}{\partial t^2} &= 0\\ \frac{\partial^2 I}{\partial z^2} - L_0 C_0 \frac{\partial^2 I}{\partial t^2} &= 0 \end{aligned}$$

传播速度

$$v = \frac{1}{\sqrt{L_0 C_0}}$$

传输线特性阻抗

$$Z=\sqrt{rac{L_0}{C_0}}$$

电压波与电流波

$$egin{align} U(z,t) &= U^+(t-rac{z}{v}) + U^-(t+rac{z}{v}) \ I(z,t) &= I^+(t-rac{z}{v}) + I^-(t+rac{z}{v}) \ I(z,t) &= rac{1}{Z_0}[U^+(t-rac{z}{v}) - U^-(t+rac{z}{v})] \ \end{split}$$