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动态位

电磁场能量

第一章

极化面电荷:

$$\sigma_{\mathbf{P}} = \mathbf{P} \cdot \mathbf{e}_n$$

极化体电荷:

$$\rho_{\mathbf{P}} = -\nabla \cdot \mathbf{P}$$

极化电荷代数和应为0:

$$0 = \oint_S \sigma_{\mathbf{P}} dS + \int_V \rho_{\mathbf{P}} dV$$

在理想电介质中:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

电通量连续性:

$$\mathbf{D}_{n2} - \mathbf{D}_{n1} = \sigma$$

电位连续性、E的切向连续性:

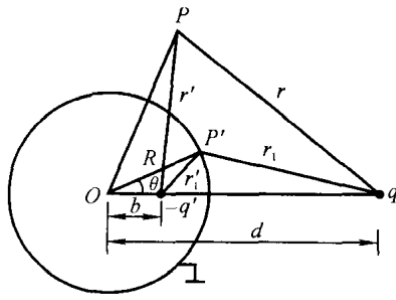
$$E_{1t} = E_{2t}$$

静电场折射定律(理想介质、分界面上无自由电荷):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

电像法

导体球



$$b = \frac{R^2}{d}$$

$$q' = \frac{R}{d}q$$

两种理想介质
在 $r_1 = r_2$ 处

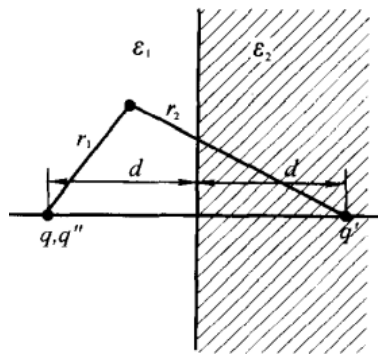


图 1 - 30 点电荷对无限大
介质分界平面的镜像

$$\begin{cases} \frac{q}{\epsilon_1} + \frac{q'}{\epsilon_2} = \frac{q''}{\epsilon_2} \\ q - q' = q'' \end{cases}$$

$$\therefore \begin{cases} q' = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} q \\ q'' = \frac{2\epsilon_2}{\epsilon_1 + \epsilon_2} q \end{cases}$$

几个电容公式

同轴夹层线缆

$$C = \frac{2\pi\epsilon}{\ln(b/a)}$$

同心夹层球

$$C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$$

孤立球

$$C = 4\pi\epsilon_0 a$$

静电场能量

介质各处均匀线性充电，容易想到

$$\begin{aligned}
 W_e &= \frac{1}{2} \int_V \rho \varphi dV \\
 &= \frac{1}{2} \int_S \sigma \varphi dS
 \end{aligned}$$

静电场能量密度

$$w_e' = \frac{1}{2} \mathbf{D} \cdot \mathbf{E}$$

虚功原理

$$dW = dW_e + f dg$$

不与电源相连

$$\begin{aligned}
 0 &= dW_e + f dg \\
 \therefore f &= - \frac{\partial W_e}{\partial g}
 \end{aligned}$$

与电源相连,各带电体电位不变

$$\begin{aligned}
 dW_e &= \frac{1}{2} \sum_k \varphi_k dq_k = \frac{1}{2} dW \\
 \therefore f &= \frac{\partial W_e}{\partial g}
 \end{aligned}$$

第二章

电流密度

$$\mathbf{J} = \rho \mathbf{v}$$

作用于垂直于 v 的 dS

$$dI = \mathbf{J} \cdot d\mathbf{S}$$

面电流密度

$$\mathbf{K} = \sigma \mathbf{v}$$

作用于垂直于 v 的 dl

$$dI = \mathbf{K} \cdot \mathbf{e}_n dl$$

线电流密度

$$I = \tau v$$

四种电流元

$$\mathbf{v} dq = \mathbf{J} dV = \mathbf{K} dS = I d\mathbf{l}$$

欧姆定律微分形式

$$\mathbf{J} = \gamma \mathbf{E}$$

γ 为电导率

焦耳定律微分形式

$$p = \frac{dP}{dV} = \mathbf{J} \cdot \mathbf{E}$$

含源欧姆定律

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{E}_e)$$

电流连续性方程

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial q}{\partial t} = 0$$

衔接条件

$$\mathbf{J}_{1n} = \mathbf{J}_{2n}, \text{体现一个电荷守恒}$$

$$\mathbf{E}_{1t} = \mathbf{E}_{2t}, \text{体现一个保守场}$$

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}, \text{谓之折射定律}$$

第三章

BS定律

$$\begin{aligned} \mathbf{B} &= \frac{\mu}{4\pi} \oint_L \frac{Id\mathbf{l} \times \mathbf{e}_R}{R^2} \\ &= \frac{\mu}{4\pi} \oint_V \frac{\mathbf{J}dV \times \mathbf{e}_R}{R^2} \\ &= \frac{\mu}{4\pi} \oint_S \frac{\mathbf{K}dS \times \mathbf{e}_R}{R^2} \end{aligned}$$

B 在场点, 其余均在源点

安培力

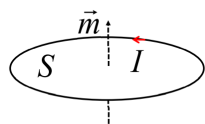
$$d\mathbf{F} = Id\mathbf{l} \times \mathbf{B}$$

安培环路定律

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} \\ \oint_L \mathbf{H} \cdot d\mathbf{l} &= \sum_k I_k \end{aligned}$$

有介质

分子磁矩



$$\mathbf{m} = I\mathbf{S}$$

磁力矩

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

磁化强度

$$\mathbf{M} = \lim_{\Delta V \rightarrow 0} \frac{\sum_i \mathbf{m}_i}{\Delta V}$$

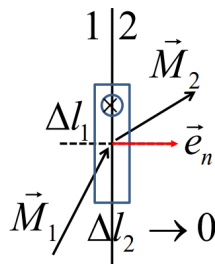
理想介质中, $\mathbf{M} = \chi_m \mathbf{H}$, χ_m 是磁化率

磁化电流

$$I_m = \oint_L \mathbf{J}_m \cdot d\mathbf{l}$$

$$\mathbf{J}_m = \nabla \times \mathbf{M}$$

磁化强度衔接条件



$$(\mathbf{M}_1 - \mathbf{M}_2) \times \mathbf{e}_n = \mathbf{K}_m$$

磁场强度

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$\mathbf{B} = \mu \mathbf{H} \text{ (理想介质)}$$

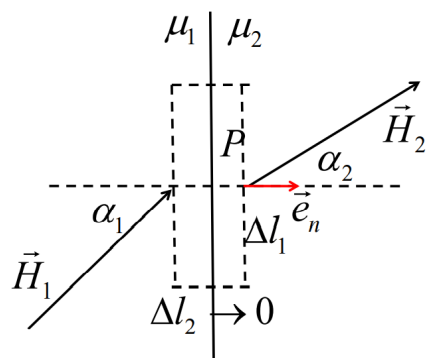
$$\nabla \times \mathbf{H} = \mathbf{J}, \text{ 自由电流密度, 安培环路定律2.0}$$

不存在磁单极子, 磁场是无源的

$$\nabla \cdot \mathbf{B} = 0 \text{ (微分形式)}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \text{ (积分形式)}$$

衔接条件、折射定律



考虑安培环路定律 $\nabla \times \mathbf{H} = \mathbf{J}$:

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{e}_n = \mathbf{K}$$

$$H_{1t} - H_{2t} = K, \text{ 分量形式}$$

磁场无源:

$$(\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{e}_n = 0$$

$$\mathbf{B}_{1n} = \mathbf{B}_{2n}, \text{ 分量形式}$$

折射定律

理想介质, 无面电流

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2}$$

磁矢位

磁场无源, 所以将 \mathbf{B} 看做一个场的旋度场(旋度无散)

$$\mathbf{B} = \nabla \times \mathbf{A}$$

称 \mathbf{A} 为磁矢位

代入安培环路定律和磁场构造方程

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

利用矢量恒等式, 并令 $\nabla \cdot \mathbf{A} = 0$ (库伦规范条件), 得到一个三维泊松方程

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

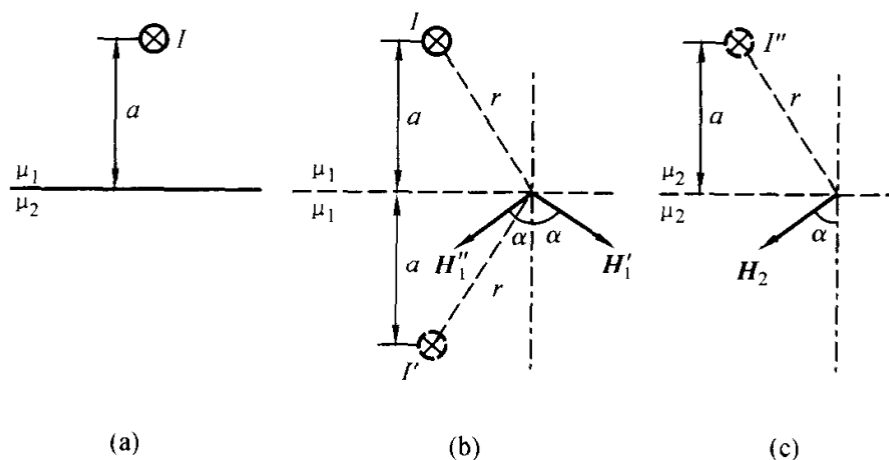
类比静电场的泊松方程的解

$$\begin{aligned} \mathbf{A} &= \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J} dV'}{R} \\ &= \frac{\mu}{4\pi} \int_{S'} \frac{\mathbf{K} dS'}{R} \\ &= \frac{\mu}{4\pi} \oint_{L'} \frac{I d\mathbf{l}'}{R} \end{aligned}$$

衔接方程

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{A}_2 \\ \frac{1}{\mu_1} \frac{\partial A_1}{\partial n} - \frac{1}{\mu_2} \frac{\partial A_2}{\partial n} &= K, \text{ 平行平面场} \end{aligned}$$

镜像法



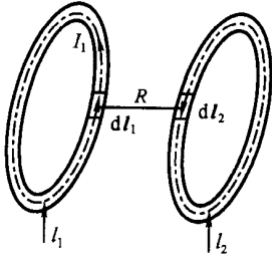
由衔接条件

$$\begin{aligned} \frac{I}{2\pi r} \sin \alpha - \frac{I'}{2\pi r} \sin \alpha &= \frac{I''}{2\pi r} \sin \alpha \\ \mu_1 \left(\frac{I}{2\pi r} \cos \alpha + \frac{I'}{2\pi r} \cos \alpha \right) &= \mu_2 \frac{I''}{2\pi r} \cos \alpha \end{aligned}$$

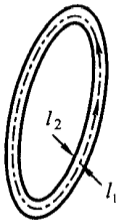
解得

$$I' = \frac{\mu_2 - \mu_1}{\mu_1 + \mu_2} I$$
$$I'' = \frac{2\mu_1}{\mu_1 + \mu_2} I$$

Neumann公式



$$M_{12} = M_{21} = \frac{N_1 N_2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl}_1 \cdot \mathbf{dl}_2}{R}$$



$$L_o = \frac{N^2 \mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\mathbf{dl}_1 \cdot \mathbf{dl}_2}{R}$$
$$L = L_i + L_o \approx L_o$$
$$L_i \approx \frac{\mu l}{8\pi}$$

第四章

含时Maxwell's Equation

涡旋电场

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B})$$

在静止媒质中 $\mathbf{v} = \mathbf{0}$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

全电流定律

位移电流

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

理想导体边界条件

导体内部(静电屏蔽)

$$\begin{aligned}\mathbf{E}_1 &= \mathbf{0} \\ \mathbf{B}_1 &= \mathbf{0}\end{aligned}$$

导体外部(衔接方程)

$$\begin{aligned}(\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{e}_n &= \mathbf{0} \Rightarrow E_{2t} = 0 \\ (\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{e}_n &= \mathbf{K} \Rightarrow B_{2t} = K \\ (\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{e}_n &= \sigma \Rightarrow D_{2n} = \sigma \\ (\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{e}_n &= 0 \Rightarrow B_{2n} = 0\end{aligned}$$

导体附近**E**线垂直于表面,**B**线平行于表面

动态位

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi$$

于是

$$\begin{aligned}\mathbf{E} &= -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} &= \nabla \times \mathbf{A}\end{aligned}$$

达朗贝尔方程

$$\begin{aligned}\square &= \nabla^2 - \mu\varepsilon \frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \\ \square \mathbf{A} &= -\mu \mathbf{J} \\ \square \varphi &= -\frac{\rho}{\varepsilon}\end{aligned}$$

洛伦兹规范条件

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

电磁场能量
