LLG Equation

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The LLG equation reads,

$$\frac{d\vec{s}}{dt} = -\frac{\gamma}{1+\lambda^2}\left[\vec{s}\times\vec{h}_{eff} + \lambda\vec{s}\times\left(\vec{s}\times\vec{h}_{eff}\right)\right]$$

where $\vec{s}$ is a unit vector representing a spin, γ is the gyromagnetic ratio, $\vec{h}_{eff}$ is the effective magnetic field acting upon the spin and λ is the Gilbert damping parameter.

The effective field can be computed out of a given hamiltonian by,

$$\vec{h}_{eff} = -\frac{1}{\mu_s}\frac{\partial\mathcal{H}}{\partial\vec{s}}$$

Heun method for the LLG Equation

Now, a popular method to solve this equation is the Heun half step method, which is a predictor corrector scheme, first you compute,

$$\vec{s}\,' = \vec{s} + \Delta\vec{s}\Delta t$$

where,

$$\Delta\vec{s} = -\frac{\gamma}{1+\lambda^2}\left[\vec{s}\times\vec{h}_{eff} + \lambda\vec{s}\times\left(\vec{s}\times\vec{h}_{eff}\right)\right]$$

Here is the time to re normalize the new vector, since this scheme does not preserve the norm, then recompute the field given the new spin prima, then apply the corrector step,

$$\vec{s}^{\,(t+\Delta t)} = \vec{s}^{\,(t)} + \frac{1}{2}\left[\Delta\vec{s} + \Delta\vec{s}\,'\right]$$

where,

$$\Delta\vec{s}\,' = -\frac{\gamma}{1+\lambda^2}\left[\vec{s}\,'\times\vec{h}'_{eff} + \lambda\vec{s}\,'\times\left(\vec{s}\,'\times\vec{h}'_{eff}\right)\right]$$

And it's worthwhile to remember to normalize the spin after this step as well.