$\gamma - \beta$ Spectrometry of 207 Bi

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$\gamma - \beta$ Spectrometry of ^{207}Bi

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This report is the work done in L'Institut de Physique Nucléaire d'Orsay for Experimental Physics course in M1 General Physics track of Master of Physics, Paris-Sud University, Orsay, France.

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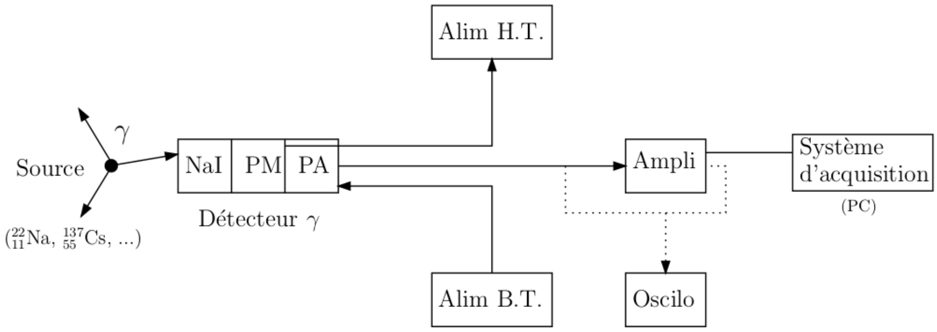
21th December 2019

1 γ Spectrometry

1.1 Experimental Setup and Steps of studying

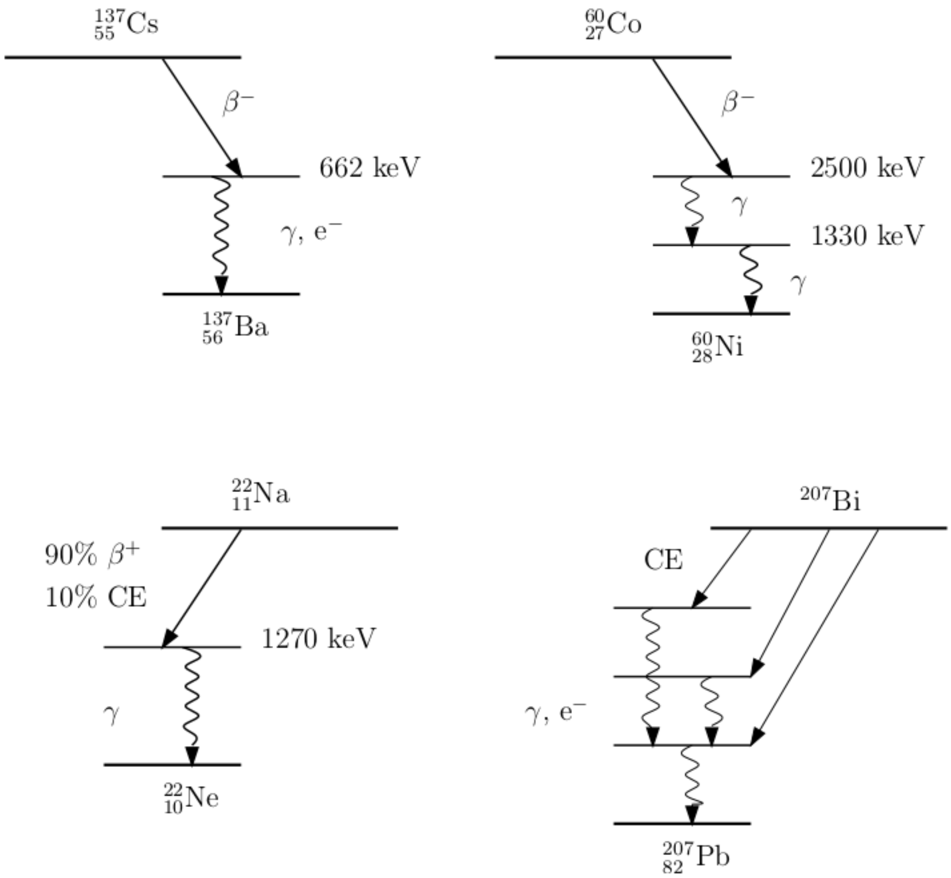
The figure below shows the experimental setup of our detector with osciloscope and system of acquisition.



The way to set it up is written in the manual:
First we have to ensure that the low voltage supply of the PA is present.
We have to check that polarity is correct and set the high voltage.
The connection from detector to PC (system of acquisition) will give us the spectrum and we will observe it in the RHB.
Important: The maximum energy signal must correspond to a maximum positive of 7 volts at the output of the amplifier, to enter the acquisiton system.

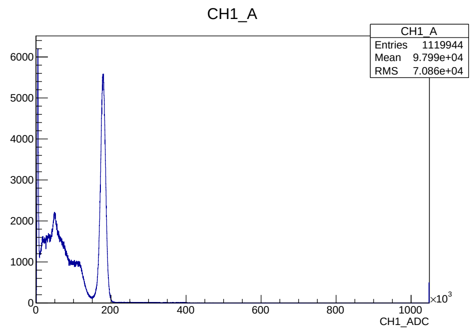
1.2 Study of γ spectra

Below are shown the decay schemes for the different sources.

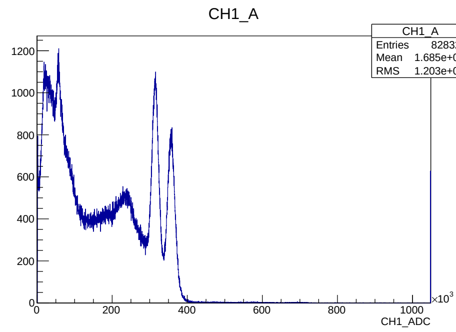


Our goal is to determine the energy of the principal γ of ^{207}Pb* which is the daughter nucleus of ^{207}Bi, and giving the relative intensities of these rays.
The **objective** in this step is to determine the relation between channel and energy of Na, Cs, Co.
Then, one can easily determine intensities for each ray for ^{207}Bi and we can conclude the difference between experimental and theoretical data.

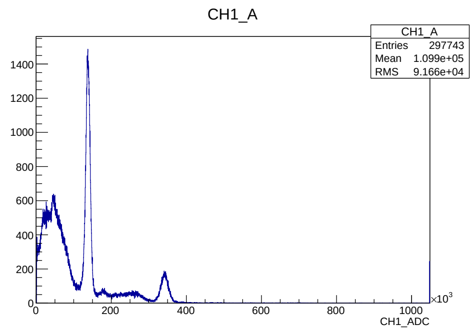
1.3 Sources spectra



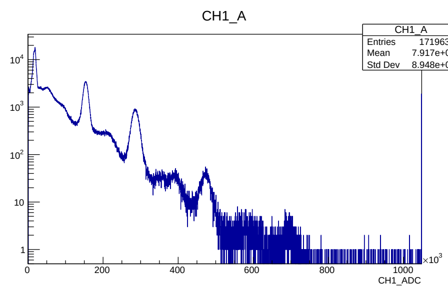
(a) $^{137}_{55}$Cs



(b) $^{60}_{27}$Co



(c) $^{22}_{11}$Na

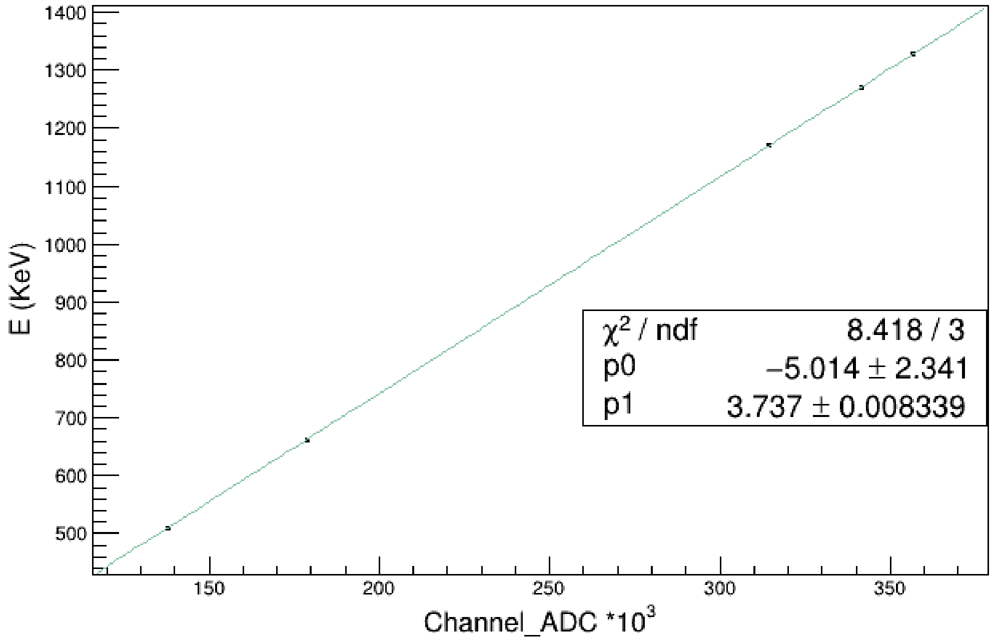


(d) ^{207}Bi

In this section we only have shown the spectrum for each source, and still we have not applied the calibration to find the relation between energy and channel.
The problem here is that ^{207}Bi spectrum should have three peaks, and we decided to present this spectrum in logy scale, so it can be easily seen by naked eye three peaks.

1.4 Calibration Curve

The most important part of this work is to produce the calibration curve, which shows the relation between energy and channel.
This calibration curve is the study of different sources and correlated in linear plot.



From the graph and from data given by Root we can easily find the relation of energy by channel. Its linear: E = A∗X+B
$A = 3.73685 \pm 0.0083$; $B = -5.01436 \pm 2.341$; X=[CHANNEL]
The formula reads as E= 3.73685∗[CHANNEL]−5.01436
This is the formula that we need for finding each peak related by the channel number and finally we can conclude on intensities of ^{207}Bi.
Channel bins will contribute to systematic errors in our experiments.

1.5 Commenting spectrums

Experimental evaluation of peaks, compton edges and backscattering compton for three first sources are:

Elements	Peak1	Peak2	Compton Edge	Backscattering
$^{137}_{55}$Cs	672 KeV	-	435 KeV	185 KeV
$^{60}_{27}$Co	1177 KeV	1338 KeV	895 KeV	215 KeV
$^{22}_{11}$Na	513 KeV	1289 KeV	986 KeV	-

In the spectrum of $^{22}_{11}$Na the first peak which has the value 513 KeV is beacuse of annihilation and it is near the mass of electron.
This comes from β^+: "$e^+ + e^- \rightarrow 2\gamma$" decay which has high probability of happening than electron capture. Sodium decays emitting a positron in stable $^{22}_{11}$Ne.

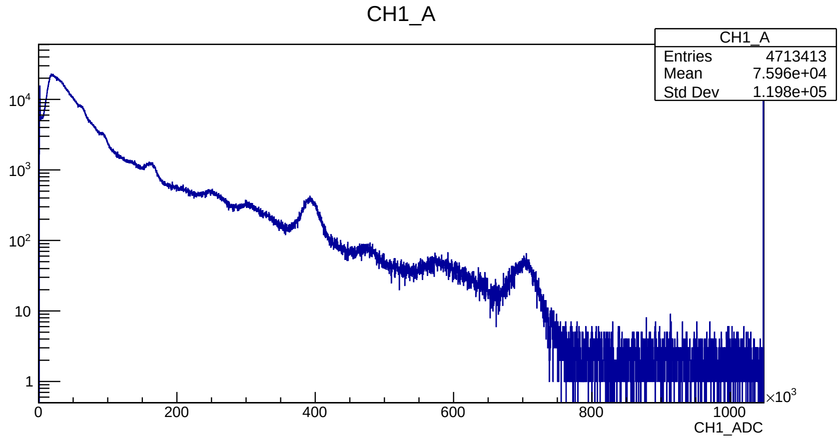
Real Values of Energies:

Elements	Peak1	Peak2	Compton Edge	Backscattering
$^{137}_{55}$Cs	662 KeV	-	477 KeV	185 KeV
$^{60}_{27}$Co	1170 KeV	1330 KeV	960 KeV	209 KeV
$^{22}_{11}$Na	511 KeV	1270 KeV	1057 KeV	-

The statistical error for these three sources are approximately 1% from the real value, for the γ energy and approximately 8% for compton edges.
It can be seen that for the backscattering the error is very small for Cs, and 2% for Co.

1.6 Background spectrum

Here is shown the background spectrum, in logarithmic scale, with one day running and a lot of entries.



It is seen that in spectra appeared three peaks:
1st Peak: 67.9 KeV, **2nd Peak**: 598 Kev, **3rd Peak**: 1464 KeV, **4th Peak**: 2634 KeV
The reasons of these peaks are as follows, first inside laboratory there are radioactive elements which perturbs the measurements of background data and gives us high energy, second, we are bombarded by cosmic rays every seconds, third, in our body we have radioactive elements(**potassium 40 and carbon 14 isotope that comes from foods - they are emitters of electron**) that our detector catches. Our environment is also surrounded by rays that are unknown to us.

1.7 Resolution of NaI (Ti) detector

The definition of energy resolution is given by:

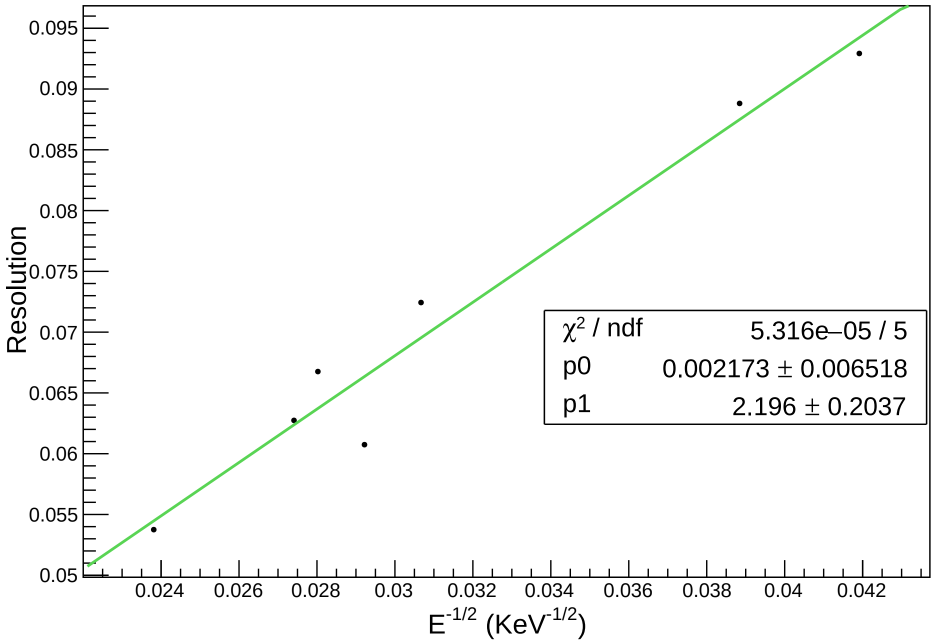
$$R = \frac{\Delta E}{E}$$

ΔE is the Full Width and Half Max of the ditribution.
For a gaussian distribution we write:

$$\Delta E = 2\sigma\sqrt{2\ln 2} = 2.35\sigma = 2.35\sqrt{N_{e^-}}$$

The energy resolution will go like:

$$R(E) = \frac{\Delta E}{E} \approx \frac{C\sqrt{N}}{N} \approx \frac{C}{\sqrt{N}} \approx \frac{C}{\sqrt{E}}$$



The formula for energy resolution will now read as:

$$R(E) = p_0 - \frac{p_1}{\sqrt{E}}$$

The value of energy resolution at 1 MeV is $R(1000KeV) = 0.071617$

1.8 Efficiency

Definition of relative efficiency for a given detector reads as:

$$\frac{\epsilon(E_\gamma^A)}{\epsilon(E_\gamma^B)} = (\frac{E_\gamma^A}{E_\gamma^B})^\alpha$$

where α is a coeffiecient that depends on the detector.
Photons in the NaI are detected through their interaction with matter in either photoelectric effect or in compton scattering. As a result, they are not detected direcly. The number of produced electrons indicates the number of incident photons. Since not all produced electrons maybe measured in the detector, the detection efficiency is not 100 %.
We first shall calculate the efficiency for $^{22}_{11}$Na for the reason that on this element we can calculate from β^+ decay the number of γ emitted:

$$\frac{\epsilon_1}{\epsilon_2} = (\frac{E_1}{E_2})^\alpha$$

$$\frac{\epsilon_1}{\epsilon_2} = \frac{N_\gamma^{(1)} detected/N_\gamma^{(1)} emitted}{N_\gamma^{(2)} detected/N_\gamma^{(2)} emitted}$$

since from β^+ decay we have only 90% and we write as follows:

$$N_{emitted}^{(2)} = 2*(\frac{90}{100})*N_{emitted}^{(1)} = 1.8N_{emitted}^{(1)}$$

which gives:

$$\frac{\epsilon_1}{\epsilon_2} = 1.8\frac{N_{detected}^{(1)}}{N_{detected}^{(2)}}$$

Now the crucial point that gives us the efficiency is as follows:

$$\frac{\epsilon_1}{\epsilon_2} = 1.8\sqrt{2\pi}\frac{(\sigma_1)(P_0)_1}{(\sigma_2)(P_0)_2} = (\frac{E_1}{E_2})^\alpha$$

and now we have all σ_1, $(P_0)_1$, σ_2, $(P_0)_2$ these values comes from fits and for energies we have the real values E_1 = 1270 KeV and E_2 = 511 KeV.
Now we now how to calculate α:

$$\alpha = \frac{\ln\left(1.8\sqrt{2\pi}\frac{(\sigma_1)(P_0)_1}{(\sigma_2)(P_0)_2}\right)}{\ln\left(\frac{E_1}{E_2}\right)}$$

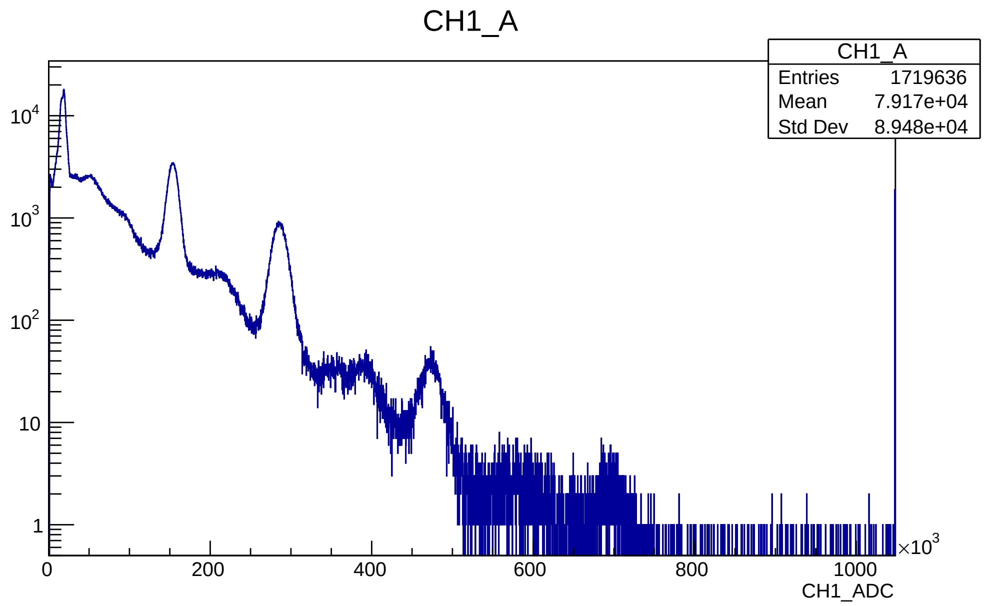
and the error itself by variation:

$$\delta\alpha = \frac{[1.8\sqrt{2\pi}\frac{\delta\sigma_1}{\sigma_1} + \frac{\delta\sigma_2}{\sigma_2} - \frac{\delta(P_0)_1}{(P_0)_1} - \frac{\delta(P_0)_2}{(P_0)2}]\ln\left(\frac{E_1}{E_2}\right) - [\frac{\delta E_1}{E_1} - \frac{\delta E_2}{E_2}]\ln\left(1.8\sqrt{2\pi}\frac{\sigma_1(P_0)_1}{\sigma_2(P_0)_2}\right)}{\ln^2(\frac{E_1}{E_2})}$$

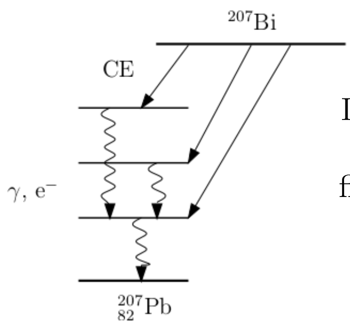
Our result is α = **-1.11473 ± 0.0086031**.

1.9 Study of $^{207}_{82}$Pb* spectrum

In this section we are going to study the energy spectrum of γ emitted by the ^{207}Bi. Below the spectrum is presented in logy scale for reasons of clarity of spectrum.



The energy values of ^{207}Bi are as follows:
1st Peak: 569.194 KeV, **2nd Peak**: 1063.1 KeV, **3rd Peak**: 1763.58 KeV
1st Compton edge: 345 KeV, **2nd Compton edge**: 761 KeV, **3rd Compton edge**: 1467 KeV
We expect for the backscattering peaks the energies **229 KeV, 301 KeV and 307 KeV**. The channel that we expect that the most probable peak of backscattering is $76.004 \cdot 10^3$.
The peak near zero channel number is from the background spectrum.



In this diagram that shows energy levels we only have electronic capture and we should find the branching ratios from the efficiency that we determined before.

1.10 Branching ratios

In our given text of practical work it says " The branching ratios to the ^{207}Bi decay toward the excited level of $^{207}_{82}$Pb* are given as follows: 10% for the lowest level, 83% for the intermediate level and 7% for the highest level in energy."
Our task in this subsection is to verify this sentence by applying our experimental values found in laboratory.
We shall define the formula to use for these branching ratios:

$$B(n) * 100\% = \frac{N_\gamma^{(n)} detected}{N_\gamma^{(3)} detected}(\frac{E_3}{E_n})^\alpha = \frac{\sigma_n (P_0)_n}{\sigma_3 (P_0)_3}(\frac{E_3}{E_n})^\alpha$$

and for error we apply the same rule as applied before.
The branching ratios for n=1, B(1) = 0.0675695 and for n=2, B(2) = 0.782755, and their error respectively ±0.034721, ±(-0.499902)..
Now the exact values of branching ratios in percentage reads as:

B(1)% = 6.75695 % ± 0.34721
B(2)% = 78.2755 % ∓ 4.99902
B(3)% = 14.96755 % ± 5.34623

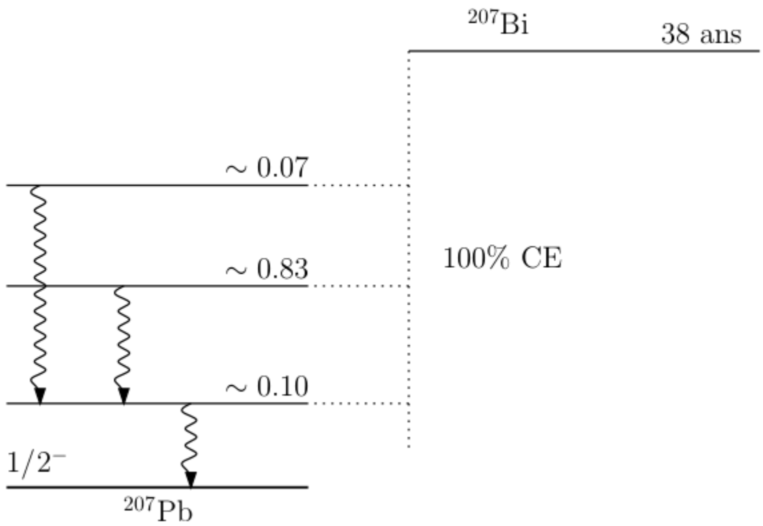
With these values we now can confirm that for the lowest level that was 10% our result is 14.9% with the error, the intermediate level for us is 78% and for the highest level it is 6.7%. In these calculations we only have included the relative errors from the real values.
There are also systematic error and they can not be calculated precisely or at all.This is the reason why we get such values and such errors.

2 β Spectrometry

The experiment uses junction detector which is a sensitive to electrons and charged particles.
Objective: Complement γ spectroscopy by observing electrons coming from the internal conversion process competing with γ emission.
We shall define new coefficient α_i as probability of emitting an electron from the layer i relatively to the emission of a γ from the nuclei:

$$\alpha_i = \frac{N_e}{N_\gamma}$$

The measurements of the partial conversion coefficients α_K and α_L allows to find spins and parities of the concerned levels.



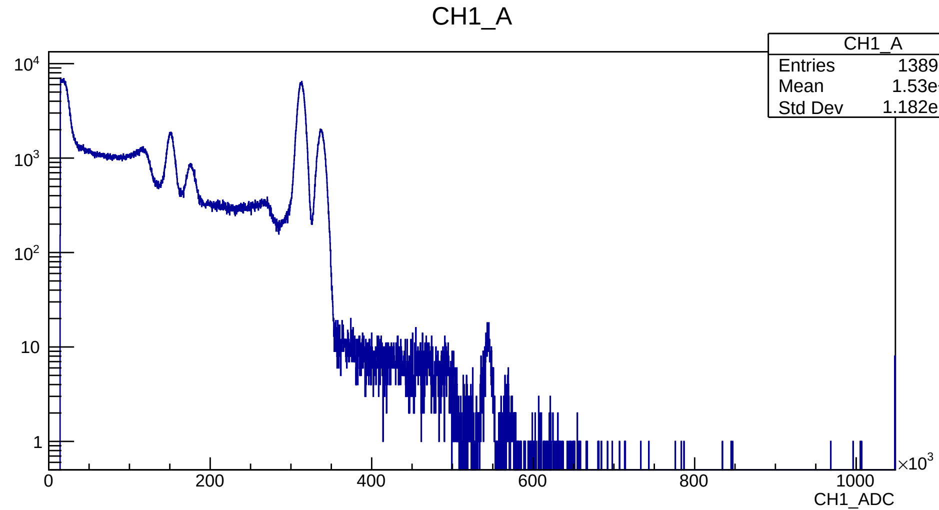
We shall get the results of previous work to complement this experiment.
Important quantity:

$$R(E, XL) = \frac{\alpha_K}{(\alpha_{L_I} + \alpha_{L_{II}} + \alpha_{L_{III}})}$$

Where E is the energy of transition, XL the multipolarity of the transition, with X=E for an electric transition or M for a magnetic transition
and L = 1,2,3 ..

2.1 Study of $^{207}_{82}$Pb* spectrum

At the begining of the 3rd day we started to record the ^{207}Bi spectrum, and we observed the spectrum indicated below. In this spectrum we see lots of peaks which is our objective to study.



Why we see these peaks? The reason behind is that in this spectrum that is produced we have recorded internal conversion.
In this spectrum we observe doublets, each two corresponds to one multipolarity nuclear transition. In the range of channel number [100-200]10^3 we have K and L peaks respectively from left to right corresponding to nuclear transition 570 KeV, and another two peak in the range of channel number [300-400]10^3 we have K and L peaks respectively from left to right corresponding to nuclear transition 1063 KeV. And finally K shell peak corresponding to E = 1763.58 KeV in the range of channel numbers [500-600]10^3.
The energy of each peak is $T = E_\gamma - E_B$ the energy of β particles where E_γ is the corresponding energy of nuclear transition and E_B is the binding energy of the electron in each shell that will go in internal conversion process.
In β spectra, we do not have enviromental background since our detector and source were in an Al box that can stop electron from the outside environment.
In this spectra we have "background" for β peaks which may correspond to K or L internal conversion electrons with high energies that did not lose all of its energy at detector at once, and it may contribute to lower energies peaks.

Previously we studied branching ratios and we know for the lowest level 10%, 83% for the intermediate level and here in the graph we see that the range in channel for the intermmediate level is roughly from [300 - 400]10^3. For lowest level 10% we are in the range of [100-200]10^3 channel number. The highest level in energy, where we have branching ratio 7% corresponds to channel [500-600]10^3, with energy 1764 KeV for the peak. The peak near 0 channel number seem to corresponds to the amplifier signal itself because it does exists anytime and we also observe in γ spectrum as in part 1, in background in part 1, and in this experiment.
Next step is to calculate

$$\frac{\alpha_K}{\alpha_L} = \frac{(N_e)_K emitted}{(N_e)_L emitted} = \frac{(N_e)_K detected}{(N_e)_L detected}$$

Now we have to give $\frac{\alpha_K}{\alpha_L}$ ratios and their uncertainties for energies, 0.57 MeV, and 1.06 MeV.
First calculation for 0.57 MeV:

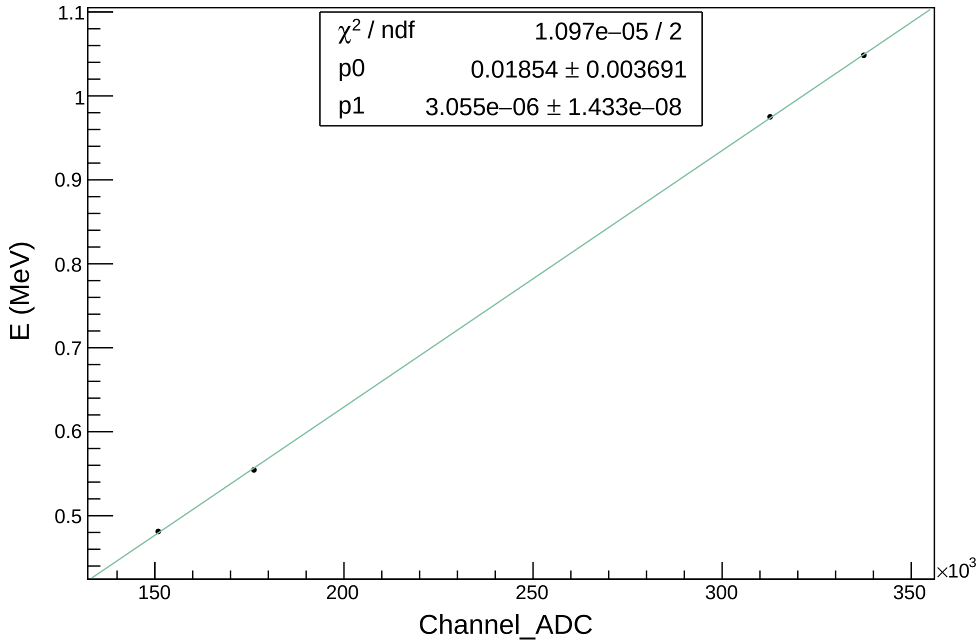
$$\frac{\alpha_K}{\alpha_L} = 2.55846 \pm 0.000924613$$

and second calculation for 1.06 MeV:

$$\frac{\alpha_K}{\alpha_L} = 3.09057 \pm 0.0100149$$

2.2 Calibration curve

The crucial point in this second part of practical work is to find the calibration curve, meaning, the relation of energy to channel which is very important because this is how we calculate the energy on peaks.



Now we can easily express energy as:

$$E = 3.055 * 10^{-6} * CHANNEL - 0.01854$$

2.3 Resolution of Si detector

When we do the measurements with our detectors they have systematic errors that we should consider before we start next steps and also they contribute in the total errors we make.
We shall now compare the resolution of NaI doped with thallium (Ti) and second detector of Si type:
Using E2 = 1063.1 KeV and ΔE2 77.0058 KeV, $R_{NaI} = \frac{\Delta E}{E} = 0.0724354$, 7.24%.
Using E2 = 975.85 KeV, and ΔE2 30.2496 KeV, $R_{Si} = \frac{\Delta E}{E} = 0.0310226$, 3.1%.

2.4 Internal Conversion Coefficients and Energies

At first we have to know how we can determine energies per given electrical or magnetic multipolarity, and we used two formulas:

$$\alpha(EL) \cong (\frac{Z}{n})^3(\frac{L}{L+1})(\frac{e^2}{4\pi\varepsilon_0\hbar c})^4(\frac{2m_e c^2}{E})^{L+\frac{5}{2}} \tag{1}$$

$$\alpha(ML) \cong (\frac{Z}{n})^3(\frac{e^2}{4\pi\varepsilon_0\hbar c})^4(\frac{2m_e c^2}{E})^{L+\frac{3}{2}} \tag{2}$$

Now we need to know what XL transition correspond to, and first trying for energies 570, and 1064 KeV and use their ratios, assuming 100% efficiency we can say that the number of detected electrons in K shell are the same as the number of emitted electron in K shell, one can write:

$$\frac{\alpha_K^{570}}{\alpha_K^{1064}} = \frac{(N_e)_K detected}{(N_e)_K detected} \cdot \frac{(N_\gamma^{1067}) emitted}{(N_\gamma^{570}) emitted}$$

and we have the values of first term which are the integrals of gaussian per peak and the second term comes form relative intensity that we calculated previously, branching ratios, and here we have the branching ratio of intermmediate level which is 78.3%, we now are able to calculate the ratio:

$$\frac{\alpha_K^{570}}{\alpha_K^{1064}} = \frac{1.4 \cdot 10^7}{6.36 \cdot 10^7} \cdot 0.783 = 0.174$$

$$\alpha_K^{1064} = \frac{\alpha_K^{570}}{0.174}$$

we find for $\frac{E_\gamma}{511} = \frac{570}{511} = 1.11546$ and from equation (1) we find coefficient taking the first transition E2 $\alpha_K^{570}(E2) \cong 0.0144$ and we were expecting 0.0182. From α_K^{1067} and α_K^{570} above relation we get $\alpha_K^{1064} = 0.0862$ and from α_K plot, it corresponds to M4.
For the third level, highest energy 1764 KeV, with branching ratio 6.77%
The ratio reads as follows:

$$\frac{\alpha_K^{1764}}{\alpha_K^{570}} = \frac{1.1 \cdot 10^5}{1.4 \cdot 10^7} \cdot \frac{1}{0.06757} = 0.120246$$

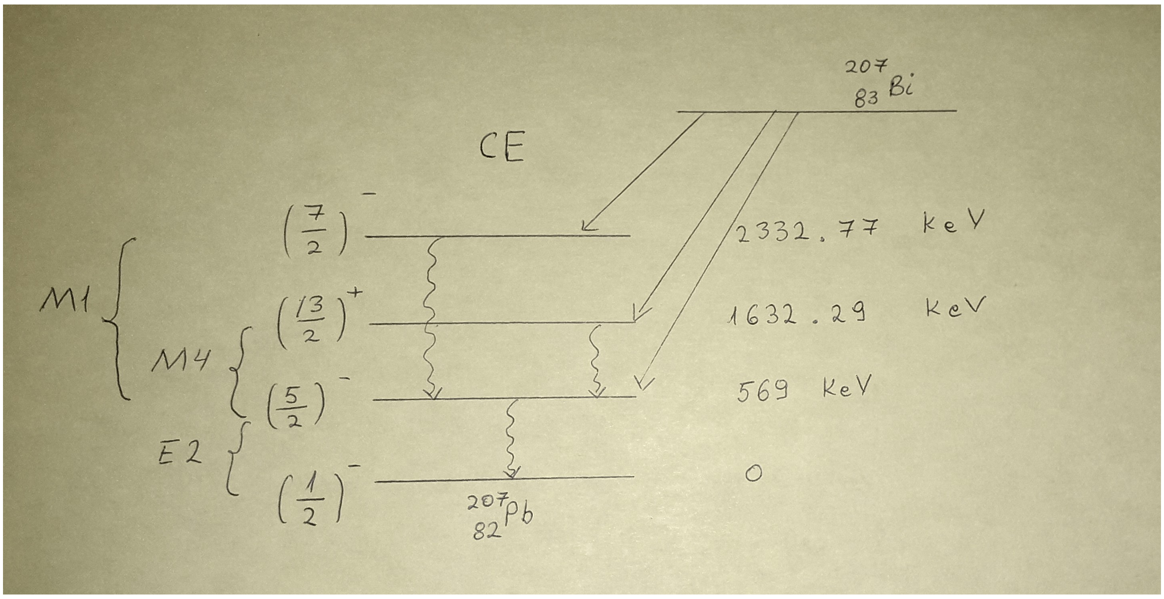
and $\alpha_K^{1764} = 0.0018$, and the expected value is 0.03421 which corresponds to M1 multipolarity[3].

2.5 Total Angular Momentum and Parity

The angular momentum l carried by photon, called multipolarity of the transition, must satisfy $\vec{l} = \vec{I_i} + \vec{I_f}$, corresponding to:

$$|I_i - I_f| \leq l \leq |I_i + I_f|$$

and for given multipolarity, we have parities π_i and π_f.
The product is $\Delta\pi = \pi_i \cdot \pi_f$, and we can now determine parities:
Electric transition, $\Delta\pi = (-1)^l$
Magnetic transition, $\Delta\pi = (-1)^{l+1}$
For example, for E = 569 KeV the corresponding transition is E2 (l=2) this means that $\Delta\pi = (-1)^2 = (+)1$ then the initial level parity is the same as the final level parity which is (-) for this transition, and the initial angular momentum is $I_i = l + I_f = 2 + \frac{1}{2} = \frac{5}{2}$ since the lowest value of l is the preferred one.
The same procedure continues for other transitions.
These is the final scheme of decay which contains transitions, parity.



From both γ and β spectrometry collected information, we were able to deduce the energy, total angular momentum and parities of $^{207}_{82}$Pb levels.

References

[1] Caroline Gaulard *Practical Work: Spectrometry of* ^{207}Bi .

[2] Kenneth S. Krane *Introductory Nuclear Physics*, 1987

[3] *Table of radionuclides*
`https://www.bipm.org/utils/common/pdf/monographieRI/Monographie_BIPM-5_Comments_Vol1-6.pdf`

Other Works Florian Millo:

[4] Observation of high-energy astrophysical neutrinos in three years of IceCube Data **Florian Millo**

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