Hyperbolic cosine and arctan double angle reductions. Probably for cosh parameterization of an ellipse.

```
$Assumptions = m > 0 && phi > 0 && b > 0 && b < 1 && a > 0 && a > b;
Cosh[m + I phi] Cosh[m - I phi] // TrigToExp // FullSimplify
```

$$\frac{1}{2}(\text{Cos}[2\,\text{phi}] + \text{Cosh}[2\,\text{m}])$$

```
$Assumptions = b > 0 && b < 1 && a > 0 && a > b;
(1 - (b / a) ^ 2) Cosh[2 ArcTanh[b / a]] // FullSimplify
```

$$1 + \frac{b^2}{a^2}$$

```
Cosh[2 ArcTanh[b / a]]
```

```
1 + Cos[2 m] // FullSimplify
```

$$\text{Cosh}\left[2\,\text{ArcTanh}\left[\frac{b}{a}\right]\right]$$

$$2\,\text{Cos}[m]^2$$

```
1 - 2 Cos[m] Cos[m] // FullSimplify
```

$$-\text{Cos}[2\,m]$$