

This is the notebook for a few bivector related illustrations. The first is two circular representations of a bivector viewed from a 3D vantage point. Another is a bivector addition figure, with two bivectors summed in 3D. That figure was confusing (but cool), and has been left out of the book. The last figure separates the space between those bivectors summed in the second figure showing the summands and the sum all distinct. The current draft of the book includes this figure, but it is still a bit confusing.

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<< peeters` ;
peeters`setGitDir[ "../project/figures/GAelectrodynamics" ]
/Users/pjoot/project/figures/GAelectrodynamics

ClearAll[circle, e1, e2, e3, o, bt];
o = {0, 0, 0};
{e1, e2, e3} = IdentityMatrix[3];
circle[u_, v_, r_, t_] := r (u Cos[t] + v Sin[t]);
bt = Style[#, FontSize → 16] &;

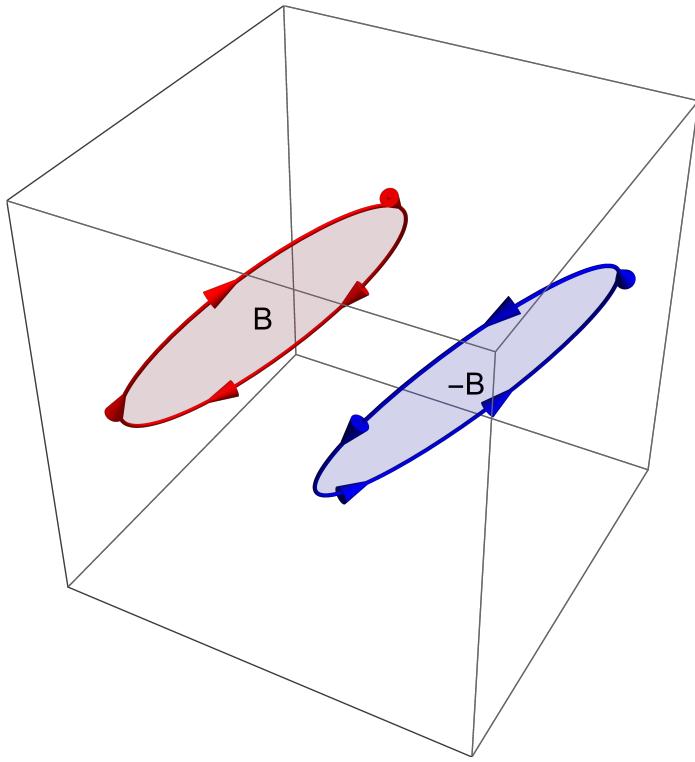
p1 = Module[{asz, ucap1, vcap1, ucap2, vcap2, rho, range},
  asz = 0.05;
  rho = 1;
  ucap1 = {1, 1, 1};
  vcap1 = Cross[ucap1, e1] // Normalize;
  ucap2 = ucap1;
  vcap2 = -vcap1;
  range = 2.2 {-rho, rho};
  {ParametricPlot3D[
    -1.2 e1 + circle[ucap1, vcap1, rho, t], {t, 0, 2 π},
    PlotStyle → Directive[Thick, Red],
    PlotRange → {range, range, range},
    Ticks → None
  ] /. Line[pts_] →
    {Arrowheads[{asz, asz, asz, asz, asz, 0}], Arrow[Tube[pts, {0.02}], {0, .2}]}
  ,
  ParametricPlot3D[
    -1.2 e1 + circle[ucap1, vcap1, r, t], {t, 0, 2 π}, {r, 0, rho},
    PlotStyle → Directive[Red // Lighter, Opacity[0.2]],
    Mesh → None
  ]
  ,
  ParametricPlot3D[
    e1 + circle[ucap2, vcap2, rho, t], {t, 0, 2 π},
    PlotStyle → Directive[Thick, Blue]
  ] /. Line[pts_] →
    {Arrowheads[{asz, asz, asz, asz, asz, 0}], Arrow[Tube[pts, {0.02}], {0, .2}]},
  ParametricPlot3D[

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e1 + circle[uCap2, vCap2, r, t], {t, 0, 2π}, {r, 0, rho},
PlotStyle → Directive[Blue // Lighter, Opacity[0.2]],
Mesh → None
],
Graphics3D[
  Text[bt["B"], -1.2 e1],
  Text[bt["-B"], e1]
]
}
] // Show

```



```

peeters`exportForLatex["circularBivectorsIn3DFig1", p1]
{circularBivectorsIn3DFig1.eps, circularBivectorsIn3DFig1pn.png}

ClearAll[step, parallelogram, parallelogramarea, p2]
step[x_] := HeavisidePi[x - 1/2];
(*Plot[step[x],{x,-2,2}]
 Plot[step[x-1],{x,-2,2}]*)
(*parameterized over [0,4]*)
parallelogram[u_, v_, t_] := u t step[t] + step[t - 1] (u + (t - 1) v) +
  step[t - 2] (u + v - u (t - 2)) + step[t - 3] (v - v (t - 3));
(*parameterized over [0,1]*)
parallelogramarea[u_, v_, p1_, p2_] := u p1 + v p2;

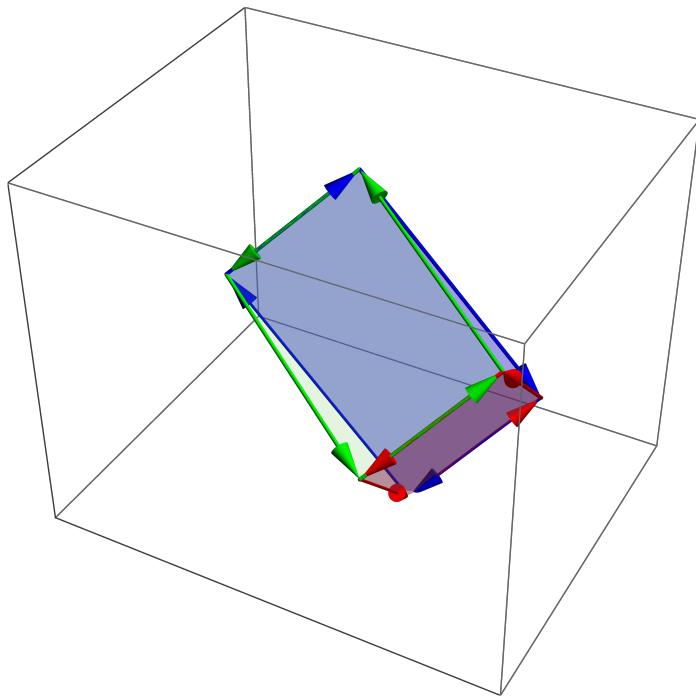
p2 = Module[{asz, u1, v1, u2, v2, us, vs, range, origin},

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asz = 0.05;
u1 = {1, 1, 1};
v1 = 1.5 (Cross[u1, e1] // Normalize);
u2 = u1;
v2 = 3 (Cross[u1, e2] // Normalize);
us = u1;
vs = v1 + v2;
origin = -(u1 + u2) / 2;
range = 2 {-1, 1};
{ParametricPlot3D[
  origin + parallelogram[v1, u1, t], {t, 0, 4},
  PlotStyle -> Directive[Thick, Red],
  PlotRange -> {{-4, 1}, range, range},
  Ticks -> None
] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
ParametricPlot3D[
  origin + parallelogramarea[u1, v1, a, b], {a, 0, 1}, {b, 0, 1},
  PlotStyle -> Directive[Red // Lighter, Opacity[0.5]],
  Mesh -> None
],
ParametricPlot3D[
  origin + v1 + parallelogram[v2, u2, t], {t, 0, 4},
  PlotStyle -> Directive[Thick, Blue]
] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
ParametricPlot3D[
  origin + v1 + parallelogramarea[u2, v2, a, b], {a, 0, 1}, {b, 0, 1},
  PlotStyle -> Directive[Blue // Lighter, Opacity[0.5]],
  Mesh -> None
],
ParametricPlot3D[
  origin + parallelogram[us, vs, t], {t, 0, 4},
  PlotStyle -> Directive[Thick, Green]
] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
ParametricPlot3D[
  origin + parallelogramarea[us, vs, a, b], {a, 0, 1}, {b, 0, 1},
  PlotStyle -> Directive[Green // Lighter, Opacity[0.15]],
  Mesh -> None
]
}
] // Show

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```

peeters`exportForLatex["AdditionOfBivectorsFig1", p2]
{AdditionOfBivectorsFig1.eps, AdditionOfBivectorsFig1pn.png}

p2a = Module[{asz, u1, v1, u2, v2, us, vs, range, origin},
  asz = 0.05;
  u1 = {1, 1, 1};
  v1 = 1.5 (Cross[u1, e1] // Normalize);
  u2 = u1;
  v2 = 3 (Cross[u1, e2] // Normalize);
  us = u1;
  vs = v1 + v2;
  origin = -(u1 + u2) / 2;
  range = 2 {-1, 1};
  {ParametricPlot3D[
    origin + parallelogram[v1, u1, t], {t, 0, 4},
    PlotStyle -> Directive[Thick, Red],
    PlotRange -> {{-4, 1}, range, range},
    Ticks -> None
  ] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
  ParametricPlot3D[
    origin + parallelogramarea[u1, v1, a, b], {a, 0, 1}, {b, 0, 1},
    PlotStyle -> Directive[Red // Lighter, Opacity[0.5]],
    Mesh -> None
  ]
}

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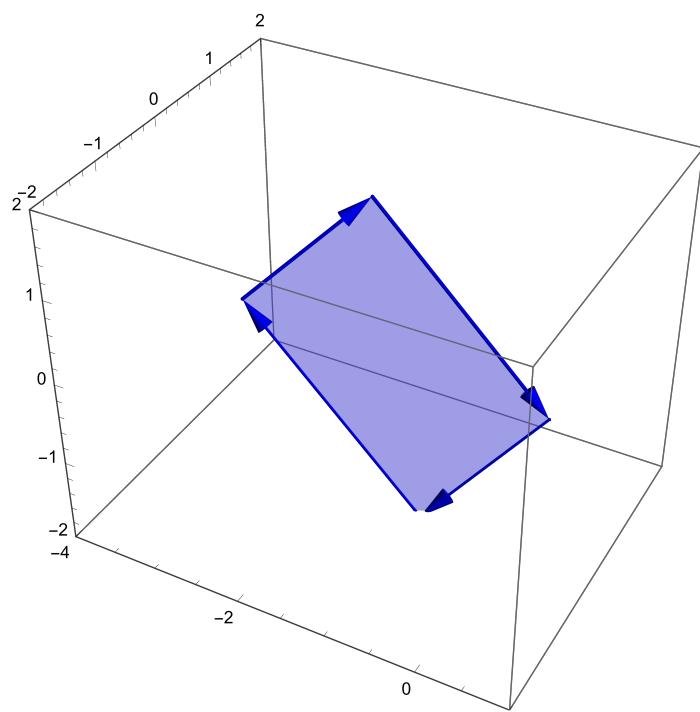
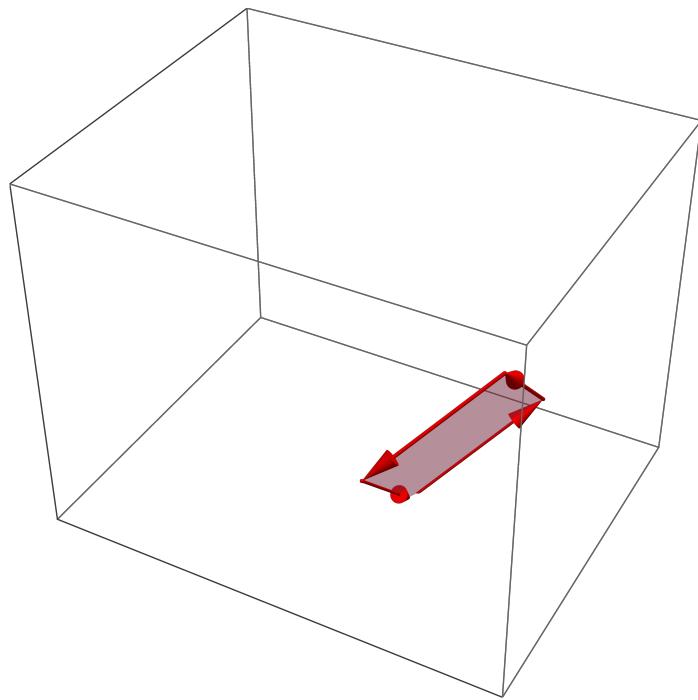
] // Show

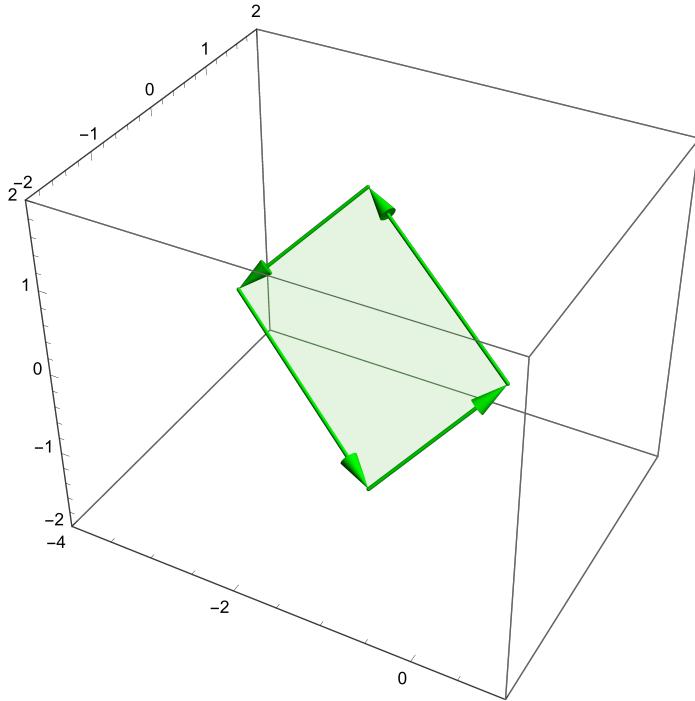
p2b = Module[{asz, u1, v1, u2, v2, us, vs, range, origin},
  asz = 0.05;
  u1 = {1, 1, 1};
  v1 = 1.5 (Cross[u1, e1] // Normalize);
  u2 = u1;
  v2 = 3 (Cross[u1, e2] // Normalize);
  us = u1;
  vs = v1 + v2;
  origin = -(u1 + u2) / 2;
  range = 2 {-1, 1};
{
  ParametricPlot3D[
    origin + v1 + parallelogram[v2, u2, t], {t, 0, 4},
    PlotStyle -> Directive[Thick, Blue],
    PlotRange -> {{-4, 1}, range, range}
  ] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
  ParametricPlot3D[
    origin + v1 + parallelogramarea[u2, v2, a, b], {a, 0, 1}, {b, 0, 1},
    PlotStyle -> Directive[Blue // Lighter, Opacity[0.5]],
    Mesh -> None
  ]
}
] // Show

p3b = Module[{asz, u1, v1, u2, v2, us, vs, range, origin},
  asz = 0.05;
  u1 = {1, 1, 1};
  v1 = 1.5 (Cross[u1, e1] // Normalize);
  u2 = u1;
  v2 = 3 (Cross[u1, e2] // Normalize);
  us = u1;
  vs = v1 + v2;
  origin = -(u1 + u2) / 2;
  range = 2 {-1, 1};
{
  ParametricPlot3D[
    origin + parallelogram[us, vs, t], {t, 0, 4},
    PlotStyle -> Directive[Thick, Green],
    PlotRange -> {{-4, 1}, range, range}
  ] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
  ParametricPlot3D[
    origin + parallelogramarea[us, vs, a, b], {a, 0, 1}, {b, 0, 1},
    PlotStyle -> Directive[Green // Lighter, Opacity[0.5]],
    PlotRange -> {{-4, 1}, range, range}
  ]
}
] // Show

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PlotStyle -> Directive[Green // Lighter, Opacity[0.15]],  
Mesh -> None  
]  
}  
] // Show
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p2 = Module[{asz, u1, v1, u2, v2, us, vs, range, origin, o2},
  asz = 0.05;
  u1 = {1, 1, 1};
  v1 = 1.5 (Cross[u1, e1] // Normalize);
  u2 = u1;
  v2 = 3 (Cross[u1, e2] // Normalize);
  us = u1;
  vs = v1 + v2;
  origin = -(u1 + u2) / 2;
  range = {-3, 3};
  o2 = e1 - e3 - e2;
  {
    ParametricPlot3D[
      origin + parallelogram[v1, u1, t], {t, 0, 4},
      PlotStyle -> Directive[Thick, Red],
      PlotRange -> {1.5 {-1, 4}, {-4, 6}, {-4, 3}},
      Ticks -> None
    ] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
    ParametricPlot3D[
      origin + parallelogramarea[u1, v1, a, b], {a, 0, 1}, {b, 0, 1},
      PlotStyle -> Directive[Red // Lighter, Opacity[0.5]],
      Mesh -> None
    ],
    ParametricPlot3D[
      2 (e1 + e2) + origin + v1 + parallelogram[v2, u2, t], {t, 0, 4},

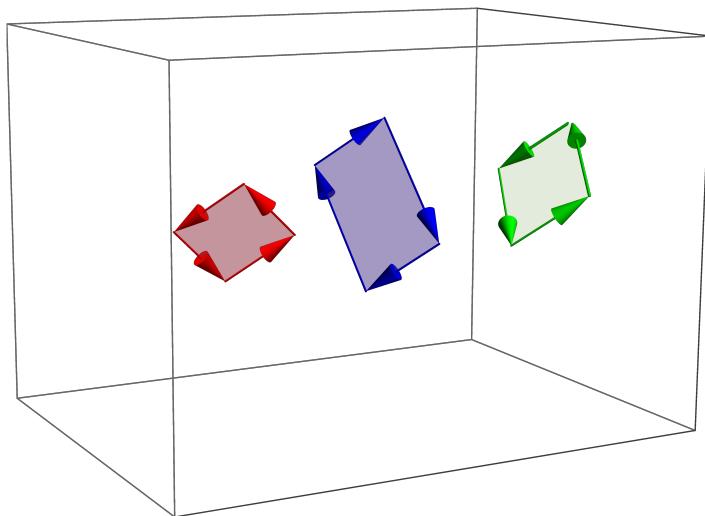
```

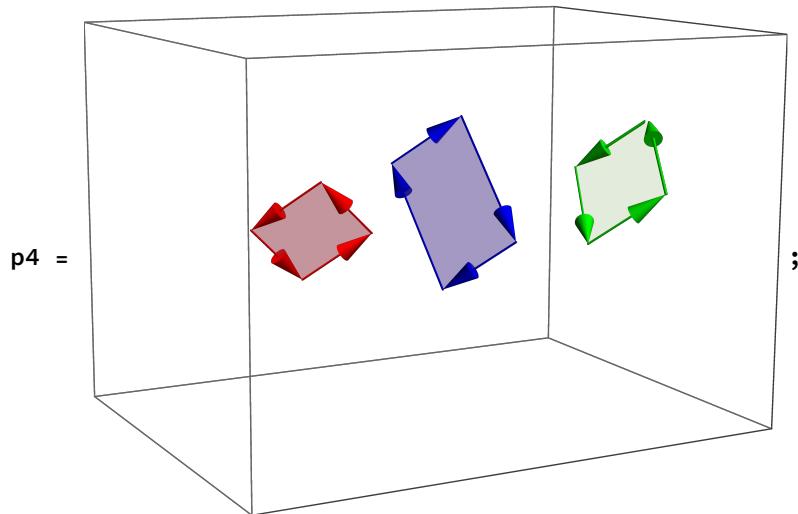
```

    PlotStyle -> Directive[Thick, Blue]
  ] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
ParametricPlot3D[

  2 (e1 + e2) + origin + v1 + parallelogramarea[u2, v2, a, b], {a, 0, 1}, {b, 0, 1},
PlotStyle -> Directive[Blue // Lighter, Opacity[0.5]],
Mesh -> None
] ,
ParametricPlot3D[
  4 e1 + 5 e2 + origin + parallelogram[us, vs, t], {t, 0, 4},
PlotStyle -> Directive[Thick, Green]
] /. Line[pts_] -> {Arrowheads[{asz}], Arrow[Tube[pts, {0.02}], {0, 0}]},
ParametricPlot3D[
  4 e1 + 5 e2 + origin + parallelogramarea[us, vs, a, b], {a, 0, 1}, {b, 0, 1},
PlotStyle -> Directive[Green // Lighter, Opacity[0.15]],
Mesh -> None
] (*,
Graphics3D[ {
  (*Arrow[Tube[{o,e1}]] ,
  Arrow[Tube[{o,e2}]],
  Arrow[Tube[{o,e3}]]*)
  Arrow[{o2,e1}] ,
  Arrow[{o2,e2}],
  Arrow[{o2,e3}]
 }]*)
} // Show
]

```





```
p4 =  
peeters`exportForLatex["AdditionOfBivectorsFig2", p4]  
{AdditionOfBivectorsFig2.eps, AdditionOfBivectorsFig2pn.png}
```