Elliptic integrals for charge/current distribution on a ring.

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$Assumptions = beta ≥ 0 && alpha ∈ Reals && (1 - beta) ^2 + alpha ^2 ≠ 0;

i1 = Integrate[ ( 1 + 4 beta Sin[u] ^2 / ((1 - beta) ^2 + alpha ^2)) ^ (-3 / 2), {u, 0, Pi}]
```

$$\left(2\sqrt{\text{alpha}^2+(-1+\text{beta})^2}\left(\sqrt{\text{alpha}^4+\left(-1+\text{beta}^2\right)^2+2\,\text{alpha}^2\left(1+\text{beta}^2\right)}\left(-i\,\text{EllipticE}\left[1+\frac{4\,\text{beta}}{\text{alpha}^2+(-1+\text{beta})^2}\right]+\text{EllipticK}\left[-\frac{4\,\text{beta}}{\text{alpha}^2+(-1+\text{beta})^2}\right]\right)+i\left(\left(\text{alpha}^2+(1+\text{beta})^2\right)\text{EllipticE}\left[1-\frac{4\,\text{beta}}{\text{alpha}^2+(1+\text{beta})^2}\right]-4\,\text{beta}\,\text{EllipticK}\left[1-\frac{4\,\text{beta}}{\text{alpha}^2+(1+\text{beta})^2}\right]\right)\right)\right)\Big/\left(\text{alpha}^2+(1+\text{beta})^2\right)^{3/2}$$

```
((i1/ (alpha^2 + (1 - beta)^2)^(3/2) / 2) // FullSimplify) /.
  {alpha → α, beta → β} // TraditionalForm
```

$$\left(i\left(\left(\alpha^2+(\beta+1)^2\right)E\left(1-\frac{4\beta}{\alpha^2+(\beta+1)^2}\right)-4\beta K\left(1-\frac{4\beta}{\alpha^2+(\beta+1)^2}\right)\right)+\sqrt{\alpha^4+2\alpha^2\left(\beta^2+1\right)+\left(\beta^2-1\right)^2}\left(K\left(-\frac{4\beta}{\alpha^2+(\beta-1)^2}\right)-i\,E\left(\frac{4\beta}{\alpha^2+(\beta-1)^2}+1\right)\right)\right)\Big/\left(\left(\alpha^2+(\beta-1)^2\right)\left(\alpha^2+(\beta+1)^2\right)^{3/2}\right)$$

```
(Integrate[ ( 1 + 4 beta Sin[u] ^2 / ((1 - beta) ^2 + alpha ^2)) ^ (-3 / 2), {u, 0, Pi}] //
    HoldForm) /. {alpha → α, beta → β} // TraditionalForm

(Integrate[ Cos[2 u] ( 1 + 4 beta Sin[u] ^2 / ((1 - beta) ^2 + alpha ^2)) ^ (-3 / 2),
    {u, 0, Pi}] // HoldForm) /. {alpha → α, beta → β} // TraditionalForm
```

$$\int_0^\pi \left(1+\frac{4\beta\sin^2(u)}{(1-\beta)^2+\alpha^2}\right)^{-3/2} du$$

$$\int_0^\pi \cos(2u)\left(1+\frac{4\beta\sin^2(u)}{(1-\beta)^2+\alpha^2}\right)^{-3/2} du$$

```
i2 = Integrate[
  Cos[2 u] ( 1 + 4 beta Sin[u] ^2 / ((1 - beta) ^2 + alpha ^2)) ^ (-3 / 2), {u, 0, Pi}]
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$$\text{ConditionalExpression}\Bigg[\frac{1}{2\,\text{beta}\left(\text{alpha}^2+(1+\text{beta})^2\right)^{3/2}}\sqrt{\text{alpha}^4+\left(-1+\text{beta}^2\right)^2+2\,\text{alpha}^2\left(1+\text{beta}^2\right)}$$
$$\Bigg(\sqrt{\text{alpha}^2+(-1+\text{beta})^2}\left(1+\text{alpha}^2+\text{beta}^2\right)\text{EllipticE}\left[-\frac{4\,\text{beta}}{\text{alpha}^2+(-1+\text{beta})^2}\right]+$$
$$\left(1+\text{alpha}^2+\text{beta}^2\right)\sqrt{\text{alpha}^2+(1+\text{beta})^2}\,\text{EllipticE}\left[\frac{4\,\text{beta}}{\text{alpha}^2+(1+\text{beta})^2}\right]-$$
$$\sqrt{\text{alpha}^2+(-1+\text{beta})^2}\,\text{EllipticK}\left[-\frac{4\,\text{beta}}{\text{alpha}^2+(-1+\text{beta})^2}\right]-$$
$$\text{alpha}^2\sqrt{\text{alpha}^2+(-1+\text{beta})^2}\,\text{EllipticK}\left[-\frac{4\,\text{beta}}{\text{alpha}^2+(-1+\text{beta})^2}\right]-$$
$$2\sqrt{\text{alpha}^2+(-1+\text{beta})^2}\,\text{beta}\,\text{EllipticK}\left[-\frac{4\,\text{beta}}{\text{alpha}^2+(-1+\text{beta})^2}\right]-$$
$$\sqrt{\text{alpha}^2+(-1+\text{beta})^2}\,\text{beta}^2\,\text{EllipticK}\left[-\frac{4\,\text{beta}}{\text{alpha}^2+(-1+\text{beta})^2}\right]-$$
$$\sqrt{\text{alpha}^2+(1+\text{beta})^2}\,\text{EllipticK}\left[\frac{4\,\text{beta}}{\text{alpha}^2+(1+\text{beta})^2}\right]-$$
$$\text{alpha}^2\sqrt{\text{alpha}^2+(1+\text{beta})^2}\,\text{EllipticK}\left[\frac{4\,\text{beta}}{\text{alpha}^2+(1+\text{beta})^2}\right]+$$
$$2\,\text{beta}\sqrt{\text{alpha}^2+(1+\text{beta})^2}\,\text{EllipticK}\left[\frac{4\,\text{beta}}{\text{alpha}^2+(1+\text{beta})^2}\right]-$$
$$\text{beta}^2\sqrt{\text{alpha}^2+(1+\text{beta})^2}\,\text{EllipticK}\left[\frac{4\,\text{beta}}{\text{alpha}^2+(1+\text{beta})^2}\right]\Bigg),\ \text{beta}>0\Bigg]$$

```
i3 = Integrate[ Cos[2 u] ( 1 + alpha ^2 ) ^ (-3 / 2), {u, 0, Pi}]
```

0

```
((i2/ (alpha^2 + (1 - beta)^2)^(3/2) / 2) // FullSimplify) /.
  {alpha → α, beta → β} // TraditionalForm
```

$$\text{ConditionalExpression}\Bigg[-\Bigg(\Bigg(-\sqrt{\alpha^2+(\beta-1)^2}\,(\alpha^2+\beta^2+1)\,E\!\left(-\frac{4\beta}{\alpha^2+(\beta-1)^2}\right)-(\alpha^2+\beta^2+1)\sqrt{\alpha^2+(\beta+1)^2}\,E\!\left(\frac{4\beta}{\alpha^2+(\beta+1)^2}\right)+$$
$$\beta^2\sqrt{\alpha^2+(\beta-1)^2}\,K\!\left(-\frac{4\beta}{\alpha^2+(\beta-1)^2}\right)+\beta^2\sqrt{\alpha^2+(\beta+1)^2}\,K\!\left(\frac{4\beta}{\alpha^2+(\beta+1)^2}\right)+$$
$$\alpha^2\sqrt{\alpha^2+(\beta-1)^2}\,K\!\left(-\frac{4\beta}{\alpha^2+(\beta-1)^2}\right)+\alpha^2\sqrt{\alpha^2+(\beta+1)^2}\,K\!\left(\frac{4\beta}{\alpha^2+(\beta+1)^2}\right)+$$
$$2\beta\sqrt{\alpha^2+(\beta-1)^2}\,K\!\left(-\frac{4\beta}{\alpha^2+(\beta-1)^2}\right)+\sqrt{\alpha^2+(\beta-1)^2}\,K\!\left(-\frac{4\beta}{\alpha^2+(\beta-1)^2}\right)-2\beta\sqrt{\alpha^2+(\beta+1)^2}$$
$$K\!\left(\frac{4\beta}{\alpha^2+(\beta+1)^2}\right)+\sqrt{\alpha^2+(\beta+1)^2}\,K\!\left(\frac{4\beta}{\alpha^2+(\beta+1)^2}\right)\Bigg)\Bigg/\left(4\beta\left(\alpha^4+2\alpha^2(\beta^2+1)+(\beta^2-1)^2\right)\right)\Bigg),\ \beta>0\Bigg]$$