Verify hand calculation from polarization.tex (also set as a problem).

```
ClearAll[ cToVec]

cToVec = {one → e1, Complex[0, 1] → -e2, Complex[0, -1] → e2};
```

## Take II

Got my hand calculation wrong.  Try this with Mathematica instead:

```
ClearAll[alpha1, alpha2, beta1, beta2, alphaR, alphaL, phi, one, e1, e2]
alphaR = alphaR1 + I alphaR2;
alphaL = alphaL1 + I alphaL2;

vecE = (alphaR Exp[I phi] + alphaL Exp[-I phi] // ExpToTrig) // Simplify
```

```
(alphaL1 + i alphaL2 + alphaR1 + i alphaR2) Cos[phi] +
 (-i alphaL1 + alphaL2 + i alphaR1 - alphaR2) Sin[phi]
```

```
((one alphaL1 + i alphaL2 + one alphaR1 + i alphaR2) Cos[phi] +
    (-i alphaL1 + one alphaL2 + i alphaR1 - one alphaR2) Sin[phi] /. cToVec) // Simplify
```

```
(alphaL1 e1 + alphaR1 e1 - (alphaL2 + alphaR2) e2) Cos[phi] +
 (alphaL2 e1 - alphaR2 e1 + (alphaL1 - alphaR1) e2) Sin[phi]
```

```
s = Solve[{alphaL1 + alphaR1 == alpha1, -(alphaL2 + alphaR2) == alpha2,
    alphaL2 - alphaR2 == -beta1, (alphaL1 - alphaR1) == -beta2},
   (*{alpha1, alpha2, beta1, beta2}*)
   {alphaR1, alphaR2, alphaL1, alphaL2}
  ] // First
```

$$\left\{\text{alphaR1} \to \frac{\text{alpha1} + \text{beta2}}{2},\ \text{alphaR2} \to \frac{1}{2}(-\text{alpha2} + \text{beta1}),\ \text{alphaL1} \to \frac{\text{alpha1} - \text{beta2}}{2},\ \text{alphaL2} \to \frac{1}{2}(-\text{alpha2} - \text{beta1})\right\}$$

Now check this solution:

```
alphaR = (alphaR1 + I alphaR2) /. s
alphaL = (alphaL1 + I alphaL2) /. s

(alphaR Exp[I phi] + alphaL Exp[-I phi] // ExpToTrig) // Simplify
```

$$\frac{1}{2} i (-\text{alpha2} + \text{beta1}) + \frac{\text{alpha1} + \text{beta2}}{2}$$

$$\frac{1}{2} i (-\text{alpha2} - \text{beta1}) + \frac{\text{alpha1} - \text{beta2}}{2}$$

$$(\text{alpha1} - i\,\text{alpha2}) \operatorname{Cos}[\text{phi}] - (\text{beta1} - i\,\text{beta2}) \operatorname{Sin}[\text{phi}]$$

Manually change from complex basis to vector

```
((one alpha1 - i alpha2) Cos[phi] - (one beta1 - i beta2) Sin[phi]) /. cToVec
```

$$(\text{alpha1 e1} + \text{alpha2 e2}) \operatorname{Cos}[\text{phi}] - (\text{beta1 e1} + \text{beta2 e2}) \operatorname{Sin}[\text{phi}]$$

Works!