Uses my GA30.m package to compute the values of the energy momentum tensor multivectors, and relate those to the conventional tensor description of the same.

```
<< GA30`;
```

Calculation of components of the energy momentum tensor T(a) = $F\, a\, F^{\dagger}\, \epsilon / 2$

```
ClearAll[e1, e2, e3, i, vecE, vecH,
 one, f, fDagger, vecJ, vecM, j, jDagger, cross]
e1 = Vector[1, 1] ;
e2 = Vector[1, 2];
e3 = Vector[1, 3];
i = e1 ** e2 ** e3;
one = Scalar[1];

vecE = "E"1 e1 + "E"2 e2 + "E"3 e3 ;
vecH = "H"1 e1 + "H"2 e2 + "H"3 e3 ;
vecJ = "J"1 e1 + "J"2 e2 + "J"3 e3 ;
vecM = "M"1 e1 + "M"2 e2 + "M"3 e3 ;
j = η ( c ρ - vecJ ) + i ** ( c ρm - vecM );
jDagger = η ( c ρ - vecJ ) - i ** ( c ρm - vecM );
cross[a_, b_] := -i ** (a ∧ b);

ClearAll[v, vv]
(*t[v_] := ϵ ( vecE ** v ** vecE + vecH ** v ** vecH +
     i ** ( vecH ** v ** vecE - vecE ** v ** vecH))/2*)
f := vecE + i ** vecH (η);
fDagger := vecE - i ** vecH (η);
t[v_] := (ϵ / 2) ( f ** v ** fDagger)
vv = { t[one], t[e1], t[e2], t[e3] };

(*vv[[1]]  // TexForm
  vv[[2]]
  vv[[3]]
  vv[[4]]
*)

Grid[ { Subscript["T", #], " = ", vv[[# + 1]]}  &/@ (Range[4] - 1)]
```

$$T_0 = \frac{1}{2}\epsilon\left(E_1^2+E_2^2+E_3^2+\eta^2 H_1^2+\eta^2 H_2^2+\eta^2 H_3^2\right)+ \epsilon\,\eta\,(-E_3 H_2+E_2 H_3)\,\mathbf{e}_1+\epsilon\,\eta\,(E_3 H_1-E_1 H_3)\,\mathbf{e}_2+\epsilon\,\eta\,(-E_2 H_1+E_1 H_2)\,\mathbf{e}_3$$

$$T_1 = \epsilon\,\eta\,(E_3 H_2-E_2 H_3)+\frac{1}{2}\epsilon\left(E_1^2-E_2^2-E_3^2+\eta^2 H_1^2-\eta^2 H_2^2-\eta^2 H_3^2\right)\mathbf{e}_1+ \epsilon\left(E_1 E_2+\eta^2 H_1 H_2\right)\mathbf{e}_2+\epsilon\left(E_1 E_3+\eta^2 H_1 H_3\right)\mathbf{e}_3$$

$$T_2 = \epsilon\,\eta\,(-E_3 H_1+E_1 H_3)+\epsilon\left(E_1 E_2+\eta^2 H_1 H_2\right)\mathbf{e}_1- \frac{1}{2}\epsilon\left(E_1^2-E_2^2+E_3^2+\eta^2 H_1^2-\eta^2 H_2^2+\eta^2 H_3^2\right)\mathbf{e}_2+\epsilon\left(E_2 E_3+\eta^2 H_2 H_3\right)\mathbf{e}_3$$

$$T_3 = \epsilon\,\eta\,(E_2 H_1-E_1 H_2)+\epsilon\left(E_1 E_3+\eta^2 H_1 H_3\right)\mathbf{e}_1+ \epsilon\left(E_2 E_3+\eta^2 H_2 H_3\right)\mathbf{e}_2-\frac{1}{2}\epsilon\left(E_1^2+E_2^2-E_3^2+\eta^2 H_1^2+\eta^2 H_2^2-\eta^2 H_3^2\right)\mathbf{e}_3$$

Same calculation for the adjoint of the energy momentum tensor \overbar{T}(a) = $F^\dagger\ a\ F\ \epsilon / 2$

```
ClearAll[t2, vv2]
t2[v_] := (ϵ / 2) ( (vecE - i ** vecH (η)) ** v ** (vecE + i ** vecH (η)))
vv2 = { t2[one], t2[e1], t2[e2], t2[e3] };

Grid[ { Subscript["T", #], " = ", vv2[[# + 1]]} &/@ (Range[4] - 1)]
Grid[ { Subscript["T", #], " = ", vv[[#+1]] - vv2[[# + 1]]} &/@ (Range[4] - 1)]
```

$$T_0 = \frac{1}{2}\epsilon\left(E_1^2+E_2^2+E_3^2+\eta^2 H_1^2+\eta^2 H_2^2+\eta^2 H_3^2\right) + \epsilon\,\eta\,(E_3 H_2 - E_2 H_3)\,\mathbf{e}_1 + \epsilon\,\eta\,(-E_3 H_1 + E_1 H_3)\,\mathbf{e}_2 + \epsilon\,\eta\,(E_2 H_1 - E_1 H_2)\,\mathbf{e}_3$$

$$T_1 = \epsilon\,\eta\,(-E_3 H_2 + E_2 H_3) + \frac{1}{2}\epsilon\left(E_1^2-E_2^2-E_3^2+\eta^2 H_1^2-\eta^2 H_2^2-\eta^2 H_3^2\right)\mathbf{e}_1 + \epsilon\left(E_1 E_2+\eta^2 H_1 H_2\right)\mathbf{e}_2 + \epsilon\left(E_1 E_3+\eta^2 H_1 H_3\right)\mathbf{e}_3$$

$$T_2 = \epsilon\,\eta\,(E_3 H_1 - E_1 H_3) + \epsilon\left(E_1 E_2+\eta^2 H_1 H_2\right)\mathbf{e}_1 - \frac{1}{2}\epsilon\left(E_1^2-E_2^2+E_3^2+\eta^2 H_1^2-\eta^2 H_2^2+\eta^2 H_3^2\right)\mathbf{e}_2 + \epsilon\left(E_2 E_3+\eta^2 H_2 H_3\right)\mathbf{e}_3$$

$$T_3 = \epsilon\,\eta\,(-E_2 H_1 + E_1 H_2) + \epsilon\left(E_1 E_3+\eta^2 H_1 H_3\right)\mathbf{e}_1 + \epsilon\left(E_2 E_3+\eta^2 H_2 H_3\right)\mathbf{e}_2 - \frac{1}{2}\epsilon\left(E_1^2+E_2^2-E_3^2+\eta^2 H_1^2+\eta^2 H_2^2-\eta^2 H_3^2\right)\mathbf{e}_3$$

$$T_0 = 2\,\epsilon\,\eta\,(-E_3 H_2 + E_2 H_3)\,\mathbf{e}_1 + 2\,\epsilon\,\eta\,(E_3 H_1 - E_1 H_3)\,\mathbf{e}_2 + 2\,\epsilon\,\eta\,(-E_2 H_1 + E_1 H_2)\,\mathbf{e}_3$$

$$T_1 = 2\,\epsilon\,\eta\,(E_3 H_2 - E_2 H_3)$$

$$T_2 = 2\,\epsilon\,\eta\,(-E_3 H_1 + E_1 H_3)$$

$$T_3 = 2\,\epsilon\,\eta\,(E_2 H_1 - E_1 H_2)$$

```
tt = (ϵ / 2) (fDagger ** j + jDagger ** f )
tt2 = - ϵ η ( ⟨vecE , vecJ⟩ + ⟨vecH , vecM⟩ ) +
  ϵ c η ( ρ vecE + ρm vecH ) + ϵ cross[vecE, vecM] + ϵ η^2 cross[ vecJ, vecH ]
tt - tt2
Simplify[tt2 /. {η → Sqrt[μ / ϵ], c -> Sqrt[1 / (μ ϵ)]}, Assumptions → μ > 0 && ϵ > 0]
```

$$-\epsilon\,\eta\,(E_1 J_1 + E_2 J_2 + E_3 J_3 + H_1 M_1 + H_2 M_2 + H_3 M_3) +$$
$$\epsilon\left(\eta\sqrt{\frac{1}{\epsilon\,\mu}}\;\rho\,E_1 + \eta^2 H_3 J_2 - \eta^2 H_2 J_3 - E_3 M_2 + E_2 M_3 + \eta\sqrt{\frac{1}{\epsilon\,\mu}}\;H_1\,\rho_m\right)\mathbf{e}_1 +$$
$$\epsilon\left(\eta\sqrt{\frac{1}{\epsilon\,\mu}}\;\rho\,E_2 - \eta^2 H_3 J_1 + \eta^2 H_1 J_3 + E_3 M_1 - E_1 M_3 + \eta\sqrt{\frac{1}{\epsilon\,\mu}}\;H_2\,\rho_m\right)\mathbf{e}_2 +$$
$$\epsilon\left(\eta\sqrt{\frac{1}{\epsilon\,\mu}}\;\rho\,E_3 + \eta^2 H_2 J_1 - \eta^2 H_1 J_2 - E_2 M_1 + E_1 M_2 + \eta\sqrt{\frac{1}{\epsilon\,\mu}}\;H_3\,\rho_m\right)\mathbf{e}_3$$

$$-\epsilon\,\eta\,(E_1 J_1 + E_2 J_2 + E_3 J_3 + H_1 M_1 + H_2 M_2 + H_3 M_3) +$$
$$\epsilon\left(\eta^2\,(H_3 J_2 - H_2 J_3) - E_3 M_2 + E_2 M_3 + \eta\sqrt{\frac{1}{\epsilon\,\mu}}\;(\rho\,E_1 + H_1\,\rho_m)\right)\mathbf{e}_1 +$$
$$\epsilon\left(\eta\sqrt{\frac{1}{\epsilon\,\mu}}\;\rho\,E_2 - \eta^2 H_3 J_1 + \eta^2 H_1 J_3 + E_3 M_1 - E_1 M_3 + \eta\sqrt{\frac{1}{\epsilon\,\mu}}\;H_2\,\rho_m\right)\mathbf{e}_2 +$$
$$\epsilon\left(\eta\sqrt{\frac{1}{\epsilon\,\mu}}\;\rho\,E_3 + \eta^2 H_2 J_1 - \eta^2 H_1 J_2 - E_2 M_1 + E_1 M_2 + \eta\sqrt{\frac{1}{\epsilon\,\mu}}\;H_3\,\rho_m\right)\mathbf{e}_3$$

$$0$$

$$-\sqrt{\epsilon\,\mu}\;(E_1 J_1 + E_2 J_2 + E_3 J_3 + H_1 M_1 + H_2 M_2 + H_3 M_3) +$$
$$(\rho\,E_1 + \mu\,H_3 J_2 - \mu\,H_2 J_3 - \epsilon\,E_3 M_2 + \epsilon\,E_2 M_3 + H_1\,\rho_m)\,\mathbf{e}_1 +$$
$$(\rho\,E_2 - \mu\,H_3 J_1 + \mu\,H_1 J_3 + \epsilon\,E_3 M_1 - \epsilon\,E_1 M_3 + H_2\,\rho_m)\,\mathbf{e}_2 +$$
$$(\rho\,E_3 + \mu\,H_2 J_1 - \mu\,H_1 J_2 - \epsilon\,E_2 M_1 + \epsilon\,E_1 M_2 + H_3\,\rho_m)\,\mathbf{e}_3$$