Some messy evaluation of integrals that end up expressed in terms of elliptic E() and F() functions. Was associated with the evaluation of the charge of a circular segment of line charge.

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ClearAll[u, t, ii, f1, f2]
$Assumptions = u > 0 && u < 1 && t > 0;
f1[p_, u_, t_] := (1 + u^2 - 2 u Sin[t] Cos[p])^(3 / 2);
f2[p_, u_, t_] := E^(I p) f1[p, u, t];
ii1 = Integrate[ f1[p, u, t], p];
ii2 = Integrate[ f2[p, u, t], p];
ii1 // TraditionalForm
ii2 // TraditionalForm
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$$\frac{1}{3\sqrt{-2u\cos(p)\sin(t)+u^2+1}}2$$

$$\Bigg(-2u\sin(p)\sin(t)\left(-2u\cos(p)\sin(t)+u^2+1\right)+4\left(u^2+1\right)\tan\left(\frac{p}{2}\right)\left(-2u\cos(p)\sin(t)+u^2+1\right)+$$

$$4\,i\left(u^2+1\right)\cos^2\left(\frac{p}{2}\right)\sqrt{\sec^2\left(\frac{p}{2}\right)}\left(-2u\sin(t)+u^2+1\right)\sqrt{\frac{\sec^2\left(\frac{p}{2}\right)\left(-2u\cos(p)\sin(t)+u^2+1\right)}{-2u\sin(t)+u^2+1}}$$

$$E\left(i\sinh^{-1}\left(\tan\left(\frac{p}{2}\right)\right)\middle|\frac{u^2+2\sin(t)u+1}{u^2-2\sin(t)u+1}\right)-i\cos^2\left(\frac{p}{2}\right)\sqrt{\sec^2\left(\frac{p}{2}\right)}\left(-8\left(u^3+u\right)\sin(t)-2u^2\cos(2t)+3u^4+8u^2+3\right)$$

$$\sqrt{\frac{\sec^2\left(\frac{p}{2}\right)\left(-2u\cos(p)\sin(t)+u^2+1\right)}{-2u\sin(t)+u^2+1}}\;F\left(i\sinh^{-1}\left(\tan\left(\frac{p}{2}\right)\right)\middle|\frac{u^2+2\sin(t)u+1}{u^2-2\sin(t)u+1}\right)\Bigg)$$

$$\frac{1}{10u}\sin(p)\csc(t)\Bigg(2\,i\csc(p)\left(-2u\cos(p)\sin(t)+u^2+1\right)^{5/2}+$$

$$\Bigg(\csc(t)\Bigg(8u^2\sin^2(p)\sin^2(t)\sqrt{\frac{1}{2u\sin(t)-u^2-1}}\sqrt{-2u\cos(p)\sin(t)+u^2+1}\left(-u\cos(p)\sin(t)+u^2+1\right)+$$

$$4\,i\,u\sin(t)\left(8\left(u^3+u\right)\sin(t)-6u^2\cos(2t)+u^4+8u^2+1\right)\sqrt{\frac{u(\cos(p)-1)\sin(t)}{-2u\sin(t)+u^2+1}}\sqrt{\frac{u(\cos(p)+1)\sin(t)}{2u\sin(t)+u^2+1}}$$

$$F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{u^2-2\sin(t)u+1}}\sqrt{u^2-2\cos(p)\sin(t)u+1}\right)\middle|\frac{u^2-2\sin(t)u+1}{u^2+2\sin(t)u+1}\right)-$$

$$2\,i\left(-6u^3\sin(3t)+2\left(u^5+11u^3+u\right)\sin(t)-6\left(u^4+u^2\right)\cos(2t)+u^6+9u^4+9u^2+1\right)$$

$$\sqrt{\frac{u(\cos(p)-1)\sin(t)}{-2u\sin(t)+u^2+1}}\sqrt{\frac{u(\cos(p)+1)\sin(t)}{2u\sin(t)+u^2+1}}$$

$$E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{u^2-2\sin(t)u+1}}\sqrt{u^2-2\cos(p)\sin(t)u+1}\right)\middle|\frac{u^2-2\sin(t)u+1}{u^2+2\sin(t)u+1}\right)\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{\sin^2(p)}\sqrt{u^2\sin^2(p)}\sqrt{\frac{1}{2u\sin(t)-u^2-1}}\right)\Bigg)$$