This notebook has transformation techniques to translate a couple of circular charge distribution integrals into their elliptic integral form. It also has plots of some of the electric and magnetic fields obtained from solving one such problem.

Transform integral to elliptic or hypergeometric forms?

https://mathematica.stackexchange.com/questions/161971/transform-integral-to-elliptic-or-hypergeometric-forms/162095#162095

a = z/R, b = r/R

```
<< peeters` ;
peeters`setGitDir[ "../project/figures/GAelectrodynamics" ]
```

```
/Users/pjoot/project/figures/GAelectrodynamics
```

```
ClearAll[int1, lim1t, lim1b, A]
int1[p_, a_, b_] = Integrate[1 / (1 + a ^ 2 + b ^ 2 - 2 b Cos[p]) ^ (3 / 2),
   p, Assumptions → a ∈ Reals && b ∈ Reals && 0 ≤ p ≤ 2 Pi];

lim1t =
  Limit[int1[p, a, b], p → 2 Pi, Direction → 1, Assumptions → a ∈ Reals && b ∈ Reals];

(*(4 EllipticE[-((4 b)/(a^2+(-1+b)^2))])/(Sqrt[a^2+(-1+b)^2] (a^2+(1+b)^2))*)

(*lim1b=Limit[int1[p,a,b],p→0,Direction→-1,Assumptions→a∈Reals&&b∈Reals]*)

(*0*)
```

```
A[z_, rho_] = (lim1t /. {a → z, b → rho}) ;

A[z, ρ] // TraditionalForm
```

```
Integrate[A[z, rho], z, Assumptions → (rho ∈ Reals && rho ≠ 1)]
Integrate[A[z, rho], {z, -Infinity, Infinity},
 Assumptions → (rho ∈ Reals && rho ≠ 1)]
```

$$4\,\text{Integrate}\left[\frac{\text{EllipticE}\left[-\frac{4\,\text{rho}}{(-1+\text{rho})^2+z^2}\right]}{\sqrt{(-1+\text{rho})^2+z^2}\,\left((1+\text{rho})^2+z^2\right)},\ z,\ \text{Assumptions}\to \text{rho}\in\mathbb{R}\ \&\&\ \text{rho}\neq 1\right]$$

```
$Aborted
```

```
ClearAll[int2, lim2t, lim2b, B]

int2[p_, a_, b_] = Integrate[
   Cos[p] / (1 + a^2 + b^2 - 2 b Cos[p])^(3 / 2), p, Assumptions → a ∈ Reals && b ∈ Reals];

lim2t =
  Limit[int2[p, a, b], p → 2 Pi, Direction → 1, Assumptions → a ∈ Reals && b ∈ Reals];

(*(2 (1+a^2+b^2) EllipticE[-((4 b)/(a^2+(-1+b)^2))]-
   2 (a^2+(1+b)^2) EllipticK[-((4 b)/(a^2+(-1+b)^2))])/
 (Sqrt[a^2+(-1+b)^2] b (a^2+(1+b)^2))*)

(*lim2b=Limit[int2[p,a,b],p→0,Direction→-1,Assumptions→a∈Reals&&b∈Reals]*)

(*0*)



B[z_, rho_] = (lim2t /. {a → z, b → rho}) ;

B[z, ρ] // TraditionalForm

Integrate[B[z, rho], z, Assumptions → (rho ∈ Reals && rho ≠ 1)]
Integrate[B[z, rho], {z, -Infinity, Infinity},
 Assumptions → (rho ∈ Reals && rho ≠ 1)]
```

$$\frac{1}{\text{rho}}\,\text{Integrate}\Big[\frac{2\left(1+\text{rho}^2+z^2\right)\text{EllipticE}\left[-\frac{4\,\text{rho}}{(-1+\text{rho})^2+z^2}\right]-2\left((1+\text{rho})^2+z^2\right)\text{EllipticK}\left[-\frac{4\,\text{rho}}{(-1+\text{rho})^2+z^2}\right]}{\sqrt{(-1+\text{rho})^2+z^2}\left((1+\text{rho})^2+z^2\right)},\ z,\ \text{Assumptions}\rightarrow \text{rho}\in\mathbb{R}\ \&\&\ \text{rho}\neq 1\Big]$$

```
$Aborted
```

```
ClearAll[e1, e2, e3]
{e1, e2, e3} = IdentityMatrix[3];
rhocap[phi_] = e1 Cos[phi] + e2 Sin[phi];
phicap[phi_] = e2 Cos[phi] - e1 Sin[phi];
zcap = e3;
Efield[rho_, phi_, z_] = z A[z, rho] e3 + rhocap[phi] ( rho A[z, rho] - B[z, rho]);
Hfield[rho_, phi_, z_] = -z A[z, rho] phicap[phi] - e3 ( rho B[z, rho] + A[z, rho]);
```

```
Efield[ρ, ϕ, z] // TraditionalForm
Hfield[ρ, ϕ, z] // TraditionalForm
```

$$\left\{\cos(\phi)\left(\frac{4\,\rho\,E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)}-\frac{2\left(\rho^2+z^2+1\right)E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)-2\left((\rho+1)^2+z^2\right)K\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\rho\,\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)}\right),\right.$$

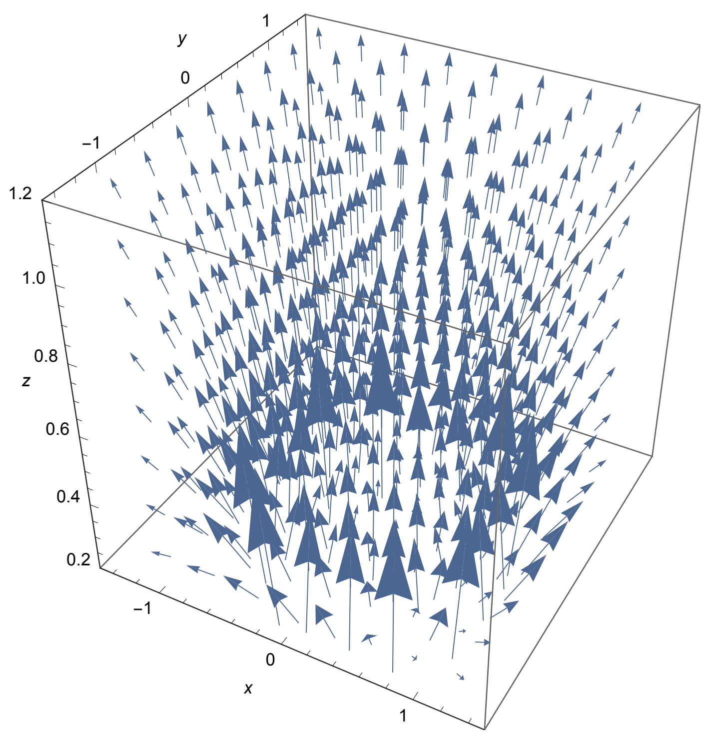
$$\sin(\phi)\left(\frac{4\,\rho\,E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)}-\frac{2\left(\rho^2+z^2+1\right)E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)-2\left((\rho+1)^2+z^2\right)K\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\rho\,\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)}\right),$$

$$\left.\frac{4\,z\,E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)}\right\}$$

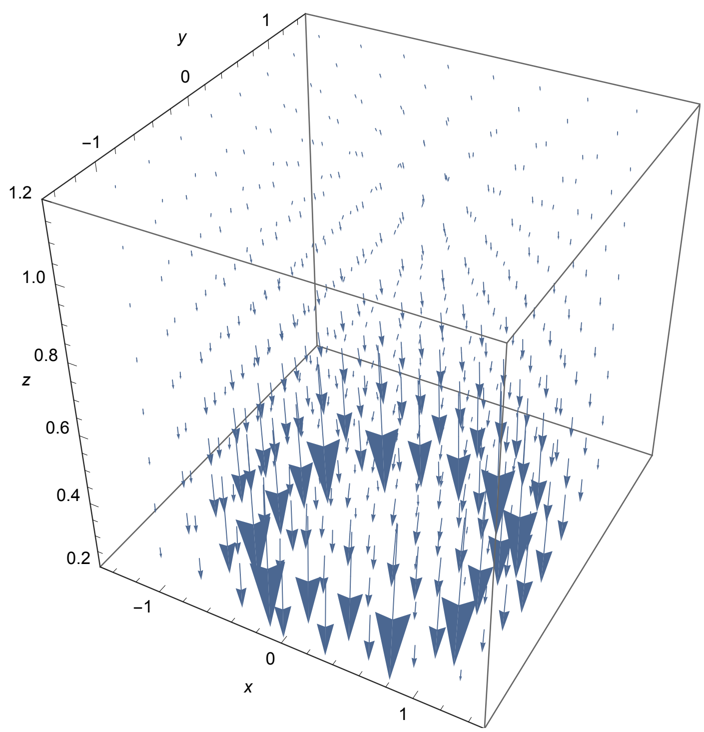
$$\left\{\frac{4\,z\sin(\phi)\,E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)},-\frac{4\,z\cos(\phi)\,E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)},\right.$$

$$\left.-\frac{2\left(\rho^2+z^2+1\right)E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)-2\left((\rho+1)^2+z^2\right)K\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)}-\frac{4\,E\!\left(-\frac{4\rho}{z^2+(\rho-1)^2}\right)}{\sqrt{(\rho-1)^2+z^2}\,\left((\rho+1)^2+z^2\right)}\right\}$$

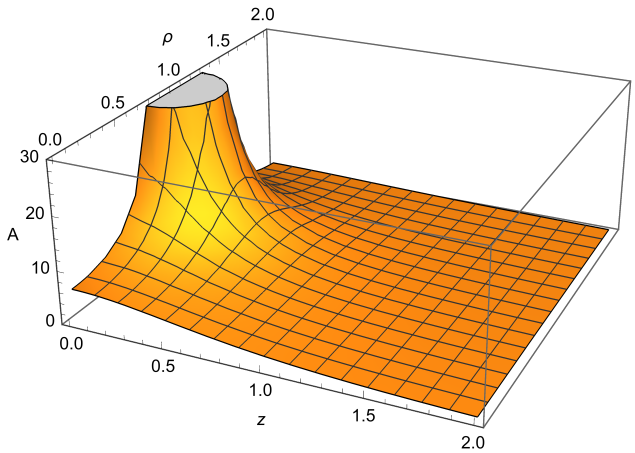
```
pe = Module[{range, zmin},
  range = 1.2;
  zmin = 0.2;
  VectorPlot3D[
   Module[{rho, phi},
    rho[x_, y_] = Sqrt[x^2 + y^2];
    phi[x_, y_] = ArcTan[x, y];
    Efield[rho[ x, y], phi[x, y], z]
   ],
   {x, -range, range},
   {y, -range, range},
   {z, zmin, range},
   PlotRange → {Full, Full, {0.2, range}},
   AxesLabel → {x, y, z}
  ]
 ]
```

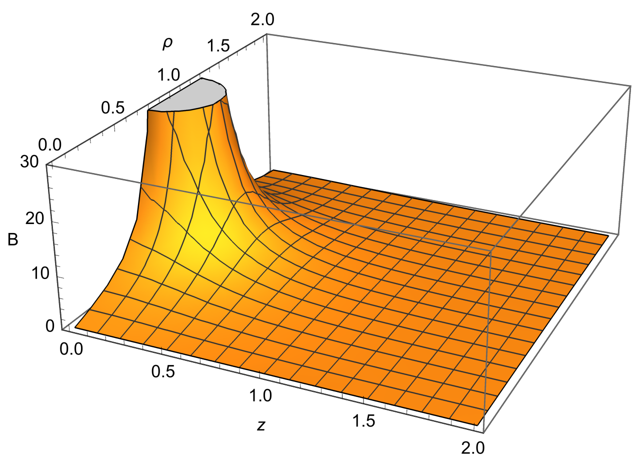


```
ph = Module[{range, zmin},
  range = 1.2;
  zmin = 0.2;
  VectorPlot3D[
   Module[{rho, phi},
    rho[x_, y_] = Sqrt[x^2 + y^2];
    phi[x_, y_] = ArcTan[x, y];
    Hfield[rho[ x, y], phi[x, y], z]
   ],
   {x, -range, range},
   {y, -range, range},
   {z, zmin, range},
   PlotRange → {Full, Full, {zmin, range}},
   AxesLabel → {x, y, z}
  ]
 ]
```



```
pa = Plot3D[A[z, rho] , {z, 0, 2}, {rho, 0, 2},
  AxesLabel → {z, ρ, "A"}, PlotRange → {Full, Full, {0, 30}}]
pb = Plot3D[B[z, rho] , {z, 0, 2}, {rho, 0, 2},
  AxesLabel → {z, ρ, "B"}, PlotRange → {Full, Full, {0, 30}}]
```






```
peeters`exportForLatex["ringFieldAFig1", pa]
peeters`exportForLatex["ringFieldBFig1", pb]
```

{ringFieldAFig1.eps, ringFieldAFig1pn.png}

{ringFieldBFig1.eps, ringFieldBFig1pn.png}

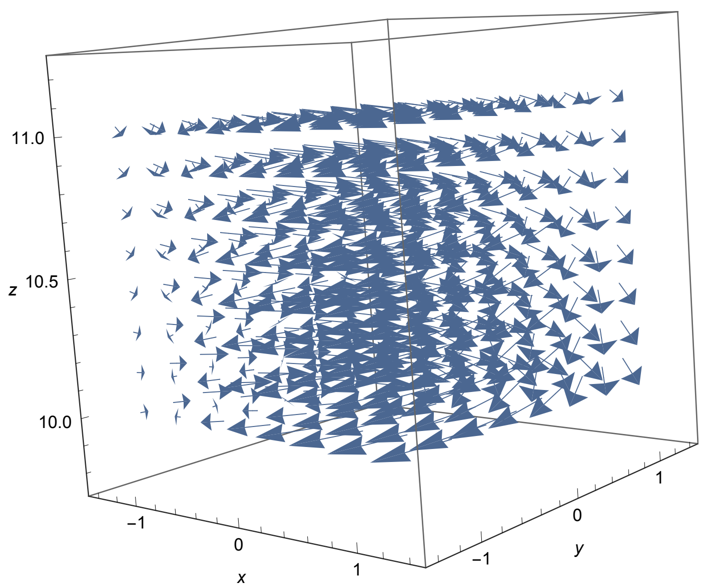
```
peeters`exportForLatex["ringFieldEFig1", pe]
peeters`exportForLatex["ringFieldHFig1", ph]
```

{ringFieldEFig1.eps, ringFieldEFig1pn.png}

{ringFieldHFig1.eps, ringFieldHFig1pn.png}

```
(*Plot[EllipticK[-x], {x,0,10}]
 Plot[EllipticE[-x], {x,0,1000}]*)
```

```
ph2 = Module[{range, zmin},
  range = 1.2;
  zmin = 0.2;
  VectorPlot3D[
   Module[{rho, phi},
    rho[x_, y_] = Sqrt[x^2 + y^2];
    phi[x_, y_] = ArcTan[x, y];
    Hfield[rho[ x, y], phi[x, y], z]
   ],
   {x, -range, range},
   {y, -range, range},
   {z, 10, 11},
   AxesLabel → {x, y, z}
  ]
 ]
```




```
peeters`exportForLatex["ringFieldHFig2", ph2]
```

```
{ringFieldHFig2.eps, ringFieldHFig2pn.png}
```