Spherical polar basis and volume element.

```
<< CliffordBasic`;
$SetSignature = {3, 0};
Import[
 "https://raw.githubusercontent.com/jkuczm/MathematicaCellsToTeX/master/NoInstall.
   m"]
```

Conventional vector algebra only

```
ClearAll[x, xr, xt, xp, e1, e2, e3, r, theta, phi]
{e1, e2, e3} = IdentityMatrix[3];

x[r_, t_, p_] = r (e1 Sin[t] Cos[p] + e2 Sin[t] Sin[p] + e3 Cos[t])
xr[r_, theta_, phi_] = D[x[a, theta, phi], a] /. {a → r, t → theta, p → phi};
xt[r_, theta_, phi_] = D[x[r, t, phi], t] /. {a → r, t → theta, p → phi};
xp[r_, theta_, phi_] = D[x[r, theta, p], p] /. {a → r, t → theta, p → phi};

{x[r, θ, ϕ],
  xr[r, θ, ϕ],
  xt[r, θ, ϕ],
  xp[r, θ, ϕ]} // Column

{xr[r, θ, ϕ].xt[r, θ, ϕ],
  xr[r, θ, ϕ].xp[r, θ, ϕ],
  xp[r, θ, ϕ].xt[r, θ, ϕ],
  xr[r, θ, ϕ].xr[r, θ, ϕ],
  xp[r, θ, ϕ].xp[r, θ, ϕ],
  xt[r, θ, ϕ].xt[r, θ, ϕ]} // FullSimplify

Det[ {xr[r, θ, ϕ],
   xt[r, θ, ϕ],
   xp[r, θ, ϕ]}] // Simplify
```

```
{r Cos[p] Sin[t], r Sin[p] Sin[t], r Cos[t]}

{r Cos[ϕ] Sin[θ], r Sin[θ] Sin[ϕ], r Cos[θ]}
{Cos[ϕ] Sin[θ], Sin[θ] Sin[ϕ], Cos[θ]}
{r Cos[θ] Cos[ϕ], r Cos[θ] Sin[ϕ], -r Sin[θ]}
{-r Sin[θ] Sin[ϕ], r Cos[ϕ] Sin[θ], 0}
```

$\{0, 0, 0, 1, r^2 \, \text{Sin}[\theta]^2, r^2\}$

$r^2 \, \text{Sin}[\theta]$

Now, only with GA.

```
ClearAll[i, j, ej, xg]
i = e[1, 2];
j[phi_] = GeometricProduct[e[3, 1], Cos[phi] + i Sin[phi]];
ej[t_, p_] = Cos[t] + j[p] Sin[t];
xg[r_, t_, p_] = r GeometricProduct[e[3], ej[t, p]];

(*Row[{"j = ", j[ϕ]}]
 Row[{"e^j = ", ej[θ,ϕ]}]*)

(*(xg[r, θ, ϕ] /. {e[1] → e1, e[2] → e2, e[3]→e3})- x[r, θ, ϕ] *)

xgr[r_, theta_, phi_] = D[xg[a, theta, phi], a] /. {a → r, t → theta, p → phi};
xgt[r_, theta_, phi_] = D[xg[r, t, phi], t] /. {a → r, t → theta, p → phi};
xgp[r_, theta_, phi_] = D[xg[r, theta, p], p] /. {a → r, t → theta, p → phi};

{xg[r, θ, ϕ],
  xgr[r, θ, ϕ],
  xgt[r, θ, ϕ],
  xgp[r, θ, ϕ]} // Column

{InnerProduct[xgr[r, θ, ϕ], xgt[r, θ, ϕ]],
  InnerProduct[xgr[r, θ, ϕ], xgp[r, θ, ϕ]], InnerProduct[xgp[r, θ, ϕ], xgt[r, θ, ϕ]],
  InnerProduct[xgr[r, θ, ϕ], xgr[r, θ, ϕ]], InnerProduct[xgp[r, θ, ϕ], xgp[r, θ, ϕ]],
  InnerProduct[xgt[r, θ, ϕ], xgt[r, θ, ϕ]]} // Simplify

OuterProduct[ xgr[r, θ, ϕ],
 xgt[r, θ, ϕ],
 xgp[r, θ, ϕ]]
```

r (Cos[θ] e[3] + Cos[ϕ] e[1] Sin[θ] + e[2] Sin[θ] Sin[ϕ])
Cos[θ] e[3] + Cos[ϕ] e[1] Sin[θ] + e[2] Sin[θ] Sin[ϕ]
r (Cos[θ] Cos[ϕ] e[1] − e[3] Sin[θ] + Cos[θ] e[2] Sin[ϕ])
r (Cos[ϕ] e[2] Sin[θ] − e[1] Sin[θ] Sin[ϕ])

$\{0, 0, 0, 1, r^2 \mathrm{Sin}[\theta]^2, r^2\}$

r^2 e[1, 2, 3] Sin[θ]

A hybrid example to use in book

```
ClearAll[i, j, ej, x, xr, xt, xp]
i = e[1, 2];
j[phi_] = GeometricProduct[e[3, 1], Cos[phi] + i Sin[phi]];
ej[t_, p_] = Cos[t] + j[p] Sin[t];
x[r_, t_, p_] = r GeometricProduct[e[3], ej[t, p]];

xr[r_, theta_, phi_] = D[x[a, theta, phi], a] /. a → r;
xt[r_, theta_, phi_] = D[x[r, t, phi], t] /. t → theta;
xp[r_, theta_, phi_] = D[x[r, theta, p], p] /. p → phi;

{x[r, θ, ϕ],
  xr[r, θ, ϕ],
  xt[r, θ, ϕ],
  xp[r, θ, ϕ]} // Column
```

```
r (Cos[θ] e[3] + Cos[ϕ] e[1] Sin[θ] + e[2] Sin[θ] Sin[ϕ])
Cos[θ] e[3] + Cos[ϕ] e[1] Sin[θ] + e[2] Sin[θ] Sin[ϕ]
r (Cos[θ] Cos[ϕ] e[1] - e[3] Sin[θ] + Cos[θ] e[2] Sin[ϕ])
r (Cos[ϕ] e[2] Sin[θ] - e[1] Sin[θ] Sin[ϕ])
```

```
OuterProduct[ xr[r, θ, ϕ],
 xt[r, θ, ϕ],
 xp[r, θ, ϕ]]

{e1, e2, e3} = IdentityMatrix[3];
jacobean = {xr[r, θ, ϕ],
    xt[r, θ, ϕ],
    xp[r, θ, ϕ]} /. {e[1] → e1, e[2] → e2, e[3] → e3};
Det[ jacobean ] // Simplify
```

```
ClearAll[x1, x2, x3]
x1 = xr[r, θ, ϕ];
x2 = xt[r, θ, ϕ];
x3 = xp[r, θ, ϕ];
MapThread[InnerProduct, {{x1, x2, x3}, {x2, x3, x1}}] // Simplify
```

```
{0, 0, 0}
```

cell output