Griffiths Problem 2.41 (generalized to all points above the square instead of just over the center)

```
ClearAll[x, y, a, f, g, u, v]
$Assumptions = a > 0 && z > 0;

f[x_, y_, z_, xp_, yp_] := z {0, 0, 1} + (x - xp) {1, 0, 0} + (y - yp) {0, 1, 0};
integrand1[u_, v_, z_] :=
 (f[x, y, z, xp, yp] / (f[x, y, z, xp, yp].f[x, y, z, xp, yp]) ^ (3 / 2) ) /.
  {x - xp → -u, y - yp → -v }
(* x - xp = -u, y - yp = -v *)

g[u, v, z]
```

$$\left\{-\frac{u}{\left(u^2+v^2+z^2\right)^{3/2}},\ -\frac{v}{\left(u^2+v^2+z^2\right)^{3/2}},\ \frac{z}{\left(u^2+v^2+z^2\right)^{3/2}}\right\}$$

```
i1[u_, v_, z_] = Integrate[ integrand1[u, v, z], v]
```

$$\left\{-\frac{u\,v}{\left(u^2+z^2\right)\sqrt{u^2+v^2+z^2}},\ \frac{1}{\sqrt{u^2+v^2+z^2}},\ \frac{v\,z}{\left(u^2+z^2\right)\sqrt{u^2+v^2+z^2}}\right\}$$

```
integrand2[u_, y_, z_] = i1[u, a / 2 - y, z] - i1[u, -a / 2 - y, z]
```

$$\left\{\frac{u\left(-\frac{a}{2}-y\right)}{\left(u^2+z^2\right)\sqrt{u^2+\left(-\frac{a}{2}-y\right)^2+z^2}}-\frac{u\left(\frac{a}{2}-y\right)}{\left(u^2+z^2\right)\sqrt{u^2+\left(\frac{a}{2}-y\right)^2+z^2}},\right.$$
$$-\frac{1}{\sqrt{u^2+\left(-\frac{a}{2}-y\right)^2+z^2}}+\frac{1}{\sqrt{u^2+\left(\frac{a}{2}-y\right)^2+z^2}},$$
$$\left.-\frac{\left(-\frac{a}{2}-y\right)z}{\left(u^2+z^2\right)\sqrt{u^2+\left(-\frac{a}{2}-y\right)^2+z^2}}+\frac{\left(\frac{a}{2}-y\right)z}{\left(u^2+z^2\right)\sqrt{u^2+\left(\frac{a}{2}-y\right)^2+z^2}}\right\}$$

```
i2[u_, y_, z_] = Integrate[ integrand2[u, y, z], u]
```

$$\left\{\text{ArcTanh}\left[\frac{\sqrt{a^2-4\,a\,y+4\,(u^2+y^2+z^2)}}{a-2\,y}\right]+\text{ArcTanh}\left[\frac{\sqrt{a^2+4\,a\,y+4\,(u^2+y^2+z^2)}}{a+2\,y}\right],\right.$$

$$\text{Log}\left[2\,u+\sqrt{a^2-4\,a\,y+4\,(u^2+y^2+z^2)}\right]-\text{Log}\left[2\,u+\sqrt{a^2+4\,a\,y+4\,(u^2+y^2+z^2)}\right],$$

$$\left.2\,z\left(\frac{\text{ArcTan}\left[\frac{u\,(a-2\,y)}{z\,\sqrt{a^2+4\,u^2-4\,a\,y+4\,y^2+4\,z^2}}\right]}{2\,z}+\frac{\text{ArcTan}\left[\frac{u\,(a+2\,y)}{z\,\sqrt{a^2+4\,u^2+4\,a\,y+4\,y^2+4\,z^2}}\right]}{2\,z}\right)\right\}$$

```
Efield[x_, y_] = (i2[a / 2 - x, y, z] - i2[-a / 2 - x, y, z]) // FullSimplify
```

$$\left\{-\text{ArcCoth}\left[\frac{a-2\,y}{\sqrt{2}\,\sqrt{a^2+2\,a\,(x-y)+2\,(x^2+y^2+z^2)}}\right]+\right.$$

$$\text{ArcCoth}\left[\frac{a-2\,y}{\sqrt{2}\,\sqrt{a^2-2\,a\,(x+y)+2\,(x^2+y^2+z^2)}}\right]-$$

$$\text{ArcCoth}\left[\frac{a+2\,y}{\sqrt{2}\,\sqrt{a^2+2\,a\,(x+y)+2\,(x^2+y^2+z^2)}}\right]+$$

$$\text{ArcCoth}\left[\frac{a+2\,y}{\sqrt{2\,a^2+4\,a\,(-x+y)+4\,(x^2+y^2+z^2)}}\right],$$

$$-\text{Log}\left[-a-2\,x+\sqrt{2}\,\sqrt{a^2+2\,a\,(x-y)+2\,(x^2+y^2+z^2)}\right]-$$

$$\text{Log}\left[a-2\,x+\sqrt{2}\,\sqrt{a^2+2\,a\,(-x+y)+2\,(x^2+y^2+z^2)}\right]+$$

$$\text{Log}\left[a-2\,x+\sqrt{2}\,\sqrt{a^2-2\,a\,(x+y)+2\,(x^2+y^2+z^2)}\right]+$$

$$\text{Log}\left[-a-2\,x+\sqrt{2}\,\sqrt{a^2+2\,a\,(x+y)+2\,(x^2+y^2+z^2)}\right],$$

$$\text{ArcTan}\left[\frac{(a+2\,x)\,(a-2\,y)}{2\,\sqrt{2}\,z\,\sqrt{a^2+2\,a\,(x-y)+2\,(x^2+y^2+z^2)}}\right]+$$

$$\text{ArcTan}\left[\frac{(a-2\,x)\,(a+2\,y)}{2\,\sqrt{2}\,z\,\sqrt{a^2+2\,a\,(-x+y)+2\,(x^2+y^2+z^2)}}\right]+$$

$$\text{ArcTan}\left[\frac{(a-2\,x)\,(a-2\,y)}{2\,\sqrt{2}\,z\,\sqrt{a^2-2\,a\,(x+y)+2\,(x^2+y^2+z^2)}}\right]+$$

$$\left.\text{ArcTan}\left[\frac{(a+2\,x)\,(a+2\,y)}{2\,\sqrt{2}\,z\,\sqrt{a^2+2\,a\,(x+y)+2\,(x^2+y^2+z^2)}}\right]\right\}$$