A hand calculation seemed to show that I had the wrong expressions for $\alpha_L,\ \alpha_R$ in my polarization notes. Here's a check of the correction of those expressions:

```
ClearAll[a1, a2, b1, b2, c1, c2, alphaL, alphaR, phi]
$Assumptions = a1 > 0 && a2 > 0 && b1 > 0 && b2 > 0 && phi > 0;

c1 = a1 + I b1;
c2 = a2 + I b2;
alphaL = (c1 + I c2) / 2
alphaR = Conjugate[(c1 - I c2) / 2] // Simplify

alphaL (Cos[phi] + I Sin[phi]) + alphaR (Cos[phi] - I Sin[phi]) // Simplify
```

$$\frac{1}{2}(a1 + i\, b1 + i\, (a2 + i\, b2))$$

$$\frac{1}{2}(a1 + i\, a2 - i\, b1 + b2)$$

```
(a1 + i a2) Cos[phi] - (b1 + i b2) Sin[phi]
```