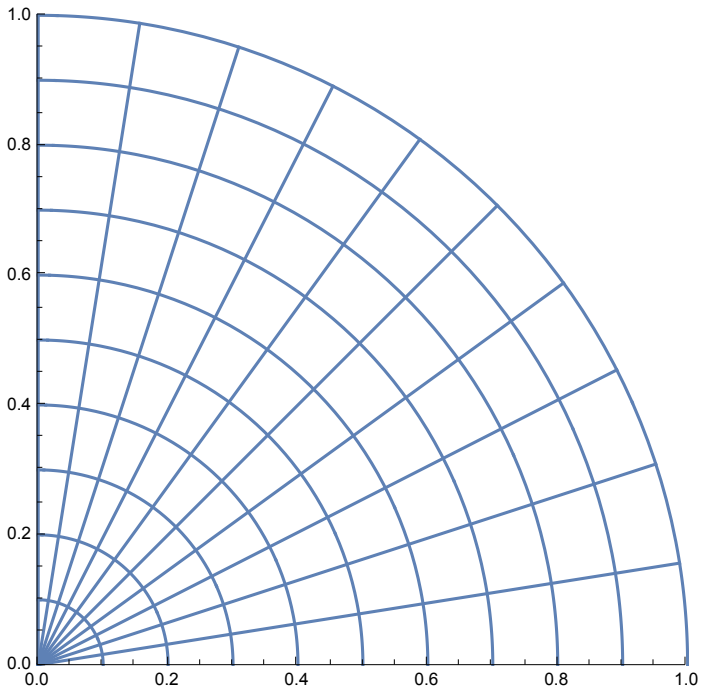
Hyperbolic parameterization of an ellipse, and contours for the associated curvilinear coordinates. ellipticalContoursFig1.eps, and ellipticalContoursFig1.eps figures.

```
<< peeters`

(*relative to ~/physicsplay*)
peeters`setGitDir[ "../project/figures/GAelectrodynamics" ]
```

```
peeters`
```

```
/Users/pjoot/project/figures/GAelectrodynamics
```

```
ClearAll[ toVector, polar]

toVector[z_] := {z // Re, z // Im};

polar[r_, t_] := r Exp[I t] // toVector;

Module[{t1, t2, tm, range},
 range = {{0, 1}, {0, 1}};
 t1 = Table[
   ParametricPlot[polar[r, t], {t, 0, Pi / 2}, PlotRange → range], {r, 0.1, 1, 0.1}] ;
 tm = Pi / 2;
 t2 = Table[ParametricPlot[polar[r, t], {r, 0, 1}, PlotRange → range],
   {t, 0.1 tm, tm, 0.1 tm}] ;
 Show[t1, t2]
]
```



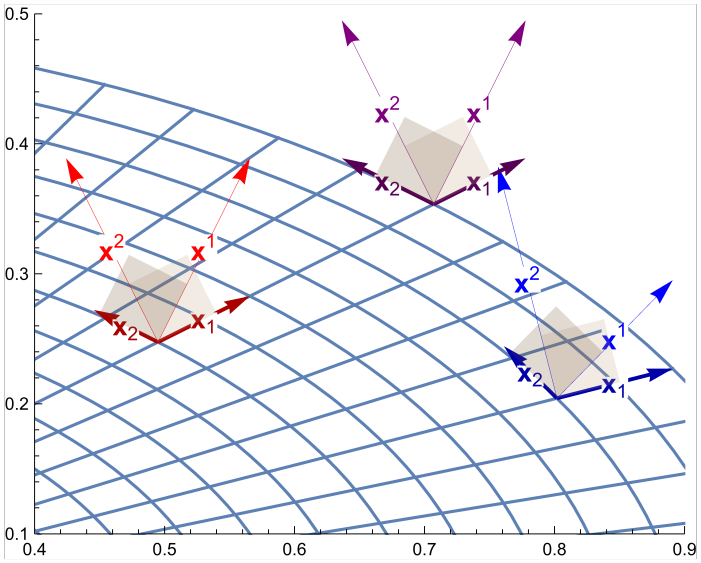

```
ClearAll[ (*elliptical,*)ellipticalH, x1, x2, r1, r2]

(*elliptical[a_, b_,t_] :=  Module[{m},
   m = ArcTanh[b/a];
   a Cosh[m+ I t]/Cosh[m] // toVector];*)

x1[a_, t_] :=  Module[{m},
   m = ArcTanh[1 / 2];
```

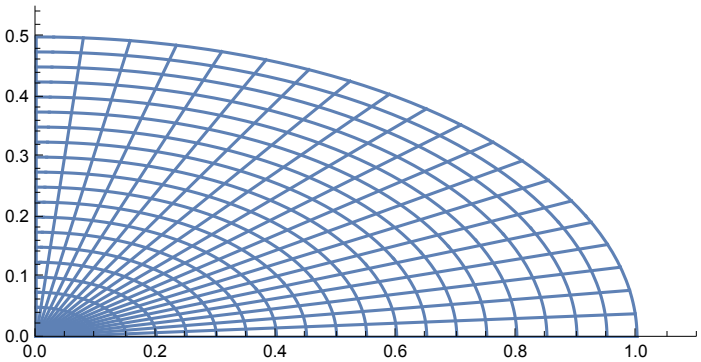
```
    Cosh[m + I t] / Cosh[m] // TrigExpand // toVector];
x2[a_, t_] := Module[{m},
    m = ArcTanh[1 / 2];
    a I Sinh[m + I t] / Cosh[m] // TrigExpand // toVector];
ellipticalH[a_, t_] := a x1[a, t];

r1[a_, t_] := Module[{m},
    m = ArcTanh[1 / 2];
    2 Sinh[m + I t] / Cosh[m] // TrigExpand // toVector];
r2[a_, t_] = Module[{m},
    m = ArcTanh[1 / 2];
    2 I Cosh[m + I t] / Cosh[m] / a // TrigExpand // toVector];


boldx = Style["x", Bold]
boldsubx = Style[Subscript[boldx, #], FontSize → 14, Background → White] &;
boldsuperx = Style[Superscript[boldx, #], FontSize → 14, Background → White] &;

p2 = Module[{t1, t2, tm, range, p1, rp1, tp1, g, p2, rp2,
    tp2, da, rp3, tp3, p3, dx11, dx21, dx12, dx22, dx13, dx23},
  (*range =1.1 {{0,1},{0,1}};*)
  range = {{0.4, 0.9}, {0.1, 0.5}};
  tm = Pi / 2;
  da = 0.1;

  t1 = Table[ParametricPlot[ellipticalH[r, t],
     {t, 0, tm}, PlotRange → range], {r, 0.1, 1, 0.05}] ;
  t2 = Table[ParametricPlot[ellipticalH[r, t], {r, 0, 1}, PlotRange → range],
    {t, 0, tm, 0.05 tm}] ;
  rp1 = 0.7;
  tp1 = 5 Pi / 20;
  p1 = ellipticalH[rp1, tp1];
  dx11 = da x1[rp1, tp1];
  dx21 = da x2[rp1, tp1];
  dr11 = da r1[rp1, tp1];
  dr21 = da r2[rp1, tp1];


  rp2 = 0.9;
  tp2 = 3 Pi / 20;
  p2 = ellipticalH[rp2, tp2];
  dx12 = da x1[rp2, tp2];
  dx22 = da x2[rp2, tp2];
  dr12 = da r1[rp2, tp2];
```

```
dr22 = da r2[rp2, tp2];

rp3 = 1.0;
tp3 = 5 Pi / 20;
p3 = ellipticalH[rp3, tp3];
dx13 = da x1[rp3, tp3];
dx23 = da x2[rp3, tp3];
dr13 = da r1[rp3, tp3];
dr23 = da r2[rp3, tp3];

g = Graphics[{
   Red // Darker,
   Thick,
   Arrow[{p1, p1 + dx11}],
   Arrow[{p1, p1 + dx21}],
   Text[ boldsubx[1], p1 + dx11 / 2],
   Text[ boldsubx[2], p1 + dx21 / 2],
   Thin,
   Red,
   Arrow[{p1, p1 + dr11}],
   Arrow[{p1, p1 + dr21}],
   Text[ boldsuperx[1], p1 + dr11 / 2],
   Text[ boldsuperx[2], p1 + dr21 / 2],
   Thick,
   Blue // Darker,
   Arrow[{p2, p2 + dx12}],
   Arrow[{p2, p2 + dx22}],
   Text[ boldsubx[1], p2 + dx12 / 2],
   Text[ boldsubx[2], p2 + dx22 / 2],
   Thin,
   Blue,
   Arrow[{p2, p2 + dr12}],
   Arrow[{p2, p2 + dr22}],
   Text[ boldsuperx[1], p2 + dr12 / 2],
   Text[ boldsuperx[2], p2 + dr22 / 2],
   Thick,
   Purple // Darker,
   Arrow[{p3, p3 + dx13}],
   Arrow[{p3, p3 + dx23}],
   Text[ boldsubx[1], p3 + dx13 / 2],
   Text[ boldsubx[2], p3 + dx23 / 2],
   Thin,
   Purple,
   Arrow[{p3, p3 + dr13}],
```

```
      Arrow[{p3, p3 + dr23}],
      Text[ boldsuperx[1], p3 + dr13 / 2],
      Text[ boldsuperx[2], p3 + dr23 / 2]



      , Opacity[0.2]
, Brown // Lighter
, Parallelogram[p1, {dx11 // Normalize, dr21 // Normalize} / 20]
, Brown // Darker
, Parallelogram[p1, {dx21 // Normalize, dr11 // Normalize} / 20]

      , Brown // Lighter
, Parallelogram[p2, {dx12 // Normalize, dr22 // Normalize} / 20]
, Brown // Darker
, Parallelogram[p2, {dx22 // Normalize, dr12 // Normalize} / 20]

      , Brown // Lighter
, Parallelogram[p3, {dx13 // Normalize, dr23 // Normalize} / 20]
, Brown // Darker
, Parallelogram[p3, {dx23 // Normalize, dr13 // Normalize} / 20]


    }];
  Show[t1, t2, g]
 ]

p1 = Module[{t1, t2, tm, range, p1, rp1, tp1, g,
    dx11, dx21, p2, rp2, tp2, da, rp3, tp3, p3, dx13, dx23},
  range = 1.1 {{0, 1}, {0, 0.5}};

  tm = Pi / 2;
  da = 0.1;

  t1 = Table[ParametricPlot[ellipticalH[r, t],
     {t, 0, tm}, PlotRange → range], {r, 0.05, 1, 0.05}] ;
  t2 = Table[ParametricPlot[ellipticalH[r, t], {r, 0, 1}, PlotRange → range],
    {t, 0, tm, 0.05 tm}] ;

  Show[t1, t2]
 ]
```

x






```
(*peeters`exportForLatex["ellipticalContoursFig1", p1]*)
peeters`exportForLatex["ellipticalContoursFig2", p2]
```

```
{ellipticalContoursFig2.eps, ellipticalContoursFig2pn.png}
```

```
$Assumptions = t > 0 && m > 0;
Simplify[Re[ Cosh[m - I t] Sinh[m + I t] /. m → ArcTanh[1 / 2]]]

r1[a, t].x1[a, t] // Simplify
r2[a, t].x2[a, t] // Simplify
r2[a, t].x1[a, t] // FullSimplify
r1[a, t].x2[a, t] // FullSimplify
```

$\frac{2}{3}$

1

1

0

0