$$\Phi(t_0) = 1 - e^{-\lambda t_0}$$

$$\beta(\cdot) : [0, \bar{\tau}] \rightarrow [0, \bar{\beta}]$$

$$\Phi(t_0|t_i) = \frac{e^{\lambda\eta} - e^{\lambda(t_i - t_0)}}{e^{\lambda\eta} - 1}$$

$$\Phi(t_0|t_i') \underset{FOSD}{\succ} \Phi(t_0|t_i) \quad \forall t_i' > t_i$$

$$T^*(t_0) = \min\{T(t_0 + \eta\kappa), t_0 + \bar{\tau}\}$$

$$T^*(t_0) = \inf\{t | s(t, t_0) \geq \kappa \text{ or } t = t_0 + \bar{\tau}\}$$

$$\Pi(t|t_i) = \int_{T^*(t_0)<t} d\Phi(t_0|t_i)$$

$$\int_{t_i}^{t} e^{-rs}[1 - \beta(s - T^{*-1}(s))]\, p(t)\, d\pi(s|t_i) \,+\, e^{-rt} p(t)[1 - \Pi(t|t_i)]$$

$$h(t|t_i) = \frac{\pi(t|t_i)}{1 - \Pi(t|t_i)} = \frac{g - r}{\beta(t - T^{*-1}(s))}$$

$$h(t_i + \tau|t_i) = \frac{\lambda}{1 - e^{-\lambda(\xi - \tau)}} = \frac{g - r}{\beta(\xi)}$$

$$\tau = \xi - \frac{1}{\lambda} ln\left(\frac{g - r}{g - r - \lambda\beta(\xi)}\right)$$

$$\xi = \min\{\eta\kappa + \tau, \bar{\tau}\}$$

$$t_0 + \underbrace{\tau + \eta\kappa}_{\xi} < t_0 + \underbrace{\bar{\tau}}_{\xi}$$

$$h(t_i + \tau | t_i) = \frac{\lambda}{1 - e^{-\lambda(\xi - \tau)}} = \frac{g - r}{\beta(\xi)}$$

$$\frac{\lambda}{1 - e^{-\lambda(\eta\kappa)}} \geq \frac{g - r}{\beta(\tau + \eta\kappa)}$$

$$\tau^* = \beta^{-1}\left(\frac{g - r}{\lambda/(1 - e^{-\lambda\eta\kappa})}\right) - \eta\kappa$$

$$t_0 + \xi^* = t_o + \beta^{-1}\left(\frac{g - r}{\lambda/(1 - e^{-\lambda\eta\kappa})}\right) < t_0 + \bar{\tau}$$

$$\beta(t - t_0) = 1 - e^{-(g-r)(t-t_0)}$$

$$s(t, t_0) := \int_{t_0}^{min\{t, t_j + \eta\}} \sigma(t, t_j)\, dt_j \tag{1}$$

$$\sigma(\cdot, t_j) : [0, t_j + \bar{\tau}] \to \{0, 1\} \tag{2}$$

$$\sigma(\cdot, t_j) = \begin{cases} 0 & \forall\, t < T(t_j) \\ 1 & \forall\, t \geq T(t_j) \end{cases} \tag{3}$$

$$Pr(\tilde{t}_0 \le t_0 | \tilde{t}_j = t_j) = \frac{Pr(\tilde{t}_j = t_j | \tilde{t}_0 \le t_0) Pr(\tilde{t}_0 \le t_0)}{Pr(\tilde{t}_j = t_j)} \tag{4}$$

$$= \frac{\int_0^{t_0} Pr(\tilde{t}_j = t_j | \tilde{t}_0 = t) Pr(\tilde{t}_0 = t)\, dt}{\int_0^{\infty} Pr(\tilde{t}_j = t_j | \tilde{t}_0 = t) Pr(\tilde{t}_0 = t)\, dt} \tag{5}$$

$$Pr(\tilde{t}_j = t_j | \tilde{t}_0 = t) = \frac{1}{\eta} \quad \text{if } t \in [t_j - \eta, t_j] \tag{6}$$

$$= 0 \quad \text{otherwise} \tag{7}$$

$$Pr(\tilde{t}_0 \le t_0 | \tilde{t}_j = t_j) = \frac{\int_{t_j-\eta}^{t_o} \frac{1}{\eta} \lambda e^{-\lambda t}\, dt}{\int_{t_j-\eta}^{t_j} \frac{1}{\eta} \lambda e^{-\lambda t}\, dt} \tag{8}$$

$$= \frac{e^{-\lambda(t_j-\eta)} - e^{-\lambda t_0}}{e^{-\lambda(t_j-\eta)} - e^{-\lambda t_j}} \tag{9}$$

$$Pr(\tilde{t}_0 \le t_0 | \tilde{t}_j = t_j) = \frac{e^{\lambda \eta} - e^{-\lambda(t_j - t_0)}}{e^{\lambda \eta} - 1} \tag{10}$$

$$\tag{11}$$

$$Pr(\tilde{t}_0 \le t_0 | \tilde{t}_j = t_j) = \Phi(t_0 | t_j) \tag{12}$$

$$= \int_{t_j-\eta}^{t_0} \phi(s | t_j) ds \tag{13}$$

$$= \frac{e^{\lambda \eta} - e^{-\lambda(t_j - t_0)}}{e^{\lambda \eta} - 1} \tag{14}$$

$$\Pi(t | t_j) = \int_{t_0 + \xi < t} d\Phi(t_0 | t_i) = \Phi(t - \xi | t_j) \tag{15}$$

$$
\begin{aligned}
\Pi(t_j+\tau|t_j) &= \Phi(t_j+\tau-\xi|t_i) && (16)\\
&= \frac{e^{\lambda\eta}-e^{-\lambda(t_j-(t_j+\tau-\xi))}}{e^{\lambda\eta}-1} && (17)\\
&= \frac{e^{\lambda\eta}-e^{-\lambda(\xi-\tau)}}{e^{\lambda\eta}-1} && (18)
\end{aligned}
$$

$$
\pi(t_j+\tau|t_j) = \frac{d}{dt}\left[\frac{e^{\lambda\eta}-e^{\lambda(\xi-\tau)}}{e^{\lambda\eta}-1}\right] = \frac{\lambda e^{\lambda(\xi-\tau)}}{e^{\lambda\eta}-1} \tag{19}
$$

$$
V(t) = \int_{t_j}^{t} e^{-(r-f)s}[1-\beta(t-T^{*-1}(s))]p(s)d\pi(s|t_j) \;+\; e^{-(r-f)t}p(t)[1-\Pi(t|t_j)] \tag{20}
$$

$$
\frac{d}{dt}V(t) = e^{t(g+f-r)}[1-\beta(t-T^{*-1}(s))]\pi(s|t_j) \;+\; \frac{d}{dt}\left[e^{t(g+f-r)} \;-\; \underbrace{e^{t(g+f-r)}}_{f}\underbrace{\Pi(t|t_j)}_{g}\right] \tag{21}
$$

$$
\frac{d}{dt}\left[e^{t(g+f-r)} \;-\; e^{t(g+f-r)}\Pi(t|t_j)\right] = (g+f-r)e^{t(g+f-r)} \;-\; (g+f-r)e^{t(g+f-r)}\Pi(t|t_j) \;+\; \pi(t|t_j)e^{t(g+f-r)} \tag{22}
$$

$$\frac{d}{dt}V(t) = e^{t(g+f-r)}[1-\beta(\cdot)]\pi(t|t_j) \;+\; e^{t(g+f-r)}\left[(g+f-r)(1-\Pi(t|t_j)+\pi(t|t_j))\right] \tag{23}$$

$$0 = (g+f-r)(1-\Pi(t|t_j)) - \beta(\cdot) \tag{24}$$

$$h(t|t_j) = \frac{\pi(t|t_j)}{1-\Pi(t|t_j)} > \frac{g+f-r}{\beta(t-T^{*-1}(t))} \tag{25}$$

$$h(t_j+\tau|t_i) = \frac{\lambda e^{(\xi-\tau)}/(e^{\lambda\eta}-1)}{1-\frac{e^{\lambda\eta}-e^{\lambda(\xi-\tau)}}{e^{\lambda\eta}-1}} \tag{26}$$

$$= \frac{\lambda}{1-e^{-\lambda(\xi-\tau)}} \tag{27}$$

$$\frac{\lambda}{1-e^{-\lambda(\xi-\tau)}} = \frac{g+f-r}{\beta(t-T^{*-1}(t))} \tag{28}$$

$$\lambda\beta(\cdot) = (g+f-r) - (g+f-r)e^{-\lambda(\xi-\tau)} \tag{29}$$

$$(g+f-r)e^{-\lambda(\xi-\tau)} = g+f-r-\lambda\beta(\cdot) \tag{30}$$

$$\frac{g+f-r}{g+f-r-\lambda\beta(\cdot)} = \frac{1}{e^{-\lambda(\xi-\tau)}} = e^{\lambda(\xi-\tau)} \tag{31}$$

$$\tau^* = \xi \;-\; \frac{1}{\lambda} ln\left(\frac{g+f-r}{g+f-r-\lambda\beta(\cdot)}\right) \tag{32}$$

$$\Phi(\cdot|\Phi(t_0|t_i) \geq \alpha) \underset{FOSD}{\succ} \Phi(\cdot|\Phi(t_0|t_j) \geq \alpha) \tag{33}$$

$$G(\cdot|G_y(Y) \geq \alpha) \underset{FOSD}{\succ} F(\cdot|F_x(X) \geq \alpha) \tag{34}$$

$$F(\cdot|F_x(X) \leq \alpha) \underset{FOSD}{\succ} G(\cdot|G_y(Y) \leq \alpha) \tag{35}$$

$$\tag{36}$$

$$\begin{aligned}\lambda(t,N) &= \lambda_L[1 - Pr(\lambda = \lambda_L|N(t) = n)] \; + \; \lambda_H Pr(\lambda = \lambda_H|N(t) = n) \\ &= \lambda_L + (\lambda_H - \lambda_L)Pr(\lambda = \lambda_H|N(t) = n) \\ &= \lambda_L + (\lambda_H - \lambda_L)\frac{Pr(\lambda = \lambda_H)Pr(N(t) = n|\lambda = \lambda_H)}{Pr(\lambda = \lambda_H)Pr(N(t) = n|\lambda = \lambda_H) + Pr(\lambda = \lambda_L)Pr(N(t) = n|\lambda_L)}\end{aligned}$$

$$\begin{aligned}\lambda(t,N) &= \lambda_L[1 - Pr(\lambda_L|N(t) = n)] \; + \; \lambda_H Pr(\lambda_H|N(t) = n) \\ &= \lambda_L + (\lambda_H - \lambda_L)Pr(\lambda_H|N(t) = n) \\ &= \lambda_L + (\lambda_H - \lambda_L)\frac{Pr(\lambda_H) \cdot Pr(N(t) = n|\lambda_H)}{Pr(\lambda_H) \cdot Pr(N(t) = n|\lambda_H) + Pr(\lambda_L) \cdot Pr(N(t) = n|\lambda_L)}\end{aligned}$$

$$\begin{aligned}Pr(\lambda = \lambda_H) &= \frac{\lambda(0) - \lambda_L}{\lambda_H - \lambda_L} \\ Pr(\lambda = \lambda_L) &= \frac{\lambda_H - \lambda(0)}{\lambda_H - \lambda_L}\end{aligned}$$

$$\lambda(t,n) = \lambda_L + \frac{\frac{\lambda(0)-\lambda_L}{\lambda_H-\lambda_L} e^{-\lambda_H t} \frac{(\lambda_H t)^n}{n!} (\lambda_H - \lambda_L)}{\frac{\lambda(0)-\lambda_L}{\lambda_H-\lambda_L} e^{-\lambda_H t} \frac{(\lambda_H t)^n}{n!} + \frac{\lambda_H-\lambda(0)}{\lambda_H-\lambda_L} e^{-\lambda_L t} \frac{(\lambda_L t)^n}{n!}}$$

$$= \lambda_L + \frac{\lambda_H - \lambda_L}{1 + \frac{\lambda_H-\lambda(0)}{\lambda_H-\lambda_L} e^{[t(\lambda_H-\lambda_L) - n(ln(\lambda_H)-ln(\lambda_L))]}}$$

$$\frac{d}{df}\left[\xi - \frac{1}{\lambda} ln\left(\frac{g+f-r}{g+f-r-\lambda\beta(\cdot)}\right)\right] = \frac{\beta(\cdot)}{(r-g-f)(\lambda\beta(\cdot)+r-g-f)}$$

$$\frac{d}{d\lambda}\left[\xi - \frac{1}{\lambda} ln\left(\frac{g+f-r}{g+f-r-\lambda\beta(\cdot)}\right)\right] = -\frac{1}{\lambda^2}\left(\frac{\lambda\beta(\cdot)}{g+f-r-\lambda\beta(\cdot)} - ln\left(\frac{g+f-r}{g+f-r-\lambda\beta(\cdot)}\right)\right)$$

$$\frac{d}{d\lambda}\left[\frac{1}{g+f-r} ln\left(\frac{\lambda}{\lambda-(g+f-r)(1-e^{\lambda\eta\kappa})}\right) + \eta\kappa\right] = \frac{1}{\lambda}\frac{1+\lambda\eta\kappa - e^{\lambda\eta\kappa}}{(g+f-r)(1+e^{\lambda\eta\kappa})}$$

$$\frac{d}{df}\left[\frac{1}{g+f-r} ln\left(\frac{\lambda}{\lambda-(g+f-r)(1-e^{\lambda\eta\kappa})}\right) + \eta\kappa\right] = -\frac{1}{(g+f-r)^2}\left[\frac{(g+f-r)(e^{-\lambda\eta\kappa}-1)}{\lambda-(g+f-r)(1-e^{-\lambda\eta\kappa})} - ln\left(\frac{\lambda}{\lambda-(g+f-r)(1-e}\right.\right.$$