# Finite Complete Suites for CSP Refinement Testing

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**Abstract.** In this paper, new contributions to testing Communicating Sequential Processes (CSP) are presented, with focus on the generation of complete, finite test suites. A test suite is complete if it can uncover every conformance violation of the system under test with respect to a reference model. Both reference models and implementation behaviours are represented as CSP processes. As conformance relation, we consider trace equivalence and trace refinement, as well as failures equivalence and failures refinement. Complete black-box test suites here rely on the fact that the SUT's true behaviour is represented by a member of a fault-domain, that is, a collection of CSP processes that may or may not conform to the reference model. We define fault domains by bounding the number of excessive states occurring in a fault domain member's representation as a normalised transition graph, when comparing it to the number of states present in the graph of the reference model. This notion of fault domains is quite close to the way they are defined for finite state machines, and these fault domains guarantee the existence of finite complete test suites.

**Keywords:** Model-based testing, CSP, Trace Refinement, Failures Refinement, Complete Test Suites

#### 1 Introduction

Motivation Model-based testing (MBT) is an active research field that is currently evaluated and integrated into industrial verification processes by many companies. This holds particularly for the embedded and cyber-physical systems domain. While MBT is applied in different flavours, we consider the most effective variant to be the one where test cases and concrete test data, as well as checkers for the expected results (test oracles), are automatically generated from a reference model: it guarantees the maximal return of investment for the time and effort invested into creating the test model. The test suites generated in this

way, however, usually have different test strength, depending on the generation algorithms applied.

For the safety-critical domain, test suites with guaranteed fault coverage are of particular interest. For black-box testing, guarantees can be given only if certain hypotheses are satisfied. These hypotheses are usually specified by a fault domain: a set of models that may or may not conform to the SUT. The so-called complete test strategies guarantee to uncover every conformance violation of the SUT with respect to a reference model, provided that the true SUT behaviour is captured by a member of the fault domain.

Generation methods for complete test suites have been developed for various modelling formalisms. In this paper, we use *Communicating Sequential Processes* (CSP) [6, 10]; this is a mature process-algebraic approach that has been shown to be well-suited for the description of reactive control systems in many publications over almost five decades. Industrial success has also been reported.

Contributions This paper presents complete black-box test suites for divergence-free<sup>4</sup> CSP processes interpreted both in the trace and the failures semantics. Our results complement work published by two of the authors in [2]. There, fault domains are specified as collections of processes refining a "most general" fault domain member. With this concept of fault domains, complete test suites may be finite or infinite. While this gives important insight into the theory of complete test suites, we are particularly interested in finite suites when it comes to their practical application.

Therefore, a complementary approach to the definition of CSP fault domains is presented in this paper. To this end, we observe that every finite-state CSP process can be semantically represented as a finite normalised transition graph, whose edges are labelled by the events the process engages in, and whose nodes are labelled by minimal acceptances or, alternatively, maximal refusals [9]. The maximal refusals express the degree of nondeterminism that is present in a given CSP process state that is in one-one-correspondence to a node of the normalised transition graph. Inspired by the way that fault-domains are specified for finite state machines, we define them here as the set of CSP processes whose normalised transition graphs do not exceed the size of the reference model's graph by more than a give constant.

Our main contributions in this paper consist in the proof that for fault domains of the described type, finite, complete test suite generation methods can be given for verifying (1) trace equivalence, (2) trace refinement, (3) failures equivalence, and (4) failures refinement.

The existence of – possibly infinite – complete test suites has been established for process algebras in general and for CSP in particular by several authors, e.g. [5, 11, 8, 7, 2]. To our best knowledge, however, this article is the first to present *finite*, complete test suites associated with this class of fault domains.

<sup>&</sup>lt;sup>4</sup> The assumption of divergence freedom is usually applied in black-box testing, since it cannot distinguish between divergence and deadlock.

It should be noted that the presented results are of a theoretical nature: despite being finite, the resulting test suite size will still be too large to be applied in practice to real-world problems. We discuss a number of promising options how the test suite size can be reduced to become practically applicable.

**Overview** In Section 2, we present the background material relevant to our work.

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## 2 Preliminaries

#### 2.1 CSP and Refinement

Communicating Sequential Processes @todo

FiXme Warning: alcc: I can make this small contribution.

Normalised Transition Graphs for CSP Processes As shown in [9], any finite-state CSP process P can be represented by a normalised transition graph

$$G(P) = (N, \underline{n}, \Sigma, t : N \times \Sigma \to N, r : N \to \mathbb{PP}(\Sigma)),$$

with nodes N, initial node  $\underline{n} \in N$ , and process alphabet  $\Sigma$ . The partial transition function t maps a node n and an event  $e \in \Sigma$  to its successor node t(n, e), if, and only if, (n, e) is in the domain of t, that is, there is a transition from n with label e. Normalisation of G(P) is reflected by the fact that t is a function.

A finite sequence of events  $s \in \Sigma^*$  is a *trace* of P, if there is a path through G(P) starting at  $\underline{n}$  whose edge labels coincide with s. The set of traces of P is denoted by  $\operatorname{trc}(P)$ . If  $s \in \operatorname{trc}(P)$ , then the process corresponding to P after having executed s is denoted by P/s. Since G(P) is normalised, there is a unique node reached by applying the events from s one by one, starting in  $\underline{n}$ . Therefore, G(P)/s is also well defined.

By  $[n]^0$  we denote the *fan-out* of n: the set of events occurring as labels in any outgoing transitions.

$$[n]^0 = \{ e \in \Sigma \mid (n, e) \in \text{dom } t \}$$

We also use this notation for CSP processes:  $[P]^0$  is the set of events P may engage into, in other words, the initials of P after the empty trace of events, that is,  $initials(P/\langle\rangle)$  as defined in [10].

The total function r maps each node n to its refulsals r(n) = Ref(n). Each element of r(n) is a set of events that the CSP process P might refuse to engage into, when in a process state corresponding to n. The number of refusal sets in Ref(P/s) specifies the degree of nondeterminism that is present in process state P/s: the more refusal sets contained in Ref(P/s), the more nondeterministic is the behaviour in state P/s. If P/s is deterministic, its refusals coincide with the set of subsets of  $\Sigma - [P/s]^0$ , including the empty set.

For finite CSP processes, since the refusals of each process state are subsetclosed [6, 10],  $\operatorname{Ref}(P/s)$  can be re-constructed by knowing the set of maximal refusals  $\operatorname{maxRef}(P/s) \subseteq \operatorname{Ref}(P/s)$ . More formally, the maximal refusals  $\operatorname{maxRef}(P/s)$  are defined as

$$\max \operatorname{Ref}(P/s) = \{ R \in \operatorname{Ref}(P/s) \mid \forall R' \in \operatorname{Ref}(P/s) - \{ R \} : R \not\subseteq R' \}$$

Conversely, with the maximal refusals at hand, we can reconstruct the refusals by subset-closure:

$$\operatorname{Ref}(P/s) = \{ R' \in \mathbb{P}(\Sigma) \mid \exists R \in \operatorname{maxRef}(P/s) : R' \subseteq R \}.$$

To see that this approach works only for finite CSP processes, consider the example where  $\Sigma$  is infinite. In this case,  $\max \operatorname{Ref}(STOP/\langle \rangle)$  is empty, and so we cannot use this set to calculate the refusals of STOP, that is,  $\operatorname{Ref}(STOP/\langle \rangle)$  as defined above. As with refusals, we also use the transition graph-oriented notation  $\max \operatorname{Ref}(n) \subseteq r(n)$  to denote the maximal refusals associated with graph state n: if n is the state reached in the transition graph by following the edge labels in trace s, then  $\max \operatorname{Ref}(n) = \max \operatorname{Ref}(P/s)$ .

Well-formed normalised transition graphs must not refuse an event of the fan-out of a state in *every* refusal applicable in this state; more formally,

$$\forall n \in N, e \in \Sigma : (n, e) \in \text{dom } t \Rightarrow \exists R \in \text{maxRef}(n) : e \notin R$$
 (1)

By construction, normalised transition graphs reflect the *failures semantics* of finite-state CSP processes: the traces s of a process are the sequences of transition associated with paths through the graph, starting at  $\underline{n}$ . The pairs (s, R) with  $s \in \text{trc}(P)$  and  $R \in r(G(P)/s)$  represent the failures failures (P) of (P).

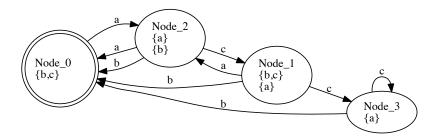


Fig. 1. Normalised transition graph of CSP process P from Example ??.

Example 1. ex:a Consider the CSP process P defined below, and that the set of events  $\Sigma$  is  $\{a, b, c\}$ .

$$P = a \rightarrow (Q \sqcap R)$$

$$Q = a \rightarrow P \sqcap c \rightarrow P$$

$$R = b \rightarrow P \sqcap c \rightarrow R$$

Its transition graph G(P) is shown in Fig. 1. Process state  $P/\langle\rangle$  is represented there as Node\_0, with  $\{b,c\}$  as the only maximal refusal, since a can never be refused, and no other events are accepted. Having engaged into a, the transition emanating from Node\_0 leads to Node\_2 representing the process state  $P/a = Q \sqcap R$ . The internal choice operator induces several refusal sets derived from Q and R. Since these processes accept their initial events in external choice,  $Q \sqcap R$  induces two maximal refusal sets  $\{b\}$  and  $\{a\}$ . Note that event c can never be refused, since it is not a member of any maximal refusal.

Having engaged into c, the next process state is represented by Node\_1. Due to normalisation, there is only a single transition satisfying  $t(\text{Node}\_2, c) = \text{Node}\_1$ . This transition, however, can have been caused by either Q or R engaging into c, so Node\_1 corresponds to process state  $Q/c \sqcap R/c = P \sqcap R$ . This is reflected by the two maximal refusals labelling Node\_1.

Similar considerations explain the other nodes and transitions in Fig. 1.

Note that the node names are generated by the FDR tool (see next paragraph). The node numbering is generated by FDR during the normalisation procedure. Therefore, the node numbers do not reflect the distance from the initial node Node\_0.

Refinement relations between finite-state CSP processes P, Q can be be expressed by means of their normalised transition graphs in the following way.

#### Lemma 1.

$$P \sqsubseteq_T Q \Leftrightarrow trc(G(Q)) \subseteq trc(G(P)) \tag{2}$$

$$P \sqsubseteq_F Q \Leftrightarrow trc(G(Q)) \subseteq trc(G(P)) \land \tag{3}$$

$$\forall s \in trc(G(Q)), R_Q \in maxRef(G(Q)/s): \tag{4}$$

$$\exists R_P \in maxRef(G(P)/s) : R_Q \subseteq R_P \tag{5}$$

$$P \sqsubseteq_F Q \Leftrightarrow trc(G(Q)) \subseteq trc(G(P)) \land \tag{6}$$

$$\forall s \in trc(G(Q)), A_Q \in minAcc(G(Q)/s): \tag{7}$$

$$\exists A_P \in minAcc(G(P)/s) : A_P \subseteq A_O \tag{8}$$

**Tool Considerations** The FDR tool [4] supports model checking and semantic analyses of CSP processes. It provides an API [3] that can be used to construct normalised transition graphs for CSP processes.

The FDR graph nodes are labelled by  $minimal\ acceptances$  instead of maximal refusals as described above. Since such a minimal acceptance set is the complement of a maximal refusal, the function r mapping states to their refusals

can be implemented by creating the complements of all minimal acceptances and then building all subsets of these complements. For practical applications, the subset closure is never constructed in an explicit way; instead, sets are checked with respect to containment in a maximal refusal.

### 2.2 Test Cases and Complete Test Suites

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#### 2.3 Minimal Hitting Sets

When testing for failures refinement, it will become apparent that the main idea of the underlying test strategy can be based on solving a *hitting set problem*. Given a collection of finite sets  $C = \{A_1, \ldots, A_n\}$  that are each subsets of a universe  $\Sigma$ , a *hitting set*  $H \subseteq \Sigma$  is a set satisfying

$$\forall A \in C : H \cap C \neq \varnothing. \tag{9}$$

A minimal hitting set is a hitting set which cannot be further reduced without losing the characteristinc property (9). The problem of determining minimal hitting sets is known to be NP-hard [1], but we will see below that it reduces the effort of testing for failures refinement from a factor of  $2^{\Sigma}$  to a factor that equals the number of minimal hitting sets.

For testing in the failures model, the following lemmas about hitting sets are required.

**Lemma 2.** Let P, Q be two finite-state CSP processes satisfying  $P \sqsubseteq_T Q$ . For each  $s \in trc(P)$ , let MINHIT(s) denote the collection of all minimal hitting sets of minAcc(P/s). Then the following statements are equivalent.

- 1.  $P \sqsubseteq_F Q$
- 2. For all  $s \in trc(P) \cap trc(Q)$  and  $H \in MINHIT(s)$ , H is a (not necessarily minimal) hitting set of minAcc(Q/s).

*Proof.* For showing "1  $\Rightarrow$  2", assume that  $P \sqsubseteq_F Q$  and suppose that  $s \in \text{trc}(P) \cap \text{trc}(Q)$ . Then Lemma 1, (6), implies that

$$\forall A_Q \in \min \mathrm{Acc}(G(Q)/s) : \exists A_P \in \min \mathrm{Acc}(G(P)/s) : A_P \subseteq A_Q$$

Therefore, every minimal acceptance set  $A_Q \in \min Acc(G(Q)/s)$  contains a minimal acceptance set from  $\min Acc(G(P)/s)$ . As a consequence, any hitting set  $H \in MINHIT(s)$  also has a non-empty intersection with every  $A_Q \in \min Acc(G(Q)/s)$ .

To prove "2  $\Rightarrow$  1", assume  $P \sqsubseteq_T Q$ , but  $P \not\sqsubseteq_F Q$ . According to Lemma 1, (6), there exists  $s \in \operatorname{trc}(P) \cap \operatorname{trc}(Q)$  such that

$$\exists\, A_Q \in \operatorname{minAcc}(G(Q)/s) : \forall\, A_P \in \operatorname{minAcc}(G(P)/s) : A_P \not\subseteq A_Q(*)$$

Define

$$\overline{H} = \bigcup_{A_P \in \min \operatorname{Acc}(G(P)/s)} (A_P \setminus A_Q).$$

Since  $A_P \setminus A_Q \neq \emptyset$  because of (\*),  $\overline{H}$  is a hitting set of minAcc(G(P)/s) which has a non-empty intersection with  $A_Q$ . Minimising  $\overline{H}$  induces the existence of a minimal hitting set  $H \in \text{MINHIT}(s)$  which is *not* a hitting set of of minAcc(G(Q)/s). This completes the proof of the lemma.

- 3 Finite Complete Test Suites for CSP Trace Refinement
- 4 Finite Complete Test Suites for CSP Failures Refinment
- 5 Applications
- 5.1 Testing for Trace Equivalence
- 5.2 Testing for Trace Refinement
- 5.3 Testing for Failures Equivalence
- 5.4 Testing for Failures Refinement
- 6 Related Work
- 7 Conclusion

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