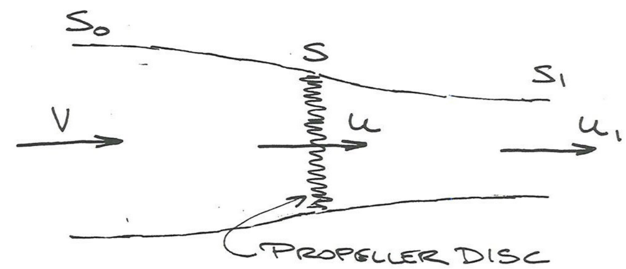
INTRODUCTION TO PROPELLER THEORY

R. H. LIEBECK

16 NOVEMBER 1984

RANKINE – FROUDE THEORY



$$\text{THRUST} = \dot{m}\,\Delta U = \rho u_1 S_1 (u_1 - V)$$

CONSIDER THE PROPELLER STATIONERY WITH FLOW THROUGH IT –

POWER ABSORBED BY PROPELLER = CHANGE IN KE PER UNIT TIME

$$KE)_{IN} = \tfrac{1}{2}\dot{m}V^2 \qquad KE)_{OUT} = \tfrac{1}{2}\dot{m}u_1^2$$

$$P = \tfrac{1}{2}\rho u_1 S_1 (u_1^2 - V^2)$$

CONSIDER PROPELLER MOVING BY WITH UPSTREAM AIR STATIONERY. LET E EQUAL KE IMPARTED TO THE AIR PER UNIT TIME

$$E = \tfrac{1}{2}\dot{m}(\Delta U)^2 = \tfrac{1}{2}\rho u_1 S_1 (u_1 - V)^2$$

WORK DONE BY THE MOVING PROPELLER IS VT. HERE THE POWER ABSORBED BY THE MOVING PROPELLER IS

$$P = VT + E$$

$$P = V\rho u_1 S_1 (u_1 - V) + \tfrac{1}{2}\rho u_1 S_1 (u_1 - V)^2$$

$$= \tfrac{1}{2}\rho u_1 S_1 (u_1^2 - V^2)$$

WHICH IS THE SAME AS THE RESULT FOR THE "STATIONERY" PROPELLER.

AGAIN FOR THE STATIONERY PROPELLER, WORK DONE ON THE AIR BY THE PROPELLER IS uT PER UNIT TIME. THIS IS ALSO EQUAL TO THE POWER ABSORBED BY THE PROPELLER.

$$P = uT$$

ALSO, $P = \tfrac{1}{2}\rho u_1 S_1 (u_1 - V)(u_1 + V) = \tfrac{1}{2}(u_1 + V) T$

$$\Rightarrow \boxed{u = \tfrac{1}{2}(u_1 + V)}$$

WHICH SAYS THE VELOCITY AT THE PROPELLER IS THE AVERAGE OF THE UPSTREAM AND DOWNSTREAM VELOCITIES.

<u>PROPULSIVE</u> <u>EFFICIENCY</u> IS DEFINED AS

$$\eta = \frac{VT}{P}$$

FROM THE PREVIOUS EXPRESSIONS FOR THRUST & POWER:

$$\eta = \frac{V\rho u_1 S_1 (u_1 - V)}{\tfrac{1}{2}\rho u_1 S_1 (u_1^2 - V^2)} = \frac{2V}{u_1 + V}$$

FOR CONVENIENCE, RE-DEFINE THE AXIAL VELOCITIES AS

$$u = V(1+a)$$

$$u_1 = V(1+b) = V(1+2a)$$

THE EFFICIENCY NOW BECOMES

$$\boxed{\eta = \frac{1}{1+a}}$$

AND THE EXPRESSIONS FOR THRUST & POWER ARE

$$T = 2S\rho V^2 (1+a)\, a$$

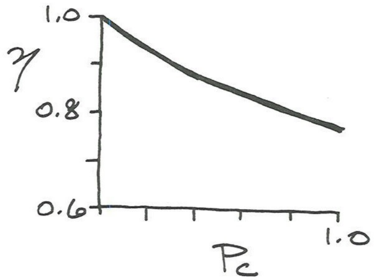
$$P = 2S\rho V^3 (1+a)^2\, a$$

DEFINE THRUST & POWER COEFFICIENTS:

$$T_c = \frac{T}{S\rho V^2}, \quad P_c = \frac{P}{S\rho V^3}$$

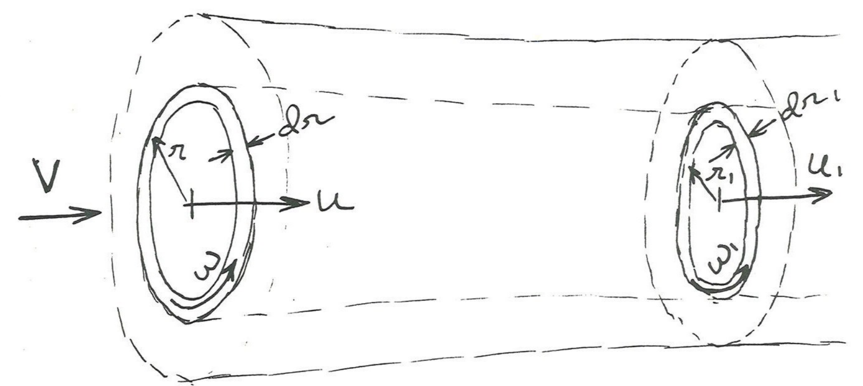
USING THE EXPRESSIONS FOR T & P & η IN TERMS OF a, THESE BECOME

$$T_c = \frac{2(1-\eta)}{\eta^2}, \quad P_c = \frac{2(1-\eta)}{\eta^3}$$



HERE THE EFFICIENCY $\eta = 1/1+a$ IS SOMETIMES CALLED "IDEAL". SINCE VISCOUS & ROTATIONAL LOSSES HAVE BEEN IGNORED, IT REPRESENTS AN UPPER BOUND ON EFFICIENCY. THIS RESULT SHOWS THAT INCREASING DISC AREA S AND/OR SPEED V WILL IMPROVE THE EFFICIENCY.

R.L. LIEBECK
10-FEB-84



PROPELLER DISC

FAR DOWNSTREAM

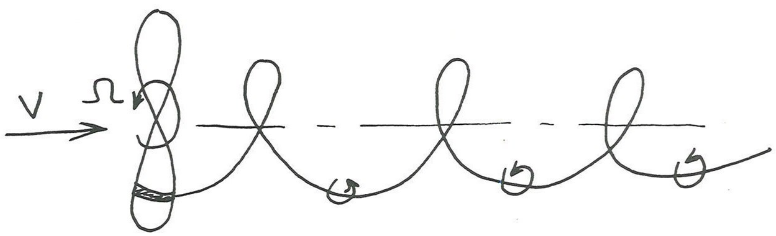
CONSIDER A PROPELLER OPERATING AT VELOCITY V WITH ROTATIONAL SPEED Ω. BEHIND THE DISC, A ROTATIONAL FLOW IS INDUCED WITH ROTATIONAL SPEED ω.

RELATING CONDITIONS FOR AN ELEMENTAL ANNULUS dr:

CONTINUITY: $u\, r\, dr = u_1 r_1 dr_1$

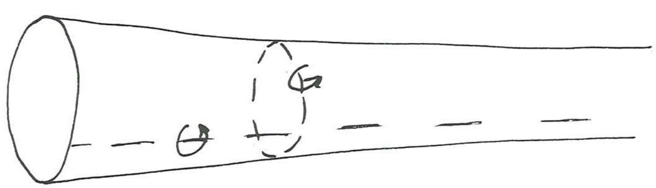
ANGULAR MOMENTUM: $\omega r^2 = \omega_1 r_1^2$

VORTEX MODEL OF A PROPELLER



AT A STATION r ON THE BLADE, A VORTEX IS SHED WHICH FOLLOWS A MORE OR LESS HELICAL PATH DOWNSTREAM TO FORM THE WAKE.

IF IT IS ASSUMED THAT THE PROPELLER HAS MANY BLADES, THE FLOW FIELD CAN BE MODELED AS A DENSE SET OF VORTEX RINGS AND VORTEX LINES.



RINGS → AXIAL INDUCED VELOCITY

LINES → ROTATIONAL INDUCED VELOCITY

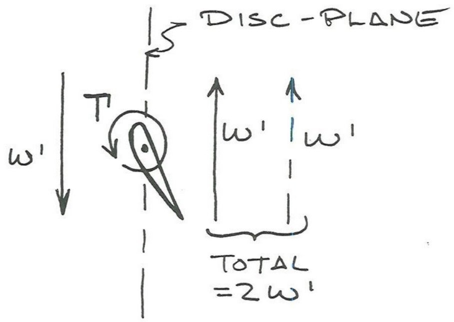
INDUCED AXIAL VELOCITY

CONSIDER THE VORTEX SYSTEM AS A SEMI-INFINITE CYLINDER. (THIS NEGLECTS WAKE CONTRACTION.)

- AT THE DISC-PLANE, THE CYLINDER EXTENDS INFINITELY FAR IN ONE DIRECTION.
- INFINITELY FAR DOWNSTREAM, THE CYLINDER EXTENDS INFINITELY IN BOTH DIRECTIONS.
- THEREFORE, THE INDUCED AXIAL VELOCITY FAR DOWNSTREAM WILL BE TWICE THAT AT THE DISC-PLANE.

$$u - V = \frac{1}{2}(u_1 - V)$$

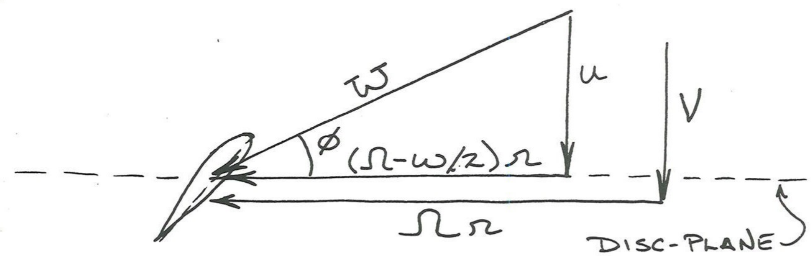
INDUCED ANGULAR VELOCITY



IRROTATIONAL FLOW UPSTREAM OF THE DISC-PLANE REQUIRES THAT THERE BE NO INDUCED ANGULAR VELOCITY IN FRONT OF PROPELLER.

ω' & $-\omega'$ ARE INDUCED BY THE BLADE CIRCULATION Γ. ω' IN FRONT OF DISC MUST BE CANCELED – THEREFORE THE TOTAL INDUCED ANGULAR VELOCITY BEHIND THE DISC IS $2\omega'$.

IF THE TOTAL INDUCED ANGULAR VELOCITY IS CALLED ω, THEN THE PORTION PROVIDED BY THE WAKE VORTICES IS $\frac{1}{2}\omega$ (AT THE DISC-PLANE).



DEFINE: $u = V(1+a)$
$\omega = 2\Omega a'$

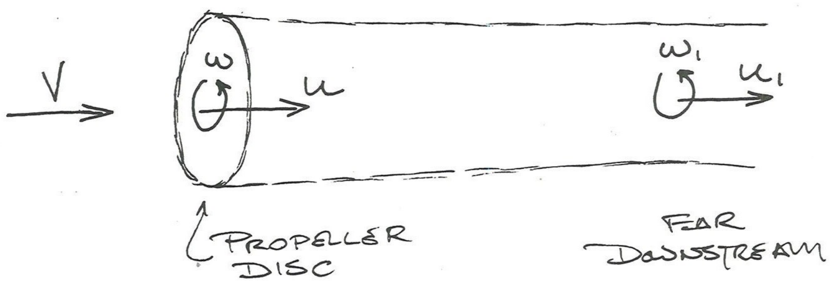
VELOCITY COMPONENTS OF W AT THE DISC-PLANE:

AXIAL $W \sin\phi = u = V(1+a)$

ROTATIONAL $W \cos\phi = (\Omega - \omega/2)\, r = \Omega r (1-a')$

$$\tan\phi = \frac{V}{\Omega r}\,\frac{1+a}{1-a'}$$

NOTE ON INDUCED VELOCITIES



DEFINE:

$u = V(1+a)$ $\quad\quad$ $u_1 = V(1+b)$

$\omega = 2\Omega a'$ $\quad\quad$ $\omega_1 = 2\Omega b'$

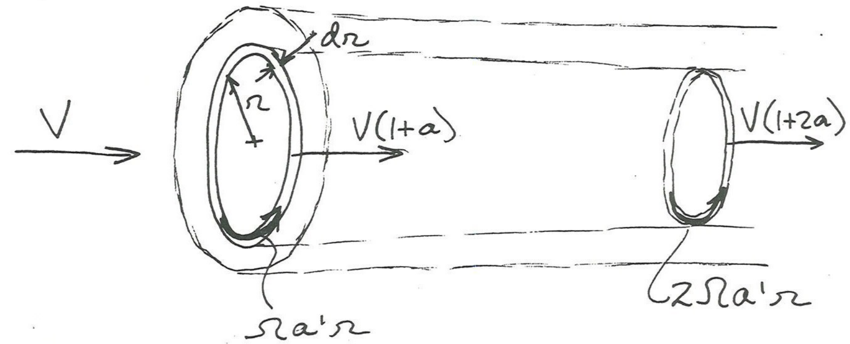
FOR THE GENERAL CASE:

$$a \neq \tfrac{1}{2} b$$

NEGLECTING WAKE CONTRACTION (LIGHT TO MODERATE LOADING):

$$a = \tfrac{1}{2} b$$

MOMENTUM RELATIONS



$$\text{FORCE} = \dot{m}\,\Delta V$$

$$d\dot{m} = 2\pi r\,dr\,\rho V(1+a)$$

$$\Delta V = V(1+2a) - V = 2aV$$

$$\boxed{\frac{dT}{dr} = 4\pi r \rho V^2 a(1+a)\cdot F}$$

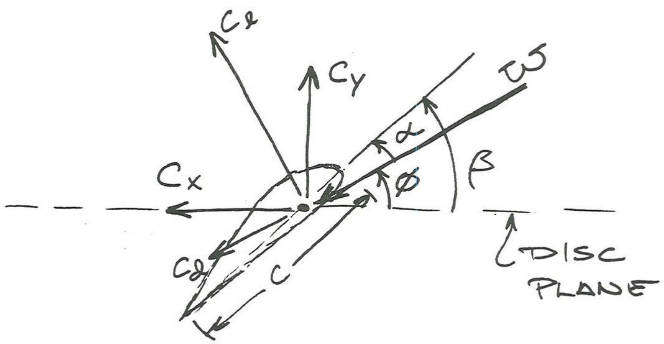
$$\text{TORQUE} = \dot{m}\, r\, \Delta\Omega\, r$$

$$\Delta\Omega = 2\Omega a' - 0 = 2\Omega a'$$

$$\boxed{\frac{1}{r}\frac{dQ}{dr} = 4\pi r^2 \rho V \Omega a'(1+a)\cdot F}$$

F = MOMENTUM LOSS FACTOR TO ACCOUNT FOR RADIAL FLOW.

Forces on a Blade Element



$$C_y = C_l \cos\phi - C_d \sin\phi$$

$$C_x = C_l \sin\phi + C_d \cos\phi$$

$$\boxed{\frac{dT}{dr} = \frac{1}{2}\rho W^2 B c C_y}$$

$$\boxed{\frac{1}{r}\frac{dQ}{dr} = \frac{1}{2}\rho W^2 B c C_x}$$

(B = NUMBER OF BLADES)

EQUATE MOMENTUM RELATIONS WITH BLADE ELEMENT ONES.

$$4\pi r \rho V^2 a(1+a)F = \frac{1}{2}\rho W^2 B c C_y$$

$$\sin\phi = \frac{V(1+a)}{W} \qquad \cos\phi = \frac{\Omega r(1-a')}{W}$$

$$\boxed{\frac{a}{1+a} = \frac{1}{4}\,\frac{Bc}{2\pi r}\,\frac{1}{F}\,\frac{C_y}{\sin^2\phi}}$$

$$4\pi r^2 \rho V \Omega a'(1+a)F = \frac{1}{2}\rho W^2 B c C_x$$

$$\boxed{\frac{a'}{1-a'} = \frac{1}{4}\,\frac{Bc}{2\pi r}\,\frac{1}{F}\,\frac{C_x}{\sin\phi\cos\phi}}$$

$$\frac{Bc}{2\pi r} = \sigma = \text{LOCAL SOLIDITY}$$

$$\tan\phi = \frac{V(1+a)}{\Omega r(1-a')}$$

ANALYSIS PROCEDURE

$$\phi = \tan^{-1}\left[\frac{V(1+a)}{\Omega r(1-a')}\right] \quad (1)$$

$$\left.\begin{aligned} \alpha = \beta - \phi, \quad W &= V(1+a)\sin\phi \\ RN &= \rho W c/\mu \end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned} C_y &= C_l \cos\phi - C_d \sin\phi \\ C_x &= C_l \sin\phi + C_d \cos\phi \end{aligned}\right\} \quad (3)$$

$$\left.\begin{aligned} a &= \frac{\sigma}{4F}\frac{C_y}{\sin^2\phi} \Big/ \left(1 - \frac{\sigma}{4F}\frac{C_y}{\sin^2\phi}\right) \\ a' &= \frac{\sigma}{4F}\frac{C_x}{\sin\phi\cos\phi} \Big/ \left(1 + \frac{\sigma}{4F}\frac{C_x}{\sin\phi\cos\phi}\right) \end{aligned}\right\} \quad (4)$$

AT EACH BLADE STATION r:

1. ASSUME AN INITIAL VALUE FOR ϕ

$$\phi_i = \tan^{-1}(V/\Omega r)$$

2. FROM (2), GET α AND CALCULATE RN.

3. USING α & RN, GET C_l & C_d FROM AIRFOIL CHARACTERISTICS.

4. OBTAIN C_y & C_x FROM (3) USING C_l & C_d.

5. CALCULATE F FROM

$$F = \frac{2}{\pi}\cos^{-1}(e^{-f}), \quad f = \frac{B}{2}\frac{1 - r/R}{\sin\phi_t}$$

WHERE ϕ_t IS OBTAINED FROM

$$R\tan\phi_t = r\tan\phi$$

6. CALCULATE a & a' FROM (4).

7. USING a & a', CALCULATE ϕ_{NEW} FROM (1).

8. REPEAT FROM STEP 2 WITH

$$\phi = \phi_{OLD} + 0.4(\phi_{NEW} - \phi_{OLD})$$

UNTIL CONVERGENCE IS OBTAINED.

DEFINE COEFFICIENTS:

THRUST $C_T = \dfrac{T}{\rho n^2 D^4}$

POWER $C_P = \dfrac{P}{\rho n^3 D^5} = \dfrac{\Omega Q}{\rho n^3 D^5}$

ADVANCE RATIO $= J = V/nD$

EFFICIENCY $= \eta = C_T J / C_P$

---

9. NUMERICALLY INTEGRATE RESULTS AT BLADE STATIONS:

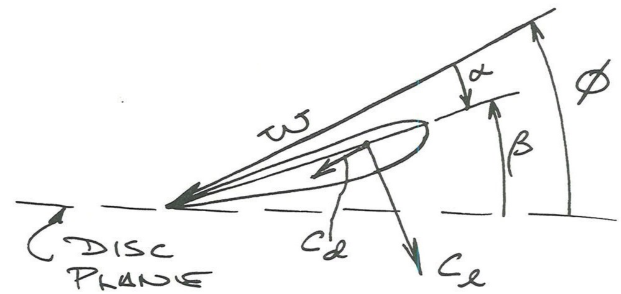
$$C_T = \frac{1}{\rho n^2 D^4}\int_{R_0}^{R} \frac{dT}{dr}\, dr$$

$$= \frac{1}{\rho n^2 D^4}\int_{R_0}^{R} \tfrac{1}{2}\rho W^2 B c C_y \, dr$$

$$C_P = \frac{1}{\rho n^3 D^5}\int_{R_0}^{R} \Omega \frac{dQ}{dr}\, dr$$

$$= \frac{1}{\rho n^3 D^5}\int_{R_0}^{R} \tfrac{1}{2}\rho W^2 B c C_x r \, dr$$

WINDMILLS



WINDMILL $\alpha = \phi - \beta$

PROPELLER $\alpha = \beta - \phi$

FOR A WINDMILL, C_l IS OF OPPOSITE SIGN TO THAT FOR A PROPELLER.

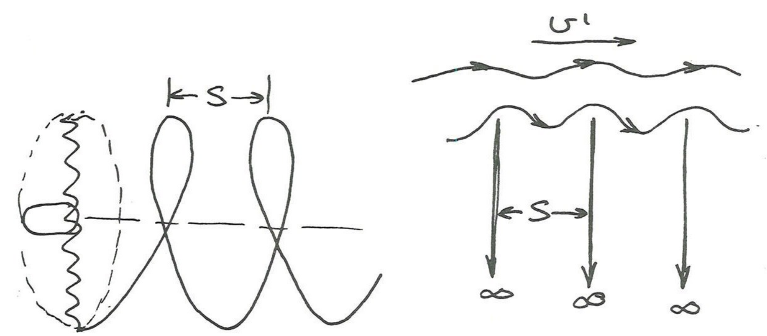
OPTIMUM DESIGN $\Rightarrow$ MINIMUM THRUST

NOTE: THE SIGNS ON T_c, P_c, a & a', AND W' WILL ALSO BE OPPOSITE TO THOSE FOR A PROPELLER.

EFFECT OF NUMBER OF BLADES

BASIC ANALYSES HAVE ASSUMED A LARGE NUMBER OF LIGHTLY-LOADED BLADES. THE RESULTING WAKE IS THEN "DENSE" WITH THE TRAILING VORTEX SHEET, AND THE RADIAL FLOW BETWEEN LAYERS OF THE HELICAL SHEET CAN BE IGNORED. HOWEVER, WHEN THE NUMBER OF BLADES IS REDUCED TO THE MORE CONVENTIONAL 2, 3, OR 4, SOME ACCOUNTING FOR THE RADIAL FLOW IS REQUIRED. A SIMPLE AND CONVENIENTLY APPLIED METHOD WAS DEVELOPED BY PRANDTL.

PRANDTL'S APPROACH WAS TO SIMULATE THE WAKE HELICAL VORTEX SHEETS BY A SERIES OF PARALLEL FLAT PLANES WHICH ARE SEMI-INFINITE IN THE DIRECTION OF THE PROPELLER AXIS. SAID PLANES ARE SPACED AT A DISTANCE EQUIVALENT TO THAT OF THE EDGES OF THE WAKE HELIX.



HELICAL WAKE

SEMI-INFINITE PLATE ANALOGY

CONSIDER THE FLOW WITH VELOCITY U' OVER THE PLATES.

- FAR INSIDE (FROM THE EDGES OF THE PLATES) THERE WILL BE NO FLOW.
- NEAR THE EDGES OF THE PLATES THERE WILL BE SOME FLOW (BOTH RADIAL AND IN THE DIRECTION OF U')
- THE FLOW IN THE DIRECTION OF U' NEAR THE EDGES REPRESENTS A LOSS IN MOMENTUM (THRUST) DUE TO THE LOCAL RADIAL VELOCITY.

FOR A PROPELLER WITH B-BLADES, THE SPACING OF THE WAKE HELIX IS

$$s = \frac{2\pi R}{B} \sin\phi_t$$

THE COMPLEX POTENTIAL FUNCTION FOR THE FLOW ABOUT THE CORRESPONDING SET OF SEMI-INFINITE PLATES IS READILY SOLVED AND THE LOCAL NORMAL VELOCITY (i.e. PARALLEL TO ω') BETWEEN THE PLATES CAN BE CALCULATED. PRANDTL DEFINED THE MOMENTUM LOSS FACTOR F TO REPRESENT THAT FRACTION OF ω' WHICH IS LOST TO FLOW ABOUT THE EDGES OF THE WAKE.

- NEAR THE HUB OF THE PROPELLER, THERE IS NO LOSS $\Rightarrow F = 1.0$
- AS THE TIP IS APPROACHED, THE LOSS INCREASES $\Rightarrow F \to 0$.

THE RELATIONS FOR CALCULATING PRANDTL'S MOMENTUM LOSS FACTOR ARE GIVEN BY

$$F = \frac{2}{\pi}\cos^{-1}\left(e^{-f}\right)$$

$$f = \frac{B}{2}(1-\xi)/\sin\phi_t\, , \quad \xi = \frac{r}{R}$$

WHERE ϕ_t IS THE FLOW ANGLE AT THE BLADE TIP.

FOR THE DESIGN PROBLEM, ϕ_t IS OBTAINED FROM

$$\tan\phi_t = \frac{V}{\Omega R}\left(1 + \zeta/2\right)$$

FOR THE ANALYSIS PROBLEM, GLAUERT SUGGESTS

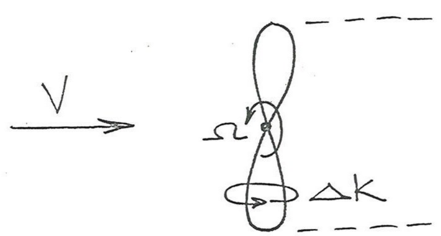
$$\sin\phi_t = \xi \sin\phi$$

INSTEAD, IT IS RECOMMENDED THAT

$$\tan\phi_t = \xi \tan\phi$$

WHICH IS EXACT WHEN ANALYZING AN OPTIMALLY DESIGNED PROPELLER.

DESIGN FOR MAXIMUM EFFICIENCY



ΔK = INCREMENT OF CIRCULATION AT SAME BLADE STATION r.

⇒ INCREASE IN THRUST ΔT & TORQUE ΔQ.

DEFINE CHANGE IN EFFICIENCY DUE TO ΔK

$$k = \frac{V \Delta T}{\Omega \Delta Q} \approx \frac{V}{\Omega} \frac{dT}{dQ}$$

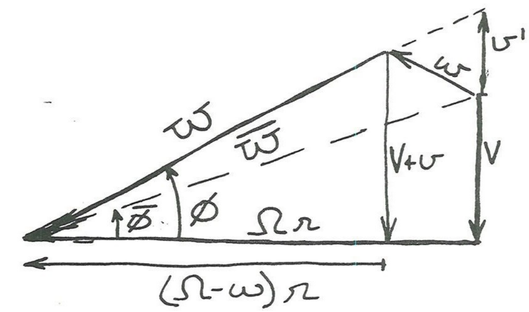
ADDING ΔK AT OTHER r WILL GIVE DIFFERENT VALUES FOR k. SET $\Omega \Delta Q$:

⇒ ADD ΔK WHERE k IS LARGE, AND REDUCE ΔK WHERE k IS SMALL, WHERE $\Omega \Delta Q$ IS FIXED FOR ALL r.

⇒ EFFICIENCY WILL BE A MAXIMUM WHEN k HAS THE SAME VALUE FOR ALL BLADE STATIONS AT WHICH ΔK IS ADDED.

⇒ $$\boxed{k = \frac{V \Delta T}{\Omega \Delta Q} = \text{CONST.}}$$

2



$\overline{W}$ = VELOCITY VECTOR OF AN ELEMENT OF THE WAKE AT RADIUS r.

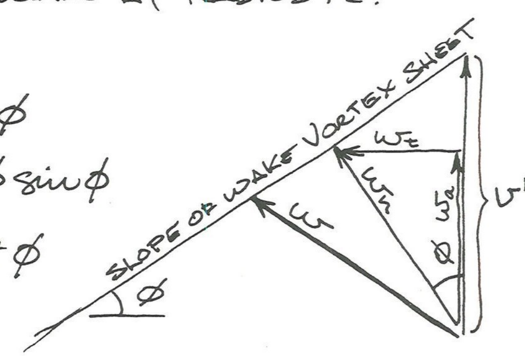
W = VELOCITY VECTOR OF AN ELEMENT OF THE WAKE RELATIVE TO THE FLUID AT RADIUS r.

w = VELOCITY VECTOR OF THE FLUID IN THE WAKE AT RADIUS r.

$$w_n = w' \cos\phi$$

$$w_t = w' \cos\phi \sin\phi$$

$$w_a = w' \cos^2\phi$$



PERFORM CALCULATION OF k IN THE WAKE –

- AVOIDS INTERFERENCE AT PROPELLER FROM ADDING ΔK.
- MUNK'S DISPLACEMENT THEOREM SAYS ADDING ΔK IN WAKE IS EQUIVALENT.

FOR AN INCREMENT OF CIRCULATION ΔK AT RADIUS r IN THE WAKE

$$\Delta T = \rho \Delta K (\Omega - \omega) r \, dr$$

$$\Delta Q = \rho \Delta K (V + v) r \, dr$$

$$k = \frac{V \Delta T}{\Omega \Delta Q} = \frac{V(\Omega - \omega)}{(V+v)\Omega} = \text{CONST. WRT TO } r \text{ FOR MAX. EFFICIENCY}$$

IN TERMS OF THE FLOW ANGLE, ϕ

$$\tan\phi = \frac{V+v}{(\Omega-\omega)r} = \frac{1}{k}\frac{V}{\Omega r}$$

CAN ALSO DEFINE

$$\tan\bar{\phi} = \frac{V}{\Omega r}, \qquad \phi \neq \bar{\phi}$$

$\tan\bar{\phi} = V/\Omega r$ DEFINES A SCREW SURFACE OF CONSTANT PITCH.

SINCE k IS CONSTANT, $\tan\phi = V/k\Omega r$ ALSO DEFINES A SCREW SURFACE OF CONSTANT (ALTHOUGH DIFFERENT) PITCH.

DEFINE VELOCITY v' BY

$$\tan\phi = \frac{V+v'}{\Omega r}$$

WHICH ACCOUNTS FOR CHANGE IN PITCH.

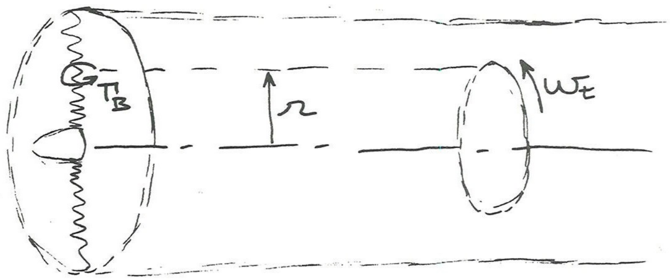
v' IS CALLED THE "DISPLACEMENT VELOCITY", AND IT REPRESENTS THE AXIAL VELOCITY OF THE WAKE VORTEX SHEET WITH RESPECT TO THE PROPELLER DISC.

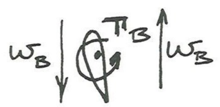
SINCE k = CONST., $\boxed{v' = \text{CONST. WRT TO } r.}$

BETZ' CONDITION FOR MINIMUM ENERGY LOSS: "TRAILING VORTICES FORM A REGULAR SCREW SURFACE"

SUMMARIZED BY THE SIMPLE RELATION –

$$\boxed{r\tan\phi = \text{CONST.}}$$





$B\Gamma_B$ = CIRCULATION ABOUT BLADE(S) AT r

$$= [w_B - (-w_B)]\, 2\pi r$$

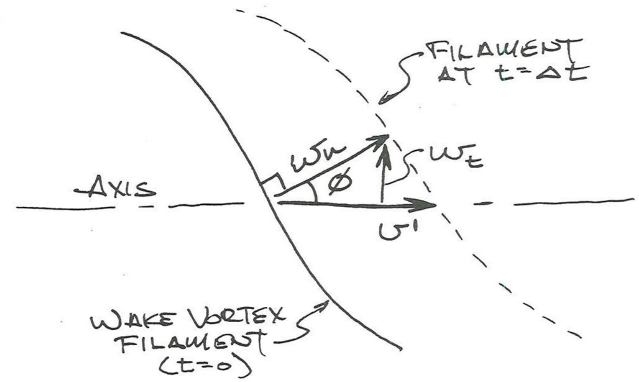
$$= 4\pi r\, w_B$$

$\Gamma_w = 2\pi r\, w_t$ = CIRCULATION IN WAKE

HOWEVER $w_B = \frac{1}{2} w_t$ FROM BASIC VORTEX THEORY ARGUEMENTS —

$$\boxed{B\Gamma = 2\pi r\, w_t}$$

(THE SUBSCRIPT ON Γ HAS BEEN DROPPED.) THIS ASSUMES A CYLINDRICAL WAKE WHICH IS APPROPRIATE FOR LIGHT AND POSSIBLY MODERATE LOADING.



MOTION OF VORTEX FILAMENTS WITH RESPECT TO THE FLUID IS NORMAL TO THE FILAMENT AND GIVEN BY VELOCITY w_n.

THE TANGENTIAL VELOCITY IS $w_t = w_n \sin\phi$.

FOR A COORDINATE SYSTEM FIXED TO THE PROPELLER DISC, THE AXIAL VELOCITY OF THE FILAMENT IS

$$U' = w_n / \cos\phi$$

THE INCREASE OF U' OVER w_n IS DUE TO THE ROTATION OF THE FILAMENT.

— ANALOGOUS TO A BARBER POLE GIVING APPARENT TRANSLATION FROM PURE ROTATION.

NON-DIMENSIONALIZE v' FOR CONVENIENCE BY DEFINING

$$\zeta = v'/V$$

w_t CAN BE EXPRESSED IN TERMS OF ζ AS

$$w_t = V\zeta \sin\phi \cos\phi$$

AND THE BLADE CIRCULATION Γ AT RADIAL STATION r BECOMES

$$B\Gamma = 2\pi r V \zeta \sin\phi \cos\phi \, F$$

(HERE THE MOMENTUM LOSS FACTOR F HAS BEEN INCLUDED.)

DEFINING THE SPEED RATIO

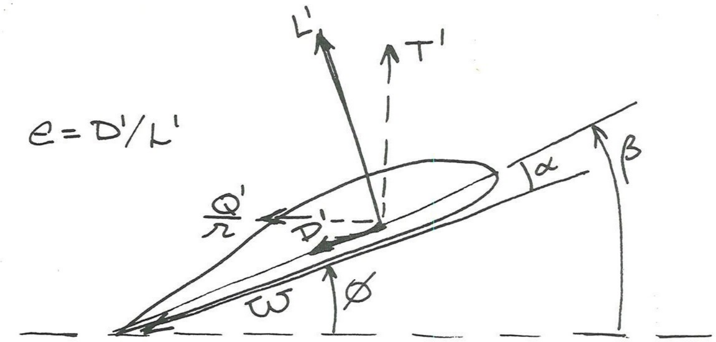
$$x = \Omega r / V$$

AND THE CIRCULATION FUNCTION

$$G = F x \sin\phi \cos\phi$$

THE EXPRESSION FOR $B\Gamma$ BECOMES

$$B\Gamma = \frac{2\pi V^2 \zeta G}{\Omega}$$



$$T' = L'\cos\phi - D'\sin\phi = L'\cos\phi(1 - \epsilon\tan\phi)$$

$$\frac{Q'}{r} = L'\sin\phi + D'\cos\phi = L'\sin\phi(1 + \epsilon/\tan\phi)$$

THE LIFT/UNIT RADIUS CAN ALSO BE EXPRESSED IN TERMS CIRCULATION

$$L' = \rho W B\Gamma$$

OR, FOR CONVENIENCE

$$L' = \frac{\rho \Omega r (1-a')}{\cos\phi} B\Gamma = \rho \frac{V(1+a)}{\sin\phi} B\Gamma$$

REPLACING L' IN THE EXPRESSIONS FOR T' & Q'

$$T' = \rho \Omega r (1-a') B\Gamma (1 - \epsilon\tan\phi)$$

$$\frac{Q'}{r} = \rho V (1+a) B\Gamma (1 + \epsilon/\tan\phi)$$

AT THIS POINT, THE EXPRESSIONS FOR T' & Q'/Ω ARE GENERAL, AND APPLY TO ANY PROPELLER. FOR THE OPTIMUM PROPELLER, THE RELATION FOR MINIMUM ENERGY BLADE LOADING IS SUBSTITUTED FOR BT!

$$T' = 2\pi r \rho V^2 \zeta G (1-a')(1-\epsilon \tan\phi)$$

$$\frac{Q'}{\Omega} = 2\pi r \rho V^3 \zeta G (1+a)(1+\epsilon/\tan\phi)$$

RECALL THE BLADE ELEMENT MOMENTUM EQUATIONS

$$T' = 4\pi r \rho V^2 (1+a)\, a\, F$$

$$\frac{Q'}{\Omega} = 4\pi r \rho V (1+a)\, \Omega a'\, F$$

SETTING THESE EXPRESSIONS EQUAL RESULTS IN THE INTERFERENCE FACTORS a & a' BEING UNIQUELY RELATED TO THE DISPLACEMENT VELOCITY.

(NOTE THAT F APPEARS IN BOTH SETS OF EQUATIONS, AND HENCE CANCELS WHEN THEY ARE EQUATED.)

THE EXPRESSIONS FOR a & a' ARE

$$a = \frac{\zeta}{2} \cos^2\phi\,(1-\epsilon \tan\phi)$$

$$a' = \frac{\zeta}{2x} \cos\phi \sin\phi\,(1+\epsilon/\tan\phi)$$

WHERE THE ϵ FACTOR ACCOUNTS FOR THE VISCOUS CONTRIBUTION.

THE ABOVE EXPRESSIONS FOR a & a' TOGETHER WITH $\tan\phi = [(1+a)/x(1-a')]$ CAN BE USED TO DERIVE (AFTER SOME MANIPULATION) THE SIMPLE BUT IMPORTANT RELATION

$$\tan\phi = \frac{1+\zeta/2}{x} = \frac{V}{\Omega r}(1+\zeta/2)$$

RECALLING THAT THE BETZ' CONDITION FOR MINIMUM ENERGY IS THAT THE WAKE FORM A REGULAR SCREW SURFACE ($r \tan\phi = \text{const.}$), AND THE EXPRESSION ABOVE SHOWS THAT IN THIS CASE, $\zeta = $ CONST. AS PREVIOUSLY DERIVED.

IT REMAINS TO DETERMINE THE VALUE OF $\mathcal{S}$ = CONST. DEFINE THE THRUST & POWER COEFFICIENTS

$$T_c = \frac{T}{\frac{1}{2}\rho V^2 \pi R^2}, \quad P_c = \frac{\Omega Q}{\frac{1}{2}\rho V^3 \pi R^2}$$

AND THE NON-DIMENSIONAL RADIUS

$$\xi = r/R$$

WHERE ξ RUNS FROM ξ_0 AT THE PROPELLER HUB TO $\xi = 1$ AT THE TIP.

THE DIFFERENTIAL FORM FOR THE THRUST COEFFICIENT BECOMES

$$dT_c = 4\xi \mathcal{S} G(1-a')(1-\epsilon\tan\phi)\,d\xi$$

WITH $\quad a' = \frac{\mathcal{S}}{2x}\sin\phi\cos\phi(1+\epsilon/\tan\phi)$

INTEGRATING WRTO ξ

$$T_c = 4\mathcal{S} I_1 - 2\mathcal{S}^2 I_2 \qquad (\mathcal{S} = \text{CONST.})$$

$$I_1 = \int_{\xi_0}^{1} G(1-\epsilon\tan\phi)\,\xi\,d\xi$$

$$I_2 = \int_{\xi_0}^{1} G(1-\epsilon\tan\phi)\,\xi\,\frac{\sin\phi\cos\phi}{x}(1+\epsilon/\tan\phi)\,d\xi$$

SIMILARLY, FOR POWER COEFFICIENT

$$dP_c = 4\xi\mathcal{S} G(1+a)(1+\epsilon/\tan\phi)\,d\xi$$

WITH $\quad a = \frac{\mathcal{S}}{2}\cos^2\phi(1-\epsilon\tan\phi)$

INTEGRATING WRTO ξ

$$P_c = 4\mathcal{S} J_1 + 2\mathcal{S}^2 J_2$$

$$J_1 = \int_{\xi_0}^{1} G(1+\epsilon/\tan\phi)\,\xi\,d\xi$$

$$J_2 = \int_{\xi_0}^{1} G(1+\epsilon/\tan\phi)\,\xi\cos^2\phi(1-\epsilon\tan\phi)\,d\xi$$

EITHER THRUST OR POWER CAN NOW BE SPECIFIED AND THE CORRESPONDING VALUE OF $\mathcal{S}$ IS THEN SET. SPECIFYING THRUST (ACTUALLY T_c) GIVES

$$\mathcal{S} = \frac{I_1}{2I_2} - \left[\left(\frac{I_1}{2I_2}\right)^2 - \frac{T_c}{I_2}\right]^{1/2}$$

AND THIS VALUE OF $\mathcal{S}$ IS USED TO GET P_c. SIMILARLY, SPECIFYING POWER (P_c) GIVES

$$\mathcal{S} = -\frac{J_1}{2J_2} + \left[\left(\frac{J_1}{2J_2}\right)^2 + \frac{P_c}{J_2}\right]^{1/2}$$

AND THE RESULTING VALUE $\mathcal{S}$ IS USED TO CALCULATE T_c.

ONCE A VALUE FOR ζ ~~HAS~~ BEEN OBTAINED FOR A SPECIFIED POWER OR THRUST, IT REMAINS TO CALCULATE THE BLADE GEOMETRY. THE LIFT/UNIT RADIUS FOR ONE BLADE CAN BE EXPRESSED AS

$$L'/B = \rho \varpi \Gamma = \tfrac{1}{2}\rho \varpi^2 c\, C_l$$

c = BLADE CHORD

C_l = SECTION LIFT COEFFICIENT

USING THE OPTIMUM LOADING EXPRESSION FOR Γ

$$\Gamma = \frac{2\pi V^2 \zeta G}{B\Omega}$$

THE PRODUCT

$$\varpi c = \frac{4\pi V^2 G \zeta}{B\Omega C_l}$$

IS OBTAINED. THE LOCAL FLOW VELOCITY ϖ IS GIVEN BY

$$\varpi = V(1+a)/\sin\phi$$

AND THE LOCAL FLOW ANGLE ϕ IS OBTAINED FROM

$$\phi = \tan^{-1}\left(\frac{1+\zeta/2}{x}\right), \quad x = \frac{\Omega r}{V}$$

THUS, FOR A GIVEN VALUE OF ζ AND SPECIFIED BLADE SECTION C_l, THE QUANTITIES ϖ, ϕ, & c MAY BE CALCULATED.

THE ANGLE OF ATTACK α AT WHICH THE CHOSEN BLADE SECTION OBTAINS C_l IS GIVEN BY THE AIRFOIL CHARACTERISTICS AND THIS DEFINES THE LOCAL BLADE CHORD ANGLE

$$\beta = \alpha + \phi$$

SINCE THE LOCAL REYNOLDS NUMBER IS

$$RN = \frac{\rho \varpi c}{\mu}$$

THE SECTION C_d AND ALSO $\epsilon = C_d/C_l$ ARE OBTAINED.

DESIGN PROCEDURE

GIVEN: DIAMETER, No. OF BLADES, VELOCITY, RPM, AIRFOIL CHAR., THRUST OR POWER

1. SELECT AN INITIAL VALUE FOR ζ. ($\zeta = 0$ WILL WORK.)

2. DETERMINE F AND ϕ AT EACH BLADE STATION.

$$F = \frac{2}{\pi} \cos^{-1}(e^{-f})$$

$$f = \frac{B}{2} \frac{1-\xi}{\sin\phi_t}$$

ϕ_t IS THE FLOW ANGLE AT THE TIP

$$\tan\phi_t = \frac{V}{\Omega R}(1+\zeta/2)$$

ALSO, SINCE $r\tan\phi = R\tan\phi_t = \text{CONST.}$, THIS ALSO GIVES THE INITIAL VALUES FOR ϕ.

3. DETERMINE Wc FROM

$$Wc = \frac{4\pi V^2 G \zeta}{B\Omega C_l}, \quad RN = \frac{\rho Wc}{\mu}$$

4. USE THE AIRFOIL CHARACTERISTICS TO DETERMINE α & $\epsilon = C_d/C_l$.

5. CALCULATE a & a' FROM

$$a = \frac{\zeta}{2}\cos^2\phi\,(1-\epsilon\tan\phi)$$

$$a' = \frac{\zeta}{2x}\cos\phi\sin\phi\,(1+\epsilon/\tan\phi)$$

6. CALCULATE W FROM

$$W = V(1+a)/\sin\phi$$

7. USING THE VALUE OF Wc FROM STEP 3, CALCULATE BLADE CHORD c, AND CALCULATE BLADE TWIST FROM $\beta = \alpha + \phi$.

8. CALCULATE THE INTEGRANDS OF THE INTEGRALS I_1, I_2, J_1, J_2 AT EACH BLADE STATION, AND NUMERICALLY INTEGRATE FROM $\xi = \xi_0$ TO $\xi = 1$.

9. IF THRUST HAS BEEN SPECIFIED, GET ζ FROM

$$\zeta = \frac{I_1}{2I_2} - \left[\left(\frac{I_1}{2I_2}\right)^2 - \frac{T_c}{I_2}\right]^{1/2}$$

$$P_c = 4\zeta J_1 + 2\zeta^2 J_2$$

17

IF POWER HAS BEEN SPECIFIED, GET γ AND T_c FROM

$$\gamma = -\frac{I_1}{2I_2} + \left[\left(\frac{I_1}{2I_2}\right)^2 + \frac{P_c}{I_2}\right]^{1/2}$$

$$T_c = 4\gamma I_1 - 2\gamma^2 I_2$$

10. IF THE VALUE OF γ CALCULATED IN STEP 9 IS NOT SUFFICIENTLY CLOSE TO THAT SELECTED IN STEP 1 (SAY, WITHIN 0.1%), START OVER AT STEP 2 USING THE NEW VALUE FOR γ.

11. DETERMINE PROPELLER EFFICIENCY $\eta = T_c/P_c$ AND OTHER FEATURES SUCH AS SOLIDITY.

12. PROPELLER GEOMETRY IS DEFINED BY THE TABLE OF VALUES OF r, $c(r)$, $\beta(r)$ AND THE AIRFOIL SECTION.

PROPELLER DESIGN PROGRAM

V	=	161.33	DT	=	5.75	BHP	=	70.00	CP	=	0.0402	SOLIDITY =	0.058
RPM	=	2400.00	DH	=	1.00	THRUST	=	207.44	CT	=	0.0498	AF =	113.92
J	=	0.701	B	=	2	ALT	=	0.0	ETA	=	0.8693	VP =	0.2046

DEFINING AIRFOIL: NACA 4415 CL = 0.7000

I	R	CHORD	BETA	PHI	CCL	L/D	RN	MACH	A	AP
1	0.50	0.3353	56.42	54.75	0.2347	59.56	0.44	0.18	0.0333	0.0626
2	0.62	0.3966	50.50	48.83	0.2776	64.02	0.56	0.20	0.0435	0.0533
3	0.74	0.4325	45.48	43.81	0.3028	67.41	0.67	0.22	0.0525	0.0452
4	0.86	0.4484	41.24	39.57	0.3139	69.92	0.77	0.24	0.0601	0.0383
5	0.97	0.4501	37.64	35.97	0.3151	71.78	0.84	0.26	0.0663	0.0326
6	1.09	0.4423	34.57	32.90	0.3096	73.15	0.90	0.29	0.0715	0.0280
7	1.21	0.4285	31.94	30.27	0.3000	74.15	0.94	0.31	0.0757	0.0241
8	1.33	0.4110	29.66	27.99	0.2877	74.85	0.97	0.33	0.0792	0.0210
9	1.45	0.3913	27.68	26.01	0.2739	75.32	0.99	0.36	0.0821	0.0183
10	1.57	0.3704	25.95	24.28	0.2593	75.56	1.00	0.38	0.0845	0.0162
11	1.69	0.3487	24.42	22.75	0.2441	75.61	1.01	0.41	0.0865	0.0143
12	1.81	0.3265	23.06	21.39	0.2286	75.51	1.00	0.43	0.0882	0.0128
13	1.92	0.3040	21.85	20.18	0.2128	75.19	0.99	0.46	0.0897	0.0114
14	2.04	0.2810	20.77	19.10	0.1967	74.66	0.96	0.48	0.0909	0.0103
15	2.16	0.2572	19.79	18.12	0.1800	73.88	0.93	0.51	0.0920	0.0093
16	2.28	0.2324	18.90	17.23	0.1627	72.78	0.88	0.53	0.0929	0.0085
17	2.40	0.2059	18.10	16.43	0.1441	71.24	0.82	0.56	0.0937	0.0078
18	2.52	0.1768	17.36	15.69	0.1238	69.08	0.73	0.58	0.0945	0.0071
19	2.64	0.1433	16.69	15.02	0.1003	65.83	0.62	0.61	0.0951	0.0066
20	2.76	0.1006	16.07	14.40	0.0704	60.27	0.45	0.64	0.0956	0.0061
21	2.87	0.0	15.50	13.83	0.0	54.72	0.0	0.66	0.0960	0.0057

PROPELLOR ANALYSIS PROGRAM

V	=	161.33	DT	=	5.75	BHP	=	70.00	CP	=	0.0402	SOLIDITY	=	0.058
RPM	=	2400.00	DH	=	1.00	THRUST	=	207.45	CT	=	0.0498	AF	=	113.92
J	=	0.701	B	=	2	ALT	=	0.0	ETA	=	0.8693	DBETA	=	0.0

I	R	CHORD	BETA	PHI	CL	L/D	RN	MACH	A	AP
1	0.50	0.3353	56.42	54.75	0.7000	59.56	0.44	0.18	0.0333	0.0626
2	0.62	0.3966	50.50	48.83	0.7000	64.02	0.56	0.20	0.0435	0.0533
3	0.74	0.4325	45.48	43.81	0.7000	67.42	0.67	0.22	0.0525	0.0452
4	0.86	0.4484	41.24	39.57	0.7000	69.92	0.77	0.24	0.0601	0.0383
5	0.97	0.4501	37.64	35.97	0.7000	71.78	0.84	0.26	0.0663	0.0326
6	1.09	0.4423	34.57	32.90	0.7000	73.15	0.90	0.29	0.0715	0.0280
7	1.21	0.4285	31.94	30.27	0.7000	74.15	0.94	0.31	0.0757	0.0241
8	1.33	0.4110	29.66	27.99	0.7000	74.85	0.97	0.33	0.0792	0.0210
9	1.45	0.3913	27.68	26.01	0.7000	75.32	0.99	0.36	0.0821	0.0183
10	1.57	0.3704	25.95	24.28	0.7000	75.56	1.00	0.38	0.0845	0.0162
11	1.69	0.3487	24.42	22.75	0.7000	75.61	1.01	0.41	0.0865	0.0143
12	1.81	0.3265	23.06	21.39	0.7000	75.52	1.00	0.43	0.0883	0.0128
13	1.92	0.3040	21.85	20.18	0.7000	75.20	0.99	0.46	0.0897	0.0114
14	2.04	0.2810	20.77	19.10	0.7000	74.66	0.96	0.48	0.0909	0.0103
15	2.16	0.2572	19.79	18.12	0.7000	73.88	0.93	0.51	0.0920	0.0093
16	2.28	0.2324	18.90	17.23	0.7000	72.78	0.88	0.53	0.0929	0.0085
17	2.40	0.2059	18.10	16.43	0.7000	71.25	0.82	0.56	0.0938	0.0078
18	2.52	0.1768	17.36	15.69	0.7000	69.08	0.73	0.58	0.0945	0.0071
19	2.64	0.1433	16.69	15.02	0.7000	65.83	0.62	0.61	0.0951	0.0066
20	2.76	0.1006	16.07	14.40	0.7000	60.28	0.45	0.64	0.0956	0.0061
21	2.88	0.0	15.50	13.83	0.0	64.59	0.0	0.66	0.0961	0.0056

I	R(I)/RT	FY(I)	FX(I)	FN(I)	FC(I)	MY(I)	MX(I)	MN(I)	MC(I)	MT(I)
1	0.1739	104.5143	46.3362	96.4058	-61.4508	-54.0749	145.4002	91.2352	125.4650	-5.4490
2	0.2152	103.7369	45.1958	100.8571	-51.3016	-48.7132	133.0888	71.7137	122.2405	-5.3763
3	0.2565	102.4699	43.7005	103.0031	-42.4286	-43.5284	120.9288	55.7115	115.8219	-5.2542
4	0.2978	100.6414	41.8933	103.2946	-34.8421	-38.5575	108.9866	42.8517	107.3708	-5.0795
5	0.3391	98.2089	39.8238	102.0885	-28.4405	-33.8316	97.3336	32.6510	97.7359	-4.8545
6	0.3804	95.1525	37.5400	99.6522	-23.0799	-29.3763	86.0435	24.6336	87.5193	-4.5846
7	0.4217	91.4692	35.0851	96.1831	-18.6118	-25.2119	75.1908	18.3794	77.1459	-4.2770
8	0.4630	87.1684	32.4969	91.8280	-14.8993	-21.3544	64.8484	13.5360	66.9186	-3.9390
9	0.5043	82.2691	29.8083	86.7004	-11.8231	-17.8157	55.0874	9.8152	57.0586	-3.5781
10	0.5457	76.7993	27.0486	80.8925	-9.2821	-14.6044	45.9755	6.9847	47.7309	-3.2014
11	0.5870	70.7954	24.2444	74.4853	-7.1917	-11.7256	37.5758	4.8573	39.0619	-2.8158
12	0.6283	64.3033	21.4207	67.5553	-5.4813	-9.1819	29.9462	3.2830	31.1496	-2.4280
13	0.6696	57.3800	18.6022	60.1811	-4.0923	-6.9724	23.1376	2.1408	24.0703	-2.0445
14	0.7109	50.0968	15.8146	52.4493	-2.9757	-5.0936	17.1927	1.3334	17.8817	-1.6720
15	0.7522	42.5413	13.0849	44.4591	-2.0903	-3.5384	12.1433	0.7817	12.6242	-1.3173
16	0.7935	34.8250	10.4439	36.3303	-1.4013	-2.2963	8.0081	0.4218	8.3202	-0.9877
17	0.8348	27.0903	7.9278	28.2128	-0.8794	-1.3524	4.7886	0.2020	4.9718	-0.6907
18	0.8761	19.5247	5.5814	20.3007	-0.4992	-0.6862	2.4637	0.0802	2.5563	-0.4345
19	0.9174	12.3858	3.4639	12.8588	-0.2387	-0.2702	0.9809	0.0229	1.0172	-0.2276
20	0.9587	6.0560	1.6623	6.2795	-0.0789	-0.0461	0.1678	0.0022	0.1740	-0.0794
21	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

EXAMPLE 1

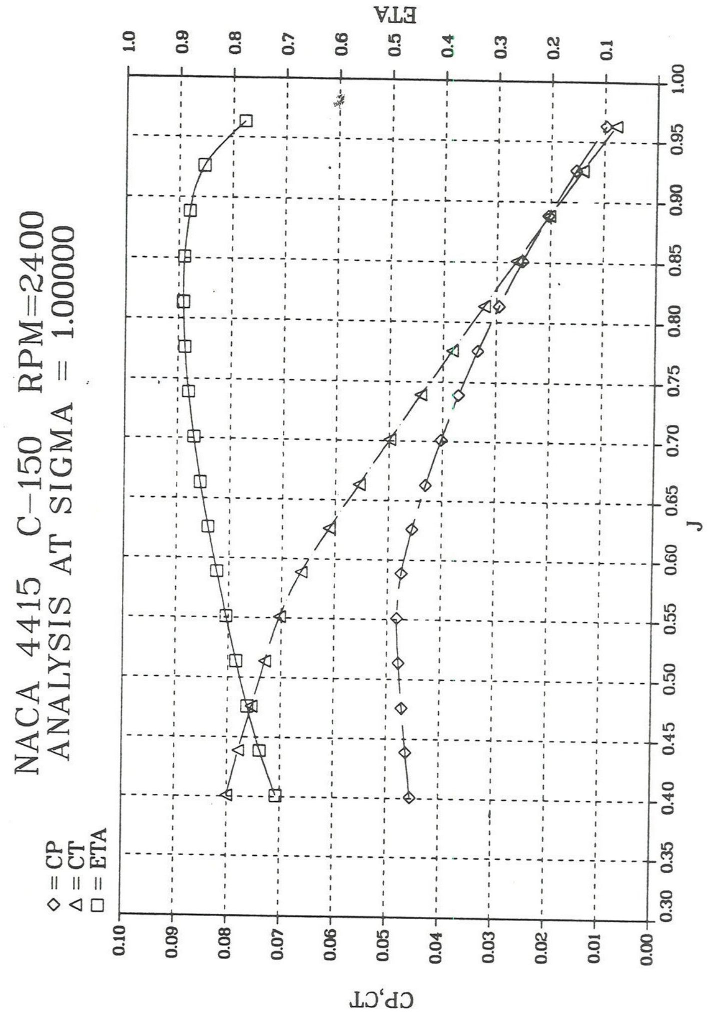
BASE 1 σ' = .2170, .2100, .2102
IDEAL 1 σ' = .2241, .2148, .2152

	BASE 1	IDEAL 1	VP 1
η_{DES}	.8773	.9029	.8654
η_{ANAL}	.8773	.8757	.8750

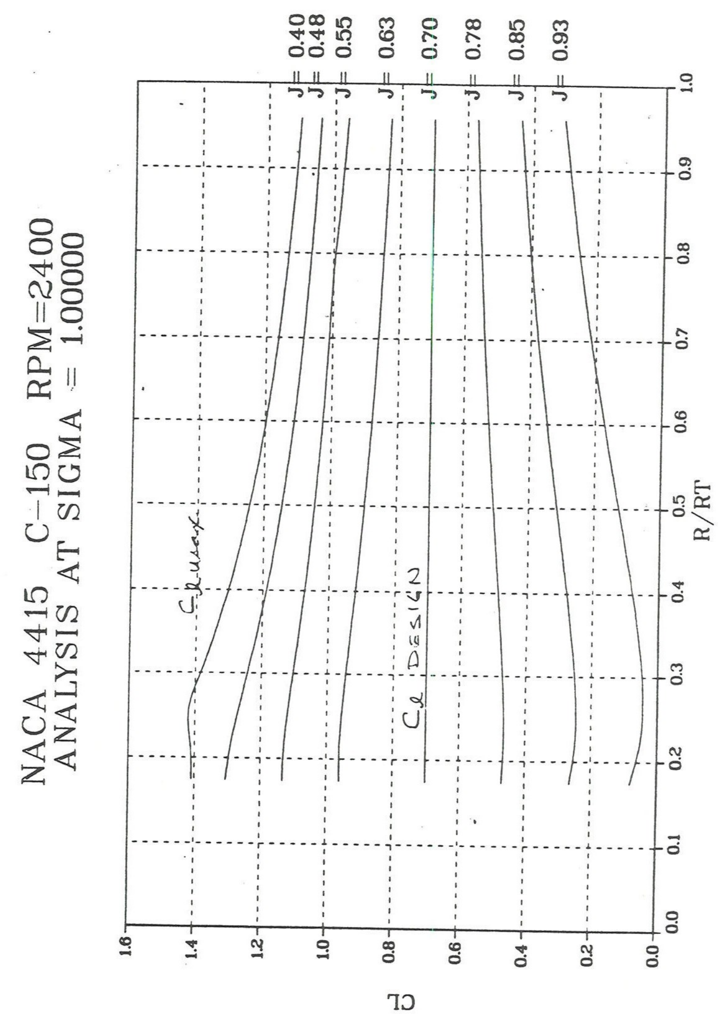
EXAMPLE 2

BASE 2 σ' = .3531, .3374, .3380
IDEAL 2 σ' = .3641, .3431, .3440

	BASE 2	IDEAL 2	VP 2
η_{DES}	.8344	.8532	.8135
η_{ANAL}	.8344	.8325	.8295

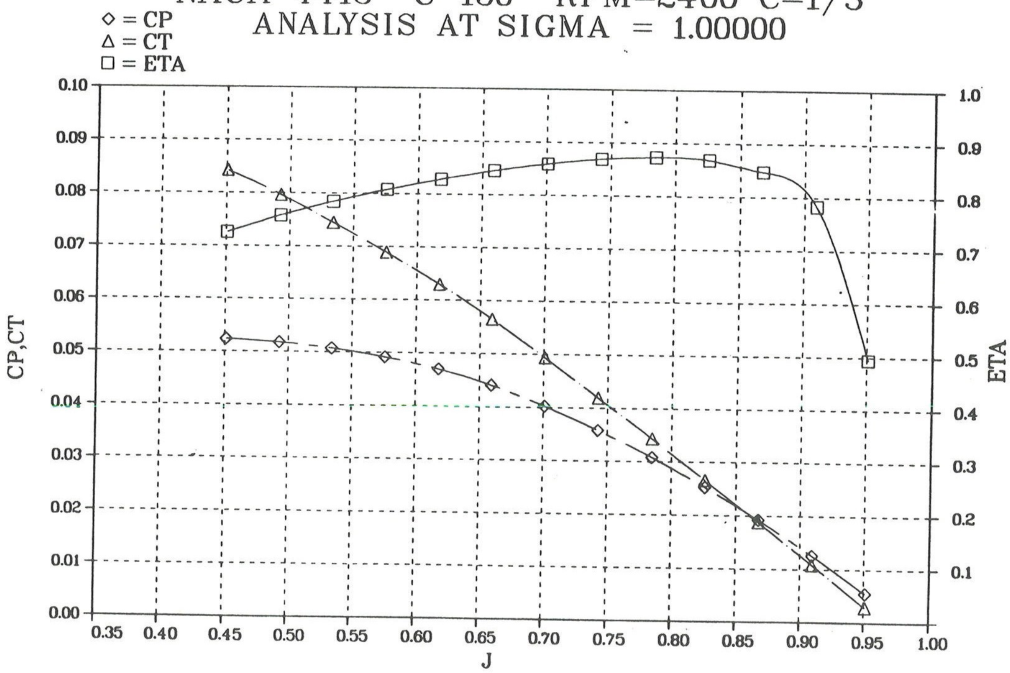


NACA 4415 C−150 RPM=2400
ANALYSIS AT SIGMA = 1.00000
◇ = CP
△ = CT
□ = ETA
CP,CT
ETA
J

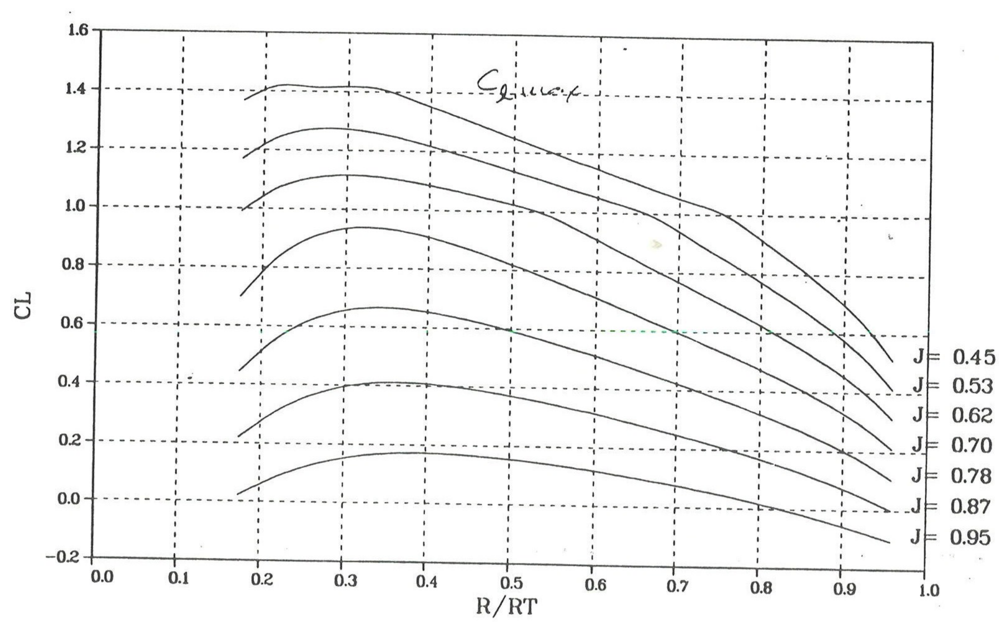


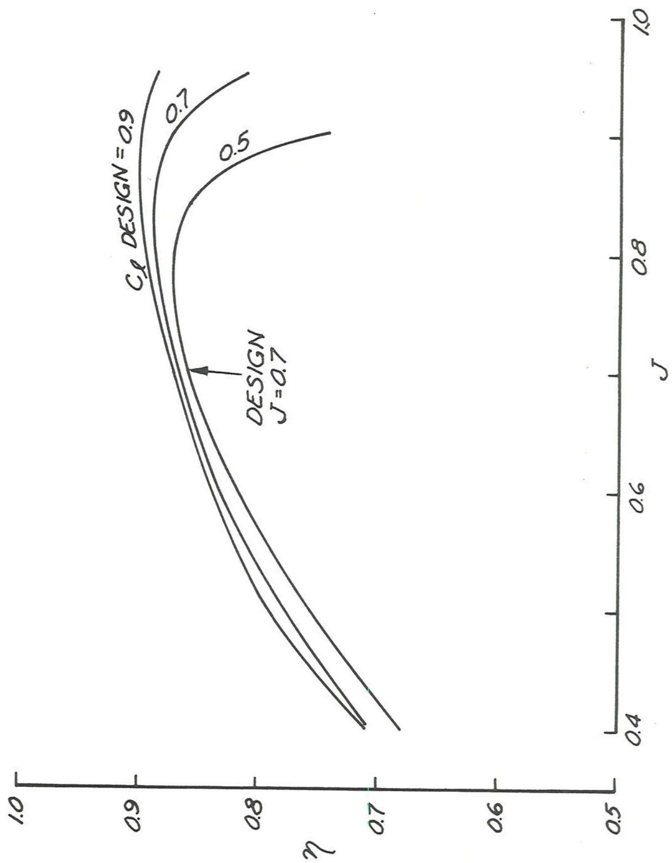
NACA 4415 C−150 RPM=2400
ANALYSIS AT SIGMA = 1.00000
J= 0.40
J= 0.48
J= 0.55
J= 0.63
J= 0.70
J= 0.78
J= 0.85
J= 0.93
Cl max
Cl DESIGN
CL
R/RT

NACA 4415 C−150 RPM=2400 C=1/3
ANALYSIS AT SIGMA = 1.00000



NACA 4415 C−150 RPM=2400 C=1/3
ANALYSIS AT SIGMA = 1.00000





46

AIAA'83

AIAA-83-0190
Design of Optimum Propellers

C.N. Adkins, Consulting Engineer, Falls Church, VA; and R.H. Liebeck, Douglas Aircraft Co., Long Beach, CA

AIAA 21st Aerospace Sciences Meeting

January 10-13, 1983/Reno, Nevada

DESIGN OF OPTIMUM PROPELLERS

Charles N. Adkins*
Falls Church, Virginia

and

Robert H. Liebeck**
Douglas Aircraft Company
Long Beach, California
and
University of Southern California
Los Angeles, California

Abstract

Improvements have been made in the equations and computational procedures for the design of propellers and wind turbines of maximum efficiency. These eliminate the small angle approximation and many of the "lightly loaded" approximations prevalent in the classical design theory. Though wake contraction is still neglected, certain viscous terms have been added to the induced velocities which are important at low Reynolds numbers or high profile drag. An iterative scheme is introduced for accurate calculation of the vortex displacement velocity and the flow angle distribution. Momentum losses due to radial flow can be estimated by either the Prandtl or Goldstein momentum loss function. For the less complex Prandtl function, the "lightly loaded" approximation can be eliminated for both design and analysis. The methods presented here now bring into exact agreement the procedures for design and analysis even when applied to cases of low Reynolds number and large disk loading. Furthermore, the exactness of this agreement makes possible an empirical verification of the Betz condition that a constant displacement velocity across the wake provides a design of maximum propeller efficiency.

Nomenclature

a = axial interference factor
a' = rotational interference factor
B = number of blades
b = axial slipstream factor
c = blade section chord
C_d = blade section drag coefficient
C_ℓ = blade section lift coefficient
C_P = power coefficient = $P/\rho\ n^3\ D^5$
C_T = thrust coefficient = $T/\rho\ n^2\ D^4$
C_x = torque force coefficient
C_y = thrust force coefficient
D = propeller diameter = 2R
D' = drag force per unit radius
F = Prandtl momentum loss factor
G = circulation function
J = advance ratio = V/nD
L' = lift force per unit radius
n = propeller revolutions per second
P = power into propeller
P_c = power coefficient = $2\ P/\rho\ V^3\ \pi\ R^2$
Q = torque
R = propeller tip radius
r = radial coordinate
T = thrust
T_c = thrust coefficient = $2\ T/\rho\ V^2\ \pi\ R^2$
V = freestream velocity
v' = vortex displacement velocity
W = local total velocity
w_n = velocity normal to the vortex sheet
w_t = tangential (swirl) velocity
x = nondimensional distance = Ω r/V
α = angle of attack
β = blade twist angle
Γ = circulation
ϵ = drag to lift ratio
ζ = displacement velocity ratio = v'/V
η = propeller efficiency
λ = speed ratio = V/Ω R
ξ = nondimensional radius = r/R = λ x
ξ_o = nondimensional hub radius
ξ_e = nondimensional Prandtl radius
ρ = fluid density
σ = local solidity = Bc/2 π r
ϕ = flow angle
ϕ_t = flow angle at the tip
Ω = propeller angular velocity

Superscripts
' (prime) = derivative with respect to r or ξ, unless otherwise noted

* Consulting Engineer
Member AIAA
**Principal Engineer, Aerodynamics Subdivision
Professor, Aerospace Engineering Department
Associate Fellow AIAA

Introduction

In 1936, a classic treatise on propeller theory was authored by H. Glauert.[1] Here a combination of momentum theory and blade element theory, when corrected for momentum loss due to radial flow, provide a good method for analysis of arbitrary designs even though contraction of the propeller wake is neglected. Though the theory is developed for low disk loading (small thrust or power per unit disk area) it works quite well for moderate loading and, in light of its simplicity, is adequate for estimating performance even for high disk loadings. The conditions under which a design would have minimum energy loss were stated by A. Betz[2] as early as 1919, however, no organized procedure for producing such a design is evident in Glauert's work. Those equations which are given make extensive use of small angle approximations and relations applicable only to light loading conditions. The momentum blade element theory for analysis took several decades to become generally accepted only as blade section data and propeller test procedures became sufficiently accurate to validate its usefulness. But the design equations, in their rudimentary form, laid fallow for more decades still, yielding to experiment and experience for the creation of new designs.

Quite recently (1979), E. Larrabee[3] resurrected the design equations and presented a straightforward procedure for optimum design. However, there are still three problems: first, small angle approximations are used; second, the solution for the displacement velocity is accurate only for vanishingly small values (light loading), though an approximate correction is suggested for moderate loading; and third, there are viscous terms missing in the expressions for the induced velocities. The purpose of this paper is to correct these difficulties and bring the design method into exact agreement with that of analysis. With this done, it is now possible to verify empirically the optimality of design.

The Momentum Equations

Detailed axial and general momentum theory is described by Glauert[1], and only a brief summary is given here to point out several important features. Consider a fluid element of mass dm far upstream moving toward the propeller disk in a thin annular stream tube with velocity V. It arrives at the disk with increased velocity V(1 + a), where a is called the axial interference factor. At the disk, dm exist in the annulus 2 π r dr and the mass rate per unit radius passing through the disk is 2 π r ρ V(1 + a), neglecting radial flow. The element dm moves downstream into the far wake increasing speed to the value V(1 + b), where b is called the axial slipstream factor. Axial momentum theory determines b to be exactly 2a, whereas the general theory (which includes rotation of the flow) determines b to be approximately 2a. Using the axial approximation, which is generally accepted, the overall change in momentum of the element is 2 V a F dm where F, the momentum loss factor, accounts for radial flow of the fluid. The thrust per unit radius T' acting on the annulus can now be expressed as

$$T' = dT/dr = 2\ \pi\ r\ \rho\ V(1 + a)(2\ V\ a\ F) \qquad (1a)$$

By similar arguments, the torque per unit radius Q' is given by

$$Q'/r = 2\ \pi\ r\ \rho\ V(1 + a)(2\ \Omega\ r\ a'\ F) \qquad (1b)$$

where Ω is the angular velocity of the propeller and a' is called the rotational interference factor. Flow geometry about a blade element at the disk is shown in Figure 1, where the total local velocity W acts on the blade element with angle of attack α and acts on the disk at the flow angle ϕ. The momentum loss factor F goes from about 1 at the hub (where the radial flow is typically negligible), to zero at the tip, and is not unlike the spanwise loading of a wing. The functional form of this factor was first estimated by Prandtl[1,2], and a more accurate (though more complex) form was determined by Goldstein[4] and Lock.[5,6,7]

The Circulation Equations

At each radial position along the blade, infinitesimal vortices are shed and move aft as a helicoidal vortex sheet. Since these vortices follow the direction of local flow, the helix angle of the spiral surface is the flow angle ϕ shown in Figure 1. The Betz condition for minimum energy loss, neglecting contraction of the wake, is for the vortex sheet to be a regular screw surface; i.e. r tanϕ must be a constant independent of radius. This optimum vortex sheet acts as an Archimedean screw, pumping fluid aft between rigid spiral surfaces.

At the blade station r, the total lift per unit radius is given by

$$L' = dL/dr = B\ \rho\ W\ \Gamma \qquad (2)$$

and in the wake, the circulation in the corresponding annulus is

$$B\ \Gamma = 2\ \pi\ r\ F\ w_t \qquad (3)$$

where w_t is the local tangential velocity in the wake and B is the number of blades. Setting the circulation Γ in (2) equal to that in (3) will ultimately determine that circulation distribution Γ(r) which minimizes the induced power of the propeller.

In order to obtain Γ (r), it is necessary to relate w_t to a more measurable quantity. Figure 2 shows the wake vortex filament at station r and the definition of the various velocity components there. The motion of the fluid must be normal to the local vortex sheet, and this normal velocity is called w_n. Therefore, the tangential velocity is given by

$$w_t = w_n \sin\phi$$

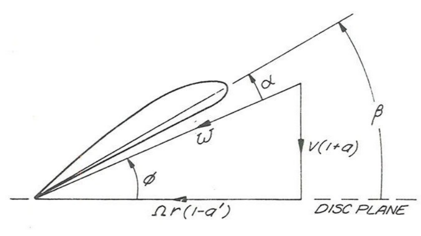


Fig. 1 Flow geometry for blade element at radial station r.

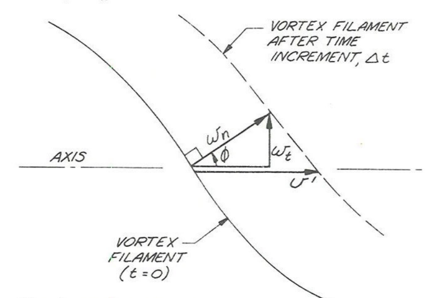


Fig. 2 Definition of the displacement velocity v' in the propeller wake.

However, for a coordinate system fixed to the propeller disk, the axial velocity of the vortex filament would be

$$v' = w_n/\cos\phi$$

where the increase in magnitude of v' over w_n is due to rotation of the filament. This is analogous to a barber pole where it appears that the stripes are translating in spite of the fact that only a rotational velocity exists. It will become clear that it is convenient to use the vortex displacement velocity v' and the corresponding displacement velocity ratio $\zeta = v'/V$. The tangential velocity is then

$$w_t = V\,\zeta\,\sin\phi\,\cos\phi$$

and the circulation of (3) can be expressed as

$$\Gamma = 2\,\pi\,V^2\,\zeta\,G/(B\,\Omega) \qquad (4)$$

where G is the circulation function

$$G = F\,x\,\cos\phi\,\sin\phi \qquad (5)$$

and x is the speed ratio given by

$$x = \Omega\,r/V.$$

The circulation equations for thrust T' and torque Q' per unit radius can be written by inspection of Figure 3 as

$$T' = L'\cos\phi - D'\sin\phi$$
$$= L'\cos\phi\,(1 - \varepsilon\tan\phi) \qquad (6a)$$

$$Q'/r = L'\sin\phi + D'\cos\phi$$
$$= L'\sin\phi\,(1 + \varepsilon/\tan\phi) \qquad (6b)$$

where ε is the drag-to-lift-ratio of the blade element. Next, using (2), the lift per unit radius L' can be replaced by Γ (r) which in turn is related to conditions in the wake by (3). Based on the flow in the wake, Γ (r) is given by (4) and (5), and T' and Q'/r are reduced to being functions of ϕ and the displacement velocity $\zeta = v'/V$. The local flow angle ϕ will clearly be a function of the radius; however, at this stage of the analysis, the optimum distribution ζ (r) is not yet determined. Several diagrams and an excellent photograph of the vortex sheet can be found in Reference 8.

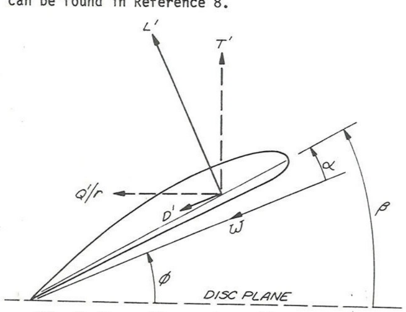


Fig. 3 Force diagram for a blade element.

The Condition for Minimum Energy Loss

At this point, a departure from Larrabee's[3] design procedure is made, and the momentum equation (1) and the circulating equations (6) are required to be exactly equal. This condition results in the interference factors being related to the displacement velocity ratio ζ by the equations

$$a = (\zeta/2)\cos^2\phi\,(1-\varepsilon\tan\phi) \qquad (7a)$$

$$a' = (\zeta/2x)\cos\phi\,\sin\phi\,(1 + \varepsilon/\tan\phi) \qquad (7b)$$

where the terms in epsilon correctly describe the viscous contribution. Equations (7), together with the geometry of Figure 1, lead to the very important simple relation

$$\tan\phi = (1 + \zeta/2)/x = (1 + \zeta/2)\lambda/\xi \qquad (8)$$

Here, λ is the speed ratio $V/\Omega R$ (a constant) and ξ is the nondimensional radius r/R which varies from ξ_0 at the hub to unity at the edge of the disk. The relation between the two nondimensional distances and the constant speed ratio is

$$x = \Omega\,r/V = (r/R)/\lambda = \xi/\lambda.$$

Recalling the Betz condition, r $\tan\phi$ = constant, (8) proves that in order for the vortex sheet to be a regular screw surface, the displacement velocity ζ must be a constant independent of radius, and this then is the condition for minimum energy loss.

The Constraint Equations

For design, it is necessary to specify either the thrust T delivered by the propeller or the power P delivered to the propeller. The nondimensional thrust and power coefficients used for design are

$$T_c = 2\,T/(\rho\,V^2\,\pi\,R^2) \qquad (9a)$$

$$P_c = 2\,P/(\rho\,V^3\,\pi\,R^2)$$
$$= 2\,Q\,\Omega/(\rho\,V^3\,\pi\,R^2) \qquad (9b)$$

and using these definitions, (6) can be written as

$$T_c' = I_1'\,\zeta - I_2'\,\zeta^2 \qquad (10a)$$

$$P_c' = J_1'\,\zeta + J_2'\,\zeta^2 \qquad (10b)$$

where the primes denote derivatives with respect to ξ, and

$$I_1' = 4\,\xi\,G\,(1 - \varepsilon\tan\phi) \qquad (11a)$$

$$I_2' = \lambda\,(I_1'/2\,\xi)(1 + \varepsilon/\tan\phi)\,\sin\phi\,\cos\phi \qquad (11b)$$

$$J_1' = 4\,\xi\,G\,(1 + \varepsilon/\tan\phi) \qquad (11c)$$

$$J_2' = (J_1'/2)(1 - \varepsilon\tan\phi)\cos^2\phi \qquad (11d)$$

Since ζ is constant for an optimum design, a specified thrust gives the constraint equations

$$\zeta = (I_1/2\,I_2) - [(I_1/2\,I_2)^2 - T_c/I_2]^{1/2} \qquad (12)$$

$$P_c = J_1\,\zeta + J_2\,\zeta^2 \qquad (13)$$

Similarly, if power is specified, the constraint relations are

$$\zeta = -(J_1/2\,J_2) + [(J_1/2\,J_2)^2 + P_c/J_2]^{1/2} \qquad (14)$$

$$T_c = I_1\,\zeta - I_2\,\zeta^2 \qquad (15)$$

where the integration has been carried out over the region $\xi = \xi_0$ to $\xi = 1$.

Blade Geometry

In the element dr of a single blade, let c be the chord and C_ℓ the local lift coefficient. Then, the lift per unit radius of one blade is

$$\rho\,W^2\,c\,C_\ell/2 = \rho\,W\,\Gamma$$

where Γ is given by (4). It follows directly that

$$Wc = 4\,\pi\,\lambda\,G\,V\,R\,\zeta/(C_\ell\,B). \qquad (16)$$

Assume for the moment that the constant displacement velocity ζ is known, then the local value of ϕ is known from (8) and the above relation is a function only of the local lift coefficient. Since the local Reynolds number is Wc divided by the kinematic viscosity, (16) plus a choice for C_ℓ will determine the Reynolds number and also the drag-to-lift ratio ε from the airfoil section data. The total velocity is then determined by Figure 1 as

$$W = V(1 + a)/\sin\phi \qquad (17)$$

where a is given by (7), and the chord is then known from (16). If the choice for C_ℓ causes ε to be a minimum, then viscous as well as momentum losses will be minimized, and overall propeller efficiency will be the highest possible value. If the blade strength is found to be insufficient at the hub, C_ℓ can be reduced to increase the chord, or a thicker section can be used in that region. For preliminary considerations, it is usually sufficient to choose one C_ℓ, the design C_ℓ, for determining blade geometry. Since angle of attack α is known from C_ℓ and Reynolds number, the blade twist with respect to the disk is $\beta = \alpha + \phi$. G is zero at the edge of the disk, and therefore the tip chord is always zero for a finite lift coefficient.

The Design Procedure

Either the Prandtl or Goldstein relation for the momentum loss function F can be selected. For simplicity, only the Prandtl relation is described as

$$F = (2/\pi)\,\mathrm{arc\,cos}(e^{-f})^* \qquad (18)$$

*For computational purposes, the form $F = (2/\pi)$ arc $\tan[(e^{2f} - 1)^{1/2}]$ is more tractable.

where

$$f = (B/2)(1 - \xi)/\sin\phi_t \qquad (19)$$

and ϕ_t is the flow angle at the tip. From (8)

$$\tan\phi_t = \lambda\,(1 + \zeta/2) \qquad (20)$$

so that a choice for ζ determines the function F as well as ϕ by

$$\tan\phi = (\tan\phi_t)/\xi \qquad (21)$$

which is simply the condition that the vortex sheet in the wake is a rigid screw surface (r $\tan\phi$ = constant).

Next, an initial estimate for the displacement velocity ratio is chosen as either zero, the lightly loaded approximation, or by the estimate

$$\zeta \simeq 0.5\,P_c/(\xi_e^2 - \xi_o^2) \simeq 0.5\,T_c/(\xi_e^2 - \xi_o^2)$$

where ξ_o is the nondimensional hub-radius and ξ_e is the effective Prandtl radius given by

$$\xi_e \simeq 1 - 1.386\,\lambda/B.$$

A nonzero estimate for ζ will only reduce computational time, not accuracy.

The design is initiated with the specified conditions of power (or thrust), hub and tip radius, rotational rate, freestream velocity, number of blades, and a finite number of stations at which blade geometry is to be determined. Also, the design lift coefficient, one for each station, if it is not constant, must be specified. The design procedure can then be described by the following steps:

1. Select an initial estimate for ζ (ζ = 0 will work).
2. Determine the values for F and ϕ at each blade station by (18-21).
3. Determine Wc and Reynolds number from (16).
4. Determine ε and α from airfoil section data.
5. If ε is to be minimized, change C_ℓ and repeat steps 3 and 4 until this is accomplished at each station.
6. Determine a and a' from (7) and W from (17).
7. Compute the chord from step 3 and the blade twist $\beta = \alpha + \phi$.
8. Determine the four derivatives in I and J from (11) and numerically integrate these from $\xi = \xi_0$ to $\xi = 1$.
9. Determine ζ and P_c from (12-13), or ζ and T_c from (14-15).
10. If this new value for ζ is not sufficiently close to the old one (say, within 0.1%) start over at step 2 using the new ζ.
11. Determine propeller efficiency as T_c/P_c, and other features such as solidity, etc.

The above steps converge rapidly with a direct substitution for ζ, seldom taking more than three trials. An accurate description of viscous losses can be obtained by performing a second design with ε equal to zero and noting the difference in propeller efficiency.

Analysis of Arbitrary Designs

The analysis method is outlined here in order to discuss problems of convergence for off design and for square-tipped propellers in general, and to point out two minor errors in Glauert's work. Figure 4 (which is simply an alternate version of Figure 3) shows the relation between the propeller force coefficients C_y and C_x and the airfoil coefficients C_ℓ and C_d. The equations are

$$C_y = C_\ell \cos\phi - C_d \sin\phi$$

$$C_x = C_\ell \sin\phi + C_d \cos\phi$$

and the relations for the thrust T' and torque Q' per unit radius are then

$$T' = (1/2)\, \rho\, W^2\, Bc\, C_y \qquad (22a)$$

$$Q'/r = (1/2)\, \rho\, W^2\, Bc\, C_x \qquad (22b)$$

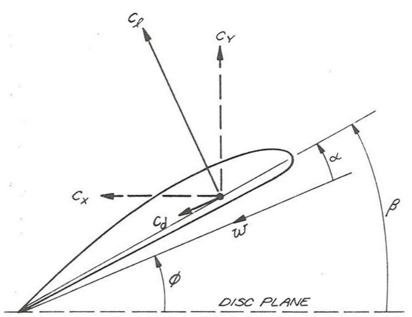


Fig. 4 Force coefficients for propeller blade element analysis.

Again, it is required that the loading (22) be exactly equal to the momentum result (1). With the use of the flow geometry in Figure 1, this requires the interference factors to be

$$a = \sigma K/(F - \sigma K) \qquad (23a)$$

$$a' = \sigma K'/(F + \sigma K') \qquad (23b)$$

where

$$K = C_y/(4 \sin^2\phi) \qquad (24a)$$

$$K' = C_x/(4 \cos\phi \sin\phi) \qquad (24b)$$

and σ is the local solidity given by

$$\sigma = Bc/(2\, \pi\, r).$$

Equations (23) correct the placement of the factor F used by Glauert in his Equations (5.5) of Chapter VII as identified by Larrabee.[3]

The relation for the flow angle is obtained from Figure 1 and (23) as

$$\tan\phi = [V(1 + a)]/[\Omega\, r(1 - a')]. \qquad (25)$$

For determining the function F in (18), Glauert suggests the relation $\sin\phi_t = \xi \sin\phi$ for use in (19). It is recommended that (21) be used instead, i.e.

$$\tan\phi_t = \xi \tan\phi$$

which is exact for the analysis of an optimumly designed propeller at the design point.

The analysis procedure requires an iterative solution for the flow angle ϕ at each radial position ξ. An initial estimate for ϕ can be obtained from (8) by setting ζ equal to zero. Since the blade twist β is known, the value for α in Figure 3 is $\beta - \phi$ and the airfoil coefficients are known from the section data. The Reynolds number is determined from the known chord and the total velocity W which is obtained from Figure 1 and (23a), and the new estimate for ϕ is then found from (25). A direct substitution of the new ϕ for the old value will cause adequate convergence for an optimum design which is being analyzed at the design point. However, for analysis off design and for non-optimum designs, some combination of the old and new values for ϕ is required to cause adequate convergence. Under some conditions (usually near the tip) convergence may not be possible at all due to large values for the interference factors a and a' in (23). Since F is zero at the tip and σ is not, for a square tip propeller, the value for a is -1 and a' is +1. Such values are physically impossible since the slipstream factors are approximately twice the values at the rotor plane. Wilson and Lissaman[9] suggest empirical relations for resolving this problem, whereas Viterna and Janetzke[10] give empirical arguments for clipping the magnitude of a and a' at the value 0.7 (σ/F at the tip is finite at the design point for an optimum propeller).

For analysis, the conventional thrust and power coefficients are

$$C_T = T/(\rho\, n^2\, D^4)$$

$$C_P = P/(\rho\, n^3\, D^5)$$

where n is revolutions per second and D is the propeller diameter. Using (22) and (24), the differential forms, with respect to ξ, are given by

$$C_T' = (\pi^3/4)\, \sigma\, C_y\, \xi^3\, F^2/[(F + \sigma K') \cos\phi]^2$$

$$C_P' = C_T'\, \pi\, \xi\, C_x/C_y.$$

When these have been integrated from the hub to the tip, the propeller efficiency is

$$\eta = C_T\, J/C_P$$

where $J = V/(nD)$ is the advance-diameter ratio.

Propeller performance is typically described by plots of C_T, C_P, and η vs. J.

Airfoil Section Data

In the previous discussions, reference has been made to the use of airfoil characteristics in the form of lift curves and drag polars. This data should include a reliable estimate of the drag variation with respect to Reynolds number over that range in which the propeller is expected to operate. In addition, a detailed knowledge of the airfoil's post-stall behavior up to an angle of attack of 90 degrees is essential for predicting off-design performance, particularly in the case of wind turbines. Such data may be difficult to obtain; however, its importance should not be underestimated.

Empirical Optimality

In Chapter VII of Glauert's work, his equation (2.20) shows that when blade friction is neglected the most favorable distribution of circulation is one where the displacement velocity is constant across the wake. Here, the term $x^2/(1 + x^2)$ is the small angle approximation of the circulation function G given by (5) of this paper. The effect of profile drag is shown by Glauert in his equation (3.5) which states that the optimum distribution for the displacement velocity ratio is

$$\zeta = \zeta_0 - \varepsilon\, x \qquad (26)$$

where the effect of profile drag on thrust has been ignored. In order to study this problem empirically, consider a general function H(x) and the two first-order terms of its Laurent series, 1/x and x, and describe the displacement velocity distribution as

$$\zeta = \zeta_0 + \delta_1\, x + \delta_2/x \qquad (27)$$

which includes the case of (26). It is desired to find values for δ_1 and δ_2 which maximize propeller efficiency subject to the constraints of (10). To solve this problem, ζ in (10) is replaced by (27). Then, a choice for δ_1 and δ_2 will enable a determination of ζ_0 and a calculation of overall propeller efficiency. A systematic study of various propeller conditions was undertaken using the design/analysis procedures of this paper. Under no conditions could nonzero values for δ_1 and δ_2 be found which caused an increase in propeller efficiency. Therefore, it was concluded that a constant displacement velocity is at least locally optimum whether profile drag is considered or not. The momentum and viscous losses are then uncoupled; the former is minimized by constant displacement velocity, the latter by choosing a C_ℓ-distribution so that the drag-lift ratio is everywhere a minimum.

Windmills

All of the analyses described in this paper are directly applicable to the windmill problem after a minor adjustment in the angle definitions of Figure 1. The corresponding flow geometry for a windmill is shown in Figure 5, where the primary distinction is that the blade section is inverted (as compared with a propeller), and the local angle of attack is measured from below the local velocity vector. Corresponding relations for the angles are:

windmill $\alpha = \phi - \beta$
propeller $\alpha = \beta - \phi$

as shown in Figures 5 and 1, respectively. Referring again to Figures 1 and 5, C_ℓ for the windmill is negative with respect to that for the propeller, and this sign change together with the angle definition will convert the propeller methods to the windmill application. For the design case, the input P_c value should be negative, and the resulting values of v' (and the interference factors a and a') and T_c will also be negative. (Thrust is of less interest for a windmill since it typically represents the tower load and is not a main performance parameter.) Similarly, the analysis results for a windmill rotor will yield negative values for both P_c and T_c.

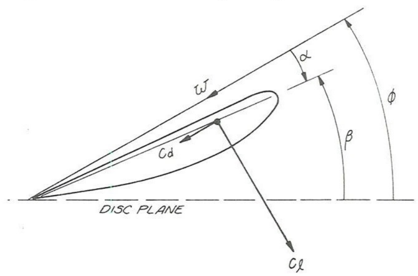


Fig. 5 Force coefficients for windmill blade element.

Example

As a sample calculation, the design of a propeller for a light airplane is considered, where the design conditions are:

hp = 70	no. of blades = 2
rpm = 2400	tip dia. = 5.75 ft.
V = 110 mph	hub dia. = 1.00 ft.
J = 0.7	NACA 4415 airfoil

The resulting design is described in Table I which gives for each radial station: blade chord, blade pitch angle, local flow angle, blade loading, blade section lift to drag ratio, local Reynolds number, local Mach number, and the interference coefficients a and a'.

This propeller geometry has in turn been analyzed at the design condition and the result is given in Table II. Agreement is virtually exact. Analysis over a range of values of the advance ratio $J = V/(nD)$ provides the typical propeller performance plots which are shown in Figure 6, and Figure 7 gives the blade lift coefficient distribution over a range of J's where the design condition is the $C_\ell = 0.7 =$ constant line at $J = 0.7$.

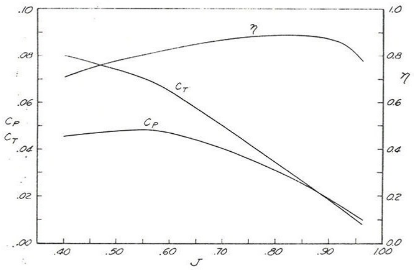


Fig. 6 Analysis results for propeller with C_ℓ = 0.7 = constant at the design condition.

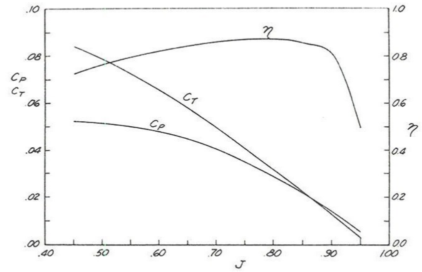


Fig. 8 Analysis results for propeller with chord = 4 in. = constant.

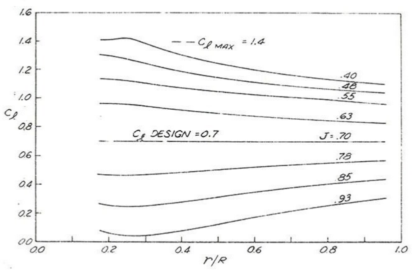


Fig. 7 Blade section C_ℓ variation for C_ℓ = 0.7 = constant at the design condition.

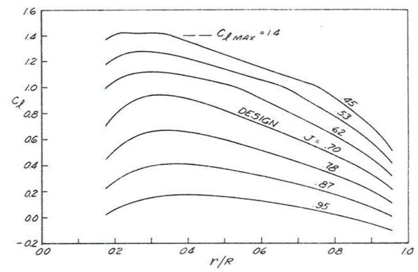


Fig. 9 Blade section C_ℓ variation for propeller with chord = 4 in. = constant.

The effect of variations in the design conditions can be readily evaluated. If, for example, the blade chord is held constant (which requires that the local blade C_ℓ varies as required by the optimum blade loading) the resulting performance is given in Figure 8. At the design point, the efficiency is only slightly reduced as compared with the constant C_ℓ design; however, the range of J over which the efficiency remains high is reduced for the constant chord design. The corresponding blade C_ℓ-distributions are shown in Figure 9 which indicates a wide range of local C_ℓ's at all values of J. This explains the reduced operational J-range since either the tip or the root of the blade quits "working" sooner than in the constant C_ℓ design.

Another interesting result is obtained by varying the design C_ℓ value, and the resulting performance for propeller designs for C_ℓ = 0.5 and C_ℓ = 0.9 is compared with that for C_ℓ = 0.7 in Figure 10. The efficiency is seen to increase with increasing C_ℓ; however the blade chord, and consequently the blade thickness for a given airfoil section, is reduced since the blade loading c C_ℓ is effectively the same for all three cases. Structural and other practical considerations will of course influence the choice of design C_ℓ, particularly near the hub.

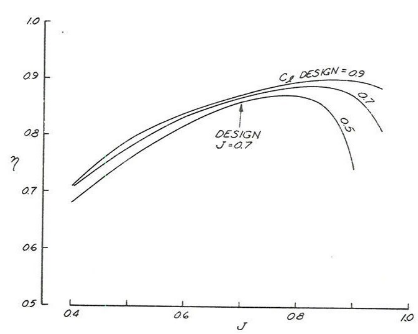


Fig. 10 Comparison of efficiencies of three propeller designs with design blade section lift coefficients of 0.5, 0.7, and 0.9.

Conclusions & Recommendations

The propeller theory of Glauert has been extended to improve the design of optimal propellers and refine the calculation of the performance of arbitrary propellers. Extensions of the theory include:

a) Elimination of the light loading and small angle assumptions in the optimal design theory.

b) Accurate calculation of the vortex displacement velocity which properly accounts for the blade section drag.

c) Elimination of the light loading and small angle assumptions in the Prandtl momentum loss function for both design and analysis.

Implementation of these extensions has brought the design and analysis procedures to exact numerical agreement within the precision of computer analysis.

The primary approximation remaining in both procedures is the use of the axial momentum equations which require the increase in wake velocities to be twice those at the disk. Under certain conditions this approximation is not good and gives rise to the unnatural conditions and convergence problems described in the analysis section. It is suggested that improvements could be made by replacing the axial momentum equations with relations more closely aligned with the general theory, particularly in those differential stream tubes in which "heavy loading" exists. Such conditions appear to be more prevalent in the analyses at off design than in the design itself and, when combined with post stall misknowledge, can lead to large analysis errors. However, for design and analysis within the conventional operating regime, both procedures are simple, accurate, and reliable.

Acknowledgement

The authors would like to acknowledge Mr. D.N. Smyth of Douglas for his helpful ideas and suggestions, and Mr. P.P. Camacho of Douglas for his development of the computer code used in the calculations.

References

1. Glauert, H., "Airplane Propellers," Ed. by Durand, W., Aerodynamic Theory, Div. L, Vol. 4, Peter Smith, Gloucester, MA, 1976, pp. 169-269.

2. Betz, A., with appendix by Prandtl, L., "Screw Propellers with Minimum Energy Loss," Göttinger Reports, 1919, pp. 193-213.

3. Larrabee, E., "Practical Design of Minimum Induced Loss Propellers," SAE, pre-print for the April 1979 Business Aircraft Meeting and Exposition, Wichita, KS.

4. Goldstein, S., "On the Vortex Theory of Screw Propellers," Royal Society Proc., Series A, Vol. 123, London, 1929, pp. 440-465.

5. Lock, C., "The Application of Goldstein's Airscrew Theory to Design," British Aeronautical Research Committee, R. and M. 1377, Nov. 1930.

6. Lock, C., "An Application of Prandtl's Theory to an Airscrew," British Aeronautical Research Committee, R. and M. 1521, Aug. 1932.

7. Lock, C. "Tables for use in an Empirical Method of Airscrew Strip Theory Calculations," British Aeronautical Research Committee, R. and M. 1674, Oct. 1934,

8. Larrabee, E. "The Screw Propeller," Scientific American, Vol. 243, No. 1, July 1980, pp. 134-148.

9. Willson, R. and Lissaman, P., "Applied Aerodynamics of Wind Power Machines," Oregon State University, NSF/RA/N-74-113, PB-238595/3, July 1974.

10. Viterna, A. and Janetzke, D., "Theoretical and Experimental Power from Large Horizontal-Axis Wind Turbines," Proceedings from the Large Horizontal-Axis Wind Turbine Conference, DOE/NASA-LeRC, July 1981.

Table I Example propeller design.

V	=	161.33	DT	=	5.75	BHP	=	70.00	CP	=	0.0402	SOLIDITY	=	0.058
RPM	=	2400.00	DH	=	1.00	THRUST	=	207.44	CT	=	0.0498	AF	=	113.92
J	=	0.701	B	=	2	ALT	=	0.0	ETA	=	0.8693	VP	=	0.2046

DEFINING AIRFOIL: NACA 4415   CL = 0.7000

I	R	CHORD	BETA	PHI	CCL	L/D	RN	MACH	A	AP
1	0.50	0.3353	56.42	54.75	0.2347	59.56	0.44	0.18	0.0333	0.0626
2	0.62	0.3966	50.50	48.83	0.2776	64.02	0.56	0.20	0.0435	0.0533
3	0.74	0.4325	45.48	43.81	0.3028	67.41	0.67	0.22	0.0525	0.0452
4	0.86	0.4484	41.24	39.57	0.3139	69.92	0.77	0.24	0.0601	0.0383
5	0.97	0.4501	37.64	35.97	0.3151	71.78	0.84	0.26	0.0663	0.0326
6	1.09	0.4423	34.57	32.90	0.3096	73.15	0.90	0.29	0.0715	0.0280
7	1.21	0.4285	31.94	30.27	0.3000	74.15	0.94	0.31	0.0757	0.0241
8	1.33	0.4110	29.66	27.99	0.2877	74.85	0.97	0.33	0.0792	0.0210
9	1.45	0.3913	27.68	26.01	0.2739	75.32	0.99	0.36	0.0821	0.0183
10	1.57	0.3704	25.95	24.28	0.2593	75.56	1.00	0.38	0.0845	0.0162
11	1.69	0.3487	24.42	22.75	0.2441	75.61	1.01	0.41	0.0865	0.0143
12	1.81	0.3265	23.06	21.39	0.2286	75.51	1.00	0.43	0.0882	0.0128
13	1.92	0.3040	21.85	20.18	0.2128	75.19	0.99	0.46	0.0897	0.0114
14	2.04	0.2810	20.77	19.10	0.1967	74.66	0.96	0.48	0.0909	0.0103
15	2.16	0.2572	19.79	18.12	0.1800	73.88	0.93	0.51	0.0920	0.0093
16	2.28	0.2324	18.90	17.23	0.1627	72.78	0.88	0.53	0.0929	0.0085
17	2.40	0.2059	18.10	16.43	0.1441	71.24	0.82	0.56	0.0937	0.0078
18	2.52	0.1768	17.36	15.69	0.1238	69.08	0.73	0.58	0.0945	0.0071
19	2.64	0.1433	16.69	15.02	0.1003	65.83	0.62	0.61	0.0951	0.0066
20	2.76	0.1006	16.07	14.40	0.0704	60.27	0.45	0.64	0.0956	0.0061
21	2.87	0.0	15.50	13.83	0.0	54.72	0.0	0.66	0.0960	0.0057

Table II Analysis of propeller of Table I at the design condition.

V	=	161.33	DT	=	5.75	BHP	=	70.00	CP	=	0.0402	SOLIDITY	=	0.058
RPM	=	2400.00	DH	=	1.00	THRUST	=	207.45	CT	=	0.0498	AF	=	113.92
J	=	0.701	B	=	2	ALT	=	0.0	ETA	=	0.8693	DBETA	=	0.0

I	R	CHORD	BETA	PHI	CL	L/D	RN	MACH	A	AP
1	0.50	0.3353	56.42	54.75	0.7000	59.56	0.44	0.18	0.0333	0.0626
2	0.62	0.3966	50.50	48.83	0.7000	64.02	0.56	0.20	0.0435	0.0533
3	0.74	0.4325	45.48	43.81	0.7000	67.42	0.67	0.22	0.0525	0.0452
4	0.86	0.4484	41.24	39.57	0.7000	69.92	0.77	0.24	0.0601	0.0383
5	0.97	0.4501	37.64	35.97	0.7000	71.78	0.84	0.26	0.0663	0.0326
6	1.09	0.4423	34.57	32.90	0.7000	73.15	0.90	0.29	0.0715	0.0280
7	1.21	0.4285	31.94	30.27	0.7000	74.15	0.94	0.31	0.0757	0.0241
8	1.33	0.4110	29.66	27.99	0.7000	74.85	0.97	0.33	0.0792	0.0210
9	1.45	0.3913	27.68	26.01	0.7000	75.32	0.99	0.36	0.0821	0.0183
10	1.57	0.3704	25.95	24.28	0.7000	75.56	1.00	0.38	0.0845	0.0162
11	1.69	0.3487	24.42	22.75	0.7000	75.61	1.01	0.41	0.0865	0.0143
12	1.81	0.3265	23.06	21.39	0.7000	75.52	1.00	0.43	0.0883	0.0128
13	1.92	0.3040	21.85	20.18	0.7000	75.20	0.99	0.46	0.0897	0.0114
14	2.04	0.2810	20.77	19.10	0.7000	74.66	0.96	0.48	0.0909	0.0103
15	2.16	0.2572	19.79	18.12	0.7000	73.88	0.93	0.51	0.0920	0.0093
16	2.28	0.2324	18.90	17.23	0.7000	72.78	0.88	0.53	0.0929	0.0085
17	2.40	0.2059	18.10	16.43	0.7000	71.25	0.82	0.56	0.0938	0.0078
18	2.52	0.1768	17.36	15.69	0.7000	69.08	0.73	0.58	0.0945	0.0071
19	2.64	0.1433	16.69	15.02	0.7000	65.83	0.62	0.61	0.0951	0.0066
20	2.76	0.1006	16.07	14.40	0.7000	60.28	0.45	0.64	0.0956	0.0061
21	2.88	0.0	15.50	13.83	0.0	64.59	0.0	0.66	0.0961	0.0056

-- NOTES --