One Step Toward Causality: Unobserved Time-Invariant Confounding in Cross-Lagged Panel Models

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Introduction

Cross-lagged panel designs, which are based on measuring the same individuals on the same variables for two or more occasions, are commonly used in psychological research to investigate relationships between multiple constructs over time. Several longitudinal SEM models have been developed for analyzing such data, of which the random intercept cross-lagged panel model (RI-CLPM, Hamaker et al., 2015; Mulder & Hamaker, 2021) is one of the most popular. The RI-CLPM, shown in Figure 1a, decomposes observed variables into stable between-person differences and temporal within-person components, and the dynamics are modeled using these within-person components. Usually, the effects of the variables on each other at later timepoints, the cross-lagged effects, are most of interest. These are often interpreted as causal effects, albeit implicitly. However, a causal interpretation presumes that both time-varying and time-invariant confounders are adequately controlled for, an assumption commonly referred to as absence of unobserved confounding.

Usami et al. (2019) show that when certain assumptions are met, the random intercept in the RI-CLPM controls for unobserved heterogeneity resulting from confounding. This requires, in particular, that the confounders are time-invariant, and that the effects of the confounders on the variables of interest are stable over time. However, it has not yet been studied how the RI-CLPM performs when the effects of unobserved time-invariant confounders are of a time-varying nature. Murayama and Gfrörer (2022) show that the Dynamic Panel Model (DPM), shown in Figure 1b, may be an alternative in this case. This model is similar to the RI-CLPM due to the inclusion of lagged effects as well as a latent component, but the dynamics are now situated between the observed variables, rather than between within-person components. While some prefer this model because it is less restrictive and it is claimed to better control for unobserved confounding (Allison et al., 2017), others would regard the absence of an explicit within-between decomposition as a downside (e.g. Hoffman & Hall, 2024). Furthermore, it is unknown how either of these models perform when the underlying causal process is more complex, for example when multiple confounders with time-varying effects are involved.

Therefore, the effect of time-invariant confounders on both the RI-CLPM and the DPM should be explored further. This research report will evaluate the extent to which unobserved time-invariant confounders affect the estimates of the RI-CLPM and DPM. Specifically, we will assess whether these estimates give a good indication of underlying causal effects when multiple unobserved time-invariant confounders are involved.

This report is structured as follows. We start with a conceptual comparison between the RI-CLPM and the DPM. Then, a hypothetical causal model that includes multiple time-invariant confounders is introduced. This model is used as the data generating mechanism for a simulation study that evaluates the performance of the RI-CLPM and the DPM under the data generating mechanism. We end with a discussion of results and implications.

A Comparison of the RI-CLPM and the DPM

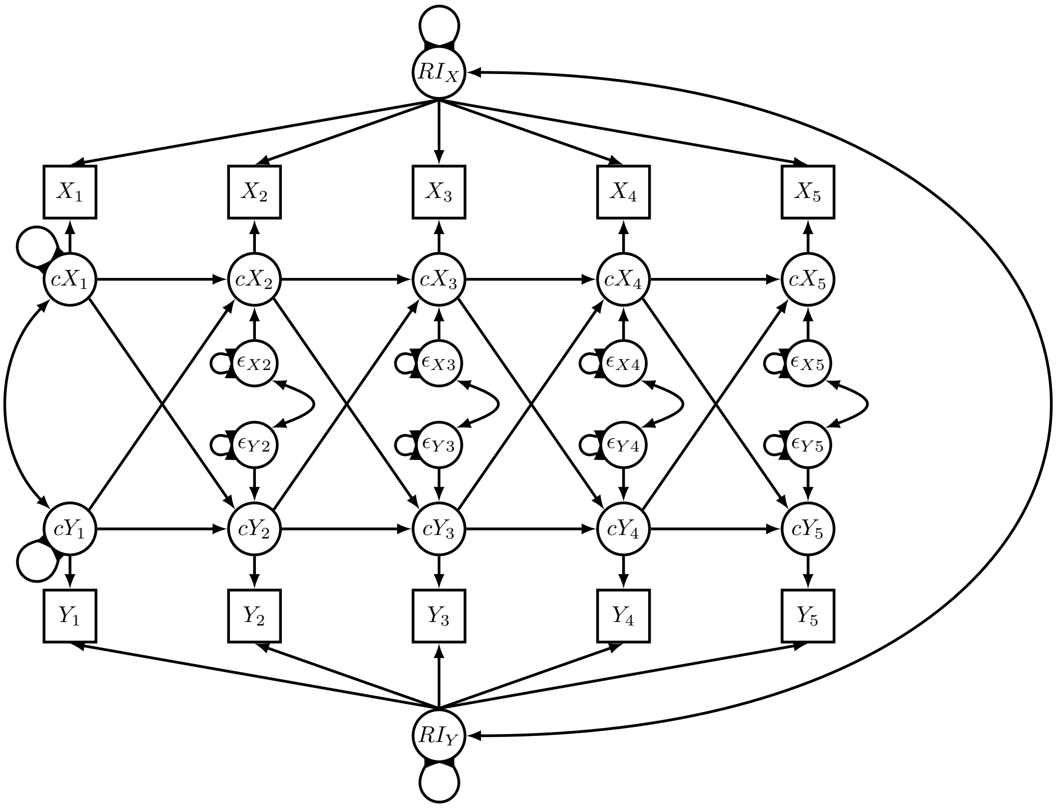
The RI-CLPM is commonly used with the goal to investigate causal relationships between variables while accounting for stable between-person differences (Hamaker et al., 2015). It decomposes the observed scores of an individual into a stable person-specific deviation from the grand mean, and a temporal deviation from this stable component by including a random intercept factor. These temporal deviations make up the 'within' part of the RI-CLPM where the dynamics are modeled. As Figure 1a shows, this factor only has direct effects on the observed variables, and no effect on the observed scores of the other variable. The random intercept of X is only related to the observed scores of Y through a covariance between the random intercepts, and vice versa.

Likewise, the DPM is also used to estimate causal relationships between variables over time. However, the DPM does this while aiming to control for unmeasured time-invariant confounders (Allison et al., 2017). It does not explicitly separate within person dynamics from stable between person differences, and lagged effects are included on the observed scores. The latent factors in the DPM are

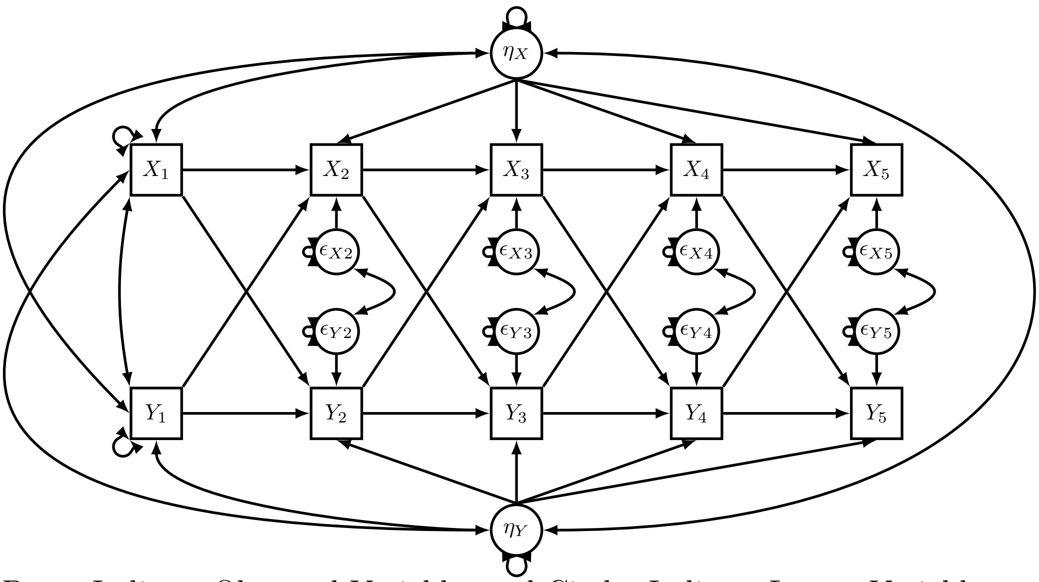
Figure 1

Structural Equation Model Representations of Two Popular Models in Panel Research

(a) *Random Intercept Cross-Lagged Panel Model (RI-CLPM)*[a]



(b) *Dynamic Panel Model (DPM)*



Note. Boxes Indicate Observed Variables and Circles Indicate Latent Variables.

[a] Observed Scores are Decomposed into a Stable Person-Specific Component (RI_X and RI_Y), and Within-Person Components (cX and cY).

sometimes called 'accumulating factors' (Usami et al., 2019), as their effects on the observed variables are both direct, as well as indirect through lagged relationships between the observed variables themselves, a property that becomes clear from Figure 1b. To account for the fact that measurements are usually sampled at a random moment in time in an ongoing process, the observations at the first timepoints are often allowed to covary freely with each other and the accumulating factors (Hamaker, 2005), which is not necessary in the RI-CLPM.

Hamaker (2005) shows that under certain circumstances, the RI-CLPM and the DPM are statistically equivalent and yield equivalent estimates of the lagged parameters. This is the case when lagged parameters are invariant over time, and the factor loadings in the DPM at the first timepoint are constrained to reflect this, rather than specified as free covariances. This also implies that when these conditions hold in a data generating mechanism, the RI-CLPM and the DPM should both yield unbiased estimates. However, it is yet unknown how the estimates behave when the underlying mechanism is more complex.

Methods

Although there is information on the behavior of the RI-CLPM and the DPM in simple cases of unobserved confounding, as is discussed in the previous section, it is yet unknown how the estimates behave when the underlying mechanism is more complex. Therefore, we present a simulation study to assess the performance of the RI-CLPM and the DPM under different patterns of unobserved confounding. We first introduce a causal model that includes multiple confounders. This serves as a general data generating mechanism for the simulations. Three different scenarios are considered, which differ with respect to the stability of the effects of the confounders.

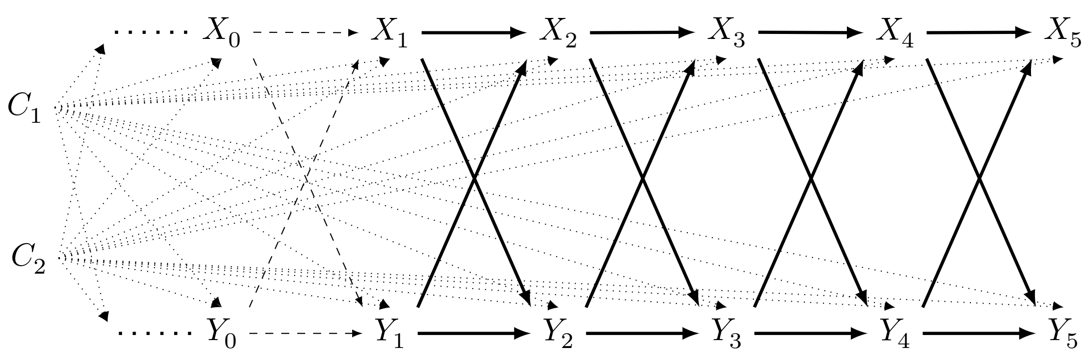
The Causal Model

To represent the causal model that we simulated, we use the causal directed acyclic graph (DAG) in Figure 2. It shows a dynamic process for $t = 1, ..., 5$ between time-varying variables X and Y and includes two time-invariant baseline confounders,

C_1 and C_2, that each have an effect on all future observations of X and Y. The structure of the DAG is most similar to the dynamic panel model, as observed values are determined by observed values at previous timepoints, but it may also represent confounding on the within-person dynamics in the RI-CLPM. However, it should be noted that this DAG is not equivalent to either model, as each confounder has effects on both X and Y, whereas the latent factors in the RI-CLPM and DPM are variable-specific.

Figure 2

Directed Acyclic Graph for Cross-Lagged Relationships Between X and Y at Multiple Timepoints. Confounders C_1 and C_2 Have Effects on Both X and Y at Each Timepoint. Dashed Lines Indicate the Process Before the Observed Timepoints



When the underlying dynamic process has stabilized around an equilibrium, and when lagged effects as well as the effects of the confounders are time-invariant, in both the RI-CLPM and the DPM the latent factors will be a linear combination of the confounders (Usami et al., 2019). Therefore, when these conditions hold, we expect both models to yield unbiased estimates of the lagged effects, even when the confounders are unobserved. However, when the effects of the confounders are not time-stable, the models may not be able to fully account for the effects of confounders, thus resulting in biased effects.

This causal model was simulated to assess the performance of the RI-CLPM and the DPM under different scenarios of confounding. These scenarios were all simulated as special cases of the data generating mechanism in Figure 2 and are represented by the following equations:

For person i at timepoint t,

$$x_{it} = \phi_{xx} x_{i,t-1} + \phi_{xy} y_{i,t-1} + \gamma_{1t} C_{1i} + \gamma_{2t} C_{2i} + \epsilon_{xit},$$

$$y_{it} = \phi_{yy} y_{i,t-1} + \phi_{yx} x_{i,t-1} + \delta_{1t} C_{1i} + \delta_{2t} C_{2i} + \epsilon_{yit}.$$

Furthermore, at the first simulated timepoint,

$$x_i = \gamma_1 C_{1i} + \gamma_2 C_{2i} + \epsilon_{xi},$$

$$y_i = \delta_1 C_{1i} + \delta_2 C_{2i} + \epsilon_{yi}.$$

In addition,

$$\epsilon_{xt} \sim \mathcal{N}(0, \psi_x),$$

$$\epsilon_{yt} \sim \mathcal{N}(0, \psi_y),$$

$$C_1 \sim \mathcal{N}(0, \psi_{C_1}),$$

$$C_2 \sim \mathcal{N}(0, \psi_{C_2}).$$

From these equations follows that lagged effects and (residual) variances are time-invariant, whereas effects of the confounders may be time-varying, as indicated by the absence or presence of a time index t for these parameters. In our simulation, for all scenarios, lagged effects were set to $\phi_{xx} = 0.2$, $\phi_{yy} = 0.3$, $\phi_{xy} = 0.15$, $\phi_{yx} = 0.1$ (based on Mulder, 2023), and (residual) variances were all set to 1.

Three scenarios were simulated. For all scenarios, at the start, $\gamma_1 = 0.3$, $\gamma_2 = 0.8$, $\delta_1 = 0.5$, and $\delta_2 = 0.2$. For scenario 1, the effects of the confounders remain stable. For scenario 2, at $t = 3$, the effects of C_1 on x and y decrease, and remain stable afterwards ($\gamma_1 = 0.1$ and $\delta_1 = 0.2$). For scenario 3, at $t = 3$ all effects of confounders change and afterwards remain stable. For C_1 its effect on x increases and its effect on y decreases ($\gamma_1 = 0.6$ and $\delta_1 = 0.2$), and vice versa for C_2 ($\gamma_2 = 0.3$ and $\delta_2 = 0.5$).

For each scenario, 1000 datasets were simulated with N=500. To allow for convergence to an equilibrium, we simulated 50 timepoints, of which 45 were used as burn-in. The remaining 5 timepoints, $t = 1, ..., 5$, were analyzed.

Analysis

To assess the performance of the RI-CLPM and the DPM when confounders are unobserved, all simulated datasets were analyzed using the RI-CLPM and the DPM, as well as versions of these models with free loadings for the latent factors, because freeing the factor loadings may, in part, capture the time-varying effects of the unobserved confounders. Models that did not converge or did not result in positive definite covariance matrices were excluded. All simulations and analyses were performed using R version 4.3.2 (R Core Team, 2022). Models were fit using the lavaan package version 0.6-16 (Rosseel, 2012).

Results

The discussion of results focuses on the $x_4 \rightarrow y_5$ effect ($\phi_{xy} = 0.1$). For all models, for each scenario, the bias, Monte-Carlo error, root mean squared error (RMSE), and coverage were computed, based on recommendations by Morris et al. (2019). The bias was computed as the difference between the mean of the empirical sampling distribution, that is, the distribution of our estimates, and the true effect. The Monte-Carlo error was computed as the standard error of this estimate, and the RMSE was computed as the root of the mean squared deviation from the true value. Setting $\alpha = 0.5$, the coverage is the proportion of times that the 95% confidence interval contains the true value. The coverage is typically seen as acceptable when it is close to 95%.

Upon initial inspection of the models, we found that for scenario 3, a high number of models resulted in a non positive-definite matrix of the latent variables. Specifically, the random intercepts or accumulating factors had a correlation higher than 1 in all of the RI-CLPM models and in most of the DPM models (91.9% and 55.8% for the DPM without and with free loadings, respectively). Therefore, for subsequent results, the models were rerun, with correlations between random intercepts or accumulating factors set to 1. It should be kept in mind, however, that this constitutes an exploratory approach as it was not based on theory. Therefore, results may not generalize well to other situations.

Table 1 shows the relative bias, RMSE, and coverage for the $x_4 \rightarrow y_5$ effect for each model, in each scenario, and Figure 3 visualizes the biases, with their corresponding Monte-Carlo errors. Table 1 and Figure 3 show that under scenario 1, as expected, both the RI-CLPM and the DPM yield unbiased estimates. Surprisingly, freeing the factor loadings results in bias. Specifically, they yield a negative bias, indicating that the effect is underestimated. Furthermore, the DPM has a lower RMSE than the RI-CLPM, indicating that under this scenario, the DPM shows less average absolute deviation from the true effect and thus may be a better choice. All coverages are relatively acceptable.

Under scenario 2, where the effects of C_1 are time-varying, both the RI-CLPM and the DPM yield biased estimates. Specifically, the RI-CLPM shows a relatively extreme underestimation of the true effect, and the DPM yields a small overestimation. Furthermore, freeing the loadings does not capture the time-varying effects of the confounders, as indicated by the extreme relative biases. The DPM has the lowest RMSE of all models. The RI-CLPM shows very low coverage, and the RI-CLPM with free loadings has coverage that may be too high.

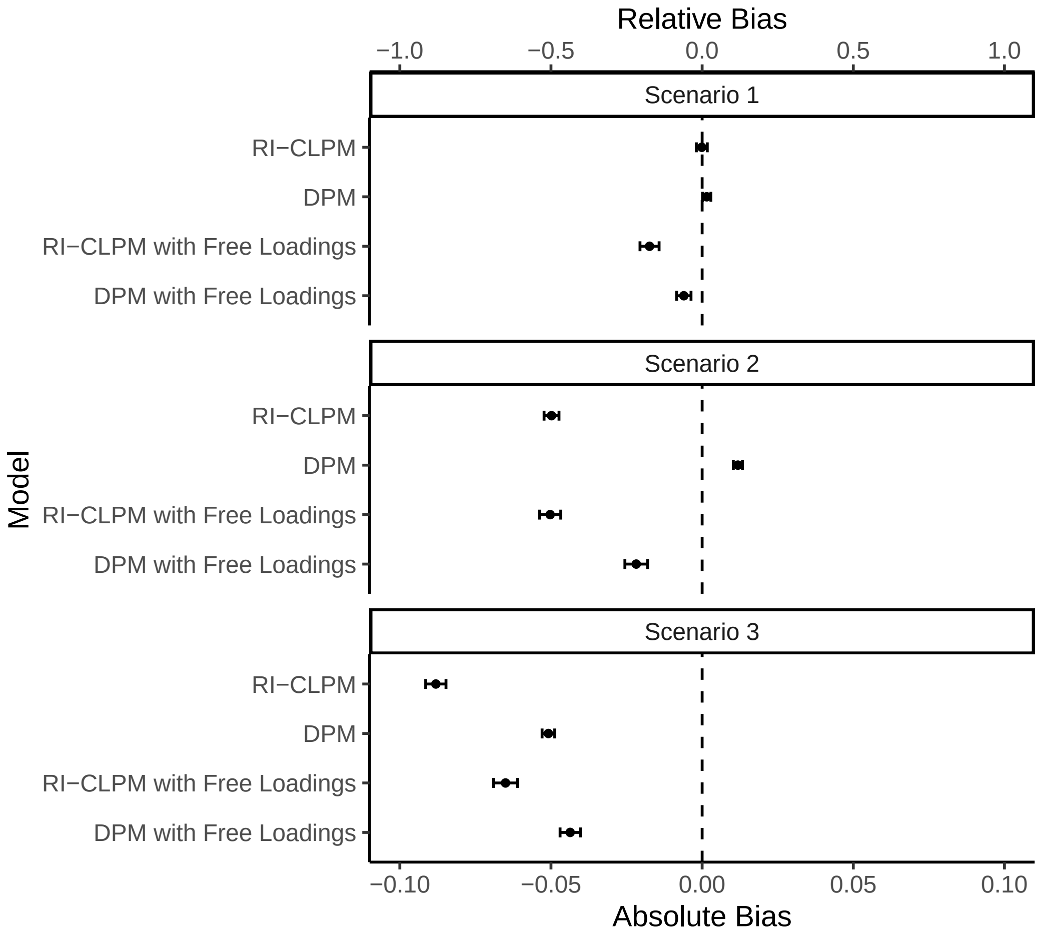
Under scenario 3, where both confounders have time-varying effects, all models show large bias. In this situation, the RI-CLPM and DPM with free factor loadings have lower bias than their counterparts with fixed loadings. Furthermore, RI-CLPM and the DPM have very low coverages, and the versions with free factor loadings have coverages that may be too high. The DPM yields the lowest RMSE.

Discussion

Previous research has suggested that when lagged effects and the effects of unobserved time-invariant confounders are stable over time, both the RI-CLPM and the DPM yield unbiased estimates of the true cross-lagged effect, even when the confounders are unobserved (e.g. Usami et al., 2019). This characteristic of both models was replicated by the discussed simulations in scenario 1. However, the extent to which

Figure 3

Biases for the Effect of x_4 on y_5



Note. The true effect is equal to $\phi_{yx} = 0.1$. The Dots indicate the (relative) bias of each model's estimates. The error bars indicate the Monte-Carlo error.

this is a reasonable causal model is debatable. For many phenomena, it may be more realistic to assume effects of confounders to vary over time.

Therefore, we also considered situations where effects of one or two confounders changed over time. In our simulations, the RI-CLPM and the DPM respectively underestimated and overestimated the cross-lagged effects when this was the case. Furthermore, freeing the factor loadings in both models did not fully capture the time-varying nature as negative bias was observed in these models as well. However, because only few scenarios were considered, we cannot yet conclude that these models

Table 1

Relative Bias, RMSE, and Covarage for Each Scenario on the Effect of x_4 on y_5. The True Effect is 0.1

Scenario	Model	Relative Bias (%)	RMSE[1]	Coverage (%)[2]
1	RI-CLPM	-0.11	0.06	94.40
	DPM	1.52	0.04	95.69
	RI-CLPM (free loadings)	-17.39	0.10	95.81
	DPM (free loadings)	-6.06	0.07	94.73
2	RI-CLPM	-49.81	0.08	88.94
	DPM	11.83	0.04	94.52
	RI-CLPM (free loadings)	-50.27	0.11	97.78
	DPM (free loadings)	-21.80	0.09	95.42
3	RI-CLPM	-88.09	0.11	71.70
	DPM	-50.86	0.07	79.10
	RI-CLPM (free loadings)	-65.04	0.12	98.00
	DPM (free loadings)	-43.64	0.10	96.65

[1] Root Mean Square Error.

[2] Inclusion frequency of the true effect for the 95% confidence interval.

generally underestimate cross-lagged effects when confounders are unobserved; other data generating mechanisms may show different patterns. Furthermore, although bias in the estimates was present in these scenarios, conclusions on the direction of the effect generally did not change, as the estimated effects were still positive.

The analyses performed in our simulation were used on simulated data with unobserved confounders. When confounders are measured, however, it may be more appropriate to model them explicitly. This can be done by including them in the model as covariates (Mulder & Hamaker, 2021), but methods specifically designed for causal inference may be more fitting (Leite et al., 2019; Schafer & Kang, 2008), especially when many confounders are involved or when effects are nonlinear. In reality, however, only a subset of confounders are likely observed, while others remain unobserved. In such situations, it could be advantageous to use a combination of methods, such as using the RI-CLPM or DPM to correct for some unobserved confounders, and use propensity score based causal inference methods to correct for observed confounders.

However, the extent to which this is possible and sensible is yet to be explored.

We should therefore investigate whether the toolbox of social science researchers can be expanded by integrating causal inference methods with cross-lagged panel models, which will be covered by the thesis. The presented results highlight the necessity of such research and emphasize the importance of acting with caution when attempting to assess cause and effect.

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