Asymptotic Properties of the Hill estimator

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Avainsanat Vastus, resistanssi, lämpötila

Preface

I want to thank Professor Pirjo Professori and my instructor Dr Alan Advisor for their good and poor guidance.

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Eddie E. A. Engineer

Contents

Symbols and abbreviations

Symbols

$x^* = sup\{x : F(x) < 1\}$	right endpoint of the distribution
γ	extreme value index

Operators

Abbreviations

cdf	cumulative distribution function
i.d.d	independent and identically distributed

1 Introduction

2 Backround

2.1 Extreme Value Theory

First approch to study behavior of extreme events would be to find limiting distribution of the sample maxima $M_n = max(X_1, X_2, ..., X_n)$. Here $X_1, X_2, ..., X_n$ are i.d.d random variables from cdf F. Function for the cdf of M_n can be derived, because $X_1, X_2, ..., X_3$ are i.d.d.

$$P(max(X_1, X_2, ..., X_n) \leq x) = P(X_1 \leq x, X_2 \leq x, ..., X_n \leq x) =$$
$$P(X_1 \leq x)P(X_2 \leq x)...P(X_n \leq x) = F^n(x).$$

Now it can be shown that this approach is not very fruitful since

$$\lim_{n\to\infty} F^n(x) = \begin{cases} 0, x < x^* \\ 1, x \geq x^*. \end{cases}$$

To achieve a nondegerate distribution it is necessary to normalize the sample maxima M_n. After normalization a nondegenate distribution is gained as stated in the Fisher-Tippett-Gnedenko Theorem [1].

Theorem 2.1 *There exists real constants $a_n > 0$ and $b_n \in \mathbb{R}$ such that*

$$\lim_{n\to\infty} F^n(a_n x + b_n) = G_\gamma(x) = \begin{cases} exp(-(1+\gamma x)^{-\frac{1}{\gamma}}), \gamma \neq 0 \\ exp(-e^{-x}), \gamma = 0, \end{cases} \tag{1}$$

for all x with $1 + \gamma x > 0$ where $\gamma \in \mathbb{R}$.

If cdf F satisfies the equation 1 for some $\gamma \in \mathbb{R}$ it is said that F is in the maximum domain of attraction of G_γ i.e $F \in D(G_\gamma)$. Concerning the Hill estimator special interest lies in the case $F \in D(G_{\gamma>0})$.

2.2 Problem

3 Consistency of the Hill estimator

4 Simulations

References

[1] A. F. Laurens De Haan. *Extreme Value Theory.* Springer, 2009.

Appendix