CS 136 Assignment 6: Sponsored Search Auctions

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Friday, March 25, 2016

1 The Balanced Bidding Agent

1. **[10 Points]** What is the average utility of a population of truthful agents? What is the average utility of a population of balanced bidding agents? *Compare the two cases and explain your findings.*

   Solution: For truthful agents, the average utility is \$345.59. For balanced bidding agents, the average utility is \$709.14. The balanced bidding agents are able to do much better because the agents usually bid less than their true values. Thus each agent pays less than when all were truthful, but they earn the same value. □

2. **[10 Points]** In addition, what is the average utility of one balanced bidding agent against 4 truthful agents, and one truthful agent against 4 balanced-bidding agents? What does this suggest about the incentives to follow the truthful vs. the balanced bidding strategy?

   Solution: The single balanced bidding agent earns on average \$564.64 when against 4 truthful agents, which earn on average \$390.67. The single truthful agent earns on average \$664.64 when against 4 balanced bidding agents, which earn on average \$676.30. This suggests that if other agents are bidding truthfully, it is advantageous to play a balanced bidding strategy. However, if other agents are playing a balanced bidding strategy, it is just as good to play balanced bidding as to play truthfully. The truthful agent will tend to bid more than balanced bidding agents and thus will both pay more and win more, effects which balance each other out. Thus all agents bidding truthfully is not a Nash Equilibrium but all agents doing balanced bidding is a Nash equilibrium. □

2 Experiments with Revenue: GSP vs VCG auctions

3. **[55 Points] Auction Design and Reserve Prices**

   (a) **[20 Points] Solution:** Code written. □

   (b) **[10 Points]** What is the auctioneer's revenue under GSP with no reserve price when all the agents use the balanced-bidding strategy? What happens as the reserve price increases? What is the revenue-optimal reserve price?

   Solution: Figure **??** plots reserve prices on the x-axis and and average revenue on the y-axis. The revenue with no reserve price is \$4383.64, and the highest revenue is \$5110.07 at $r = \$0.75$. The optimal reserve price for GSP is thus between \$0.50 and \$1.00, somewhere near \$0.75. □

(c) **[10 Points]** What is the auctioneer's revenue under VCG with no reserve price when all agents are truthful? What happens as the reserve price increases? Explain your findings and compare with the results of part (b).

Solution: See Figure 1. The revenue with no reserve price is \$4231.64, and the highest revenue is \$4997.27 at $r = \$1.00$. The optimal reserve price for VCG is thus between \$0.70 and \$1.25, somewhere near \$1.00. The VCG revenues are comparable to the GSP revenues. They are lower with lower reserve prices and higher with higher reserve prices. The maximum revenue is higher under GSP than under VCG, suggesting that GSP is desirable if high revenue is the goal, as discussed in the reading. □

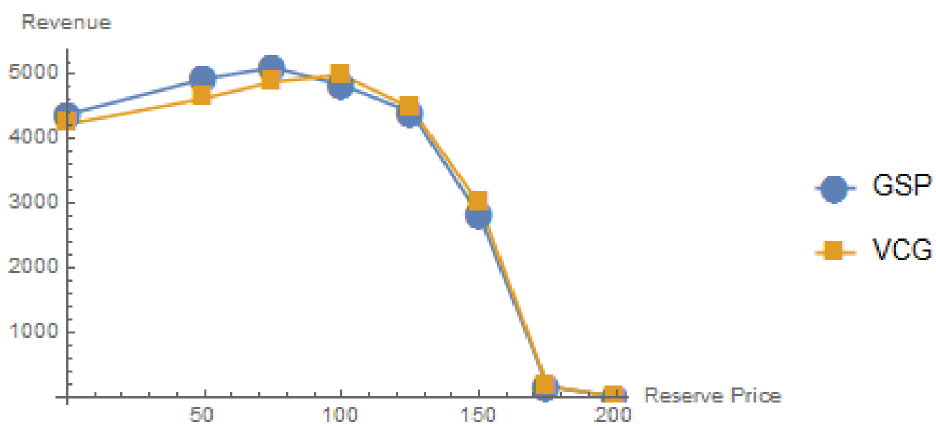

Figure 1: Revenues and Reserve Prices

(d) **[10 Points]** Fix the reserve price to zero. Explore what might happen if a search engine switched over from the GSP to VCG design. For this, run the balanced-bidding agents in GSP, and at period 24, switch to VCG, by using the `--mech=switch` parameter. What happens to the revenue?

Solution: The revenue is \$3949.34 when switching from GSP to VCG halfway through the day. This is approximately equal to the average of the GSP revenue \$4383.64) and VCG revenue (\$3488.17) under the same auction parameters. □

(e) **[5 Points]** In one paragraph, state what you learned from these exercises about agent design, auction design, and revenue? (There is no specific right answer.)

Solution: We gained deep familiarity with the mechanics of the VCG and GSP mechanisms as well as with the balanced bidding strategy. We gained empirical evidence that the balanced bidding strategy performs better than the truthful strategy when other agents are truthful and that having some reserve price can increase revenue. Finally, we honed our CS skill of implementing in an algorithm in software. □

3 The Competition

4. **[30 Points] Budget constraints**

(a) **[25 Points]** Describe in a few sentences how your agent works, why it is designed this way, and how you expect it to perform in the class competition.

Solution: The expected number of clicks is given by the function

$$c_j^t = 0.75^{(j-1)} * \text{round}(30\cos(\frac{\pi t}{24}) + 50)$$

This is plotted in Figure 2 below.

```
Plot3D[(30 Cos[Pi t / 24] + 50) (3 / 4)^n, {t, 0, 47}, {n, 0, 5}, MeshFunctions → (#3 &)]
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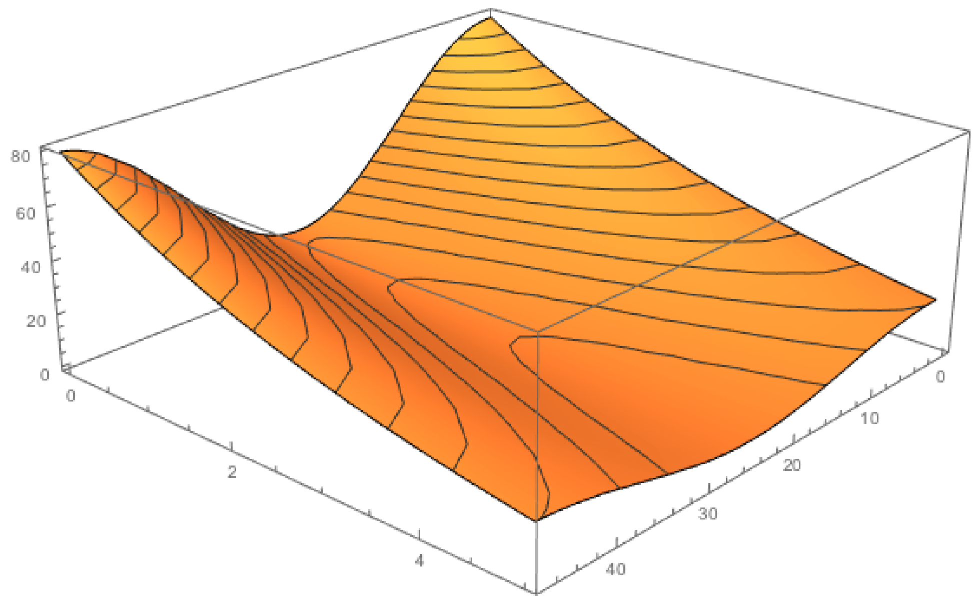

Figure 2: Expected Clicks

The level curves here tell us that the expected number of clicks at the highest-traffic areas of the day in the lowest slot are roughly equal to the the loweset-traffic areas of the day in the highest slot. In essence, it's just as valuable to be in the lowest slots at the beginning and end of the day as it is to be in the highest slots in the middle of the day. Furthermore, the drop in overall number of clicks (and therefore utility) is less (in overall magnitude) at the low-traffic areas of the day compared to the higher traffic areas. In other words, the top slots at the highest traffic areas are worth much more than the high slots at other times of the day. To capitalize on this, we devised a system to over-bid at this important times, so as to ensure a higher slot. Over-bidding is an effective strategy because balanced bidding assumes other people keep their bets the same, which would not necessarily be the case. In order to bet more when more clicks are expected, we calculated our bid according to the standard balanced bidding strategy, and then multiplied our bid by the factor

$$f_t = \frac{c_1^t}{50}$$

Which averages to 1 throughout the day but overbids in higher traffic times.

We did not see our agent consistenly running into strong budget constraints, and we think that this modification of the bidding strategy will help the agent preserve funds during the low-traffic section of the day. We therefore expect this agent to far outperform standard balanced-bidding agents under budget constraints, but perhaps be weaker than more sophisticated budgeting strategies.