

# Hierarchical Reinforcement Learning for Multi-modal Continuous Control Environments with Sparse Rewards

by

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and have found that it is complete and satisfactory in all respects,  
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the thesis examination committee have been made.

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# Acknowledgments

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# Hierarchical Reinforcement Learning for Multi-modal Continuous Control Environments with Sparse Rewards

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Abstract

# Chapter 1

## Introduction

### 1.1 Overview

Deep reinforcement learning methods have gained much attention from researchers and the public since the recent advances such as DQN [1] and AlphaGo [2]. The method of deep reinforcement learning combine deep neural network models and reinforcement learning algorithms, enabling the potential for solving general control problems in fully autonomous manner. However, the application of contemporary deep reinforcement learning methods are still limited to a finite class of problems. The reinforcement learning problem either has a fully known model like the Go game, or has to be problems with low complexity like Atari game [1] or primary robot control tasks [3]. The real-world robot control problems are far from being solved without labour-intensive domain specific engineering.

In this study, we propose a hierarchical reinforcement learning method that can learn to solve increasingly complex tasks in several realistic robot control environments.

### 1.2 Background

#### 1.2.1 Reinforcement Learning (RL)

Reinforcement learning [4] is learning a policy (map from situations to actions) in order to maximize a numerical reward signal. The learner must discover the optimal policy by trying them. Reinforcement learning is different from supervised learning that the learner is not provided any labelled examples by any knowledgeable external supervisor. Reinforcement learning is also different from unsupervised learning, whose objective is finding hidden structures in unlabeled data.

Most reinforcement learning study is based on the problem formulation of Markov Decision Processes (MDPs). MDP is a framework defining the problem of learning from interaction. The learner is called the *agent*. Everything outside that interacts with the agent is called the *environment*. The agent and environments interact during a finite sequence of discrete time steps,  $t = 0, 1, 2, 3, \dots$ . At each time step  $t$ , the agent receives an observation of the environment's *state*,  $s \in S$  and selects an *action*  $a \in A$ . The agent receives a numerical reward  $r$ , and also the next state  $s'$ . A finite-horizon discounted Markov decision process (MDP) is defined by the tuple  $(S, A, P, r, \rho_0, \gamma)$  where the  $\rho_0 : S \mapsto \mathbb{R}$  is the initial state distribution,  $P(s'|s, a) : S \times A \mapsto \mathbb{R}$  is the distribution describing the state transition dynamics,  $r : S \times A \mapsto \mathbb{R}$  is the expected reward function, and  $\gamma \in (0, 1]$  is the discount factor.

Let  $\pi : S \times A \mapsto [0, 1]$  denote a policy, following are definitions of the action value function  $Q_\pi$ , the value function  $V_\pi$  and the advantage function  $A_\pi$ :

$$Q_\pi(s_t, a_t) = \mathbb{E}_{s_{t+1}, a_{t+1}, \dots} \left[ \sum_{l=0}^{\infty} \gamma^l r(s_{t+l}) \right],$$

$$V_\pi(s_t) = \mathbb{E}_{a_t, s_{t+1}, a_{t+1}, \dots} \left[ \sum_{l=0}^{\infty} \gamma^l r(s_{t+l}) \right],$$

$$A_\pi(s, a) = Q_\pi(s, a) - V_\pi(s), \text{ where } a_t \sim \pi(a_t|s_t), \text{ for } t \geq 0$$

### 1.2.2 Hierarchical reinforcement learning based on transferred policies from easy tasks

This study focus on a class of reinforcement learning problems, where the objective is to solve a *target task* given a set of trained *source tasks*. The policies that can solve the source tasks, namely the *actuator policies*, are available for the agent. It is known that target task can be solved by a sequential execution of a subset of the source-task policies. The agent needs to learn a *scheduler policy*, which chooses the source-task policy to be executed and decides when to switch the currently running source-task policy.

This paradigm is related to hierarchical reinforcement learning (HRL) [5], which focuses on learning and utilizing closed-loop partial policy / skills to solve a complex task. However, our case is different from the standard setting of HRL where the agent needs to extract the useful closed-loop partial policy from the original task. However, we focus on extracting and reusing partial policies from other tasks.

This paradigm is also related to transfer learning in reinforcement learning [6], because the source-task policies are reused by the high-level scheduler agent. However, conventional transfer learning paradigms usually don't assume a hierarchical relationship between the target task and the source tasks.

## 1.3 Scope of study

Real-world robot control problems involves many sources of complexity. However, we only focus on some critical properties of real-world robot control problems. This study focus on the reinforcement learning problems with multi-modality state space, continuous action space and sparse reward in a continual learning setting.

### 1.3.1 Multi-modality state space

The reinforcement learning problems with multi-modality state space refers to the environments whose state space is a combination of signals with different modality.

Specifically, the study focus on realistic robot control environments whose state space is a combination of a low-dimensional state sensor and a image sensor observation.

The low-dimensional state input provides accurate information about the agent's physical system. The image input provides information about the external environment and has a datatype that is in much higher dimension.

Most recent works in reinforcement learning for continuous control focus on the RL tasks with low-dimensional state input only [3]. There are also works [7] trying to handle some tasks with image input only, but these image-based tasks are very simple to solve.

However, the proposed multi-modality environment is the most common case in real-world robotic problems, and is considerably more complex and realistic than solving a low-dimensional state input only task or an image only task.

### 1.3.2 Sparse reward function

The problem of sparse / delayed reward signal has been one of the major problems to be tackled in reinforcement learning. Many methods are proposed to solve the sparse reward problem in environments with discrete action spaces. However, the problem is much more difficult for the environments with continuous action spaces, and few works have focused

on the class of system. By the time of writing, we haven't seen any method that has been proposed to effectively solve continuous control tasks with sparse rewards.

Studies have shown that smooth and informative reward function can generally make the training of reinforcement learning agents easier. Theoretically, any reinforcement learning problems when given a reward function that is good enough.

However, the reward function of a realistic robot control environment is usually discontinuous and sparse. Reward shaping techniques may improve the reward function, but it usually relies on domain-specific engineering. In our study, we try to solve some realistic robot problems without reward shaping, which essentially indicates the problems have sparse reward functions.

### **1.3.3 Assumption on hierarchical relationship of tasks**

We focus on solving the reinforcement learning setting described in section 1.2.2 . We assume that the agent has already learned the optimal policies for a set of source tasks before solving the target problem, and we also assume that the target task can be solved by executing a subset of the source task policies in according to some "high level policy".

However, we don't pose any other assumption on the relationship between the source tasks and the target task. We don't know whether any specific source task is useful. A source task policy might be useful in some cases, or it could be completely useless. We also don't make assumption on how long an actuator policy should be executed in its term. The scheduler agent need to learn it instead. An actuator policy might be executed for only 1 time-step before the agent switch to another one, or it could be executed for a longer time, or until a specific condition.

This assumption is the only assumption that is involved in domain-specific design. However, we observe that many real-world complex robot control problems can usually be decomposed to primary tasks. Classical robot control methods also organize the abstract problems (such as trajectory planning) and low level problems (such as model control) in hierarchical structures.

If this hierarchical task relationship assumption was removed, the problem becomes the fully autonomous hierarchical reinforcement learning problem, where the agents need to learn not only a task structure but also all actuator policies from scratch. This problem is considered in-feasible to solve for general robot control problems [5].

## 1.4 Research Questions

We will propose several reinforcement learning environments for continuous control in section 3.1 according to the stated scope and assumption. The study will mainly focus on solving the proposed problems. The primary research question that we focus on is:

*Can we develop an agent that can achieve good performance (in terms of total reward) in the reinforcement learning environments defined in section 3.1 within reasonable training time, with an RL model that doesn't rely on domain-specific techniques (including reward shaping, manual definition of policy hierarchy, manual / heuristic methods to define sub-goals, or manual criteria on the switching of actuator policies)?*

We will first answer the question on whether the previous work already solve the problem: *Can the contemporary flat reinforcement learning methods achieve good performance in the proposed environments?*

We will then answer the question on whether our proposed model can solve this: A model that solves the primary question needs to address two issues: reusing the policies learned in source tasks and solving the target task. To solve the first issue, we need to answer the question of *"Can a trained actuator policy achieve a good performance (in terms of the expected total return of the original problem), with or without fine-tuning methods, in a target environment which has different initial state distribution?"*.

To address the second issue, we will answer the question of *"Can the target scheduler agent learn the optimal policy given the trained sub-task policies, without any domain-specific design defined previously?"*.



## Chapter 2

# Related works

## 2.1 Policy Gradient Methods

Policy gradient method is a class of reinforcement learning methods that solve the problem by optimizing a parametrized policy model  $\pi_\theta(a|s), a \in A, s \in S$ , so that the expected return:

$$J(\theta) = \mathbb{E}\left[\sum_{t=0}^T \gamma^t r_t\right]$$

is maximized, where  $\gamma$  is a discount factor and  $T$  is the episode length.

Policy gradient methods maximize the expected return iteratively by estimating the gradient  $g := \nabla \mathbb{E}\left[\sum_{t=0}^T \gamma^t r_t\right]$ , which has the form [8]:

$$g = \mathbb{E}_{t,s_t,a_t}\left[\Psi_t \frac{\nabla_\theta \pi_\theta(a_t|s_t)}{\pi_\theta(a_t|s_t)}\right] = \mathbb{E}_{t,s_t,a_t}\left[\Psi_t \nabla_\theta \log \pi_\theta(a_t|s_t)\right] \quad (2.1)$$

$\Psi_t$  may be one of the following:

1.  $\sum_{t=0}^{\infty} r_t$  expected total return
2.  $\sum_{t'=t}^{\infty} r_{t'}$  expected return following  $a_t$
3.  $\sum_{t'=t}^{\infty} [r_{t'} - b(s_t)]$
4.  $Q_\pi(s_t, a_t)$
5.  $A_\pi(s_t, a_t)$
6.  $\hat{A}_t^{(1)} := \delta_t = r_t + V_\pi(s_{t+1}) - V_\pi(s_t)$  : 1-step TD residual
7.  $\hat{A}_t^{(2)} := \delta_t + \gamma \delta_{t+1} = r_t + \gamma r_{t+1} + \gamma^2 V_\pi(s_{t+2}) - V_\pi(s_t)$  : 2-step TD residual

8.  $\hat{A}_t^{(k)} := \sum_{l=0}^{k-1} \gamma^l \delta_{t+l} = r_t + \gamma r_{t+1} + \dots + \gamma^{k-1} r_{t+k-1} + \gamma^k V(s_{t+k}) - V(s_t)$  : k-step TD residual
9.  $GAE(\lambda) = \frac{1}{(1+\lambda+\lambda^2+\dots)} (\hat{A}_t^{(1)} + \lambda \hat{A}_t^{(2)} + \lambda^2 \hat{A}_t^{(3)} + \dots) \approx \sum_{l=0}^{\infty} (\gamma \lambda)^l \delta_{t+l}$ , where  $\lambda \in [0, 1]$   
: the generalized advantage estimator proposed by [8]

Several optimization method for updating the policy parameters given the policy gradient have been proposed. The following section will discuss some state-of-art methods.

### 2.1.1 Trust Region Policy Optimization

The method proposed by [9], namely Trust Region Policy Optimization (TRPO) is one of the early works in deep reinforcement learning for continuous control. The method formulates the policy optimization problem as a constraint optimization problem:

$$\begin{aligned} & \underset{\theta}{\text{maximize}} && J(\theta) \\ & \text{subject to} && \overline{D_{KL}}(\pi_{\theta_{old}} \parallel \pi_{\theta}) \leq \delta_{KL} \end{aligned} \tag{2.2}$$

where  $\pi_{\theta_{old}}$  is the old policy parameter before performing parameter update, and  $\overline{D_{KL}}$  is the average KL divergence over the samples,  $\delta_{KL}$  is the hyper-parameter that control the step size of updates.

The method solves this problem through 4 steps. The first step computes  $g$  following the Equation 2.1. The second step computes the Riemannian metric tensor of the parameter space of the policy model  $A = H_{\theta} \left( \overline{D_{KL}}(\pi_{\theta_{old}} \parallel \pi_{\theta}) \right)$ , where  $H_{\theta}(\cdot)$  denotes the Hessian matrix with respect to  $\theta$ . The third step obtains which is the natural gradient update direction  $s = A^{-1}g$  by solving  $g = As$  through the conjugate gradient algorithm. The fourth step computes the maximum step-size  $\beta = \sqrt{2\delta_{KL}/s^T As}$  and performs a line search to ensure the improvement of the objective function.

The method is able to solve some primary robot control tasks in [10] within a few number of training samples. However, both the computation of the Hessian matrix and the conjugate gradient algorithm are too expensive for deep neural network models.

### 2.1.2 Kronecker-factored Trust Region

The work of [7] proposes to reduce the computation time solving the natural gradient by Kronecker-factored approximation method. The Fisher Information Matrix  $F =$

$\mathbb{E}[\nabla_{\theta} L \nabla_{\theta} L^T]$ , where  $L = \log \pi(a|s)$  is used here to approximate the Riemannian metric tensor. The matrix is approximated by:

$$F = \mathbb{E}[\nabla_{\theta} L \nabla_{\theta} L^T] = \mathbb{E}[aa^T \otimes \nabla_s L \nabla_s L^T] \approx \mathbb{E}[aa^T] \otimes \mathbb{E}[\nabla_s L \nabla_s L^T]$$

where  $a$  is the input activation vector corresponding to the neural network weight parameters and  $s$  is the corresponding preactivation vector, The natural gradient is then solved by:

$$s = A^{-1}g \approx F^{-1}g \approx \left( \mathbb{E}[aa^T] \otimes \mathbb{E}[\nabla_s L \nabla_s L^T] \right)^{-1} g = \mathbb{E}[aa^T]^{-1} g \mathbb{E}[\nabla_s L \nabla_s L^T]^{-1} \quad (2.3)$$

The proposed method further speeds up the optimization by reusing recently computed statistic matrices in the previous steps and performing asynchronous computation, and finally achieves a computation time comparable to ordinary gradient decent algorithms. The method manages to computationally efficiently perform trust region natural policy gradient optimization without sacrificing the reinforcement learning performance.

### 2.1.3 Proximal Policy Gradient Method

The author of [11] proposed a first-order policy optimization method, namely proximal policy gradient with clipping (PPO). The method clips the surrogate objective according to the following equation:

$$L_{CLIP}(\theta) = \mathbb{E} \left[ \min \left( r_t(\theta) A_t, \text{clip}(r_t(\theta), 1 - \epsilon, 1 + \epsilon) A_t \right) \right] \quad (2.4)$$

where  $r_t(\theta) = \pi_{\theta} / \pi_{\theta_{old}}$  is the likelihood ratio of the current policy over the sample policy, and  $\epsilon \in [0, 1]$  is a hyperparameter.

The algorithm performs minibatch stochastic optimization at each batch of collected agent's experience, for a number of epochs. Therefore it is actually an off-policy method since the policy has already deviated from the sampling policy once updated for the first minibatch.

The method can achieve a relatively fast improvement rate in the early phase (within 1 million training steps) of training in several continuous control environments. However, the method is more difficult to tune compared to trust region methods, and might become increasingly unstable in the late phase of training. The method might also converge too early and stuck at a local minimum point, because updating the parameter too many times on a single batch might lead to overfitting.

### 2.1.4 Other Methods in Deep Reinforcement Learning for Continuous Control

Other methods related to deep reinforcement learning for continuous control include the Deep Deterministic Policy Gradient Method (DDPG) [12], Actor-Critic with Experience Replay (ACER) [13], and Trust-PCL [14]. These methods are first order off-policy algorithm, and are not the focus of our study because they're not reported to significantly outperform the above mentioned methods in terms of training time, final performance and stability.

## 2.2 Option framework

The work of [15] is among the earliest studies of hierarchical reinforcement learning, which uses the notion of *option* to define the partial policies / subroutines. An option consists of a policy  $\pi$ , a termination condition  $\beta$  and an input set  $I$  that indicates whether the partial policy is available at the current state. Once an option is executed, then actions are chosen according to  $\pi$  until  $\beta(s)$  outputs a termination signal.

An option is Markov if its policy is Markov. Semi-Markov options, on the other hand, are options whose policies are based on the entire history since the option was initiated. Semi-Markov options include options that terminates after a specific number of time steps, which could be particularly useful when considering policies over options.

A policy over options  $\mu$  selects option  $o$  in state  $s$  with probability  $\mu(s, o)$ .

A concept of multi-step model is proposed as a generalization of single-step models. For any option  $o$ , let  $E(o, s, t)$  denote the event of the option  $o$  initialized in state  $s$  at time  $t$ . Then the reward of the multi-step model is defined as:

$$R(s, o) = E\{r_{t+1} + \gamma r_{t+2} + \dots + \gamma^{t+\tau} r_{t+\tau} | E(o, s, t)\}$$

where  $t + \tau$  is the termination time of  $o$ . The state transition model for the option is:

$$P(s'|s, o) = \sum_{\tau=1}^{\infty} p(s', \tau) \gamma^{\tau}$$

for all states  $s' \in S$ , where  $p(s', \tau)$  denotes the probability that the option terminates after  $\tau$  steps and results in the state  $s'$ .

A generalized form of the Bellman optimality equation is then proposed:

$$V_O^*(s) = \max_{o \in O_s} \left[ R(s, o) + \sum_{s'} P(s'|s, o) V_O^*(s') \right] \quad (2.5)$$

In conclusion, this work proposes a formulation of hierarchical reinforcement learning and connects it to SMDP theory. The authors also propose the method for training the policy over options. However, the question on how the hierarchy of options are developed are not answered.

## 2.3 Modular hierarchical architecture

The work of [16] is among the first studies that applies hierarchical reinforcement learning on robot continuous control problems. They propose a two-level hierarchical agent architecture, where there is a high-level policy and a low-level policy. The high-level policy outputs a modulation signal which the low level agent then takes as part of its input. They train the hierarchical agent in a pretraining-transfer manner, where the low-level policy is trained in a simple task first, and the high level agent policy is then trained in the target task. The proposed method achieves reasonable performance on a few realistic robot control tasks. However, the hierarchical relationships between the two agent, including the pre-training task definition and modulation model, need to be manually predefined. This involves a large amount of domain-specific engineering and cannot be applied to general tasks.

## 2.4 Learning a hierarchical model by multi-task training

Another work [17], namely meta learning shared hierarchies, proposes a two-level hierarchical reinforcement learning agent as well as a optimization algorithm. There is one high level policy and several low level policies. The high level policy make decisions at fixed intervals, which selects the corresponding low level policy to be executed.

The method trains the high level policy and the low level policies simultaneously. However, the agent is trained for different tasks in parallel with different high level policy parameters but shared low level policy parameters. Both the high level policy and the low

level policy are trained on the original reward function of the corresponding task. The method achieves good performance in several simple control environments. However the tasks are actually simplified version of the control tasks in [3].

The method predefines the temporal relationship between the high level policy and low level policy. Apart from that, the training of the low level policy extremely relies on a near-optimal high level policy, which is not possible to be obtained without a set of good low level policies.

Therefore, this method not only needs domain-specific design, but is also in-feasible for general realistic environments.

## 2.5 Goal-directed behaviour learning method

Several works [18], [19] have tried to solve sparse-reward robot control environments through the learning of auxiliary tasks, which are defined by reaching specific states, namely "goals".

A representative work is [18], namely Scheduled Auxiliary Control. The method proposes a two-level hierarchical reinforcement learning agent, which consists of one high level policy and a predefined number ( $K$ ) of low level policies. The low level policies are trained in auxiliary MDPs based on predefined goal states.

The reward function of a auxiliary task with goal state  $g$  is defined as:

$$r_g(s, a) = \begin{cases} \delta_g(s), & \text{if } d(s, g) \\ 0, & \text{else} \end{cases} \quad (2.6)$$

The method first train the low level policies in their corresponding auxiliary MDPs, then train the high level policy according to SMDP theory.

The method manage to solve several challenging robot control tasks. However, the method heavily relies on the design of goal states, which is involves domain-specific engineering and is infeasible in tasks with high-dimensional states.

## 2.6 Strategic Attentive Writer

The work of [20] proposes a method that can learn macro-actions, which are open-loop partial policies with flexible length of execution. The method is not scalable to realistic

problems due to the lack of closed-loop control. However, one contribution of the work is that it proposes a model that can learn high-level, temporally abstracted macro-actions of varying length in an end-to-end manner.

## Chapter 3

# Methodology

### 3.1 TODO: Target Environments

#### 3.1.1 Ant-based environments

##### Summary of Environments

The source environments and target environments are summarized in table 3.1.1.

| Task name | Goal                               | Reward  | Description   |
|-----------|------------------------------------|---|---|
| move0     | velocity: $g_0 = (1, 0)$           | $v_g + 1 - c_p - c_c$   | move toward east direction                              |
| move1     | velocity: $g_1 = (-1, 0)$          | $v_g + 1 - c_p - c_c$   | move toward west direction                              |
| move2     | velocity: $g_2 = (0, 1)$           | $v_g + 1 - c_p - c_c$   | move toward north direction                             |
| move3     | velocity: $g_3 = (0, -1)$          | $v_g + 1 - c_p - c_c$   | move toward south direction                             |
| move1d    | velocity: $\{g_0, g_1\}$           | $v_g + 1 - c_p - c_c$   | each episode has a random sampled goal direction        |
| move2d    | velocity: $\{g_0, g_1, g_2, g_3\}$ | $v_g + 1 - c_p - c_c$   | there are 4 possible goal directions compared to move1d |
| dynamic2d | velocity: $\{g_0, g_1, g_2, g_3\}$ | $v_g + 1 - c_p - c_c$   | the goal direction is reset with probability 0.01       |
| reach2d   | position: $\{g_0, g_1, g_2, g_3\}$ | $I(\text{The agent's distance to the goal position is within } 0.5) - 0.01$ | move to a specific location instead of a direction      |

Table 3.1: Summary of Ant-based environments



Dynamic2D environment

Reach2D environment

### 3.1.2 TODO: Robot manipulation environments

## 3.2 Wasserstein Actor Critic Kronecker-factored Trust Region Policy Optimization method

As is reported in [21], the result of the current state-of-art deep reinforcement learning methods for continuous control, including TRPO, ACKTR, PPO and DDPG are hard to be reproduced, due to they're heavily influenced by a variety of factors including random seed, neural network architecture, activation functions in neural network and software implementation. This means that the state-of-art methods are lack of stability in the agent's final performance and robustness to minor change in parameters. That has greatly diminishes the reliability of contemporary deep reinforcement learning methods.

We propose a new policy optimization method, namely Wasserstein Actor Critic Kronecker-factored Trust Region Policy Optimization (W-KTR). We claim that the proposed method outperform other state-of-art methods in terms of agent's final performance, total training time and reproducibility.

The proposed W-KTR method focus on the problem scope of deep reinforcement learning for continuous control, specifically robot control problems. The action of these environments usually stands for mechanics-related physical quantity, such as motor torque and target motor phase. However, the traditional KL-divergence based algorithms are not suitable for these problems. Because the KL-divergence cannot represent the "deviation of policy" of the problem well, because a small perturbation in the mean value of a policy will lead to a large KL-divergence when the variance is small.

Therefore, we propose to use another criteria, which is the Wasserstein metric, to measure the deviation of the policy updates. We reformulate the trust region policy optimization problem as follows:

$$\begin{aligned} & \underset{\theta}{\text{maximize}} && J(\theta) \\ & \text{subject to} && \overline{W_2}(\pi_{\theta_{old}}, \pi_{\theta}) \leq \delta_W \end{aligned} \tag{3.1}$$

where  $\overline{W_2}(\pi_{\theta_{old}}, \pi_{\theta})$  is the average Wasserstein-2 metric between the old policy and the

current policy.

The Wasserstein-2 metric is defined by the following equation [22]:

$$W_2(P, Q) = \inf_{\Gamma \in \mathbb{P}(X \sim P, Y \sim Q)} \mathbb{E}_{X, Y \sim \Gamma} [\|X - Y\|_2^2]^{1/2} \quad (3.2)$$

where  $\mathbb{P}(X \sim P, Y \sim Q)$  is the set of all joint distribution of  $X, Y$  with marginals  $P, Q$  respectively.

Specifically, when  $P$  and  $Q$  are Gaussian distributions with mean  $m_1, m_2$  and covariance matrix  $\Sigma_1$  and  $\Sigma_2$ , the squared value of Wasserstein-2 distance  $W_2^2(.)$  is defined as the following equation [23] :

$$W_2^2(P, Q) = \|m_1 - m_2\|_2^2 + \text{Tr} \left( \Sigma_1 + \Sigma_2 - 2(\Sigma_1^{1/2} \Sigma_2 \Sigma_1^{1/2})^{1/2} \right) \quad (3.3)$$

The problem is solved by performing gradient updates iteratively following the natural gradient with the Rianmanian metric as  $W_2$  metric:

$$s = H_\theta \left( \overline{W_2^2}(\pi_{old}, \pi) \right)^{-1} g := A^{-1} g \quad (3.4)$$

We use the approximation of the Rianmanian metric tensor  $A$ :

$$A \approx \mathbb{E}_{st} \left[ \nabla_\theta W_2^2(\pi_{old}, \pi) \left( \nabla_\theta W_2^2(\pi_{old}, \pi) \right)^T \right] \quad (3.5)$$

The solution of  $s$  is then computed using Kronecker-factor approximation technique [7].

The policy is then updated following the gradient direction  $s$  with ADAM stochastic optimization algorithm [24]. The step size is adjusted in an adaptive manner according to the resulting  $W_2$  distance at each training step.

### 3.3 Efficient Exploration Through Exceptional Advantage Regularization

One of the major reasons that multi-modality robot environments are challenging is because the features in different modalities have different complexities.

In the target case, where there are two modality: the locomotion state vector and the image input, the agent is very likely to get stuck at a local minimum after having learned the features of the locomotion state vectors while haven't learned the image features.

However, the policy is at a much lower entropy at the phase when the agent starts to learn image features.

As far as we know, the distribution type of the continuous stochastic policy in reinforcement learning is always chosen to be normal distribution with diagonal covariance matrices in the works on policy gradient methods. We have observed a phenomenon in the training process of policy gradient methods that, the variance of the policy is extremely unlikely to be increased, even when the agent has just escaped from a local minimum. Therefore, a regulation method becomes necessary to encourage the exploration of the agent when it finds that its policy has converged too fast at a sub-optimal point.

A conventional regularization method for encouraging exploration is entropy regularization:

$$g' = g + \beta_{ent} \nabla_{\theta} \mathbb{E}[H(\pi_{\theta}(a|s))] \quad (3.6)$$

Where  $\beta_{ent}$  is the weight controlling the penalty on low entropies. The entropy of a normal distribution  $\mathcal{N}(\mu, \Sigma)$  is defined as:

$$H(\pi) = \frac{1}{2} \ln \det(2\pi e \Sigma) \quad (3.7)$$

When the policy distribution is a normal distribution with diagonal covariance matrix, the entropy regularization basically introduces a constant bias in the gradient of the logarithm of variance parameters. As a result, the entropy regularization method is usually hard to tune, and leads to degradation on learning performance.

Here we propose a novel method that can efficiently encourage exploration without large penalty on the agent's learning performance. We add a loss, namely Exceptional Advantage Regularization loss (EAR), to the original gradient of the variance parameters:

$$g'_{\Sigma} = g_{\Sigma} + \beta_{exc} \mathbb{E} \left[ \max \left( 0, I(\hat{A}^{GAE} > 0) \hat{A}^{GAE} \nabla_{\Sigma} \log \pi(a|s) \right) \right] \quad (3.8)$$

where  $\beta_{exc}$  is the weight controlling the bias on exploration, and  $I(.)$  is the indicator function, which returns 1 if the expression is true, otherwise 0.

The EAR loss add weights for the positive gradients of the variance parameters of the samples with positive advantage values. The reason behind the loss is that we want to add more importance to the samples which produce exceptionally positive advantage

value, but with low sampling likelihood. Therefore we first only focus on points with positive advantage values. The problem remaining is to identify the points with low likelihood, namely "exceptional" points. We simply choose these points that produce positive gradients any of the variance parameters.

### 3.4 Efficient Exploration Through Robust Concentric Mixture Gaussian Policy

Apart from the EAR method for exploration, we propose to use an alternative probability distribution type, namely Concentric Mixture Gaussian, instead of a diagonal Gaussian policy.

The proposed policy distribution, namely Robust Concentric Mixture Gaussian (RCMG) Policy, is a mixture of two Gaussian distributions:

$$\pi(a|s) = (1 - \alpha_{ex})\mathcal{N}(\mu, \Sigma) + \alpha_{ex}\mathcal{N}(\mu, q_{ex}\Sigma) \quad (3.9)$$

where the constant  $\alpha_{ex}$  is the weight of second distribution, which for example can be set at 0.05, and the constant  $q_{ex} > 1$ , for example  $q_{ex} = 5$ , controls the standard deviation of the second deviation.

Although the KL divergence between the two RCMG policy doesn't have a analytical form, the Wasserstein-2 distance between two RCMG policy can be given by:

$$\begin{aligned} W_2(\pi_0(a|\mu_0, \Sigma_0), \pi_1(a|\mu_1, \Sigma_1)) = \\ (1 - \alpha_{ex})W_2(\mathcal{N}(\mu_0, \Sigma_0), \mathcal{N}(\mu_1, \Sigma_1)) + \alpha_{ex}W_2(\mathcal{N}(\mu_0, q_{ex}\Sigma_0), \mathcal{N}(\mu_1, q_{ex}\Sigma_1)) \end{aligned} \quad (3.10)$$

The RCMG policy is much more robust than an ordinary Gaussian policy because it has much longer tails.

### 3.5 Hierarchical reinforcement learning architecture

We propose to solve the reinforcement learning problem by a two-level hierarchical model.

The hierarchical model consists of a top-level decider agent and a set of bottom-level actuator agents. The actuator agents' policies are trained in the source task environments.

The decider agent takes an action at every time-step. It may either decide which actuator-policy should be executed, or simply skip and continue current actuator-policy. Therefore, assume there are  $n_a$  sub-policies, the action space of the root agent is an  $(n_a + 1)$ -discrete action space.

The observation space of the root agent consists of 2 parts, original statn (motion-sensor observation, image observation) and the meta state. The meta observation state the current sub-policy being executed and the number of time-steps since the last decision has been made.

Empirically, the decider agent is parameterized by two policy networks,  $\theta_s$  and  $\theta_d$ . The network  $\theta_s$ , namely switcher network, outputs a binary action that decides whether the agent should simply continue using the current acting actuator policy, or switch to another policy based on the current state. The agent

The network  $\theta_d$ , namely decider network, outputs an  $n_a$ -discrete action space, that select the acting agent

The selected leaf agent executes the corresponding sub-policy and computes the primary actions the agent should take for the original environment.

The overall decision-making process of the decider agent is shown in Algorithm 1.

---

**Algorithm 1** The decider agent mechanism

---

```

function DECIDERACT(self,  $s_t$ )
     $a_{decider} \sim \pi_{decider}(s_t)$ 
    if  $a_{decider} \neq 0$  then
         $self.currentActuator \leftarrow self.allActuators[a_{decider} - 1]$ 
     $a_{actuator} \leftarrow self.currentActuator.act(s_t)$ 
    return  $a_{actuator}$ 

```

---

## 3.6 Generalized advantage estimation for hierarchical reinforcement learning agents

We propose a generalized advantage estimation method for decider agents hierarchical reinforcement learning agents.

Assume that a decider agent makes decisions at time  $t_1, t_2, \dots$ , then the execution length of the decisions are  $l_i = t_{i+1} - t_i, i = 1, 2, \dots$ .

Then the definition of reward of the decider action at  $t_i$  is given by:

$$\bar{r}_{t_i} := \sum_{l=0}^{t_{i+1}-t_i-1} \gamma^l r_{t_i+l} \quad (3.11)$$

Define the TD residual  $\delta_{t_i}^V$  for  $i = 0, 1, \dots$  by:

$$\delta_{t_i}^V := -V(s_{t_i}) + \bar{r}_{t_i} + \gamma^{t_{i+1}-t_i} V(s_{t_{i+1}}) \quad (3.12)$$

Then the k-step advantage estimation is given by:

$$\hat{A}_{t_i}^{(1)} := \delta_{t_i}^V = -V(s_{t_i}) + \bar{r}_{t_i} + \gamma^{t_{i+1}-t_i} V(s_{t_{i+1}}) \quad (3.13)$$

$$\hat{A}_{t_i}^{(2)} := \delta_{t_i}^V + \gamma^{t_{i+1}-t_i} \delta_{t_{i+1}}^V = -V(s_{t_i}) + \bar{r}_{t_i} + \gamma^{t_{i+1}-t_i} \bar{r}_{t_{i+1}} + \gamma^{t_{i+2}-t_i} V(s_{t_{i+2}}) \quad (3.14)$$

$$\hat{A}_{t_i}^{(3)} := \delta_{t_i}^V + \gamma \delta_{t_{i+1}}^V + \gamma^2 \delta_{t_{i+2}}^V = -V(s_{t_i}) + \bar{r}_{t_i} + \gamma \bar{r}_{t_{i+1}} + \gamma^2 \bar{r}_{t_{i+2}} + \gamma^3 V(s_{t_{i+3}}) \quad (3.15)$$

$$\begin{aligned} \hat{A}_{t_i}^{(k)} &:= \sum_{d=0}^{k-1} \gamma^{t_{i+d}-t_i} \delta_{t_{i+d}}^V \\ &= -V(s_{t_i}) + \bar{r}_{t_i} + \gamma^{t_{i+1}-t_i} \bar{r}_{t_{i+1}} + \dots + \gamma^{t_{i+k-1}-t_i} \bar{r}_{t_{i+k-1}} + \gamma^{t_{i+k}-t_i} V(s_{t_{i+k}}) \end{aligned} \quad (3.16)$$

We can define the unnormalized generalized advantage estimator as a exponentially-weighted sum of these k-step advantage estimators [8]:

$$\begin{aligned} \hat{A}_{t_i}^{GAE_{unnorm}(\lambda)} &:= \hat{A}_{t_i}^{(1)} + \lambda^{t_{i+1}-t_i} \hat{A}_{t_i}^{(2)} + \lambda^{t_{i+2}-t_i} \hat{A}_{t_i}^{(3)} + \dots + \lambda^{t_{i+k-1}-t_i} \hat{A}_{t_i}^{(k)} \\ &= \delta_{t_i}^V + \lambda^{t_{i+1}-t_i} (\delta_{t_i}^V + \gamma^{t_{i+1}-t_i} \delta_{t_{i+1}}^V) \end{aligned} \quad (3.17)$$

$$\begin{aligned} &+ \lambda^{t_{i+2}-t_i} (\delta_{t_i}^V + \gamma^{t_{i+1}-t_i} \delta_{t_{i+1}}^V + \gamma^{t_{i+2}-t_i} \delta_{t_{i+2}}^V) + \dots \\ &+ \lambda^{t_{i+k-1}-t_i} \sum_{d=0}^{k-1} \gamma^{t_{i+d}-t_i} \delta_{t_{i+d}}^V \end{aligned} \quad (3.18)$$

$$\begin{aligned} &= (\delta_{t_i}^V \sum_{b=0}^{k-1} \lambda^{t_{i+k-1}-t_i} + \gamma^{t_{i+1}-t_i} \delta_{t_{i+1}}^V \sum_{b=1}^k \lambda^{t_{i+b}-t_i} \\ &+ \gamma^{t_{i+2}-t_i} \delta_{t_{i+2}}^V \sum_{b=2}^k \lambda^{t_{i+b}-t_i} + \dots \end{aligned} \quad (3.19)$$

$$\begin{aligned} &+ \gamma^{t_{i+k-1}-t_i} \delta_{t_{i+k-1}}^V \lambda^{t_{i+k-1}-t_i}) \\ &= \sum_{d=0}^{k-1} \delta_{t_i}^V \gamma^{t_{i+d}-t_i} \sum_{b=0}^{k-1} \lambda^{t_{i+b}-t_i} \end{aligned} \quad (3.20)$$

The normalized generalized advantage estimator is then given by:

$$\hat{A}_{t_i}^{GAE(\lambda)} = \frac{\hat{A}_{t_i}^{GAE_{unnorm}(\lambda)}}{\sum_{b=0}^{k-1} \lambda^{t_{i+b}-t_i}} \quad (3.21)$$

However, the unnormalized GAE estimator is usually used in practice instead of the normalized one, with a postprocessing step of batch normalization to adjust the scale of the advantages. This practical method usually lead to large advantage scales for experience data at the beginning of the episodes and small scales for the experience data near episode ends.

## Chapter 4

# Experiments

### 4.1 Solution to the basic environment: *move0*

In this section, we compare the proposed W-KTR method with other state-of-art algorithms on the basic *move0* environment.

The performance of W-KTR algorithm is shown in Figure 4.1:

It can be shown that the W-KTR agent has a stable final performance, which is not much dependent on the hyper-parameter  $\delta_W^2$ .

The performance of ACKTR agents is shown in Figure 4.1: It can be shown that the ACKTR agent can achieve a faster rate of improvement at the first 10 million steps, but the improvement rate becomes slow as the training proceeds. The agents tend to stuck at sub-optimal policies since the policies converge too fast and cannot effectively adjust once the STD becomes small.

We also run experiments on tuning a PPO agent for the task *move0*, which is shown in Figure 4.1. The original PPO algorithms has 40 minibatch updates in each batch, which converges too early. We only perform one gradient update over the whole batch in this experiment. The performance of the PPO agent appears to be sensitive to the hyper-parameter, and the agent can't avoid the drop of performance in the late phase of training. The method also needs to trade-off between the early improvement rate and the final performance. In conclusion, the PPO method is not a robust algorithm.

All the method above use a batch size of 4000 timesteps produced by 20 parallel agents.



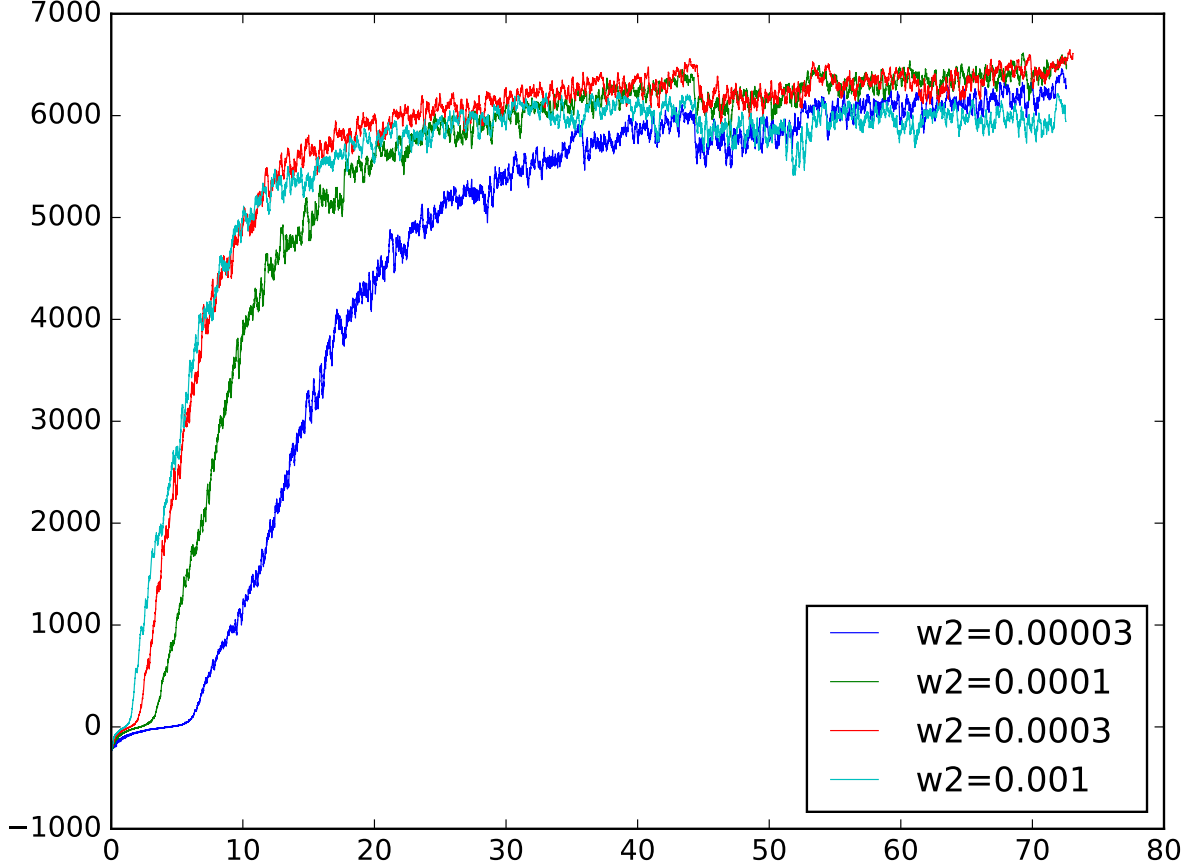


Figure 4.1: Performance of move0 W-KTR agent with different hyper-parameter  $\delta_W^2$ , the x-axis is the number of million timesteps and the y-axis is the total episode reward averaged over the last 200 episodes

## 4.2 Flat solution to a simple Multi-Modality environment: *move1d*

In the *move1d* environment, the agent needs to learn from the image input to decide whether to move forward or backward, as well as control according to the states. We have tried to solve the problem using ACKTR method, presented in Figure 4.2, shows that the agent can easily get stuck at the score of 1000, and cannot improve much even if the agent breaks the bottleneck because the average STD parameter already drops to below 0.04.

We also did a single experiment using the proposed W-KTR method, and is shown in Figure 4.2. The W-KTR agent manages to achieve a fast improvement rate and high final performance after getting stuck at the bottleneck score 1000 for a while.

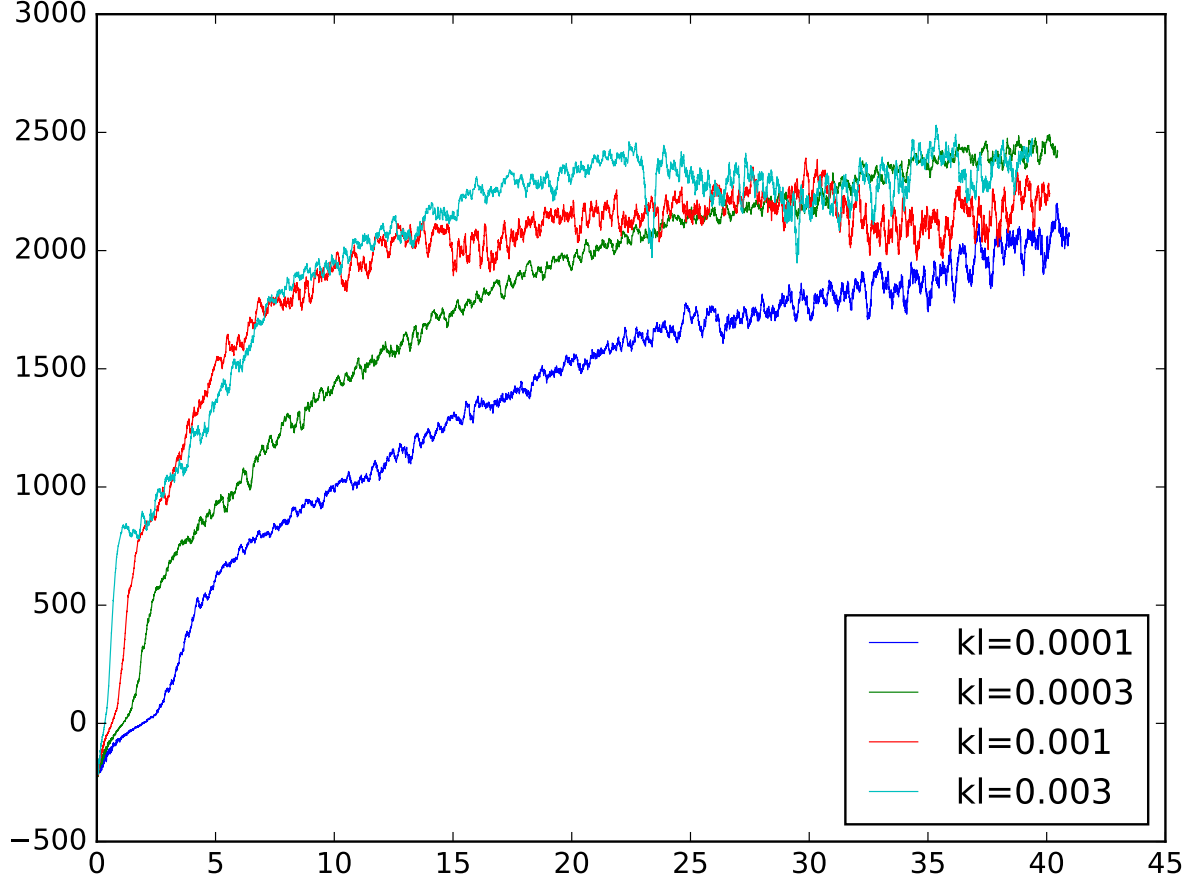


Figure 4.2: Performance of move0 PPO agent with different hyper-parameter  $\delta_{kl}$ , the x-axis is the number of million timesteps and the y-axis is the total episode reward averaged over the last 200 episodes

This proves that the W-KTR agent is better at handle local minimum and re-adjust the agent's policy even when the policy STD becomes low.

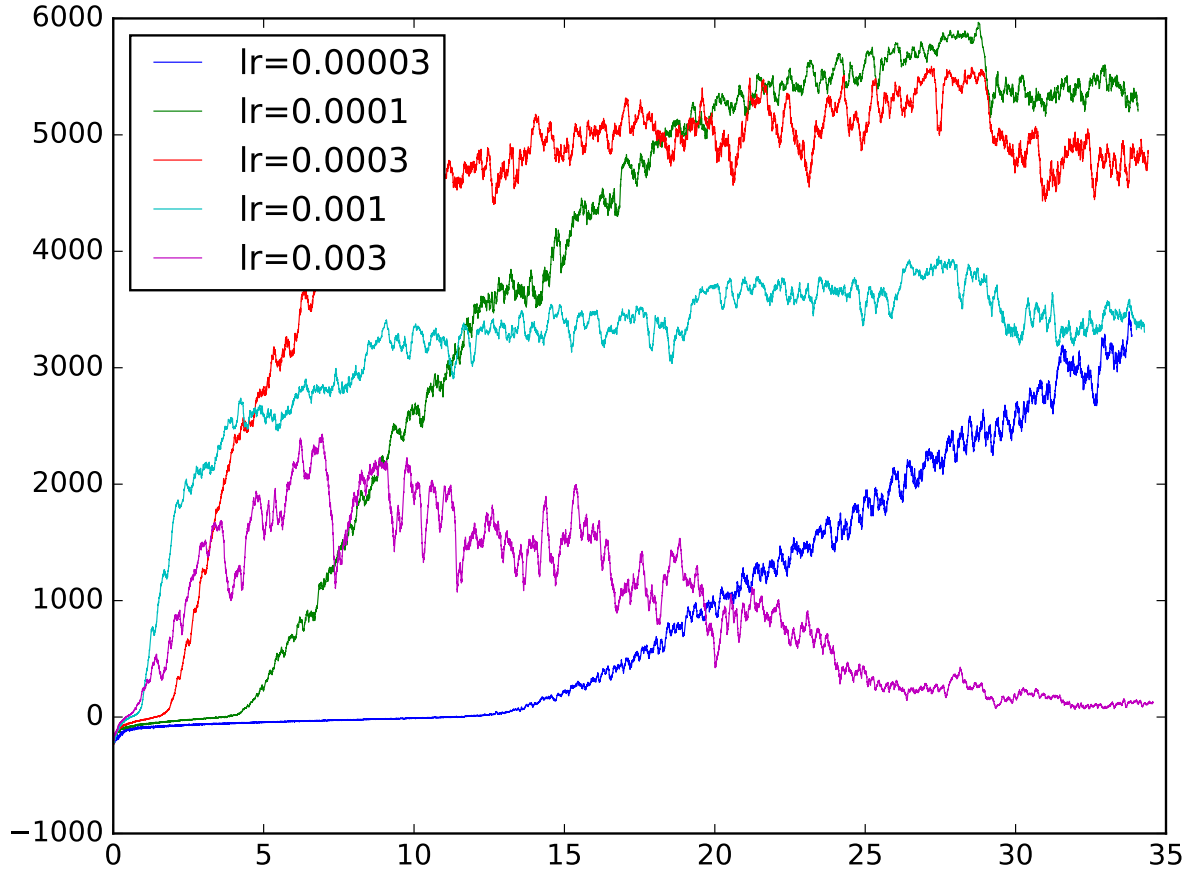


Figure 4.3: Performance of move0 PPO agent with different hyper-parameter *learningrate*, the x-axis is the number of million timesteps and the y-axis is the total episode reward averaged over the last 200 episodes

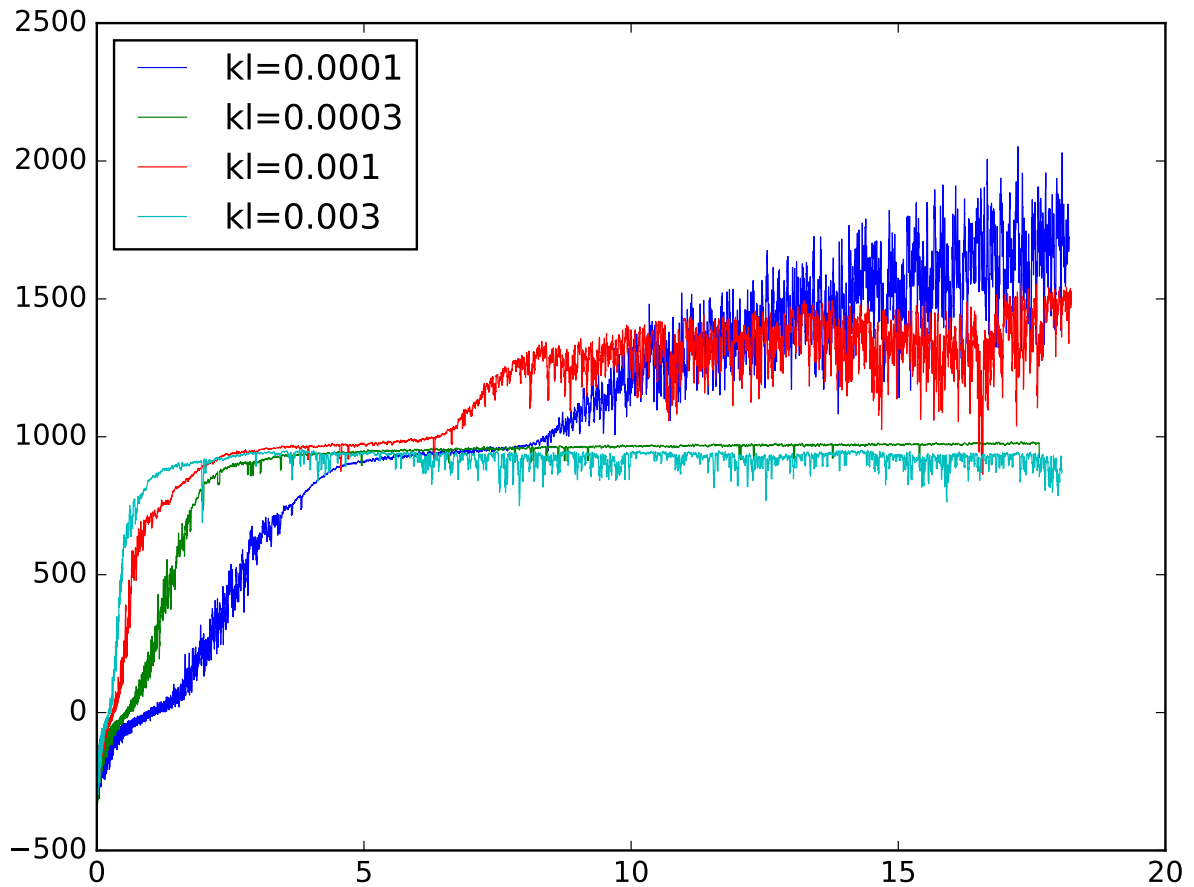


Figure 4.4: Performance of move1d ACKTR agent with different hyper-parameter *learningrate*, the x-axis is the number of million timesteps and the y-axis is the total episode reward averaged over the last 20 episodes

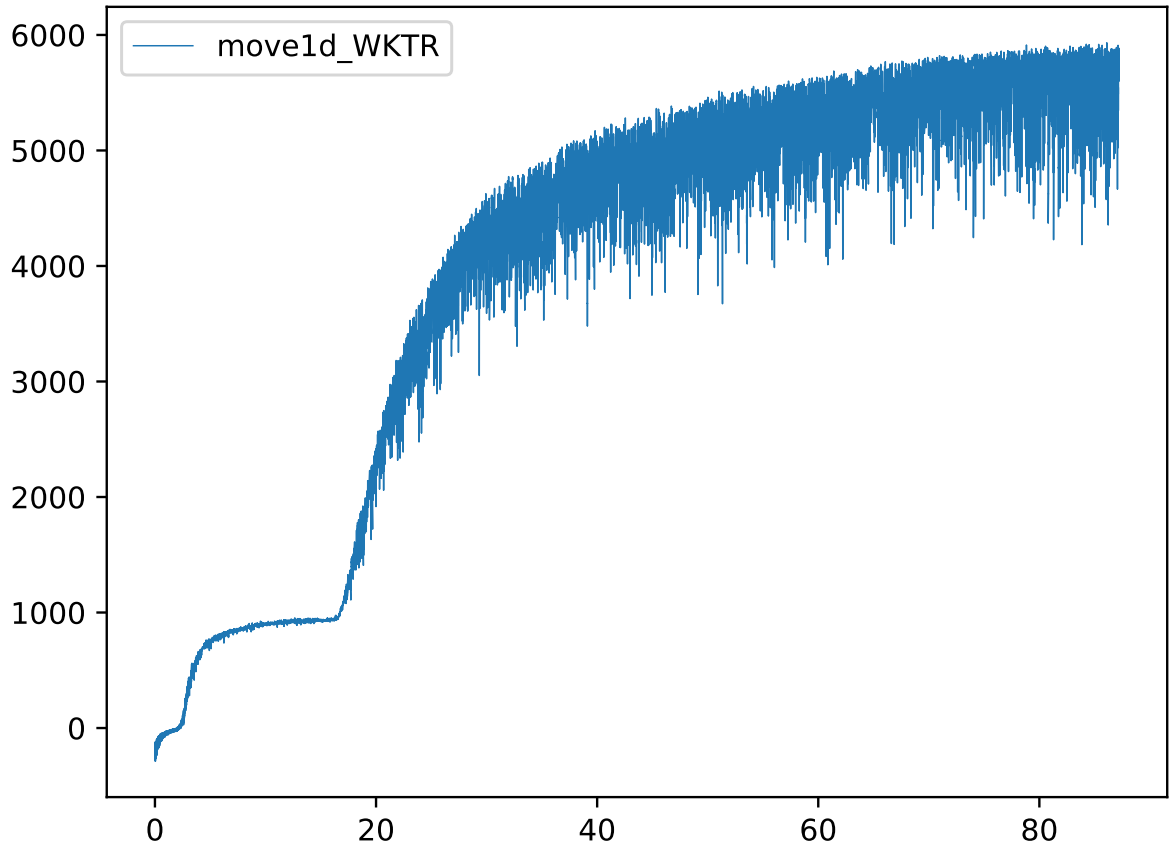


Figure 4.5: Performance of move1d W-KTR agent with hyper-parameter  $\delta_W^2 = 0.0001$ , the x-axis is the number of million timesteps and the y-axis is the total episode reward averaged over the last 20 episodes

## Chapter 5

# Discussion

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## Appendix A

### Appendix 1