Fuzzy Control of an Inverted Pendulum

Inverted Pendulum Dynamics Modeling

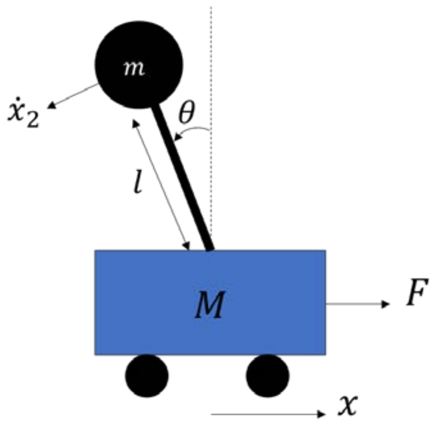

Figure 1 Pendulum model

Below is the parameters used for this project.

M	2 kg
m	0.8 kg
b_x	0.005 kg/s
θ_x	0.0005 kgm^2/s
l	0.25 m

Table 1 Pendulum model parameter

Lagrange equation

$$\begin{aligned} L &= T - V \\ &= \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2 - mgl\cos\theta \end{aligned}$$

where,

$$v_1^2 = \dot{x}^2$$

$$v_2^2 = \left(\frac{d}{dt}\left(x - l\sin\theta\right)\right)^2 + \left(\frac{d}{dt}\left(l\cos\theta\right)\right)^2$$

$$= \dot{x}^2 - 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2$$

We can substitute v_1^2, v_2^2 into L to get below equation.

$$L = \frac{1}{2}\left(M+m\right)\dot{x}^2 - ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$$

From the definition of Lagrange equation,

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) - \frac{dL}{dx} = F - b_x\dot{\theta} \quad \rightarrow ①$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) - \frac{dL}{d\theta} = b_\theta\dot{\theta} \quad \rightarrow ②$$

Here, b_x, b_θ is the coefficient of friction from the cart and the pendulum, respectively.

We can rewrite ① and ② to get the below equation

$$\left(M+m\right)\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta = F - b_x\dot{x} \quad \rightarrow ③$$

$$-\ddot{x}\cos\theta + l\ddot{\theta} - g\sin\theta = -b_\theta\dot{\theta} \quad \rightarrow ④$$

③ and ④ can be expressed as $\ddot{x}$ and $\ddot{\theta}$ to be used in ODE45's state.

From ④, $\ddot{x} = \frac{1}{\cos\theta}\left(l\ddot{\theta} - g\sin\theta + b_\theta\dot{\theta}\right)$. If you substitute this equation into ③, you can find the equation about $\ddot{\theta}$ as below.

$$\ddot{\theta} = \frac{1}{\frac{\left(M+m\right)}{\cos\theta}l - ml\cos\theta}\left\{\left(M+m\right)g\tan\theta - \frac{\left(M+m\right)}{\cos\theta}b_\theta\dot{\theta} - ml\dot{\theta}^2\sin\theta + F - b_x\dot{x}\right\}$$

Again from ④, $\ddot{\theta} = \frac{1}{l}\left(\ddot{x}\cos\theta + g\sin\theta - b_\theta\dot{\theta}\right)$. If you substitute this equation into ③, you can find the equation about $\ddot{x}$ as below.

$$\ddot{x} = \frac{1}{M + m - m\cos^2\theta}\left(mg\sin\theta\cos\theta - mb_\theta\dot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta + F - b_x\dot{x}\right)$$

The state of ODE45 is defined as below.

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}^T$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{M + m - m\cos^2 x_3}\left(mg\sin x_3 \cos x_3 - mb_\theta \cos x_3 \cdot x_4 - mlx_4^2 \sin x_3 + F - b_x x_2 \right)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{1}{\frac{M+m}{\cos x_3} l - ml\cos x_3}\left\{ (M+m) g \tan x_3 - \frac{M+m}{\cos x_3} b_\theta x_4 - mlx_4^2 \sin x_3 + F - b_x x_2 \right\}$$