Fully Distributed Flocking with a Moving Leader for Lagrange Networks with Parametric Uncertainties ⋆

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Abstract

This paper addresses the leader-follower flocking problem with a moving leader for networked Lagrange systems with parametric uncertainties under a proximity graph. Here a group of followers move cohesively with the moving leader to maintain connectivity and avoid collisions for all time and also eventually achieve velocity matching. In the proximity graph, the neighbor relationship is defined according to the relative distance between each pair of agents. Each follower is able to obtain information from only the neighbors in its proximity, involving only local interaction. We consider two cases: i) the leader moves with a constant velocity, and ii) the leader moves with a varying velocity. In the first case, a distributed continuous adaptive control algorithm accounting for unknown parameters is proposed in combination with a distributed continuous estimator for each follower. The relative position and relative velocity information between each follower and its neighbors are used in the control design. In the second case, a distributed discontinuous adaptive control algorithm and estimator are proposed. Here both the one-hop and two-hop neighbors' information are used. Then the algorithm is extended to be fully distributed with the introduction of a gain adaptation law. In both cases, flocking is achieved as long as the connectivity and collision avoidance are ensured at the initial time and the control gains are designed properly. Numerical simulations are presented to illustrate the theoretical results.

Key words: Flocking, Cooperative Control, Lagrange Dynamics, Multi-agent Systems.

1 Introduction

A multi-agent system is defined as a collection of autonomous agents which are able to interact with each other or with their environments to solve problems that are difficult or impossible for an individual agent. In a multi-agent system, the agents often act in a distributed manner to complete global tasks cooperatively with only local information from their neighbors so as to increase flexibility and robustness.

The collective behavior can be observed in nature like flock of birds, swarm of insects, and school of fish. In [1], three heuristic rules are characterized for the flocking of multi-agent systems, namely, flock centering, collision avoidance and velocity matching. In [2], a flocking algorithm is introduced for a group of agents when there is no leader. A theoretical framework is proposed in [3] to address the flocking problem with a leader, which has a constant velocity and is a neighbor of all followers. Ref. [4] considers both cases where the leader has a constant and a varying velocity. When the leader has a constant velocity, [4] relaxes the constraint that the leader is a neighbor of all followers. However, in the case where the leader has a varying velocity, it still requires that the leader be a neighbor of all followers. Unfortunately, this is an unrealistic restriction on the distributed control design, especially when the number of the followers becomes large. In [5], distributed control algorithms for swarm tracking are studied via a variable structure approach, where the moving leader is a neighbor of only a subset of the followers. In [6], the flocking control and communication optimization problem is considered for multi-agent systems in a realistic communication environment and the desired separation distances between neighboring agents is calculated in real time.

Note that all above references focus on linear multi-agent systems with single- or double-integrator dynamics. However, in reality, many physical systems are inherently nonlinear and cannot be described by linear equations. Among the nonlinear systems, Lagrange models can be used to describe a large class of physical systems of practical interest such as autonomous vehicles, robotic manipulators, and walking robots [7]. But due to the existence of nonlinear terms with parametric uncertainties, the algorithms for linear models cannot be directly used to solve the coordination problem

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for multi-agent systems with Lagrange dynamics and there are significant challenges involved.

Recent results on distributed coordination of networked Lagrange systems focus on the consensus without a leader [8–11], coordinated tracking with one leader [12–14], containment control with multiple leaders [15, 16], and flocking or swarming without or with a leader [17–19]. Ref. [17] proposes a control algorithm based on potential functions for networked Lagrange systems to achieve collision avoidance and velocity matching simultaneously in both time-delay and switching-topology scenarios. However, parametric uncertainties are not considered and there is no leader. Significant additional challenges would exist when there exist parametric uncertainties and a moving leader in the context of only local interaction. Ref. [18] presents a region-based shape controller for a swarm of Lagrange systems. By utilizing potential functions, the authors design a control scheme that can force multiple robots to move as a group inside a desired region with a common velocity while maintaining a minimum distance among themselves. However, the algorithm relies on the strict assumption that all followers have access to the information of the desired region and the common velocity. A leader-follower swarm tracking framework is established in [19] in the presence of multiple leaders. However, only a compromised result can be obtained when the group dispersion, cohesion, and containment objectives are considered together. In the proposed algorithms, the variables of the estimators must be communicated among the followers even in the case of leaders with constant velocities. Furthermore, more information is used in the controller design, for example, the second-order derivatives of the potential functions.

In this paper we focus on the distributed leader-follower flocking problem with a moving leader for networked Lagrange systems with unknown parameters under a proximity graph defined according to the relative distance between each pair of agents. Here a group of followers move cohesively with the moving leader to maintain connectivity and avoid collisions for all time and also eventually achieve velocity matching. The leader can be a physical or virtual vehicle, which encapsulates the group trajectory. We consider two cases: i) the leader moves with a constant velocity, and ii) the leader moves with a varying velocity. In the first case, a distributed continuous adaptive control algorithm accounting for unknown parameters and a distributed continuous estimator are proposed for each follower. Here the relative position and relative velocity information with respect to neighbors are used in the control design. In the second case, we first propose a distributed discontinuous adaptive control algorithm and estimator, where both the one-hop and two-hop neighbors' information are used. In this algorithm we use a common control gain that is sufficiently large for all followers, and hence the system is not completely distributed. We then improve the algorithm by further proposing a gain adaption scheme to implement a fully distributed algorithm. In both cases, flocking is achieved as long as the connectivity and collision avoidance are ensured at the initial time and the control gains are designed properly.

Comparison with the Existing Literature. This paper provides a distributed solution to flocking (connectivity maintenance, collision avoidance, and velocity matching) for multiple nonlinear Lagrange systems with parametric uncertainties in the presence of a *moving* leader with only *local interaction*. In particular, the leader's information is available to only followers in its proximity and the followers can interact with only neighbors in its proximity. While the flocking problem is addressed for single- or double-integrator agents [2, 3, 5, 6], there are significant challenges involved when the agents are nonlinear Lagrange systems with parametric uncertainties. The existence of a moving leader whose information is available to only neighbors in its proximity introduces further complexities. It is not clear how the results of these existing references can be used to deal with these new challenges. While the coordinated tracking problem for Lagrange systems also involves a moving leader [12–14], the objective is only on tracking of the leader's trajectory. With the introduction of the connectivity maintenance and collision avoidance behaviors, the distributed flocking problem with a moving leader is more complicated and it is nontrivial to consider connectivity maintenance, collision avoidance, and velocity matching in a unified manner. As shown in [5], even for double-integrator agents, the extension from coordinated tracking (Theorem 4.1) to leader-follower flocking (Theorem 4.11) is nontrivial, not to mention nonlinear Lagrange systems. While the flocking or swarming problem is addressed in [17, 18] for Lagrange systems, either the parametric uncertainties and the existence of a moving leader are not considered or it is assumed that the leader's information is available to all followers (against the local interaction nature of the problem). In the absence of a leader, it is possible to directly extend the result on leaderless synchronization to leaderless flocking. Unfortunately, this is no longer the case when there exists a moving leader with the local interaction constraint. The coexistence of the constraints, namely, nonlinear Lagrange dynamics with parametric uncertainties, a moving leader, flocking behavior, and local interaction results in significant challenges. It is also worthwhile to mention that the focus or contribution of this paper is not on proposing new potential functions. Instead, the contribution is on providing distributed control design to address the leader-follower flocking problem in the presence of the challenges, namely, a moving leader with only local interaction and nonlinear Lagrange dynamics with parametric uncertainties.

Notations: Let $\mathbf{1}_n$ denote the $n \times 1$ column vector of all ones. Let $\lambda_{\max}(.)$ and $\lambda_{\min}(.)$ denote, respectively, the maximum and minimum eigenvalue of a square real matrix with real eigenvalues. Let $\mathrm{diag}(z_1, \ldots, z_p)$ be the diagonal matrix with diagonal entries z_1 to z_p. For symmetric square real matrices A and B with the same order, $A > B$ or equivalently $B < A$ (respectively, $A \geq B$ or equivalently $B \leq A$) means that $A - B$ is symmetric positive definite (respectively, semi-definite). Throughout the paper, we use $||\cdot||$ to denote the Euclidean norm, $\otimes$ to denote the Kronecker product, and sgn$(\cdot)$ to denote the signum function defined componentwise. For a vector function $f(t) : \mathbb{R} \mapsto \mathbb{R}^m$, it is said that $f(t) \in \mathbb{L}_2$ if $\int_0^\infty f(\tau)^T f(\tau)\mathrm{d}\tau < \infty$ and $f(t) \in \mathbb{L}_\infty$ if for each element of $f(t)$, noted as $f_i(t)$, $\sup_{t\geq 0} |f_i(t)| < \infty$, $i = 1, \ldots, m$.

2 Background

2.1 Lagrange Dynamics

Suppose that there exist $n+1$ agents (e.g., autonomous vehicles) consisting of one leader and n followers. The leader is labeled as agent 0 and the followers are labeled as agent 1 to n. The n followers are described by Lagrange equations of the form [7]

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = u_i, \qquad i = 1, \ldots, n, \tag{1}$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates [1], $M_i(q_i)$ is the $p \times p$ symmetric inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i$ is the Coriolis and centrifugal force, $g_i(q_i)$ is the vector of gravitational force, and u_i is the control input. The dynamics of the Lagrange systems satisfy the following properties:

(P1) There exist positive constants $k_{\underline{M}}, k_{\overline{M}}, k_{\overline{C}}, k_{\overline{g}}$ such that $k_{\underline{M}} I_p \leq M_i(q_i) \leq k_{\overline{M}} I_p$, $||C_i(q_i, \dot{q}_i)\dot{q}_i|| \leq k_{\overline{C}}||\dot{q}_i||$ and $||g_i(q_i)|| \leq k_{\overline{g}}$.
(P2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.
(P3) The left-hand side of the Lagrange dynamics can be parameterized, i.e., $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + g_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\theta_i$, $\forall x, y \in \mathbb{R}^p$, where $Y_i \in \mathbb{R}^{p \times p_\theta}$ is the regression matrix and $\theta_i \in \mathbb{R}^{p_\theta}$ is the unknown but constant parameter vector.

In this paper, the leader can be a physical or virtual vehicle, which encapsulates the group trajectory. The leader's position and velocity are denoted by, respectively, $q_0 \in \mathbb{R}^p$ and $\dot{q}_0 \in \mathbb{R}^p$.

2.2 Graph Theory

With k agents in a team, a graph is used to characterize the interaction topology among the agents. A graph is a pair $G = (V, E)$, where $V = \{1, \ldots, k\}$ is the node set and $E \subseteq V \times V$ is the edge set. In a directed graph, an edge $(j, i) \in E$ means that node i can obtain information from node j but not necessarily vice versa. Here node j is a neighbor of node i. In an undirected graph $(i, j) \in E \Leftrightarrow (j, i) \in E$. A directed path in a directed graph is an ordered sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots$, where $i_j \in V$. A subgraph of G is a graph whose node set and edge set are subsets of those of G.

[1] In the context of autonomous vehicles, q_i denotes the position of agent i.

The adjacency matrix $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{k\times k}$ of the graph G is defined such that the edge weight $a_{ij} = 1$ if $(j, i) \in E$ and $a_{ij} = 0$ otherwise. For an undirected graph, $a_{ij} = a_{ji}$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{k\times k}$ associated with $\mathbf{A}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. For an undirected graph, L is symmetric positive semi-definite [20].

In this paper, we assume that the neighbor relationship among the leader and the followers is based on their relative distance and hence the graph characterizing the interaction topology is a *proximity* graph. We also assume that the leader has no neighbor and its motion is not necessarily dependent on the followers. In particular, followers i and j are neighbors of each other if $||q_i - q_j|| < R$ and the leader is a neighbor of follower i if $||q_i - q_0|| < R$, where R denotes the communication or sensing radius of the agents. Let G_F be the proximity graph characterizing the interaction among the n followers with the associated Laplacian matrix L_F. Note that by definition G_F is undirected and hence L_F is symmetric positive semi-definite. Let $\overline{G}$ be the directed graph characterizing the interaction among the leader and the n followers corresponding to G_F. Also let the edge weight $a_{i0} = 1$ if the leader is a neighbor of follower i and $a_{i0} = 0$ otherwise. Define the *leader-follower topology matrix* associated with the graph $\overline{G}$ as $H = L_F + \mathrm{diag}(a_{10}, \ldots, a_{n0})$. It is obvious that H is symmetric positive semi-definite. Before moving on, we need the following lemmas.

Lemma 2.1 *[21] If the leader has directed paths to all followers, the matrix H is symmetric positive definite.*

Lemma 2.2 *Let H^a and H^b be the leader-follower topology matrix associated with, respectively the graph $\overline{G}^a$ and $\overline{G}^b$. If $\overline{G}^a$ is a subgraph of $\overline{G}^b$, then $H^a \leq H^b$.*

Proof: When $\overline{G}^a$ is a subgraph of $\overline{G}^b$, H^b can be written as $H^b = H^a + P$, where P is a positive semi-definite matrix. Therefore, it can be concluded that $H^a \leq H^b$.

3 Main Results

In this section, we study the leader-follower flocking problem for networked Lagrange systems. The goal is to design u_i for each follower to achieve leader-follower flocking. That is, the followers move cohesively with the leader (connectivity maintenance) and avoid collisions for all time and eventually achieve velocity matching with the leader ($||\dot{q}_i(t) - \dot{q}_0(t)|| \to 0$) in the presence of unknown parameters under only local interaction defined by the proximity graph. Before moving on, the following auxiliary variables are defined:

$$s_i = \dot{q}_i - v_i, \qquad \tilde{q}_i = q_i - q_0, \qquad \tilde{v}_i = v_i - \dot{q}_0, \tag{2}$$

where v_i is agent i's estimate of the leader's velocity to be designed later. Note that

$$s_i = \dot{\tilde{q}}_i - \tilde{v}_i. \tag{3}$$

3.1 Flocking when the leader has a constant velocity

In this subsection, we consider the case where the leader has a constant velocity. We propose the following distributed control algorithm

$$u_i = \hat{u}_i + Y_i(q_i, \dot{q}_i, \dot{v}_i, v_i)\hat{\theta}_i, \tag{4}$$

$$\hat{u}_i = -\sum_{j=0}^{n} \frac{\partial V_{ij}}{\partial q_i} - \sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j), \tag{5}$$

$$\dot{v}_i = -\sum_{j=0}^{n} \frac{\partial V_{ij}}{\partial q_i} - \sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j), \tag{6}$$

$$\dot{\hat{\theta}}_i = -\Gamma_i Y_i^T(q_i, \dot{q}_i, \dot{v}_i, v_i)s_i, \tag{7}$$

where $a_{ij}(t)$ is the edge weight associated with the proximity graph $\overline{G}$ defined in Section II-B, V_{ij} is the potential function between agents i and j to be designed, $\hat{\theta}_i$ is the estimate of the unknown but constant parameter θ_i, s_i is defined in (2), and Γ_i is a symmetric positive-definite matrix.

The potential function V_{ij} is defined as follows (see [5])

(1) When $||q_i(0) - q_j(0)|| \geq R$, V_{ij} is a differentiable nonnegative function of $||q_i - q_j||$ satisfying the conditions:
  i) $V_{ij} = V_{ji}$ achieves its unique minimum when $||q_i - q_j||$ is equal to its desired value $\overline{d}$, where $\overline{d} < R$.
  ii) $V_{ij} \to \infty$ as $||q_i - q_j|| \to 0$.
  iii) $\frac{\partial V_{ij}}{\partial(||q_i - q_j||)} = 0$ if $||q_i - q_j|| \geq R$.
  iv) $V_{ii} = c, i = 1, \ldots, n$, where c is a positive constant.

(2) When $||q_i(0) - q_j(0)|| < R$, V_{ij} is defined as above except that condition iii) is replaced with the condition that $V_{ij} \to \infty$ as $||q_i - q_j|| \to R$.

The motivation for the definition of V_{ij} is to maintain the initial connectivity pattern and to avoid collision.

In the control algorithm (4)-(7), the term $-\sum_{j=0}^{n} \frac{\partial V_{ij}}{\partial q_i}$ is used for collision avoidance and connectivity maintenance while the term $-\sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j)$ is used for velocity matching. The control algorithm (4)-(7) is distributed in the sense that each agent uses only its own position and velocity and the relative position and relative velocity between itself and its neighbors.

Theorem 3.1 *Suppose that at the initial time $t = 0$, the leader has directed paths to all followers and there is no collision among the agents. Using the control law* (4)-(7) *for* (1)*, the leader-follower flocking is achieved.*

Proof: By using the property (P3) of the Lagrange dynamics (1), it follows that $M_i(q_i)\dot{v}_i + C_i(q_i, \dot{q}_i)v_i + g_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{v}_i, v_i)\theta_i$. Then using (1), (2) and (4), we have

$$M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i = \hat{u}_i - Y_i(q_i, \dot{q}_i, \dot{v}_i, v_i)\tilde{\theta}_i, \tag{8}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. We first define the following nonnegative function

$$V_1 = \frac{1}{2}\sum_{i=1}^{n} s_i^T M_i(q_i)s_i + \frac{1}{2}\sum_{i=1}^{n} \tilde{\theta}_i^T \Gamma_i^{-1}\tilde{\theta}_i. \tag{9}$$

Note that $\dot{\tilde{\theta}}_i = -\dot{\hat{\theta}}_i$, since θ_i is constant. The derivative of V_1 is given as

$$\begin{aligned}
\dot{V}_1 =& \sum_{i=1}^{n}[s_i^T M_i(q_i)\dot{s}_i + \frac{1}{2}s_i^T \dot{M}_i(q_i)s_i - \tilde{\theta}_i^T \Gamma_i^{-1}\dot{\hat{\theta}}_i] \\
=& \sum_{i=1}^{n}[-s_i^T C_i(q_i, \dot{q}_i)s_i + s_i^T \hat{u}_i - s_i^T Y_i(q_i, \dot{q}_i, \dot{v}_i, v_i)\tilde{\theta}_i \\
&+ \frac{1}{2}s_i^T \dot{M}_i(q_i)s_i - \tilde{\theta}_i^T \Gamma_i^{-1}\dot{\hat{\theta}}_i] \\
=& \sum_{i=1}^{n} s_i^T \hat{u}_i,
\end{aligned} \tag{10}$$

where we have used (8) to obtain the second equality and the property (P2) and (7) to obtain the last equality. We then define

$$V_2 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} V_{ij} + \sum_{i=1}^{n} V_{i0}. \tag{11}$$

It follows that

$$\begin{aligned}
\dot{V}_2 =& \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(\dot{q}_i^T \frac{\partial V_{ij}}{\partial q_i} + \dot{q}_j^T \frac{\partial V_{ij}}{\partial q_j}) + \sum_{i=1}^{n}(\dot{q}_i^T \frac{\partial V_{i0}}{\partial q_i} + \dot{q}_0^T \frac{\partial V_{i0}}{\partial q_0}) \\
=& \sum_{i=1}^{n}\sum_{j=1}^{n} \dot{q}_i^T \frac{\partial V_{ij}}{\partial q_i} + \sum_{i=1}^{n}(\dot{q}_i^T \frac{\partial V_{i0}}{\partial q_i} - \dot{q}_0^T \frac{\partial V_{i0}}{\partial q_i}) \\
=& \sum_{i=1}^{n}\sum_{j=1}^{n}(\dot{q}_i - \dot{q}_0)^T \frac{\partial V_{ij}}{\partial q_i} + \sum_{i=1}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{i0}}{\partial q_i} \\
=& \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i},
\end{aligned}$$

where we have used Lemma 3.1 in [5] and the fact that $\frac{\partial V_{ij}}{\partial q_i} = -\frac{\partial V_{ij}}{\partial q_j}$ to obtain the second equality, and the fact that $\sum_{i=1}^{n}\sum_{j=1}^{n} \dot{q}_0^T \frac{\partial V_{ij}}{\partial q_i} = \dot{q}_0^T \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial V_{ij}}{\partial q_i} = 0$, because $\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial V_{ij}}{\partial q_i} = 0$.

Now consider the following Lyapunov function candidate

$$V = V_1 + \frac{1}{2}\sum_{i=1}^{n} \tilde{v}_i^T \tilde{v}_i + V_2. \tag{12}$$

Then the derivative of V is given as

$$\dot{V} = \sum_{i=1}^{n} s_i^T \hat{u}_i + \sum_{i=1}^{n} \tilde{v}_i^T \dot{\tilde{v}}_i + \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i},$$

Since the leader's velocity $\dot{q}_0$ is constant, we have $\dot{\tilde{v}}_i = \dot{v}_i = \hat{u}_i$ according to (5) and (6). It follows that

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n} \dot{\tilde{q}}_i^T \hat{u}_i + \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial \tilde{q}_i} \\ =& -\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}(t)\dot{\tilde{q}}_i^T(\dot{\tilde{q}}_i - \dot{\tilde{q}}_j),\end{aligned} \tag{13}$$

where we have used (3) to obtain the first equality and (5) and the fact that $q_i - q_j = \tilde{q}_i - \tilde{q}_j$ to obtain the second equality. Eq. (13) can be written in a compact form as

$$\dot{V} = -\dot{\tilde{q}}^T [H(t) \otimes I_p]\dot{\tilde{q}}, \tag{14}$$

where $\tilde{q}$ is a column stack vector of $\tilde{q}_i$, $i = 1, \ldots, n$, and $H(t)$ is the leader-follower topology matrix at time t defined in Section II-B. Note that $H(t)$ is symmetric positive semi-definite. It follows that $\dot{V}$ is negative semi-definite. Therefore, from $V \geq 0$ and $\dot{V} \leq 0$, it can be concluded that V is bounded and thus $s_i, \tilde{\theta}_i, \tilde{v}_i, V_{ij} \in \mathbb{L}_\infty$. Since V_{ij} is bounded, it is guaranteed that there is no collision and no edge in the graph $\overline{G}(0)$ will be lost. In other words, for any pair of agents i, j, there exist positive constants $0 < R_{\min} \leq R_{\max} < R$, such that

$$\begin{aligned}&\|q_i(t) - q_j(t)\| \in [R_{\min}, R_{\max}], \quad \text{if } \|q_i(0) - q_j(0)\| < R, \\ &\|q_i(t) - q_j(t)\| \in [R_{\min}, (n-1)R_{\max}], \quad \text{otherwise.}\end{aligned} \tag{15}$$

Hence, we can conclude that the graph $\overline{G}(0)$ is a subgraph of the graph $\overline{G}(t)$ for all $t \geq 0$. It follows from Lemma 2.2 that $H(0) \leq H(t)$. Therefore, we can get from (14) that

$$\dot{V} \leq -\dot{\tilde{q}}^T [H(0) \otimes I_p]\dot{\tilde{q}}. \tag{16}$$

Since in $\overline{G}(0)$ the leader has directed paths to all followers, it follows from Lemma 2.1 that $H(0)$ is symmetric positive definite. Integrating both sides of (16), we can obtain that $\dot{\tilde{q}} \in \mathbb{L}_2$. Note that $\dot{q}_0$ is constant and hence bounded. Combining the above boundedness arguments we can get from (2) that $\dot{q}_i, \dot{\tilde{q}}_i, v_i \in \mathbb{L}_\infty$. Since V_{ij} is continuously differentiable, we can get from (15) that $\frac{\partial V_{ij}}{\partial q_i} \in \mathbb{L}_\infty$. From (5) and (6), we have $\hat{u}_i, \dot{v}_i \in \mathbb{L}_\infty$. Then from (8) and the property (P1), it can be concluded that $\dot{s}_i \in \mathbb{L}_\infty$. By noting that $\dot{s}_i = \ddot{q}_i - \dot{v}_i$, it follows that $\ddot{q}_i \in \mathbb{L}_\infty$. Overall, we have $\dot{\tilde{q}}_i \in \mathbb{L}_\infty \bigcap \mathbb{L}_2$ and $\ddot{\tilde{q}}_i \in \mathbb{L}_\infty$. From Barbalat's lemma [22], we can conclude that $\dot{\tilde{q}}_i \to 0$, that is, $||\dot{q}_i - \dot{q}_0|| \to 0$ asymptotically. ■

Remark 3.2 *As it can be seen, by using the control law* (4)-(7) *for Eq.* (1)*, the followers can track the leader with the same velocity while avoiding collision and maintaining the initial connectivity. Note that with our algorithm design, as long as at the initial time the connectivity is maintained and there is no collision, the connectivity maintenance and collision avoidance are ensured for all time. The proposed algorithm is continuous and accounts for unknown parameters of the agents' dynamics. Also in the proposed control input the relative positions and relative velocities with respect to neighbors are used besides each agent's own position and velocity.*

3.2 Flocking when the leader has a varying velocity

In this subsection, we consider the case when the leader moves with a varying velocity. In this case, the problem is more difficult to tackle since all followers must track the leader while the leader's velocity changes over time and the leader is a neighbor of only a subset of the followers in its proximity. In the remainder of the paper, we have the following assumption on the leader.

Assumption 3.3 *The leader's velocity* $\dot{q}_0$ *and acceleration* $\ddot{q}_0$ *are both bounded. It is assumed that* $||\mathbf{1}_n \otimes \ddot{q}_0|| \leq \sigma_l$*, where* σ_l *is a positive constant.*

We propose the following distributed control algorithm

$$u_i = \hat{u}_i + Y_i(q_i, \dot{q}_i, \dot{v}_i, v_i)\hat{\theta}_i, \tag{17}$$

$$\hat{u}_i = -\sum_{j=0}^{n} \frac{\partial V_{ij}}{\partial q_i} - \gamma \sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j) - \beta\chi_i - \alpha \text{sgn}(s_i), \tag{18}$$

$$\dot{v}_i = -\sum_{j=0}^{n} \frac{\partial V_{ij}}{\partial q_i} - \gamma \sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j) - \beta\chi_i, \tag{19}$$

$$\dot{\hat{\theta}}_i = -\Gamma_i Y_i^T(q_i, \dot{q}_i, \dot{v}_i, v_i)s_i, \tag{20}$$

where γ, α and β are positive constants, $\chi_i = \sum_{j=0}^{n} a_{ij}(t) \times \{\text{sgn}[\sum_{k=0}^{n} a_{ik}(t)(\dot{q}_i - \dot{q}_k)] - \text{sgn}[\sum_{k=0}^{n} a_{jk}(t)(\dot{q}_j - \dot{q}_k)]\}$, and $a_{ij}(t)$, V_{ij}, s_i and Γ_i are defined in Section III-A with the additional assumption that $a_{0i} = 0$, $i = 1, \ldots, n$.

Theorem 3.4 *Suppose that at the initial time* $t = 0$*, the leader has directed paths to all followers and there is no collision among the agents. Using the control law* (17)-(20) *for* (1)*, if* $\alpha \geq \sigma_l$ *and* $\beta \geq \frac{\sigma_l}{\lambda_{\min}[H(0)]}$ [2]*, then the leader-*

[2] Since at the initial time $t = 0$, the leader has directed paths to all followers, we can get from Lemma 2.1 that $\lambda_{\min}[H(0)] > 0$, and thus the term $\frac{\sigma_l}{\lambda_{\min}[H(0)]}$ is well defined.

follower flocking is achieved.

Proof: Consider the same Lyapunov function candidate V defined in (12). Note that using (17) for (1), where $\hat{u}_i$ is given by (18), both (8) and (10) still hold. The derivative of V is given as

$$\dot{V} = \sum_{i=1}^{n} s_i^T \hat{u}_i + \sum_{i=1}^{n} \tilde{v}_i^T \dot{\tilde{v}}_i + \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i}.$$

Note from (2) that $\dot{\tilde{v}}_i = \dot{v}_i - \ddot{q}_0$ and from (18) and (19) that

$$\dot{v}_i = \hat{u}_i + \alpha \mathrm{sgn}(s_i).$$

It follows that

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n} s_i^T \hat{u}_i + \sum_{i=1}^{n} \tilde{v}_i^T [\hat{u}_i + \alpha \mathrm{sgn}(s_i) - \ddot{q}_0] \\ &+ \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i}.\end{aligned} \tag{21}$$

Note from (3) that $\tilde{v}_i = \dot{\tilde{q}}_i - s_i$. Therefore, it follows that

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n} \dot{\tilde{q}}_i^T [\hat{u}_i + \alpha \mathrm{sgn}(s_i) - \ddot{q}_0] - \sum_{i=1}^{n} s_i^T [\alpha \mathrm{sgn}(s_i) - \ddot{q}_0] \\ &+ \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i}.\end{aligned} \tag{22}$$

Note that $\sum_{i=1}^{n} \dot{\tilde{q}}_i^T \chi_i = \sum_{i=1}^{n} \dot{\tilde{q}}_i^T \{\sum_{j=0}^{n} a_{ij} sgn([H_i \otimes I_p]\dot{\tilde{q}}) - \sum_{k=1}^{n} a_{ik} sgn([H_k \otimes I_p]\dot{\tilde{q}})\} = \dot{\tilde{q}}^T \{H(t) \otimes \mathrm{sgn}([H(t) \otimes I_p]\dot{\tilde{q}})\}$, where $\tilde{q}$ is a column stack vector of all $\tilde{q}_i$'s, $i = 1, \ldots, n$. Substituting $\hat{u}_i$ defined in (18) to (22), we get

$$\begin{aligned}\dot{V} =& -\gamma \dot{\tilde{q}}^T [H(t) \otimes I_p]\dot{\tilde{q}} - \beta \|[H(t) \otimes I_p]\dot{\tilde{q}}\|_1 - \dot{\tilde{q}}^T (\mathbf{1}_n \otimes \ddot{q}_0) \\ &- \alpha ||s||_1 + s^T (\mathbf{1}_n \otimes \ddot{q}_0) \\ \leq& -\gamma \dot{\tilde{q}}^T [H(t) \otimes I_p]\dot{\tilde{q}} - \beta \|[H(t) \otimes I_p]\dot{\tilde{q}}\| + ||\mathbf{1}_n \otimes \ddot{q}_0|| \cdot ||\dot{\tilde{q}}|| \\ &- \alpha ||s||_1 + ||\mathbf{1}_n \otimes \ddot{q}_0|| \cdot ||s||_1,\end{aligned}$$

where s is a column stack vector of all s_i's, $i = 1, \ldots, n$ and we have used the fact that $\|\cdot\| \leq \|\cdot\|_1$ for any vector to obtain the second inequality. Since $||\mathbf{1}_n \otimes \ddot{q}_0|| \leq \sigma_l$, we have

$$\begin{aligned}\dot{V} \leq& -\gamma \dot{\tilde{q}}^T [H(t) \otimes I_p]\dot{\tilde{q}} - \beta \|[H(t) \otimes I_p]\dot{\tilde{q}}\| + \sigma_l ||\dot{\tilde{q}}|| \\ &+ (\sigma_l - \alpha)||s||_1 \\ \leq& -\gamma \dot{\tilde{q}}^T [H(t) \otimes I_p]\dot{\tilde{q}} - \beta \lambda_{\min}[H(t)]||\dot{\tilde{q}}|| + \sigma_l ||\dot{\tilde{q}}|| \\ &+ (\sigma_l - \alpha)||s||_1.\end{aligned}$$

where we have used the fact that $H(t)$ is positive semi-definite to obtain the last inequality.

Note that at the initial time $t = 0$, the leader has directed paths to all followers. We can get from Lemma 2.1 that $H(0)$ is symmetric positive definite and thus $\lambda_{\min}[H(0)] > 0$. Since the parameters α and β satisfy $\alpha \geq \sigma_1$ and $\beta \geq \frac{\sigma_1}{\lambda_{\min}[H(0)]}$, we have that at time $t = 0$,

$$\dot{V}(t) \leq -\gamma \dot{\tilde{q}}^T [H(0) \otimes I_p]\dot{\tilde{q}} \leq 0. \tag{23}$$

Note that although the control input u_i is discontinuous, the positions of the agents are continuous and $H(t)$ changes according to the relative positions among the agents. If $H(t)$ changes at some time, there exists $t_1 > 0$ such that, $H(t) = H(0)$ for $t \in [0, t_1)$ and $H(t_1) \neq H(0)$. Therefore, we have

$$\dot{V}(t) \leq -\gamma \dot{\tilde{q}}^T [H(0) \otimes I_p]\dot{\tilde{q}} \leq 0, \qquad t \in [0, t_1),$$

which implies that for $t \in [0, t_1)$, $V_{ij} \in \mathbb{L}_\infty$ for all pairs of $q_i(t)$ and $q_j(t)$. Since V_{ij} is continuous, we can conclude that $V_{ij} \in \mathbb{L}_\infty$ when $t = t_1$. From the definition of V_{ij}, it follows that there is no collision and also no edge in the graph $\overline{G}(0)$ will be lost for $t \in [0, t_1]$. Therefore, the only possibility that $H(t)$ changes at $t = t_1$ is that, some edges are added in the graph. It implies that $\overline{G}(0)$ is a subgraph of $\overline{G}(t_1)$. We can then get from Lemma 2.2 that $H(0) \leq H(t_1)$ and thus $\lambda_{\min}[H(0)] \leq \lambda_{\min}[H(t_1)]$. Therefore, at time $t = t_1$,

$$\begin{aligned}\dot{V}(t) \leq& -\gamma \dot{\tilde{q}}^T [H(t_1) \otimes I_p]\dot{\tilde{q}} - \beta \lambda_{\min}[H(t_1)]||\dot{\tilde{q}}|| + \sigma_l ||\dot{\tilde{q}}|| \\ \leq& -\gamma \dot{\tilde{q}}^T [H(0) \otimes I_p]\dot{\tilde{q}} - \beta \lambda_{\min}[H(0)]||\dot{\tilde{q}}|| + \sigma_l ||\dot{\tilde{q}}|| \\ \leq& -\gamma \dot{\tilde{q}}^T [H(0) \otimes I_p]\dot{\tilde{q}} \leq 0.\end{aligned}$$

Following the same argument, if $H(t)$ changes at $t = t_i > t_1$, $i = 2, \ldots$, we can get that V_{ij} will always be bounded. Hence there is no collision and no edge in the graph $\overline{G}(0)$ will be lost. This in turn implies that for all $t \in [t_i, t_{i+1})$, $\overline{G}(0)$ is a subgraph of $\overline{G}_i(t)$. It thus follows that for all $t \in [t_i, t_{i+1})$, $H(0) \leq H_i(t)$ and $\lambda_{\min}[H(0)] \leq \lambda_{\min}[H_i(t)]$. That is, $\beta \geq \frac{\sigma_1}{\lambda_{\min}[H(0)]} \geq \frac{\sigma_1}{\lambda_{\min}[H_i(t)]}$ for all $t \geq 0$. Hence (23) holds for all $t \geq 0$. We then can get that $s_i, \tilde{\theta}_i, \tilde{v}_i \in \mathbb{L}_\infty$. Integrating both sides of (23), we can obtain that $\dot{\tilde{q}} \in \mathbb{L}_2$. Note from Assumption 3.3 that both $\dot{q}_0$ and $\ddot{q}_0$ are bounded. Combining the above boundedness arguments, we can get from (2) that $\dot{q}_i, \dot{\tilde{q}}_i, v_i \in \mathbb{L}_\infty$. Following the same statements from the proof of Theorem 3.1, we can conclude that $\frac{\partial V_{ij}}{\partial q_i} \in \mathbb{L}_\infty$. From (18) and (19), we have $\hat{u}_i, \dot{v}_i \in \mathbb{L}_\infty$. Then from the closed loop dynamic for each follower and (P1), we have $\dot{s}_i \in \mathbb{L}_\infty$. By noting that $\dot{s}_i = \ddot{q}_i - \dot{v}_i$, it follows that $\ddot{q}_i \in \mathbb{L}_\infty$ and thus $\ddot{\tilde{q}}_i \in \mathbb{L}_\infty$. Overall, we have $\dot{\tilde{q}}_i \in \mathbb{L}_\infty \bigcap \mathbb{L}_2$ and $\ddot{\tilde{q}}_i \in \mathbb{L}_\infty$. From Barbalat's lemma [22], we can conclude that $\dot{\tilde{q}}_i \to 0$. That is, $||\dot{q}_i - \dot{q}_0|| \to 0$ asymptotically. ■

Remark 3.5 *As it can be seen, the proposed algorithm* (17)-(20) *guarantees that the leader-follower flocking is achieved when the leader has a varying velocity in the presence of unknown parameters. Therefore, despite the hard restrictions* *such as nonlinear Lagrange dynamics, unknown models' parameters, and the existence of a moving leader with a varying velocity, the control input* (17)-(20) *solves the flocking problem.*

Remark 3.6 *Due to the existence of the signum function, the closed-loop dynamics of* (1) *using* (17) *is discontinuous. The solution should be investigated in terms of differential inclusions. Note that the signum function is measurable and locally essentially bounded. Therefore, from the nonsmooth analysis in [23], the Filippov solutions for the closed-loop dynamics always exist. Because the Lyapunov function candidate in the proof of Theorem 3.4 is continuously differentiable and the set-valued Lie derivative of the Lyapunov function is a singleton at the discontinuous point, the proof of Theorem 3.4 still holds. To avoid symbol redundancy, we do not use the differential inclusions in the proof.*

Remark 3.7 *The case of a leader with a constant velocity is a special case of a leader with a varying velocity. Hence we can also use the algorithm* (17)-(20) *for the leader-follower flocking problem when the leader has a constant velocity. However, the algorithm* (4)-(7) *is continuous and each agent needs its own position and velocity, and the relative position and relative velocity with respect to its neighbors, which can be implemented by local sensing in the absence of communication. In contrast, the algorithm* (17)-(20) *is discontinuous and each agent needs the relative position and the relative velocity with respect to its neighbors as well as two-hop neighbors' information, which necessitates the existence of communication capabilities besides local sensing. Therefore, when the leader has a constant velocity, the algorithm* (4)-(7) *is more favorable than the algorithm* (17)-(20)*.*

3.3 Fully distributed flocking when the leader has a varying velocity

In the previous section, all agents use common gains in their control inputs and the gains should be above certain bounds which are actually determined by the global information $(\lambda_{\min}[H(0)], \sigma_l)$. Therefore, the algorithm (17)-(20) is not fully distributed. In this section, the previous algorithm is extended to be fully distributed and a gain adaptation law is introduced. Here the control input for each follower is designed as

$$u_i = \hat{u}_i + Y_i(q_i, \dot{q}_i, \dot{v}_i, v_i)\hat{\theta}_i, \tag{24}$$

$$\hat{u}_i = -\sum_{j=0}^{n} \frac{\partial V_{ij}}{\partial q_i} - \sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j) - \chi_i - \alpha_i \mathrm{sgn}(s_i), \tag{25}$$

$$\dot{v}_i = -\sum_{j=0}^{n} \frac{\partial V_{ij}}{\partial q_i} - \sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j) - \chi_i, \tag{26}$$

$$\dot{\alpha}_i = ||\sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j)||_1 + ||s_i||_1, \tag{27}$$

$$\dot{\hat{\theta}}_i = -\Gamma_i Y_i^T(q_i, \dot{q}_i, \dot{v}_i, v_i)s_i, \tag{28}$$

where $\chi_i = \sum_{j=0}^{n} a_{ij}(t)\{\alpha_i \mathrm{sgn}[\sum_{k=0}^{n} a_{ik}(\dot{q}_i - \dot{q}_k)] - \alpha_j \mathrm{sgn}[\sum_{k=0}^{n} a_{jk}(\dot{q}_j - \dot{q}_k)]\}$, and $a_{ij}(t)$, V_{ij}, s_i and Γ_i are defined in Section III-A.

Theorem 3.8 *Suppose that at the initial time $t = 0$, the leader has directed paths to all followers and there is no collision among the agents. Using the control input* (24)-(28) *for* (1)*, the leader-follower flocking is achieved.*

Proof: Define $V_3 = \frac{1}{2}\sum_{i=1}^{n}(\alpha_i - \bar{\alpha})^2$, where $\bar{\alpha}$ is chosen such that $\bar{\alpha} \geq \max\{\frac{\sigma_l}{\lambda_{\min}[H(0)]}, \sigma_l\}$. The derivative of V_3 is given as

$$\begin{aligned}\dot{V}_3 =& \sum_{i=1}^{n}(\alpha_i - \bar{\alpha})\dot{\alpha}_i \\ =& \sum_{i=1}^{n}(\alpha_i - \bar{\alpha})[||\sum_{j=0}^{n} a_{ij}(t)(\dot{q}_i - \dot{q}_j)||_1 + ||s_i||_1].\end{aligned} \tag{29}$$

Now we introduce the following Lyapunov function candidate

$$V = V_1 + \frac{1}{2}\sum_{i=1}^{n} \tilde{v}_i^T \tilde{v}_i + V_2 + V_3,$$

where V_1 is defined in (9) and V_2 is defined in (11). Following the proof of Theorem 3.4, the derivative of V is given as

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n} s_i^T \hat{u}_i + \sum_{i=1}^{n} \tilde{v}_i^T \dot{\tilde{v}}_i + \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i} + \dot{V}_3 \\ =& \sum_{i=1}^{n} s_i^T \hat{u}_i + \sum_{i=1}^{n} \tilde{v}_i^T [\hat{u}_i + \alpha_i \mathrm{sgn}(s_i) - \ddot{q}_0] \\ &+ \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i} + \dot{V}_3 \\ =& \sum_{i=1}^{n} \dot{\tilde{q}}_i^T [\hat{u}_i + \alpha_i \mathrm{sgn}(s_i) - \ddot{q}_0] - \sum_{i=1}^{n} s_i^T [\alpha_i \mathrm{sgn}(s_i) - \ddot{q}_0] \\ &+ \sum_{i=1}^{n}\sum_{j=0}^{n} \dot{\tilde{q}}_i^T \frac{\partial V_{ij}}{\partial q_i} + \dot{V}_3.\end{aligned} \tag{30}$$

where we have used $\tilde{v}_i = \dot{\tilde{q}}_i - s_i$ to obtain the third equality. Note that $\sum_{i=1}^{n} \dot{\tilde{q}}_i^T \chi_i = \sum_{i=1}^{n} \alpha_i [\sum_{j=0}^{n} a_{ij}(t)(\dot{\tilde{q}}_i - \dot{\tilde{q}}_j)]^T \mathrm{sgn}[\sum_{j=0}^{n} a_{ij}(t)(\dot{\tilde{q}}_i - \dot{\tilde{q}}_j)]$. Substituting (25) and (29) to (30), we can get

$$\begin{aligned}\dot{V} =& -\dot{\tilde{q}}^T [H(t) \otimes I_p]\dot{\tilde{q}} - \dot{\tilde{q}}^T(\mathbf{1}_n \otimes \ddot{q}_0) + s^T(\mathbf{1}_n \otimes \ddot{q}_0) \\ &- \bar{\alpha}||[H(t) \otimes I_p]\dot{\tilde{q}}||_1 - \bar{\alpha}||s||_1 \\ \leq& -\dot{\tilde{q}}^T [H(t) \otimes I_p]\dot{\tilde{q}} + ||\mathbf{1}_n \otimes \ddot{q}_0|| \cdot ||\dot{\tilde{q}}|| + ||\mathbf{1}_n \otimes \ddot{q}_0|| \cdot ||s||_1 \\ &- \bar{\alpha}||(H(t) \otimes I_p)\dot{\tilde{q}}|| - \bar{\alpha}||s||_1,\end{aligned}$$

where $\tilde{q}$ and s are, respectively, column stack vectors of $\tilde{q}_i$ and s_i, $i = 1, \ldots, n$. Since $||\mathbf{1}_n \otimes \ddot{q}_0|| \leq \sigma_l$, we have

$$\begin{aligned}\dot{V} \leq & -\dot{\tilde{q}}^T [H(t) \otimes I_p] \dot{\tilde{q}} + \sigma_l ||\dot{\tilde{q}}|| - (\bar{\alpha} - \sigma_l) ||s||_1 \\ & - \bar{\alpha} \lambda_{\min} [H(t)] ||\dot{\tilde{q}}||.\end{aligned}$$

Again there exists $t_1 > 0$ such that, $H(t) = H(0)$ for $t \in [0, t_1)$. Since at time $t = 0$ the parameter $\bar{\alpha}$ satisfy $\bar{\alpha} \geq \max\{\frac{\sigma_l}{\lambda_{\min}[H(0)]}, \sigma_l\}$, we have

$$\dot{V}(t) \leq -\dot{\tilde{q}}^T [H(t) \otimes I_p] \dot{\tilde{q}} \leq 0, \qquad t \in [0, t_1)$$

Similar to the statements in the proof of Theorem 3.4, it can be proved that for $t \in [0, t_1]$, V_{ij} is bounded for all i, j and there is no collision and also no edge in the graph $\overline{G}(0)$ will be lost. It can also be proved that $\dot{V}(t) \leq 0$ for all $t \geq 0$. Then it can be concluded that $\dot{\tilde{q}}_i \in \mathbb{L}_\infty \bigcap \mathbb{L}_2$ and $\ddot{\tilde{q}}_i \in \mathbb{L}_\infty$. Hence from Barbalat's lemma, we conclude that $||\dot{q}_i - \dot{q}_0|| \to 0$ asymptotically.

Remark 3.9 *In the fully distributed algorithm* (24)-(28) *an adaptive gain scheme is introduced, where the neighbors' gains are used in the control input. This algorithm guarantees that the leader-follower flocking is achieved when the leader has a varying velocity in the presence of unknown parameters and the followers do not require global information for parameter determination. Therefore, despite the hard restrictions described in Section II-B and the existence of a moving leader with a varying velocity, the fully distributed control input* (24)-(28) *solves the flocking problem.*

Remark 3.10 *In [5], the distributed flocking problem with a moving leader has been solved for multi-agent systems with single- or double-integrator dynamics. Here in this paper, we address the problem for networked nonlinear Lagrange systems with parametric uncertainties, which is more challenging. The algorithms in [5] cannot deal with nonlinear Lagrange dynamics and account for fully distributed gain design. Even in the case where the leader has a constant velocity, the control algorithm in [5] relies on both one-hop and two-hop neighbors' information, while only one-hop neighbors' information is required in* (4)*. In [13] a distributed coordinated tracking problem is studied for Lagrange systems with parametric uncertainties. However, the algorithms in [13] cannot deal with the flocking behavior and it is nontrivial to address the distributed leader-follower flocking problem as shown above.*

4 Simulation

In this section, numerical simulation results are given to illustrate the effectiveness of the theoretical results obtained in Section III. It is assumed that there exist one leader and four followers $(n = 4)$. Let $q_i = [q_{ix}, q_{iy}]^T \in \mathbb{R}^2$ denotes the position of agent i. The system dynamics are given by

$$m_i \ddot{q}_i + \beta_i \dot{q}_i = u_i,$$

where m_i and β_i represent, respectively, the constant but unknown mass and damping constants of the ith follower, and u_i denotes the control input. In the simulations, we let $m_i = (i+1) \times 0.5$ and $\beta_i = 0.4 + i \times 0.1$, $i = 1, \ldots, 4$. To guarantee the collision avoidance and connectivity maintenance, we adopt the potential function partial derivatives as Eqs. (36) and (37) in [5], where $R = 5$ and $d_{ij} = \bar{d} = 0.1$.

In the first case, we simulate the case where the leader has a constant velocity under the control algorithm (4)-(7). The initial states of the followers are chosen as $q_1(0) = [3, 3]^T$, $q_2(0) = [7, 5]^T$, $q_3(0) = [5, 0]^T$, $q_4(0) = [9, 2]^T$, $\dot{q}_i(0) = [0.05 \times i - 0.2, -0.05 \times i + 0.2]^T$, $i = 1, \ldots, 4$, and the leader's trajectory is chosen as $q_0(0) = [9, 0]^T$ and $\dot{q}_0 = [1, 2]^T$. The initial values for the estimates of the leader's velocity are $v_i(0) = [0.5, 0.5]^T$, $i = 1, \ldots, 4$. The control parameter is chosen as $\Gamma_i = 20 \times I_2$, $i = 1, \ldots, 4$. Fig. 1 shows the trajectories of the leader and the followers. Clearly, all followers move cohesively with the leader without colliding with each other. Fig. 2 shows the velocity of the followers and the leader. It can be seen that the velocities of the followers converge to that of the leader and all agents move with the same velocity.

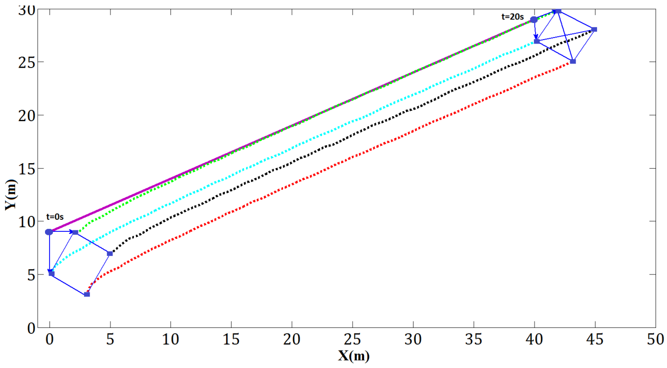

Fig. 1. The trajectories of the followers and the leader in the first case. The leader is represented as a dot while the followers are represented as squares. An edge between two followers denotes that the two are neighbors, and an arrow from the leader to a follower denotes that the leader is a neighbor of the follower.

In the second case, we simulate the case where the leader has a varying velocity under the control algorithm (17)-(20). The initial states of the followers are chosen as above and the leader's velocity is chosen as $\dot{q}_0(t) = [-\sin(\frac{2\pi}{60}t), \cos(\frac{2\pi}{60}t)]^T$. The control parameters are chosen as $\alpha = 1$, $\gamma = 50$, $\beta = 0.1$, and $\Gamma_i = 10 \times I_2$, $i = 1, \ldots, 4$. Fig. 3 shows the trajectories of the followers and the leader. The agents maintain the initial connectivity while avoiding collisions. Fig. 4 shows that each follower eventually moves with the same velocity as the leader.

In the third case, we simulate the case where the leader has a varying velocity under the fully distributed control algorithm

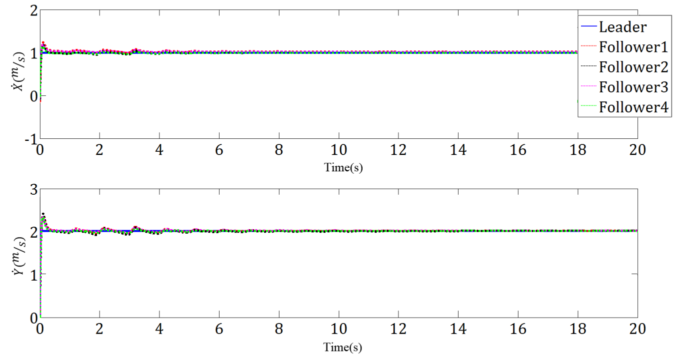

Fig. 2. The velocities of the followers and the leader in the first case

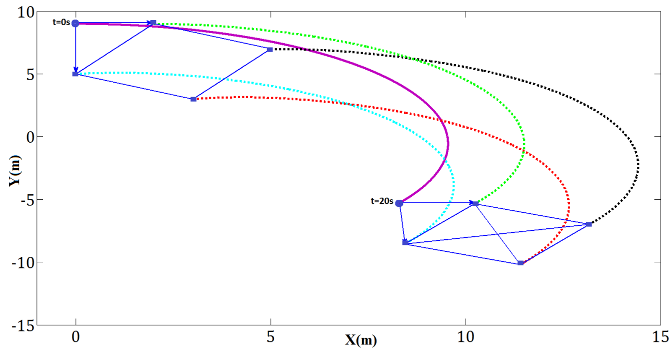

Fig. 3. The trajectories of the followers and the leader in the second case

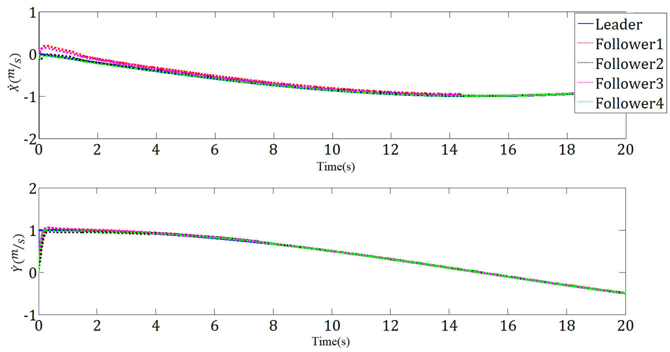

Fig. 4. The velocities of the followers and the leader in the second case

(24)-(28). Here the initial states and the leader's trajectory are chosen as the second case. The control parameter is chosen as $\Gamma_i = 1 \times I_2$, $i = 1, \ldots, 4$. Fig. 5 shows the trajectories while Fig. 6 shows the velocities of the followers and the leader. The leader-following flocking is achieved in this case.

5 CONCLUSIONS

In this paper, the distributed leader-follower flocking problem has been studied. The agents' models are described by Lagrange dynamics with unknown but constant parameters.

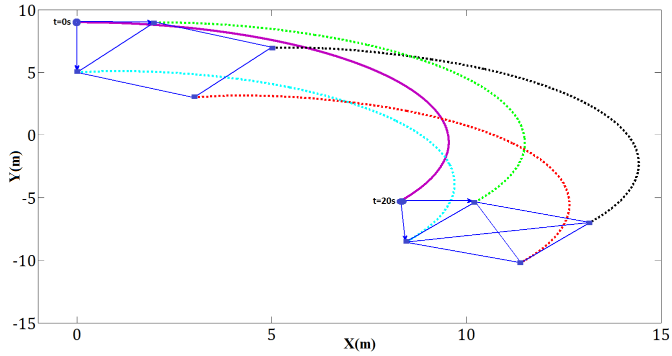

Fig. 5. The trajectories of the followers and the leader in the third case

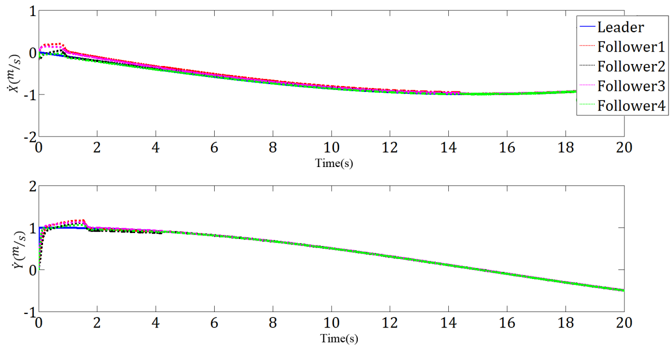

Fig. 6. The velocities of the followers and the leader in the third case

Two cases for the leader have been considered: i) the leader has a constant velocity, and ii) the leader has a varying velocity. In both cases the leader is a neighbor of only a group of followers and the followers interact with only their neighbors defined by a proximity graph. In the second case we also relaxed the assumption of global information for parameter determination and proposed a fully distributed control algorithm. All proposed distributed control algorithms have been shown to achieve connectivity maintenance, collision avoidance, and velocity matching, where only neighbors' information was used in the control design. Numerical simulations have been also presented to illustrate the theoretical results.

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