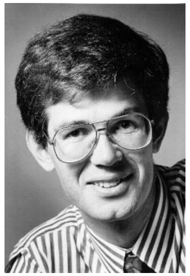
# Multi-Carrier Transmission over Mobile Radio Channels



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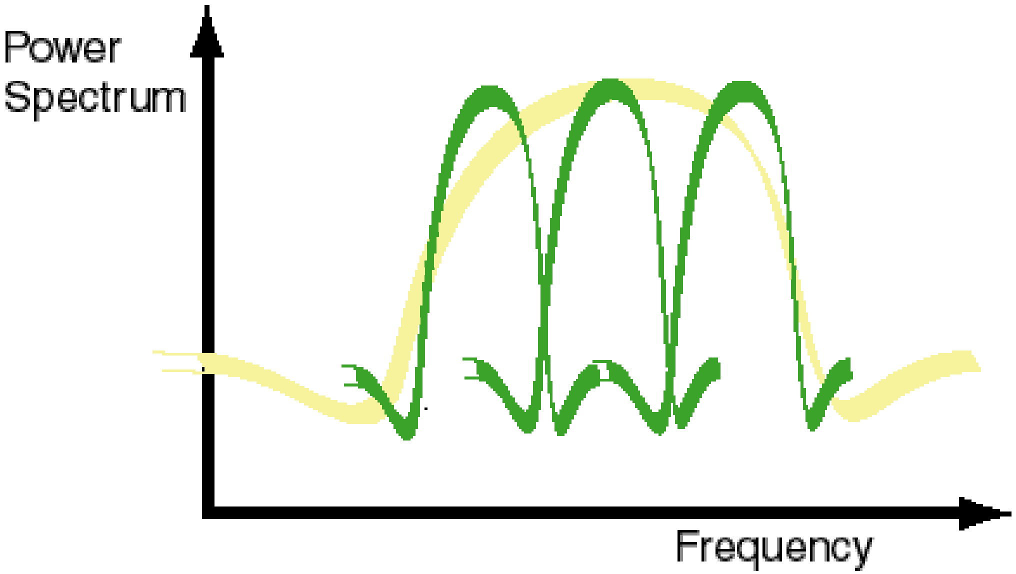
# Outline

- Introduction to OFDM
- Discussion of receivers for OFDM and MC-CDMA
- Intercarrier Interference, FFT Leakage
- New receiver designs
- Simulation of Performance
- Conclusions

# OFDM




OFDM: a form of MultiCarrier Modulation.

- Different symbols are transmitted over different subcarriers
- Spectra overlap, but signals are orthogonal.
- Example: Rectangular waveform -> Sinc spectrum

# Applications

Fixed / Wireline:

- ADSL Asymmetric Digital Subscriber Line

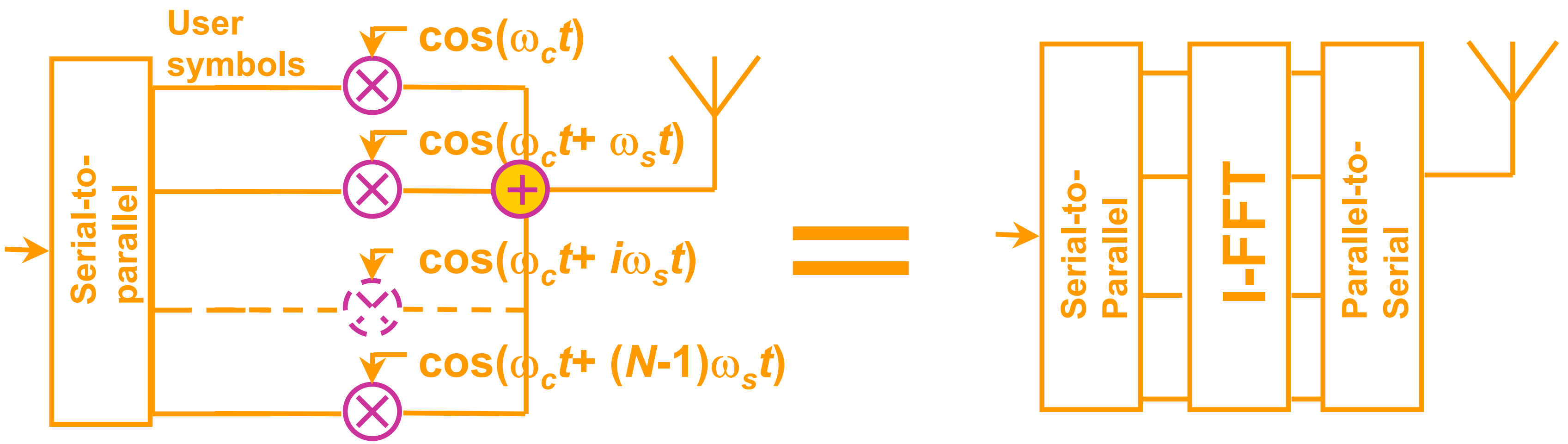
Mobile / Radio:

- Digital Audio Broadcasting (DAB)
- Digital Video Broadcasting - Terrestrial (DVB-T)
- Hiperlan II
- Wireless 1394
- 4G (?)

# I-FFT: OFDM Transmission

Transmission of QAM symbols on parallel subcarriers

Overlapping, yet orthogonal subcarriers



# I-FFT: OFDM Transmission

Transmission of QAM symbols on parallel subcarriers

Overlapping, yet orthogonal subcarriers

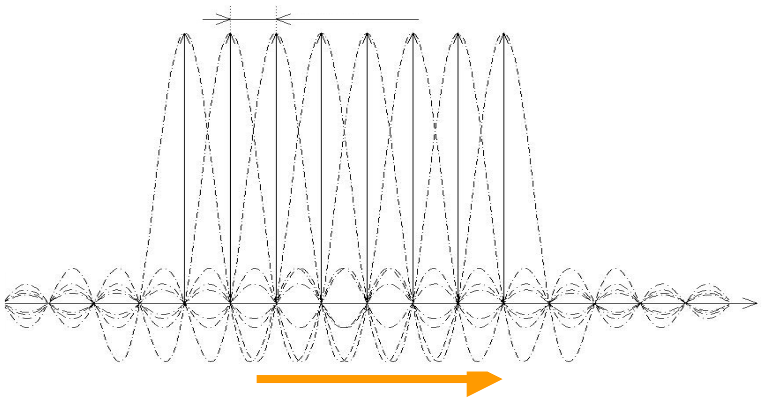
$$\psi_k(t) = \begin{cases} \dfrac{1}{\sqrt{T_s}} e^{j\omega_k t} & t \in [0, T_s] \\ 0 & \text{otherwise} \end{cases}$$

$$\text{with} \quad \omega_k = \omega_0 + k\omega_s \, ; \quad k = 0,1,\ldots,N_c - 1$$

Although the subchannels overlap, they do not interfere with each other at $f = f_k$; ($k$ = 0, 1,...,$N_c$-1). Indeed, they are orthogonal:

$$\int_0^{T_s} \psi_k(t)\psi_l^*(t)dt = \delta(k-l)$$

# OFDM Subcarrier Spectra




Symbol duration : inverse of subcarrier spacing plus cyclic prefix

Sampling rate : inverse of transmit bandwidth

Pulse shape in time domain: rectangle $\Pi(t / NT_s)$

$$s(t)=\sum_{n=0}^{N-1} a_n \Pi\left(\frac{t}{\left(N+N_{cp}\right)T_s}\right) \stackrel{F}{\longleftrightarrow} \mathrm{S(f)}=\sum_{n=0}^{N-1} a_n \operatorname{sinc}\left(f\left(N+N_{cp}\right)Ts\right)=\frac{\sin\left(f\left(N+N_{cp}\right)Ts\right)}{f\left(N+N_{cp}\right)Ts}$$

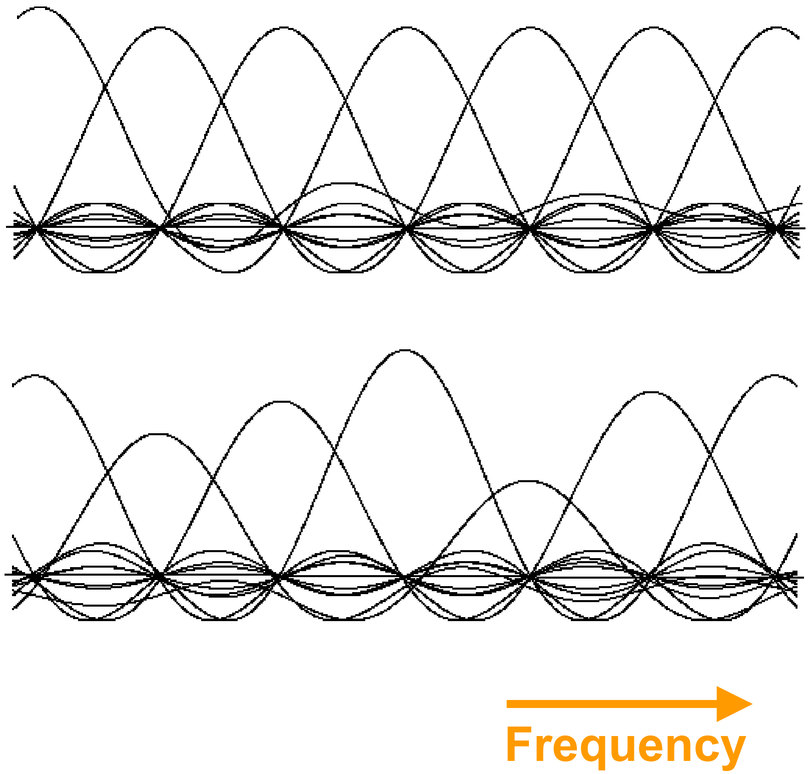
# OFDM Subcarrier Spectra

OFDM signal strength versus frequency.

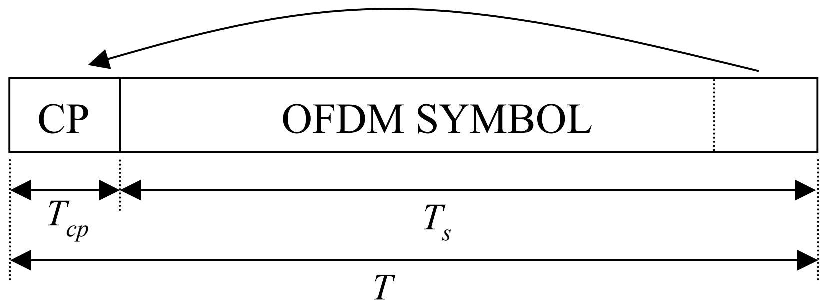
Rectangle <- FFT -> Sinc

before channel

after channel



# Cyclic Prefic / Cyclix postfix




The length of the cyclic prefix should be made longer than the experienced impulse response to avoid ISI and ICI. However, the transmitted energy increases with the length of the cyclic prefix. The *SNR* loss due to the insertion of the CP is given by

$$SNR_{loss} = -10\log_{10}\left(1 - \frac{T_{cp}}{T}\right)$$

where $T_{cp}$ denotes the length of the cyclic prefix and $T=T_{cp}+T_s$ is the length of the transmitted symbol.

# Coded OFDM

Received signal at a subcarrier is not affected by transmitted symbols in any other subcarrier

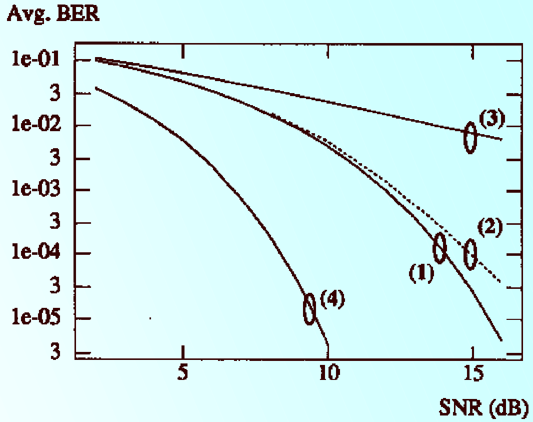
$$\begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} H_0 & & 0 \\ & \ddots & \\ 0 & & H_{N-1} \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_{N-1} \end{bmatrix} + \begin{bmatrix} n_0 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

Symbol data can be recovered using simple single tap data estimation

$$H_m = \sum_l h_l e^{-j2\pi(mf_s)\tau_l}$$

Viterbi decoder helps recover bits from subcarriers in deep fade

$$\begin{bmatrix} \hat{a}_0 \\ \vdots \\ \hat{a}_{N-1} \end{bmatrix} = \begin{bmatrix} H_0^{-1} & & 0 \\ & \ddots & \\ 0 & & H_{N-1}^{-1} \end{bmatrix} \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix}$$




**Ave BER curves: slope ~ degree of diversity on fading channel**

# Single Frequency Networks

OFDM is robust against delay spread

We can “mis”use this by transmitting a synchronous signal from two transmit sites

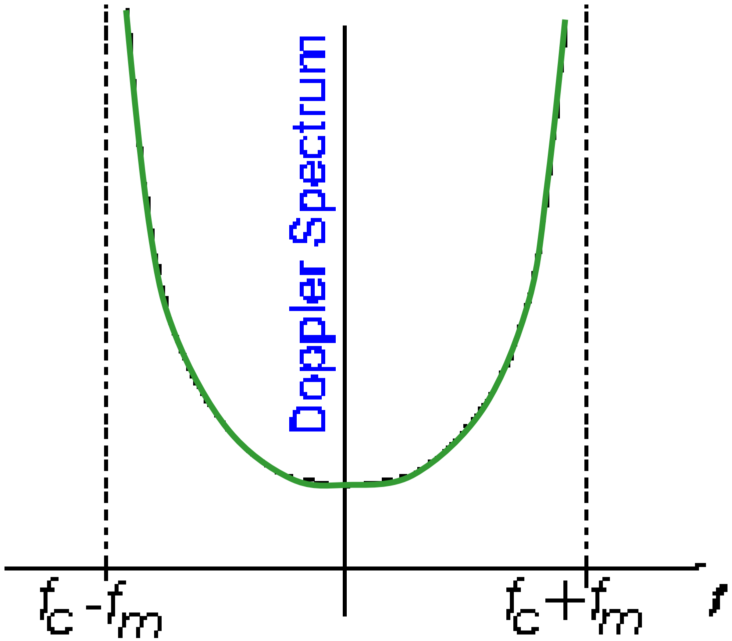
DVB
Digital Video Broadcasting

# The Wireless Multipath Channel

# OFDM and MC-CDMA in a rapidly time-varying channel

Doppler spread is the Fourier-dual of a delay spread

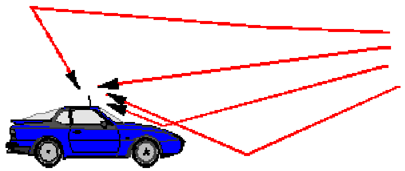
# Mobile Multipath Channel




**Doppler Spectrum**

Collection of reflected waves, each with

- random angle of arrival
- random delay

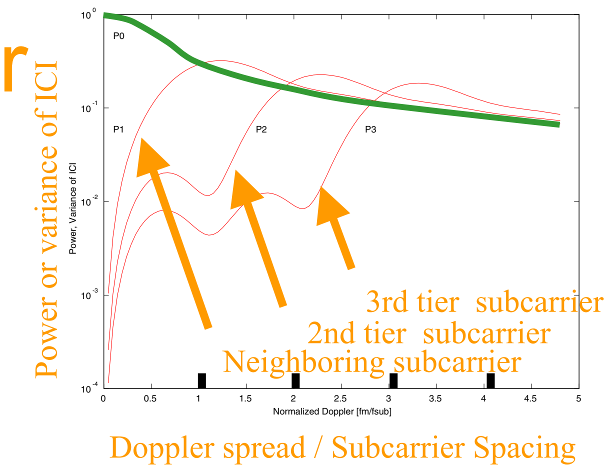


Angle of arrival is uniform

Doppler shift is cos(angle)

U-shaped power density spectrum

# Crosstalk β caused by Doppler




$\boldsymbol{Y} = [y_0, y_1, .., y_{N-1}]^{\mathrm{T}}$, with $y_m = \Sigma_n a_n \beta_{m,n} T_s$

$\beta_{m,n}$ is the 'transfer' for a signal transmitted at subcarrier *n* and received at subcarrier *m*,

$$\beta_{m,n} = \sum_{i=0}^{I_w - 1} \frac{D_i}{2} \operatorname{sinc}\left( n - m + \frac{\omega_i}{\omega_s} \right)$$

$$\exp\{-j(\omega_c + \omega_i + n\omega_s)T_i - \tfrac{1}{2} j\omega_i T_s + j\pi(n-m)\}$$

For uniform angles of arrivals of waves, ICI power spilled from transmit subcarrier *n* into received subcarrier *m* = *n* + Δ equals

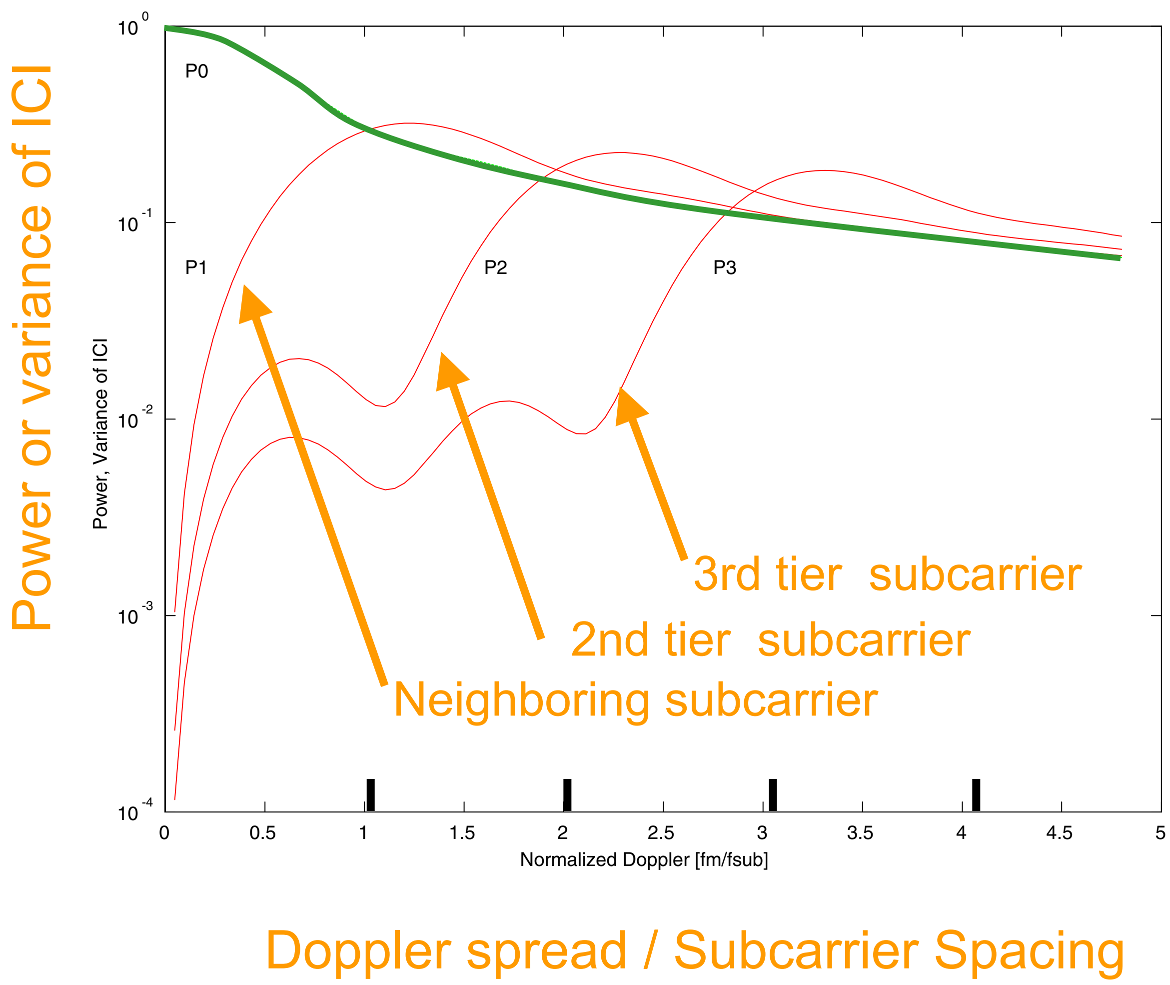
$$P_\Delta = \mathrm{E}_{ch}\, \beta_{n+\Delta,n} \beta^*{}_{n+\Delta,n} = \frac{P_T}{8\pi} \int_{-1}^{1} \frac{\operatorname{sinc}^2\left( \Delta + \frac{f_\Delta}{f_s} x \right) dx}{\sqrt{(1-x^2)}}$$

where

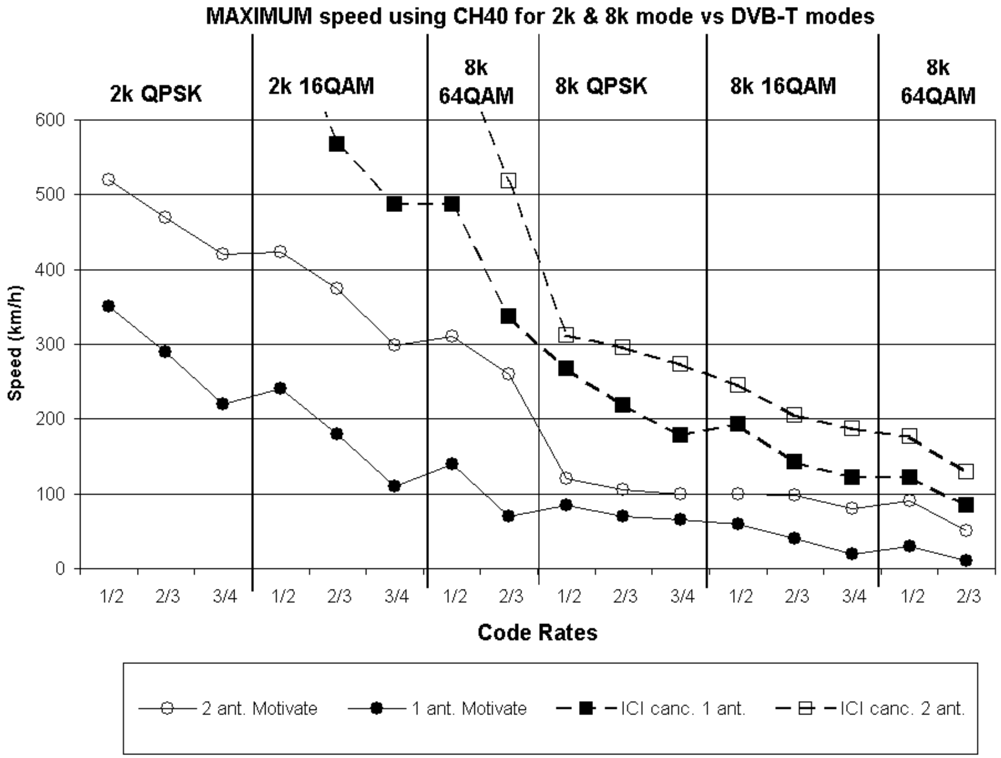
$f_\Delta$ is the maximum Doppler shift, and

$P_T$ the local mean received power, per subcarrier

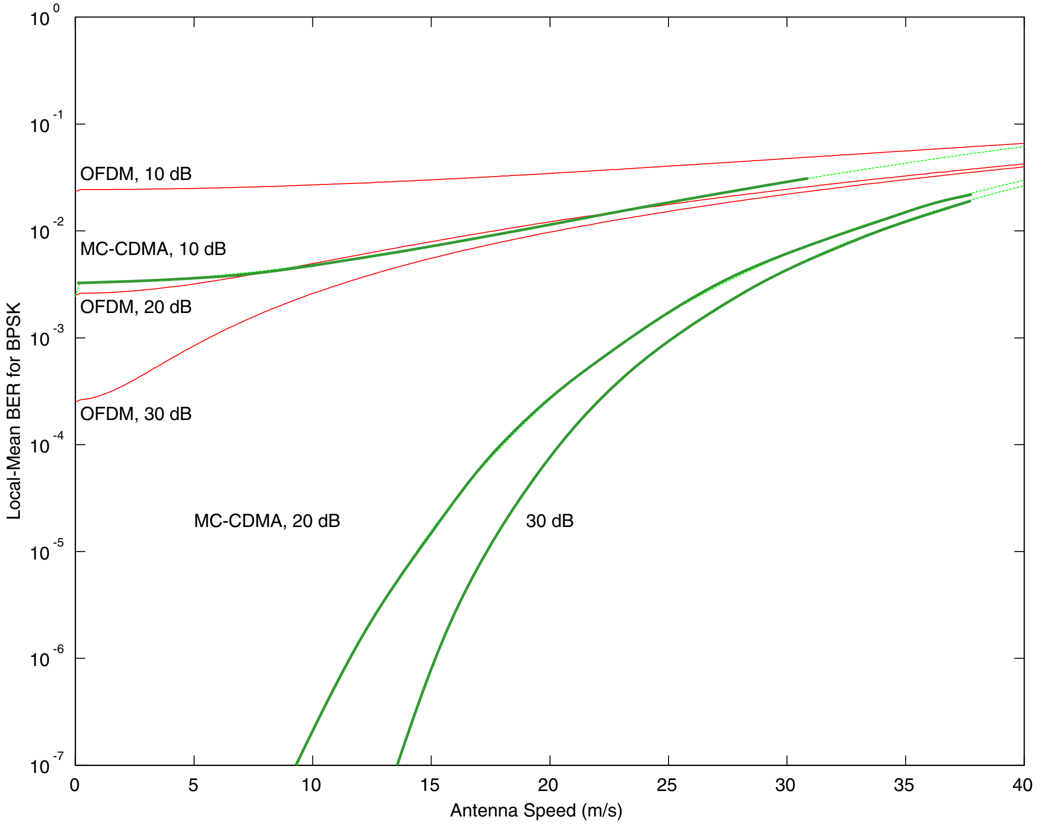
# ICI caused by Doppler



# Maximum attainable speed



# BER in a mobile channel




Antenna Speed [m/s]

- Local-mean BER for BPSK, versus antenna speed.
- Local mean SNR of 10, 20 and 30 dB.
- Comparison between MC-CDMA and uncoded OFDM for $f_c$ = 4 GHz
- Frame duration $T_s$ = 896 $\mu$s
- FFT size: $N$ = 8192.
- Sub. spacing $f_s$ = 1.17 kHz
- Data rate 9.14 Msymbol/s.

# Mobile OFDM weakness: InterCarrier Interference

## An effective description of ICI

$$\underline{y} = [\underbrace{\underline{H}}_{\text{desired}} + \underbrace{\Xi\, \underline{H}'}_{\text{self-interference}}]\, \underline{a} + \underline{n}$$

where:

- $\underline{H}$ is a diagonal matrix with the complex transfer function per subcarrier
- $\underline{H}'$ is the temporal derivative of $\underline{H}$: $\underline{H}' = d\underline{H}/dt$
- $\Xi$ is the ICI spreading matrix: a system value, fixed!
- $\underline{a}$ is a vector of transmitted data
- $\underline{n}$ is vector of white Gaussian noise

# Random Complex-Gaussian Amplitude

It can be shown that for *p* + *q* is even

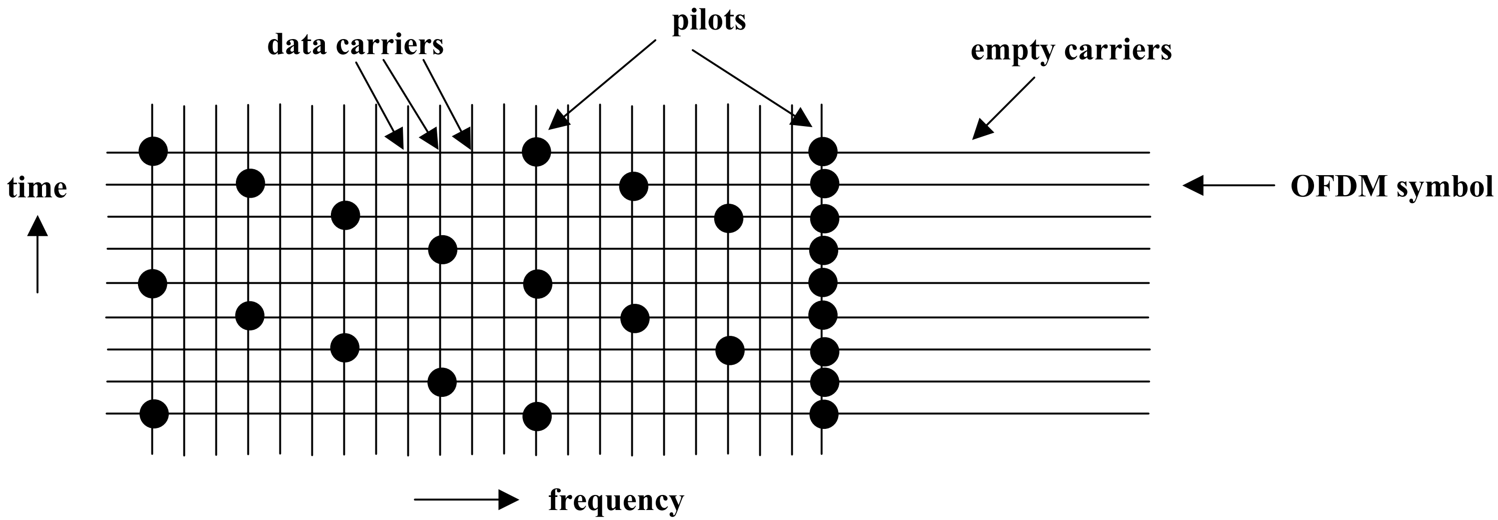
$$\mathrm{E}\, H_n^{(p)} H_m^{*(q)} = \left(2\pi f_D\right)^{p+q} \frac{(p+q-1)!!}{(p+q)!!} \frac{(-1)^q\, j^{p+q}}{1 + j(n-m)T_{rms}\omega_s}$$

and 0 for *p* + *q* is odd.

- This defines the covariance matrix of subcarrier amplitudes and derivatives,
- allows system modeling and simulation between the input of the transmit I-FFT and output of the receive FFT.

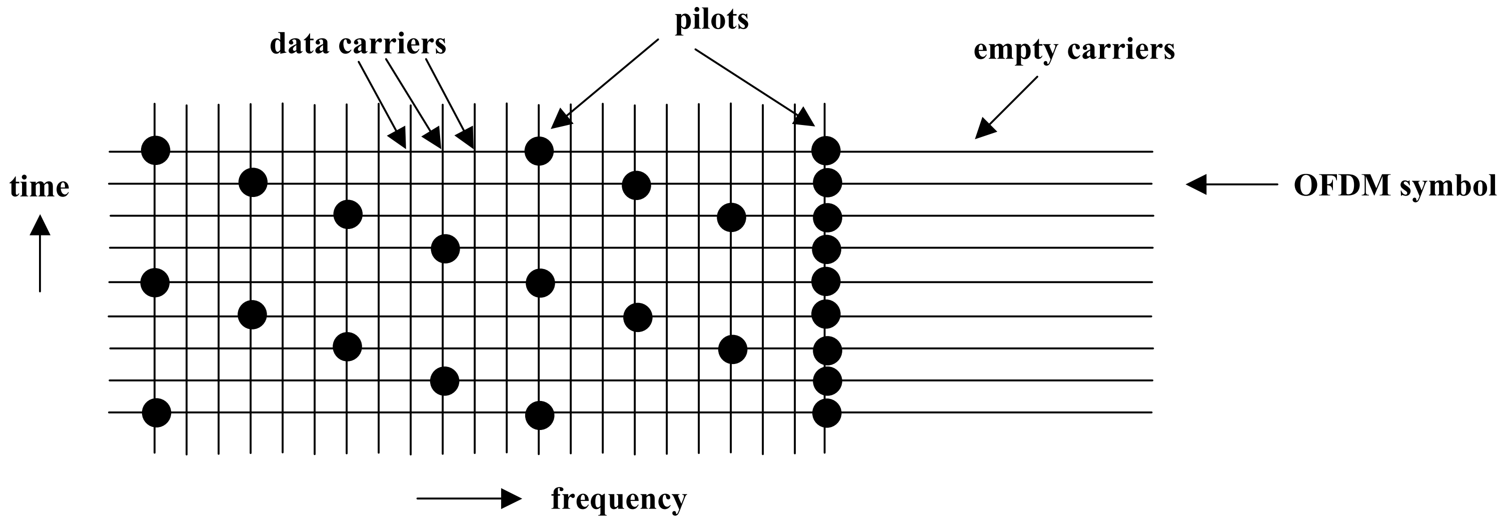
# Channel estimation DVB-T

*H* estimation based on pilots in frequency domain



# Estimation of *H*

DVB-T pilots



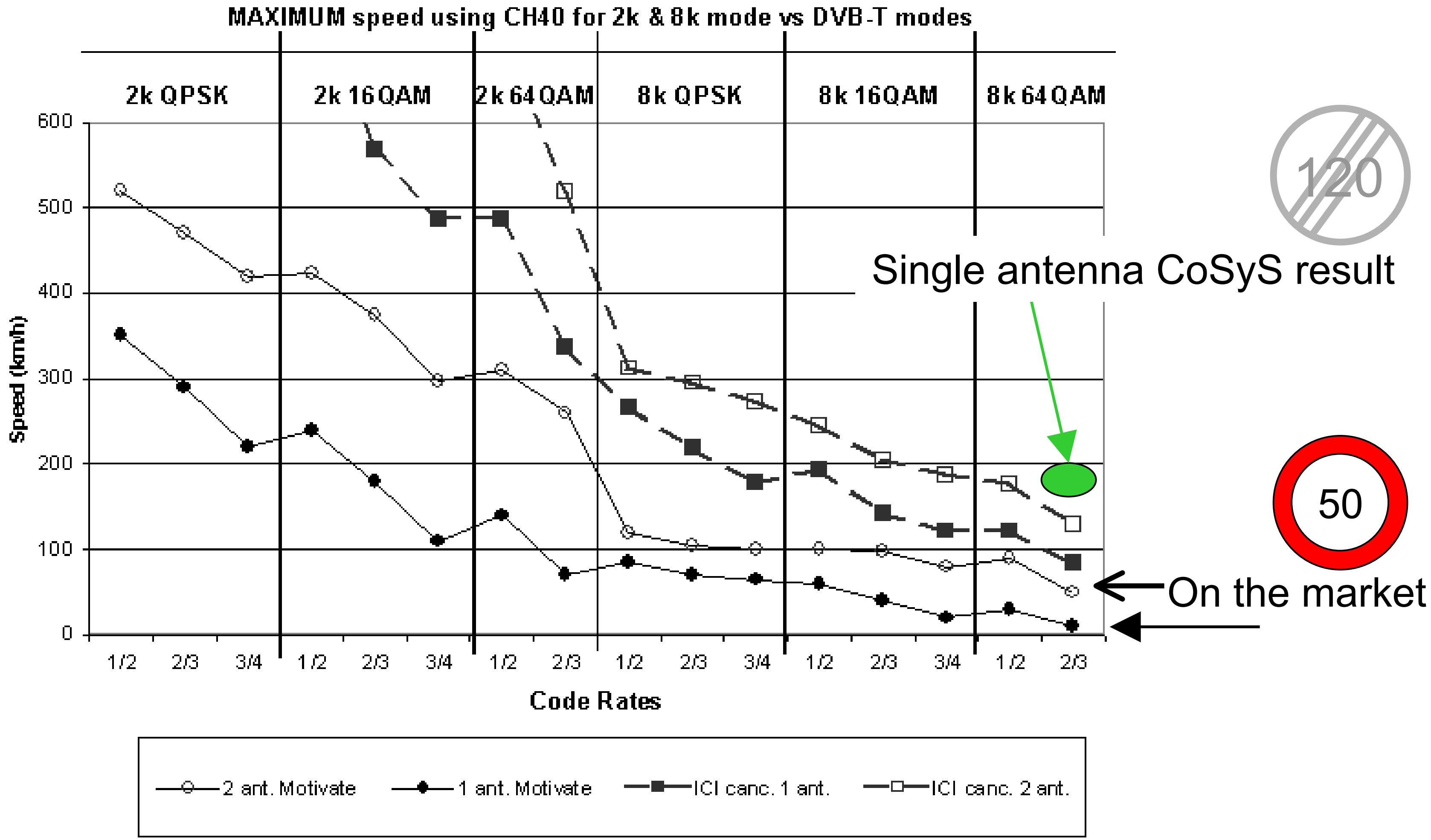

In every OFDM symbol, initial estimates of H at every pilot subcarriers can be obtained.

In every OFDM symbol, estimate of H at each subcarrier is obtained:

- Spectral Wiener Filtering
- Input: Initial estimates of H at several (scattered) pilot positions .

# Wiener Filtering for OFDM

# Achievable car speeds with BB signal processing



# Receiver 1: MMSE Matrix Inversion

Receiver sees $\boldsymbol{Y} = \boldsymbol{Q}\,\boldsymbol{A} + \boldsymbol{N}$, with $\boldsymbol{Q}=\text{DIAG}(\underline{\boldsymbol{H}})+ \Xi\,\text{DIAG}(\underline{\boldsymbol{H}}^{(1)})$

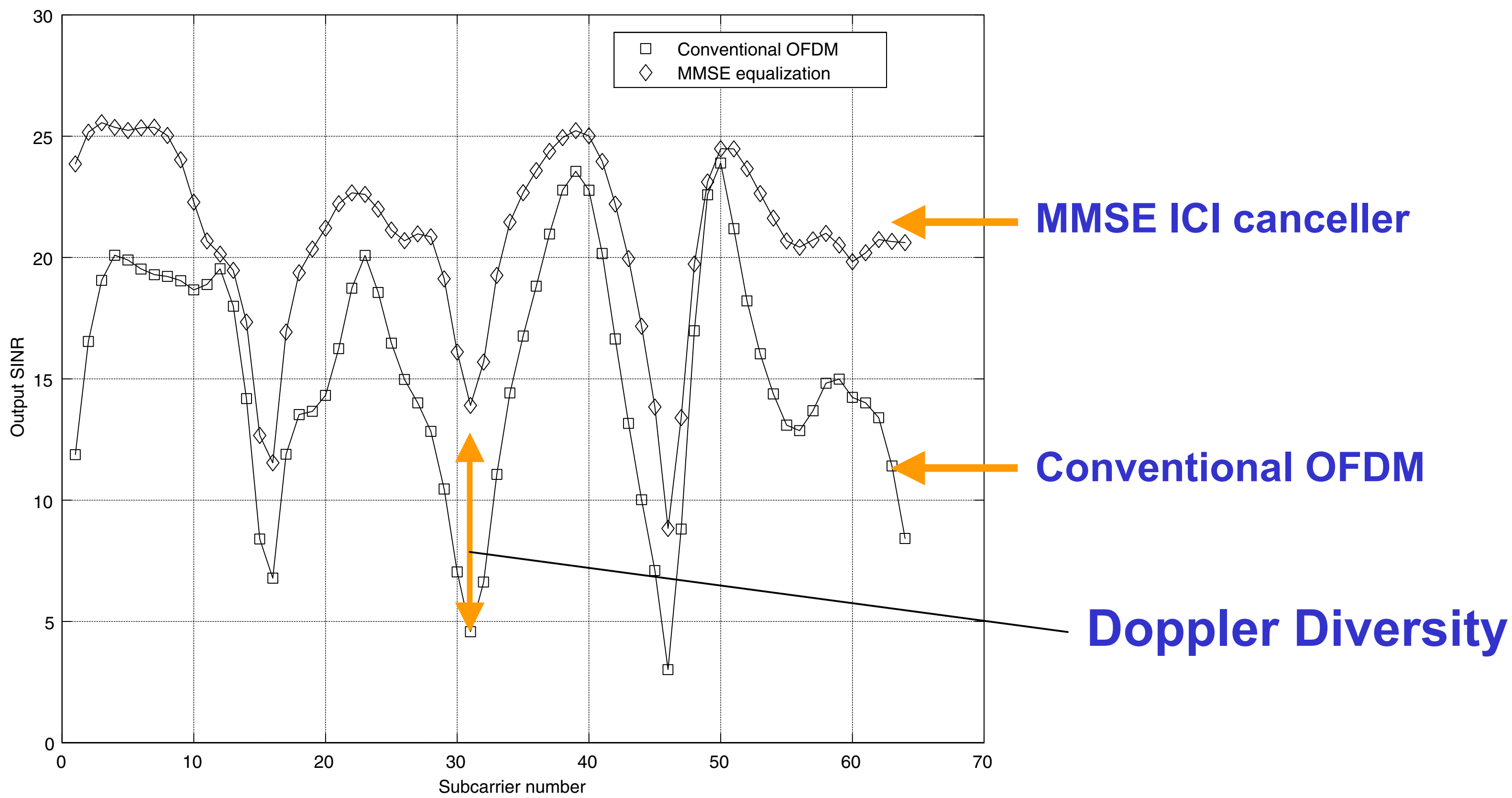
- Calculate matrix $\underline{\boldsymbol{Q}} = \text{DIAG}(\underline{\boldsymbol{H}})+ \Xi\,\text{DIAG}(\underline{\boldsymbol{H}}^{(1)})$
- Compute MMSE filter $\boldsymbol{W} = \underline{\boldsymbol{Q}}^{H}\,[\underline{\boldsymbol{Q}}\,\underline{\boldsymbol{Q}}^{H} + \sigma_n^{2}\,I_N]^{-1}$.

**Performance evaluation:**

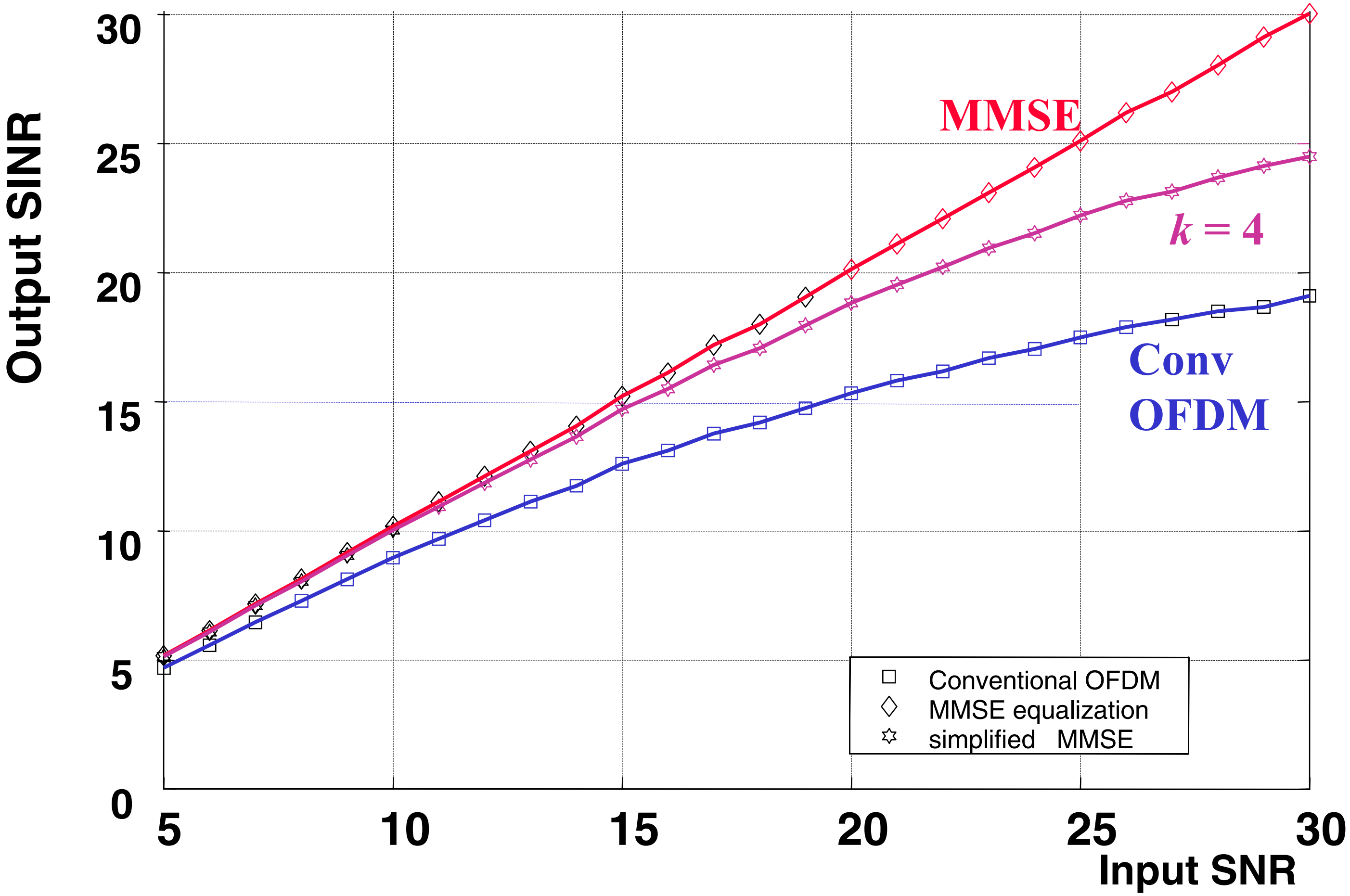
- Signal power per subcarrier
- Residual ICI and Noise enhancement from ***W***

# ICI Handling: Brute force Matrix Inversion

SNR of decision variable. Simulation for *N* = 64, MMSE Wiener filtering to cancel ICI



# Performance of (Blocked Band) Matrix Inversion




$N$ = 64, $v$ = 200 km/h, $f_c$ = 17 GHz, $T_{RMS}$ = 1 μs, sampling at $T$ = 1μs.

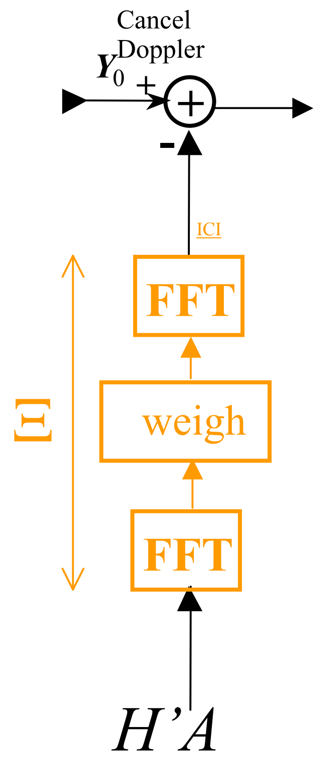
$f_{Doppler}$ = 3.15 kHz, Subc. spacing $f_{sr}$ = 31.25 kHz:

**Compare to DVB-T**: $v$ = 140 km/h, $f_c$ = 800MHz: $f_{doppler}$ = 100 Hz while $f_{sr}$ = 1.17 kHz

# ICI cancellation

If $H'_n$ and $a_n$ are known for all $n$, ICI can be removed almost completely from any subcarrier.

$$y'_m = y_m - \sum_{n=0}^{N-1} \Xi_{m,n} H'_n a_n = H_m a_m + n_k$$




Implement Ξ as "FFT - Ramp - FFT"

# ICI cancellation

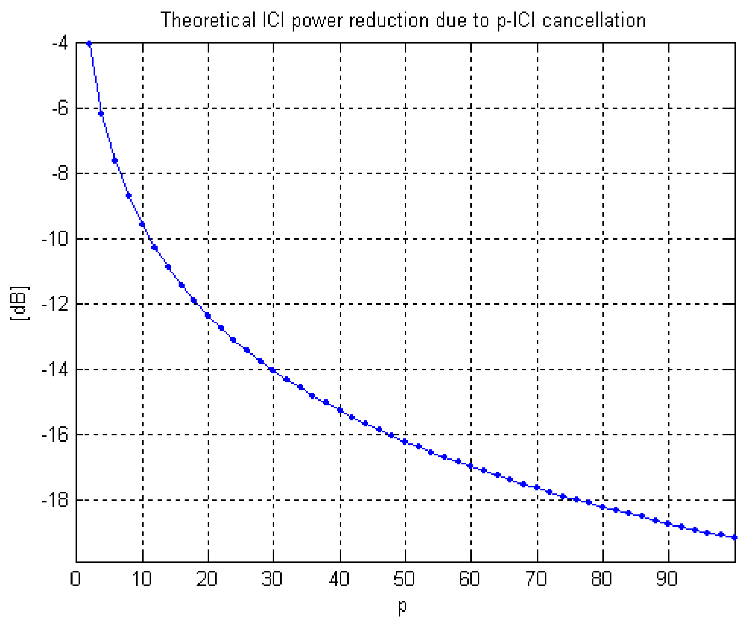
However, $H'$ and $a$ are not known to the receiver: They have to be estimated

- Full cancellation

$$y'_m = y_m - \sum_{n=0}^{N-1} \Xi_{m,n} \hat{H}'_n \hat{a}_n$$

- Partial cancellation: $\hat{a}_n$ is used only to cancel interference it caused to $p$ closest subcarriers

$$y'_m = y_m - \sum_{n=m-p/2}^{m+p/2} \Xi_{m,n} \hat{H}'_n \hat{a}_n$$

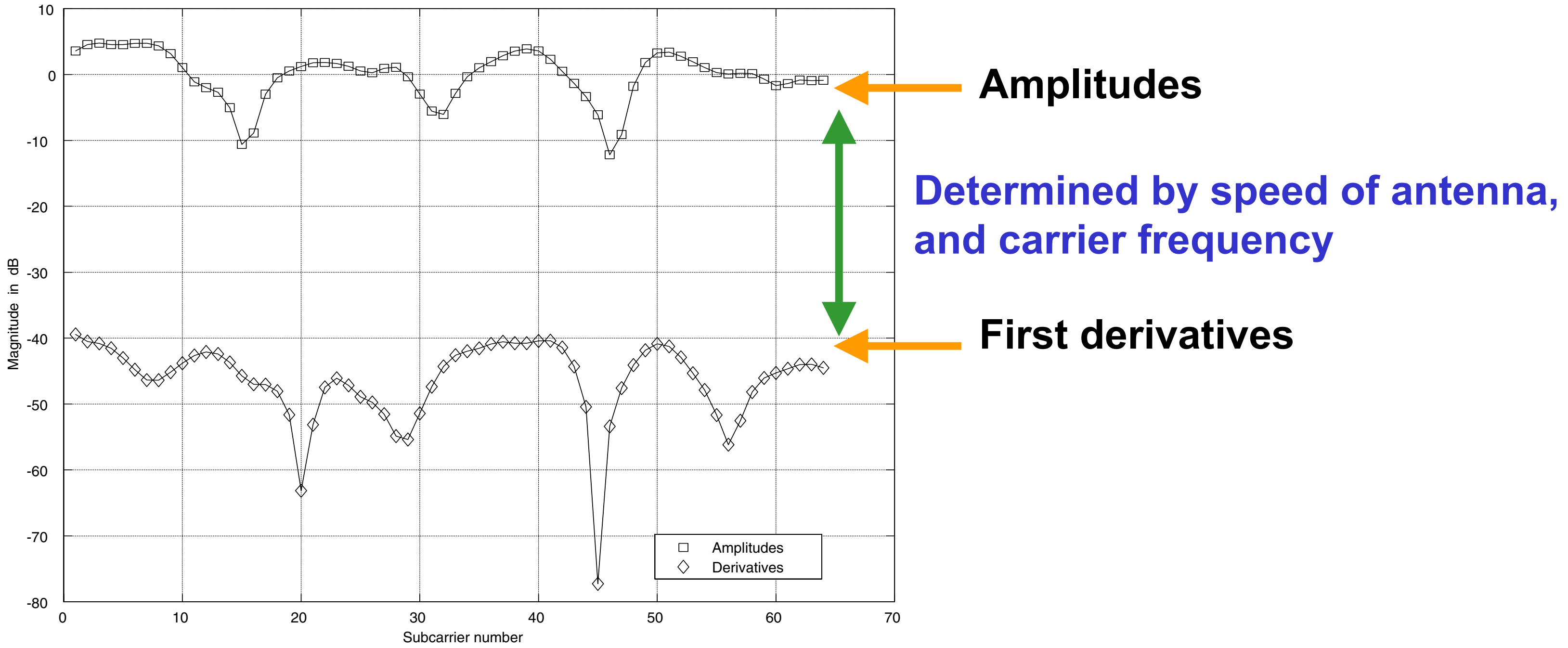



On average:

Canceling ICI originating from 4 closest subcarriers reduced the ICI power by 6 dB

Canceling ICI originating from 10 closest subcarriers reduced the ICI power by 10 dB

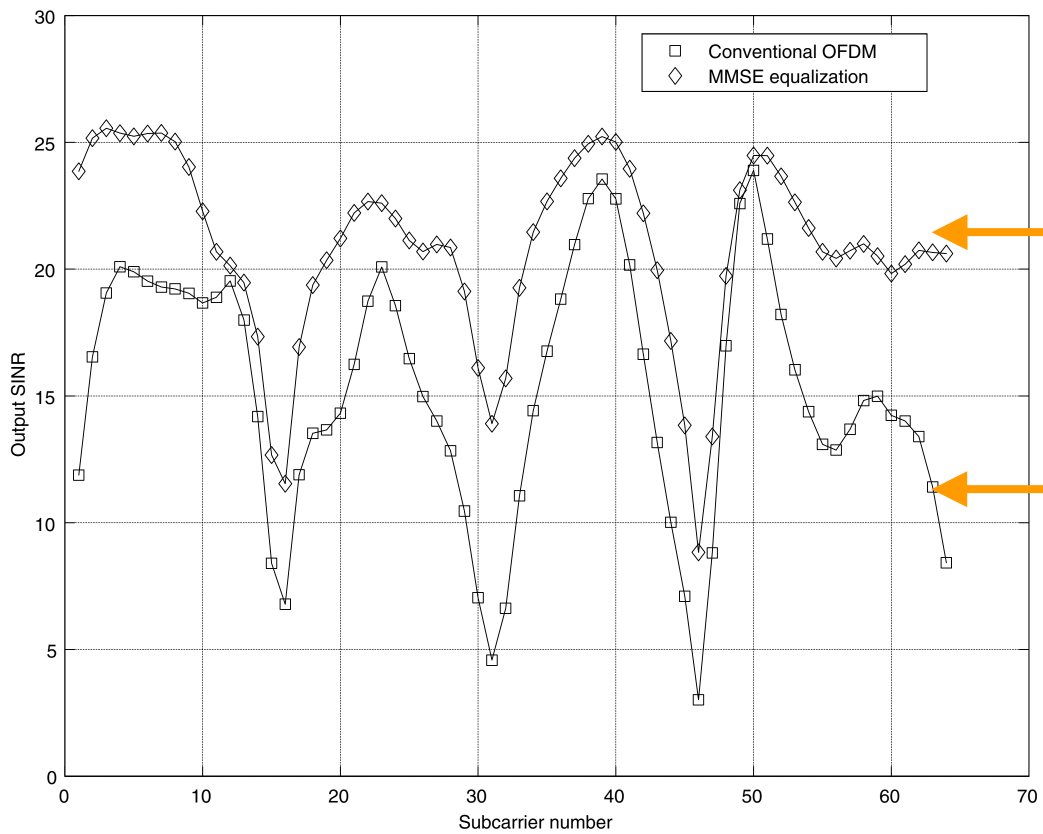
# Receiver 1: Matrix Inversion




Simulation of channel for $N$ = 64, $v$ = 200 km/h $f_c$ = 17 GHz, $T_{RMS}$ = 1 μs, sampling at $T$ = 1μs. $f_{Doppler}$ = 3.14 kHz, Subcarrier spacing $f_{sr}$ = 31.25 kHz, signal-to-ICI = 18 dB

# Receiver 1: Matrix Inversion

SNR of decision variable. Simulation for N = 64, MMSE Wiener filtering to cancel ICI




**MMSE ICI canceller**

**Conventional OFDM**

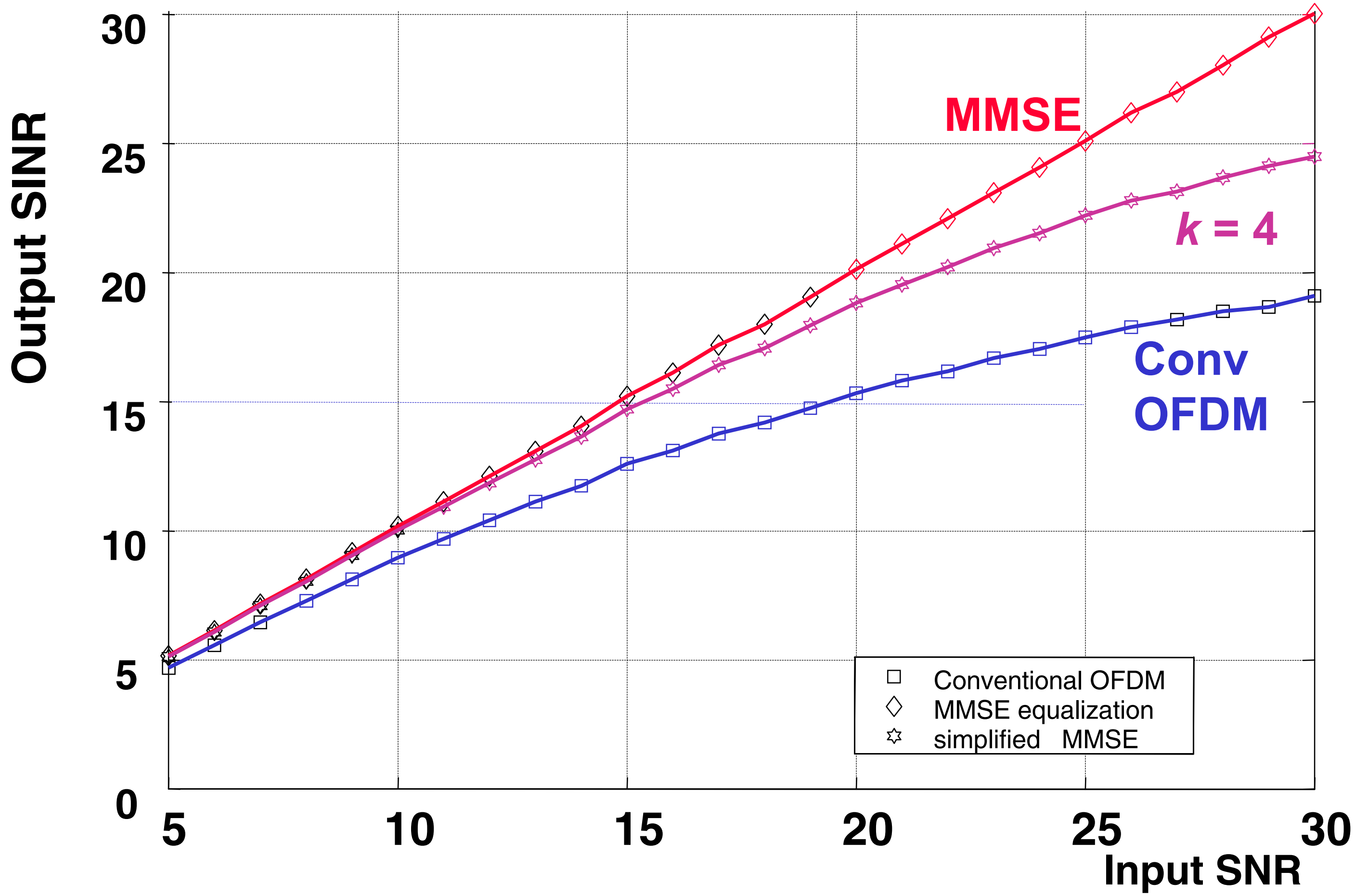
# Simplified Matrix Inversion

Rationale

- ICI diminishes with increasing subcarrier difference
- Approximate $\Xi$ by band matrix with $2k$+1 non-zero diagonals
- Matrix Q is approximately Q = $[I + \Delta]\, \Lambda$
  - $\Delta$ small, $\Delta \sim \Xi$ diag($\boldsymbol{V}^{(1)}$ ./ $\boldsymbol{V}$)
  - $\Lambda$ diagonal of amplitudes $\boldsymbol{V}$
- Approximate $Q^{-1} = [I - \Delta]\, \Lambda^{-1}$

Complexity ~$2kN$

# Performance of (Simplified) Matrix Inversion

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**Compare to DVB-T**: $v$ = 140 km/h, $f_c$ = 800MHz: $f_{\text{doppler}}$ = 100 Hz while $f_{sr}$ = 1.17 kHz

# Multi-Carrier CDMA

# Multi-Carrier CDMA

Various different proposals.

- (1) DS-CDMA followed by OFDM
- (2) OFDM followed by DS-CDMA
- (3) DS-CDMA on multiple parallel carriers

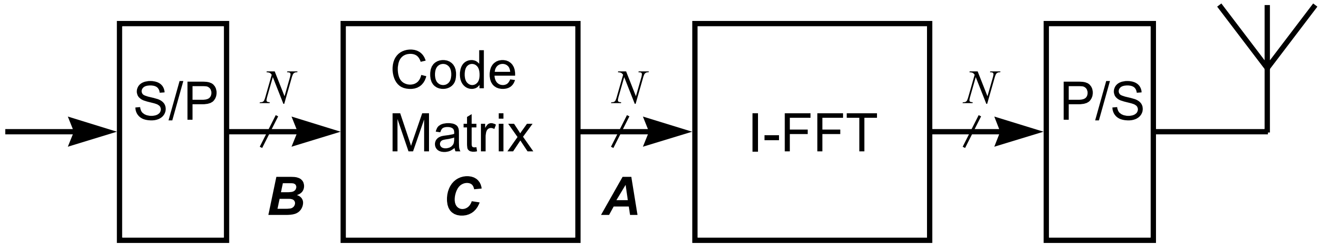
First research papers on system (1) in 1993:

- Fettweis, Linnartz, Yee (U.C. Berkeley)
- Fazel (Germany)
- Chouly (Philips LEP)

System (2): Vandendorpe (LLN)

System (3): Milstein (UCSD); Sourour and Nakagawa

# Multi-Carrier CDM Transmitter



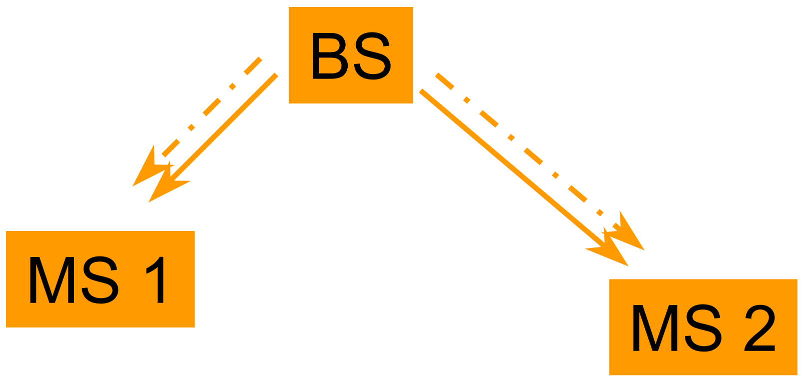

## What is MC-CDMA (System 1)?

- a form of Direct Sequence CDMA, but after spreading a Fourier Transform (FFT) is performed.
- a form of Orthogonal Frequency Division Multiplexing (OFDM), but with an orthogonal matrix operation on the bits.
- a form of Direct Sequence CDMA, but the code sequence is the Fourier Transform of the code.
- a form of frequency diversity. Each bit is transmitted simultaneously (in parallel) on many different subcarriers.

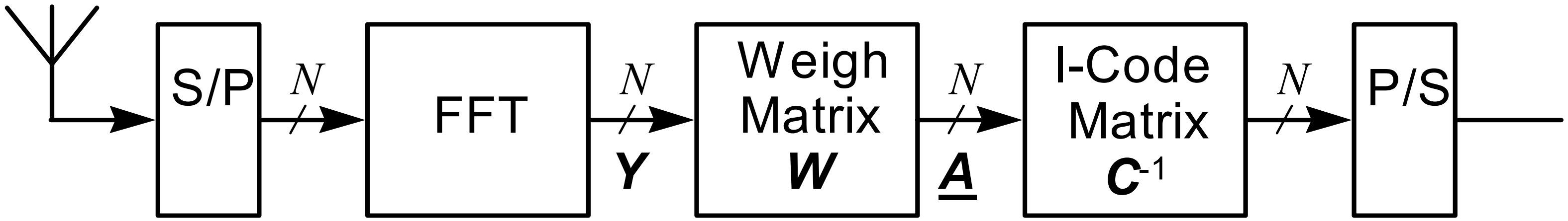
# MC-CDM (Code Division Multiplexing) in Downlink

In the 'forward' or downlink (base-to-mobile): all signals originate at the base station and travel over the same path.

One can easily exploit orthogonality of user signals. It is fairly simple to reduce mutual interference from users within the same cell, by assigning orthogonal Walsh-Hadamard codes.



# Synchronous MC-CDM receiver




The MC-CDM receiver

- separates the various subcarrier signals (FFT)
- weights these subcarriers in ***W***, and
- does a code despreading in $\boldsymbol{C}^{-1}$:

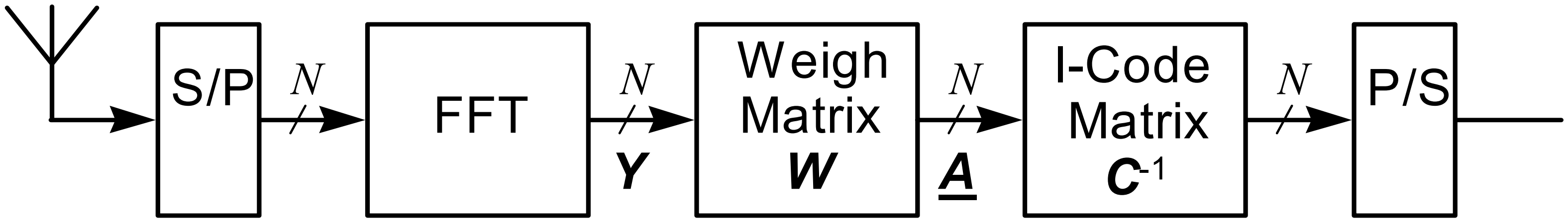
  (linear matrix over the complex numbers)

Compare to C-OFDM:

***W*** := equalization or AGC per subcarrier

$\boldsymbol{C}^{-1}$ := Error correction decoder (non-linear operation)

# Synchronous MC-CDM receiver

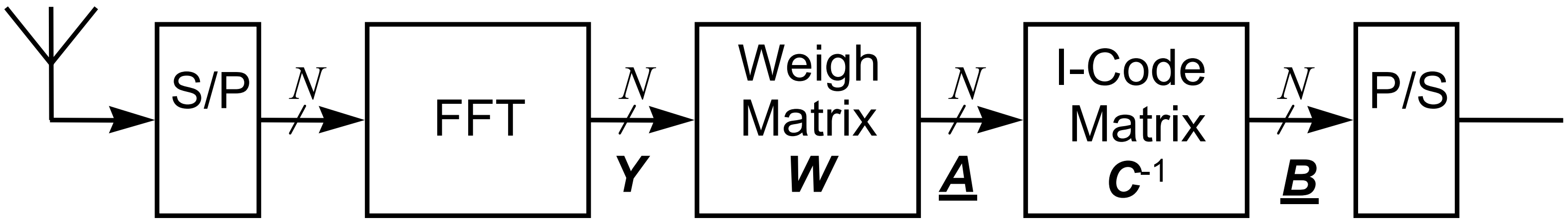



Receiver strategies (How to pick ***W*** ?)

- equalization (MUI reduction) $w = 1/\beta$
- maximum ratio combining (noise reduction) $w = \beta$
- Wiener Filtering (joint optimization) $w = \beta/(\beta^2 + c)$

**Next step:** ***W*** can be reduced to an automatic gain control, per subcarrier, if no ICI occurs

# Synchronous MC-CDM receiver




- Optimum estimate per symbol $\underline{\boldsymbol{B}}$ is obtained from $\underline{\boldsymbol{B}} = \mathrm{E}\boldsymbol{B}|\boldsymbol{Y}$ $= \boldsymbol{C}^{-1}\mathrm{E}\boldsymbol{A}|\boldsymbol{Y} = \boldsymbol{C}^{-1}\underline{\boldsymbol{A}}$.
- Thus: optimum linear receiver can implement FFT - $\boldsymbol{W}$ - $\boldsymbol{C}^{-1}$
- Orthogonality Principle: $\mathrm{E}(\boldsymbol{A}-\underline{\boldsymbol{A}})\boldsymbol{Y}^{H} = 0_N$, where $\underline{\boldsymbol{A}} = \boldsymbol{W}\boldsymbol{Y}^{H}$
- Wiener Filtering: $\boldsymbol{W} = \mathrm{E}\,\boldsymbol{A}\boldsymbol{Y}^{H}\,(\mathrm{E}\boldsymbol{Y}\boldsymbol{Y}^{H})^{-1}$
- $\mathrm{E}\boldsymbol{A}\boldsymbol{Y}^{H}$ diagonal matrix of signal power
- $\mathrm{E}\boldsymbol{Y}\boldsymbol{Y}^{H}$ diagonal matrix of signal plus noise power
- $\boldsymbol{W}$ can be reduced to an AGC, per subcarrier

$$w = \frac{\beta^*}{\beta\ \beta^* + \dfrac{N_0}{T_s}}$$

# MC-CDM BER analysis

Rayleigh fading channel

- Exponential delay spread
- Doppler spread with uniform angle of arrival

Perfect synchronisation

Perfect channel estimation, no estimation of ICI

Orthogonal codes

Pseudo MMSE (no cancellation of ICI)

# Composite received signal

Wanted signal

$$x_0 = b_0 \frac{T_s}{N} \left[ \sum_{n=0}^{N-1} \beta_{n,n} w_{n,n} + \sum_{m \neq 0} \sum_{n=0}^{N-1} \beta_{m,n} w_{n,n} c_0[n] c_0[n-m] \right]$$

Multi-user Interference (MUI)

$$x_{MUI} = T_s \sum_{k=1}^{N-1} b_k \left[ \sum_{n=0}^{N-1} \beta_{n,n} w_{n,n} c_0[n] c_k[n] \right]$$

Intercarrier interference (ICI)

$$x_{ICI} = T_s \sum_{n=0}^{N-1} a_{\mathrm{n}} \sum_{\Delta \neq 0} \beta_{\mathrm{n}+\Delta,n} w_{\mathrm{n}+\Delta,\mathrm{n}+\Delta} c_0[n+\Delta]$$

# Composite received signal

## Wanted signal

$$x_0 = b_0 \frac{T_s}{N} \sum_{n=0}^{N-1} \beta_{n,n} w_{n,n}$$

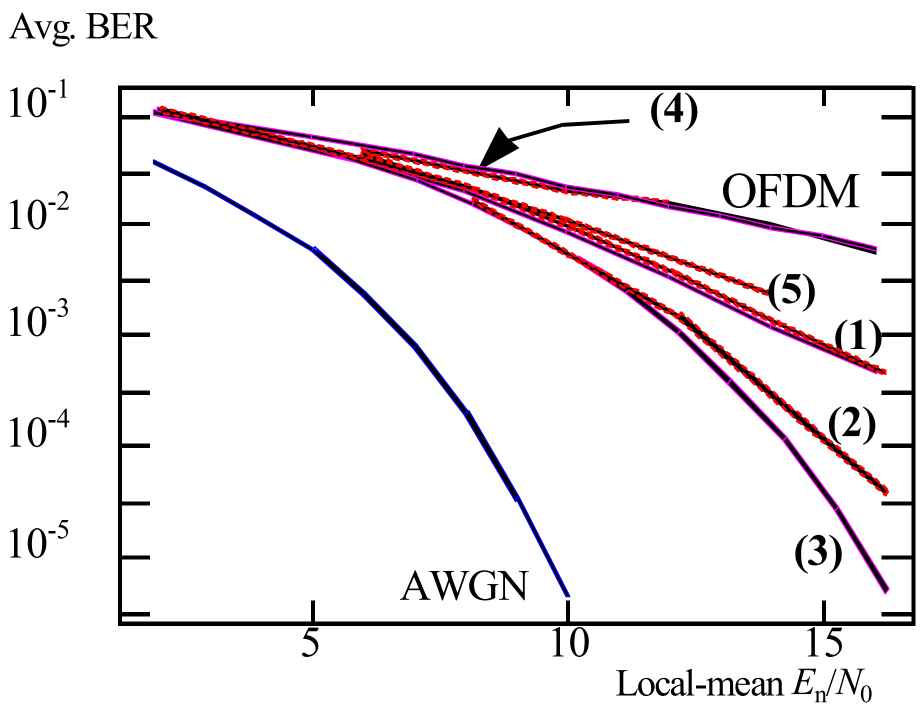
## Multi-User Interference (MUI)

$$\sigma_{MUI}^2 = \mathrm{E}_{\mathrm{ch}} \mathrm{E} \; x_{MUI} x_{MUI}^* = \frac{T_s^2}{N^2} \mathrm{E} \sum_{k=1}^{N-1} b_k^2 \quad \mathrm{E}_{\mathrm{ch}} \left[ \sum_{n \in A+} \beta_{n,n} w_{n,n} - \sum_{n \in A-} \beta_{n,n} w_{n,n} \right]^2$$

## Intercarrier interference (ICI)

$$\sigma_{ICI}^2 = \frac{1}{N} \mathrm{E} \sum_{k=1}^{N-1} b_k^2 \qquad \mathrm{E} \left[ \sum_{\Delta \neq 0} \left[ c_0(n) c_k(n-m) \right]^2 \mathrm{E}_{\mathrm{ch}} \sum_{n=0}^{N-1} \left| \beta_{m,n} \right|^2 \mathrm{E}_{\mathrm{ch}} \sum_{n=0}^{N-1} \left| w_{n,n} \right|^2 \right]$$

# BER for MC-CDMA




Local-mean $E_b/N_0$

BER for BPSK versus $E_b/N_0$

(1) 8 subcarriers

(2) 64 subcarriers

(3) infinitely many subcarriers

(4) 8 subc., short delay spread

(5) 8 subc., typical delay spread

# Capacity

## relative to non-fading channel

### Coded-OFDM

same as *N* fading channels

$$C_{OFDM} = 2\int_0^{\infty} \frac{N_0}{P_0 T_s} \exp\left( -\frac{N_0}{P_0 T_s} x \right) \tfrac{1}{2}\log_2(1+2x)\,dx$$

$$C_{OFDM} = \frac{1}{\ln 2} \exp\left( \frac{N_0}{2 P_0 T_s} \right) E_1\left( \frac{N_0}{2 P_0 T_s} \right)$$

For large $P_0 T_s / N_0$ on a Rayleigh fading channel, OFDM has 0.4 bit less capacity per dimension than a non-fading channel.

### MC-CDM

Data Processing Theorem:

$$C_{\mathrm{OFDM}} = C_{\mathrm{MC\text{-}CDM}}$$

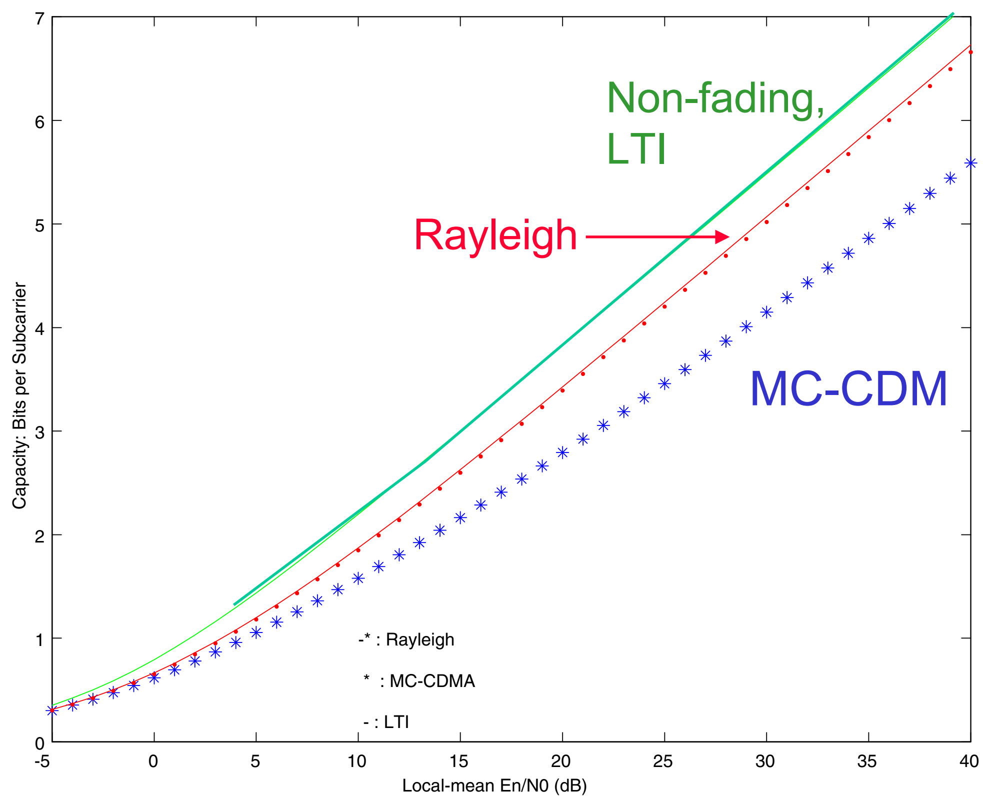
In practise, we loose a little.

In fact, for infinitely many subcarriers,

$$C_{\mathrm{MC\text{-}CDM}} = \tfrac{1}{2} \log_2(1 + \varsigma P_0 T_s / N_0).$$

where $\varsigma$ is MC-CDM figure of merit, typically -4 .. -6 dB.

# Capacity




- Capacity per dimension versus local-mean $E_N/N_0$, no Doppler.

# Advantages

Simpler user separation than with DS-CDMA

Higher capacity than DS-CDMA in downlink: elegant frame work for doing simultaneous anti-multipath and interference rejection

FFT instead of rake: simpler training of receiver

# MC-CDMA in uplink

In the 'reverse' or uplink (mobile-to-base), it is technically difficult to ensure that all signals arrive with perfect time alignment at the base station.

Frame mis-alignments cause severe interference

Different Doppler spectra for each signal

Different channels for different signals

Power control needed