Alignment Methods: Static vs. In-Motion

Part 3 of the series on Fault Tolerant Low-Cost Inertial Navigation Systems

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I. Introduction

An Inertial Navigation System (INS) algorithm generates velocity, position and attitude information based on the inputs of measured accelerometer and gyro outputs. Attitude is the result of integration of the IMU measured angular rate, while velocity and position are the result of transformation and integration of the IMU specific force acceleration, Coriolis acceleration and modeled gravity relative to the desired navigation frame.

Initial misalignment is one of the major error sources in the INS. Because the velocity, position and attitude values are integrated from initial conditions, it is important for the initial alignment errors to be small. Thus, accurate initial conditions and alignment must be performed first and is crucial for successful operation of the INS. This will be the focus of this paper.

II. Alignment of Low-Cost INS

Alignment is referred to as the procedure to initialize the INS: in particular, the attitude information between the body-frame (b-frame hereafter) and the navigation-frame (n-frame hereafter). Several algorithms are available and can be classified in different ways. If the criterion is the amount of attitude error that the algorithm has to deal with, then these are classified as either coarse alignment of fine alignment methods. Typically, the threshold of the attitude errors between the two categories can reach a few degrees. Is, on the other hand, the criterion is the dynamics of the vehicle upon initialization, the alignment methods can be classified as stationary alignment or in-motion alignment methods.

Typically, coarse alignment of an INS is done in stationary mode using leveling (by accelerometers) followed by gyro-compassing or, alternatively, an analytical methods solving the two-vector measurement problem using the gravity and the Earth rotation measurements in one step as follows, [1],

$$\mathbf{C}_b^n = \begin{bmatrix} \frac{-\tan\psi}{g} & \frac{1}{\omega_e \cos\psi} & 0 \\ 0 & 0 & \frac{-1}{g\omega_e \cos\psi} \\ \frac{-1}{g} & 0 & 0 \end{bmatrix} \begin{bmatrix} (\mathbf{f}^b)^\top \\ (\boldsymbol{\omega}_{ib}^b)^\top \\ (\mathbf{f}^b \times \boldsymbol{\omega}_{ib}^b)^\top \end{bmatrix}, \tag{1}$$

where g is gravity. However due to their large biases and the low signal-to-noise ratio of low-cost gyroscopes, gyro-compassing and the analytical coarse alignment cannot be applied in stationary mode. Therefore only the roll and pitch can be determined from the accelerometer measurements as follows:

$$\phi = \text{sign}(f_z)\sin^{-1}(f_y/g), \tag{2}$$
$$\theta = \text{sign}(f_z)\sin^{-1}(f_y/g), \tag{3}$$

where $\mathbf{f}^b = [f_x \ f_y \ f_z]^\top$ and $\text{sign}(\cdot)$ denotes the sign of a value. For the z-channel, only the sign is used because the gravity error is smaller than the bias of low-cost IMUs. The heading has to be determined from other sensors such as a multi-antenna GNSS, a magnetic compass, or 3-axis magnetometer with magnetic declination corrections from Magnetic North to True North.

If the IMU is installed in a customer vehicle, we cannot expect the user to wait until the alignment is finished. Hence, in-motion alignment techniques need to be considered. The GNSS-derived velocity can be used for coarse in-motion alignment is the forward axis is parallel to the velocity vector, which holds approximately for most land vehicle navigation systems, but not for example for helicopters or quad-copters which can move orthogonal to their forward orientation. Within mechanical alignment uncertainty, the roll can be initialized to zero with a $\pm 5°$ uncertainty, in most cases for land vehicles. The pitch and heading can be initialized as follows:

$$\theta = \tan^{-1}(-v_D/\sqrt{v_N^2 + v_E^2}), \tag{4}$$
$$\psi = \tan^{-1}(v_E/v_N), \tag{5}$$

where v_N, v_E and v_D are the north, east and down velocities, respectively.

For aircraft or ships, however, lateral or vertical velocity components can exist due to wind gusts or maneuvers. Therefore eqns. (4 & 5) cannot be applied. If the heading is completely unknown, then the EKF with large heading uncertainty (LHU) model must be used for the coarse alignment, which was discussed in Part 1 of this series.

Once the coarse alignment is achieved, the fine alignment is usually applied using the EKF with a small heading uncertainty (SHU) model. Table (I) summarizes the alignment methods which can be categorized into two classes with subsequent subclasses.

III. Literature Review

A. Self-Alignment Models

One of the ground alignment methods is to obtain the INS initial attitude through the use of external reference

TABLE I
ALIGNMENT METHODS

	Static	In-Motion
Coarse	· Leveling · Analytic	· GPS Velocity · EKF with LHU model
Fine	· Gyro-Compassing	
	EKF with SHU model	

by optical techniques, magnetic heading sensors or other external means. Most ground based applications consist of two phases: leveling (for roll and pitch) and azimuth alignment (for heading) [11]. Two self-alignment methods have been considered: gyrocompass alignment and analytic alignment [11]- [20]. The analytic alignment method is used for coarse alignment, while the gyrocompass method is used for fine alignment [11], [19].

The analytic coarse alignment determines the transformation matrix which relates vectors in the desired navigation frame to the same vectors expressed in the geographic-frame (g-frame hereafter) or in other equivalent frames using the knowledge of the gravity g and the earth rate ω_{ie} vectors.

Define a vector v as

$$v = g \times \omega_{ie}, \tag{6}$$

the alignment matrix $\mathbf{C}_g^n$ is given by

$$\mathbf{C}_b^n = \begin{bmatrix} \frac{-\tan\psi}{g} & \frac{1}{\omega_e \cos\psi} & 0 \\ 0 & 0 & \frac{-1}{g\omega_e \cos\psi} \\ \frac{-1}{g} & 0 & 0 \end{bmatrix} \begin{bmatrix} (\mathbf{f}^b)^\top \\ (\boldsymbol{\omega}_{ib}^b)^\top \\ (\mathbf{f}^b \times \boldsymbol{\omega}_{ib}^b)^\top \end{bmatrix}, \tag{7}$$

where the gravity vector g and the earth rate ω_{ie} in the desired navigation-frame (n-frame hereafter) and the g-frame are g^n, g^g, ω_{ie}^n and ω_{ie}^g.

Britting [1] analyzed the errors of the analytic coarse alignment scheme by taking into account the effects of the instrument uncertainties and that the base motions are not readily amenable to analytic methods. This method requires the vectors g and ω_{ie} to be measured.

For a gimbaled INS, physical gyrocompass alignment consists of finding the coordinate transformation between local geographic axes and the navigation axes by physical gyrocompassing. The transformation matrix $\mathbf{C}_g^n$ is physically driven to be the unit matrix.

For strapdown INS, the initial alignment matrix represents the transformation from the b-frame to the n-frame. This analytic alignment method can be used for high accuracy applications in only the most benign of environments, since the performance deteriorates because of angular disturbance vibrations and accelerations. The measurement of the gravity and the earth rate could also be corrupted. Some filtering is introduced in order to reduce the effects of these vibrations. When a low-pass filter is used to obtain the average values of the measured quantities, the instantaneous position of the b-frame can vary considerably from its average position. A significant misalignment could exist when the system is switched to the navigation operation mode. A self-corrective alignment scheme is introduced to refine the initial estimate of the transformation matrix by using the error angles between the known reference frame and the corresponding c-frame.

Many gyrocompassing algorithms for both gimbaled INSs and strapdown INSs have appeared in literature. In a 1967 paper by Jurenka [19], they developed an optimum controller for driving the alignment. Huang [18] presented self-alignment techniques for an IMU considering the fine alignment of and INS platform whose base is subject to vibration and whose sensors are subject to noise. THe observability of INS ground alignment was analyzed by Jiang [14]. In the covariance analysis of strapdown INS considering gyrocompassing characteristics by in [18], it was found that the cross-couplling terms in the gyrocompass alignment errors can significantly influence the strapdown INS error propagation. The initial heading error has a close correlation with the east component of gyro bias error, while initial level tilt errors are closely linked to the accelerometer bias error. Heung [18] also developed a multi-position alignment method for INS stationary alignment on the ground.

Bar-Itzhack summarized INS error models for the ground alignment in 1988 [11]. All these models in the literature assume that the initial misalignments are small angles with errors of only a few degrees. The measurement of the earth rate is indispensable and an essential input for the ground alignment.

B. In-Motion Alignment Models

If the initial alignment is carried out when the vehicle is in motion, the alignment is called in-motion alignment. The initial condition can be obtained by transfer alignment.

Transfer alignment is the operation of aligning a slave INS witgh a master INS comparing quantities computed by both INSs [21]. Direct transfer of the master INS navigation states would not account for the fact that the slave INS is not pointed in the same direction as the master INS. In the transfer alignment stage, the velocity computed by the master INS is compared with the one computed by the slave INS and the difference, which is indicative of the slave misalignment with respect to the master INS, is processed by a Kalman filer. This yields misalignment data as well as the slave gyro and accelerometer error data. There are some other initial measurement matching methods such as velocity matching alignment and position update alignment which achieve in-motion alignment by comparing estimates of the velocity and position generated by the aligning INS with estimates of the dame quantities provided by the master INS.

There are many issues concerning the transfer alignment algorithm design [27], [21], [22], Bar-Itzhack investigated the azimuth observability enhancement that was accomplished by subjecting the INS to accelerations generated by maneuvers pf a combat aircraft [27]. The effect of acceleration switching during INS in-motion alignment was also analyzed [23].

To obtain high accuracy and a highly robust navigation system, inertial navigation aided by other navigation references like a star scanner, GPS, laser, radar, sighting devices, and Doppler have been widely used [24], [25], [26]. Complementary filtering is the one mutual aiding method implemented in the frequency domain [26]. It is applied to the fusion of a compass-aided heading gyro, Doppler-inertial ground speed estimation and a baro-inertial altimeter. In the time domain, Kalman filtering with INS error models has been the main tool for the implementation of aided INS in-motion alignment, navigation and calibration stages. The assumption of small misalignment angles is implied in those filtering and models.

C. Large Misalignment Models

Ground alignment in the stationary stage using gyrocompassing, analytic alignment, aiding attitude sensors and in-motion alignment using reference navigation information have been the main methods for INS initial alignment. With the assumption of small initial misalignments, Kalman filter mechanizations using the INS error propagation models have been designed and implemented for many years.

Ground self-alignment is limited to high resolution IMU's which are able to measure the vectors of gravity and the Earth rate. For low-cost IMU's, with resolution lower than the quantities of gravity and Earth rate, self-alignment principles will not work. External sensors have to be employed to obtain the initial attitude [3]. If external sensors are not available, the initial misalignment will be large. Several attempts have been made to solve this problem.

Jiang [28] suggested an error estimation algorithm in the true frame for ground alignemnt for an arbitrary azimuth. The misalignment propagation was not modeled.

Pham [10] introduced a Kalman filter mechanization for strapdown INS air-start with an unknown azimuth. He assumed two small tilt misalignment errors and one large heading error. Heading misalignment α is considered as a wanter angle in a wander angle frame and separated into two states $\sin\alpha$ and $\cos\alpha$ in the filter. The earth rate is still the required input in the filter process model. The misalignment was not modeled.

Scherzinger [7], [9] developed inertial navigator error models for large heading uncertainty. The expected application was for alignment with an unknown heading. The computer-frame (c-frame hereafter) approach was used. Again, two states were used to track one heading misalignment variable. An extended misalignment vector ψ_e was defined as

$$\boldsymbol{\psi}_e = [\psi_x,\ \psi_y,\ \sin\psi_z,\ \cos\psi_z - 1], \tag{8}$$

where the ψ-angle $[\psi_x,\ \psi_y,\ \psi_z]$ is the misalignment of the c-frame and the platform-frame (p-frame hereafter). Scherzinger's model has a similar form of the ψ-angle model developed in the literature by Benson [6] and Bar-Itzhack [13]. In 1997, Dmitriyev published a nonlinear filter method application in INS alignment which discussed the problem of coarse alignment [29]. Error equations with a nonlinear characteristic have been obtained with a considerable level of *a priori* coarse uncertainty. The error model is designed in the t-frame with the assumption of two small tile misalignments and one large heading misalignment. The nonlinear characteristic of the problem was considered over a short period. As a result, the heading misalignment rate was assumed zero. In the misalignment error models, the rates of misalignment angles were coupled with the velocity errors and the velocity errors are coupled with the misalignment angles.

IN 1997 Rogers [8] proposed another in-motion alignment method without the benefit of attitude initialization. Similar to the methods of Pham and Scherzinger, Rogers' filter contained two sine and coseine states of the heading error. Small tilts and large heading error were assumed.

The ψ-angle models and quaternion models for three large misalignment error angles in the c-frame developed in the previous paper provide a solution to the unknown initial attitude problem. The additional two tilt angles and one heading angle could be all large using the proposed models.

IV. Coarse Alignment

An INS commonly uses the analytic alignment and the gyrocompass principle to perform self-alignment on the ground. For a low-cost IMU which cannot measure the Earth rate vector, external sensors must be used.

In this paper we will consider a common-off-the-shelf (COTS) IMU by InvenSense, the MPU 6050.

TABLE II

Stationary error based on raw data from the MPU-6050.

IMU	Stationary Error
Gyro	$1.0779 \times 10^4 rad/sec$
Accelerometer	$0.0024 m/s^2$

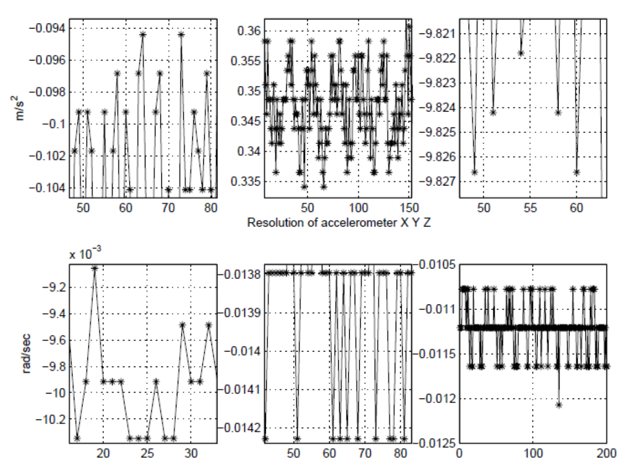

Fig. 1. Stationary error based on raw data from the MPU-6050

The quantity of the Earth rate vector is 7.2722×10^{-5} rad/sec. Usually, the resolution of accelerometers is high enough to measure the gravity vector. For this kind of IMU, the coarse alignment can be partially performed using the acceleration data to detect the tilt information. The coarse alignment is introduced here. The raw data process which determines the average turn-on biases of the accelerometers and gyros is included in the coarse alignment stage.

A. Raw Data Process

Once the IMU is turned on, there are random biases in the accelerometers and gyros. They have a crucial effect on the attitude, velocity and position. The integration of the biases of the accelerometers cause the position errors to increase quadratically over time. The attitude, direction cosine matrices and quaternions will also have important errors from integrating the gyro biases. Therefore, the gyro biases will accumulate the position errors over time proportional to the cub of the time index.

The following table gives examples of the effects on velocity and position by the MPU-6050.

TABLE III

AVERAGE BIAS FOR THE MPU-6050 IMU OVER 30 SECONDS.

axis(IMU b-frame)	gyro bias (rad/sec)	accel. bias (m/sec^2)
x	0.004	0.008
y	0.008	0.2
z	0.003	0.03

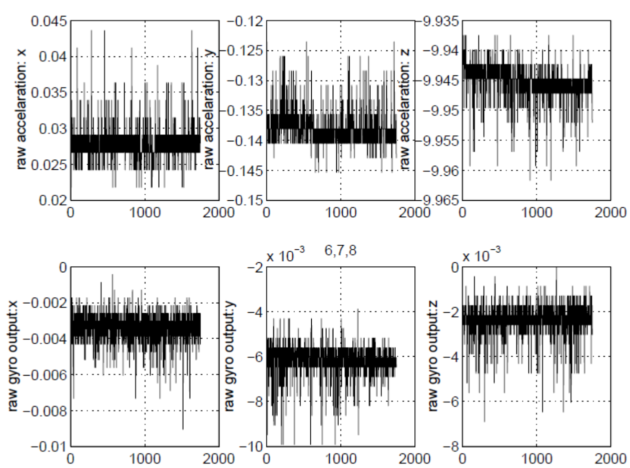

Fig. 2. Average biases of the MPU-6050 over a 30 second period.

Figure 3 shows the quaternion errors due to the gyro biases in a stationary position over 30 seconds. The four curves of the four elements $[Q_0,\ Q_1,\ Q_2,\ Q_3]$ show linear divergence over time.

The first, second and third order effects over time of the IMU torn-on biases are shown in Figure 4. The acceleration shows a large linear divergence over time. The velocity show a quadratic divergence over time squared. The position shows a third-order relationship with time.

The gyro biases of 0.008 rad/sec and the accelerometer bias of 0.2 m/s^2 will cause a velocity error of 20 m/s and a position error of 200 m over 30 seconds.

These results demonstrate that before the raw data can be used, it is essential to remove the turn-on biases. The methods of bias determination is discussed here.

The bank and elevation information are defined as teh angles with respect to the local level frame. The average tilt angles from the tilt gyros are used to generate the coarse alignment information.

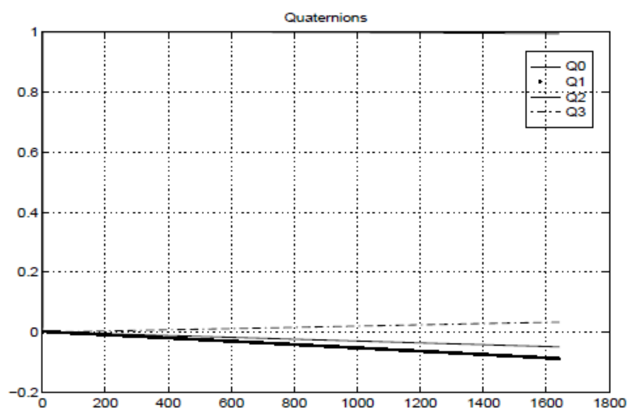

Fig. 3. Quaternion errors over a 30 second period.

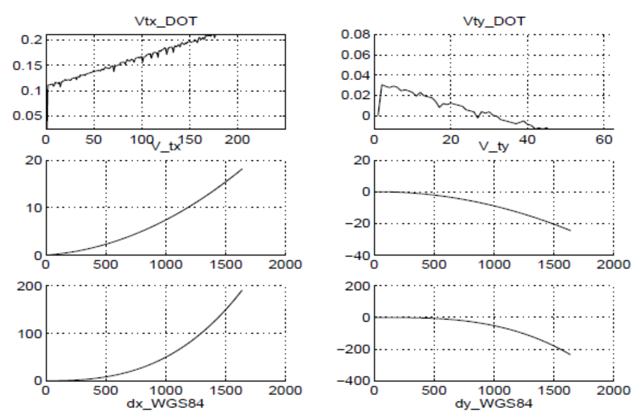

Fig. 4. Errors of acceleration, velocity, and position due to turn-on biases of the IMU. y-axes top-to-bottom are m/s^2, m/s, m respectively.

B. Turn-on Biases of the Accelerometers

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V. IN-MOTION ALIGNMENT

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VI. Conclusion

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Part 3 in this series will cover INS error sensitivities for low-cost Inertial Navigation Systems.

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