Regularized and Simplified Monte Carlo Joint Probabilistic Data Association Filter for Multi-Target Tracking in Wireless Sensor Networks

M. A. Tinati, T. Yousefi Rezaii, M. J. Museviniya
Faculty of Electrical and Computer Engineering
University of Tabriz
Tabriz, Iran
{tinati, yousefi & niya}@tabrizu.ac.ir

Abstract - *In this paper we propose to use Regularized Monte Carlo-Joint Probabilistic Data Association Filter (RMC-JPDAF) to the classical problem of multiple target tracking in a cluttered area. We have used the Monte Carlo methods in order to the fact that they have the ability to model any state-space with nonlinear and non-Gaussian models for target dynamics and measurement likelihood. To encounter with the data association problem that arises due to unlabeled measurements in the presence of clutter, we have used the Joint Probabilistic Data Association Filter (JPDAF). Due to the resampling stage in the MC-JPDAF, the sample impoverishment phenomenon is unavoidable and the tracking performance will decrease. So we propose to use the Regularized resampling stage instead, to counteract this effect. Finally we have used the target dynamics model as the proposal distribution in MC-JPDAF, in order to decrease the computational cost while the performance of the tracking system is nearly maintained.*

Keywords - *multiple target tracking;data association; JPDAF, regularization*

I. INTRODUCTION

Increasingly, for many application areas, it is becoming important to include elements of nonlinearity and non-Gaussianity in order to model accurately the underlying dynamics of a physical system. In [1] and [2], several methods are introduced to encounter with such models, such as the Extended Kalman Filter, approximate Grid-based methods and Particle filters. They demonstrate that in the case of nonlinear and non-Gaussian models for dynamic and likelihood distributions, the particle filtering method represents the best performance and is most reliable, in spite of its higher computational cost.

Multiple Target Tracking (MTT) is not a trivial extension of single target tracking but rather a challenging topic of research. In MTT scenarios, there is a combinatorial explosion in the space of possible multiple target trajectories due to the uncertainty in the association of observed measurements with known targets in each step. The main challenge of realizing an MTT system is to manage the computational complexity of the problem while still providing the reasonable tracking performance.

Multiple Hypothesis Tracking (MHT) and JPDAF are the two major approaches to deal with MTT problems. We have used the JPDAF approach due to its lower computational complexity and online implementation. In this way, at each time step infeasible hypotheses are pruned away using a gating procedure. A filtering estimate is then computed for each of the remaining hypotheses, and combined in proportion to the corresponding posterior hypothesis probabilities. The main shortcoming of the JPDAF is that, to maintain the tractability, the final estimate is collapsed to a single Gaussian, thus discarding pertinent information. However the performance of these algorithms degrades as the non-linearities become more severe. More recently, strategies have been proposed to combine the JPDAF with Particle techniques to accommodate general nonlinear and non-Gaussian models [3], [4]. The MC-JPDAF developed in [4], can be considered as the first comprehensive algorithm that uses the Monte Carlo methods to implement the MTT while efficiently taking into account the data association problem. So, the proposed method can be used for the general case of nonlinear and non-Gaussian dynamics and measurement models. The main shortcoming of the MC-JPDAF is the sample impoverishment due to the resampling stage and high computational complexity.

In this paper we have used the Regularized resampling, developed in [5] to overcome the sample impoverishment problem. Furthermore, in order to reduce the computation complexity of MC-JPDAF, we propose to use the prior density (dynamics model) as the proposal density function instead of the proposal density introduced in [4]. This leads to considerable reduction in computational complexity, because in each time instance, the particles are sampled from the simple and known prior distribution and no further computation is needed to construct the proposal distribution. This simplification is achieved with no considerable reduction in tracking performance.

II. MODEL DESCRIPTION

In this section, several models used in MTT scenario are described. The evolution of the joint state space of the K slowly maneuvering targets in the xy plane, is considered to be of the form described in [6] (the near constant velocity model). Also the state evolution of each target is assumed to be independent of the others. So the state of the k^{th} target in the xy plane at time t, comprises its position and velocity,

$$\mathbf{x}_{k,t} = (x_{k,t}, \dot{x}_{k,t}, y_{k,t}, \dot{y}_{k,t}) \tag{1}$$

So the matrix form of the state transition equation is

$$\mathbf{x}_{k,t} = \mathbf{A}\mathbf{x}_{k,t-1} + \mathbf{v}_{k,t} \tag{2}$$

where $\mathbf{A}$ is the state transition matrix and $\mathbf{v}_{k,t}$ is the process noise and assumed to be zero mean Gaussian distributed with known covariance matrix as defined in [4]. The measurements are assumed to be available from N_o observer sensors at locations $P_o^i, i = 1,...,N_o$. So the joint measurement vector from N_o observers at each time interval is

$$\mathbf{z} = (\mathbf{z}^1 ... \mathbf{z}^{N_o}), \text{ where } \mathbf{z}^i = (\mathbf{z}_1^i ... \mathbf{z}_{M^i}^i) \tag{3}$$

M^i is the total number of measurements in the i[th] observer, and defined as

$$M^i = M_T^i \,(\text{target measurements}) + M_C^i \,(\text{clutter measurements})$$

To deal with the data association problem, we consider the association variables presented in [4].

Measurement to Target association hypothesis:

$$\boldsymbol{\lambda} = (\boldsymbol{\lambda}^1 ... \boldsymbol{\lambda}^{N_o}) \text{ where } \boldsymbol{\lambda}^i = (\mathbf{r}^i, M_C^i, M_T^i)$$

The elements of the association vector $\mathbf{r}^i$ are given as

$$r_j^i = \begin{cases} 0 & \text{if measurement is due to clutter} \\ k \in \{1...K\} & \text{if measurement stems from target k} \end{cases}$$

The target to measurement association hypothesis:

$$\tilde{\boldsymbol{\lambda}} = (\tilde{\boldsymbol{\lambda}}^1 ... \tilde{\boldsymbol{\lambda}}^{N_o}) \text{ where } \tilde{\boldsymbol{\lambda}}^i = (\tilde{\mathbf{r}}^i, M_C^i, M_T^i)$$

$$\tilde{r}_k^i = \begin{cases} 0 & \text{if target } k \text{ is undetected at observer } i \\ j \in \{1...M^i\} & \text{if target } k \text{ generated measurement } j \end{cases}$$

The two set of above hypotheses are equivalent. Considering that the observer sensors measure the range R_j^i and bearing θ_j^i from the observer to the target, the individual measurements at the i[th] observer are $\mathbf{z}_j^i = (R_j^i, \theta_j^i),\ j = 1,...,M^i$. If the range and bearing are assumed to be corrupted by independent Gaussian noise, the likelihood for the target j[th] measurement, under the hypothesis that it is associated with the k[th] target

$$p_T^i(\mathbf{z}_j^i \mid \mathbf{x}_k) = \mathrm{N}(\mathbf{z}_j^i \mid \hat{\mathbf{z}}_j^i, \boldsymbol{\Sigma}_z^i) \tag{4}$$

where $\boldsymbol{\Sigma}_y^i = diag(\sigma_{R^i}^2, \sigma_{\theta^i}^2)$ is the fixed and known diagonal covariance with the individual noise variances. In order to estimate the unknown association hypothesis within a Bayesian framework, definition of a prior distribution as described in [4] and [6], for these hypotheses is necessary. For the measurement to target association hypothesis it is assumed that the prior factorizes over the observers, i.e.

$$p(\boldsymbol{\lambda}) = \prod_{i=1}^{N_o} p(\boldsymbol{\lambda}^i) \tag{5}$$

and so for each of the observers, the prior is assumed to be

$$p(\boldsymbol{\lambda}^i) = p(\mathbf{r}^i \mid M_C^i, M_T^i)\, p(M_C^i)\, p(M_T^i) \tag{6}$$

The prior for the association vector is assumed to be uniform over all valid hypotheses, the number of clutter measurements is assumed to follow a Poisson distribution with rate parameter $\boldsymbol{\lambda}_C^i$ and the prior for the target measurements is assumed to follow a binomial distribution.

III. MC-JPDAF FRAMEWORK

As mentioned previously, JPDAF approach, due to its simplicity and low computational complexity in contrast with MHT approach, is the most widely applied method in MTT problems. So we have also chosen this approach to solve the data association problem in this paper. Several implementation strategies for JPDAF method have been proposed in literature, according to the application area. Since the target dynamics and measurement likelihood models in target tracking applications are nonlinear and non-Gaussian in general, the selected Bayesian framework should have the ability to estimate and track such nonlinear and non-Gaussian models. For this reason, the Monte Carlo implementation of JPDAF presented in [4] is used in our MTT system. The main idea of JPDAF is to recursively update the marginal filtering distributions for each of the targets $p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1,t-1}),\ k = 1,...,K$, through the Bayesian sequential estimation recursion,

$$p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1,t-1}) = \int p_k(\mathbf{x}_{k,t} \mid \mathbf{x}_{k,t-1}) p_k(\mathbf{x}_{k,t-1} \mid \mathbf{z}_{1,t-1}) d\mathbf{x}_{k,t-1} \tag{7}$$

Due to the data association uncertainty, the filtering step cannot be performed independently. In JPDAF the likelihood for the k-th target is assumed to be

$$p_k(\mathbf{z}_t \mid \mathbf{x}_{k,t}) = \prod_{i=1}^{N_o} \left[\beta_{jk}^i + \sum_{j=1}^{M^i} \beta_{jk}^i p_T^i(\mathbf{z}_{j,t}^i \mid \mathbf{x}_{k,t}) \right] \tag{8}$$

where $\beta_{jk}^i = p(\tilde{r}_{k,t}^i = j \mid \mathbf{z}_{1:t}),\ j = 1...M^i$, is the posterior probability that the k[th] target is associated with j[th] measurement in the i[th] observer with β_{0k}^i, the posterior probability that the k[th] target is undetected. Furthermore, the likelihood is assumed to be independent over the observers. With the definition of the likelihood as in (4), the filtering step is as follows

$$p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1:t}) \propto p_k(\mathbf{z}_t \mid \mathbf{x}_{k,t}) p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1:t-1}) \tag{9}$$

All that remains is to compute the posterior probabilities of the marginal associations β_{jk}^i as,

$$\beta_{jk}^i = p(\tilde{r}_{k,t}^i = j \mid \mathbf{z}_{1:t}) = \sum_{\{\tilde{\boldsymbol{\lambda}}_t^i \in \tilde{\Lambda}_t^i : \tilde{r}_{k,t}^i = j\}} p(\tilde{\boldsymbol{\lambda}}_t^i \mid \mathbf{z}_{1:t}) \tag{10}$$

where $\tilde{\Lambda}_t^i$ is the set of all joint target to measurement association hypotheses for the data at the i[th] observer.

The posterior probability for the joint association hypothesis $p(\tilde{\boldsymbol{\lambda}}_t^i \mid \mathbf{z}_{1:t})$ can be expressed as,

$$p(\tilde{\boldsymbol{\lambda}}_t^i \mid \mathbf{z}_{1:t}) \propto p(\tilde{\boldsymbol{\lambda}}_t^i)(V^i)^{-M_C^i} \prod_{j \in \mathrm{I}^i} p_{r_{j,t}^i}(\mathbf{z}_{j,t}^i \mid \mathbf{z}_{1:t-1}) \tag{11}$$

where the expression $p(\tilde{\boldsymbol{\lambda}}_t^i)$ is the target to measurement association hypothesis, V^i is the volume of the measurement space for the i[th] observer defined as $V^i = 2\pi R_{\max}^i$, where $R_{\max}^i$ is the maximum range of the i[th] observer, $\mathrm{I}^i = \{j \in \{1 \ldots M^i\} : r_j^i \neq 0\}$.

The expression $p_{r_{j,t}^i}(\mathbf{z}_{j,t}^i \mid \mathbf{z}_{1:t-1})$ is the predictive likelihood for the j[th] measurement at the i[th] observer using the information from the k[th] target, given in the standard form by

$$p_k(\mathbf{z}_{j,t}^i \mid \mathbf{z}_{1:t-1}) = \int p_T^i(\mathbf{z}_{j,t}^i \mid \mathbf{x}_{k,t}) p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1:t-1}) d\mathbf{x}_{k,t} \tag{12}$$

In Monte Carlo frame work, the predictive likelihood in (12) is approximated using the Monte Carlo samples from the proposal distribution. It is assumed that for the k[th] target the set of samples $\{w_{k,t-1}^{(n)}, \mathbf{x}_{k,t-1}^{(n)}\}_{n=1}^N$ is available, approximately distributed according the marginal filtering distribution at the previous time step $p_k(\mathbf{x}_{k,t-1} \mid \mathbf{z}_{1:t-1})$. At the current time step new samples for the target state are generated from a suitably constructed proposal distribution, i.e.

$$\mathbf{x}_{k,t}^{(n)} \sim q_k(\mathbf{x}_{k,t} \mid \mathbf{x}_{k,t-1}^{(n)}, \mathbf{z}_t), \; n = 1 \ldots N \tag{13}$$

As mentioned previously, we propose to use the prior distribution as the proposal function for k-th target, i.e.

$$q_k(\mathbf{x}_{k,t} \mid \mathbf{x}_{k,t-1}^{(n)}, \mathbf{z}_t) = p_k(\mathbf{x}_{k,t}^{(n)} \mid \mathbf{x}_{k,t-1}^{(n)}) \tag{14}$$

Using these Monte Carlo samples, the predictive likelihood in (12) can be approximated as

$$p_k(\mathbf{z}_{j,t}^i \mid \mathbf{z}_{1:t-1}) \approx \sum_{n=1}^N \alpha_{k,t}^{(n)} p_T^i(\mathbf{z}_{j,t}^i \mid \mathbf{x}_{k,t}^{(n)}) \tag{15}$$

where the predictive weights are given by

$$\alpha_{k,t}^{(n)} \propto w_{k,t-1}^{(n)} \frac{p_k(\mathbf{x}_{k,t}^{(n)} \mid \mathbf{x}_{k,t-1}^{(n)})}{q_k(\mathbf{x}_{k,t}^{(n)} \mid \mathbf{x}_{k,t-1}^{(n)}, \mathbf{z}_t)}, \quad \sum_{n=1}^N \alpha_{k,t}^{(n)} = 1 \tag{16}$$

Considering the proposal distribution given in (14), the predictive weights will be the same as the importance weights, i.e.

$$\alpha_{k,t}^{(n)} = w_{k,t}^{(n)}, \; n = 1, \ldots, N \tag{17}$$

The approximation to the predictive likelihood can now straightforwardly be substituted into (11) to obtain the approximation for the joint association posterior probabilities, from which approximations for the marginal target to measurement association posterior probabilities can be computed according to (10). These approximations can be used in (9) to approximate the target likelihood. Finally, setting the new importance weights to

$$w_{k,t}^{(n)} \propto w_{k,t-1}^{(n)} \frac{p_k(\mathbf{z}_t \mid \mathbf{x}_{k,t}^{(n)}) p_k(\mathbf{x}_{k,t}^{(n)} \mid \mathbf{x}_{k,t-1}^{(n)})}{q_k(\mathbf{x}_{k,t}^{(n)} \mid \mathbf{x}_{k,t-1}^{(n)}, \mathbf{z}_t)}, \quad \sum_{n=1}^N w_{k,t}^{(n)} = 1 \tag{18}$$

leads to the sample set $\{w_{k,t}^{(n)}, \mathbf{x}_{k,t}^{(n)}\}_{n=1}^N$ being approximately distributed according to the marginal filtering distribution at the current time step $p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1:t})$.

Considering the prior distribution as the proposal distribution, the eq. (18) simplifies as follows

$$w_{k,t}^{(n)} \propto w_{k,t-1}^{(n)} p_k(\mathbf{z}_t \mid \mathbf{x}_{k,t}^{(n)}), \quad \sum_{n=1}^N w_{k,t}^{(n)} = 1 \tag{19}$$

IV. REGULARIZED MC-JPDAF

The resampling stage is introduced to reduce the degeneracy problem, which is prevalent in particle filters. However, the resampling, in turn causes the problem of loss of diversity among the particles, called sample impoverishment. This arises due to the fact that in the resampling stage, samples are drawn from a discrete distribution rather than a continuous one. If this problem is not addressed properly, it may lead to "particle collapse," which is a severe case of sample impoverishment, where all N particles occupy the same point in the state space, giving a poor representation of the posterior density [1]. The Regularized Particle Filter (RPF) is proposed in [5] to address this problem. The RPF resamples from a continuous approximation of the posterior density $p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1:t})$.

$$p_k(\mathbf{x}_{k,t} \mid \mathbf{z}_{1:t}) \approx \sum_{i=1}^N w_{k,t}^{(n)} \mathbf{K}_h(\mathbf{x}_{k,t} - \mathbf{x}_{k,t}^{(n)}) \tag{20}$$

where $\mathbf{K}_h(\mathbf{x}) = \frac{1}{h^{n_x}}\mathbf{K}(\frac{\mathbf{x}}{h})$, is the rescaled kernel density, $h > 0$ is the kernel band-width, n_x is the dimension of the state vector $\mathbf{x}$. In the special case of all the samples having the same weight, the optimal choice of the kernel density is the Epanechnikov kernel,

$$\mathbf{K}_{opt} = \begin{cases} \frac{n_x + 2}{2c_{n_x}}(1 - \|\mathbf{x}\|^2) & \text{if } \|\mathbf{x}\| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

where c_{n_x} is the volume of the unit hypersphere in $\Re^{n_x}$. Furthermore when the underlying density is Gaussian with a unit covariance matrix, the optimal choice for the band-width is [5],

$$h_{opt} = AN^{1/(n_x+4)} \tag{22}$$

$$\mathrm{A}=[8\mathrm{c}_{c_{n_x}}^{-1}(\mathrm{n}_x+4)(2\sqrt{\pi})^{n_x}]^{1/(n_x+4)} \tag{23}$$

Although the results of (21), (22) and (23) are optimal only in the special case of equally weighted particles and underlying Gaussian density, these results can still be used in the general case to obtain a suboptimal filter.

V. SIMULATION RESULTS

In this section, the simulation results for a two target tracking system in the presence of clutter and false alarms, using both the MC-JPDAF and RMC-JPDAF methods, is presented. In the both algorithms, the proposal distribution is assumed to be the prior density function. In what follows, all location and distance measures are in meters, all angle measures in radians, all time measures in seconds and all velocity measures in meter per second. As depicted in Fig. 1, two observers are place in locations $(-45,-45)$ and $(45,45)$ in the xy plane, with $\sigma_R = 5$, $\sigma_\theta = 0.05$ and $R_{\max} = 150$ being the independent Gaussian noise variances for range and bearing measurements and the maximum range of the observers, respectively.

We model the target dynamics with the near constant velocity model in [7] with $\sigma_x = \sigma_y = 0.05$ being the process noise variances along x and y axes. The discretization time step for the system is set to $T = 1$. The initial states of targets are $(-50,1,50,-1.5)$ and $(-50,1,0,-0.5)$. Target detection probability for each observer is assumed to be $P_D = 0.5$ and the clutter measurements are assumed to have Poisson distribution with rate $\lambda_c = 0.8$ for both sensors. In order to further reduce the computational cost, a kind of gating procedure is conducted in order to prune away the infeasible hypotheses in each time step. A suitable validation region is obtained by setting $\varepsilon = 40$. Parameter ε is defined in [4]. The simulations are performed for 100 time steps. Fig. 1 shows the true and the estimated target trajectories for MC-JPDAF and RMC-JPDAF methods, where the number of particles N is set to 100. It is obvious that the performance and tracking ability of the proposed RMC-JPDAF method is considerably high due to the regularization stage used, while the computational complexity and so the execution time are significantly reduced.

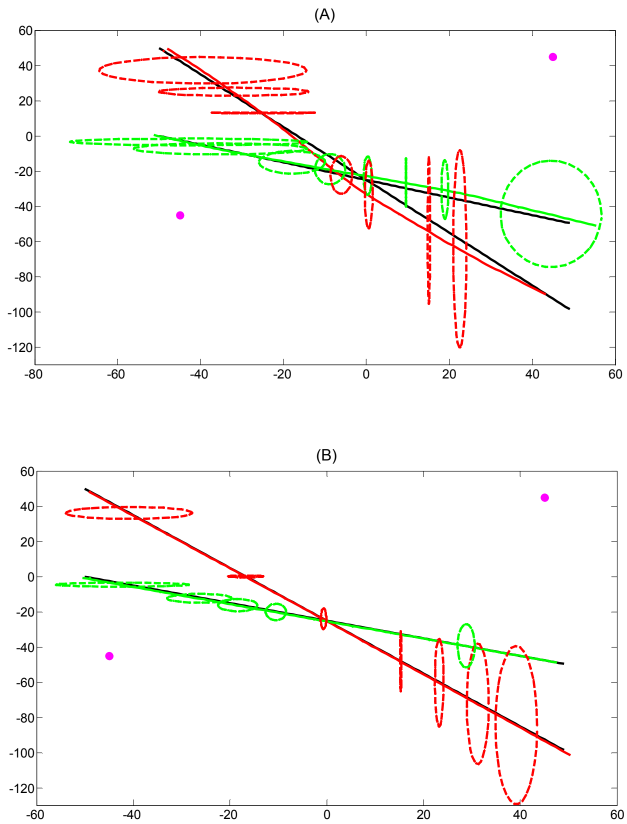

Fig. 1. Trajectories for two targets. $N = 100$, two magenta points are the observers, solid black lines are the true target locations, solid red and green lines are the estimated trajectories of targets and ellipses are the estimated 2σ regions of the estimate covariances. (A): MC-JPDAF, (B): RMC-JPDAF

However, due to the fact that the number of particles drawn in each time step from the proposal distribution is finite, the computed 2σ regions are somewhat large. This can be compensated by increasing the number of particles. In Fig. 2, the number of particles is set to 500, and considerable reduction in 2σ regions is obtained. In this case, the higher performance of the proposed method versus MC-JPDAF is obvious.

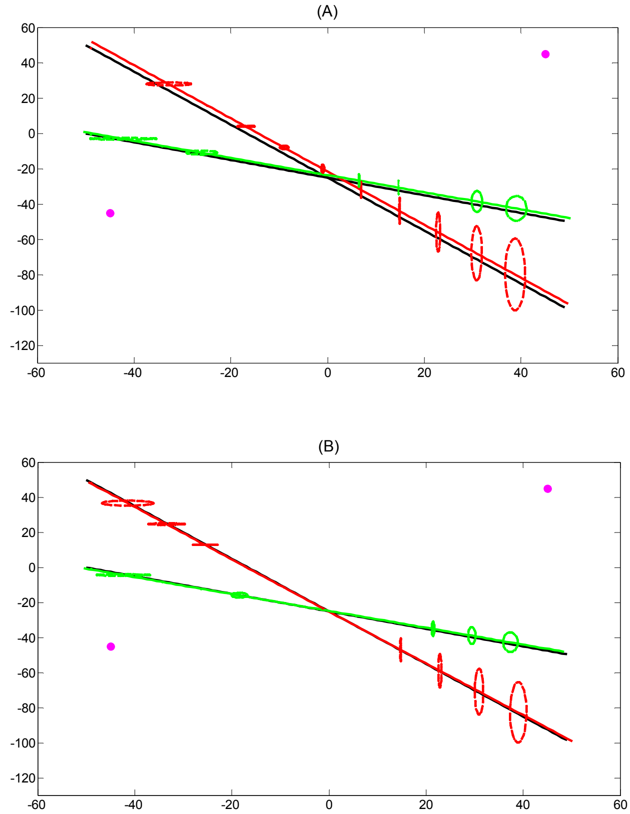

Fig. 2. Trajectories for two targets. $N = 500$, (A): MC-JPDAF, (B): RMC-JPDAF.

In Fig. 3, the well known Root Mean Square Error (RMSE), versus different values for particles number is plotted for MC-JPDAF and RMC-JPDAF. As it is obvious, the RMSE and so the estimation variance decrease as the number of particles increases for both methods. Furthermore, the RMSE for the proposed method has smaller values than MC-JPDAF method, for different particle number values. Finally the average execution time for a single time step (cycle) of the simulation program versus the number of particles, is depicted in Fig .4. As it was expected, the average execution time increases nearly linearly as the number of particles increases.

VI. CONCLUSION REMARKS

In this paper, a kind of robust MTT system is implemented using Regularized and simplified MC-JPDAF tracker. The regularization step is performed to overcome the sample impoverishment problem due to the resampling step in MC-JPDAF, and the prior distribution in used as the proposal distribution in order to significantly reduce the computational cost of the tracking system. Finally, the simulation results for the proposed method prove the simplicity robustness of the proposed method versus MC-JPDAF.

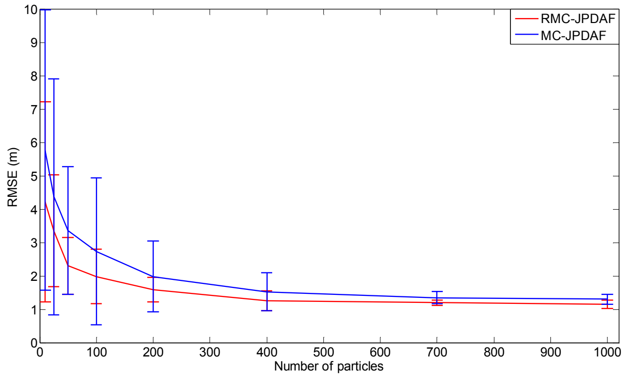

Fig. 3. The RMSE in meters for both methods.

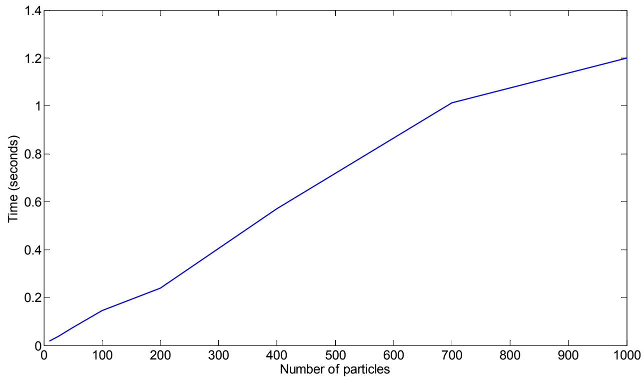

Fig. 4. Average execution time for a single time step.

ACKNOWLEDGEMENT

This work was partially supported by Iran Telecommunication Research Center.

REFERENCES

[1] S. Arulampalan, S. Maskell, N. Gordon, and T. Clapp, “A Tutorial on Particle Filters for On-line Non-linear/Non-Gaussian Bayesian Tracking,” IEEE Transactions on Signal Processing, vol. 50, no. 2, Feb. 2002, pp. 174-188.

[2] Z. Chen, “Bayesian Filtering: From Kalman Filters to Particle Filters and Beyond,” Technical report, McMaster University, 2003.

[3] D. Schuls, W. Burgard, D. Fox, “People Tracking with Mobile Robots Using Sample-Based Joint Probabilistic Data Association Filters,” International Journal Of Robotics Research, vol. 22, no. 2, 2003.

[4] J. Vermaak, S. J. Godsill, and P. Perez, “Monte Carlo Filtering for Multi-Target Tracking and Data Association,” IEEE Transactions on Aerospace and Electronic Systems, vol. 41, no. 1, Jan 2005, pp. 309-322.

[5] C. Musso, N. Oudjane, and F. LeGland, “Improved Regularized Particle Filters,” in Sequential Monte Carlo Methods in Practice, A. Doucet, J. F. G. Freitas, and N. J. Gordon, Eds. New York, Springer-Verlag, 2001.

[6] N. J. Gordon, D. J. Salmond, and D. Fisher, “Bayesian Target Tracking after Group Pattern Distortion,” in Signal and Data Processing of Small Targets, SPIE 3163, O. E. Drummond, Ed., 1997, pp. 238-248.

[7] Y. Bar-Shalom and T. E. Fortmann, *Tracking and Data Association*. Academic Press, 1998.