Proceedings of the 28th Conference
on Decision and Control
Tampa, Florida • December 1989

BAYESIAN ADAPTIVE FILTERS FOR MULTIPLE MANEUVERING TARGET TRACKING WITH MEASUREMENTS OF UNCERTAIN ORIGIN

B. TOMASINI
Compagnie des Signaux
et d'Equipements Electroniques
230, rue Marcelin Berthelot
83087 TOULON Cédex - FRANCE

M. GAUVRIT
Centre d'Etudes et de Recherche de TOULOUSE
CERT/ONERA
2, Avenue Edouard Belin, BP 4025
31500 TOULOUSE Cédex - FRANCE

B. SIFFREDI
Direction des Constructions et Armes Navales
C.A.P.CA.
Les Oursinières
83800 TOULON Naval - FRANCE

KEYWORDS: PDA Filter : Multiple Maneuvering Target Tracking: Cluttered Environment : Bayesian Filtering: Adaptive Control

Abstract

The Probabilistic Data Association method (P.D.A.) has been successfully used for tracking targets in the presence of source uncertainty and measurement inaccuracy. In this field and using this technic, the problem of maneuvering target tracking will be considered. The aim of this paper concerns the description of three adaptive methods, intresecally different, to track single target and/or multiple targets. These methods called Adaptive Control Probabilistic Data Association Filter (ACPDAF). Adaptive Joint Probabilistic Data Association Filter (AJPDAF) and Adaptive Control Joint Probabilistic Data Association Filter (ACJPDAF) estimate the state of each target in a cluttered environment for abrupt or slowly changes of the target parameters.

1 - Introduction

The problem is to estimate the state of a target in the presence of source uncertainty and measurement inaccuracy. The source uncertainty is due to the origin of the measurements ; any report which is used in this tracking procedure might not have originated from the target of interest. The foremost difficulty is to associate measurements with the tracks when there are reports missing and the proliferation of the false reports generated by clutter.

The paper is structured as follows : in the first part, a summary of the Adaptive Control Probabilistic Data Association (ACPDA) method is given; in the second part the Bayesian adaptive procedure called Adaptive Joint Probabilistic Data Association (AJPDA) is developed; In the third part, the Adaptive Control Joint Probabilistic Data Association (ACJPDA) developpements are given. In conclusion, a figure synthesis of the main techniques is given.

2 - Description of the ACPDA Filter

In this algorithm, the data association problem is handled by computing the joint posterior probabilities of measurement associations with a single target in clutter. These probabilities are then used to update a set of Gaussian distributions representing (suboptimal) Bayesian estimates of the target locations.

2.1 - Problem Formulation

The dynamics of the target is modelized by the following state vector equation :

$$\underline{x}_{k+1} = \phi_k \underline{x}_k + \Gamma_k \left[\underline{w}_k + \underline{u}_k^j \right] \quad [2.1]$$

for k = 0, 1, ...

$$\hat{\underline{x}}_{k+1/k} = \sum_{j=1}^{N}\sum_{i=1}^{m_k} \int \underline{x}_{k+1}\, P(\underline{x}_{k+1}, \underline{\theta}_j, \xi_{k,i}/Z_k, m_k)\, d\underline{x}_{k+1} \quad [2.2]$$

Assuming that $\underline{\theta}_j$ is discrete or suitable quantified to a finite number of grid points, with known or assumed *a priori* probability for each $\underline{\theta}_j$, using Baye's rule we can obtain the estimated state:

$$\hat{\underline{x}}_{k+1/k} = \phi_k \hat{\underline{x}}_{k/k} + \Gamma_k \hat{\underline{u}}_k \quad [2.3]$$

where

$$\hat{\underline{u}}_k = \sum_{j=1}^{N} \left[\underline{u}_k^j\right] P(\underline{\theta}_j/Z_k, m_k) \quad [2.4]$$

is the estimated control to be identify in real time and:

$$\sum_{j=1}^{N} P(\underline{\theta}_j/Z_k, m_k) = 1 \quad [2.5]$$

Considering the conditional density $P(\underline{\theta}_j/Z_k, m_k)$ and using BAYES 's rule, we obtain:

$$P(\underline{\theta}_j/Z_k, m_k) = \frac{P(\underline{\theta}_j/Z_{k-1})\, P(Z_k^V, m_k/Z_{k-1}, \underline{\theta}_j)}{P(Z_k^V, m_k/Z_{k-1})} \quad [2.6]$$

The fundamental recursive equation of the conditional probabilities can be written as the following expression:

$$P(\underline{\theta}_j / Z_k, m_k) = \frac{C_k^j \sum_{n=1}^{N} \theta_{jn} P(\underline{\theta}_n / Z_{k-1}, m_{k-1})}{\sum_{j=1}^{N} C_k^j \sum_{n=1}^{N} \theta_{jn} P(\underline{\theta}_n / Z_{k-1}, m_{k-1})} \quad [2.7]$$

with

$$C_k^j = b_k + \sum_{i=1}^{m_k} f_k\left(z_{k,i}, \underline{\theta}_j\right) \quad [2.8]$$

$$f_k(z_{k,i}, \underline{\theta}_j) = N\left(z_{k,i}, \hat{Z}^j_{k/k-1}, \tilde{Z}_k\right) \quad [2.9]$$

$$b_k = \frac{m_k\left(\alpha_1 + \alpha_2 - \alpha_1\alpha_2\right)}{\left(1 - \alpha_1\right)\left(1 - \alpha_2\right) V_k} \quad [2.10]$$

$$\theta_{jn} = \begin{cases} p & \text{si } j = n \\ \frac{1-p}{N-1} & \text{si } j \neq n \end{cases} \quad [2.11]$$

The relations [2.3] to [2.11] describe the probabilistic part of the Adaptive Control Probabilistic Data Association Filter. The filter equations will be done on paragraph 3 with t equals to 1.

3 - Description of the ACJPDA Filter

The method previously described, is suitable to a monotarget problem to estimate the state vector of a single maneuvering target having abrupt change parameters. Now, a generalization of this method for the multiple target tracking problem is given.

3.1 - An overview of the algorithm

The dynamics of the t^{th} target is modelized by the following state vector equation :

$$\underline{x}_{k+1,t} = \Phi_k \underline{x}_{k,t} + \Gamma_k \left[\underline{w}_k + \underline{u}^j_{k,t} \right] \quad \text{for } j=1 \text{ to } N \qquad [3.1]$$

for $t = 1, \ldots, n$ and $k = 0, 1, \ldots$

for each t^{th} target. Assuming that $\underline{\theta}_{j,t}$ is discrete or suitable quantified to a finite number of grid points, with known or assumed *a priori* probability for each $\underline{\theta}_{j,t}$ of each t^{th} target, using Baye's rule we can obtain the estimated state:

$$\hat{\underline{x}}_{k+1/k,t} = \sum_{j=1}^{N} \sum_{i=1}^{m_k^t} \sum_{\xi \in X_{it}} \int \underline{x}_{k+1}\, P(\underline{x}_{k+1}, \underline{\theta}_{j,t}, \xi / Z_k, \Omega)\, d\underline{x}_{k+1} \qquad [3.2]$$

Using Baye's rule, and doing developments of [3.3], we obtain:

$$\hat{\underline{x}}_{k+1/k,t} = \Phi_k \hat{\underline{x}}_{k/k,t} + \Gamma_k \hat{\underline{u}}_{k,t} \qquad [3.3]$$

where

$$\hat{\underline{u}}_{k,t} = \sum_{j=1}^{N} \left[\underline{u}^j_{k,t} \right] P(\underline{\theta}_{j,t} / Z_k, \Omega) \qquad [3.4]$$

is the estimated control to be identify in real time.
Considering the joint conditional density and using Baye's rule we obtain :

$$P(Z_k^V, \Omega / Z_k, \underline{\theta}_{j,t}) = \sum_{i=0}^{m_k^t} \sum_{\xi \in X_{it}} P(Z_k^V / Z_{k-1}, \underline{\theta}_{j,t}, \xi, \Omega) \qquad [3.5]$$

$$P(\Omega / Z_{k-1}, \underline{\theta}_{j,t}, Z_k^V, \xi) P(\xi / Z_{k-1}, \underline{\theta}_{j,t})$$

The three terms composing the relation [3.9] are exactly the same terms which have been used to calculate the probability $P(\xi / Z_k, \Omega)$ in the expression [3.8] conditioning to the j^{th} control.
The fundamental recursive equation of the conditional probabilities can be written as the following expression:

$$P(\underline{\theta}_{j,t} / Z_k, \Omega_k) = \frac{C^j_{k,t} \sum_{l=1}^{N} \underline{\theta}_{jl,t} P(\underline{\theta}_{l,t} / Z_{k-1}, \Omega_{k-1})}{\sum_{j=1}^{N} C^j_{k,t} \sum_{l=1}^{N} \underline{\theta}_{jl,t} P(\underline{\theta}_{l,t} / Z_{k-1}, \Omega_{k-1})} \qquad [3.6]$$

with

$$C^j_{k,t} = \sum_{i=0}^{m_k^t} \beta_{it} \qquad [3.7]$$

$$\beta_{k,i} = \sum_{\xi \in X_{it}} \frac{\lambda^{\Phi(\xi)}}{C'\, q_\delta} \prod_{a=1}^{m_v} (\Phi_a!)^{-1} \prod_{i=1}^{m_k^t} f_t(\underline{z}_{k,i}, \underline{\theta}_{j,t}) \qquad [3.8]$$

$$\prod_{t:\delta_t(\xi)=1} (1 - \alpha_{2_t}) \prod_{t:\delta_t(\xi)=0} \left[1 - (1 - \alpha_{2_t})(1 - \alpha_{1_t}) \right]$$

and

$$f_t(\underline{z}_{k,i}, \underline{\theta}_{j,t}) = N\left(\underline{z}_{k,i}, \hat{\underline{Z}}^j_{k/k-1,t}, \tilde{Z}_{k,t} \right) \qquad [3.9]$$

$$\theta_{jl,t} = \begin{cases} p & \text{si } j = l \\ \dfrac{1-p}{N-1} & \text{si } j \neq l \end{cases} \qquad [3.10]$$

Using [3.6], we can compute on the a posteriori conditional probability. Referring to [3.1], the estimated state is obtained by the following Adaptive Control J.P.D.A. Filter equations:

$$\hat{\underline{x}}_{k/k,t} = \hat{\underline{x}}_{k/k-1,t} + K_{k,t}\, \underline{v}^u_{k,t} \qquad [3.11]$$

$$\underline{v}^u_{k,t} = \sum_{i=1}^{m_k^t} \beta_{k,it}\, \underline{v}^u_{k,it} \qquad [3.12]$$

$$\underline{v}^u_{k,t} = \underline{z}_{k,i} - \hat{\underline{Z}}^u_{k/k-1,t} \qquad [3.13]$$

$$\hat{\underline{Z}}^u_{k/k-1,t} = h\left(\hat{\underline{x}}_{k/k-1,t} + \Gamma_k \hat{\underline{u}}_{k,t} \right) \qquad [3.14]$$

The complementary equations of this method are identical to the J.P.D.A. Filter expressions. The relations [3.6] and [3.11] to [3.14] associated with [3.3] describe the Adaptive Control Joint Probabilistic Data Association Filter called A.C.J.P.D.A.F..

4 - Description of the AJPDA Filter

The method previously described, is suitable to a multitarget problem to estimate the state vector of maneuvering targets having abrupt change parameters. GAUVRIT (1984) has proposed an adaptive estimation of the state vector in a single target having slowly variable parameters. Now, a generalization of this method for the multiple target tracking problem is given.

4.1 - An overview of the algorithm

The statistical model is specified up to a set of unknown parameters denoted by the vector $\underline{\Psi}_t = \{Q_t, R_t\}$ for each t^{th} target. We have:

$$\underline{\Psi}_{jl,t} = \left\{ Q_{j,t}, R_{l,t}, \text{ for } j = 1 \text{ to } p,\ l = 1 \text{ to } q \text{ and } t = 1 \text{ to } c \right\}$$

The state vector of the t^{th} target is given by the following expression:

$$\hat{\underline{x}}_{k/k,t} = \sum_{j=1}^{p} \sum_{l=1}^{q} \sum_{i=1}^{m_k^t} \sum_{\xi \in X_{it}} \int \underline{x}_k\, P(\underline{x}_k, \underline{\Psi}_{jl,t}, \xi / Z_k, \Omega)\, d\underline{x}_k \qquad [4.1]$$

Using BAYES's rule, this last equation becomes:

$$\hat{\underline{x}}_{k/k,t} = \sum_{j=1}^{p} \sum_{l=1}^{q} \hat{\underline{x}}^{jl}_{k/k,t}\, P(\underline{\Psi}_{jl,t} / Z_k, \Omega) \qquad [4.2]$$

with

$$\sum_{j=1}^{p} \sum_{l=1}^{q} P(\underline{\Psi}_{jl,t} / Z_k, \Omega) = 1 \qquad [4.3]$$

$$P(\underline{\Psi}_{jl,t} / Z_k, \Omega_k) = \frac{A^{jl}_{k,t} P(\underline{\Psi}_{jl,t} / Z_{k-1}, \Omega_{k-1})}{\sum_{j=1}^{p} \sum_{l=1}^{q} A^{jl}_{k,t} P(\underline{\Psi}_{jl,t} / Z_{k-1}, \Omega_{k-1})} \qquad [4.4]$$

$$A^{jl}_{k,t} = \sum_{i=0}^{m_k^t} \beta^{jl}_{it} \qquad [4.5]$$

$$\hat{\underline{x}}^{jl}_{k/k,t} = \hat{\underline{x}}^{jl}_{k/k-1,t} + K^{jl}_{k,t}\underline{v}^{jl}_{k,t} \qquad [4.6]$$

$$\underline{v}^{jl}_{k,t} = \sum_{i=1}^{m_k^t} \beta^{jl}_{k,it}\,\underline{v}^{jl}_{k,it} \qquad [4.7]$$

$$\underline{v}^{jl}_{k,t} = \underline{z}_{k,i} - \hat{\underline{Z}}^{jl}_{k/k-1,t} \qquad [4.8]$$

$$\hat{\underline{Z}}^{jl}_{k/k-1,t} = h\left(\hat{\underline{x}}^{jl}_{k/k-1,t}, k-1\right) \qquad [4.9]$$

$$K^{jl}_{k,t} = P^{jl}_{k/k-1,t}\, H^{jl\,T}_{k,t}\, \tilde{Z}^{jl\,-1}_{k,t} \qquad [4.10]$$

$$\tilde{Z}^{jl}_{k,t} = H_{n,k,t}\, P^{jl}_{k/k-1,t}\, H^{jl\,T}_{k,t} + R^{l}_{k,t} \qquad [4.11]$$

$$P^{jl}_{k/k,t} = P^{jl\,0}_{k/k,t} + K^{jl}_{k,t}\left[\sum_{i=1}^{m_k^t} \beta^{jl}_{k,it}\,\underline{v}^{jl}_{k,it}\,\underline{v}^{jl\,T}_{k,it} - \underline{v}^{jl}_{k,t}\,\underline{v}^{jl\,T}_{k,t}\right] K^{jl\,T}_{k,t} \qquad [4.12]$$

where

$\hat{\underline{x}}^{jl}_{k/k,t}$ is the estimated state vector of the t^{th} target given by the jl^{th} J.P.D.A. Filter.

$K^{jl}_{k,t}$ is the associated weighting matrix.

$\underline{v}^{jl}_{k,t}$ is the innovation vector.

$$P^{jl}_{k/k,t} = \beta^{jl}_{k,0t}\, P^{jl}_{k/k-1,t} + \left(1 - \beta^{jl}_{k,0t}\right) P^{jl\,*}_{k/k,t} \qquad [4.13]$$

$$P^{jl\,*}_{k/k,t} = \left(I - K^{jl}_{k,t} H^{jl}_{k,t}\right) P^{jl}_{k/k-1,t} \qquad [4.14]$$

$$P^{jl}_{k/k-1,t} = \phi_{k,k-1,t}\, P^{jl}_{k/k,t}\, \phi^{T}_{k,k-1,t} + Q^{j}_{k,t} \qquad [4.15]$$

The relations [4.2] to [4.15] describe the Adaptive Joint Probabilistic Data Association Filter called A.J.P.D.A.F..

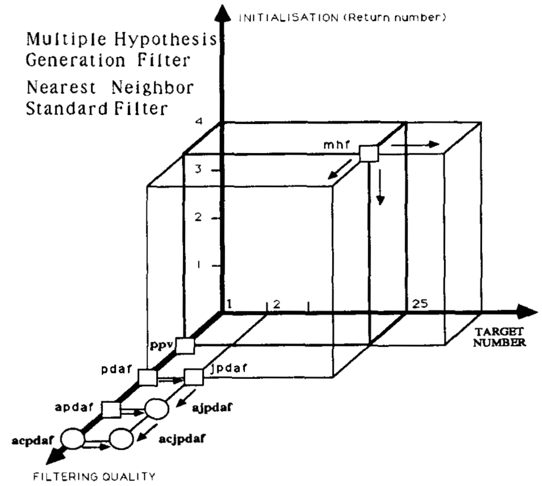
5 - Conclusion

This paper develops three Bayesian Adaptive procedures based on the Probabilistic Data Association technic introduced by Y. BAR-SHALOM for treating the multiple maneuvering target tracking problem.

These methods identify on line, first and/or second order errors of the model of each target. Simulation results[8], [9] and [10] show that the quality of the tracking has been improved. Concerning the multiple maneuvering target problem, it can be seen that the Adaptive techniques avoid missed or false choices for both target tracks. For the implementation of these algorithms, parallel processing techniques can be used to solve this estimation problem. A classification of the different methods is given by the following figure.

mhf: Multiple Hypothesis Generation Filter

ppv: Nearest Neighbor Standard Filter



ACKNOWLEDGEMENTS

The authors are particulary grateful to P. RAMACCIOTTI, Director, and M. FERRIE of Centre d'Analyse, de Programmation et de CAlculs (C.A.P.CA.), for their support of this work.

This study has been sponsored by :

- Direction des Constructions et Armes Navales (DCAN)-TOULON
- Ministère de la Défense - FRANCE.

REFERENCES

[1] BAR-SHALOM Y. (September 1974). Extension of the Probabilistic Data Association Filter in MultiTarget Tracking. *Proceeding of the Fifth Symposium on Non-Linear Estimation*, pp16-21. SAN DIEGO.

[2] BOZZO C. (1982) *Le Filtrage et ses Applications*. Edition Lavoisier "Techniques & Documentation". PARIS.

[3] GAUVRIT M. (1984). *Bayesian Adaptive Filter for Tracking with Measurements of Uncertain Origin*. Automatica. Vol. 20 no 2, pp 217-224.

[4] GAUVRIT M. (1985). *Amélioration de la robustesse du filtre de Kalman Etendu dans le cas d'une poursuite 3D*. Rapport DERA/CAPCA no 1/7446.

[5] MOOSE R.L., VALANDINGHAM H.F. and Mc CABE D.H. *Improved Modeling Techniques for Adaptive Target Tracking* Electrical Engineering Department V.P.I. 6 SU BLAKSBURG VIRGINIA 24061.

[6] MOOSE R.L. (June 1975). *An Adaptive State Estimation Solution to the Manoeuvring Target Problem*. IEEE Transactions on Automatic Control.

[7] TOMASINI B., GAUVRIT M. and SIFFREDI B. (August 1987). *Bayesian Adaptive Control Filter for Maneuvering Target Tracking with Measurements of Uncertain Origin*. Proceeding of the First International Symposium on Signal Processing and its Applications. pp 473-479. BRISBANE, AUSTRALIA.

[8] TOMASINI B., GAUVRIT M. and SIFFREDI B. (March 1989). *Bayesian Adaptive Filter for Multiple Maneuvering Target Tracking with Measurements of Uncertain Origin*. S.P.I.E. Proceedings Vol 1096. ORLANDO, U.S.A..

[9] TOMASINI B. (March 1989). *Poursuite Multicible Adaptative dans un Environnement très Perturbé. Application à un Problème de Veille*. Thèse de Doctorat en Sciences. Université d'AIX-MARSEILLE III, FRANCE.