Technical Note: Summary of Baro Pressure for GPS-INS Vertical Channel Stability

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I. Introduction

Note: the text that I use most for a sensors reference is by Kayton and Fried [1]. Here you will find the equations for Pressure altitude and airspeed. The text that I use most for state estimation with Pressure sensors is by Groves [2].

II. Pressure Altitude Calculation

Pressure is a measure of force per unit area or

$$p = \frac{F}{A}. \tag{1}$$

The static pressure at a particular altitude is determined by the force exerted by a column of air at that altitude:

$$p = \frac{m_{\text{column}} g}{A}, \tag{2}$$

where m_{column} is the mass of the column of air, $g = 9.81\ m/s^2$ is the acceleration due to gravity, and A is the area upon which the column is exerting pressure. The density of the is the mass per unit volume. Since the volume is given by the area times the height we get

$$p = \rho h g, \tag{3}$$

where ρ is the density of air, and h is the altitude [1].

We are interested in the altitude or heights above a ground station. Suppose that at the ground station, or start point, the pressure sensor is calibrated to read

$$p_{\text{ground}} = \rho h_{\text{ground}} g. \tag{4}$$

The output of the static pressure sensor is given by [1]

$$\begin{aligned} y_{\text{static pres}} &= \rho g h + \eta_{\text{static pres}}(t) && (5)\\ &= \rho g (h - h_{\text{ground}}) + \rho g h_{\text{ground}} + \eta_{\text{static pres}}(t), && (6) \end{aligned}$$

where h_{ground} is the altitude of the ground station. We will assume that the autopilot is calibrated to determine h_{ground}. Therefore, we can model the output of the static pressure sensor in the simulator as

$$y_{\text{static pres}} = \rho g (h - h_{\text{ground}}) + \eta_{\text{static pres}}(t). \tag{7}$$

where $\eta_{\text{static pres}}(t)$ is the sensor bias.

The height above the ground station can be computed as

$$\begin{aligned} h - h_{\text{ground}} &= \frac{p}{\rho g} - \frac{p_{\text{ground}}}{\rho g} \\ &= \frac{p - p_{\text{ground}}}{\rho g}. \end{aligned} \tag{8}$$

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III. Density of air

The density of air ρ is dependent on the air temperature T and air pressure p_s.

The air density is given by

$$\rho = \frac{p_s}{RT}, \tag{9}$$

where $R = 287.05[J/kgK]$ is the specific gas constant. Notice that in this formula, temperature is expressed in units of Kelvin. The conversion from Fahrenheit to Kelvin is given by

$$T[K] = \frac{5}{9}(T[F] - 32) + 273.15. \tag{10}$$

Pressure is expressed in N/m^2. Typical weather data reports pressure in inches of Mercury, *in.Hg*. The conversion factor is

$$p_s = 3385 p_m, \tag{11}$$

where p_m is the pressure in inches of Mercury. Therefore, to get accurate measurements of altitude (and airspeed), we need to know air temperature and air pressure, which can be obtained from a hand-held weather station, or from the Internet.

IV. Barometric Altitude Calculation

Barometric altitude is calculated assuming standard day conditions using a third order curve fit. Barometric altitude is limited to lie between $-1000m$ and $18,288m$. Barometric altitude in meters, h_p, is given by:

$$\begin{aligned} h_p = &-2.1558e^{-11} p_s^3 + 5.0154e^{-6} p_s^2 \\ &- 4.8540e^{-1} p_s + 1.9866e^{+4}, \end{aligned} \tag{12}$$

where static pressure, p_s, in units of Pascals (Pa), is the raw measurement from the barometric pressure sensor.

This calculation assumes that no installation calibrations or corrections are applied to the raw input values of pressure. Pressure measurement errors such as bias and scale factor are also not corrected.

Inexpensive barometric pressure sensor tend to have spikes in the data, so a measurement rejection algorithm is recommended, with a threshold of 5 Pa.

V. Kollsman Correction

The altitude, calculated in eqn. (12), is the height above Mean Sea Level (MSL) based on ambient static pressure, as referenced to the U.S. Standard Atmosphere model. To correct for non-standard day conditions, the pressure correction, known as the Kollsman correction, can be used to correct the sensed barometric pressure with the following expression:

$$h_{corr} = h_{std} + \frac{\partial h}{\partial p_{sl}}(p_k - p_{std}), \tag{13}$$

where h_{std} is the standard day height, p_{std} is the standard day pressure, p_{sl} is the pressure at sea-level, p_k is the Kollsman pressure correction, and h_{corr} is the corrected barometric pressure altitude.

That is, the corrected altitude is the altitude determined by the standard-atmosphere model based on ambient pressure, plus an offset determined by the difference between the actual sea-level air pressure and standard sea-level pressure, multiplied by the gradient of altitude with pressure at sea level. In standard day conditions, p_k is $101,325\ Pa$ (29.92 $in.Hg$) and this offset will be zero.

In EKF applications, the pressure correction will be estimated as a barometric pressure bias, thus the Kollsman correction may be unnecessary.

VI. Baro Altitude in an EKF

Consider, for example, a GPS-INS with an EKF. Let $\boldsymbol{x} \in \mathbb{R}^{n_s}$ denote the 6DOF rover state vector contained in the set of real numbers, $\mathbb{R}$. For example, the state vector

$$\boldsymbol{x} = [\mathbf{p}^\intercal, \mathbf{v}^\intercal, \mathbf{q}^\intercal, \mathbf{b}_a^\intercal, \mathbf{b}_g^\intercal, b_{cb}, b_{cd}, b_{b_b}, b_{b_{sf}}]^\intercal \in \mathbb{R}^{n_s}, \tag{14}$$

is comprised of 3D position, velocity, attitude (e.g., quaternion), accelerometer bias and gyroscope bias, GPS receiver clock bias and clock drift, baro bias and scale factor, where $n_s = 20$.

The baro altitude is useful when fewer than four GPS satellites are available, and can be used to stabilize the unstable vertical channel in an INS. Depending on the filter design, the baro altitude measurement can be used continuously, or only when GPS altitude is unreliable.

For EKF measurement update time, t_k, the baro altitude residual is calculated as

$$\begin{aligned} \mathbf{z}_{k_{baro}} &= h_p \\ &\quad - (h - \delta P_d - (v_d + \delta v_d) * dt_b - b_{b_b} - b_{b_{sf}} h_p) \end{aligned} \tag{15}$$

where h_p is the calculated baro altitude from eqn. (12), h is the INS calculated altitude, δP_d is the EKF estimated altitude error, v_d is the INS vertical velocity, δv_d is the EKF estimated vertical velocity error, dt_b is the time difference between the Baro measurement time and the INS state propagation time, and $\mathbf{z}_{k_{baro}}$ is the measurement input to the EKF.

Observation matrix $\mathbf{H}$ for baro measurement is,

$$\mathbf{H}_{baro} = [0, 0, -1, 0, 0, -dt_b, 0, \cdots, 0, -1, -h_m] \tag{16}$$

The baro measurement noise $\sigma_b^2 = 1.0\ m$.

References

[1] M. Kayton and W. R. Fried, *Avionics Navigation Systems*. Wiley-Interscience, 2nd Ed., 1998.

[2] P. D. Groves, *Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems*. Artech House, 2013.