Technical Note: Magnetometer and Baro Pressure Equations

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I. Introduction

The text that I use most for a sensors reference is by Kayton and Fried [1]. Here you will find the equations for Pressure sensors and Magnetometers.

The text that I use most for state estimation with Pressure sensors and Magnetometers is by Groves [2].

This paper assumes that the application is on a small quad-rotor UAV or airplane equipped with an electric motor.

II. Pressure Sensors

A. Altitude Sensor

Pressure is a measure of force per unit area or

$$P = \frac{F}{A}. \quad (1)$$

We will be concerned with one type of pressure, static pressure, though two are common in UAV applications: 1) static pressure due to altitude, and 2) dynamic pressure due to airspeed.

The static pressure at a particular altitude is determined by the force exerted by a column of air at that altitude:

$$P = \frac{m_{\text{column}} g}{A}, \quad (2)$$

where m_{column} is the mass of the column of air, g is the gravitational constant, and A is the area upon which the column is exerting pressure. The density of the is the mass per unit volume. Since the volume is given by the area times the height we get

$$P = \rho h g, \quad (3)$$

where ρ is the density of air, and h is the altitude [1].

We are interested in the altitude or heights above a ground station. Suppose that at the ground station, or start point, the pressure sensor is calibrated to read

$$P_{\text{ground}} = \rho h_{\text{ground}} g. \quad (4)$$

The output of the static pressure sensor is given by [1]

$$\begin{aligned} y_{\text{static pres}} &= \rho g h + \eta_{\text{static pres}}(t) & (5)\\ &= \rho g (h - h_{\text{ground}}) + \rho g h_{\text{ground}} + \eta_{\text{static pres}}(t), & (6) \end{aligned}$$

where h_{ground} is the altitude of the ground station. We will assume that the autopilot is calibrated to determine h_{ground}. Therefore, we can model the output of the static pressure sensor in the simulator as

$$y_{\text{static pres}} = \rho g (h - h_{\text{ground}}) + \eta_{\text{static pres}}(t). \quad (7)$$

where $\eta_{\text{static pres}}(t)$ is the sensor bias.

The height above the ground station can be computed as

$$\begin{aligned} h - h_{\text{ground}} &= \frac{P}{\rho g} - \frac{P_{\text{ground}}}{\rho g} \\ &= \frac{P - P_{\text{ground}}}{\rho g}. \quad (8) \end{aligned}$$

B. Air Speed Sensor

When the UAV is in motion, the atmosphere exerts dynamic pressure on the UAV parallel to the direction of flow. The dynamic pressure is given by [1]

$$P_I = \frac{1}{2} \rho V^2, \quad (9)$$

where P_I is the indicated pressure, and V is the airspeed of the UAV. Bernoulli's theorem states that [1]

$$P_s = P_I + P_O, \quad (10)$$

where P_s is the total pressure, and P_O is the static pressure.

Therefore, the output of the differential pressure sensor is

$$\begin{aligned} y_{\text{diff pres}} &= P_s - P_O + \eta_{\text{diff pres}} \\ &= \frac{1}{2} \rho V_a^2. \quad (11) \end{aligned}$$

Therefore we have

$$V = \sqrt{2(P_s - P_0)/\rho}. \quad (12)$$

C. Density of air

The density of air ρ is dependent on the air temperature T and air pressure p_s.

The air density is given by

$$\rho = \frac{p_s}{RT}, \quad (13)$$

where $R = 287.05[J/kgK]$ is the specific gas constant. Notice that in this formula, temperature is expressed in units of Kelvin. The conversion from Fahrenheit to Kelvin is given by

$$T[K] = \frac{5}{9}(T[F] - 32) + 273.15. \quad (14)$$

Pressure is expressed in N/m^2. Typical weather data reports pressure in inches of Mercury. The conversion factor is

$$p_s = 3385H, \quad (15)$$

where H is the pressure in inches of Mercury. Therefore, to get accurate measurements of altitude and airspeed, we need to know air temperature and air pressure, which can be obtained from a hand-held weather station, or from the Internet.

III. Magnetometers

The earth's magnetic field is three dimensional. There is a component that points to magnetic north, but there is also a vertical component. A three axis magnetometer measures the strength of the magnetic field along each of its axes. Let $\mathbf{m}_0$ be the magnetic field vector that is fixed in the inertial or vehicle frame. Given the orientation of the airframe, the strength of the magnetic field along the body axes of the UAV are given by

$$\begin{aligned}\mathbf{m}(t) &= R_v^b \mathbf{m}_0 \\ &= \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix} \mathbf{m}_0.\end{aligned} \tag{16}$$

In the presence of ferrous metals or magnets like those in an electric motor, the magnetometer will sense a change in the local magnetic field.

Furthermore, when the electric motor of the UAV is activated, an additional magnetic field is activated due to the rotation of the motor coils. Fortunately, the magnetic field is sinusoidal, with a frequency approximately equal to the frequency of the rotating rotor ω_{motor}, and can be removed with a low-pass filter. We will model the effect due to the motor as

$$\mathbf{m}_{\text{motor}} = \sin(\omega_{\text{motor}} t)\mathbf{m}_{\text{motor}} \tag{17}$$

which is measured in the body frame.

Including bias β_{mag} and white noise η_{mag} terms, the output of the magnetometers is given by

$$\begin{aligned}\mathbf{y}_{\text{mag}} = &\begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix} \mathbf{m}_0 \\ &+ \sin(\omega_{\text{motor}} t)\mathbf{m}_{\text{motor}} + \begin{pmatrix} \beta_{mag,x} \\ \beta_{mag,y} \\ \beta_{mag,z} \end{pmatrix} + \begin{pmatrix} \eta_{mag,x} \\ \eta_{mag,y} \\ \eta_{mag,z} \end{pmatrix}.\end{aligned} \tag{18}$$

The magnetometers are sampled by the microcontroller and therefore run at about 20 Hz.

References

[1] M. Kayton and W. R. Fried, *Avionics Navigation Systems*. Wiley-Interscience, 2nd Ed., 1998.

[2] P. D. Groves, *Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems*. Artech House, 2013.