Kalman Filter Mechanization for INS Airstart

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ABSTRACT

A strapdown mechanization and associated Kalman filter are developed to provide both ground align and airstart capabilities for inertial navigation systems using Doppler velocity and position fixes, while not requiring an initial heading estimate. Position update during coarse mode is possible by defining sine and cosine of wander angle as filter states, and modeling the position error in geographic frame while integrating velocity in the wander frame. Position measurements may come from sources such as GPS, fly-over updates, and sighting/ranging systems. INS/GPS differential position due to GPS antenna moment arm can aid heading convergence during hover turns in helicopter applications. Azimuth error state in the fine mode of the filter is defined as wander angle error to provide continuous estimation of navigational states, as well as inertial/aiding sensor errors, across the coarse-to-fine mode transition. Though motivated by a tactical helicopter application, the design can be applied to other vehicles. Advantages to conventional systems in addition to the airstart capability include robustness and versatility in handling many different operational conditions.

INTRODUCTION

Inertial Navigation Systems (INS) commonly employ the principle of gyrocompassing or earth rate sensing for self-initialization of attitude or alignment, given only initial position. Traditionally, a ground alignment mode is employed during which the vehicle must be stationary. Prior to the development of Kalman state estimation, classical control loops were used to physically align gimballed platforms.[1,2,3] The utility of the Kalman filter for gyrocompassing was quickly recognized and applied after its inception[4], and today practically all strapdown systems incorporate a Kalman filter for ground and in-flight alignment, calibration and integration of navigation sub-systems in aided operation.

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Robust ground alignment methods using Kalman filters have been designed and implemented. The unknown initial azimuth is typically handled by modeling the heading error with two states. One design represents the heading error ψ_{err} with two error states $\sin \psi_{err}$ and $(1 - \cos \psi_{err})$. Another method models the level earth rate components Ω_x, and Ω_y as Kalman states[5,6]. The latter has been extended to permit airstart gyrocompassing using Doppler velocity as a Kalman update[5]. However, a problem remains with both methods; if the vehicle is in motion during coarse alignment (ground align interruption or airstart), position cannot be updated because the wander angle is not sufficiently known to transform wander-frame velocity to an earth-fixed frame for integration. In addition, the Kalman filter cannot accept position fixes as measurement updates in coarse align mode.

There have been many papers published on the use of Kalman state estimation for ground, inflight, or transfer alignment[4,5,7,8,9]. However, most papers started from the premise that the azimuth angle has been predetermined to within a few degrees of accuracy using an unspecified coarse alignment mode, permitting the use of small angle approximations in the error propagation equations of a linearized extended Kalman filter. A "Fast" reaction mode is offered in most systems by initializing to the heading stored prior to the last system shut-down, or by using a flux valve for an initial heading estimate. Airstart capability is sometimes provided, but also requires magnetic heading, or GPS ground track, or another already aligned inertial system for an initial heading estimate. On some helicopters, the only magnetic heading sensor is a backup "whiskey compass." GPS ground track cannot be reliably used in a vehicle that can fly backward. In low speed forward flight, the helicopter may have large slide slip even without a cross-wind.

In contrast to previous works, this paper describes a Kalman filter formulation that, in addition to ground gyrocompassing, provides fast reaction and airstart capability without an initial heading estimate. This is accomplished by use of a Doppler velocity sensor and a position source such as GPS, manual fly-over update or target sighting systems. Filter transition from coarse to fine align mode is accomplished without disrupting the estimation of the inertial instrument or aiding sensor errors, by defining the azimuth error state as wander angle error, and using a simple, but effective, manipulation of the filter covariance matrix.

GPS, if available, is an ideal reference for inflight alignment. Ring Laser Gyro (RLG) inertial systems with practically no warm-up requirements should be capable of near zero reaction time when aided by GPS. With a reliable airstart capability, such systems may not require battery back-up to safeguard against power interruption, except for some applications such as stability augmentation of fly-by-wire systems. The flux valve may also be eliminated.

BACKGROUND

Wander Frame Mechanization Equations

The familiar mechanization equations for a local level INS using a wander azimuth navigation frame are as follows. Attitude with respect to the wander frame is determined from a direction cosine C_b^w which is updated with

$$\dot{C}_b^w = C_b^w [\omega_{ib}^b \times] - [\omega_{iw}^w \times] C_b^w \tag{1}$$

$$\omega_{iw}^w = \omega_{ie}^w + \omega_{ew}^w \tag{2}$$

where

- ω_{ib}^b = spatial rate of vehicle, in body frame, as measured by strapdown gyros
- ω_{iw}^w = spatial rate of wander frame, expressed in wander frame
- ω_{ie}^w = spatial rate of the earth, expressed in wander frame
- ω_{ew}^w = transport rate, or rotational rate of wander frame with respect to the earth, expressed in wander frame

and the cross-product form $[wx]$ of a vector w is the skew-symmetric matrix:

$$[\omega\times] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

The velocity in wander frame V^w is updated with

$$\dot{V}^w = C_b^w f^b - (2\omega_{ie}^w + \omega_{ew}^w) \times V^w + g^w \tag{3}$$

where

- f^b = body-axis specific force sensed by strapdown accelerometers
- g^w = gravity in wander frame

Geographic position is given by the direction cosine C^e_w relating the wander frame to the earth-fixed frame:

$$\dot{C}_w^e = C_w^e [\omega_{ew}^w \times] \tag{4}$$

For a mechanization using a wander frame that is oriented at an azimuth angle α from the North-East-Down frame, the level components p_x, p_y of the transport rate w^w_{ew} are computed from the level components V_x , V_y of wander velocity V^w with

$$\omega_{ew}^w = \{ \rho_x, \rho_y, 0 \} \tag{5}$$

$$\rho_x = V_y \left(\frac{c^2\alpha}{R_n} + \frac{s^2\alpha}{R_m} \right) + V_x c\alpha\, s\alpha \left(\frac{1}{R_n} - \frac{1}{R_m} \right) \tag{6}$$

$$\rho_y = -V_x \left(\frac{c^2\alpha}{R_m} + \frac{s^2\alpha}{R_n} \right) + V_y c\alpha\, s\alpha \left(\frac{1}{R_m} - \frac{1}{R_n} \right) \tag{7}$$

where the meridional radius of curvature R_m and the principal normal radius R_n are computed from the earth ellipticity e^2, major axis α and geodetic, latitude L

$$R_n = a/(1 - e^2 \sin^2 L)^{1/2}$$
$$R_m = a(1 - e^2)/(1 - e^2 \sin^2 L)^{3/2}$$

INS MODES OF OPERATION

The strapdown INS or AHRS (Attitude and Heading Reference System) typically goes through three modes after being powered up on ground: leveling, coarse alignment, then fine alignment. The leveling mode roughly establishes level attitude by sensing the direction of gravity with respect to the vehicle body axes. Next, the coarse alignment mode determines azimuth to within a few degrees by sensing the rotation of the gravity vector with respect to the inertial space. The azimuth is further refined in fine alignment mode, which is usually continued into flight using navigation aids. Some airstart capability may also be provided after an inflight power loss, the simplest being an "Attitude" mode, whose operation is not unlike that of the old vertical and directional gyros (VG/DG)[10,11]. Interrupted ground alignment is an auxiliary mode activated manually, or by the use of a motion detection scheme, to suspend the "zero velocity" update[11] while the aircraft is taxiing.

Part of an INS design is the definition of manual mode selection and logic for automatic mode transitions. This varies with the application, and depends on availability of aiding sensors and the level of integration of the avionic suite. This paper suggests techniques to ease the transitioning between the three basic modes.

LEVELING MODE

The leveling mode, though simple in function, is important because it establishes initial attitude and velocity for the Kalman filter, which assumes small errors. The design for airstart capability must begin with an examination of this mode. It should be noted that if it weren't for the airstart scenario, the leveling mode could be bypassed altogether, and its job delegated to the coarse align mode. Vehicle attitude as large as 30 degrees on ground can be accommodated by the coarse align Kalman filter, aided by some filter tuning.

Leveling mode implementations vary in complexity depending on the application. The simplest form assumes the aircraft is stationary, so that specific force components sensed by the accelerometers are

$$f^b = g\,\{\ \sin\theta,\quad -\cos\theta\sin\phi,\quad -\cos\theta\cos\phi\ \}$$

where Θ and ϕ are the Euler pitch and roll angles. Thus, level attitude can theoretically be determined from an instantaneous sample of the accelerometer outputs. This scheme would obviously be foiled by high vibration levels. The accelerometer outputs can be integrated over a short interval to attenuate the high frequency content[6], but this still does not work inflight if the vehicle is maneuvering. Inflight leveling can be accommodated by incorporating both the angular rates from the gyros, and the accelerometer outputs, aided by a velocity measurement, in an erection loop.

Inflight leveling using airspeed has been used in many AHRS's to avoid erection to the false vertical when the aircraft speed changes. Though working well in fix-wing applications, this method is unreliable for helicopters because of the maneuverability at low speed, and both longitudinal and lateral speed are needed. The three-dimensional Doppler velocimeter is a perfect source of speed reference for leveling. Tactical helicopters may also have an omni-airspeed system, used primarily for wind estimation for fire control solutions. Such two-dimensional body-axis speed measurements are also suitable for inflight leveling. Earth-referenced velocity such as from GPS cannot be used because the aircraft heading is unknown.

Inflight leveling using a velocity reference can be accomplished with second-order leveling loops as shown in Fig. 1. The attitude direction cosine (or its quaternion counterpart) is initialized to the identity matrix, implying that $\hat{\phi} = \hat{\Theta} = 0$. This direction cosine, though initially erroneous, is used to transform body acceleration to the local level frame for integration. The resulting inertial velocity is compared to the velocity reference transformed to the local level frame. The X and Y velocity error components are then used to correct the attitude direction cosine.

During leveling mode, a complementary filter (Fig. 2) can be used to blend inertial vertical acceleration, the vertical component from the velocity reference, and baro altitude. The complementary action of the filter generates good vertical velocity from the baro altitude and the inertial acceleration, even when the vertical speed input is absent, such as the case when Doppler is inoperative. Even for applications where the vertical channel is included in the Kalman filter, this baro filter should be implemented to serve as a preparatory state estimator to provide the Kalman filter with initial state values. Because the filter starts prior to the leveling loop convergence, relatively short time constants in the order of a few seconds should be used for fast settling. In contrast, normal INS baro loops may use time constants up to 30 seconds[12] for more effective smoothing of altitude.

When the two-dimensional omni-airspeed is used for leveling, it can be supplemented by the altitude rate from the vertical filter to obtain the reference level frame velocity. The

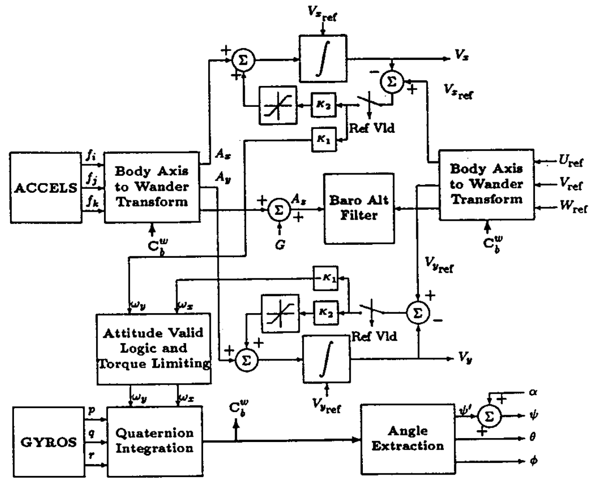

Fig. 1. Second Order Leveling

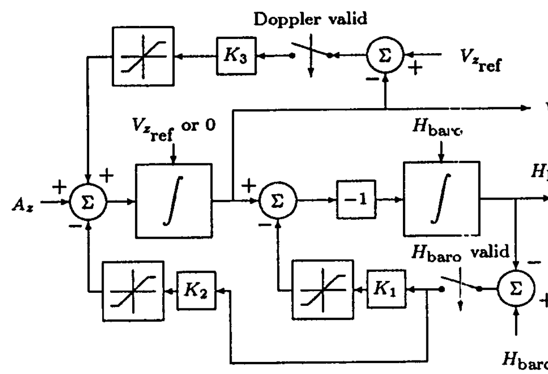

Fig. 2. Baro Filter

transformation still has singularities at $\pm 90°$ pitch and roll, where a body-axis airspeed measurement coincides with vertical velocity, requiring erection cutout at high altitudes.

When no velocity sources exist, a conventional first-order leveling mode is used (Fig. 3). Because Doppler sensors are typically unable to sense zero or very small Doppler shifts, this mode is normally used for ground start.. It operates quite similarly to the conventional VG erection loop by assuming that the vehicle level acceleration averages to zero. Different erection rate limits should be used for ground and air modes. The low-pass filters in the feedback path keep the airframe vibration from being aliased by the low sampling rate of the erection loop. Transitioning to the second-order mode is easily achieved upon availability of a speed reference.

After leveling mode completion, if either Doppler or GPS is available, the inflight alignment can commence. Without GPS, this initial position can be prompted from the pilot. If only airspeed is available, a degraded navigation mode can be implemented with a flux valve, or by manual entry of a whiskey compass reading. Knowledge of latitude also allows azimuth stabilization of the wander axis by torquing the Z axis of C_b^w to match the vertical component of the earth rotation.

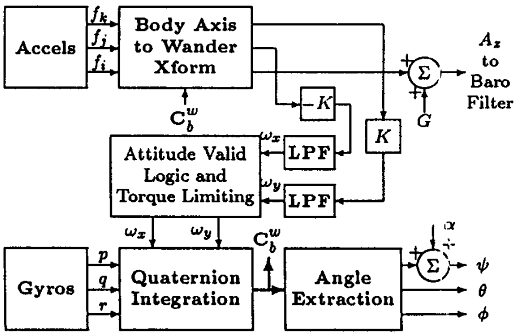

Fig. 3. First Order Leveling

COARSE ALIGN MODE

Motivation for Use of Sine and Cosine of Wander Angle as States

When leveling is complete, an arbitrary wander frame is established relative to the body frame by C_b^w. True north heading is obtained by using a Kalman filter to observe the wander angle α between this frame and the NED (North-East-Down) frame. The rationale for choosing sine and cosine of wander angle ($s\alpha$, $c\alpha$) as states is that the error equations, though non-linear in heading error, are linear in $s\alpha$ and $c\alpha$, permitting the design of an extended Kalman filter. Actually, products of error states still exist, and the case where these cannot be ignored will be discussed later. When the equation for the spatial rate of the wander frame is examined, i.e.,

$$\omega_{iw}^{w} = \Omega \{ cL\ c\alpha,\ -cL\ s\alpha,\ -sL \} + \{ \rho_x,\ \rho_y,\ 0 \}$$

it appears that the filter formulation is the same as the one using the two level earth rates[5]

$$\begin{aligned} \Omega_x &= \Omega\, cL\ c\alpha \\ \Omega_y &= -\Omega\, cL\ s\alpha \end{aligned}$$

for ground alignment, where the transport rates p_x and p_y are zero. The two state choices differ only by the scale factor $\Omega\, cL$, where earth rotation rate, Ω and latitude L are known. Thus, the two filters behave similarly for ground alignment, and airstart gyrocompassing using Doppler. Note that, for inflight airstart, the wander frame transport rates p_x and p_y can be computed since V_x and V_y are known. The only difference from the fine align mode is that, since the azimuth is unknown, the radii of curvature along the wander level axes and the radius of torsion cannot be computed. The first can be approximated by an average of the meridional radius R_m and the principal normal radius R_n. The torsion can be assumed zero. Errors caused by this approximation are insignificant in this mode.

Recognition of $c\alpha$ and $s\alpha$ as states, however, allows position time integration and modeling of position error for Kalman updates. This is obvious when one considers that the rate of change of position is

$$\dot{L} = \frac{1}{R_m} (V_x\, c\alpha - V_y\, s\alpha) \tag{8}$$

$$\dot{\lambda} = \frac{1}{R_n \cos L} (V_x\, s\alpha + V_y\, c\alpha) \tag{9}$$

If wander frame velocity components V_x and V_y have been established by Doppler in an airstart, or by inertial acceleration integration from a known initial condition (such as ground alignment), the velocity errors will be relatively small. The position errors are then linear functions of $c\alpha$ and $s\alpha$. Note that this is true only if position error states are modeled in geographic frame. Systems using the wander azimuth mechanization traditionally model the position error in the wander frame, which requires that the azimuth be approximately known before the geographic position difference between aiding sensors and the inertial system can be expressed as near-linear combinations of Kalman states.

Ground alignment is made by applying "zero velocity" Kalman update. Interrupted ground alignment mode described earlier can continue to gyrocompass with Doppler, the latter typically becoming valid soon after the aircraft starts moving. This means the vehicle can be moved any time during the ground align phase without serious degradation of later navigation performance.

Coarse Align Mechanization

In coarse align mode, the attitude direction cosine (or its quaternion algorithm counterpart) is updated with Eqs. 1 and 2, where

$$\begin{aligned} \omega_{ie}^{w} &= \{ \hat{\Omega}_x,\ \hat{\Omega}_y,\ \Omega_z \} \\ &= \Omega \{ \cos L\, \widehat{c\alpha},\ -\cos L\, \widehat{s\alpha},\ -\sin L \} \\ \omega_{ew}^{w} &= \{ \rho_x,\ \rho_y,\ 0 \} \end{aligned}$$

where $\hat{c}\alpha$, $\hat{s}\alpha$ are the estimates of sine and cosine of wander angle. Normally, the initial values of $c\alpha$ and $s\alpha$ are zero. The transport rates p_x, p_y are computed from wander frame velocity using

$$\begin{aligned} \rho_x &= \frac{V_y}{(R_m + R_n)/2} \\ \rho_y &= -\frac{V_x}{(R_m + R_n)/2} \end{aligned}$$

The wander frame velocity is updated with Eq. 3. The position is updated with Eqs. 8 and 9, using the estimates $\hat{c}\alpha$, and $\hat{s}\alpha$:

$$\begin{aligned} \dot{L} &= \frac{1}{R_m} (V_x\, \widehat{c\alpha} - V_y\, \widehat{s\alpha}) \\ \dot{\lambda} &= \frac{1}{R_n \cos L} (V_x\, \widehat{s\alpha} + V_y\, \widehat{c\alpha}) \end{aligned}$$

Eq. 10 describes the error propagation for the filter model in coarse mode. Additional states must be added to this equation to model the inertial sensor errors. Accelerometer errors transformed to level frame with $C_b{}^w$ drive the velocity errors,

$$
\begin{array}{c}
\begin{array}{cccccccccc}
\delta N & \delta E & \delta H & \delta V_x & \delta V_y & \delta V_z & \phi_x & \phi_y & \delta c\alpha & \delta s\alpha
\end{array}
\\
\begin{bmatrix} \delta\dot{N} \\ \delta\dot{E} \\ \delta\dot{H} \\ \delta\dot{V}_x \\ \delta\dot{V}_y \\ \delta\dot{V}_z \\ \dot{\phi}_x \\ \dot{\phi}_y \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & \widehat{c\alpha} & -\widehat{s\alpha} & 0 & 0 & 0 & V_x & -V_y \\
0 & 0 & 0 & \widehat{s\alpha} & \widehat{c\alpha} & 0 & 0 & 0 & V_y & V_x \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\Omega_z & -(2\hat{\Omega}_y+\rho_y) & 0 & -f_z & 0 & 2\Omega\, cL\, V_z \\
0 & 0 & 0 & -2\Omega_z & 0 & 2\hat{\Omega}_x+\rho_x & f_z & 0 & 2\Omega\, cL\, V_z & 0 \\
0 & 0 & -2\frac{g}{R_e} & 2\hat{\Omega}_y+\rho_y & -(2\hat{\Omega}_x+\rho_x) & 0 & -f_y & f_x & -2\Omega\, cL\, V_y & -2\Omega\, cL\, V_x \\
\frac{\Omega_z \widehat{c\alpha}}{R_e} & 0 & 0 & 0 & \frac{1}{R_e} & 0 & 0 & \Omega_z & \Omega\, cL & 0 \\
-\frac{\Omega_z \widehat{s\alpha}}{R_e} & 0 & 0 & -\frac{1}{R_e} & 0 & 0 & -\Omega_z & 0 & 0 & -\Omega\, cL
\end{bmatrix}
\begin{bmatrix} \delta N \\ \delta E \\ \delta H \\ \delta V_x \\ \delta V_y \\ \delta V_z \\ \phi_x \\ \phi_y \\ \delta c\alpha \\ \delta s\alpha \end{bmatrix}
\end{array}
\qquad (10)
$$

while the level gyro drifts transformed from body frame drive the tilt states. Aiding sensor errors are similarly included, but they appear in the observation matrices of the filter. The filter measurement corrections for Doppler, GPS, and landmark sighting are derived in the following paragraphs as examples.

Doppler Aiding

Doppler-measured body-frame velocity is transformed to wander frame, then subtracted from the inertial velocity. The observation matrix for the velocity difference is obtained from

$$
\begin{aligned}
\Delta V^w &= V^w_{\text{INS}} - C^w_b V^b_{\text{DOPP}} \\
&= \delta V^w_{\text{INS}} - [\delta C^w_b]\, V^b - C^w_b \delta V^b_{\text{DOPP}} \\
&= \begin{bmatrix} \delta V_x \\ \delta V_y \\ \delta V_z \end{bmatrix} - \begin{bmatrix} 0 & -V_z \\ V_z & 0 \\ -V_y & V_x \end{bmatrix} \begin{bmatrix} \phi_x \\ \phi_y \end{bmatrix} \\
&\quad -C^w_b \delta V^b_{\text{DOPP}}
\end{aligned}
\qquad (11)
$$

The doppler error $\delta V_{dopp}{}^b$ may be expanded into biases, scale factor errors, and non-orthogonalities, depending on the application.

GPS Aiding—Antenna Moment Arm

GPS pseudo-range aiding is straightforward since the position error states are in NED frame. Benefits of this coarse align mechanization include the use of GPS antenna moment arm to the INS to observe the heading in a hover or taxiing turn. Compensation for this lever arm normally requires the use of heading, which is unknown in an airstart yet a helicopter is capable of rapid heading changes. By correlating the mismatch between the INS and GPS measurements with the vehicle yawing motion, the Kalman filter obtains a measurement of $s\alpha$ and $c\alpha$. The quality of the heading obtained depends on the length of the moment arm, and the GPS measurement noise. This effect is most noticeable for a hover turn, but it may never be deliberately exercised because much better heading convergence will be obtained by vehicle speed change, or heading change in flight. However, it enhances system robustness; filter divergence will occur if the moment arm effect is not accounted for, and a hover or ground taxi turn is executed before a heading estimate has been obtained. This problem is not severe with fix-wing applications, because rapid heading convergence occurs with speed change and turning in flight.

The coarse align filter accepts pseudo-range measurements from any number of GPS satellites to solve for azimuth. It should be noted that, early in the coarse mode, the INS velocity in geographic frame is not available, and the GPS receiver must operate in unaided mode. The derivation of the GPS pseudo-range update follows. Using the satellite ECEF (Earth-Centered-Earth-Fixed frame) position $[X_s, Y_s, Z_s]$ and the INS ECEF position $[X, Y, Z]$, the LOS (line-of-sight) unit vector U^e_{GPS} and the range $\hat{R}$ are computed:

$$
\hat{R} = \sqrt{(X_s - X)^2 + (Y_s - Y)^2 + (Z_s - Z)^2}
$$

$$
U^e_{\text{GPS}} = \frac{1}{\hat{R}} \{\, X_s - X,\ Y_s - Y,\ Z_s - Z \,\}
$$

The LOS unit vector U_{GPS} is transformed from ECEF to NED because the position error states are modeled in that frame:

$$
U^n_{\text{GPS}} = \{\, U_n,\ U_e,\ U_d \,\} = C^n_e U^e_{\text{GPS}}
$$

$$
C^n_e = \begin{bmatrix} -sL\, c\lambda & -sL\, s\lambda & cL \\ -s\lambda & c\lambda & 0 \\ -cL\, c\lambda & -cL\, s\lambda & -sL \end{bmatrix}
$$

The lever arm in NED is estimated using $c\alpha$, $s\alpha$, and $[L_i, L_j, L_k]$, the latter being the lever arm from the INS to the GPS antenna, expressed in body frame:

$$
\{\, L_n,\ L_e,\ L_d \,\} = \begin{bmatrix} \widehat{c\alpha} & -\widehat{s\alpha} & 0 \\ \widehat{s\alpha} & \widehat{c\alpha} & 0 \\ 0 & 0 & 1 \end{bmatrix} \{\, L_x,\ L_y,\ L_z \,\}
$$

$$
\{\, L_x,\ L_y,\ L_z \,\} = C^w_b \{\, L_i,\ L_j,\ L_k \,\}
$$

The filter measurement is computed as the difference between the measured pseudo-range R_{meas}, and the computed range $\hat{R}$, compensated for the antenna lever arm:

$$
\delta R = R_{\text{meas}} - \hat{R} + (L_n U_n + L_e U_e + L_d U_d)
$$

The observation vector $\mathbf{H}_{\delta R}$ is

$$
\begin{array}{c}
\begin{array}{ccccccccc}
\delta N & \delta E & \delta H & \cdots & \delta c\alpha & \delta s\alpha & \cdots & Clk & \cdots
\end{array}
\\
\mathbf{H}_{\delta R} = [\, -U_n \ \ -U_e \ \ U_d \ \cdots \ H_{\delta c\alpha} \ \ H_{\delta s\alpha} \ \cdots \ -1 \ \cdots \,]
\end{array}
$$

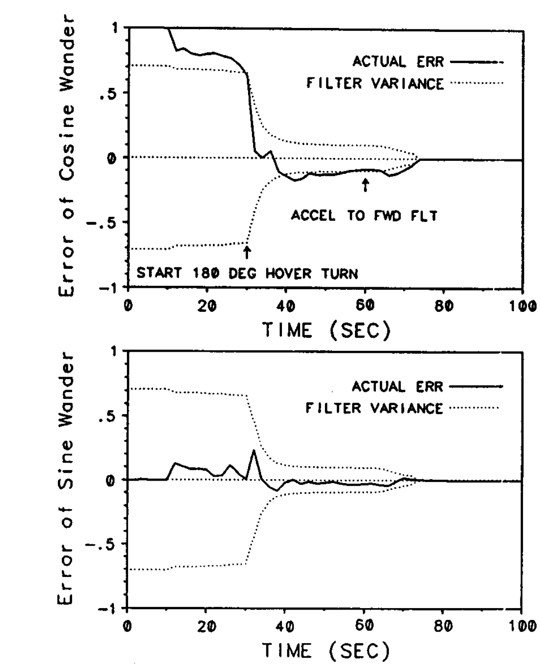

Fig. 4. GPS Antenna Moment Arm Effect in Hover Turn

where

$$
\begin{aligned}
H_{\delta c\alpha} &= L_x U_n + L_y U_e \\
H_{\delta s\alpha} &= L_y U_n - L_x U_e
\end{aligned}
$$

Fig. 4 shows convergence of $c\alpha$ and $s\alpha$ in a hover turn with an antenna lever arm of 40 ft, and GPS pseudo-range measurements from 4 satellites, available at 2-second intervals with a noise of 12 ft, 1 α . Faster convergence can be obtained with GPS range rate updates.

Target Sighting Data

The information from a sighting system consists of the LOS unit vector U^b_{LOS}, coordinated in aircraft body axis, and the slant range r to a known landmark at geographic latitude L_{LM}, and longitude λ_{LM}. This target sighting data normally requires the use of the vehicle attitude to compute the vehicle geographic position from the landmark geographic coordinate. The computed vehicle position error is correlated to the INS heading error, which is easily modeled. However, the transformation of the slant range to geographic frame cannot be made when the azimuth is totally unknown. The coarse filter formulation described here uses the correlation between north and east position errors and sine and cosine of the wander angle to solve for both heading and position simultaneously, when the sighting measurement is combined with other Kalman measurements. The update can then be accepted to aid alignment, rather than being disallowed in coarse mode.

The Kalman update is as follows, with compensation for the lever arm between the INS and the sight system, $L^b_{Sight/INS}$. The range vector from the sight system to the landmark is first expressed in wander frame:

$$
\{\; r_x, \;\; r_y, \;\; r_z \;\} = \mathbf{C}^w_b \left(r U^b_{\mathrm{LOS}} - \mathbf{L}^b_{\mathrm{Sight/INS}} \right)
$$

The estimated North/East components of the range are

$$
\begin{aligned}
r_n &= r_x \,\widehat{c\alpha} - r_y \,\widehat{s\alpha} \\
r_e &= r_y \,\widehat{c\alpha} + r_x \,\widehat{s\alpha}
\end{aligned}
$$

The Kalman filter measurement is the difference between the INS position and the landmark geographic position, L_{LM}, λ_{LM}, offset by the range estimate

$$
\begin{aligned}
\Delta N &= R_m (L - L_{\mathrm{LM}}) + r_n \\
\Delta E &= R_n \cos L (\lambda - \lambda_{\mathrm{LM}}) + r_e
\end{aligned}
$$

The observation matrix of the position error is

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \delta N & \delta E & & \delta c\alpha & \delta s\alpha & \end{array} \\
\mathbf{H} \;=\; & \left[\begin{array}{cccccc} 1 & 0 & \cdots & r_x & -r_y & \cdots \\ 0 & 1 & \cdots & r_y & r_x & \cdots \end{array} \right]
\end{array}
$$

In a simulated helicopter quick reaction scenario, the aircraft immediately lifted off after the INS and Doppler were powered up (no GPS). An initial position fix was entered (using stored landing pad coordinates). A short flight of 2.5 km was made, after which a known landmark 2.8 km away was sighted. With an uncertainty of 10 m (1 δ , each axis) for both position fixes, the heading was solved to 1 degree (1 δ), then further refined by inflight gyrocompassing. Alternatively, if a subsequent sighting were made to another landmark, the heading error would be reduced immediately to 0.28 deg. Without these position fixes, inflight gyrocompassing with just Doppler requires several minutes to reach the same heading uncertainty. These sighting fixes can be replaced by fly-over updates with the same results. Similar to the GPS moment arm treatment, this feature enhances the system versatility, although it may not be normally exercised.

Flux Valve—Heading Input

Earlier, a cited benefit of this mechanization is that the flux valve is no longer needed. However, some applications may require the flux valve for other reasons. If the magnetic heading is brought into the INS, the Doppler-only airstart can be accelerated somewhat by initializing $c\alpha$ and $s\alpha$. Better performance will be obtained if the variances and correlation of $\delta c\alpha$ and $\delta s\alpha$ are also computed to properly initialize the filter covariance matrix. For fix-wing aircraft, ground track from GPS can be treated the same way, except that the correlations between $\delta c\alpha$, $\delta s\alpha$, δV_x, and δV_y are now non-zero, and should be computed for covariance matrix initialization. It should be emphasized that the use of an initial heading estimate is helpful with GPS aiding only when Doppler is not available. Armed with the initial heading estimate, the filter can then solve for large wander frame velocity errors, such as those caused by inflight leveling with just airspeed.

Limitations of the Coarse Mode

The use of this coarse align formulation in an airstart is not without limitations. The first is that, because of the zero initial values of $\hat{c}\alpha$ *and* $\hat{s}\alpha$, the errors in V_x and V_y are effectively unaccounted for at the beginning of the mode. When velocity errors δV_x and δV_y are large relative to the true velocity V_x and V_y, the position error rate is dominated by the product of velocity errors and errors of $c\alpha$ and $s\alpha$. This condition can only arise if the Doppler measurement is not available. Filter noise model Q's can be used to compensate for this modeling error. It is more severe with helicopters in hover or low-speed flight, or for carrier aircraft. In the latter case, a non-linear estimation method has been used as a precursor to the Kalman filter[13].

The second problem is caused by modeling of $c\alpha$ and $s\alpha$ as unknown constants. In an airstart, the wander angle changes at the rate

$$\dot{\alpha} = \dot{\lambda} \sin L = \frac{V_e \tan L}{R_n} \qquad (12)$$

which depends on east velocity and increases rapidly near the poles. A technique has been developed to account for this wander rate, permitting airstart using Doppler at speeds up to Mach 1, and at latitudes up to 75 degrees.

Thirdly, the vertical gyro drift, coming mostly from the Z gyro bias, is not modeled. This error causes the wander angle to drift, hence $c\alpha$ and $s\alpha$ change with time. It is not serious because the coarse align phase (down to 1 degree of heading) lasts less than 10 minutes even with noisy Doppler aiding inflight, and much less for ground align. During this short time, gyro drifts of up to 1 deg/hr integrate to only a small portion of heading error, compared to the total uncertainty. A technique exists to compensate for this, if it is so desired. However, for modern RLG's with drifts of less than 0.01 deg/hr, this error is truly insignificant.

FINE ALIGN MECHANIZATION

In fine align mode, the two states $\delta c\alpha$ and $\delta s\alpha$ are replaced by a single azimuth error state $\delta\alpha$. The mechanization equation for inertial updating of attitude and velocity is the same as for coarse mode, except that the transport rates can now be computed more accurately with the correct earth radii of curvature and torsion. The position integration may be continued with Eqs. 8 and 9, as long as Eq. 12 is also integrated. Alternatively, the traditional method of updating Long/Lat/Wander via a direction cosine C_w^e (Eq. 4) can be invoked.

Fine align Kalman filters have been implemented using the error equations based on the "phi" or "psi" angle formulation[14]. Kalman filter mechanizations often reinitialize the rows and columns of the covariance matrix associated with the switched states, causing loss of the state correlation built-up in coarse mode. State convergence is slowed down after this transition. The fine align filter described here is designed as a continuation from the coarse mode by modifying the "phi" angle formulation. The error equation for fine mode is shown in Eq. 13, where yaw angle from C_b^w is considered "perfect," and azimuth error is modeled as error in α. Note that, in this equation, this state is defined as $-\delta\alpha$ to maintain the same sense as the traditional Z-tilt state. Similar to the coarse align error equation, the inertial instrument errors to be added depend on the application.

In the wander azimuth mechanization, the heading is obtained as the sum of the azimuth angle ψ' from C_b^w and the wander angle α from C_w^e. The "phi" angle formulation assigns the error states ϕ_z to C_b^w and Θ_z to C_w^e. The first is driven by gyro errors, while the latter's rate-of-change comes from transport rate errors and initial azimuth error. A Kalman filter has been implemented using both ϕ_z and Θ_z as states[9]. However there is only one "physical" heading, hence the use of two states is redundant. The assumption that the azimuth of C_b^w is "perfect" can be compensated by an error term of the form $w \times V$, since ϕ_z is driven by gyro drifts. Reference 9 alludes to a single azimuth state mechanization in a footnote, but, from the brief discussion that is provided, it appears to be a different technique.

Modeling azimuth error as wander angle error requires that the Kalman state correction be made by torquing C_w^e. This direction cosine is typically integrated using only a first-order algorithm[15]. The justification for this is that the transport rate magnitude is only in the order of degrees per hour, in contrast to the vehicle angular rate inputs to C_b^w that can be in the hundreds of degrees per second. However, correction impulses from the Kalman filter can cause serious numerical degradation by loss of orthonormality. Reference 9 noted this problem when

$$\begin{bmatrix} \delta\dot{N} \\ \delta\dot{E} \\ \delta\dot{H} \\ \delta\dot{V}_x \\ \delta\dot{V}_y \\ \delta\dot{V}_z \\ \dot{\phi}_x \\ \dot{\phi}_y \\ -\delta\dot{\alpha} \end{bmatrix} = \begin{array}{c} \begin{array}{ccccccccc} \delta N & \delta E & \delta H & \delta V_x & \delta V_y & \delta V_z & \phi_x & \phi_y & -\delta\alpha \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & c\alpha & -s\alpha & 0 & 0 & 0 & V_E \\ 0 & 0 & 0 & s\alpha & c\alpha & 0 & 0 & 0 & -V_N \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\Omega_z & -(\Omega_y+\omega_y) & 0 & -f_z & 0 \\ 0 & 0 & 0 & -2\Omega_z & 0 & \Omega_x+\omega_x & f_z & 0 & 0 \\ 0 & 0 & -2\frac{g}{R_e} & \Omega_y+\omega_y & -(\Omega_x+\omega_x) & 0 & -f_y & f_x & 0 \\ \frac{\Omega_z c\alpha}{R_e} & 0 & 0 & 0 & \frac{1}{R_e} & 0 & 0 & \Omega_z & -\Omega_y \\ -\frac{\Omega_z s\alpha}{R_e} & 0 & 0 & -\frac{1}{R_e} & 0 & 0 & -\Omega_z & 0 & \Omega_x \\ -(\frac{\Omega cL}{R_e}+\frac{\dot{\lambda}}{R_e cL}) & 0 & 0 & -\frac{tL\, s\alpha}{R_e} & -\frac{tL\, c\alpha}{R_e} & 0 & \omega_y & -\omega_x & \frac{V_n tL}{R_e} \end{bmatrix} \end{array} \begin{bmatrix} \delta N \\ \delta E \\ \delta H \\ \delta V_x \\ \delta V_y \\ \delta V_z \\ \phi_x \\ \phi_y \\ -\delta\alpha \end{bmatrix} \qquad (13)$$

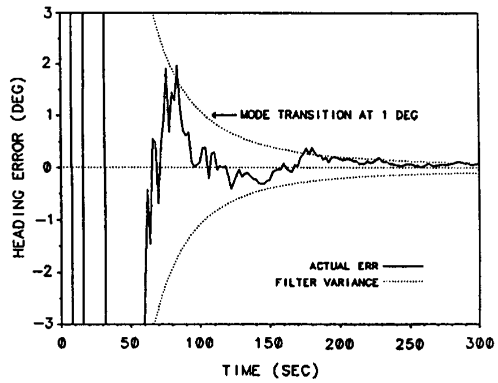

Fig. 5. Heading Convergence Across Mode Transition

it was caused by position correction. In the mechanization described here, azimuth correction impulses with GPS aiding can be of much higher angular values than latitude and longitude corrections. Simulation results have shown a second-order direction cosine integration algorithm to be satisfactory, although a third-order integration may give some safety margin. Alternatively, the azimuth "torquing" can be rate limited, and the remaining error propagated in the Kalman state vector.

Kalman update with aiding inputs from Doppler, GPS range and range rate, baro, and position fixes including target sighting are made by differencing INS states and the aiding data. Forming the residuals and their observation matrices follows standard perturbation techniques, therefore is not included here. The results are somewhat different from the conventional filter due to the different definition of heading error. For example, the Doppler update takes the same form as in coarse align; there is no Z-tilt state in Eq. 11, due to the definition of the azimuth error.

Coarse-to-Fine Mode Transition

Fine align mode is activated when the heading uncertainty falls below an arbitrary small value such as one degree. This heading variance is computed in coarse mode from the variances and correlation of $\delta s\alpha$ and $\delta c\alpha$

$$\sigma_\alpha^2 = (\widehat{c\alpha})^2\sigma_{s\alpha}^2 + (\widehat{s\alpha})^2\sigma_{c\alpha}^2 - \widehat{c\alpha}\,\widehat{s\alpha}\,\sigma_{c\alpha,s\alpha}$$

At mode transition, sin α and cos α are "squared up" using the identity "$\sin^2\alpha = \cos^2\alpha = 1$." The normalization takes the form of a Kalman "measurement" correction:

$$\begin{aligned} z &= 1 - (\widehat{c\alpha})^2 - (\widehat{s\alpha})^2 \\ &\approx -2\,\widehat{c\alpha}\,\delta c\alpha - 2\,\widehat{s\alpha}\,\delta s\alpha \end{aligned}$$

A small value of R should be used as "measurement noise" to allow for the linearization error. The correlation of α to a state x, including inertial and aiding sensor error states, is then computed from the correlation of $\delta s\alpha$ and $\delta c\alpha$ to the same state:

$$\begin{aligned} \sigma_{x,\delta\alpha} &= E[x\,\delta\alpha] \\ &= \widehat{c\alpha}\,\sigma_{x,\delta s\alpha} - \widehat{s\alpha}\,\sigma_{x,\iota c\alpha} \end{aligned}$$

Thus, the correlation between states is preserved across the mode transition, and the rate of state convergence is continuous. Fig. 5 illustrates this effect in a ground alignment.

CONCLUSION

Compared to traditional INS Kalman filter mechanizations, the design described here provides more capabilities without much additional complexity. Though motivated by a helicopter application, it can be applied to other vehicles. The major limitation of this approach involves cases where products of the defined error states are not negligible. However, this difficulty is inherent to any linearized Kalman filter, and can only be solved by a non-linear estimation method. A practical solution to this problem is being researched.

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