Performance Comparison of a Multiple-Detection Probabilistic Data Association Filter with PCRLB

B. K. Habtemariam, R. Tharmarasa, M. Mallick and T. Kirubarajan

Abstract— **Most target tracking algorithms assume that at most one measurement is generated by a target in a scan. However, there are tracking problems where this assumption is not valid. For example, multiple detections from a target can arise due to multipath propagation where different signals scattered from a target arrive at the sensor via different paths. With multiple target-originated measurements, most multitarget trackers will fail or become ineffective due to the violation of the one-to-one assumption. For example, the joint probabilistic data association (JPDA) filter is capable of using multiple measurements for a single target through weighted measurement-to-track association, but its fundamental assumption is still one-to-one. In order to rectify this shortcoming, we developed a new algorithm in our previous work, the multiple-detection probabilistic data association (MD-PDA) filter, which is capable of handling multiple detections from a target in a scan, in the presence of false alarm and probability of detection less than unity. In this paper, the performance of this MD-PDA filter is compared with the posterior Cramér-Rao lower bound (PCRLB), which is explicitly derived for the multiple-detection scenario. Furthermore, experimental results show multiple-detection pattern based probabilistic data association improves the state estimation accuracy and reduces the total number of false tracks.**

I. INTRODUCTION

Most target tracking algorithms assume that at most one measurement is generated by a target in a scan, with sensor probability of detection less than unity. Thus, given a set of measurements for a single target tracking problem, at most one of them is from the target and the rest are false alarms (FAs). This basic assumption leads to formulation of tracking algorithms as one-to-one measurement-to-track association. For example, in the probabilistic data association (PDA) filter [1], [10] and its multitarget version, the joint probabilistic data association (JPDA) filter [4], [14], [16], the weights are assigned to measurements based on a Bayesian assumption that only one of the measurements is from the target and the rest are FAs. In the multiple hypothesis tracker (MHT) [15], [11], [3] using the multiframe assignment (MFA) algorithm [5], measurement-to-track association is evaluated as a one-to-one combinatorial optimization in the best global hypothesis. In all these cases, the one-to-one assumption is fundamental for the correct measurement-to-track associations and accurate target state estimation.

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However, in certain tracking problems, a target can generate multiple measurements in a scan. For example, multiple detections from a target can arise due to multipath propagation, where signals scattered from a target can arrive at a sensor via different propagation paths. When multiple detections from a target fall within the association gate, the PDA and JIPDA filters tend to apportion the association probabilities, but still the fundamental assumption is that at most one of them is target-generated. When the measurements are not close to each other, as in the case of multipath detections, the PDA and JIPDA filters initialize multiple tracks for the same target.

Such ad-hoc handling of multiple detections has undesirable side effects. An algorithm that explicitly considers multiple detections from a target in a scan is appropriate such that all useful information in the received measurements about the target state is extracted with the correct assumption. Multiple detections increase the complexity of tracking algorithms due to uncertainty in the number of target-originated measurements, which can vary from time to time due to probability of detection less than unity, in addition to the measurement origin uncertainty. However, estimation accuracy can be improved and the number of false tracks can be reduced using the correct assumption with multiple detections.

A reviewer brought to our attention that a random finite set (RFS) based algorithm for handling multiple detections from a target in a scan has been developed in [18]. This paper uses the nearly constant velocity (NCV) model in 2D and position-measurement model with FAs and sensor probability of detection less than unity. Additional multiple target-originated measurements are generated by a Poisson RFS with linear Gaussian intensity. A multiple-detection probabilistic data association (MD-PDA) filter was developed in [6], where the algorithm was demonstrated using the 2D NCV motion and 2D radar range and azimuth measurements. In this paper, we have developed the algorithm for the posterior Cramér-Rao lower bound (PCRLB) for multiple detections and compare the performance of the MD-PDA filter with the PCRLB.

The remainder of the paper is organized as follows. Section II discusses the multiple-detection pattern, the basic input to MD-PDA filter. The MD-PDA filter is presented in Section III where theoretical development of the algorithm is discussed. The PCRLB for multiple detections is also presented in this section. Simulation results are presented in Section IV, which are based on target tracking in clutter where observations are made with a 2D radar sensor that

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returns multiple detections. The performance of the MD-PDA filter is also compared with the PCRLB in Section IV. Finally, in Section V, concluding remarks are given along with future directions of research.

II. Multiple-Detection Pattern

Whenever multiple detections from the same target fall within the association gate, a measurement or a set of measurements might be associated to a target. Data association uncertainty corresponding to a number of target-originated measurements as well as measurement source can be resolved by generating a multiple-detection pattern. The multiple-detection pattern will consider all possible events for many-to-one measurement set-to-track association.

We assume that the target follows a linear dynamic model [2]

$$x(k) = F(k)x(k-1) + w(k-1), \tag{1}$$

where $x(k)$ represents target state at time t_k, $F(k)$ is the state transition matrix for the time interval $[t_{k-1}, t_k]$, and $w(k-1)$ is a zero-mean white Gaussian process noise with covariance $Q(k-1)$. The nonlinear measurement model is given by [2]

$$z(k) = h(x(k)) + v(k), \tag{2}$$

where h is the nonlinear measurement function and $v(k)$ is a zero-mean white Gaussian measurement noise with covariance $R(k)$. The process noise and measurement noise are assumed to be independent.

After signal processing, thresholding and detection, gating has to be applied for measurement-to-track association. For $m(k)$ measurements inside the validation gate, φ out of $m(k)$ association events are evaluated where φ runs from one to the maximum number of target originated measurements. Then

$$N_a = \sum_{i=0}^{\varphi_{max}} \text{comb}(i, m(k)), \tag{3}$$

where N_a is the total number of measurement set-to-track association events and

$$\text{comb}(x, y) = \begin{cases} \frac{y!}{x!(y-x)!}, & 1 \leq x \leq y, \\ 1, & x = 0. \end{cases} \tag{4}$$

The total association event count, N_a, represents all possible events from zero target-originated measurement to all of the measurements being target-originated. An illustrative example for a multiple-detection pattern based on four measurements can be found in [6].

III. MD-PDA

The approach of the standard PDA filter is to calculate the association probabilities for each measurement that falls in the validation region around the predicted measurement at the current time to the target of interest [1]. If two of the measurements are target-originated, the algorithm apportions the total weight among the validated measurements with more weight to target-originated measurements, with the assumption that only one of them is target-originated. It is not an efficient approach especially when there are false alarms in the validation gate. This is because the weight assigned to false alarms become significant compared to the divided weight assigned to target-originated measurements.

The MD-PDA filter evaluates the association probabilities of the events generated by the multiple-detection pattern. These event probabilities are calculated based on probabilistic inference made on number of validated measurements, number of target-originated measurements and measurements locations.

The MD-PDA filter is formulated under the following assumptions:

- Among the validated measurements, a measurement or set of measurements can originate from a target.
- The target detections occur independently with known probabilities.
- Clutter is uniform/Possion distributed within the measurement validation gate.
- There is only one target of interest whose state evolves according to a dynamic equation driven by process noise as stated in (1). This assumption can be relaxed to multiple widely separated targets provided that the validation region of each target does not overlap with another. The MD-PDA approach can be extended to the case with overlapping gates.
- One- or two-point initialization technique can be used for track initialization.

The MD-PDA filter calculates the probability that each set of measurements, rather than a single measurement, is attributable to the target of interest. The sets of measurement candidates for association are generated from multiple-detection pattern discussed in Section II. This probabilistic (Bayesian) information based on the candidate set of measurements is used in a tracking filter that updates the target states.

Accordingly, based on the multiple-detection pattern presented in Section II, the measurement set-to-target association events are given as

$$\theta_{\varphi,n_\varphi}(k) = \begin{cases} \varphi \text{ out of } m(k) \text{ are target-originated,} \\ \qquad\qquad n_\varphi = 1, ..., c_{\varphi m}(k), \\ \text{none of the measurements is target-originated,} \\ \qquad\qquad n_\varphi = 0, \end{cases} \tag{5}$$

where $c_{\varphi m}(k)$ is φ combinations out of $m(k)$ measurements. The number of association events grow rapidly for $\varphi > 2$. However, for practical implementation the expected number of target originated measurement can be used as a priori to reduce the number of association events. For example for multisensor fusion with N-sensors the maximum number of target-originated measurements will correspond to the number of sensors.

Thus the conditional mean is given by

$$\begin{aligned}\hat{x}(k|k) &= E(x(k)|Z^k) \\ &= \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} E(x(k)|\theta_{\varphi,n_\varphi}(k), Z^k) p(\theta_{\varphi,n_\varphi}(k)|Z^k) \\ &= \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} \hat{x}_{\varphi,n_\varphi}(k|k)\beta_{\varphi,n_\varphi}(k). \end{aligned} \tag{6}$$

The estimate conditioned on n_φ^{th} combination of φ measurements being correct is

$$\hat{x}_{\varphi,n_\varphi}(k|k) = \hat{x}(k|k-1) + W_{\varphi,n_\varphi}(k)\nu_{\varphi,n_\varphi}(k),$$

where the corresponding innovation is

$$\nu_{\varphi,n_\varphi}(k) = \begin{bmatrix} (z(k) - \hat{z}(k|k-1))' \\ \vdots \\ (z(k) - \hat{z}(k|k-1))' \end{bmatrix}, \tag{7}$$

and the Kalman gain $W_{\varphi,n_\varphi}(k)$ is given as

$$W_{\varphi,n_\varphi}(k) = P(k|k-1)H_{\varphi,n_\varphi}(k)'S_{\varphi,n_\varphi}(k)^{-1}, \tag{8}$$

where $H_{\varphi,n_\varphi}(k)$ is the Jacobian corresponding to the nonlinear measurement function that correspond to measurement set $z_{\varphi,n_\varphi}(k)$ and

$$\begin{aligned} S_{\varphi,n_\varphi}(k) &= H_{\varphi,n}(k)P(k|k-1)H_{\varphi,n_\varphi}(k)' \\ &+ R_{\varphi,n_\varphi}(k), \end{aligned} \tag{9}$$

$$H_{\varphi,n_\varphi}(k) = [H_1(k), \cdots, H_\varphi(k)]', \tag{10}$$

$$R_{\varphi,n_\varphi}(k) = \begin{bmatrix} R_1(k) & 0 & \dots & 0 \\ 0 & R_2(k) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & R_\varphi(k) \end{bmatrix}. \tag{11}$$

Here $\beta_{\varphi,n_\varphi}(k) \propto p(\theta_{\varphi,n_\varphi}(k)|Z^k)$ is the conditional probability of the event where the probabilistic inference is made on the number of validated measurements ($m(k)$), number of target-originated measurements (φ) and measurements locations. Then,

$$\begin{aligned}\beta_{\varphi,n_\varphi}(k) = \frac{1}{c} p(Z^k|\theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}) \\ \times p(\theta_{\varphi,n_\varphi}(k)|m(k), \varphi). \end{aligned} \tag{12}$$

The first term in (12) refers to the joint density of the pdf of the correct measurement is given in (13) where P_G is the factor that accounts for restricting the normal density to the validation gate. Thus,

$$p(Z^k|\theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}) = \begin{cases} \frac{1}{P_G} \times V(k)^{-m(k)+1}\mathcal{N}(\nu_{\varphi,n_\varphi}(k); 0, S_{\varphi,n_\varphi}(k)), & n_\varphi = 1, ..., c_{\varphi m}(k), \\ V(k)^{-m(k)}, & n_\varphi = 0. \end{cases} \tag{13}$$

The second term in (12) is the probability of the association events conditioned only on $m(k)$ and φ. Here

$$p(\theta_{\varphi,n_\varphi}(k)|m(k), \varphi) = \begin{cases} \frac{1}{m(k)} \frac{P_{D\varphi}P_G\mu(m(k)-\varphi)}{\sum_{\varphi=1}^{m(k)} P_{D\varphi}P_G\mu(m(k)-\varphi)+(1-P_D P_G)\mu(m(k))}, & n_\varphi = 1, ..., c_{\varphi m}(k), \\ \frac{(1-P_D P_G)\mu(m(k))}{\sum_{\varphi=1}^{m(k)} P_{D\varphi}P_G\mu(m(k)-\varphi)+(1-P_D P_G)\mu(m(k))}, & n_\varphi = 0. \end{cases} \tag{14}$$

The state update equation is given by

$$\begin{aligned}\hat{x}(k|k) &= \hat{x}(k|k-1) \\ &+ \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} W_{\varphi,n_\varphi}(k)\beta_{\varphi,n_\varphi}(k)\nu_{\varphi,n_\varphi}(k) \end{aligned} \tag{15}$$

and the covariance associated with the updated state is

$$P(k|k) = E\{[x(k) - \hat{x}(k|k)][x(k) - \hat{x}(k|k)]'|Z^k\} \tag{16}$$

A. Posterior Cramér-Rao Lower Bound (PCRLB)

The posterior Cramér-Rao lower bound (PCRLB) [17] provides a theoretical lower bound that can be used as a benchmark for estimation performance evaluation. Note that the standard PCRLB also makes the one-to-one assumption, which necessitates the new derivation of the modified PCRLB that accounts for multiple detections.

For the random state vector $\mathbf{x}(k)$ to be estimated and for the unbiased estimate $\hat{\mathbf{x}}(k)(\mathbf{z}(k))$ based on the measurement data $\mathbf{z}(k)$, the PCRLB is given by the inverse of the FIM, J_k, as the lower bound of the error covariance matrix [17]. That is,

$$C(k) = E\left[(\hat{\mathbf{x}}(k)(\mathbf{z}(k)) - \mathbf{x}(k))(\hat{\mathbf{x}}(k)(\mathbf{z}(k)) - \mathbf{x}(k))'\right] \geq J_k^{-1}. \tag{17}$$

A recursive formula for the evaluation of the posterior FIM [17] is given by

$$J_{k+1} = J_{k+1}^{\mathbf{x}} + J_{k+1}^{\mathbf{z}}, \tag{18}$$

where

$$J_{k+1}^{\mathbf{x}} = D_k^{22} - D_k^{21}(J_k + D_k^{11})^{-1}D_k^{12}, \tag{19}$$

$$D_k^{11} = E\left[-\frac{\partial^2}{\partial\mathbf{x}(k)\partial\mathbf{x}(k)}\ln p(\mathbf{x}(k+1)|\mathbf{x}(k))\right], \tag{20}$$

$$\begin{aligned} D_k^{12} &= (D_k^{21})', \\ &= E\left[-\frac{\partial^2}{\partial\mathbf{x}(k+1)\partial\mathbf{x}(k)}\ln p(\mathbf{x}(k+1)|\mathbf{x}(k))\right], \end{aligned} \tag{21}$$

$$D_k^{22} = E\left[-\frac{\partial^2}{\partial\mathbf{x}(k+1)\partial\mathbf{x}(k+1)}\ln p(\mathbf{x}(k+1)|\mathbf{x}(k))\right], \tag{22}$$

and the measurement contribution factor is given by

$$J_{k+1}^{\mathbf{z}} = E\left[-\frac{\partial^2}{\partial\mathbf{x}(k+1)\partial\mathbf{x}(k+1)}\ln\, p(\mathbf{z}(k+1)|\mathbf{x}(k))\right]. \tag{23}$$

In order to consider the effect of multiple detections, let $m(k)$ be the total number of measurements from sensor at time k. Thus,

$$\mathbf{z}(k) = \{z_i(k)\}_{i=1}^{m(k)}. \tag{24}$$

Under the assumption that false alarms are uniformly distributed in the measurement space and the number of false alarms is Poisson distributed, the probability of getting $m(k)$ number of measurements out of which φ are target-originated is

$$p(m(k),\varphi) = (1 - P_{D\varphi})\frac{(\lambda V)^{m(k)} e^{-\lambda V}}{m(k)!} + P_{D\varphi}\frac{(\lambda V)^{m(k)-\varphi} e^{-\lambda V}}{(m(k)-\varphi)!}. \qquad (25)$$

$P_{D\varphi}$ is the probability of detecting a target φ times per scan and V is the gated volume of the measurement space. The probability that φ measurements are target generated is then given by

$$\epsilon(m(k),\varphi) = \frac{P_{D\varphi}}{p(m(k),\varphi)} \frac{(\lambda V)^{m(k)-\varphi} e^{-\lambda V}}{(m(k)-\varphi)!}. \qquad (26)$$

With the assumption of more than one target-originated measurements, the measurement information matrix is given by

$$J_z(k) = \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} p(m(k),\varphi) J_{z_{\varphi,n_\varphi}}(k), \qquad (27)$$

where

$$J_{z_{\varphi,n_\varphi}}(k) = E\left[-\frac{\partial^2}{\partial \mathbf{x}(k+1)\partial \mathbf{x}(k+1)} \ln p(z_{\varphi,n_\varphi}|\mathbf{x}(k), m(k), \varphi)\right]. \qquad (28)$$

Here, $p(z_{\varphi,n_\varphi}|\mathbf{x}(k), m(k), \varphi)$ is given by

$$p(z_{\varphi,n_\varphi}|\mathbf{x}(k), m(k), \varphi) = \frac{\varphi - \epsilon(m(k),\varphi)}{V^{m(k)}} + \frac{\epsilon(m(k),\varphi)}{m(k)V^{m(k)-\varphi}} \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} p(z_{\varphi,n_\varphi}|\mathbf{x}(k)), \qquad (29)$$

where $p(z_{\varphi,n_\varphi}|\mathbf{x}(k))$ is the pdf of the measurement set originated from a target.

IV. Simulations

In this section, the characteristics of the MD-PDA filter with respect to event probabilities and its comparison with PDA filter in terms of estimation accuracy is studied. The simulation is performed using a 2D sensor that returns both single target-originated detections and multiple target-originated detections per scan. The measurement-to-track association event probabilities for both filters are presented for these conditions.

A. 2D Sensor Scenario

A surveillance region covering a $1500\,\mathrm{m} \times 1500\,\mathrm{m}$ is considered. In this region, a target that starts from $[700\,\mathrm{m}, 700\,\mathrm{m}]$ and heads northeast with a constant speed of 15 m/s as shown in Figure 1. Target initialization is done using the two-point target initialization method. The scan interval (sampling period) is $1\,\mathrm{s}$ and that dataset consists of 30 scans.

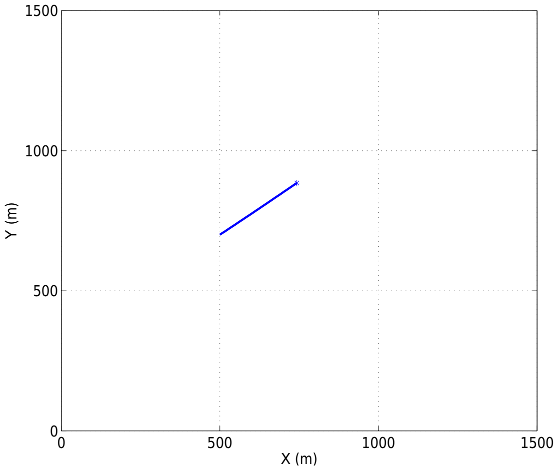

Fig. 1. Target trajectory ($*$—initial point, $+$—end point)

1) Single-Detection Sensor:

Single detection sensor refers to a sensor that returns at most one target-originated measurement per scan. A 2D radar that returns range and bearing measurements is considered with probability of detection $P_D = 0.95$, and with an average of 5 false alarms per scan. The standard deviations of the range and bearing measurement errors are $\sigma_r = 5\,\mathrm{m}$ and $\sigma_\theta = 0.1^o$, respectively.

In the case of at most one target-originated measurement per scan, the MD-PDA will reduce to PDA. Figure 2 shows the three highest $(\beta_1, \beta_2, \beta_3)$ measurement-to-track association probabilities based on probabilistic inference made on the number of measurements and measurements location. From the figure, it can be seen that the highest event probability is close to one, which corresponds to a measurement most likely to have originated from a target.

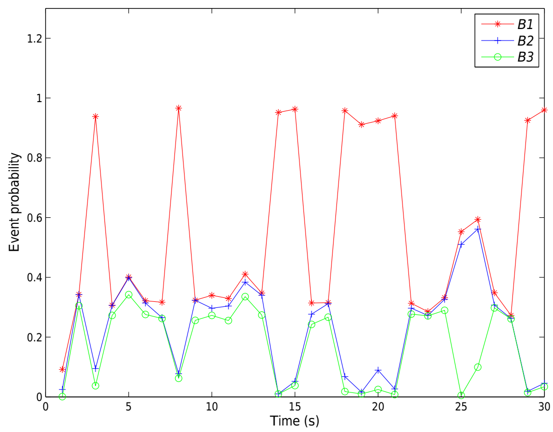

Fig. 2. Measurement-to-track association event probabilities.

2) Multiple-Detection Sensor:

A multiple-detection sensor refers to a sensor that returns more than one target-originated measurement per scan. A 2D radar with the following properties is considered.

- $P_{D1} = 0.05$ is probability of detecting a target once per scan of the measurement data
- $P_{D2} = 0.9$ is probability of detecting a target twice per scan of the measurement data
- $P_D = P_{D1} + P_{D2} = 0.95$ which is total probability of detecting a target in a scan of the measurement data (i.e, P_D used for PDA)
- A false alarm rate of 5 false alarms per scan

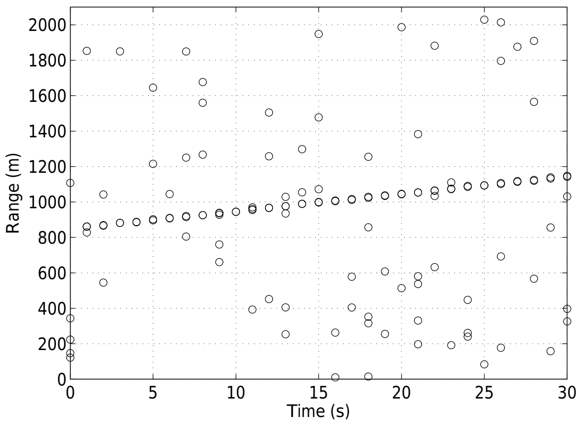

Fig. 3. Range measurements in a single run.

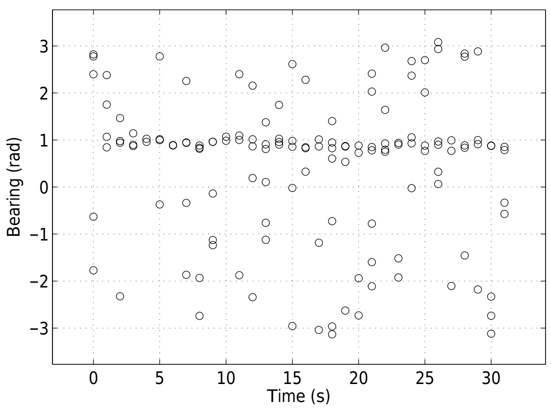

Fig. 4. Bearing measurements in a single run.

Similar to the single detection sensor, the standard deviation of the range and bearing measurement errors are $\sigma_r = 5\,\mathrm{m}$ and $\sigma_\theta = 0.1^o$, respectively. Figure 3 and Figure 4 show the range and bearing measurements from a multiple-detection sensor. In the more than one target-originated measurements case, the probabilities of detection used with the MD-PDA filter are P_{D1} and P_{D2} while $P_D = P_{D1} + P_{D2}$ is the total probability of detecting a target in PDA filter.

Figure 5 shows the measurement-to-track association event probabilities for the PDA filter. As shown in the figure, the highest probability is significantly below one since there are most likely two target-originated measurements. In this case,

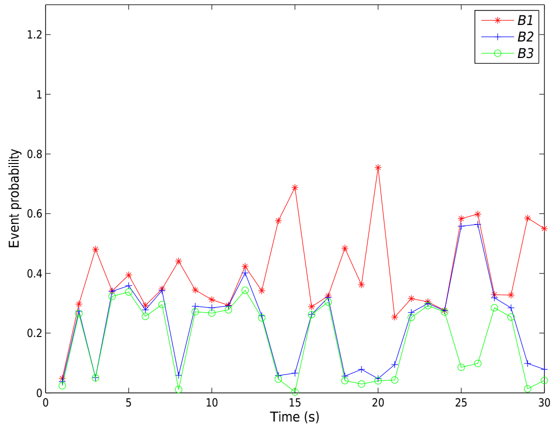

Fig. 5. Measurement-to-track association event probabilities for PDA filter with multiple detections.

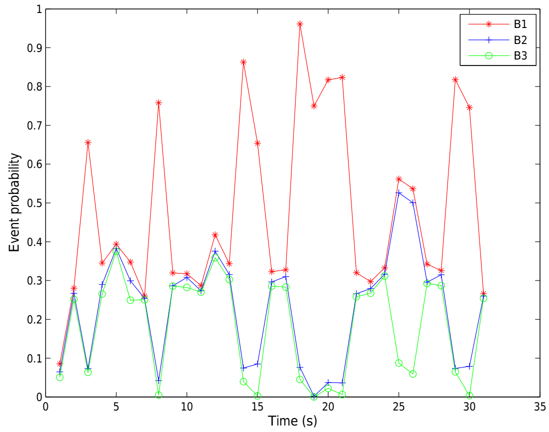

Fig. 6. Measurement-to-track association event probabilities for MD-PDA filter with multiple detections.

the wights of the false alarms are close to those of the target-originated measurements. Under the same setting, the event probabilities for the MD-PDA filter are shown in Figure 6. As the figure demonstrates, with the MD-PDA filter, the highest event association probability, which belongs to a measurement set most likely to have originated from a target, is close to one.

Figure 7 shows the Root Mean Squared Error (RMSE) for position estimation, which furthermore demonstrates the improved performance of the multiple-detection approach over the classic probability data association. While the PDA tends to apportion the weights among the target originated measurement, MD-PDA assigns weights to measurement sets, rather than to a single measurement, that are originated from a target. From the same figure it can be also seen that the estimation accuracy of MD-PDA meets the PCRLB that was derived for multiple detections.

The performance evaluation results on 100 Monte Carlo runs, are presented in Table I. Estimation accuracy measures

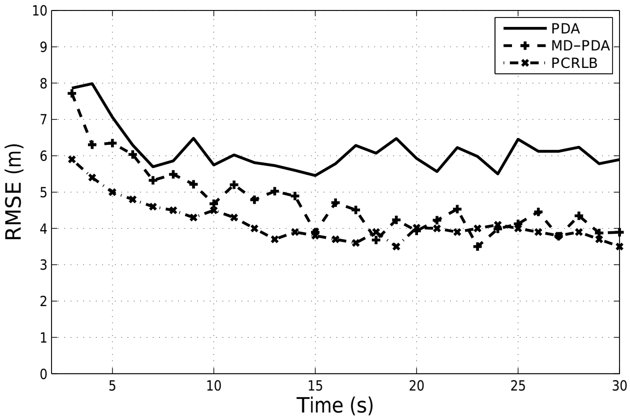

Fig. 7. Position RMSE evaluation for MD-PDA vs. PDA.

like average Euclidean error, average harmonic error and average geometric error [12] with position and velocity estimation are used for performance comparison between the two filters. It can be seen that the MD-PDA yields significantly better estimation accuracies.

TABLE I
ACCURACY MEASURES FOR MD-PDA VS. PDA

Performance Measure	PDA	MD-PDA
Average RMSE (Position)	6.793 m	4.802 m
Average RMSE (Velocity)	2.165 m/s	1.298 m/s
Average Euclidean Error (Position)	5.762 m	3.562 m
Average Euclidean Error (Velocity)	2.114 m/s	1.104 m/s
Average Harmonic Error (Position)	3.435 m	2.025 m
Average Harmonic Error (Velocity)	2.040 m/s	1.132 m/s
Average Geometric Error (Position)	4.639 m	2.969 m
Average Geometric Error (Velocity)	2.073 m/s	1.678 m/s

V. CONCLUSIONS

In this paper, we presented a brief overview of our recently developed multiple-detection probabilistic data association (MD-PDA) filter [6] for handling multiple detections from a target in a scan. When multiple detections from a target fall within the association gate, the conventional PDA filter produces degraded state estimates due to violation of the one measurement per scan assumption. In the MD-PDA filter, modified association probabilities are calculated with the explicit assumption of multiple detections from a target in a scan. We have developed a new algorithm for calculating the posterior Cramér-Rao lower bound (PCRLB) for multiple detections from a target in a scan. We performed Monte Carlo simulations using the 2D nearly constant velocity model for the target and range and azimuth measurements from a 2D radar sensor in the presence of clutter and sensor probability of detection less than unity. Our results show that the RMSE for position from the MD-PDA filter is close to the corresponding PCRLB, after processing 20 scans of measurements. The RMSE for position from the conventional PDA filter is consistently higher than that from the MD-PDA filter after 20 scans.

A. Future Works

Our future work will consider the integration of multiple-detection pattern into other multitarget tracking algorithms to handle multiple targets using the JPDA filter, MHT, and RFS based algorithm. Additional work will be done to initialize targets with multiple detections including target birth, continuity, and death probabilities.

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