Pressure Sensor Uses in Altitude and Velocity Estimation

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Abstract

Pressure sensors can be used to estimate altitude and velocity for a low-flying aircraft, such as a small UAV. This paper will address the implementation of automotive-grade pressure sensors as altitude and velocity sensors. Knowledge of the expected speed and altitude of the aircraft will help to optimize the implementation for low-power microproccessors.

1 Absolute Pressure – Altitude

The problem of measuring altitude can, at times, be tricky. Most of the difficulty comes when defining what, exactly, altitude is. Models of the shape of the earth are often used to calculate height above some geodic shape (GPS recievers use the model found in 1984 called the WGS-84) [1]. For low-flying aircraft (below 36,000 ft.), height above mean sea level is most often defined as altitude. In this range, pressure is a good indicator of altitude.

Pressure changes with altitude, temperature, and humidity. A model that takes into account humidity and temperature can be found in [2]. Assuming dry air and a temperature of 15° C on the ground, NASA determined a standard atmospheric model that indicates how pressure changes with altitude [3]. In standard units, the equation relating altitude (h in meters) and pressure (P in kPa) is

$$P = P_0 \left(\frac{T_0 - 0.00649h}{288.08} \right)^{5.256} \tag{1}$$

where P_0 and T_0 are the pressure and temperature under standard conditions at sea level ($P_0 = 101.29$ kPa, $T_0 = 288.15°$ K). Solving for altitude in terms of pressure gives

$$h = \frac{T_0}{0.00649} - \frac{288.08}{0.00649} \left(\frac{P}{P_0} \right)^{\frac{1}{5.256}} \tag{2}$$

A low-cost light weight absolute pressure sensor that is used at BYU to determine altitude is the Motorola MPX4115 [4]. The datasheet shows that voltage and pressure are related by the transfer function

$$V_{\text{out}} = V_s(0.009P - 0.095) \tag{3}$$

which can be rearranged as

$$P = \frac{V_{\text{out}}}{0.009 V_s} + \frac{0.095}{0.009} \tag{4}$$

The range of this sensor is shown as 15 to 115 kPa. For a small UAV senario, limits on the maximum and minimum altitude can be proposed which correspond through Equation (1) to maximum and minimum pressures, which in turn can be related through Equation (3) to maximum and minimum voltages. With this type of information, an operational amplifier can be used to gain up the output voltage of the pressure sensor to swing over a 5 volt range when the UAV is in the maximum and minimum altitude range. In this way, when the analog voltage is sampled with an analog-to-digital converter, more precision is obtained.

To determine the operational amplifier setup to maximize the number of bits used on the analog-to-digital converter, consider the inverting op amp configuration in Figure 1.

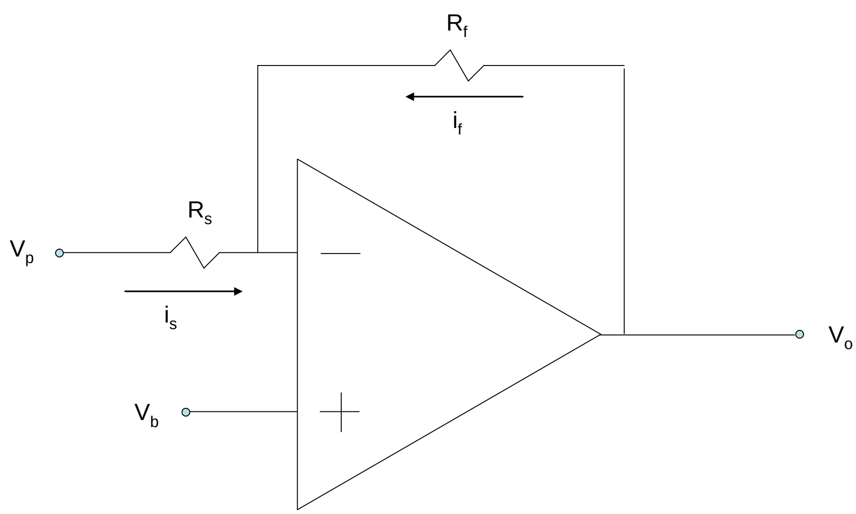

Figure 1: Operational Amplifier Setup.

From Ohm's law and assuming the amplifier is in it's linear range,

$$\begin{aligned} i_s &= \frac{V_p - V_b}{R_s} && (5) \\ i_f &= \frac{V_o - V_b}{R_f} && (6) \\ i_s &= -i_f && (7) \end{aligned}$$

Equations (5) - (7) combine to give

$$V_o = \alpha(V_b - V_p) + V_b \tag{8}$$

where $\alpha = R_f/R_s$. We want to choose α and V_b to maximize the the number of bits used in the analog-to-digital converter. In other words, we want

$$V_o = 5 \quad \text{when} \quad V_p = V_{p_{\min}} \tag{9}$$

$$V_o = 0 \quad \text{when} \quad V_p = V_{p_{\max}} \tag{10}$$

Note that $V_{o_{\max}}$ corresponds to $V_{p_{\min}}$ to ensure that α is a positive number.

Using Equations (9) and (10) along with Equation (8), the following are obtained,

$$5 = \alpha(V_b - V_{p_{\min}}) + V_b \tag{11}$$

$$0 = \alpha(V_b - V_{p_{\max}}) + V_b \tag{12}$$

solving for α and V_b yeilds

$$\alpha = \frac{5}{V_{p_{\max}} - V_{p_{\min}}} \tag{13}$$

$$V_b = \frac{5V_{p_{\max}}}{V_{p_{\max}} - V_{p_{\min}} + 5} \tag{14}$$

To realize this in hardware, simply choose resistor values R_s and R_f so that their ratio equals α. Then construct a voltage divider to put V_b on the positive pin of the op amp.

Now, to go from the reading on the analog-to-digital converter all the way to altitude, do the following,

1. Read value on analog-to-digital converter (N_{bits}). Convert to voltage V_o by the following relation

$$V_o = N_{\text{bits}} \frac{5}{N_{\text{bits}_{\max}}} \tag{15}$$

2. Calculate voltage from pressure sensor (V_p) using

$$V_p = \frac{1}{\alpha}(V_b - V_o) + V_b \tag{16}$$

3. Calculate pressure (P) from Equation (4).

4. Use Equation (2) to obtain altitude.

2 Differential Pressure – Velocity

The basis of velocity calcuation from differential pressure is the equation [5]

$$P_{\text{diff}} = \frac{1}{2}\rho V^2 \tag{17}$$

where ρ is the density of air, P_{diff} is the differential pressure in Pascals and V is the velocity (m/s). Velocity is easily determined if ρ and P_{diff} are known

$$V = \sqrt{\frac{2P_{\text{diff}}}{\rho}} \tag{18}$$

However, ρ changes with static pressure and temperature [6]

$$\rho = m\frac{P}{RT} \tag{19}$$

where m is the molar mass of the gas, P is the pressure of the gas in Pascals, R is the universal gas constant, and T is the temperature in Kelvin.

Following [5], we can define an effective airspeed under standard conditions

$$V_e = \sqrt{\frac{2P_{\text{diff}}}{\rho_0}} \tag{20}$$

where ρ_0 is the density of dry air, i.e. $\rho_0 = 1.225$ Kg/m^3.

To relate actual velocity and equivalent velocity, a ratio using Equation (18) under actual and standard conditions is formed

$$\frac{V}{V_e} = \frac{\sqrt{\frac{2P_{\text{diff}}}{\rho}}}{\sqrt{\frac{2P_{\text{diff}}}{\rho_0}}} = \sqrt{\frac{\rho_0}{\rho}} \tag{21}$$

So

$$V = V_e\sqrt{\frac{\rho_0}{\rho}} \tag{22}$$

which when using Equation (19) becomes

$$\begin{aligned}
V &= V_e\sqrt{\frac{\frac{mP_0}{RT_0}}{\frac{mP}{RT}}} \\
&= V_e\sqrt{\frac{P_0}{T_0}}\sqrt{\frac{T}{P}} \\
&= \sqrt{\frac{2P_{\text{diff}}}{\rho_0}}\sqrt{\frac{P_0}{T_0}}\sqrt{\frac{T}{P}} \\
&= \sqrt{\frac{2P_0}{\rho_0 T_0}}\sqrt{\frac{TP_{\text{diff}}}{P}} \\
&= \gamma\sqrt{\frac{TP_{\text{diff}}}{P}} \\
&\ \text{where} \\
\gamma &= \sqrt{\frac{2P_0}{\rho_0 T_0}}
\end{aligned}$$

and P_0 is the pressure at sea level (101290 Pa), T_0 is standard temperature 288.15° K, P is the current pressure of the air, and T is the current temperature.

A low-cost differential pressure sensor is available from Motorola, the MPX4006G [6]. This sensor has transfer function

$$V_{\text{out}} = V_s(0.1533P + 0.045) \tag{23}$$

which can be solved for P

$$P = \frac{V_{\text{out}}}{0.1533V_s} - \frac{0.045}{0.1533} \tag{24}$$

Using the same notation as the previous section, an operation amplifier setup can be designed to maximize the bits used in the analog-to-digital converter. This measurement of differential pressure along with the absolute pressure (measured to get altitude) and an estimate or measurement of temperature give a complete solution to getting velocity from differential pressure.

1. Read value on analog-to-digital converter (N_{bits}). Convert to voltage V_o by the following relation

$$V_o = N_{\text{bits}} \frac{5}{N_{\text{bits}_{\max}}} \tag{25}$$

2. Calculate voltage from differential pressure sensor (V_p) using

$$V_p = \frac{1}{\alpha}(V_b - V_o) + V_b \tag{26}$$

3. Calculate differential pressure (P_{diff}) from Equation (24).

4. Solve for velocity (V) using pressure from absolute pressure sensor (P) and temperature estimate T

$$V = \gamma\sqrt{\frac{TP_{\text{diff}}}{P}} \tag{27}$$

with γ as defined above.

References

[1] http://mtp.jpl.nasa.gov/notes/altitude/altitude.html.

[2] http://www.atmosphere.mpg.de/enid/791.

[3] http://www.grc.nasa.gov/WWW/K-12/airplane/atmosmet.html.

[4] http://e-www.motorola.com/files/sensors/doc/data_sheet/MPX4115A.pdf.

[5] http://web.usna.navy.mil/~dfr/flying/airspeed_wide.pdf.

[6] http://scienceworld.wolfram.com/physics/IdealGasLaw.html.