Inertial Measurement Unit (IMU) Selection: A Method for Allan Variance to Stochastic State Space Modeling

Part 1 of the series on Fault Tolerant Low-Cost Inertial Navigation Systems

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***Abstract*— IMU selection criteria for an Inertial Navigation System (INS) is motivated by the measurement, calibration, and nonlinearity errors inherent in all sensors. What is not clear in the literature is the translation from IMU manufacturer data sheets to application in a stochastic state space model like a Kalman Filter. This document seeks to bridge this gap and provide an understanding of IMU nomenclature, common sensor values used in manufacturer data-sheets, define the translation from Allan Variance to an Optimal Estimator system noise covariance matrix [Q], as well as present a short case study for three common-off-the-shelf (COTS) IMU's.**

I. Introduction

The IMU is a triad (i.e. 3-axes) angular rate sensor (gyroscope) and a triad linear acceleration sensor (accelerometer). Thus providing measurements related to three-dimensional (3D) body-fixed spatial behavior. We assume that the IMU contains integrated electronics for calibration, compensation and digital signal processing, as shown in Figure 1.

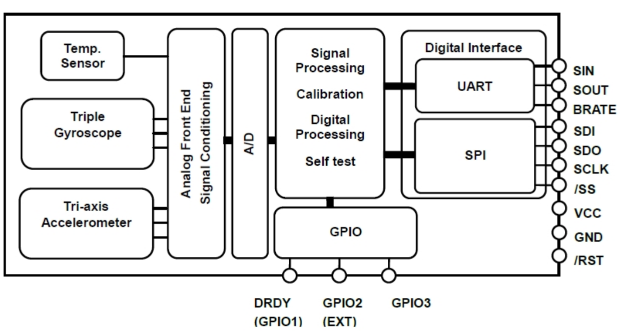

Fig. 1. Typical IMU with front-end [1]

When evaluating an IMU for a specific INS application, one must consider a few key IMU parameters which have an effect on the system-level position, velocity and attitude accuracy. The general form of the equation which represents a single axis sensor (e.g. single-axis gyro), is:

$$y_i(t) = (1 + \epsilon_k) \cdot [u_i(t) + \omega_i(t) + \eta_{MA} + \eta_Q + \cdots] \tag{1}$$

where $y_i(t)$ is the result of input $u_i(t)$, which is multiplied by some Scale Factor ϵ_k, and summed with a random bias $\omega_i(t)$ as well as other errors like Misalignment η_{MA}, Quantization Noise η_Q, Rate Random Walk η_{RRW}, Rate Ramp η_{RR}, etc.. Errors like Scale Factor and Misalignment are fixed, given by the manufacturer, and are typically part of the sensor calibration. Whereas, Random Walk and Bias Instability (both are contained in $\omega_i(t)$) are random, will cause errors to accumulate over time, and thus need to be characterized and modeled. For the remainder of this document we will address the modified form of the above equation:

$$y_i(t) = u_i(t) + \omega_i(t) \tag{2}$$

to determine a sensor which best fits the requirements. The biases, with units which are commonly found on manufacturer data sheets [1] and Allan Variance values defined in [4], are summarized in Table II, while Figure 2 provides an intuitive flow chart of the errors, with the parameters defined in Table I.

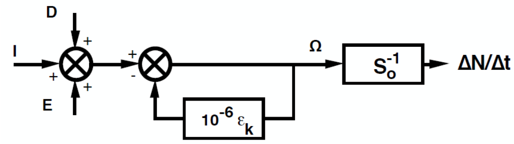

Fig. 2. Generic IMU Model [4]

TABLE I

Generic IMU Model parameters [4]

$S_0(\Delta N/\Delta t)$	is $[I + E + D](1 + 10e^{-6}\epsilon_k)^{-1}$
S_0	is nominal scale factor
$\Delta N/\Delta t$	is output pulse rate
I	is inertial input terms
E	is environmentally sensitive terms
D	is drift terms
ϵ_k	is scale factor error terms (ppm)
Ω	is equivalent gyro rate output

Note: this document can be easily extended to a 3-axis gyro and accelerometer by following the same methodology on each axis for both sensors.

Requirements: This paper considers a single-axis gyro navigation problem, however, the methods apply to any IMU application. We seek an IMU for integration within a GPS-aided INS which employs some form of optimal estimation (e.g. Extended Kalman Filter, Maximum A Posteriori [2]). For acceptable performance, under nominal operating conditions, the INS built upon the selected IMU must be capable of Dead-Reckoning for 60 seconds with 2 meter X-Y-Z position error 1σ and cost must be minimized.

TABLE II

SENSOR ERRORS

Symbol	Interpretation	$S_{\tilde{Y}}(j\omega)$	Units
Gyro Channels			
g_{bo}	Initial Bias	$B_o^2\delta(f)$	deg/hr
g_b	In-run Bias Stability	$\frac{B^2}{2\pi f}$	deg/hr
g_{RW}	Random Walk	N^2	deg/$\sqrt{hr}$
g_{SF}	Scale Factor	-	ppm
g_{MA}	Misalignment	-	deg
g_Q	Quantization	$\frac{4Q^2}{\tau}sin^2\pi f\tau$	-
g_{RRW}	Rate Random Walk	$\frac{K^2}{(2\pi f)^2}$	-
g_{RR}	Ramp Instability	$\frac{R^2}{(2\pi f)^3}$	-
Accelerometer Channels			
a_{bo}	Initial Bias	$B_o^2\delta(f)$	milli-g
a_b	In-run Bias Stability	$\frac{B^2}{2\pi f}$	μ-g
a_{RW}	Random Walk	N^2	μ-g/$\sqrt{hr}$
a_{SF}	Scale Factor	-	ppm
a_{MA}	Misalignment	-	deg
a_Q	Quantization	$\frac{4Q^2}{\tau}sin^2\pi f\tau$	-
a_{RRW}	Rate Random Walk	$\frac{K^2}{(2\pi f)^2}$	-
a_{RR}	Ramp Instability	$\frac{R^2}{(2\pi f)^3}$	-

The sections are organized as follows: First we review the literature in Section II, the IMU equations are derived in Section III, then the Allan Variance method is discussed in Section IV, followed by a method to convert the Allan Variance to the State Space model in Section V, the case study for a 3-D INS which uses three different COTS IMUs is given in Section VI, and the conclusions and future work are discussed in Section VII.

II. LITERATURE REVIEW

Common industry terms place IMUs into four major categories which stem from 1980's military requirements: Strategic, Tactical, Industrial and Consumer (see figure 3 with COTS IMUs listed). Strategic typically refers to systems which are designed for free-inertial navigation, with stability which will maintain system accuracy for weeks to months. These are typically used on ships and submarines, but also apply to ICBM missiles. Tactical systems are also designed for free-inertial (unaided) navigation, but operate on the order of seconds to minutes, and are typically used on missiles, smart bombs, and unmanned aerial vehicles (UAVs). Industrial systems span the range above and below the ability to sense Earth Rate (15 deg/hr), and are typically used in a variety of robotics applications. Below Earth Rate, it is possible to use a gyro for detecting true heading, without the aid of a magnetometer or GPS, however the In-run Bias Stability must be significantly below 15 deg/hr to make a good estimation of Earth Rate, typically <0.1 deg/hr will suffice. Above 40 deg/hr is considered consumer or automotive grade, and consists of rate gyros which cannot be used to determine heading or roll/pitch angles, but are sufficient to measure angular velocity such as yaw-rate.

While the explanation above is common in industry, we propose a new definition of IMU selection using Random Walk Error instead of In-run Bias Stability. In many applications, it is sufficient to assume that a system will

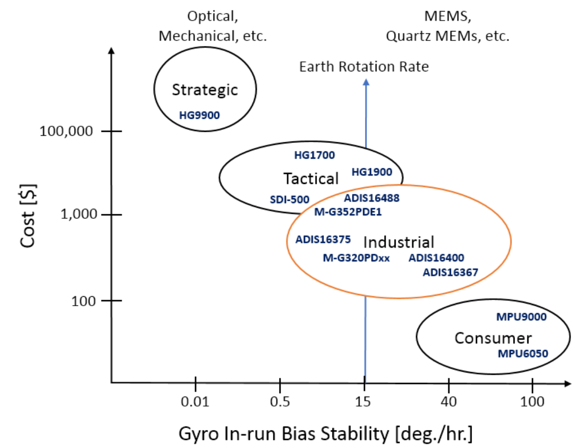

Fig. 3. Gyro Cost vs. Bias

operate under free-inertial conditions for periods up to 60 seconds, before and after-which the system is aided by a GPS receiver and the In-run Bias Stability errors are minimized by modeling them in an optimal estimator. Based on this assumption, we can define a new graph as shown in Figure 4. Notice that the Tactical and Industrial regions expand, and the expensive HG1700 IMU is in-line with the relatively cheap M-G320PDxx IMU. Based on the previous assumption, it can be shown that the two IMU's are nearly equivalent when using an Optimal Estimator, as shown in Figure 11, with a positional difference of <2 meters and the cost savings of nearly $30,000.

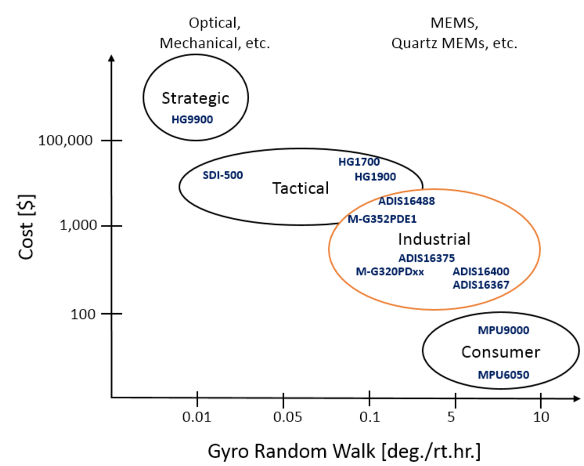

Fig. 4. Gyro Cost vs. Random Walk

III. GOVERNING EQUATIONS

The following equation, from Section 4.9.3 of [3], describe the growth of INS errors with respect to time:

$$\mathbf{P}_x(t) = \mathbf{\Phi}(t)\mathbf{P}_x(0)\mathbf{\Phi}^T(t) + \int_0^t \mathbf{\Phi}(\tau)\mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^T\mathbf{\Phi}^T(\tau)d\tau \quad (3)$$

For a 1-D INS, the result is:

$$\mathbf{\Phi}(t)\mathbf{P}_x(0)\mathbf{\Phi}^T(t) = \tag{4}$$

$$\begin{bmatrix} P_{p0} + P_{v0}t^2 + P_{b0}\frac{t^4}{4} & P_{v0}t + P_{b0}\frac{t^3}{2} & P_{b0}\frac{t^2}{2} \\ P_{v0}t + P_{b0}\frac{t^3}{2} & P_{v0} + P_{b0}\frac{t^2}{2} & P_{b0}t \\ P_{b0}\frac{t^2}{2} & P_{b0}t & P_{b0} \end{bmatrix}$$

and:

$$\int_0^t \mathbf{\Phi}(\tau)\mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^T\mathbf{\Phi}^T(\tau)d\tau = \tag{5}$$

$$\begin{bmatrix} \frac{\sigma^2_{\omega_v}t^3}{3} + \frac{\sigma^2_{\omega_b}t^5}{20} & \frac{\sigma^2_{\omega_v}t^2}{2} + \frac{\sigma^2_{\omega_b}t^4}{8} & \frac{\sigma^2_{\omega_b}t^3}{6} \\ \frac{\sigma^2_{\omega_v}t^2}{2} + \frac{\sigma^2_{\omega_b}t^4}{8} & \sigma^2_{\omega_v}t + \frac{\sigma^2_{\omega_b}t^3}{3} & \frac{\sigma^2_{\omega_b}t^2}{2} \\ \frac{\sigma^2_{\omega_b}t^3}{6} & \frac{\sigma^2_{\omega_b}t^2}{2} & \sigma^2_{\omega_b}t \end{bmatrix}$$

In these equations, the error state vector is defined as $x = [p, v, b]$, P_{p_o} is the initial position error covariance, P_{v_o} is the initial velocity error covariance, P_{b_o} is the initial bias covariance, σ_{ω_v} is the standard deviation of random walk, σ_{ω_b} is the standard deviation of the random walk rate.

Neglecting attitude error effects and axis cross-coupling, Equation 3 shows that the single-axis position error covariance is:

$$P_P(t) = P_{p_o} + P_{v_o}t^2 + P_{b_o}\frac{t^4}{4} + \sigma^2_{\omega_v}\frac{t^3}{3} + \sigma^2_{\omega_b}\frac{t^5}{20} \tag{6}$$

The first three terms are the initial condition uncertainty terms. When aiding signals are available and the vehicle is maneuvering, the terms p_o, v_o, b_o are estimated, reducing P_{p_o}, P_{v_o}, P_{b_o} from their initial (possibly large) values.

The last two terms are due to Random Walk and Random Walk Rate. The random errors are inherent in the sensor and the values of σ_{ω_v} and σ_{ω_b} cannot be reduced by estimation or on-line calibration.

While nature integrates:

$$\dot{b_a} = -\lambda b_a + \omega_{b_a} \tag{7}$$

$$P_{b_a}(t) = P_{b_a}(0) + \sigma^2_{\omega_{ba}}t \tag{8}$$

where b_a is the Bias Stability, with a correlation time $\lambda > 0$, $P_{b_a}(0)$ is the initial error bias, and $\sigma^2_{\omega_{ba}}t$ is the Bias Stability for a correlation time $\lambda = 0$, the error between INS and nature integration yields the single-axis velocity error covariance:

$$P_v(t) = P_{v_o} + P_{b_o}\frac{t^2}{2} + \sigma^2_{\omega_v}t + \sigma^2_{\omega_b}\frac{t^3}{3} \tag{9}$$

IV. Allan Variance Computation

It is common to determine the sensor error values using the Allan Variance [4], or more specifically the Allan Deviation (square root of the Allan Variance). According to [4], the general form of the Allan Variance of length τ is:

$$\sigma^2_\Omega(\tau) = \frac{1}{2}(\Omega_{k+1} \bar{-} \Omega_k)^2 = \frac{1}{2}\langle(\bar{\Omega}_{k+1} - \bar{\Omega}_k)^2\rangle \tag{10}$$

where:

$$\theta(t) = \int^t \Omega(t')dt' \tag{11}$$

$$\bar{\Omega}_k(\tau) = \frac{\theta(t_k + \tau) - \theta(t_k)}{\tau} \tag{12}$$

Processing of sensor data requires: (insert text here...)

***Remark** 4.1:* While the Allan Variance curves can be calculated from user collected data, and both Random Walk and In-run Bias Stability can be determined, it is strongly discouraged. To generate reliable data, proper laboratory equipment and methods must be followed to guarantee that the unit under test remains motionless for several days, the temperature remain constant prior to and during the test period, and both power sources and data collection systems do not introduce additional noise into the sensor data.

A. Initial Bias Error

Bias Error consists of two components: initial offset called *Initial Bias* (also called turn-on to turn-on bias), and long-term random drift called *In-run Bias Stability* (also called Flicker Noise). Because Initial Bias is calibrated from the manufacturer, and not determined from the Allan Variance, we need only consider In-run Bias Stability.

B. In-run Bias Stability Error

In-run Bias Stability is the random variation in the bias. This parameter provides the benchmark of the best that is achievable for a selected sensor in terms of bias variation for a fully modeled system without aiding from other sensors. In-run Bias Stability is an indication of angle, or velocity, error which increases proportionally with time. The resulting angle error (gyro) introduces a misalignment of the IMU orientation, affecting the projection of the gravity vector and linear acceleration. The resulting velocity error (accelerometer) effects both linear acceleration as well as the sensors ability to measure the gravity vector. These results combine in velocity errors which increase with t^2, and position error which increase with t^3.

Example 4.2: Referring to Figure 5, the minimum point on the graph, where the slope = 0, is the Gyro In-run Bias Stability. From Figure 5, the square-root of the Allan Deviation is $\approxeq$ 3 deg/hr (as read from the vertical axis of Figure 5), this corresponds to a maximum correlation time of 10 seconds.

***Remark** 4.3:* In this unique case, no equations are needed as the Allan Variance parameter can be read directly from the graph.

C. Random Walk Error

Random Walk is the unwanted signal generated from internal electronics that interfere with the measurement of the desired signal. The noise level will determine the minimum sensor output which is distinguishable from the background noise of the sensor, or noise floor. Random Walk is a

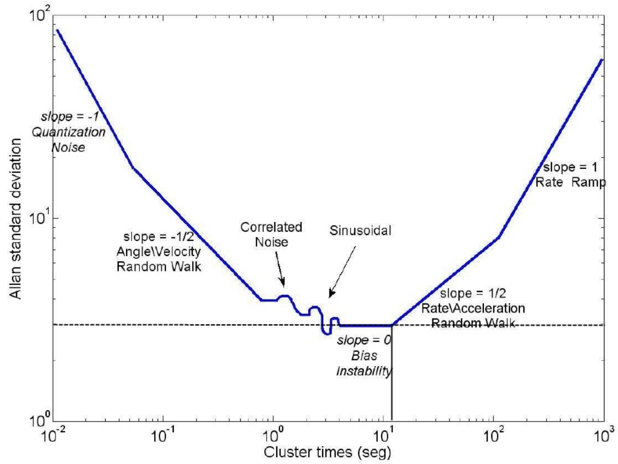

Fig. 5. Bias Stability from Allan Deviation [4]

common specification used to quantify sensor white noise output for a given sensor bandwidth. Random Walk will affect the projection of the gravity vector, as well as the position and velocity estimates. From Equation 10, we can define the formula for the gyro ARW:

$$\sigma^2(\tau) = \frac{N^2}{\tau} \tag{13}$$

Re-arranging the above equation for N, we find:

$$N = \sigma(\tau)\sqrt{\tau} \tag{14}$$

The coefficient N is the ARW and can be determined from the Allan Deviation plot by choosing the value which corresponds to $\tau = 1$.

***Remark** 4.4:* Note that different manufacturers and industries might use different units, either $\frac{deg}{\sqrt{hr}}$ or $\frac{deg/sec}{\sqrt{Hz}}$. These units are related as follows:

$$\begin{aligned}\left[\frac{\frac{o}{sec}}{\sqrt{Hz}}\right] &= \left[\frac{\frac{o}{sec}}{\sqrt{\frac{1}{sec}}}\right] = \left[\frac{o\,\frac{1}{\sqrt{sec}\sqrt{sec}}}{\frac{1}{\sqrt{sec}}}\right] \\ &= \left[o\,\frac{\sqrt{sec}}{\sqrt{sec}\sqrt{sec}}\right] = \left[\frac{o}{\sqrt{sec}}\right] \\ &= \left[\frac{o}{hr}\sqrt{hr}\right] = \left[\frac{o}{\sqrt{hr}}\right]\end{aligned}$$

Example 4.5: To determine the Gyro Random Walk error from the Allan Deviation graph, use the line with slope = -1/2, and the value which corresponds to $\tau = 0$ seconds. In Figure 6, assume the y-axis unit is deg/hr, x-axis unit is seconds. The square-root of the Allan Deviation which corresponds to $\tau = 0$ is $\approxeq$ 3.1 deg/hr [white noise assumed]. This leads to the value of Angular Random Walk:

$$\begin{aligned}N = \sigma(\tau)\sqrt{\tau} &= \left[3.1\frac{o}{hr}\sqrt{sec}\right] = \left[3.1\frac{o}{\sqrt{hr}\sqrt{hr}}\sqrt{sec}\right] \\ &= \left[3.1\frac{o}{\sqrt{3600sec}\sqrt{hr}}\sqrt{sec}\right] = \left[3.1\frac{o}{60\sqrt{hr}}\right] \\ &= \left[0.0516\frac{o}{\sqrt{hr}}\right] = \left[8.61 * 10^{-4}\frac{o}{\sqrt{sec}}\right] \\ &= \left[8.61 * 10^{-4}\frac{\frac{o}{sec}}{\sqrt{Hz}}\right]\end{aligned}$$

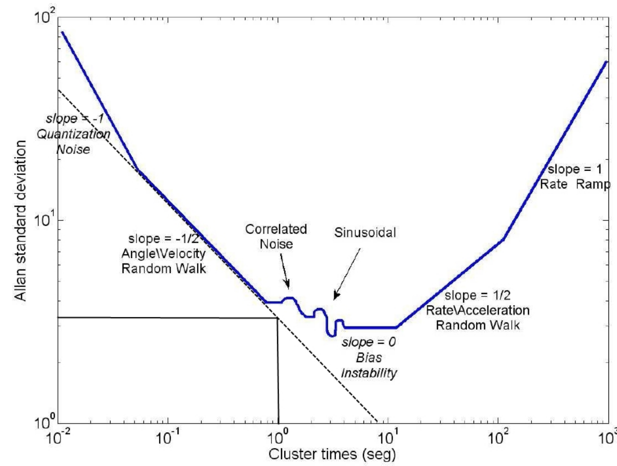

Fig. 6. ARW from Allan Deviation [4]

V. Allan Variance to State Space

From Equation 3 we can calculate gyro Angular Random Walk contributions to position error by modeling a single-axis gyro as follows:

$$\begin{bmatrix}\delta\dot{p}\\ \delta\dot{v}\\ \delta\dot{\psi}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & g\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\delta p\\ \delta v\\ \delta\psi\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \omega(t)\end{bmatrix}$$

where δp, δv, and $\delta\psi$ is the x-position, x-velocity and x-tilt angle error respectively. g represents the gravity. $\omega(t)$ is the additive white noise whose spectral level (determined in Example 4.5) $Q = 0.0516\frac{o}{\sqrt{hr}}$. The solution to the differential equation is:

$$\begin{bmatrix}\delta p(t)\\ \delta v(t)\\ \delta\psi(t)\end{bmatrix} = \begin{bmatrix}1 & t & \frac{gt^2}{2}\\ 0 & 1 & gt\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\delta p(0)\\ \delta v(0)\\ \delta\psi(0)\end{bmatrix} + \begin{bmatrix}\frac{gt^2}{2}\\ gt\\ 1\end{bmatrix}\omega(t)$$

Assume that the Initial Bias errors are zero by calibration, then we can calculate the variance of the state vector as:

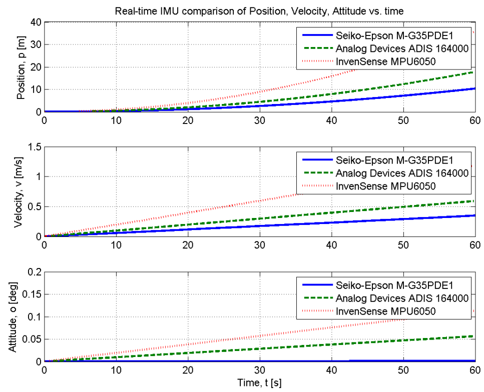

Fig. 7. 60 seconds of Real-time INS Error vs. Position, Velocity & Orientation Comparison of Industrial-Grade to Consumer-Grade IMUs

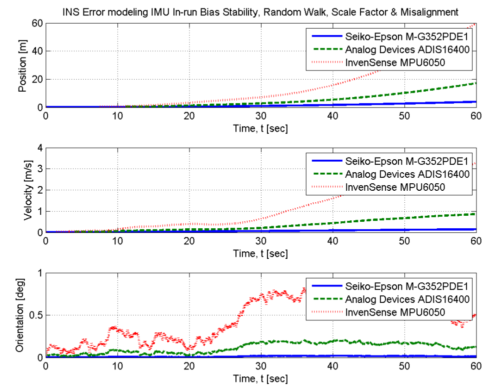

Fig. 8. 60 seconds of Simulated INS Error vs. Position, Velocity & Orientation Comparison of Industrial-Grade to Consumer-Grade IMUs

$$
\begin{aligned}
&E\left(\begin{bmatrix}\delta p(t)\\ \delta v(t)\\ \delta\psi(t)\end{bmatrix}\begin{bmatrix}\delta p(t)\\ \delta v(t)\\ \delta\psi(t)\end{bmatrix}^T\right)\\
&=\int_0^t\int_0^t\left(\begin{bmatrix}\frac{g(t-\tau)^2}{2}\\ g(t-\tau)\\ 1\end{bmatrix}E\{\omega(\tau)\omega(\lambda)\}\begin{bmatrix}\frac{g(t-\lambda)^2}{2}\\ g(t-\lambda)\\ 1\end{bmatrix}^2 d\tau d\lambda\right)\\
&=Q\begin{bmatrix}\frac{g^2t^5}{20} & \frac{g^2t^4}{8} & \frac{gt^3}{6}\\ \times & \frac{g^2t^3}{3} & \frac{gt^2}{2}\\ \times & \times & t\end{bmatrix}
\end{aligned}
$$

Therefore, after 60 seconds, the variance of the total position error due to Angular Random Walk is:

$$
\begin{aligned}
E\{\delta^2_{p_{total}}\} &= 2E\{\delta^2_p\}\\
&= 2\cdot\left(\frac{0.0516\pi}{180\cdot\sqrt{3600}}\right)^2\cdot\frac{9.81^2\cdot 60^5}{20}\\
&\approx 1.68m^2\\
&= 1.29m
\end{aligned}
$$

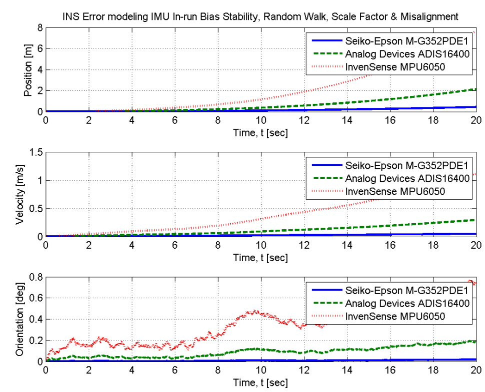

Fig. 9. 20 seconds of Simulated INS Error vs. Position, Velocity & Orientation Comparison of Industrial-Grade to Consumer-Grade IMUs

VI. Experimental Results

This analysis assumes that on-line Optimal Estimation has reduced the initial covariance to $P_{p_o} = (0.1m)^2$, $P_{v_o} = (0.1m/s)^2$, $P_{b_o} = (0.1deg)^2$, then consider error growth for different IMUs.

Applying the equations from Section III with information available for COTS IMUs of various quality, we can map sensor errors to INS errors, as shown in Table III. It is clear from Sections 2 & 3 of Table III that the dominant errors are related to the In-run Bias Stability. Yet, as it was pointed out in Section III, modeling of these biases in the Optimal Estimator will reduce the dominant error to those related to Random Walk, which is demonstrated in Figure 7. Assuming we can drive the In-run Bias Stability to zero, Figure 8 results, and the values are tabulated in Section 3 of Table III.

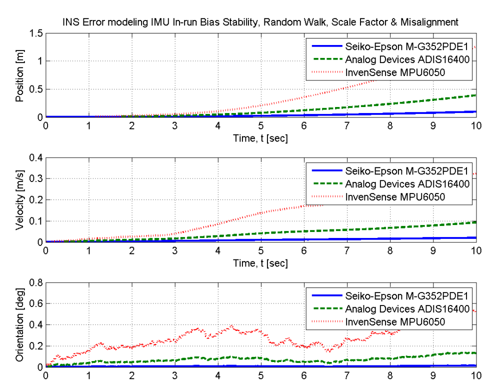

Fig. 10. 10 seconds of Simulated INS Error vs. Position, Velocity & Orientation Comparison of Industrial-Grade to Consumer-Grade IMUs

TABLE III

SENSOR TO INS ERRORS

	Seiko-Epson M-G352PDE1	Analog Devices ADIS16400	InvenSense MPU6050
cost [$]	400	450	40
Gyro Channels			
g_{bo} [deg/hr]	10,800	10,800	72,000
g_b [deg/hr]	4.0	25.2	72,000
g_{RW} [deg/$\sqrt{hr}$]	0.1	2.0	8.0
g_{SF} [ppm]	150	300	500
Accelerometer Channels			
a_{bo} [milli-g]	15.0	50.0	80.0
a_b [μ-g]	100	200	80,000
a_{RW} [μ-g/$\sqrt{hr}$]	0.05	0.02	0.23
a_{SF} [ppm]	200	300	500
INS Error, static for 60 seconds			
P_e [m]	10.04	60.01	1.43e4
V_e [m/s]	0.50	3.27	4.92e2
O_e [deg]	0.11	0.70	76.14
INS Error (In-Run Bias removed), static for 60 seconds			
P_e [m]	3.87	17.13	59.68
V_e [m/s]	0.14	0.86	3.26
O_e [deg]	0.01	0.12	0.81
INS Error (In-Run Bias removed), static for 20 seconds			
P_e [m]	0.44	2.14	7.66
V_e [m/s]	0.04	0.29	1.10
O_e [deg]	0.01	0.18	0.75
INS Error (In-Run Bias removed), static for 10 seconds			
P_e [m]	0.09	0.39	1.25
V_e [m/s]	0.02	0.09	0.32
O_e [deg]	0.01	0.13	0.52

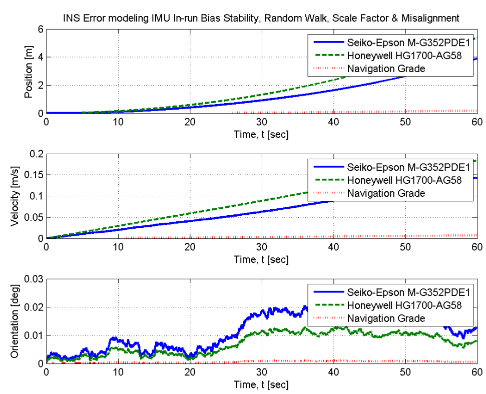

Fig. 11. 60 seconds of Simulated INS Error ARW comparison of Industrial-Grade IMU to High-end Tactical-Grade and Navigation-Grade IMUs

VII. CONCLUSIONS AND FUTURE WORK

This document has presented the equations which govern the biases of a single-axis gyro, which is easily extended to a 3-axis IMU, as well as the methods by which to obtain the PSD values for State Space modeling from the Allan Variance. Based on the case study presented in Section VI, we find that the requirements stated in Section I map to the Seiko-Epson M-G320PDxx IMU, or its equivalent.

The Appendix contains plots of the random walk process, standard deviation, and variance based on Ch. 6.2 of [9].

VIII. ACKNOWLEDGEMENTS

This research builds on the technical conversations with David Kelley. These technical collaborations are greatly appreciated.

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IX. APPENDIX

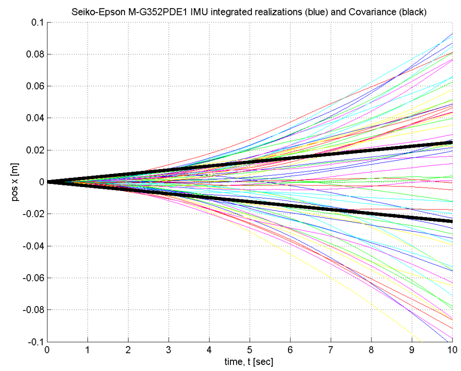

Fig. 12. Stationary Seiko-Epson M-G352PDE1 IMU Position Divergence due to VRW (blue) and the expected VRW std.dev. ($\sigma^2_{\omega_v}$) selected for Q (black)

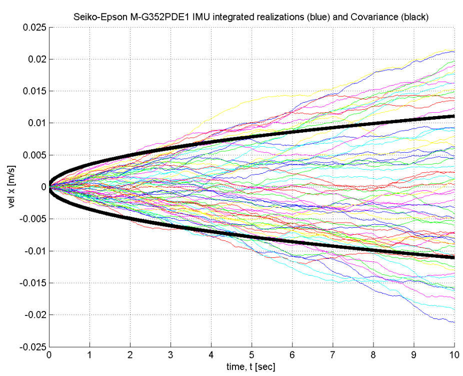

Fig. 13. Stationary Seiko-Epson M-G352PDE1 IMU Velocity Divergence due to VRW (blue) and the expected VRW std.dev. ($\sigma^2_{\omega_v}$) selected for Q (black)

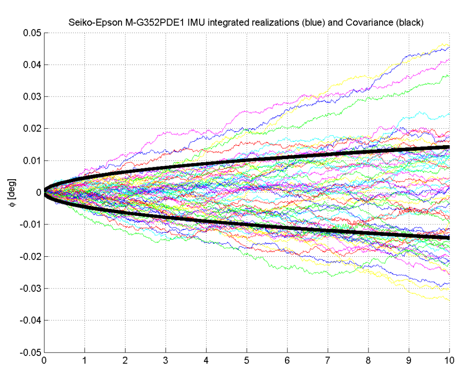

Fig. 14. Stationary Seiko-Epson M-G352PDE1 IMU Attitude Divergence due to VRW (blue) and the expected ARW std.dev. ($\sigma^2_{\omega_b}$) selected for Q (black)

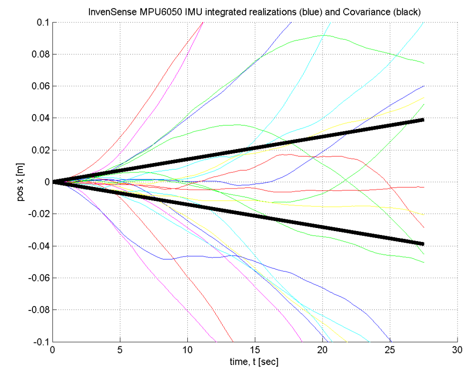

Fig. 15. Stationary InvenSense MPU6050 IMU Position Divergence due to VRW (blue) and the expected VRW std.dev. ($\sigma^2_{\omega_v}$) selected for Q (black)

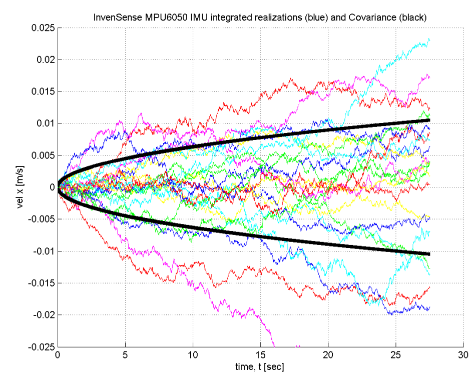

Fig. 16. Stationary InvenSense MPU6050 IMU Velocity Divergence due to VRW (blue) and the expected VRW std.dev. ($\sigma^2_{\omega_v}$) selected for Q (black)

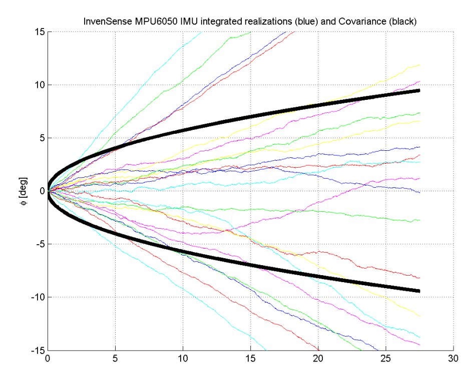

Fig. 17. Stationary InvenSense MPU6050 IMU Attitude Divergence due to VRW (blue) and the expected ARW std.dev. ($\sigma^2_{\omega_b}$) selected for Q (black)