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0 前言

> 自然界中沒有哪怕一個真正的單粒子系統，甚至連少體系統都沒有。虛粒子對和的出現和漲落告訴我們，粒子數不變的日子一去不復返了…… —— Viki Weisskopf

- 很多時候我們的成長都付出了相當沉重的代價，例如犧牲掉我們過去所認爲是相當美的東西，最終使得我們無比懷念的日子都一去不復返了。這一切都值得嗎？物理學的經驗告訴我們，值得，至少是大多數情況下。

這個前言兼目錄把之前零散的文章匯集起來，并且做一說明。

一個有志於從事廣義相對論相關領域研究的年輕人爲什麼會轉來讀這本講量子場論的講義呢？實際上只是因爲讀完了一本書后不想立即讀下一本了，加之這本講義的長度比較適合換腦子的時候讀，我便選擇了它。

我不說明的話，可能很多人不知道我每篇都在寫一些什麼東西。好吧，其實它們是我對於原作者（David Tong，我親切地稱之爲“佟大爲”）省去的或描述了但是語焉不詳的一些步驟的補充。這個補充的過程是充滿樂趣的，即使有了這幾篇文章，我希望後來的讀者仍然應該首先嘗試自己進行推導。我曾經犯過這樣一個天大的錯誤，即覺得書上那些推導自己看一遍就會了。這個壞習慣從某種意義上說毀掉了我，我希望說出這句話之後會有更少的人避免他，會有更少的人不再繼續欺騙自己。

同時, 有幾處細節我自己仍然不敢肯定, 或者還沒有來得及找相關的文獻研究, 歡迎讀者補充 :)

1 經典場論

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} + & & & \\ & - & & \\ & & - & \\ & & & - \end{pmatrix}$$

$$\partial_\mu = (\frac{\partial}{\partial t}, \nabla),\ \partial^\mu = (\frac{\partial}{\partial t}, -\nabla)$$

1.1 場的動力學

$$\varphi_a(\vec{x}, t)$$

$$L = \int \mathrm{d}^3 x \mathcal{L}(\varphi_a, \partial_a \varphi)$$

$$S = \int_{t_1}^{t_2} \mathrm{d}t \int \mathrm{d}^3 x \mathcal{L} = \int \mathrm{d}^4 x \mathcal{L}$$

$$\begin{aligned} 0 = \delta S &= \int \mathrm{d}^4 x [\frac{\partial \mathcal{L}}{\partial \varphi_a} \delta \varphi_a + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} \delta(\partial_\mu \varphi_a)] \\ &= \int \mathrm{d}^4 x [\frac{\partial \mathcal{L}}{\partial \varphi_a} - \partial_\mu (\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)})] \delta \varphi_a + \partial_\mu (\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} \delta \varphi_a) \end{aligned}$$

$$\partial_\mu (\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)}) - \frac{\partial \mathcal{L}}{\partial \varphi_a} = 0$$

$$\frac{\partial}{\partial t}(\frac{\partial \mathcal{L}}{\partial(\frac{\partial}{\partial t}\phi_a)}) - \nabla(\frac{\partial \mathcal{L}}{\partial(\nabla \phi_a)}) - \frac{\partial \mathcal{L}}{\partial \phi_a} = 0$$

1.2 Klein-Gordon 方程

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi, \ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi = (\dot{\phi}, -\nabla \phi)$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \Rightarrow \partial_\mu \partial^\mu \phi + \frac{\partial V}{\partial \phi} = 0$$

1.3 一階拉格朗日量

$$\mathcal{L} = \frac{\mathrm{i}}{2}(\psi^\star \dot{\psi} - \dot{\psi}^\star \psi) - \nabla \psi^\star \cdot \nabla \psi - m \psi^\star \psi$$

將 ψ 與 $\psi^\star$ 視爲獨立對象

$$\frac{\partial \mathcal{L}}{\partial \psi^\star} = \frac{\mathrm{i}}{2}\dot{\psi} - m\psi, \ \frac{\partial \mathcal{L}}{\partial \dot{\psi}^\star} = -\frac{\mathrm{i}}{2}\psi, \ \frac{\partial \mathcal{L}}{\partial(\nabla \psi^\star)} = -\nabla \psi$$

$$\frac{\partial}{\partial t}(-\frac{\mathrm{i}}{2}\psi) - \nabla(-\nabla \psi) - (\frac{\mathrm{i}}{2}\dot{\psi} - m\psi) = 0 \Rightarrow \frac{\mathrm{i}}{2}\psi = \nabla^2 \psi + m\psi$$

1.4 Maxwell 方程

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) + \frac{1}{2}(\partial_\mu A^\mu)^2 = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) + \frac{1}{2}(\eta^{\mu\nu}\partial_\mu A_\nu)^2$$

$$\frac{\partial \mathcal{L}}{\partial A^\mu} = 0$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -\partial^\mu A^\nu + \eta^{\mu\nu}(\eta^{\mu\nu}\partial_\mu A_\nu) = -\partial^\mu A^\nu + \eta^{\mu\nu}(\partial_\rho A^\rho)$$

$$\partial_\mu(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)}) = -\partial^2 A^\nu + \partial^\nu(\partial_\rho A^\rho) = -\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) \equiv -\partial_\mu F^{\mu\nu}$$

- 三維形式:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, A^\mu(\vec{x}, t) = (\phi, \vec{A}), \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}, \vec{B} = \nabla \times \vec{A}$$

$$\nabla \cdot \vec{B} = \nabla \cdot (\nabla \times \vec{A}) = 0,\ \nabla \times \vec{E} = \nabla \times (-\nabla\phi - \frac{\partial \vec{A}}{\partial t}) = -\frac{\partial}{\partial t}(\nabla \times \vec{A}) = -\frac{\partial \vec{B}}{\partial t}$$

$$\partial_\mu F^{\mu\nu} \Rightarrow \nabla \cdot \vec{E} = 0,\ \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B}$$

- 等價形式:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ &= -\frac{1}{4}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) + (\partial_\nu A_\mu)(\partial^\nu A^\mu)] \\ &\quad + \frac{1}{4}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) + (\partial_\nu A_\mu)(\partial^\mu A^\nu)]\end{aligned}$$

$$(\partial_\mu A_\nu)(\partial^\mu A^\nu) + (\partial_\nu A_\mu)(\partial^\nu A^\mu) = 2(\partial_\mu A_\nu)(\partial^\mu A^\nu)$$

$$\begin{aligned}(\partial_\mu A_\nu)(\partial^\nu A^\mu) + (\partial_\nu A_\mu)(\partial^\mu A^\nu) &\overset{\text{phy}}{=\!=} -A_\nu(\partial_\mu\partial^\nu A^\mu) - A_\nu(\partial_\nu\partial^\mu A^\nu) \\ &= -2A_\mu\partial^\mu\partial_\nu A^\nu \\ &= -2[\partial^\mu(A_\mu\partial_\nu A^\nu) - (\partial^\mu A_\mu)(\partial_\nu A^\mu)] \\ &\overset{\text{phy}}{=\!=} 2(\partial^\mu A_\mu)^2\end{aligned}$$

1.5 Lorentz 不變性

$$\Lambda^\mu{}_\sigma \eta^{\sigma\tau} \Lambda^\nu{}_\tau = \eta^{\mu\nu}$$

$$x^\mu \to (x')^\mu = \Lambda^\mu{}_\nu x^\nu,\ \phi(x) \to \phi'(x) = \phi(\Lambda^{-1}x)$$

- Klein-Gordon 方程

$$\phi(x) \to \phi'(x) = \phi(y),\ y = \Lambda^{-1}x$$

$$(\partial_\mu \phi)(x) \to (\Lambda^{-1})^\nu{}_\mu (\partial_\nu \psi)(y)$$

$$\mathcal{L}_{\text{deriv}}(x) = \partial_\mu\phi(x)\partial_\nu\phi(x)\eta^{\mu\nu} \to (\Lambda^{-1})^\rho{}_\mu(\partial_\rho\phi)(y)(\Lambda^{-1})^\sigma{}_\nu(\partial_\sigma\phi)(y)\eta^{\mu\nu} = (\partial_\rho\phi)(y)(\partial_\sigma\phi)(y)\eta^{\rho\sigma} = \mathcal{L}_{\text{deriv}}(y)$$

- 一階的動力學：不是 Lorentz 不變的
- Maxwell 方程：是

1.6 對稱性

1.6.1 Noether 定理

拉格朗日量的任一連續對稱性給出一個守恆荷。

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi_a}\delta\phi_a + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)}\partial_\mu(\delta\phi_a) = \partial_\mu(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)}\delta\phi_a)$$

稱變換 $\delta\phi_a(x) = X_a(\phi)$ 對稱，若同時 $\delta\mathcal{L} = \partial_\mu F^\mu$；則:

$$\partial_\mu j^\mu = 0,\ j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)X_a(\phi)} - F^\mu(\phi)$$

1.6.2 平移和能量動量張量

$$\Lambda:\ x^\mu \to x^\nu - \epsilon^\nu \Rightarrow \Lambda^{-1}:\ x^\nu \to x^\nu + \epsilon^\nu$$

$$\phi_a(x) \to \phi'_a(x) = \phi_a(\Lambda^{-1}x) = \phi_a(x^\nu + \epsilon^\nu) \xlongequal{\text{Taylor}} \phi_a(x) + \epsilon^\nu\partial_\nu\phi_a(x),\ \mathcal{L}(x) \to \mathcal{L}(x) + \epsilon^\nu\partial_\nu\mathcal{L}(x)$$

$$\delta\phi_a(x) = \epsilon^\nu\partial_\nu\phi_a(x),\ \delta\mathcal{L} = \epsilon^\mu\partial_\mu\mathcal{L}(x),\ F^\mu = \epsilon^\mu\mathcal{L}(x)$$

$$(j^\mu)_\nu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)}\partial_\nu\phi_a(x) - \delta^\mu_\nu\mathcal{L} \equiv T^\mu{}_\nu$$

對於 $L\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$:

$$T^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)}\partial^\nu\phi_a - \eta^{\mu\nu}\mathcal{L} = \partial^\mu\phi\partial^\nu\phi - \eta^{\mu\nu}\mathcal{L}$$

$$\begin{aligned}\partial_\mu T^{\mu\nu} &= \partial_\mu(\partial^\mu\phi\partial^\nu\phi) - \partial^\nu(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2)\\ &= \partial^2\phi\partial^\nu\phi + m^2\phi\partial^\nu\phi + \partial^\mu\phi\partial_\mu\partial^\nu\phi - \partial^\nu\partial_\mu\phi\partial^\mu\phi = 0\end{aligned}$$

1.6.3 Lorentz 變換和角動量

$$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu$$

$$\begin{aligned}\eta^{\mu\nu} &= (\delta^\mu{}_\sigma + \omega^\mu{}_\sigma)\eta^{\sigma\tau}(\delta^\nu{}_\tau + \omega^\nu{}_\tau)\\ &= \eta^{\mu\nu} + \omega^\mu{}_\sigma\eta^{\sigma\tau}\delta^\nu{}_\tau + \delta^\mu{}_\sigma\eta^{\sigma\tau}\omega^\nu{}_\tau \Rightarrow \omega^{\mu\nu} + \omega^{\nu\mu} = 0\\ &= \eta^{\mu\nu} + \omega^{\mu\nu} + \omega^{\nu\mu}\end{aligned}$$

$$\phi(x) \to \phi'(x) = \phi(\Lambda^{-1}x) = \phi(x^\mu - \omega^\mu{}_\nu x^\nu) \xlongequal{\text{Taylor}} \phi(x^\mu) - \omega^\mu{}_\nu x^\nu \partial_\mu \phi(x)$$

$$\delta\phi = -\omega^\mu{}_\nu x^\nu \partial_\mu \phi$$

$$\delta\mathcal{L} = -\omega^\mu{}_\nu x^\nu \partial_\mu \mathcal{L} \xlongequal{\omega^\mu{}_\nu(\partial_\mu x^\nu) = \omega^\mu{}_\nu \delta_\mu{}^\nu = 0} \partial_\mu[-\omega^\mu{}_\nu x^\nu \mathcal{L}]$$

$$j^\mu = -\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\omega^\rho{}_\nu x^\nu \partial_\rho\phi + \omega^\mu{}_\nu x^\nu \mathcal{L} = -\omega^\rho{}_\nu[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial_\rho\phi - \delta^\mu{}_\rho\mathcal{L}]x^\nu = -\omega^\rho{}_\nu T^\mu{}_\rho x^\nu$$

$$(\mathcal{J}^\mu)^{\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho}$$

$$\partial_\mu(\mathcal{J}^\mu)^{\rho\sigma} = (\delta_\mu{}^\rho T^{\mu\sigma} + x^\rho\partial_\mu T^{\mu\sigma}) - (\delta_\mu{}^\sigma T^{\mu\rho} + x^\sigma\partial_\mu T^{\mu\rho})$$

1.6.4 内稟對稱性:

考慮一複場: $\phi(x) \equiv (\phi_1(x) + \mathrm{i}\phi_2(x))/\sqrt{2}$, $\mathcal{L} = \partial_\mu\phi^\star\partial^\mu\psi - V(|\psi|^2)$, $|\psi|^2 = \psi^\star\psi$, 自由度為 2, 視 ψ 與 $\psi^\star$ 獨立:

$$\partial_\mu\partial^\mu\psi + \frac{\partial V(\psi^\star\psi)}{\partial\psi^\star} = 0$$

$$\psi \to \mathrm{e}^{\mathrm{i}\alpha}\psi = (1 + \mathrm{i}\alpha)\psi,\ \delta\psi = \mathrm{i}\alpha\psi,\ \delta\psi^\star = -\mathrm{i}\alpha\psi,\ \delta\mathcal{L} = 0$$

$$j^\mu = \mathrm{i}(\partial^\mu\psi^\star)\psi - \mathrm{i}\psi^\star(\partial^\mu\psi)$$

之後將會看到, 這一守恆量是電荷或粒子數。

1.7 Hamiltonian 理論

路徑積分形式需要用到拉格朗日量，而正則量子化需要用到哈密頓量。我們選擇後者。

$$\pi^a(x) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}_a}$$

$$\mathcal{H} = \pi^a(x)\dot{\phi}_a(x) - \mathcal{L}(x),\ H = \int \mathrm{d}^3x\ \mathcal{H}$$

$$\dot{\psi}(\vec{x},t) = \frac{\partial \mathcal{H}}{\partial \pi(\vec{x},t)},\ \dot{\pi}(\vec{x},t) = \frac{\partial \mathcal{H}}{\partial \phi(\vec{x},t)}$$

2 自由場

一位年輕的理論物理學家的職業生涯就是在不斷提高的的抽象層次中處理諧振子。——Sidney Coleman

從這篇文章開始，不再單純地抄書，主要記錄一些補充的推導過程。

2.1 熱身

$$[q_a, q_b] = [p^a, p^b] = 0,\ [q_a, p^b] = \mathrm{i}\delta_a^b$$

類比：

$$[\phi_a(\vec{x}), \phi_b(\vec{y})] = [\pi^a(\vec{x}), \pi^b(\vec{y})] = 0, [\phi_a(\vec{x}), \pi^b(\vec{y})] = \mathrm{i}\delta^{(3)}(\vec{x}-\vec{y})\delta_a^b$$

$$\partial_\mu\partial^\mu\phi(\vec{x},t) + m^2\phi(\vec{x},t) = 0$$

$$\phi(\vec{x},t) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}\phi(\vec{p},t)$$

$$\begin{aligned}
0 &= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\{\partial_\mu\partial^\mu[\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}\phi(\vec{p},t)] + m^2\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}(\vec{p},t)\} \\
&= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\{(\frac{\partial^2}{\partial t^2} - \nabla^2)[\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}\phi(\vec{p},t)] + m^2\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}(\vec{p},t)\} \\
&= \mathrm{e}^{i\vec{p}\cdot\vec{x}}\frac{\partial^2}{\partial t^2}\phi(\vec{p},t) + \mathrm{e}^{i\vec{p}\cdot\vec{x}}\vec{p}^2\phi(\vec{p},t) + \mathrm{e}^{i\vec{p}\cdot\vec{x}}m^2\phi(\vec{p},t) \\
0 &= (\frac{\partial^2}{\partial t^2} + \vec{p}^2 + m^2)\phi(\vec{p},t)
\end{aligned}$$

$$\omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

2.2 自由標量場正則量子化的一些細節

$$\phi(\vec{x}) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{\vec{p}}}}(a_{\vec{p}}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}} + a^\dagger_{\vec{p}}\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}})$$
$$\pi(\vec{x}) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}(-\mathrm{i})\sqrt{\frac{\omega_{\vec{p}}}{2}}(a_{\vec{p}}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}} - a^\dagger_{\vec{p}}\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}})$$

2.2.1 反粒子竟是我自己?!

$$\phi(\vec{x}) = \phi^\dagger(\vec{x})$$

2.2.2 場算符的對易關係和產生湮滅算符的對易關係的關係?

$$[\phi(\vec{x}),\phi(\vec{y})] = [\pi(\vec{x}),\pi(\vec{y})] = 0,\ [\phi(\vec{x}),\pi(\vec{y})] = \mathrm{i}\delta^{(3)}(\vec{x}-\vec{y})$$
$$\Leftrightarrow [a_{\vec{p}},a_{\vec{q}}] = [a^\dagger_{\vec{p}},a^\dagger_{\vec{q}}] = 0,\ [a_{\vec{p}},a^\dagger_{\vec{p}}] = (2\pi)^2\delta^{(3)}(\vec{p}-\vec{q})$$

$$\begin{aligned}
[a_{\vec{p}},a^\dagger_{\vec{p}}] &= (2\pi)^2\delta^{(3)}(\vec{p}-\vec{q}) \quad \Rightarrow \\
[\phi(\vec{x}),\phi(\vec{y})] &= \int \frac{\mathrm{d}^3p\mathrm{d}^3q}{(2\pi)^6}\frac{-\mathrm{i}}{2}\sqrt{\frac{\omega_{\vec{q}}}{\omega_{\vec{p}}}}\left([a_{\vec{p}},a_{\vec{p}}]\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\vec{q}\cdot\vec{y}} - [a_{\vec{p}},a^\dagger_{\vec{q}}]\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\vec{q}\cdot\vec{y}} + [a^\dagger_{\vec{p}},a_{\vec{q}}]\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\vec{q}\cdot\vec{y}} - [a^\dagger_{\vec{p}},a^\dagger_{\vec{q}}]\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\vec{q}\cdot\vec{y}}\right) \\
&= \int \frac{\mathrm{d}^3p\mathrm{d}^3q}{(2\pi)^6}\frac{-\mathrm{i}}{2}\sqrt{\frac{\omega_{\vec{q}}}{\omega_{\vec{p}}}}\left(-[a_{\vec{p}},a^\dagger_{\vec{q}}]\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\vec{q}\cdot\vec{y}} + [a^\dagger_{\vec{p}},a_{\vec{q}}]\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\vec{q}\cdot\vec{y}}\right) \\
&= \int \frac{\mathrm{d}^3p\mathrm{d}^3q}{(2\pi)^6}\frac{-\mathrm{i}}{2}\sqrt{\frac{\omega_{\vec{q}}}{\omega_{\vec{p}}}}\left(-(2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\vec{q}\cdot\vec{y}} - (2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\vec{q}\cdot\vec{y}}\right) \\
&= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{-\mathrm{i}}{2}\left[-\mathrm{e}^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} - \mathrm{e}^{\mathrm{i}\vec{p}\cdot(\vec{y}-\vec{x})}\right] \\
&\xlongequal{\delta^{(3)}(\vec{x})=\int\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}} \mathrm{i}\delta^{(3)}(\vec{x}-\vec{y})
\end{aligned}$$

2.2.3 計算哈密頓量中的“三重積分”

$$\begin{aligned}
H =& \frac{1}{2}\int d^3x\pi^2 + (\nabla\phi)^2 + m^2\phi^2 \\
=& \frac{1}{2}\int \frac{d^3xd^3pd^3q}{(2\pi)^6}\left[-\frac{\sqrt{\omega_{\vec{p}}\omega_{\vec{q}}}}{2}\left(a_{\vec{p}}e^{i\vec{p}\cdot\vec{x}} - a^\dagger_{\vec{p}}e^{-i\vec{p}\cdot\vec{x}}\right)\left(a_{\vec{q}}e^{i\vec{q}\cdot\vec{x}} - a^\dagger_{\vec{q}}e^{-i\vec{q}\cdot\vec{x}}\right)\right. \\
&+ \frac{1}{2\sqrt{\omega_{\vec{p}}\omega_{\vec{q}}}}\left(i\vec{p}a_{\vec{p}}e^{i\vec{p}\cdot\vec{x}} - i\vec{p}a^\dagger_{\vec{p}}e^{-i\vec{p}\cdot\vec{x}}\right)\cdot\left(i\vec{q}a_{\vec{q}}e^{i\vec{q}\cdot\vec{x}} - i\vec{q}a^\dagger_{\vec{q}}e^{-i\vec{q}\cdot\vec{x}}\right) \\
&\left.+\frac{m^2}{2\sqrt{\omega_{\vec{p}}\omega_{\vec{q}}}}\left(a_{\vec{p}}e^{i\vec{p}\cdot\vec{x}} + a^\dagger_{\vec{p}}e^{-i\vec{p}\cdot\vec{x}}\right)\left(a_{\vec{q}}e^{i\vec{q}\cdot\vec{x}} + a^\dagger_{\vec{q}}e^{-i\vec{q}\cdot\vec{x}}\right)\right]
\end{aligned}$$

實際上，這很爽。

1. 展開：

$$\begin{aligned}
&\left(a_{\vec{p}}e^{i\vec{p}\cdot\vec{x}} - a^\dagger_{\vec{p}}e^{-i\vec{p}\cdot\vec{x}}\right)\left(a_{\vec{q}}e^{i\vec{q}\cdot\vec{x}} - a^\dagger_{\vec{q}}e^{-i\vec{q}\cdot\vec{x}}\right) \\
=& a_{\vec{p}}a_{\vec{q}}\mathrm{e}^{\mathrm{i}(\vec{p}+\vec{q})\cdot\vec{x}} + a^\dagger_{\vec{p}}a^\dagger_{\vec{q}}\mathrm{e}^{-\mathrm{i}(\vec{p}+\vec{q})\cdot\vec{x}} - a^\dagger_{\vec{p}}a_{\vec{q}}\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{p})\cdot\vec{x}} - a_{\vec{p}}a^\dagger_{\vec{q}}\mathrm{e}^{\mathrm{i}(\vec{p}-\vec{q})\cdot\vec{x}}
\end{aligned}$$

2.

$$\int \frac{\mathrm{d}^3 x}{(2\pi)^3} \mathrm{e}^{\mathrm{i}(\vec{p}+\vec{q})\cdot\vec{x}} = \delta^{(3)}\left(\vec{p}+\vec{q}\right)$$

3.

$$\int \mathrm{d}^3 q \sqrt{\omega_{\vec{p}}\omega_{\vec{q}}} a_{\vec{p}} a_{\vec{q}} \delta^{(3)}\left(\vec{p}+\vec{q}\right) = \omega_{\vec{p}} a_{\vec{p}} a_{-\vec{p}}$$

$$H = \frac{1}{2}\int \frac{\mathrm{d}^3 p}{(2\pi)^3} \omega_{\vec{p}} \left[a_{\vec{p}} a_{\vec{p}}^\dagger + a_{\vec{p}}^\dagger a_{\vec{p}}\right] = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \omega_{\vec{p}} \left[a_{\vec{p}}^\dagger a_{\vec{p}} + \frac{1}{2}(2\pi)^3 \delta^{(3)}(0)\right]$$

normal ordering:

$$H = \frac{1}{2}(\omega q - \mathrm{i}p)(\omega q + \mathrm{i}p) \Rightarrow H = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \omega_{\vec{p}} a_{\vec{p}}^\dagger a_{\vec{p}},\ H\left|0\right\rangle = 0$$

2.3 看見真空就趕緊對易子!

$$\vec{P} \equiv -\int \mathrm{d}^3 x\ \pi \nabla \phi = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \vec{p} a_{\vec{p}}^\dagger a_{\vec{p}},\ \left|\vec{p}\right\rangle = a_{\vec{p}}^\dagger \left|0\right\rangle$$

$$\begin{aligned}
\vec{P}\left|\vec{p}\right\rangle &= \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \vec{p} a_{\vec{p}}^\dagger a_{\vec{p}} a_{\vec{p}}^\dagger \left|0\right\rangle \\
&\xlongequal{a_{\vec{p}} a_{\vec{p}}^\dagger = a_{\vec{p}}^\dagger a_{\vec{p}} + (2\pi)^3 \delta^{(3)}(0)} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \vec{p} a_{\vec{p}}^\dagger a_{\vec{p}}^\dagger a_{\vec{p}} \left|0\right\rangle + \int \mathrm{d}^3 p\ \vec{p} \delta^{(3)}(0) a_{\vec{p}}^\dagger \left|0\right\rangle \\
&\xlongequal{a_{\vec{p}}\left|0\right\rangle = 0} \vec{p}\left|\vec{p}\right\rangle
\end{aligned}$$

2.4 在證明相對論性歸一化因子的過程中:

$$\begin{aligned}
\delta[f(x)] &= \sum_{\{x_i | f(x_i)=0 (\text{要求單根!})\}} \frac{1}{|f'(x_i)|} \delta(x - x_i) \\
\delta(p_0^2 - \vec{p}^2 - m^2) &= \delta(p_0^2 - E_{\vec{p}}^2) \\
&= \frac{\delta(p_0 - E_{\vec{p}}) + \delta(p_0 + E_{\vec{p}})}{2E_{\vec{p}}} \\
\int \mathrm{d}^4 p\ \delta(p_0^2 - \vec{p}^2 - m^2)\Big|_{p_0>0} &= \int \frac{\mathrm{d}^3 p}{2p_0}\Big|_{p_0 = E_{\vec{p}}}
\end{aligned}$$

$$\left|p\right\rangle = \sqrt{2E_{\vec{p}}}\left|\vec{p}\right\rangle$$

2.5 Heisenberg 運動方程的計算

這裏只寫出一個罷:

$$\begin{aligned}
\dot{\pi} = \mathrm{i}[H,\pi] &= \mathrm{i}\left[\frac{1}{2}\int \mathrm{d}^3 y\ \pi(y)^2 + \nabla\phi(y)^2 + m^2\phi(y)^2, \pi(x)\right] \\
&= \frac{\mathrm{i}}{2}\int \mathrm{d}^3 y\ \left([\nabla\phi(y)^2, \pi(x)] + m^2[\phi(y)^2, \pi(x)]\right) \\
&= \frac{\mathrm{i}}{2}\int \mathrm{d}^3 y\ \left(2\nabla_y\phi(y)\nabla_y[\phi(y), \pi(x)] + m^2\ 2\phi(y)[\phi(y), \pi(x)]\right)
\end{aligned}$$

其中有這麼一步:

$$\int \mathrm{d}^3y \left(\nabla_y \delta^{(3)}(\vec{x}-\vec{y})\right) \nabla_y \phi(y) = \int \mathrm{d}^3y \nabla_y \left(\delta^{(3)}(\vec{x}-\vec{y}) \nabla_y \phi(y)\right) - \int \mathrm{d}^3y \ \delta^{(3)}(\vec{x}-\vec{y}) \left(\nabla_y^2 \phi(y)\right)$$
$$= -\nabla^2 \phi(x)$$

2.6 計算 Feynman 傳播子的一種形式

$$D(x-y) \equiv \int \frac{\mathrm{d}^3}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

$$\Delta_F(x-y) = \langle 0|\, T\phi(x)\phi(y)\, |0\rangle = \begin{cases} D(x-y) & x^0 > y^0 \\ D(y-x) & y^0 > x^0 \end{cases},\ T\phi(x)\phi(y) = \begin{cases} \phi(x)\phi(y) & x^0 > y^0 \\ \phi(y)\phi(x) & y^0 > x^0 \end{cases}$$

證: $\Delta_F(x-y) = \int \frac{\mathrm{d}^4}{(2\pi)^4} \frac{\mathrm{i}}{p^2-m^2} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$

實際上只不過是複變函數的留數定理，只需注意:

1. 注意選擇積分路徑的方向。爲了使得 $\mathrm{e}^{-\mathrm{i}p^0(x^p y^p)} \to 0$，選擇實軸和包圍下半平面的大圓。
2. 注意繞過實軸上的奇點的方向。

$$\Delta_F(x-y) = \pi\mathrm{i}\left(-\mathrm{Res}f(+E_{\vec{p}}) + \mathrm{Res}f(-E_{\vec{p}})\right)$$

$$f(p_0) = \int \frac{\mathrm{d}^3p}{(2\pi)^4} \frac{i}{(p_0+E_{\vec{p}})(p_0-E_{\vec{p}})} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

由於兩個都是單機點:

$$\mathrm{Res}f(z_0) = \lim_{z\to z_0}\left[(z-z_0)f(z)\right]$$

2.7 關於回到非相對論量子力學的情形

關於回到非相對論量子力學的情形，只需注意到幾個定義，則相應的推導問題不大:

$$\psi^\dagger(\vec{x}) = \int \frac{\mathrm{d}^3p}{(2\pi)^3} a_{\vec{p}}^\dagger \mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}}$$

$$\vec{P} = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\ \vec{p} a_{\vec{p}}^\dagger a_{\vec{p}},\ \vec{X} = \int \mathrm{d}^3x\ \vec{x}\psi^\dagger(\vec{x})\psi(\vec{x})$$

$$\vec{P}\,|\vec{p}\rangle = \vec{p}\,|\vec{p}\rangle\,,\ \vec{X}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$$

坐標表象下:

$$|\varphi\rangle = \int \mathrm{d}^3x\ \varphi(\vec{x})\,|\vec{x}\rangle$$

唉，似乎一些數學公式顯示效果並不好。

3 相互作用場

3.1 本章思路

1. 相互作用繪景

2. 求解相互作用繪景下的運動方程 $\rightarrow$ Dyson 公式/演化算符

$$\left|\psi(t)\right\rangle_I = U(t,t_0)\left|\psi(t,t_0)\right\rangle_I,\ U(t,t_0) = T\exp\left(-\mathrm{i}\int_{t_0}^{t} H_I(t')\mathrm{d}t'\right)$$

3. 希望得到的形式：$\left\langle f\right| a_1^\dagger a_2^\dagger \dots a_1 a_2 \left|i\right\rangle$ 即 normal ordering $\rightarrow$ Wick 定理

4. 推导出 Feynman 规则

3.2 使用微擾理論

由量綱分析知，對於 $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \sum_{n\leq 3}\frac{\lambda_n}{n!}\phi^n$，在低能情況下，只有 $n=3$ or 4 是需要考慮的。

3.3 相互作用繪景

$$H = H_0 + H_{\text{int}}$$

$$H_{\text{int}} \equiv (H_{\text{int}})_I = \mathrm{e}^{\mathrm{i}H_0 t}(H_{\text{int}})_S\mathrm{e}^{-\mathrm{i}H_0 t}$$

$$\mathrm{i}\frac{\mathrm{d}\left|\psi\right\rangle_I}{\mathrm{d}t} = H_I(t)\left|\psi\right\rangle_I$$

3.4 第一次求解振幅

複標量場和實標量場耦合：$H_{\text{int}} = g\int\mathrm{d}^3x\ \psi^\dagger\psi\phi$

$$\left|i\right\rangle = \sqrt{2E_{\vec{p}}}\ a_{\vec{p}}^\dagger\left|0\right\rangle,\ \left|f\right\rangle = \sqrt{4E_{\vec{q}_1}E_{\vec{q}_2}}\ b_{\vec{q}_1}^\dagger c_{\vec{q}_2}^\dagger\left|0\right\rangle$$

計算到一階項：

$$\begin{aligned}
\left\langle f\right| U(+\infty,-\infty)\left|i\right\rangle &\equiv \left\langle f\right| 1-\mathrm{i}\int\mathrm{d}t\ H_I(t)\left|i\right\rangle = \left\langle f\right| -\mathrm{i}\int\mathrm{d}t\ H_I(t)\left|i\right\rangle \\
&= -\mathrm{i}\left\langle f\right|\int\mathrm{d}t\ H_I(t)\left|i\right\rangle \\
&= -\mathrm{i}\left\langle f\right|\int\mathrm{d}t\ d\int\mathrm{d}^3x\ \psi^\dagger(x)\psi(x)\phi(x)\left|i\right\rangle \\
&= -\mathrm{i}g\left\langle f\right|\int\mathrm{d}^4x\ \psi^\dagger(x)\psi(x)\phi(x)\left|i\right\rangle \\
&= -\mathrm{i}g\left\langle f\right|\int\mathrm{d}^4x\ \psi^\dagger(x)\psi(x)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\sqrt{2E_{\vec{p}}}}{\sqrt{2E_{\vec{k}}}}a_{\vec{k}}a_{\vec{p}}^\dagger\mathrm{e}^{-\mathrm{i}k\cdot x}\left|0\right\rangle
\end{aligned}$$

爲什麼只寫出來半個 $\phi(x)$ 的展開式? 因爲另外半個:

$$
\begin{aligned}
&\langle f|\int \mathrm{d}^4x\ \psi^\dagger(x)\psi(x)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\sqrt{2E_{\vec{p}}}}{\sqrt{2E_{\vec{k}}}}a^\dagger_{\vec{k}}a^\dagger_{\vec{p}}\mathrm{e}^{\mathrm{i}k\cdot x}\,|0\rangle\\
=&\langle 0|\,a^\dagger_{\vec{k}}\sqrt{4E_{\vec{q}_1}E_{\vec{q}_2}}b_{\vec{q}_1}c_{\vec{q}_2}\int \mathrm{d}^4x\ \psi^\dagger(x)\psi(x)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\sqrt{2E_{\vec{p}}}}{\sqrt{2E_{\vec{k}}}}a^\dagger_{\vec{p}}\mathrm{e}^{\mathrm{i}k\cdot x}\,|0\rangle\text{，直接移過去了，因爲 a、b、c 之間的對易子為}\\
=&0
\end{aligned}
$$

回來繼續:

$$
\begin{aligned}
&-\mathrm{i}g\,\langle f|\int \mathrm{d}^4x\ \psi^\dagger(x)\psi(x)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\sqrt{2E_{\vec{p}}}}{\sqrt{2E_{\vec{k}}}}a_{\vec{k}}a^\dagger_{\vec{p}}\mathrm{e}^{-\mathrm{i}k\cdot x}\,|0\rangle\\
\xlongequal{a_{\vec{k}}a^\dagger_{\vec{p}}=a^\dagger_{\vec{p}}a_{\vec{k}}+(2\pi)^3\delta^{(3)}(\vec{p}-\vec{k})}&-\mathrm{i}g\,\langle f|\int \mathrm{d}^4x\ \psi^\dagger(x)\psi(x)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\sqrt{2E_{\vec{p}}}}{\sqrt{2E_{\vec{k}}}}(2\pi)^3\delta^{(3)}(\vec{p}-\vec{k})\mathrm{e}^{-\mathrm{i}k\cdot x}\,|0\rangle\\
=&-\mathrm{i}g\,\langle f|\int \mathrm{d}^4x\ \psi^\dagger(x)\psi(x)\mathrm{e}^{-\mathrm{i}p\cdot x}\,|0\rangle\\
=&-\mathrm{i}g\,\langle 0|\int\frac{\mathrm{d}^4x\mathrm{d}^3k_1\mathrm{d}^3k_2}{(2\pi)^6}\frac{\sqrt{E_{\vec{q}_1}E_{\vec{q}_2}}}{\sqrt{E_{\vec{k}_1}E_{\vec{k}_2}}}b_{\vec{q}_1}c_{\vec{q}_2}c^\dagger_{\vec{k}_1}b^\dagger_{\vec{k}_2}\,\mathrm{e}^{\mathrm{i}(k_1+k_2-p)\cdot x}\,|0\rangle\\
=&-\mathrm{i}g\,\langle 0|\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}(q_1+q_2-p)\cdot x}\,|0\rangle\\
=&-\mathrm{i}g(2\pi)^4\delta^{(4)}(q_1+q_2-p)
\end{aligned}
$$

3.5 Wick 定理計算核子散射 $\psi\psi\to\psi\psi$

$$
|i\rangle=\sqrt{2E_{\vec{p}_1}}\sqrt{2E_{\vec{p}_2}}b^\dagger_{\vec{p}_1}b^\dagger_{\vec{p}_2}\,|0\rangle\equiv|p_1,p_2\rangle\,,\ |f\rangle=\sqrt{2E_{\vec{p}'_1}}\sqrt{2E_{\vec{p}'_2}}b^\dagger_{\vec{p}'_1}b^\dagger_{\vec{p}'_2}\,|0\rangle\equiv|p'_1,p'_2\rangle
$$

振幅:

$$
\frac{(-\mathrm{i}g)^2}{2}\int \mathrm{d}^4x_1\mathrm{d}^4x_2 T\left(\psi^\dagger(x_1)\psi(x_1)\phi(x_1)\psi^\dagger(x_2)\psi(x_2)\phi(x_2)\right)
$$

由 Wick 定理做替換，之後會留下的非零項只有 $:\psi^\dagger(x_1)\psi(x_1)\psi^\dagger(x_2)\psi(x_2):\overbrace{\phi(x_1)\phi(x_2)}$，因爲先要湮滅初態的兩個核子，再生成模態的兩個核子，對於其他的項，會產生介子 which 不是我們想要的。

注 🐖：那個式子 🦁 上方的大括號是 `\overbrace`

其他的就正常運算應該沒什麼問題吧，有這麼一項 $\langle 0|\,\psi(x)\,|p\rangle=\mathrm{e}^{-\mathrm{i}p\cdot x}$，它來自:

$$
\langle 0|\,\psi(x)\,|p\rangle=\langle 0|\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\left(b_{\vec{p}}\mathrm{e}^{-\mathrm{i}p\cdot x}+c^\dagger_{\vec{p}}\mathrm{e}^{+\mathrm{i}p\cdot x}\right)\sqrt{2E_{\vec{p}}}b^\dagger_{\vec{p}}\,|0\rangle
$$

後面不用我繼續寫了罷。還想指出一點，抽象出來就是這麼個東西:

$$
\begin{aligned}
&\int_{-\infty}^{\infty} \mathrm{d}x\ f(x)\delta(x-x_1)\delta(x-x_2) \\
\xlongequal{F(x)\equiv f(x)\delta(x-x_1)} &\int_{-\infty}^{\infty} \mathrm{d}x\ F(x)\delta(x-x_2) \\
=&F(x_2) \\
=&f(x_2)\delta(x_2-x_1) = f(x_1)\delta(x_2-x_1)
\end{aligned}
$$

3.6 Feynman 圖

像近年來的硅芯片一樣，Feynman 圖為大衆帶來了計算。—— Julian Schwinger

$$H_{\text{int}} = g\int \mathrm{d}^3x\ \psi^\dagger\psi\phi$$

3.6.1 怎麽畫

“怎麽畫”指怎麽將想要計算的物理過程轉化爲 Feynman 圖，以運用相應的規則寫出散射振幅表達式。

1. 初態、末態的每個粒子對應一條入射、出射綫（外部綫）；
2. 介子（from 實標量場）用點綫，核子（from 複標量場）用實綫；（這裏的介子和核子不是現實生活中的那個）
3. 為每條綫假設一個動量（大小和方向），類似於電路中的“參考方向”；
4. 實綫（核子）上標注箭頭以指明其荷：
   - 初態 ψ 用入射箭頭，初態 $\bar{\psi}$ 用出射箭頭；
   - 末態 ψ 用出射箭頭，末態 $\bar{\psi}$ 用入射箭頭；
   - 總是，ψ 是比較自然的，$\bar{\psi}$ 是反著的；
5. 對於現在的情況，用三價的頂點將他們連接起來。

在畫圖過程中請考慮對稱性。

3.6.2 怎麽算：Feynman 法則

“怎麽算”指在已經存在一個 Feynman 圖之後怎麽將它轉化爲對應物理過程的散射振幅。

1. 每條内部綫添加一個動量 $\vec{k}$；（内部綫指非入射、出射的綫）
2. 在每個頂點上，可以寫下因子 $(-\mathrm{i}g)(2\pi)^4\delta^{(4)}\left(\sum_i k_i\right)$，其實就是動量守恆；
3. 對於每個内部的點綫，我們知道這是一個動量為 $\vec{k}$ 的介子，可以寫下因子 $\int \frac{\mathrm{d}^2k}{(2\pi)^4}\frac{\mathrm{i}}{k^2-m^2+\mathrm{i}\epsilon}$；對於核子（實綫），仍然有這個因子，但記得用核子的質量。
4. 有圈的情況，進行積分 $\int \mathrm{d}^2k/(2\pi)^4$

這是對於我們現在要處理的哈密頓量的 Feynman 法則，對於以後會見到的其它粒子，會有額外的規則。

注 🐖：不考慮非連通圖，不考慮外部綫上有圈的非截斷圖。

- 介紹完 Feynman 圖之後就是一些介紹性的內容，客觀上來説其中一些東西説的是不夠清楚的。無可指摘，因爲他們還有下一個學期的課程 “AQFT（高等量子場論）”，而我讀這本講義也是只是在學習 GR 之間做一個調劑。更詳細的內容理應不在這裏出現。

3.7 一個積分

佟大爲在用一些奇妙的方法，我不管了，我直接梁昆淼了:

$$
\begin{aligned}
&\int_0^\infty \mathrm{d}k\ \frac{k}{k^2+m^2}\sin(kr)\\
=&\pi\times\left\{\frac{k}{k^2+m^2}\ \mathrm{e}^{\mathrm{i}kr}\ \text{在上半平面所有奇點的留數之和}\right\}\\
=&\pi\times\mathrm{Res}\left[\frac{k}{(k+\mathrm{i}m)(k-\mathrm{i}m)}\ \mathrm{e}^{\mathrm{i}kr}\right]\Bigg|_{k=\mathrm{i}m}\\
=&\pi\times\lim_{k\to\mathrm{i}m}\left[(k-\mathrm{i}m)\cdot\frac{k}{(k+\mathrm{i}m)(k-\mathrm{i}m)}\ \mathrm{e}^{\mathrm{i}kr}\right]\\
=&\pi\times\frac{1}{2}\mathrm{e}^{-mr}
\end{aligned}
$$

- 下一章介紹 Dirac 方程，估計代數內容會比較多，我很期待。😄

4 Dirac 方程

> Dirac 方程中隐藏的东西比作者 1928 年写下它时预想的要多得多。Dirac 本人在一次谈话中说，他的方程式比作者更聪明。然而，应该补充的是，Dirac 发现了大部分额外的见解。—— Weisskopf 評 Dirac

4.1 定義和性質

$$\{\gamma^\mu,\gamma^\nu\}\equiv 2\eta^{\mu\nu}\mathbf{1}$$

- $$(\gamma^0)^2=1,\ (\gamma^i)^2=-1$$

- $$\gamma^0=\begin{pmatrix}0&\mathbf{1}\\\mathbf{1}&0\end{pmatrix},\ \gamma^i=\begin{pmatrix}0&\sigma^i\\-\sigma^i&0\end{pmatrix}$$

- $$\sigma^1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \sigma^2=\begin{pmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{pmatrix},\ \sigma^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\ \{\sigma^i,\sigma^j\}=2\delta^{ij}$$

- $$S^{\rho\sigma}=\frac{1}{4}[\gamma^\rho,\gamma^\sigma]=\frac{1}{2}\gamma^\rho\gamma^\sigma-\frac{1}{2}\eta^{\rho\sigma}$$
$$[S^{\mu\nu},\gamma^\rho]=\gamma^\mu\eta^{\nu\rho}-\gamma^\nu\eta^{\rho\mu}$$
$$[S^{\mu\nu},S^{\rho\sigma}]=\eta^{\nu\rho}S^{\mu\sigma}-\eta^{\mu\rho}S^{\nu\sigma}+\eta^{\mu\sigma}S^{\nu\rho}-\eta^{\nu\sigma}S^{\mu\rho}$$
- Dirac spinor:
$$\phi^\alpha(x)\to S[\Lambda]^\alpha{}_\beta\psi^\beta(\Lambda^{-1}x),\ \Lambda=\exp\left(\frac{1}{2}\Omega_{\rho\sigma}\mathcal{M}^{\rho\sigma}\right),\ S[\Lambda]=\exp\left(\frac{1}{2}\Omega_{\rho\sigma}S^{\rho\sigma}\right)$$
- $$(\gamma^\mu)^\dagger=\gamma^0\gamma^\mu\gamma^0,\ (S^{\mu\nu})^\dagger=-\gamma^0S^{\mu\nu}\gamma^0,\ S[\Lambda]^\dagger=\gamma^0S[\Lambda]^{-1}\gamma^0$$

4.2 Dirac 方程的另一半

$$\mathcal{L}=\bar{\psi}(\mathrm{i}\gamma^\mu\partial_\mu-m)\psi$$

$$\Rightarrow(\mathrm{i}\gamma^\mu\partial_\mu-m)\psi,\ (\mathrm{i}\partial_\mu\bar{\psi}\gamma^\mu+m)\bar{\psi}=0$$

4.3 Dirac spinor 滿足 Klein-Gordon 方程

$$\begin{aligned}&(\mathrm{i}\gamma^\mu\partial_\mu-m)\psi=0\\ \Rightarrow&(\mathrm{i}\gamma^\nu\partial_\nu+m)(\mathrm{i}\gamma^\mu\partial_\mu-m)\psi=0\\ \Rightarrow&-(\gamma^\nu\gamma^\mu\partial_\nu\gamma_\mu+m^2)\psi=0\end{aligned}$$

$$\begin{aligned}&\gamma^\nu\gamma^\mu\partial_\nu\partial_\mu\xlongequal{\text{輪換指標}}\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu\xlongequal{\text{偏導數可交換順序}}\gamma^\mu\gamma^\nu\partial_\nu\partial_\mu\\ =&\frac{1}{2}\left(\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu\right)\partial_\mu\partial_\nu=\frac{1}{2}\left\{\gamma^\mu,\gamma^\nu\right\}\partial_\mu\partial_\nu\\ =&\eta^{\mu\nu}\partial_\mu\partial_\nu=\partial_\mu\partial^\mu\end{aligned}$$

$$-(\partial_\mu\partial^\mu+m^2)\psi=0$$

注 🐖：我的 ‘markdown‘ 編輯器似乎不支持 ‘slashed‘ 宏包？總之，我使用 ‘cancel‘ 來表示 Diarc slash：$A_\mu\gamma^\mu\equiv\cancel{A}$。

4.4 Weyl 方程的導出

發現 Lortenz 群的 Dirac 旋量表示是可約的，可以分解爲只作用在貳份量旋量上的兩個不可約表示。

$$\psi=\begin{pmatrix}u_+\\u_-\end{pmatrix}$$

$$
\begin{aligned}
&(\mathrm{i}\not{\partial}-m)\psi=0 \quad \Rightarrow \\
\mathcal{L}=0=&\bar{\psi}(\mathrm{i}\not{\partial}-m)\psi=\mathrm{i}\bar{\psi}\not{\partial}\psi-m\bar{\psi}\psi \\
=&\mathrm{i}\psi^\dagger\gamma^0\gamma^\mu\partial_\mu\psi-m\psi^\dagger\gamma^0\psi=\mathrm{i}\psi^\dagger\gamma^0(\gamma^0\partial_0+\gamma^i\partial_i)\psi-m\psi^\dagger\gamma^0\psi \\
=&\mathrm{i}\begin{pmatrix}u_+^\dagger & u_-^\dagger\end{pmatrix}\begin{pmatrix}0 & \mathbf{1}\\ \mathbf{1} & 0\end{pmatrix}\begin{pmatrix}0 & \mathbf{1}\\ \mathbf{1} & 0\end{pmatrix}\begin{pmatrix}\partial_0 u_+\\ \partial_0 u_-\end{pmatrix} \\
&-\mathrm{i}\begin{pmatrix}u_+^\dagger & u_-^\dagger\end{pmatrix}\begin{pmatrix}0 & \mathbf{1}\\ \mathbf{1} & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^i\\ -\sigma_i & 0\end{pmatrix}\begin{pmatrix}\partial_i u_+\\ \partial_i u_-\end{pmatrix} \\
&-m\begin{pmatrix}u_+^\dagger & u_-^\dagger\end{pmatrix}\begin{pmatrix}0 & \mathbf{1}\\ \mathbf{1} & 0\end{pmatrix}\begin{pmatrix}u_+\\ u_-\end{pmatrix} \\
\xlongequal{\sigma^\mu=(1,\sigma^i),\bar{\sigma}^\mu=(1,-\sigma^i)}&\mathrm{i}u_-^\dagger\sigma^\mu\partial_\mu u_-+\mathrm{i}u_+^\dagger\bar{\sigma}^\mu\partial_\mu u_+-m(u_+^\dagger u_-+u_-^\dagger u_+) \\
\Rightarrow\xLongrightarrow{m=0}&\mathrm{i}\bar{\sigma}^\mu\partial_\mu u_+=0,\ \mathrm{i}\sigma^\mu\partial_\mu u_-=0
\end{aligned}
$$

4.5 γ^5

- $$\gamma^5=-\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$$
- $$\{\gamma^5,\gamma^\mu\}=0,\ (\gamma^5)^2=+1$$
- $$[S_{\mu\nu},\gamma^5]=0$$
- $$P_\pm=\frac{1}{2}(1\pm\gamma^5),\ P_\pm^2=P_\pm,\ P_+P_-=0$$
- 分清誰是標量誰是矢量誰是張量誰是贗標量誰是軸矢量

4.6 關於 charge conjugate 的一個證明

$$(\gamma^0)^\dagger=\gamma^0,\ (\gamma^i)^\dagger=-\gamma^i$$

$$\psi^{(c)}=C\psi^\star,\ C^\dagger C=1,\ C^\dagger\gamma^\mu C=-(\gamma^\mu)^\star$$

$$C^\dagger C=1\Rightarrow\xLongrightarrow{?\ \text{我還不知道這是否一般地成立}}CC^\dagger=1$$

$$
\begin{aligned}
C^\dagger S^{\mu\nu} C =& C^\dagger \frac{1}{4}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)C \\
=& \frac{1}{4}C^\dagger\gamma^\mu C C^\dagger \gamma^\nu C - \frac{1}{4}C^\dagger\gamma^\nu C C^\dagger\gamma^\mu C \\
=& \frac{1}{4}(\gamma^\mu)^\star(\gamma^\nu)^\star - \frac{1}{4}(\gamma^\nu)^\star(\gamma^\mu)^\star \\
=& (\frac{1}{4}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu))^\star = (S^{\mu\nu})^\star
\end{aligned}
$$

$$
C^\dagger S[\Lambda] C = (S[\Lambda])^\star \Rightarrow \cancel{CC^\dagger} S[\Lambda] C = C(S[\Lambda])^\star
$$

4.7 螺旋度（Helicity）

竟然一筆帶過了……

$$
h = \frac{\mathrm{i}}{2}\epsilon_{ijk}\hat{p}^i S^{jk} = \frac{1}{2}\hat{p}_i \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}
$$

5 量子化 Dirac 場

這一次，狄拉克比邏輯更接近真理。—— Pauli 評 Dirac

5.1 指標運算和矢量運算的一個小 tip

$$
\begin{aligned}
&\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left((\not{p}+m)\gamma^0 \mathrm{e}^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} + (\not{p}-m)\gamma^0\mathrm{e}^{-\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}\right) \\
=&\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left((p_0\gamma^0 + p_i\gamma^i + m)\gamma^0 + (p_0\gamma^0 - p_i\gamma^i - m)\gamma^0\right)\mathrm{e}^{+\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}
\end{aligned}
$$

因爲:

$$
\begin{aligned}
&\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(\not{p}-m\right)\gamma^0\mathrm{e}^{-\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} \\
=&\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(p_0\gamma^0 + p_i\gamma^i - m\right)\gamma^0\mathrm{e}^{-\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} \\
=&\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(p_0\gamma^0 - \vec{p}\cdot\vec{\gamma} - m\right)\gamma^0\mathrm{e}^{-\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} \\
\xlongequal{\vec{p}\to-\vec{p}}&\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(p_0\gamma^0 + \vec{p}\cdot\vec{\gamma} - m\right)\gamma^0\mathrm{e}^{+\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} \\
=&\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(p_0\gamma^0 - p_i\gamma^i - m\right)\gamma^0\mathrm{e}^{+\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}
\end{aligned}
$$

其中:

$$
\not{p} = p_\mu\gamma^\mu = p_0\gamma^0 + p_i\gamma^i
$$

$$
\vec{p}\cdot\vec{\gamma} = \sum_{i=1}^{3} p^i\gamma^i = -p_i\gamma^i
$$

同理:

$$\partial_i \mathrm{e}^{+\vec{p}\cdot\vec{x}} = \partial_i \mathrm{e}^{-p_i x^i} = -p_i \mathrm{e}^{-p_i x^i}$$

- 有一點, 佟大爲很難讓我滿意, 即: 賦予場算符對易關係之後求產生湮滅算符的對易關係時, 他從來都是先告訴你正確的結果, 然後展示一下正確的結果（產生湮滅算符的對易關係）確實能給出他的條件（場算符的對易關係）。這可不能稱作是證明。

5.2 Dyson-Wick 方法求解散射振幅

- 這是我自己起的名字, 意思就是第三章中的那種方法, 先用 Dyson 公式得到演化算符, 再將其 Taylor 展開, 用 Wick 定理和物理過程進行化簡, 最後用對易關係和 δ 函數的定義及性質進行化簡。

5.2.1 費米子的傳播子 $S_F(x-y)$

$$\begin{aligned}\langle 0|\,\psi_\alpha(x)\bar{\psi}_\beta(y)\,|0\rangle = \langle 0| \sum_{s,r} \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\left[b^s_{\vec{p}}u^s(\vec{p})\mathrm{e}^{-\mathrm{i}p\cdot x} + c^{s\dagger}_{\vec{p}}v^s(\vec{p})\mathrm{e}^{+\mathrm{i}p\cdot x}\right] \\ \times \int \frac{\mathrm{d}^3q}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{q}}}}\left[b^{r\dagger}_{\vec{q}}\bar{u}^r(\vec{q})\mathrm{e}^{+\mathrm{i}q\cdot y} + c^r_{\vec{q}}\bar{v}^r(\vec{q})\mathrm{e}^{-\mathrm{i}q\cdot y}\right]|0\rangle\end{aligned}$$

由于后面的 $|0\rangle$，$c^r_{\vec{q}}$ 没了；由于前面的 $\langle 0|$，$c^{s\dagger}_{\vec{p}}$ 没了。

$$\begin{aligned}\langle 0|\,\psi_\alpha(x)\bar{\psi}_\beta(y)\,|0\rangle &= \langle 0| \sum_{s,r}\int \frac{\mathrm{d}^3p\mathrm{d}^3q}{(2\pi)^6}\frac{1}{\sqrt{4E_{\vec{p}}E_{\vec{q}}}}\left(b^s_{\vec{p}}b^{r\dagger}_{\vec{q}}u^s(\vec{p})\bar{u}^r(\vec{q})\mathrm{e}^{-\mathrm{i}p\cdot x}\mathrm{e}^{+\mathrm{i}q\cdot y}\right)|0\rangle \\ &\xlongequal{b^s_{\vec{p}}b^{r\dagger}_{\vec{q}}=b^{r\dagger}_{\vec{q}}b^s_{\vec{p}}+(2\pi)^3\delta^{sr}\delta^{(3)}(\vec{p}-\vec{q})} \langle 0| \sum_{s,r}\int \frac{\mathrm{d}^3p\mathrm{d}^3q}{(2\pi)^6}\frac{1}{\sqrt{4E_{\vec{p}}E_{\vec{q}}}}\left((2\pi)^3\delta^{sr}\delta^{(3)}(\vec{p}-\vec{q})u^s(\vec{p})\bar{u}^r(\vec{q})\mathrm{e}^{-\mathrm{i}p\cdot x}\mathrm{e}^{+\mathrm{i}q\cdot y}\right)|0\rangle \\ &= \sum_s \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}u^s(\vec{p})\bar{u}^s(\vec{p})\mathrm{e}^{-\mathrm{i}p\cdot(x-y)} \\ &\xlongequal{\sum^s u^s(\vec{p})\bar{u}^s(\vec{p})=\not{p}+m} \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}(\not{p}+m)_{\alpha\beta}\ \mathrm{e}^{-\mathrm{i}p\cdot(x-y)}\end{aligned}$$

同理:

$$\langle 0|\,\bar{\psi}_\beta(y)\psi_\alpha(x)\,|0\rangle = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}(\not{p}-m)_{\alpha\beta}\ \mathrm{e}^{+\mathrm{i}p\cdot(x-y)}$$

$$S_F(x-y) = \langle 0|\,T\psi(x)\bar{\psi}(y)\,|0\rangle \equiv \begin{cases}\langle 0|+\psi(x)\bar{\psi}(y)\,|0\rangle & x^0 > y^0 \\ \langle 0|-\psi(x)\bar{\psi}(y)\,|0\rangle & x^0 < y^0\end{cases}$$

$$S_F(x-y) = \mathrm{i}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p\cdot(x-y)}\frac{\gamma\cdot p+m}{p^2-m^2+\mathrm{i}\epsilon}$$

$$(\mathrm{i}\not{\partial}_x - m)S_F(x-y) = \mathrm{i}\delta^{(4)}(x-y)$$

$$\overbrace{\psi(x)\bar{\psi}(y)} = T(\psi(x)\bar{\psi}(y)) - :\psi(x)\bar{\psi}(y): = S_F(x-y)$$

5.2.2 注意順序

在處理費米子時有很多需要注意的順序，剛開始時容易忘記的有:

- $$|f\rangle = \sqrt{4E_{\vec{p}'}E_{\vec{q}'}}b_{\vec{p}'}^{s'\dagger}b_{\vec{q}'}^{r'\dagger}|0\rangle \to \langle f| = \langle 0| b_{\vec{p}'}^{r'}b_{\vec{q}'}^{s'}\sqrt{4E_{\vec{p}'}E_{\vec{q}'}}$$

- 因爲 $\{b, c^\dagger\}$ 這樣的東西 $= 0$，所以交換順序時一定要留意負號——我甚至在這裏卡了一會兒……

- 我看不起費米子，儘管我曾經覺得費米子很酷。我認爲費米子沒有中國傳統精神——它總是不合群、總是特立獨行，更要緊的是，它總是傷害無辜的人。

- 我對費米子的成見與費米先生無關，恰恰相反，我對費米先生的評價和楊振寧先生的一致：他是最後一個理論和實驗都特別强的物理學家。

5.2.3 爲什麼計算中沒有出現 c 和 $c^\dagger$

We may ignore the $c^\dagger$ pieces in ψ since they give no contribution at order λ^2.

我們正在處理 $\psi\psi \to \psi\psi$，如果數學的過程中有 $c^\dagger$ 以及 c 參與，那麼對應的物理過程至少是 $\psi\psi \to \psi\psi\psi\bar{\psi} \to \psi\psi$，這已經不是我們考慮的最低階的情況辣!

5.2.4 計算 $b_{\vec{k}_1}^m b_{\vec{k}_2}^n b_{\vec{p}}^{s\dagger} b_{\vec{q}}^{r\dagger}|0\rangle$

看到這裏我知道這對你來說已經很簡單了，但是我還是打算寫出來，因爲我覺得它很爽 :)

$$\left\{b_{\vec{p}}^r, b_{\vec{q}}^{s\dagger}\right\} = (2\pi)^3\delta^{rs}\delta^{(3)}(\vec{p}-\vec{q}) \Rightarrow b_{\vec{p}}^r b_{\vec{q}}^{s\dagger} = -b_{\vec{q}}^{s\dagger}b_{\vec{p}}^r + (2\pi)^3\delta^{rs}\delta^{(3)}(\vec{p}-\vec{q})$$

$$\begin{aligned}
&b_{\vec{k}_1}^m b_{\vec{k}_2}^n b_{\vec{p}}^{s\dagger} b_{\vec{q}}^{r\dagger}|0\rangle \\
=&b_{\vec{k}_1}^m\left(-b_{\vec{p}}^{s\dagger}b_{\vec{k}_2}^n + (2\pi)^3\delta^{ns}\delta^{(3)}\left(\vec{k}_2-\vec{p}\right)\right)b_{\vec{q}}^{r\dagger}|0\rangle \\
=&-\left(-b_{\vec{p}}^{s\dagger}b_{\vec{k}_1}^m + (2\pi)^3\delta^{ms}\delta^{(3)}(\vec{k}_1-\vec{p})\right)\left(-b_{\vec{q}}^{r\dagger}b_{\vec{k}_2}^n + (2\pi)^3\delta^{nr}\delta^{(3)}(\vec{k}_2-\vec{q})\right)|0\rangle \\
&+(2\pi)^3\delta^{ns}\delta^{(3)}\left(\vec{k}_2-\vec{p}\right)\left(-b_{\vec{q}}^{r\dagger}b_{\vec{k}_1}^m + (2\pi)^3\delta^{mr}\delta^{(3)}(\vec{q}-\vec{k}_1)\right)|0\rangle \\
=&-(2\pi)^6\delta^{ms}\delta^{nr}\delta^{(3)}(\vec{k}_1-\vec{p})\delta^{(3)}(\vec{k}_2-\vec{q})|0\rangle + (2\pi)^6\delta^{ns}\delta^{mr}\delta^{(3)}(\vec{k}_2-\vec{p})\delta^{(3)}(\vec{k}_1-\vec{q})|0\rangle
\end{aligned}$$

- 個人感覺，推導出來 Feynman 規則之後在應用時最要緊的還是對稱性的要求!
- 然而，在 Feynman 規則推導出之後，似乎理論已經結束了。

6 量子電動力學

6.1 Lorentz 規範無法唯一決定勢

假設現在已有 $\partial_\mu A^\mu = f(x)$，慾滿足 Lorentz 規範條件，有:

$$A'_\mu = A_\mu + \partial_\mu \lambda,\ A''_\mu = A_\mu + \partial_\mu \lambda + \partial_\mu \lambda'$$

其中:

$$\partial_\mu \partial^\mu \lambda = -f,\ \partial_\mu \partial^\mu \lambda' = 0$$

可得到:

$$\partial_\mu (A')^\mu = 0,\ \partial_\mu (A'')^\mu = 0$$

都滿足 Lorentz 規範。有需要时可以进一步选择 Coulomb 规范:

$$\nabla \cdot \vec{A} = 0$$

无源情况下，它进一步导出：$A_0 = 0$。

6.2 計算正則動量

$\pi^0 = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_0)} = 0$ 是顯然的，因爲 $\mathcal{L}$ 中根本沒有出現 $\partial_0 A_0$。

$$\begin{aligned}
\pi^i =& \frac{\partial \mathcal{L}}{\partial(\partial_0 A_i)} = \frac{\partial}{\partial(\partial_0 A_i)} \left[-\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \right] \\
=& -\frac{1}{4} \frac{\partial}{\partial(\partial_0 A_i)} \left[(\partial_0 A_i - \partial_i A_0)(\partial^0 A^i - \partial^i A^0) + (\partial_i A_0 - \partial_0 A_i)(\partial^i A^0 - \partial^0 A^i) \right] \\
=& -\frac{1}{2} \frac{\partial}{\partial(\partial_0 A_i)} \left[(\partial_0 A_i)(\partial^0 A^i) - (\partial_0 A_i)(\partial^i A^0) - (\partial_i A_0)(\partial^0 A^i) \right] \\
=& -\frac{1}{2} \frac{\partial}{\partial(\partial_0 A_i)} \left[\eta^{00} \eta^{ii} (\partial_0 A_i)(\partial_0 A_i) - (\partial_0 A_i)(\partial^i A^0) - \eta^{00} \eta^{ii} (\partial_i A_0)(\partial_0 A_i) \right] \\
=& -\frac{1}{2} \left[2\eta^{00} \eta^{ii} (\partial_0 A_i) - (\partial^i A^0) - \eta^{00} \eta^{ii} (\partial_i A_0) \right] \\
=& -\frac{1}{2} \left[2(\partial^0 A^i) - 2(\partial^i A^0) \right] = -(\partial^0 A^i - \partial^i A^0)
\end{aligned}$$

6.3 不同規範下的量子化

- Coulomb 規範下給正則坐標和正則動量賦予的對易關係屬實有點怪了噢:
  - $$[A_i(\vec{x}), E_j(\vec{y})] = \mathrm{i}(\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2}) \delta^{(3)}(\vec{x} - \vec{y})$$
  - 脚注中提到了 Dirac 的那本小書，我還沒來得及去查 :)
  - 這將導致極化矢量奇怪的歸一化：$\sum_{r=1}^{2} \epsilon_r^i(\vec{p}) \epsilon_r^j(\vec{p}) = \delta^{ij} - \frac{p^i p^j}{|\vec{p}|^2}$
- 對於 Lorentz 規範的情況，我覺得佟大爲處理的不夠簡潔了

6.4 永遠不要忘記分部積分/Leibnitz 法則!

6.4.1 一个例子

$$\int \mathrm{d}^3x\ \frac{1}{2}(\dot{\vec{A}}+\nabla A_0)^2=\int \mathrm{d}^3x\ \frac{1}{2}\dot{\vec{A}}^2+\frac{1}{2}(\nabla A_0)^2$$

因爲:

$$\begin{aligned}&\int \mathrm{d}^3x\ \dot{\vec{A}}\cdot\nabla A_0\\=&\int \mathrm{d}^3x\ \nabla\cdot(\dot{\vec{A}}A_0)-A_0\frac{\mathrm{d}}{\mathrm{a}t}(\nabla\cdot\vec{A})\\\xlongequal{Gauss定理\ +\ \nabla\cdot\vec{A}=0}&0\end{aligned}$$

6.4.2 另一个例子

$$\begin{aligned}&\int \mathrm{d}^3x\ (\nabla A_0)\cdot(\nabla A_0)\\=&\int \mathrm{d}^3x\ \nabla\cdot(A_0\nabla A_0)-A_0\nabla\cdot(\nabla A_0)\\\xlongequal{Gauss定理}&-\int \mathrm{d}^3x\ A_0\nabla^2A_0\\\xlongequal{\nabla^2A_0=-ej_0}&\int \mathrm{d}^3x\ A_0\cdot ej^0\\=&e^2\int \mathrm{d}^3x\ \int \mathrm{d}^3x'\ \frac{j_0(\vec{x})j_0(\vec{x}')}{4\pi|\vec{x}-\vec{x}'|}\end{aligned}$$

有個東西呢，它看起來有點眼花繚亂，但是其實很好記，立即推!:（張宇語氣 😁

$$\nabla^2\varphi=\frac{\rho}{\varepsilon_0}\Leftrightarrow\varphi(\vec{x},t)=\int \mathrm{d}^3x'\ \frac{\rho(\vec{x}',t)}{4\pi\varepsilon_0|\vec{x}-\vec{x}'|}$$

6.5 求解 Compton 散射時的一步化簡

1. q 是光子的動量，則有：$\not{q}\not{q}=q_\mu\gamma^\mu q_\nu\gamma^\nu=q_\mu q_\nu\eta^{\mu\nu}=q_\mu q^\mu=0$
2. $(\not{p}+m)u^s(\vec{p})=[2\not{p}-(\not{p}-m)]u^s(\vec{p})\xlongequal{(\not{p}-m)u(\vec{p})=0}2\not{p}u^s(\vec{p})$
3. $\not{p}\not{q}=p\cdot q$
4. $(p+q)^2-m^2=(p^2-m^2)+q^2+p\cdot q\xlongequal{p^2=m^2,\ q^2}2p\cdot q$

6.6 譯：後記

- 是這樣的，由於上午時間不夠了，我只好在食堂一邊吃午飯一邊看完這本講義的最後三頁，看完這篇後記時，不禁潸然淚下，感慨良多，特此意譯做紀。

在這門課程中，我們學習了量子場論的基本框架。我們所見的大部分都是十九世紀三十年代中期就已經被那些偉大的先驅發展出了。

然而到了三十年代末期，物理學家們準備放棄量子場論。原因在於微擾理論中的下一項 —— 我們在課程中沒有去計算他們 —— 都與 Feynman 圖中的圈有關，而這通常給出發散的結果。十年的奮鬥和失敗之後，人們都覺得應該轉投其他理論。Dirac 在 1937 年說：

> 由於其異常的複雜性，大多數物理學家都很樂意看到 QED 的終結。

當時的領袖人物都放棄得太快了。新的一代戰後的物理學家再次轉向量子場論，並馴服了無限 —— 這段故事我們將在下個學期的課程中講述……

- 我寫完這段翻譯，再次淚流滿面：永遠不要放棄自己，永遠不要放棄自己，永遠不要放棄自己……