量子力学复习手稿

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2022 年 12 月 9 日

1 题目

1.1 1.5

$\Psi(x,t) = A\mathrm{e}^{-\lambda|x|}\mathrm{e}^{-\mathrm{i}\omega t}$

(a)

$$1 = \int |\Psi|^2 \,\mathrm{d}x = \int \Psi^* \Psi \,\mathrm{d}x \Rightarrow A = \sqrt{\lambda}$$

(b) 注意利用函数的奇偶性。

$$\langle x \rangle = \int \Psi^* x \Psi \,\mathrm{d}x$$

$$\langle x^2 \rangle = \int \Psi^* x^2 \Psi \,\mathrm{d}x$$

1.2 1.9

$\Psi(x,t) = A\mathrm{e}^{-a[mx^2/\hbar + \mathrm{i}t]}$

(a)

$$1 = \int \Psi^* \Psi \,\mathrm{d}x \Rightarrow A = \left(\frac{2am}{\pi\hbar}\right)^{1/4}$$

(b)

$$V(x) = \frac{\mathrm{i}\hbar \frac{\partial \Psi}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2}}{\Psi}$$

(c)

$$
\begin{aligned}
\langle x\rangle &= \int \Psi^* x \Psi \,\mathrm{d}x \\
\langle x^2\rangle &= \int \Psi^* x^2 \Psi \,\mathrm{d}x \\
\langle p\rangle &= \int \Psi^* \left(\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x}\right) \Psi \,\mathrm{d}x\,,\ \text{or}\ \ \langle p\rangle = m\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} \\
\langle p^2\rangle &= \int \Psi^* \left(\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x}\right)^2 \Psi \,\mathrm{d}x
\end{aligned}
$$

(d)

$$
\sigma_x^2 = \langle x^2\rangle - \langle x\rangle^2\,,\ \sigma_p^2 = \langle p^2\rangle - \langle p\rangle^2\,,\ \sigma_x\sigma_p \geq \frac{\hbar}{2}
$$

1.3 2.5

(a)

$$
1 = \int \Psi^*\Psi \,\mathrm{d}x = |A| \int |\psi_1|^2 + |\psi_2|^2 \,\mathrm{d}x \Rightarrow A = \frac{1}{\sqrt{2}}
$$

(b)

$$
\begin{aligned}
\Psi(x,t) = &\sum_{n=1}^{\infty} c_n \psi_n \mathrm{e}^{-\mathrm{i}\frac{n^2\pi^2\hbar^2}{2ma^2}\frac{t}{\hbar}} \\
|\Psi|^2 = &\Psi^*\Psi
\end{aligned}
$$

(c)

$$
\langle x\rangle = \int \Psi^* x \Psi \,\mathrm{d}x
$$

(d)

$$
\langle p\rangle = m\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t}
$$

1.4 例题 2.4

$$
\psi_1(x) = A_1 a_+ \psi_0(x) = A_1 \frac{-\hbar\frac{\mathrm{d}}{\mathrm{d}x} + m\omega x}{\sqrt{2\hbar m\omega}} \left[\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \mathrm{e}^{-\frac{m\omega}{2\hbar}x^2}\right]
$$

之后再进行归一化。

1.5 例题 2.5

见 Problem 2.12。

1.6 2.12

$\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, $\langle p^2\rangle$, $\langle T\rangle$ for the nth stationary state of the harmonic oscillator

$$x=\sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-),\ p=\mathrm{i}\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$
$$a_+\psi_n=\sqrt{n+1}\psi_{n+1},\ a_-\psi_n\sqrt{n}\psi_{n-1}$$

$$\begin{aligned}
\langle x\rangle &= \int \psi_n^* x\psi\,\mathrm{d}x=\sqrt{\frac{\hbar}{2m\omega}}\int \psi_n^*(a_+ + a_-)\psi_n\,\mathrm{d}x\\
\langle p\rangle &= m\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t}\\
\langle x^2\rangle &= \int \psi_n^* x^2\psi\,\mathrm{d}x=\frac{\hbar}{2m\omega}\int \psi_n^*(a_+ + a_-)^2\psi_n\,\mathrm{d}x=\frac{\hbar}{2m\omega}\int \psi_n^*(a_+^2+a_+a_-+a_-a_++a_-^2)\psi_n\,\mathrm{d}x\\
\langle p^2\rangle &= \int \psi_n^* p^2\psi\,\mathrm{d}x=-\frac{\hbar m\omega}{2}\int \psi_n^*(a_+ - a_-)^2\psi_n\,\mathrm{d}x=-\frac{\hbar m\omega}{2}\int \psi_n^*(a_+^2-a_+a_--a_-a_++a_-^2)\psi_n\,\mathrm{d}x\\
\langle T\rangle &= \left\langle\frac{p^2}{2m}\right\rangle
\end{aligned}$$

$$\sigma_x^2=\langle x^2\rangle-\langle x\rangle^2,\ \sigma_p^2=\langle p^2\rangle-\langle p\rangle^2,\ \sigma_x\sigma_p\ge\frac{\hbar}{2}$$

1.7 2.13

$\Psi(x,0)=A[3\psi_0(x)+4\psi_1(x)]$

(a)

$$1=|A|^2[3^2+4^2]\Rightarrow A=\frac{1}{5}$$

(b)

$$\Psi(x,t)=\sum_{n=0}^{\infty}c_n\psi_n\mathrm{e}^{-\mathrm{i}\frac{E_n}{\hbar}t},\ E_n=(n+\frac{1}{2})\hbar\omega$$

(c)

$$\langle x\rangle=\int\Psi^*x\Psi\,\mathrm{d}x,\ \langle p\rangle=m\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t}$$
$$\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}=-\left\langle\frac{\partial V}{\partial x}\right\rangle=-m\omega^2\langle x\rangle$$

1.8 2.21

free particle $\Psi(x,0) = A\mathrm{e}^{-a|x|}$

(a)

$$1 = \int \Psi^* \Psi \,\mathrm{d}x$$

(b)

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x,0)\mathrm{e}^{-\mathrm{i}kx} \,\mathrm{d}x$$

要用留数定理。（悲）

(c)

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi(k)\mathrm{e}^{\mathrm{i}\left(kx - \frac{\hbar k^2}{2m}t\right)} \,\mathrm{d}k$$

应该还要用留数定理。（大悲）

1.9 2.37

三倍角公式。（怒）

1.10 3.31

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle Q\rangle = \frac{\mathrm{i}}{\hbar}\left\langle\left[\hat{H},\hat{Q}\right]\right\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$$

$$\left[\hat{A},\hat{B}\hat{C}\right] = \hat{B}\left[\hat{A},\hat{C}\right] + \left[\hat{A},\hat{B}\right]\hat{C},\ \left[\hat{A}\hat{B},\hat{C}\right] = \hat{A}\left[\hat{B},\hat{C}\right] + \left[\hat{A},\hat{C}\right]\hat{B}$$

$$\left[\hat{f},\hat{g}\right] = \mathrm{i}\hbar\{f,g\} = \mathrm{i}\hbar\left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial x}\right)$$

1.11 3.37

$$H = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix},\ H\left|s\right\rangle = E\left|s\right\rangle$$

$$\left|s_1\right\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},\ E_1 = c;\ \left|s_2\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\ E_2 = a+b;\ \left|s_3\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix},\ E_3 = a-b$$

$$\begin{aligned} \left|s(t=0)\right\rangle &= c_1\left|s_1\right\rangle + c_2\left|s_2\right\rangle + c_3\left|s_3\right\rangle \\ \left|s(t)\right\rangle &= c_1\left|s_1\right\rangle \mathrm{e}^{-\mathrm{i}E_1 t/\hbar} + c_2\left|s_2\right\rangle \mathrm{e}^{-\mathrm{i}E_2 t/\hbar} + c_3\left|s_3\right\rangle \mathrm{e}^{-\mathrm{i}E_3 t/\hbar} \end{aligned}$$

1.12 3.38

$$H|h_1\rangle = h_1|h_1\rangle$$

$$|s(0)\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \Rightarrow \langle s(0)| = \begin{pmatrix} c_1^* & c_2^* & c_3^* \end{pmatrix}$$

$$\langle H\rangle = \langle s(0)|H|s(0)\rangle\,,\ \langle A\rangle = \langle s(0)|A|s(0)\rangle\,,\ \langle B\rangle = \langle s(0)|B|s(0)\rangle$$

$$P(h_1) = \langle h_1|s(t)\rangle$$

1.13 4.18

$$L_\pm = (L_\mp)^\dagger$$

$$L_\pm L_\mp = L^2 - L_z^2 \pm \hbar L_z$$

$$\begin{aligned}\langle f_l^m|L_\pm L_\mp f_l^m\rangle =& \langle f_l^m|(L^2 - L_z^2 \pm \hbar L_z) f_l^m\rangle \\ =& \langle f_l^m|\left[l(l+1)\hbar^2 - m^2\hbar^2 \pm m\hbar^2\right]|f_l^m\rangle \\ =& l(l+1)\hbar^2 - m^2\hbar^2 \pm m\hbar^2 \\ \langle f_l^m|L_\pm L_\mp f_l^m\rangle =& \langle L_\mp f_l^m|L_\mp f_l^m\rangle = |A_l^m|^2\end{aligned}$$

$$L_\pm f_l^m = \hbar\sqrt{l(l+1) - m(m\pm 1)} f_l^{m\pm 1}$$

1.14 4.19

$$[R^a, P^b] = \mathrm{i}\hbar\delta^{ab}$$

$$L^c = \varepsilon_{ab}{}^c R^a P^b$$

$$[L^a, R^b] = [\varepsilon_{cd}{}^a R^c P^d, R^b] = \varepsilon_{cd}{}^a R^c [P^d, R^b] = -\mathrm{i}\hbar\varepsilon_{cd}{}^a R^c \delta^{bd} = \mathrm{i}\hbar\varepsilon_c{}^{ab} R^c$$

$$[L^a, P^b] = [\varepsilon_{cd}{}^a R^c P^d, P^b] = \varepsilon_{cd}{}^a [R^c, P^b] P^d = \mathrm{i}\hbar\varepsilon_{cd}{}^a \delta^{bc} P^d = \mathrm{i}\hbar\varepsilon^{ab}{}_d P^d$$

$$[L^a, L^b] = \mathrm{i}\hbar\varepsilon^{ab}{}_c L^c$$

$$\begin{aligned}[L^a, R^2] =& [\varepsilon_{bc}{}^a R^b P^c, \delta_{de} R^d R^e] \\ =& \varepsilon_{bc}{}^a \delta_{de} R^b (R^d [P^c, R^e] + [P^c, R^d] R^e) \\ =& -2\mathrm{i}\hbar\varepsilon_{bc}{}^a R^b R^c = -2\mathrm{i}\hbar\varepsilon_{[bc]}{}^a R^{(b} R^{c)} = 0\end{aligned}$$

$$\begin{aligned}[L^a, P^2] =& [\varepsilon_{bc}{}^a R^b P^c, \delta_{de} P^d P^e] \\ =& \varepsilon_{bc}{}^a \delta_{de} (P^d [R^b, P^e] + [R^b, P^d] P^e) P^c \\ =& 2\mathrm{i}\hbar\varepsilon_{bc}{}^a P^b P^c = 2\mathrm{i}\hbar\varepsilon_{[bc]}{}^a P^{(b} P^{c)} = 0\end{aligned}$$

1.15 例题 4.2

$$\chi = \frac{1}{\sqrt{6}}\begin{pmatrix}1+\mathrm{i}\\2\end{pmatrix}$$

测 S_z:

$$P\left(S_z = +\frac{\hbar}{2}\right) = \left|\frac{1+\mathrm{i}}{\sqrt{6}}\right|^2 = \frac{1}{3},\ P\left(S_z = -\frac{\hbar}{2}\right) = \left|\frac{2}{\sqrt{6}}\right|^2 = \frac{2}{3}$$

测 S_x:

$$|\chi\rangle = c_1\left|\chi_+^{(x)}\right\rangle + c_2\left|\chi_-^{(x)}\right\rangle$$

$$c_1 = \left\langle\chi_+^{(x)}\middle|\chi\right\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1+\mathrm{i}}{\sqrt{6}}\\\frac{2}{\sqrt{6}}\end{pmatrix} = \frac{3+\mathrm{i}}{2\sqrt{3}},\ P\left(S_x = +\frac{\hbar}{2}\right) = |c_1|^2 = \frac{5}{6}$$

$$c_2 = \left\langle\chi_-^{(x)}\middle|\chi\right\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1+\mathrm{i}}{\sqrt{6}}\\\frac{2}{\sqrt{6}}\end{pmatrix} = \frac{\mathrm{i}-1}{2\sqrt{3}},\ P\left(S_x = -\frac{\hbar}{2}\right) = |c_2|^2 = \frac{1}{6}$$

1.16 4.27

$$\chi = A\begin{pmatrix}3\mathrm{i}\\4\end{pmatrix}$$

(a)

$$1 = \chi^\dagger\chi = |A|^2\begin{pmatrix}-3\mathrm{i} & 4\end{pmatrix}\begin{pmatrix}3\mathrm{i}\\4\end{pmatrix} \Rightarrow A = \frac{1}{5}$$

(b)

$$\langle S_x\rangle = \langle\chi|S_x|\chi\rangle = \chi^\dagger S_x\chi$$

(c)

$$\langle S_x^2\rangle = \langle S_y^2\rangle = \langle S_z^2\rangle = \frac{1}{3}\langle S^2\rangle = \frac{\hbar^2}{3}$$

$$\sigma_{S_x} = \sqrt{\langle S_x^2\rangle - \langle S_x\rangle^2}$$

(d)

$$\sigma_{S_x}\sigma_{S_y} \geq \frac{\hbar}{2}|\langle S_z\rangle|^2,\ \sigma_{S_y}\sigma_{S_z} \geq \frac{\hbar}{2}|\langle S_x\rangle|^2,\ \sigma_{S_z}\sigma_{S_x} \geq \frac{\hbar}{2}|\langle S_y\rangle|^2$$

1.17 4.31

$$\chi_+ = \begin{pmatrix}1\\0\\0\end{pmatrix},\ \chi_0 = \begin{pmatrix}0\\1\\0\end{pmatrix},\ \chi_- = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$S_z\chi_+ = +\hbar\chi_+,\ S_z\chi_0 = 0,\ S_z\chi_- = -\hbar\chi_-$$

$$S_+\chi_+ = 0,\ S_+\chi_0 = \sqrt{2}\hbar\chi_0,\ S_+\chi_- = \sqrt{2}\hbar\chi_0$$
$$S_-\chi_+ = \sqrt{2}\hbar\chi_0,\ S_-\chi_0 = \sqrt{2}\hbar\chi_-,\ S_-\chi_- = 0$$
$$S_\pm = \left(S_x \pm \mathrm{i}S_y\right)$$

1.18 4.43

hydrogen

(a)

$$\psi_{nlm} = R_{nl}Y_{lm}$$

(b)

$$\int \psi^\dagger_{nlm}\psi_{nlm} r^2 \,\mathrm{d}r \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi$$

(c)

$$\langle r^s \rangle = \int \psi^\dagger_{nlm} r^s \psi_{nlm} r^2 \,\mathrm{d}r \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi = \int_0^\infty r^s |R_{32}(r)|^2 r^2 \,\mathrm{d}r$$

1.19 4.49

1.20 4.55

(h)

$$R_{21}Y_{10}\chi_+ = \left|l=1, l_z=0, s=\frac{1}{2}, s_z=\frac{1}{2}\right\rangle,\ R_{21}Y_{11}\chi_- = \left|l=1, l_z=1, s=\frac{1}{2}, s_z=-\frac{1}{2}\right\rangle$$

学会读 CG 系数表:

$$\left|l=1, l_z=0, s=\frac{1}{2}, s_z=\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|l=1, s=\frac{1}{2}, j=\frac{3}{2}, j_z=\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|l=1, s=\frac{1}{2}, j=\frac{1}{2}, j_z=\frac{1}{2}\right\rangle$$

$$\left|l=1, l_z=1, s=\frac{1}{2}, s_z=-\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|l=1, s=\frac{1}{2}, j=\frac{3}{2}, j_z=\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|l=1, s=\frac{1}{2}, j=\frac{1}{2}, j_z=\frac{1}{2}\right\rangle$$

(h)

$$|\psi\rangle = \sqrt{\frac{1}{3}}\left|n=2, l=1, m=0, s=\frac{1}{2}, s_z=\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|n=2, l=1, m=1, s=\frac{1}{2}, s_z=-\frac{1}{2}\right\rangle$$

$$P = \left|\left\langle s=\frac{1}{2}, s_z=\frac{1}{2}\middle|\psi\right\rangle\right|^2 = \int \frac{1}{3}|R_{21}|^2|Y_{10}|^2 \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi = \frac{1}{3}|R_{21}|^2$$

1.21 4.58

2 试卷

2.1 2017

2.1.1 1D infinite square well

(a)

```
Solve[Integrate[(A*Sin[Pi/a*x]^3)^2, {x, 0, a}] == 1, A]
(* -> {{A -> -(4/(Sqrt[5] Sqrt[a]))}, {A -> 4/(Sqrt[5] Sqrt[a])}} *)
4/(Sqrt[5] Sqrt[a])*Sin[Pi/a*x]^3 // TrigReduce // Expand
(* -> (3 Sin[(\[Pi] x)/a])/(Sqrt[5] Sqrt[a]) - Sin[(3 \[Pi] x)/a]/(Sqrt[5] Sqrt[a]) *)
```

$$1 = \int_0^a \varphi^* \varphi \,\mathrm{d}x \Rightarrow A = \frac{4}{\sqrt{5a}}$$

$$\varphi = \frac{3}{\sqrt{10}}\psi_1 + \frac{1}{\sqrt{10}}\psi_3$$

$$\varphi(x,t) = \frac{3}{\sqrt{10}}\psi_1 \mathrm{e}^{-\mathrm{i}\frac{E_1}{\hbar}t} + \frac{1}{\sqrt{10}}\psi_3 \mathrm{e}^{-\mathrm{i}\frac{E_3}{\hbar}t},\ E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

(b)

$$\langle E\rangle = \left(\frac{3}{\sqrt{10}}\right)^2 \times E_1 + \left(\frac{1}{\sqrt{10}}\right)^2 \times E_3$$

2.1.2 Hamiltonian

(a)

```
Eigensystem[{{1, a, 0}, {a, 3, 0}, {0, 0, a - 2}}]
(* -> {{-2 + a, 2 - Sqrt[1 + a^2], 2 + Sqrt[1 + a^2]}, {{0, 0, 1}, {-((1 + Sqrt[1 + a^2])/a), 1, 0}, {-((1 - Sqrt[1 + a^2])/a), 1, 0}}} *)
```

$$\mathcal{H}\psi_i = E_i\psi_i$$

$$E_1 = a-2, \psi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; E_2 = 2-\sqrt{1+a^2}, \psi_2 = \begin{pmatrix} 1+\sqrt{1+a^2} \\ -a \\ 0 \end{pmatrix}; E_3 = 2+\sqrt{1+a^2}, \psi_3 = \begin{pmatrix} 1-\sqrt{1+a^2} \\ -a \\ 0 \end{pmatrix}$$

(b)

```
Solve[-2 + a == 2 - Sqrt[1 + a^2], a]
(* -> {{a -> 15/8}} *)
Solve[2 - Sqrt[1 + a^2] == 2 + Sqrt[1 + a^2], a]
(* -> {{a -> -I}, {a -> I}} *)
Solve[2 + Sqrt[1 + a^2] == -2 + a, a]
(* -> {} *)
```

$$a = \frac{15}{8}$$

2.1.3 Hydrogen atom

$$|\varphi\rangle = \frac{2}{\sqrt{5}}|3,1,1\rangle - \frac{1}{\sqrt{5}}|4,1,0\rangle$$

(a)

$$P(l_z = 1) = \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5},\ P(\_z = 0) = \left(\frac{1}{\sqrt{5}}\right)^2$$

(b) L_y 的本征矢

$$L_y\left[\frac{1}{2\mathrm{i}}|1,1\rangle + \frac{\sqrt{2}}{2}|1,0\rangle - \frac{1}{2\mathrm{i}}|1,-1\rangle\right] = +\hbar\left[\frac{1}{2\mathrm{i}}|1,1\rangle + \frac{\sqrt{2}}{2}|1,0\rangle - \frac{1}{2\mathrm{i}}|1,-1\rangle\right]$$

$$L_y\left[\frac{1}{\sqrt{2}}|1,1\rangle + 0|1,0\rangle + \frac{1}{\sqrt{2}}|1,-1\rangle\right] = 0\hbar\left[\frac{1}{\sqrt{2}}|1,1\rangle + 0|1,0\rangle + \frac{1}{\sqrt{2}}|1,-1\rangle\right]$$

$$L_y\left[\frac{\mathrm{i}}{2}|1,1\rangle + \frac{\sqrt{2}}{2}|1,0\rangle - \frac{\mathrm{i}}{2}|1,-1\rangle\right] = -\hbar\left[\frac{\mathrm{i}}{2}|1,1\rangle + \frac{\sqrt{2}}{2}|1,0\rangle - \frac{\mathrm{i}}{2}|1,-1\rangle\right]$$

$$P(l_y = +1) = \left|\begin{pmatrix} -\frac{1}{2\mathrm{i}} & \frac{1}{\sqrt{2}} & \frac{1}{2\mathrm{i}} \end{pmatrix}\begin{pmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{pmatrix}\right|^2 = \frac{3}{10}$$

$$P(l_y = 0) = \left|\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{pmatrix}\right|^2 = \frac{4}{10}$$

$$P(l_y = -1) = \left|\begin{pmatrix} -\frac{\mathrm{i}}{2} & \frac{1}{\sqrt{2}} & \frac{\mathrm{i}}{2} \end{pmatrix}\begin{pmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{pmatrix}\right|^2 = \frac{3}{10}$$

2.1.4 Uncertainty relationships

$$\varphi(x,0) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\sqrt{\frac{2m\omega}{\hbar}}x\mathrm{e}^{-\frac{m\omega}{2\hbar}x^2}$$
$$\varphi(x,t) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\sqrt{\frac{2m\omega}{\hbar}}x\mathrm{e}^{-\frac{m\omega}{2\hbar}x^2}\mathrm{e}^{-\mathrm{i}\frac{3}{2}\omega t}$$

(a)

$$\langle x\rangle = \int \varphi^* x\varphi\,\mathrm{d}x \sim \int x^3\mathrm{e}^{-\frac{m\omega}{\hbar}x}\,\mathrm{d}x = 0$$
$$\langle p\rangle = m\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = 0$$
$$\langle x^2\rangle = \frac{2}{m\omega^2}\langle V(x)\rangle = \frac{2}{m\omega^2}\frac{1}{2}(n+\frac{1}{2})\hbar\omega = \frac{3}{2}\frac{\hbar}{m\omega}$$
$$\langle p^2\rangle = 2m\langle T\rangle = 2m\frac{1}{2}(n+\frac{1}{2})\hbar\omega = \frac{3}{2}m\hbar\omega$$

(b)

$$\sigma_x \sigma_p = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \frac{3}{2}\hbar \geq \frac{1}{2}\hbar$$

2.1.5 Spin Entanglements

(a)

$$\hat{\mathcal{H}}\chi = \left[\hat{S}^2 - B_0 \cdot \hat{S}_z\right] |10\rangle = [1 \times (1+1)\hbar - B_0 \times 0 \times \hbar] |10\rangle = 2\hbar |10\rangle$$

(b)

$$P(s_{z1} = +\frac{1}{2} \& s_{z2} = -\frac{1}{2}) = \frac{1}{2}$$
$$P(s_{z1} = -\frac{1}{2} \& s_{z2} = +\frac{1}{2}) = \frac{1}{2}$$

$$\chi = \frac{1}{\sqrt{2}} \left[\frac{\chi_+^{1(x)} + \chi_-^{1(x)}}{\sqrt{2}} \frac{\chi_+^{2(x)} - \chi_-^{2(x)}}{\sqrt{2}} + \frac{\chi_+^{1(x)} - \chi_-^{1(x)}}{\sqrt{2}} \frac{\chi_+^{2(x)} + \chi_-^{2(x)}}{\sqrt{2}} \right] = \frac{\chi_+^{1(x)}\chi_+^{2(x)} - \chi_-^{1(x)}\chi_-^{2(x)}}{\sqrt{2}}$$

$$P(s_{x1} = +\frac{1}{2} \& s_{x2} = +\frac{1}{2}) = \frac{1}{2}$$
$$P(s_{x1} = -\frac{1}{2} \& s_{x2} = -\frac{1}{2}) = \frac{1}{2}$$

2.2 2018

2.2.1 1D infinite square well

(a)

```
Solve[Integrate[(A*Sin[Pi/a*x]*Sin[(2*Pi)/a*x])^2, {x, 0, a}] == 1, A]
(* -> {{A -> -(2/Sqrt[a])}, {A -> 2/Sqrt[a]}} *)
```

$$A = \frac{2}{\sqrt{a}}$$

(b)

$$c_n = \sqrt{\frac{2}{a}} \int_0^a \sin\left(\frac{n\pi}{a}x\right) \times \frac{2}{\sqrt{a}} \sin\left(\frac{\pi}{a}x\right) \sin\left(\frac{2\pi}{a}x\right) \mathrm{d}x = -\frac{8\sqrt{2}n[\cos(\pi n) + 1]}{\pi\left(n^4 - 10n^2 + 9\right)}$$

2.2.2 Harmonic Oscillator

(a)

$$E_n = (n_x + \frac{1}{2})\hbar\omega_x + (n_y + \frac{1}{2})\hbar\omega_y + (n_z + \frac{1}{2})\hbar\omega_z = \left[n_x + n_y + 2n_z + 2\right]\hbar$$

(b)

基态

$$\left|n_x = 0, n_y = 0, n_z = 0\right\rangle$$

第一激发态

$$\left|n_x = 1, n_y = 0, n_z = 0\right\rangle \& \left|n_x = 0, n_y = 1, n_z = 0\right\rangle$$

第二激发态

$$\left|n_x = 2, n_y = 0, n_z = 0\right\rangle \& \left|n_x = 1, n_y = 1, n_z = 0\right\rangle \& \left|n_x = 0, n_y = 2, n_z = 0\right\rangle \& \left|n_x = 0, n_y = 0, n_z = 1\right\rangle$$

2.2.3 Hydrogen Atom

(a)

$$\begin{aligned}P(s_z = +\frac{1}{2} \& l_z = 1) =& \frac{2}{3}\\ P(s_z = -\frac{1}{2} \& l_z = 1) =& \frac{1}{3}\end{aligned}$$

(b)

```
Eigensystem/@PauliMatrix/@Range[4]
(* -> {{{-1, 1}, {{-1, 1}, {1, 1}}}, {{-1, 1}, {{I, 1}, {-I, 1}}}, {{-1, 1}, {{0, 1}, {1, 0}}}, {{1, 1}, {{0, 1}, {1, 0}}}} *)
```

$$\begin{aligned}S_x \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} &= +\frac{\hbar}{2} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, & S_x \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} &= -\frac{\hbar}{2} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}\\ S_y \begin{pmatrix} -\mathrm{i}/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} &= +\frac{\hbar}{2} \begin{pmatrix} -\mathrm{i}/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, & S_y \begin{pmatrix} \mathrm{i}/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} &= -\frac{\hbar}{2} \begin{pmatrix} \mathrm{i}/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}\end{aligned}$$

2.2.4 Commutation

(a)

$$\left[\hat{L}_+, \hat{L}_x\right] = \left[\hat{L}_x + \mathrm{i}\hat{L}_y, \hat{L}_x\right] = \mathrm{i}\left[\hat{L}_y, \hat{L}_x\right] = \hbar \hat{L}_z$$

(b)

$$\left[\hat{P}_x + \hat{P}_z, \hat{L}_z\right] = \left[\hat{P}_x + \hat{P}_z, \hat{X}\hat{P}_y - \hat{Y}\hat{P}_x\right] = \left[\hat{P}_x, \hat{X}\right]\hat{P}_y = -\mathrm{i}\hbar \hat{P}_y$$