Fourier Transform and Digital Watermark

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Fourier transform turns various kinds of signals from time space into frequency space, including audio and even images. By modification of the signal in frequency space and inverse Fourier transform, we can hide information in the noise background of the original signal with appreciable robustness and invisibility to humans, which brings about the digital watermarking technology to protect the copyright of media files.

I. INTRODUCTION

With the dramatic development in the Internet nowadays, everyone can create, publish, and share multi-media works of their own, including but not limited to audio sounds, images, and videos. However, unlike editing a text message, it's not an easy task to deal with the massive amount of information contained in the media files. Typically, there are two major aspects in which lie difficulties.

A. Media Processing

Since the storage and computing devices like a computer cannot store and process infinite many data, media files normally exist in a discrete form, in which a large number of points are sampled and recorded. For audio, the sound is sampled with small time intervals; and for images, the picture is sampled with small squares called pixels.

Although it is not difficult for computers to compute with lots of data, it is very hard for them to deal with data with some "patterns". For example, A computer can crop, encode or decode the audio and image quite well and fast, but it's not easy for them to detect and deduce the noise in the sound and dirt in the picture, given a bunch of discrete amplitude or pixel data.

B. Media Copyright Protection

After a document is signed in the real world, everyone can acknowledge who produced it and who owns it from the signature. But media files in the digital world can be modified and copied with a computer and then transferred freely through the Internet, which brings barriers for source tracing and copyright protection.

In the past years, famous companies like Apple Inc. went through several serious information leakage [1] which lead to "negatively impact sales of the current model; give rival companies more time to begin on a competitive response; and lead to fewer sales of that new product when it arrives."

Besides information leakage, piracy activities such as slight modification and reproduction of works can cause great damage to the authors in many fields, including Software, Art, and public media.

For these reasons, a method called "digital watermark" is developed to mark and sign a multimedia file without significant influences on the original work in order to protect the copyright of media files.

II. MATERIALS AND METHODS

The key mathematical method for Media processing and copyright protection lies in the Fourier transform, with some related algorithms specialized for computing.

A. Fourier Transform

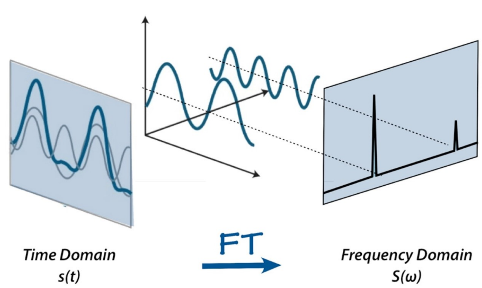

FIG. 1. Fourier transform turns signals from time domains into frequency domains. [2]

The Fourier Transform is developed by Joseph Fourier, who showed that some functions can be decomposed with the basis of trigonometric functions, also known as the Fourier Series.

For non-periodic functions, we need every possible frequency. With the Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

we can change the basis into the exponential functions $e^{i2\pi\omega}$ and finally get the Fourier transform for functions

$$F(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-iux}dx \tag{1}$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(u)e^{iux}du \tag{2}$$

Fourier transform is a critical part of our course Phys 101 at the University of California, Santa Barbara, for the reason that it is an important part of complex analysis and a powerful technique for Physics study.

The key concept of the Fourier transform is the switch between time domain and frequency domain, as illustrated in Fig 1. For a signal (function) describing the amplitude with respect to time, Fourier transform can calculate the function of amplitude with respect to frequency which is more flexible and useful to deal with patterns in the signal.

Fourier transform has an intensive influence in various fields, such as computer science, communication, and electric engineering. Both theoretic and engineering researches utilize Fourier transform. With the rapid development of computation and communication, Fourier transform serves as a technological base for our daily life with increasing significance.

B. 2D Fourier Transform

Fourier transform can be generalized into higher dimensions. For example, the 2-dimension Fourier transform can convert a double-variable function into its frequency domain [3].

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy \tag{3}$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}dudv \tag{4}$$

Rather than only determine the amplitude with respect to a frequency, we can also obtain the information of direction from the 2D Fourier transform. If we plot the real part of the transformed function, every point on the graph represents the amplitude of a given frequency (the distance from the origin) and the direction of that frequency (the direction of that point).

C. Discrete Fourier Transform

Although Fourier transform and its 2D version provide us a good tool to deal with audio and image signals, problems still exist. Computers store the data of signals in a discrete way, consisting of a large number of sample points. The continuous functions fail to work with sequences of data, which brings about the Discrete Fourier transform.

DFT transforms a complex valued sequence x_n into another complex valued sequence X_n, where $n = 0, 1, 2, \cdots, N-1$ by

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N} \tag{5}$$

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{i2\pi kn/N} \tag{6}$$

It is very similar to the Fourier transform, but sequences take the place of functions, which make it possible to deal with discrete data.

For further optimization for computing, we take the fundamental frequency $\omega_n = e^{-i2\pi/n}$ and convert the DFT formulae into a matrix multiplication.

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega_n & \omega_n^2 & \cdots & \omega_n^{n-1} \\ 1 & \omega_n^2 & \omega_n^4 & \cdots & \omega_n^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_n^{n-1} & \omega_n^{2(n-1)} & \cdots & \omega_n^{(n-1)^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \tag{7}$$

D. Fast Fourier Transform

A specialized algorithm, known as Fast Fourier transform, is developed to accelerate the computing of Discrete Fourier transform on computers and other electric devices.

If the data sequence has the length $N = 2^r$, we can rearrange the DFT matrix in Eq. 7 and obtain the following formula[4]

$$\boldsymbol{X} = \begin{bmatrix} I_{N/2} & D_{N/2} \\ I_{N/2} & -D_{N/2} \end{bmatrix} \begin{bmatrix} DFT_{N/2} & 0 \\ 0 & DFT_{N/2} \end{bmatrix} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix} \tag{8}$$

where $I_{N/2}$ is the identity matrix and $DFT_{N/2}$ is the Discrete Fourier Transform matrix with length $N/2$, and

$$D_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \omega_N & 0 & \cdots & 0 \\ 0 & 0 & \omega_N^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \omega_N^{(n-1)} \end{bmatrix}$$

Notice that in this way, we convert a problem that multiplied by DFT_N into the computation of $DFT_{N/2}$. If we continue this process recursively, we can obtain the Fast Fourier Transform algorithm. For example, we can convert computation of DFT_{512} into computation of DFT_{256}, then DFT_{128} and finally to DFT_2 which is super simple to compute.

Especially, Fast Fourier Transform takes advantage of the computing system with base 2 [5] to determine the permutation of the signal data sequence. Therefore, FFT can do DFT significantly faster than the traditional computing methods, which make the Fourier Transform more useful for signal processing.

III. RESULTS

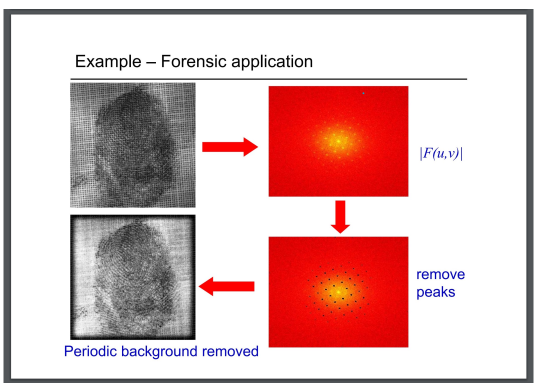

FIG. 2. An Example of Image Processing with Fourier transform [3]. After removing some signal peaks in the frequency domain, noises with definite patterns in the fingerprint image are removed with little damage to the valuable data.

With the methods introduced in the previous section, we can process the signal with the Fourier Transform to get the frequency spectrum, and then modify the spectrum and get the processed signal by inverse Fourier transform.

For example, in Fig. 2, the original picture has grids, which is very hard to process with the traditional image processing. However, after converting to the frequency spectrum, the grid pattern turns into some peaks in the frequency. By removing the peaks and inverse transform, we can get a much clearer picture.

Similarly, in the audio files, sometimes the background noise appears with definite patterns. The frequency spectrum is used to clear these noises and sharpen a particular sound in the audio files.

A. Digital Audio Watermark

With such a method, we can hide information in the audio files with the following steps.

Firstly, obtain the frequency spectrum with Fourier transform. Then add encoded watermarks into the background noise frequency. After that, inverse Fourier transform the signal to get a normal audio file.

Humans are not able to distinguish any difference caused by the slight change in the frequency of background noise. But with Fourier transform, programs can detect the watermarks embedded in the frequency spectrum, even if the original audio file undergoes cropping or limiting modification.

B. Digital Image Watermark

In the same way, computers can apply the discrete version of the 2D Fourier transform for images and then embed watermark messages in the frequency domain. After the inverse Fourier transform, we can get a watermarked picture [6].

Usually, considering the symmetry of the spectrum, we embed the watermark at two corners in the frequency spectrum symmetrically, which will make little difference to the picture and make the watermark indistinguishable by human eyes. If we calculate the residue between the watermarked data and the original data, we can find that the embedded information mainly hides in the background pixel noise.

C. Practical Experiment

Interestingly, a code repository [7] on Github offers a chance for a practical experiment.

After picking a normal picture and a message, the program can provide a watermarked image. Under various kinds of attacks such as random paintings and brightness adjustment, the frequency spectrum with the watermark still works pretty well.

Fig. 3 to Fig. 6 are some demonstrations with the code, the watermarked picture and the spectrum are displayed.

FIG. 3. Original Picture before digitally watermarking.

FIG. 4. No Attack. "WATERMARK" is embedded into the frequency domain (right) of the original picture. It's difficult for human eyes to distinguish any difference between watermarked picture (left) and the original picture Fig 3.

IV. DISCUSSION

From previous experiments and the methods of digital watermarking, digital watermarks generally have better properties of robustness and invisibility, compared to traditional simple watermark.

Attackers can modify the source in various ways like cropping and painting. Although the quality and full information may lose, the embedded digital watermark stands in the source.

A. Various Technologies

Besides the digital watermarks based on the Fourier transform, many other kinds of watermarks exist.

The simple watermarks have been developed since the era of paper letters. This kind of visible watermark has been applied to the digital version of files as well. For example, video service platforms may add visible icons to TV series to mark the copyright and source of the media works, and sometimes overwhelming icons and logos with lower opacity float all over the images. This technology requires much less computing. As a result, it is widely used in video and stream which demands the processing of massive data in a short period. But the visible simple watermarks have disadvantages. They not only influence the sources but also and attract more attention. Specialized algorithms involving machine learning are developed to watermark, and a small icon itself in the corner of the video has nothing to do with attacking like cropping.

Besides, there are more complicated methods for digital watermarks in video and stream, and even for 3D object data in VR. And more methods and algorithms with better quality and efficiency remain under research.

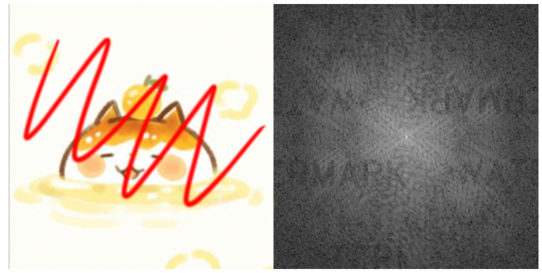

FIG. 5. Attack the watermarked picture by some random painting (left) to simulate slight modification. The embedded watermark is still very clear in the frequency domain (right) of the painted picture.

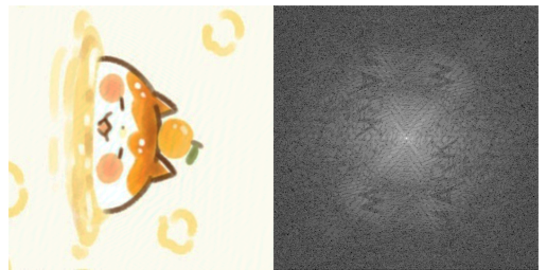

FIG. 6. Attack the watermarked picture by rotation and brightness adjustment (left) to simulate expert modification. The watermark is blurred, but still distinguishable in the frequency domain (right)

B. Social Significance

The technology of digital watermark serves as a kind of enforcement of copyright in the real world and even the legal system. For example in the Netherlands, digital watermarks that are properly added and detected can serve as legal evidence against copyright infringement [8].

Even in our daily life, when we send out pictures or media works through social media, we can perform digital watermarking to our works with the identity of the author as well as the distributing sources in case of any ways of piracy and crime with the leaked information.

Whenever needed, with the proper way of Fourier transform as detection, we can present the hidden information to prove the copyright or discover the source of the information leakage.

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[1] M. Gurman, "Apple warns employees to stop leaking information to media," (2018), `https://www.bloomberg.com/news/articles/2018-04-13/apple-warns-employees-to-stop-leaking-information-to-media` (visited in 2020).

[2] K. Chaudhary, "Understanding Audio data, Fourier Transform, FFT and Spectrogram features for a Speech Recognition System," (2020), `https://towardsdatascience.com/understanding-audio-data-fourier-transform-fft-spectrogram-and-speech-recognition-a4072d228520` (visited in 2020).

[3] A. Zisserman, "Lecture 2: 2D Fourier transforms and applications," (2014), lecture note `http://www.robots.ox.ac.uk/~az/lectures/ia/lect2.pdf` (visited in 2020).

[4] C. V. Loan, "The FFT Via Matrix Factorizations," (2010), lecture note `https://www.cs.cornell.edu/~bindel/class/cs5220-s10/slides/FFT.pdf` (visited in 2020).

[5] P. Heckbert, "Fourier Transforms and the Fast Fourier Transform (FFT) Algorithm," (1995), lecture note `http://www.cs.cmu.edu/afs/andrew/scs/cs/15-463/2001/pub/www/notes/fourier/fourier.pdf` (visited in 2020).

[6] H. O. Nasereddin, "Digital watermarking a technology overview," IJRRAS **6** (2011).

[7] Y. (zy445566) Zhang, "node-digital-watermarking," (2017), github Repository `https://github.com/zy445566/node-digital-watermarking` (visited in 2020).

[8] M. Schellekens, "DIGITAL WATERMARKS AS LEGAL EVIDENCE," Digital Evidence and Electronic Signature Law Review **8** (2014).