

SVT3 RW Radiation $\Omega(x)$ $k < 0$ Polar

1 Background

1.1 Comoving $a(t)$

First, determine the form of $a(t)$ for $\rho = 3p$ radiation in comoving coordinates

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) = -dt^2 + a(t)^2 \tilde{g}_{ij} dx^i dx^j \quad (1.1)$$

$$T_{\mu\nu} = p(4U_\mu U_\nu + g_{\mu\nu}), \quad U_\mu = \delta_\mu^0 \quad (1.2)$$

$$T_{00} = 3p, \quad T_{ij} = a^2(t)p\tilde{g}_{ij} \quad (1.3)$$

$$G_{00} = -3ka^{-2} - 3\dot{a}^2 a^{-2}, \quad G_{ij} = \tilde{g}_{ij}(k + \dot{a}^2 + 2a\ddot{a}) \quad (1.4)$$

$$\Delta_{\mu\nu} = G_{\mu\nu} + T_{\mu\nu} = 0 \quad (1.5)$$

$$\Delta_{00} = 3(p - ka^{-2} - \dot{a}^2 a^{-2}), \quad \Delta_{ij} = \tilde{g}_{ij}(a^2 p + k + \dot{a}^2 + 2a\ddot{a}) \quad (1.6)$$

$$\rightarrow \boxed{p = ka^{-2} + \dot{a}^2 a^{-2}} \quad \boxed{0 = k + \dot{a}^2 + a\ddot{a}} \quad (1.7)$$

With $k = -1/L^2$, we will follow APM (B1) and take

$$a^2(t) = \frac{d^2}{L^2} \left(1 + \frac{t^2}{d^2}\right) \quad (1.8)$$

$$p = -\frac{1}{d^2(1 + t^2/d^2)^2} = -\frac{d^2}{L^4 a^4} \quad (1.9)$$

1.2 Conformal T, R Coordinates

Given $a(t)$ in the form (1.8), we may transform the metric from

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (1.10)$$

to the conformal flat form

$$ds^2 = \Omega^2(X)(-dT^2 + dR^2 + R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2), \quad (1.11)$$

via the coordinate transformation below with the corresponding scale factors:

$$T = \left(\frac{t}{d} + \sqrt{\left(\frac{t}{d}\right)^2 + 1}\right) \left(1 + \left(\frac{r}{L}\right)^2\right)^{1/2} \quad R = \left(\frac{t}{d} + \sqrt{\left(\frac{t}{d}\right)^2 + 1}\right) \frac{r}{L} \quad (1.12)$$

$$X^2 = T^2 - R^2 = \left(\frac{t}{d} + \sqrt{\left(\frac{t}{d} \right)^2 + 1} \right)^2 \quad (1.13)$$

$$a^2(X) = \frac{d^2 (X^2 + 1)^2}{L^2 4X^2} \quad (1.14)$$

$$\Omega^2(X) = L^2 \frac{a^2(X)}{X^2} = \frac{d^2 (1 + X^2)^2}{4 X^4} = d^2 \frac{(1 + t^2/d^2)}{(t/d + (1 + t^2/d^2)^{1/2})^2} \quad (1.15)$$

$$\frac{t}{d} = \pm \frac{(X^2 - 1)}{2X}, \quad \frac{r}{L} = \begin{cases} RX, & \text{for } u = -\frac{(X^2 - 1)}{2X} \\ \frac{R}{X}, & \text{for } u = +\frac{(X^2 - 1)}{2X} \end{cases} \quad (1.16)$$

Transformation Matrix (taking + in (1.16)):

$$x'^\mu = (T, R, \theta, \phi), \quad x^\mu = (t, r, \theta, \phi) \quad (1.17)$$

$$A'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} A_{\alpha\beta} \frac{\partial x^\beta}{\partial x'^\nu} = J^T A J \quad (1.18)$$

$$J = \frac{\partial x^\sigma}{\partial x'^\rho} = \begin{pmatrix} \frac{dT(X^2+1)}{2X^3} & -\frac{dR(X^2+1)}{2X^3} & 0 & 0 \\ -\frac{LRT}{X^3} & \frac{LT^2}{X^3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1.19)$$

$$A'_{\mu\nu} = \quad (1.20)$$

$$\begin{pmatrix} \frac{T^2(d^2 A_{00}(X^2+1)^2 + 4LR(LRA_{11} - d(X^2+1)A_{01}))}{4X^6} & \frac{T(2L(d(-R^4+R^2+T^4+T^2)A_{01} - 2LRT^2 A_{11}) - d^2 R(X^2+1)^2 A_{00})}{4X^6} & \frac{T(d(X^2+1)A_{02} - 2LRA_{12})}{2X^3} & \frac{T(d(X^2+1)A_{03} - 2LRA_{13})}{2X^3} \\ \frac{T(2L(d(-R^4+R^2+T^4+T^2)A_{01} - 2LRT^2 A_{11}) - d^2 R(X^2+1)^2 A_{00})}{4X^6} & \frac{4L(LT^2 A_{11} - dR(X^2+1)A_{01})T^2 + d^2 R^2(X^2+1)^2 A_{00}}{4X^6} & -\frac{dR(X^2+1)A_{02} - 2LT^2 A_{12}}{2X^3} & -\frac{dR(X^2+1)A_{03} - 2LT^2 A_{13}}{2X^3} \\ \frac{T(d(X^2+1)A_{02} - 2LRA_{12})}{2X^3} & -\frac{dR(X^2+1)A_{02} - 2LT^2 A_{12}}{2X^3} & A_{22} & A_{23} \\ \frac{T(d(X^2+1)A_{03} - 2LRA_{13})}{2X^3} & -\frac{dR(X^2+1)A_{03} - 2LT^2 A_{13}}{2X^3} & A_{23} & A_{33} \end{pmatrix}$$

1.3 $T'_{\mu\nu}(T, R)$

$$T'_{\mu\nu} = p(4U'_\mu U'_\nu + g'_{\mu\nu}) \quad (1.21)$$

$$p = -d^2 \Omega^{-4} X^{-4} \quad (1.22)$$

$$\begin{aligned} U'_\mu &= \frac{\partial x^\alpha}{\partial x'^\mu} U_\alpha = U J \\ &= \Omega \left(-\frac{T}{X}, \frac{R}{X}, 0, 0 \right) \end{aligned} \quad (1.23)$$

$$g'_{\mu\nu} = \Omega^2 \text{diag}(-1, 1, R^2, R^2 \sin^2 \theta) \quad (1.24)$$

2 Fluctuations

$$ds^2 = \Omega^2(x)(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \quad (2.1)$$

$$\tilde{g}_{ij} = \text{diag}(-1, R^2, R^2 \sin^2 \theta) \quad (2.2)$$

$$f_{00} = -2\phi, \quad f_{0i} = \tilde{\nabla}_i B + B_i, \quad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i \tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \quad (2.3)$$

2.1 $\delta G_{\mu\nu}$

$$\begin{aligned} \delta G_{00} = & 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tilde{\nabla}_a \tilde{\nabla}^a \psi + 4\phi\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega \\ & + 4\psi\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega + 4\Omega^{-1}\tilde{\nabla}_a \dot{\Omega}\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a B - 2\Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \psi \\ & - 2\phi\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega - 2\psi\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b \tilde{\nabla}_a E \\ & + 2\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}_a E\tilde{\nabla}^b \Omega - 4\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b \tilde{\nabla}^a E \\ & + 4B^a \Omega^{-1}\tilde{\nabla}_a \dot{\Omega} - 2B^a \dot{\Omega}\Omega^{-2}\tilde{\nabla}_a \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b E_a + 2\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}_b \Omega\tilde{\nabla}^b E^a \\ & - 4\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b E^a - 4E^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega + 2E_{ab}\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}^b \Omega \end{aligned} \quad (2.4)$$

$$\begin{aligned} \delta G_{0i} = & -\dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_i B + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_i B + \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_i B - 2\tilde{\nabla}_i \dot{\psi} \\ & - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \phi + 2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_i \tilde{\nabla}_a \dot{E} - B_i \dot{\Omega}^2 \Omega^{-2} + 2B_i \dot{\Omega}\Omega^{-1} \\ & + \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a B_i - \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E}_i - 2B_i \Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega + \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a B_i - \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \dot{E}_i \\ & + B_i \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega - \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}_i B^a - \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}_i \dot{E}^a - 2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a \Omega \end{aligned} \quad (2.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\dot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2 \tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2 \tilde{g}_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\ & - 4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} \\ & - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \phi + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \psi - 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \dot{\Omega}\tilde{\nabla}^a B + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a B \\ & - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \dot{B} - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \phi + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b \tilde{\nabla}_a E \\ & - 2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}_a E\tilde{\nabla}^b \Omega + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b \tilde{\nabla}^a E + 2\Omega^{-1}\tilde{\nabla}_i \Omega\tilde{\nabla}_j \psi \\ & + 2\Omega^{-1}\tilde{\nabla}_i \psi\tilde{\nabla}_j \Omega + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i B + \tilde{\nabla}_j \tilde{\nabla}_i \dot{B} - \tilde{\nabla}_j \tilde{\nabla}_i \ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{E} \\ & - 2\dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_j \tilde{\nabla}_i E + 4\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i E - 4\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i E + 2\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i E \\ & + \tilde{\nabla}_j \tilde{\nabla}_i \phi - \tilde{\nabla}_j \tilde{\nabla}_i \psi - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i \tilde{\nabla}_a E \\ & - 4B^a \tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \dot{\Omega} + 2B^a \dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega - 2\dot{B}^a \tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \Omega + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b E_a \\ & - 2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}_b \Omega\tilde{\nabla}^b E^a + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b E^a + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \frac{1}{2}\tilde{\nabla}_i \dot{B}_j - \frac{1}{2}\tilde{\nabla}_i \ddot{E}_j \\ & - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \dot{E}_j - \dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_i E_j + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j - 2\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_i E_j \\ & + \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_i E_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \frac{1}{2}\tilde{\nabla}_j \dot{B}_i - \frac{1}{2}\tilde{\nabla}_j \ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \dot{E}_i - \dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_j E_i \\ & + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_j E_i + \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i E_a \\ & - \ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{E}_{ij}\Omega^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{ij} - 4E_{ij}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega \\ & + 2\Omega^{-1}\tilde{\nabla}_a E_{ij}\tilde{\nabla}^a \Omega + 2E_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega + 4E^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega \\ & - 2E_{ab}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}^b \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_i E_{ja} - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_j E_{ia} \end{aligned} \quad (2.6)$$

$$\begin{aligned} g^{\mu\nu} \delta G_{\mu\nu} = & \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\ = & 6\dot{\Omega}^2 \phi\Omega^{-4} + 6\dot{\Omega}^2 \psi\Omega^{-4} - 6\dot{\phi}\dot{\Omega}\Omega^{-3} - 18\dot{\psi}\dot{\Omega}\Omega^{-3} - 12\ddot{\Omega}\phi\Omega^{-3} - 12\ddot{\Omega}\psi\Omega^{-3} - 6\ddot{\psi}\Omega^{-2} \\ & - 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\Omega^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + 2\Omega^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} + 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} \end{aligned}$$

$$\begin{aligned}
& -2\dot{\Omega}^2\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^aE + 4\ddot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 4\phi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\
& -4\psi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega - 16\Omega^{-3}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^aB + 8\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB - 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} \\
& -6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi + 2\phi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega + 2\psi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega \\
& +2\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE - 4\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE\tilde{\nabla}_b\tilde{\nabla}^b\Omega + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE \\
& -8\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega + 16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE - 16B^a\Omega^{-3}\tilde{\nabla}_a\dot{\Omega} \\
& +8B^a\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega - 6\dot{B}^a\Omega^{-3}\tilde{\nabla}_a\Omega + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a - 8\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a \\
& +16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a + 16E^{ab}\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega - 8E_{ab}\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{2.7}$$

2.2 $\delta T_{\mu\nu}$

From $\delta(g^{\mu\nu}U_\mu U_\nu) = 0$ we find

$$\begin{aligned}
U^\mu\delta U_\mu &= \frac{1}{2}(h_{\mu\nu}U^\mu U^\nu) \\
\rightarrow T\delta U_0 + R\delta U_r &= \Omega X^{-1} \left[-\phi T^2 + (-\psi + \tilde{\nabla}_r\tilde{\nabla}_rE + \tilde{\nabla}_rE_r + E_{rr})R^2 + (\tilde{\nabla}_rB + B_r)TR \right]
\end{aligned} \tag{2.8}$$

$$p = -d^2\Omega^{-4}X^{-4}, \quad U_\mu = \Omega \left(-\frac{T}{X}, \frac{R}{X}, 0, 0 \right), \quad U^\mu = \Omega^{-1} \left(\frac{T}{X}, \frac{R}{X}, 0, 0 \right) \tag{2.9}$$

$$\delta T_{\mu\nu} = \delta p(4U_\mu U_\nu + \Omega^2\tilde{g}_{\mu\nu}) + p(4\delta U_\mu U_\nu + 4U_\mu\delta U_\nu + \Omega^2f_{\mu\nu}) \tag{2.10}$$

$$\delta T_{00} = \Omega^2\delta p(4T^2X^{-2} - 1) - 8\Omega TX^{-1}p\delta U_0 - 2\Omega^2p\phi \tag{2.11}$$

$$\delta T_{0i} = -4\Omega TX^{-1}U_i\delta p + 4pU_i\delta U_0 - 4\Omega TX^{-1}p\delta U_i + \Omega^2p(\tilde{\nabla}_iB + B_i) \tag{2.12}$$

$$\begin{aligned}
\delta T_{ij} &= 4U_iU_j\delta p + \Omega^2\tilde{g}_{ij}\delta p + 4pU_i\delta U_j + 4pU_j\delta U_i \\
&+ \Omega^2p(-2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_jE + \tilde{\nabla}_iE_j + \tilde{\nabla}_jE_i + 2E_{ij})
\end{aligned} \tag{2.13}$$

$$g^{\mu\nu}\delta T_{\mu\nu} = 8\Omega^{-1}TX^{-1}p\delta U_0 + 8pU^i\delta U_i + p(2\phi - 6\psi + 2\tilde{\nabla}_a\tilde{\nabla}^aE) \tag{2.14}$$