1D Kinematics

1501 Physics for Engineers I

University of Connecticut

May 18, 2020

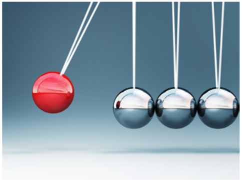
Overview

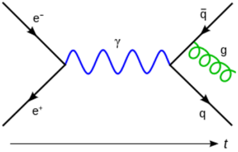
- Physics Theory
- $1D$ Kinematics

Effective Physics Theories

- General Relativity

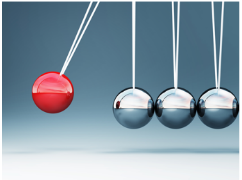
Length and Energy Scale

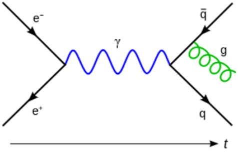




- General Relativity
  - Geometric theory of gravitation

Length and Energy Scale





Effective Physics Theories

- General Relativity
  - Geometric theory of gravitation
  - Relativity of time

Length
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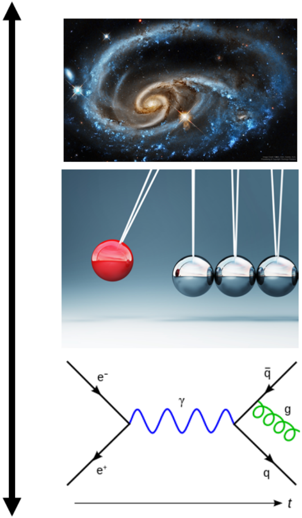
Energy
Scale



Effective Physics Theories

- General Relativity
    - Geometric theory of gravitation
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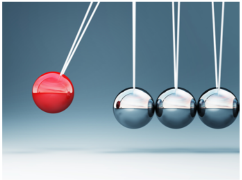

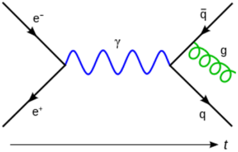

- General Relativity
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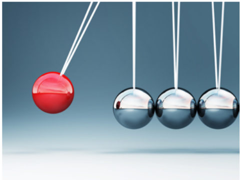

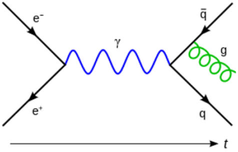

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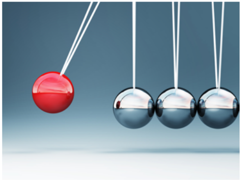

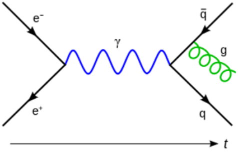

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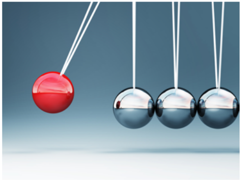

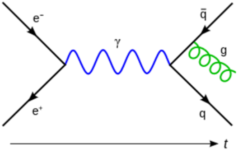

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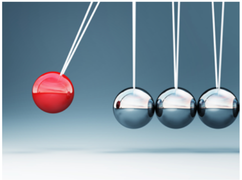

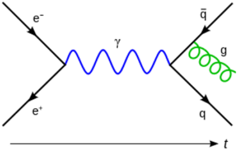

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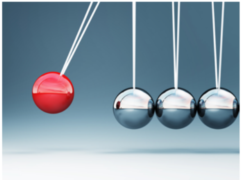
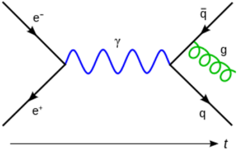
Length and Energy Scale





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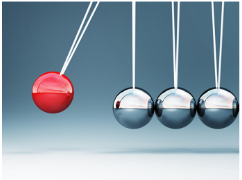

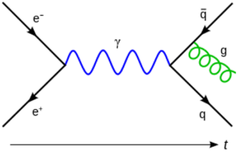

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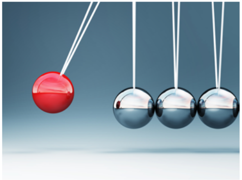
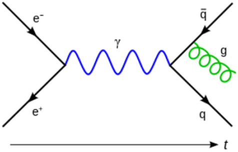
  - Quantum Mechanics
  - Quantum Field Theory

Length and Energy Scale





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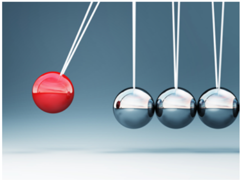
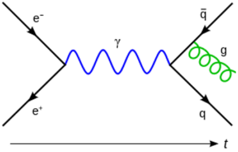
- Quantum Theory
  - Quantum Mechanics
  - Quantum Field Theory
  - Fundamental Particles

Length and Energy Scale





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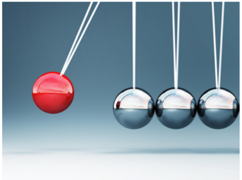
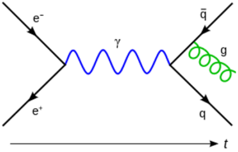
  - Quantum Mechanics
  - Quantum Field Theory
  - Fundamental Particles
  - Probabilistic flow of quantum states

Length and Energy Scale





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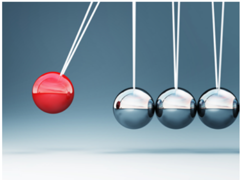
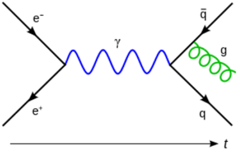
- Quantum Theory
  - Quantum Mechanics
  - Quantum Field Theory
  - Fundamental Particles
  - Probabilistic flow of quantum states
  - Discrete physics properties

Length and Energy Scale





1D Kinematics

- Independent of mass, force

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- Restrict to $D = 1$

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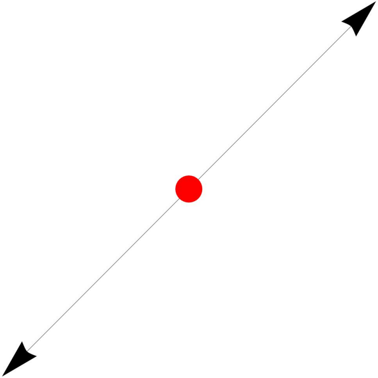
- Independent of mass, force
- Restrict to $D = 1$
- $\frac{d\mathbf{F}}{dt} = 0$

1D Motion: Analysis



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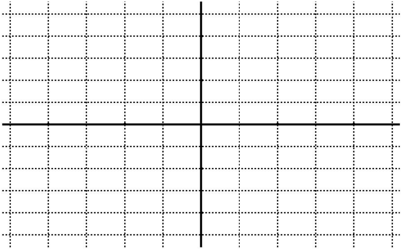

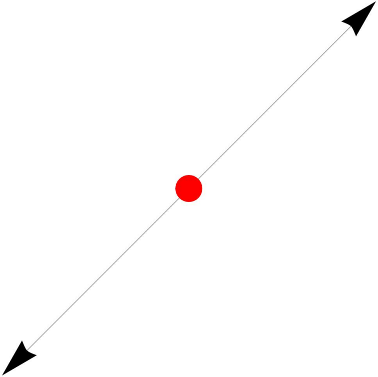

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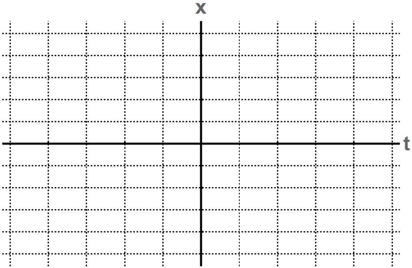

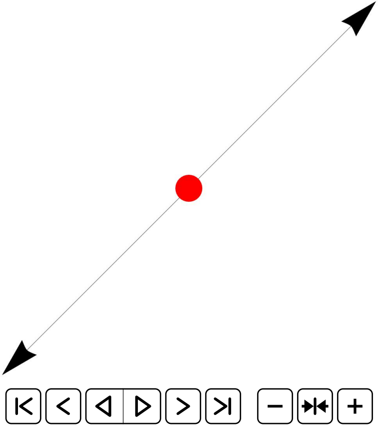

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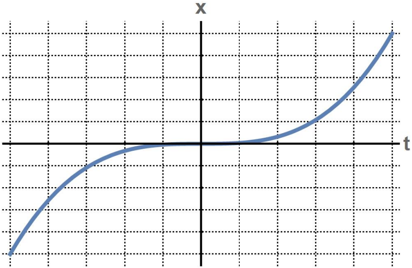

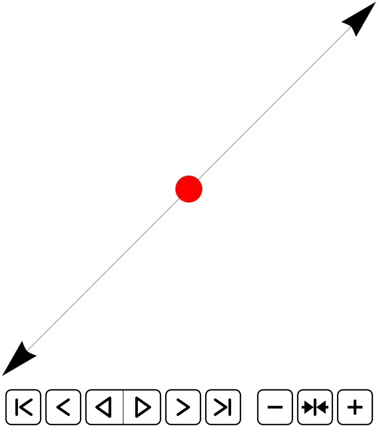

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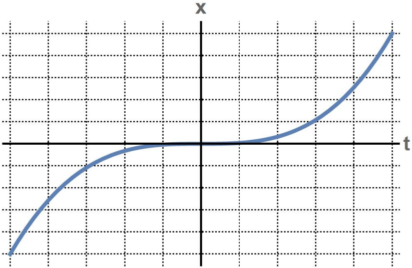
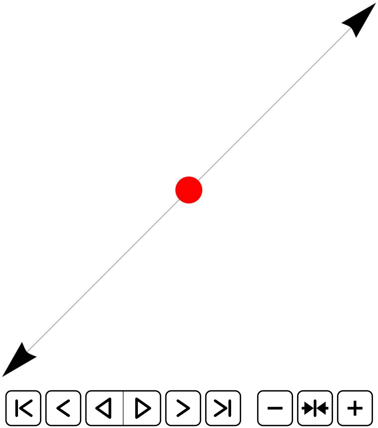
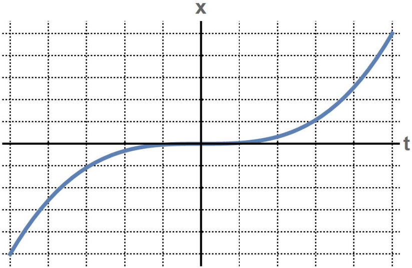


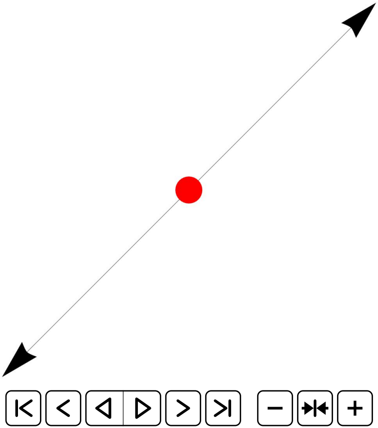
- Distance



1D Motion: Analysis



- Distance
- Position (Displacement)



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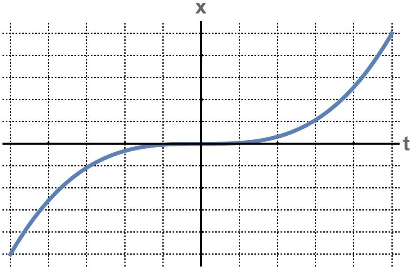
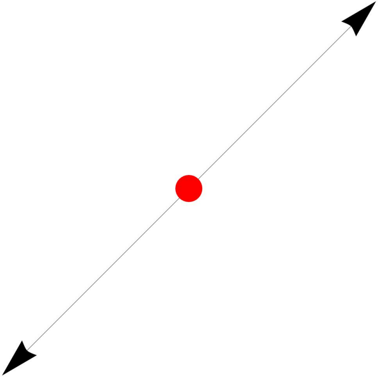
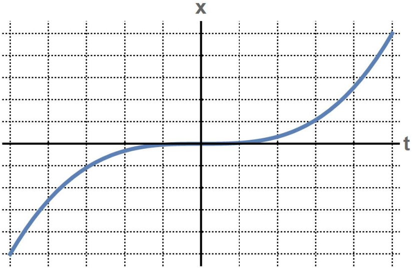


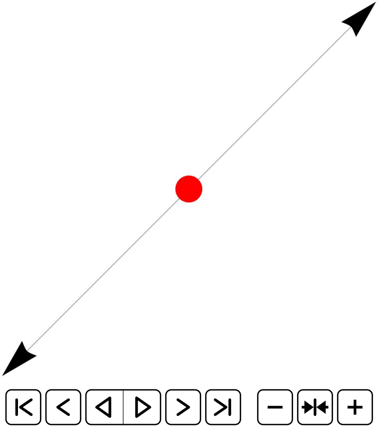


- Distance
- Position (Displacement)
- Speed

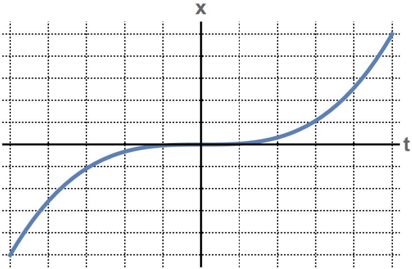
1D Motion: Analysis



- Distance
- Position (Displacement)
- Speed
- Velocity

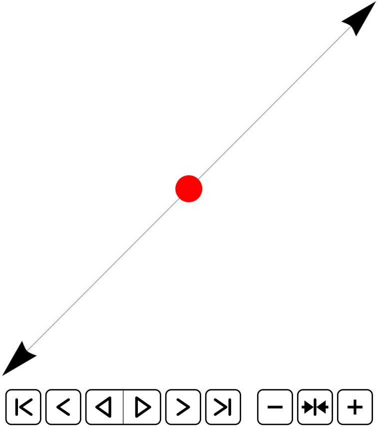


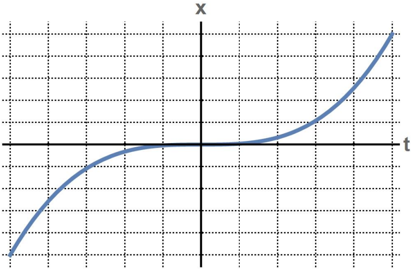
1D Motion: Average Velocity



$$v_{avg} = \frac{\Delta x}{\Delta t}$$

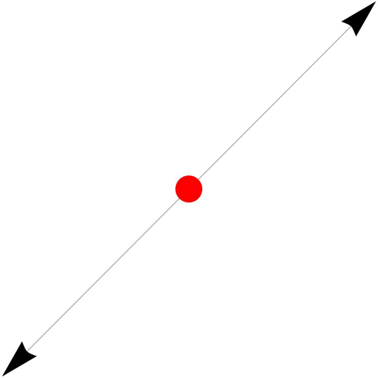
$|v| = $ speed



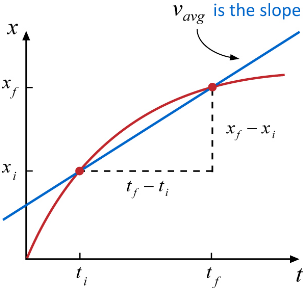


$$v_{avg} = \frac{\Delta x}{\Delta t} \qquad |v| = \text{speed}$$

What about $v(t)$?



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(t_f) - x(t_i)}{t_f - t_i}$$

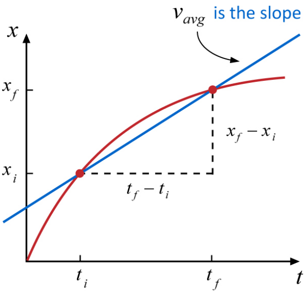


1D Motion: Instantaneous Velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(t_f) - x(t_i)}{t_f - t_i}$$

$$t_f = t_i + \Delta t$$

$$v_{avg} = \frac{x(t_i + \Delta t) - x(t_i)}{\Delta t}$$



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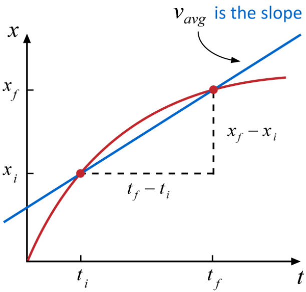
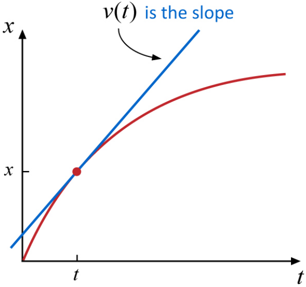
$$v_{avg} = \frac{x(t_i + \Delta t) - x(t_i)}{\Delta t}$$

$$v(t_i) = \lim_{\Delta t \to 0} \frac{x(t_i + \Delta t) - x(t_i)}{\Delta t} = \left.\frac{dx}{dt}\right|_{t=t_i}$$

$$v = \frac{dx}{dt}$$

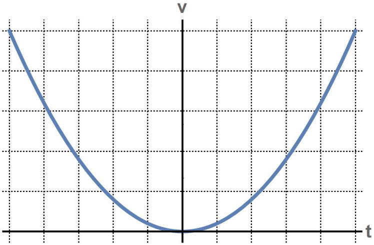




1D Motion: Average Acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(t_f) - v(t_i)}{t_f - t_i}$$

What about $a(t)$?



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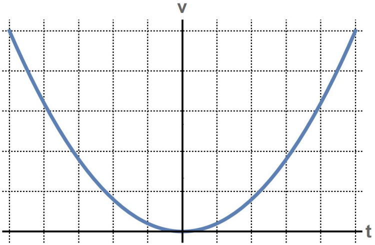

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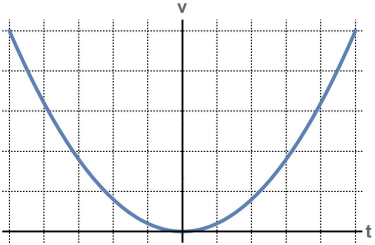

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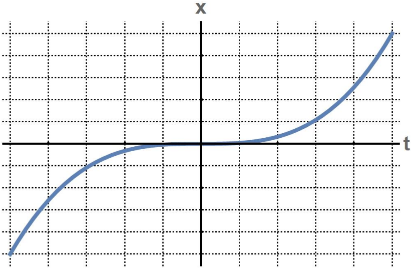
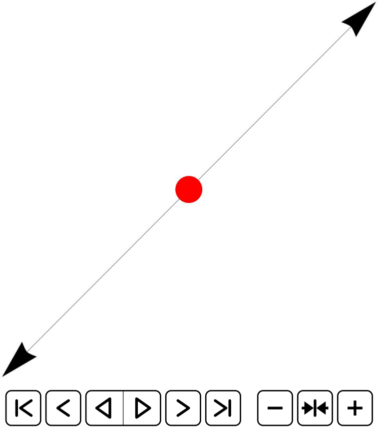
$$a = \frac{dv}{dt}$$



- Distance
- Displacement
- Speed
- Average Velocity
- Instantaneous Velocity
- Average Acceleration
- Instantaneous Acceleration



iClicker 1

A particle's trajectory is given by the function

$$x(t) = (21 + 22t - 6t^2),$$

with t, x in SI units. What is the average velocity from $t = 1$ to $t = 3$?

(A) $2\,\mathrm{m/s}$

(B) $-4\,\mathrm{m/s}$

(C) $-2\,\mathrm{m/s}$

(D) $-8\,\mathrm{m/s}$

(E) $8\,\mathrm{m/s}$

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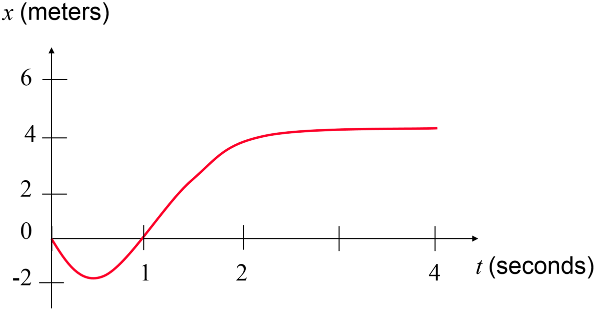
(D) $-8\,\mathrm{m/s}$

(E) $8\,\mathrm{m/s}$

$$\begin{aligned} v_{avg} &= \frac{x(3) - x(1)}{3 - 1} \\ &= \frac{33 - 37}{2} \\ &= -2\,\mathrm{m/s} \end{aligned}$$

What is the velocity (instantaneous) at $t = 4$?

(A) $4\,\mathrm{m/s}$
(B) $0\,\mathrm{m/s}$
(C) $1\,\mathrm{m/s}$
(D) Not enough information



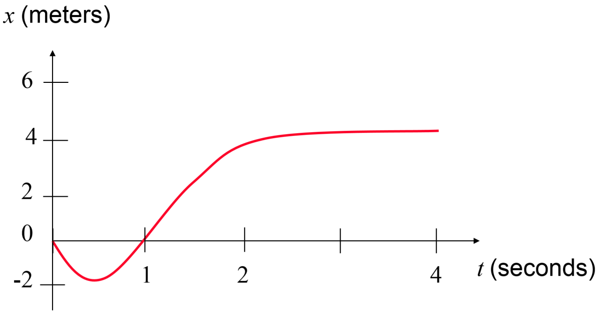
What is the velocity (instantaneous) at $t = 4$?

(A) $4\,\mathrm{m/s}$

(B) $\boxed{0\,\mathrm{m/s}}$

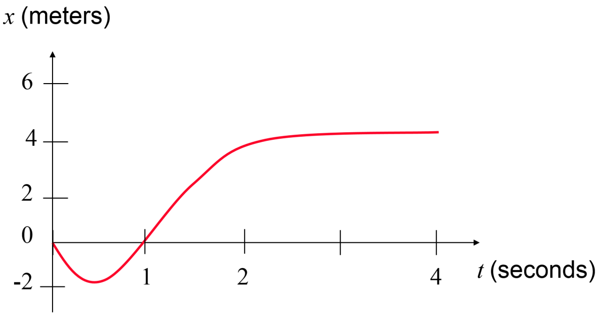
(C) $1\,\mathrm{m/s}$

(D) Not enough information



What is the average velocity over the first 4 seconds?

(A) $-2\,\mathrm{m/s}$
(B) $4\,\mathrm{m/s}$
(C) $1\,\mathrm{m/s}$
(D) Not enough information



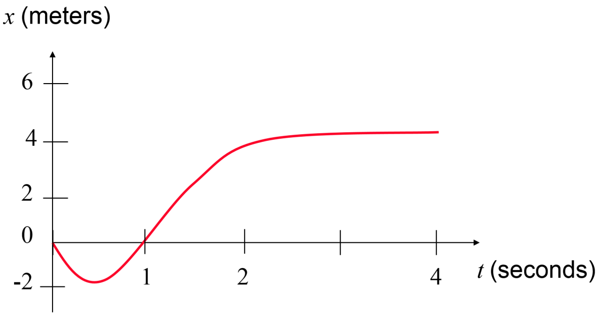
What is the average velocity over the first 4 seconds?

(A) $-2\,\mathrm{m/s}$

(B) $4\,\mathrm{m/s}$

$\boxed{\text{(C) } 1\,\mathrm{m/s}}$

(D) Not enough information



Calculus: Derivatives

Given $f(x)$,

$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$

$$f(x) = x^n \qquad \frac{df}{dx} = nx^{n-1}$$

$$f(x) = \sin(x) \qquad \frac{df}{dx} = \cos x$$

$$f(x) = \cos(x) \qquad \frac{df}{dx} = -\sin x$$

Calculus: Integrals

Definite:

$$\int_a^b f(x)\ dx$$

$$\int_a^b x^n\ dx = \frac{x^{n+1}}{n+1}\bigg|_b^a$$
$$= \frac{a^{n+1}}{(n+1)} - \frac{b^{n+1}}{(n+1)}$$

Indefinite:

$$\int f(x)\ dx$$

$$\int x^n\ dx = \frac{x^{n+1}}{(n+1)} + C$$

1D Kinematic Equations: Derivation

- Fundamental Theorem of Calculus
- $\mathbf{a} =$ constant
- Boundaries

1D Kinematic Equations

Arbitrary Time Interval:

$$v(t_f) - v(t_i) = a(t_f - t_i)$$

$$x(t_f) - x(t_i) = v(t_i) + \frac{1}{2}a(t_f - t_i)^2$$

1D Kinematic Equations

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Initial Time Interval:

$$t_i = 0, \quad t_f = t$$
$$v(0) = v_0, \quad x(0) = x_0$$

$$\boxed{v(t) = v_0 + at}$$

$$\boxed{x(t) = x_0 + v_0 t + \frac{1}{2}at^2}$$

1D Kinematic Equations

Arbitrary Time Interval:

$$v(t_f) - v(t_i) = a(t_f - t_i)$$

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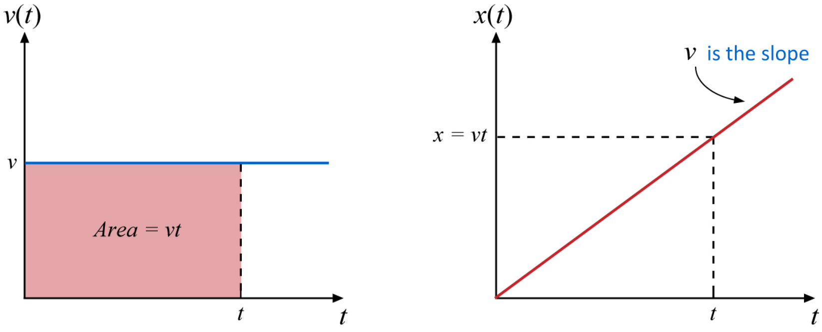
$$\boxed{v(t) = v_0 + at}$$

$$\boxed{x(t) = x_0 + v_0 t + \frac{1}{2}at^2}$$

Solve for common t:

$$\boxed{2a(x - x_0) = v^2 - v_0^2}$$

Example: Velocity Integration



$$\int_a^b v(t)\ dt = \int_a^b \left(\frac{dx}{dt}\right)\ dt$$
$$= x(b) - x(a)$$

The position of a particle is given by

$$x(t) = 6t - 3t^2.$$

Determine the x position at which the particle's velocity is zero.

(A) $3\,\mathrm{m}$
(B) $2\,\mathrm{m}$
(C) $-1\,\mathrm{m}$
(D) $-3\,\mathrm{m}$

iClicker 4

The position of a particle is given by

$$x(t) = 6t - 3t^2.$$

Determine the x position at which the particle's velocity is zero.

$$v = \frac{dx}{dt} = 6 - 6t$$

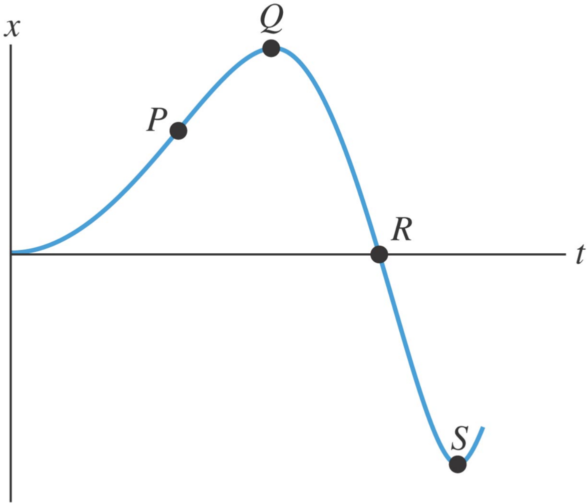
$$6 - 6t = 0$$

$$t = 1$$

$$x(1) = 3\,\mathrm{m}$$

(A) $\boxed{3\,\mathrm{m}}$
(B) $2\,\mathrm{m}$
(C) $-1\,\mathrm{m}$
(D) $-3\,\mathrm{m}$

At which point is the speed the greatest?



(A) P
(B) Q
(C) R
(D) S

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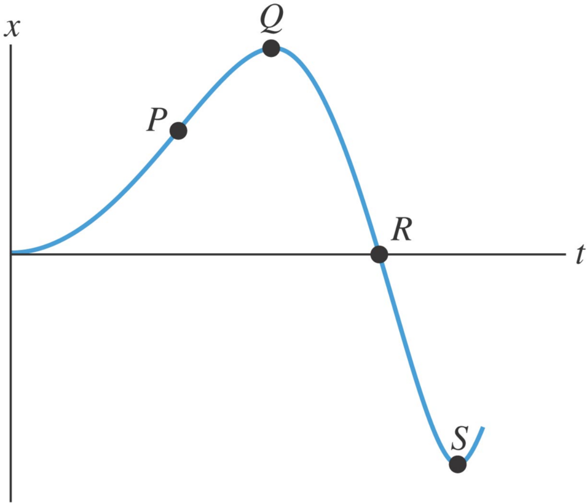
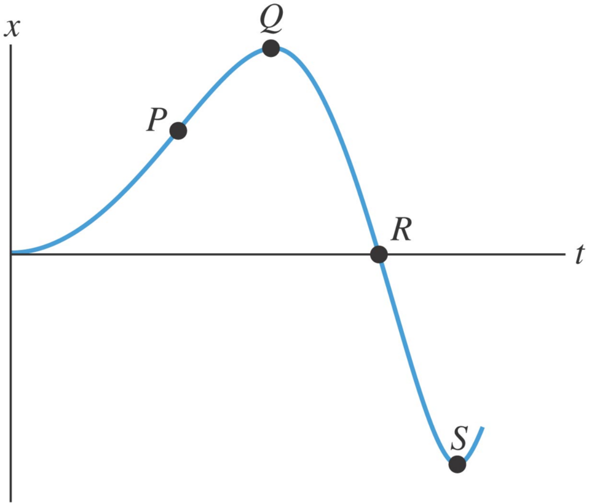
At which point is the speed the greatest?

(A) P
(B) Q
(C) R
(D) S



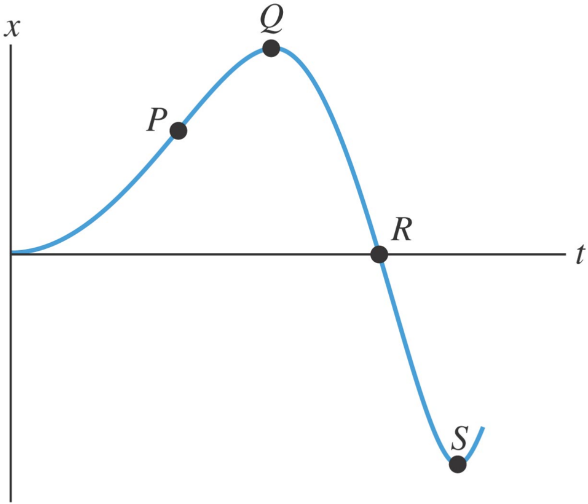
At which point does the object have maximum positive velocity?



(A) P
(B) Q
(C) R
(D) S

At which point does the object have maximum positive velocity?

(A) P
(B) Q
(C) R
(D) S



Example: Vertical Ball Throw

A ball is thrown upward vertically from an initial height of $5\,\mathrm{m}$ and with an initial velocity of $10\,\mathrm{m/s}$. Determine $x(t)$, $v(t)$, and the maximum height the ball reaches.

What time does it hit the ground?

Reminders

- 1D Kinematics Homework due Tuesday Sep. 4 (11:59PM)
- Vectors and 2D Kinematics Prelecture and Checkpoint due Wednesday Sep. 5 (11:25 AM)
- Lab begins week of Sep. 10
    - Lab Notebook
    - Safety Glasses
    - Ruler/Calculator