RW SVT3 $k < 0$ v3

1 T, R Coordinates

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 + r^2/L^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{1.1}$$

$$p = \frac{\tau}{L}, \quad \sinh\chi = \frac{r}{L}, \quad \tau = \int \frac{dt}{a(t)} \tag{1.2}$$

$$ds^2 = L^2 a^2(p)(-dp^2 + d\chi^2 + \sinh^2\chi d\theta^2 + \sin^2\chi \sin^2\theta d\phi^2) \tag{1.3}$$

$$T = e^p \cosh\chi, \quad R = e^p \sinh\chi, \quad X^2 \equiv T^2 - R^2 = e^{2p} \tag{1.4}$$

$$\sinh\chi = \frac{R}{X}, \quad \cosh\chi = \frac{T}{X} \tag{1.5}$$

$$ds^2 = L^2 \frac{a^2(X)}{X^2}(-dT^2 + dR^2 + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2) \tag{1.6}$$

$$\Omega(X) = L\frac{a(X)}{X} \tag{1.7}$$

2 Background

2.1 Comoving $a(t)$

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) = -dt^2 + a(t)^2 \tilde{g}_{ij} dx^i dx^j \tag{2.1}$$

$$G_{00} = -3ka^{-2} - 3\dot{a}^2 a^{-2}, \qquad G_{ij} = \tilde{g}_{ij}(k + \dot{a}^2 + 2a\ddot{a}) \tag{2.2}$$

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu}, \qquad U_\mu = -\delta^0_\mu \tag{2.3}$$

$$T_{00} = \rho, \qquad T_{ij} = a^2(t)p\tilde{g}_{ij} \tag{2.4}$$

$$\Delta^{(0)}_{\mu\nu} = G_{\mu\nu} + T_{\mu\nu} = 0 \tag{2.5}$$

$$\Delta^{(0)}_{00} = \rho - 3ka^{-2} - 3\dot{a}^2 a^{-2}, \qquad \Delta^{(0)}_{ij} = \tilde{g}_{ij}(a^2 p + k + \dot{a}^2 + 2a\ddot{a}) \tag{2.6}$$

$$\rightarrow \qquad \boxed{\rho = 3ka^{-2} + 3\dot{a}^2 a^{-2}} \qquad \boxed{p = -a^{-2}k - \dot{a}^2 a^{-2} - 2\ddot{a}a^{-1}} \tag{2.7}$$

2.2 Conformal $\Omega(X)$

$$ds^2 = \Omega^2(X)\tilde{g}_{\mu\nu}dx^\mu dx^\nu, \qquad \tilde{g}_{\mu\nu} = \mathrm{diag}\left(-1,1,r^2,r^2\sin^2\theta\right) \tag{2.8}$$

$$G_{00} = -3\dot{\Omega}^2\Omega^{-2} + 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega - \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega \tag{2.9}$$

$$G_{0i} = 2\Omega^{-1}\tilde{\nabla}_i\dot{\Omega} - 4\dot{\Omega}\Omega^{-2}\tilde{\nabla}_i\Omega \tag{2.10}$$

$$G_{ij} = -\dot{\Omega}^2\tilde{g}_{ij}\Omega^{-2} + 2\ddot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - 4\Omega^{-2}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\Omega$$
$$+2\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\Omega \tag{2.11}$$

$$g^{\mu\nu}G_{\mu\nu} = 6\ddot{\Omega}\Omega^{-3} - 6\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \tag{2.12}$$

$$U'_\mu(T,R) = \frac{\partial x^\alpha}{\partial x'^\mu}\left(-a(\tau)\delta^0_\alpha\right) = -\frac{\partial\tau}{\partial x'^\mu}a(X) \tag{2.13}$$

$$U_\mu(T,R) = \Omega\left(-\frac{T}{X},\frac{R}{X},0,0\right) \tag{2.14}$$

$$T_{\mu\nu} = (\rho+p)U_\mu U_\nu + p\Omega^2\tilde{g}_{\mu\nu}, \qquad U_\mu = -\Omega\delta^0_\mu \qquad \text{[Evaluated in (2.8)]} \tag{2.15}$$

$$\Delta^{(0)}_{00} = -3k - 3\dot{\Omega}^2\Omega^{-2} + \Omega^2\rho \tag{2.16}$$

$$\rightarrow \quad \boxed{\rho = 3k\Omega^{-2} + 3\dot{\Omega}^2\Omega^{-4}} \tag{2.17}$$

$$\Delta^{(0)}_{ij} = k\tilde{g}_{ij} - \dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} + 2\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij} + \Omega^2 p\tilde{g}_{ij} \tag{2.18}$$

$$\rightarrow \quad \boxed{p = -k\Omega^{-2} + \dot{\Omega}^2\Omega^{-4} - 2\ddot{\Omega}\Omega^{-3}} \tag{2.19}$$

$$\nabla_\mu T^{\mu 0} = \Omega^{-5}\left(\tilde{g}^{ab}T_{ab}\dot{\Omega} + T_{00}\dot{\Omega} + \dot{T}_{00}\Omega - \Omega\tilde{\nabla}_a T_0{}^a\right)$$
$$= 3\dot{\Omega}\Omega^{-3}p + 3\dot{\Omega}\Omega^{-3}\rho + \Omega^{-2}\dot{\rho} \tag{2.20}$$

$$\nabla_\mu T^{\mu i} = \Omega^{-5}\left(-2T_0{}^i\dot{\Omega} - \dot{T}_0{}^i\Omega + \Omega\tilde{\nabla}_a T^{ia}\right)$$
$$= 0 \tag{2.21}$$

3 Fluctuations

$$ds^2 = \Omega^2(\tau)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu \tag{3.1}$$

$$\tilde{g}_{\mu\nu} = \mathrm{diag}\left(-1,\frac{1}{1-kr^2},r^2,r^2\sin^2\theta\right) \tag{3.2}$$

$$f_{00} = -2\phi, \qquad f_{0i} = \tilde{\nabla}_i B + B_i, \qquad f_{ij} = -2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{3.3}$$

$$\delta T_{00} = \Omega^2\delta\rho + (6k + 6\dot{\Omega}^2\Omega^{-2})\phi \tag{3.4}$$

$$\delta T_{0i} = (-k + \dot{\Omega}^2\Omega^{-2} - 2\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_i B$$
$$+(-4\dot{\Omega}^2\Omega^{-3} + 2\ddot{\Omega}\Omega^{-2} - 2k\Omega^{-1})\tilde{\nabla}_i V + (-k + \dot{\Omega}^2\Omega^{-2} - 2\ddot{\Omega}\Omega^{-1})B_i$$
$$+(-4\dot{\Omega}^2\Omega^{-3} + 2\ddot{\Omega}\Omega^{-2} - 2k\Omega^{-1})V_i \tag{3.5}$$

$$\delta T_{ij} = \Omega^2\delta p\tilde{g}_{ij} + (2k\tilde{g}_{ij} - 2\dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} + 4\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij})\psi$$
$$+(-2k + 2\dot{\Omega}^2\Omega^{-2} - 4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_i\tilde{\nabla}_j E + (-k + \dot{\Omega}^2\Omega^{-2} - 2\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_i E_j$$
$$+(-k + \dot{\Omega}^2\Omega^{-2} - 2\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_j E_i + (-2k + 2\dot{\Omega}^2\Omega^{-2} - 4\ddot{\Omega}\Omega^{-1})E_{ij} \tag{3.6}$$

$$g^{\mu\nu}\delta T_{\mu\nu} = 3\delta p - \delta\rho + (-6\dot{\Omega}^2\Omega^{-4} - 6k\Omega^{-2})\phi + (-6\dot{\Omega}^2\Omega^{-4} + 12\ddot{\Omega}\Omega^{-3} + 6k\Omega^{-2})\psi$$
$$+(2\dot{\Omega}^2\Omega^{-4} - 4\ddot{\Omega}\Omega^{-3} - 2k\Omega^{-2})\tilde{\nabla}_a\tilde{\nabla}^a E \tag{3.7}$$

$$\delta G_{00} = 6\dot{\Omega}\Omega^{-1}\dot{\psi} - 6k\phi - 6k\psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a \dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi \tag{3.8}$$

$$\delta G_{0i} = (3k - \dot{\Omega}^2\Omega^{-2} + 2\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_i B - 2k\tilde{\nabla}_i\dot{E} - 2\tilde{\nabla}_i\dot{\psi} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + (2k - \dot{\Omega}^2\Omega^{-2} + 2\ddot{\Omega}\Omega^{-1})B_i$$
$$-k\dot{E}_i + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a \dot{E}_i \tag{3.9}$$

$$\delta G_{ij} = -2\ddot{\psi}\tilde{g}_{ij} - 2\dot{\Omega}\Omega^{-1}\dot{\phi}\tilde{g}_{ij} - 4\dot{\Omega}\Omega^{-1}\dot{\psi}\tilde{g}_{ij} + (2\dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} - 4\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij})\phi$$
$$+(2\dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} - 4\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij})\psi - 2\dot{\Omega}\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a \dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a \ddot{E}$$
$$+2\dot{\Omega}\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a \dot{E} - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E}$$
$$-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} + (2k - 2\dot{\Omega}^2\Omega^{-2} + 4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_j\tilde{\nabla}_i E + \tilde{\nabla}_j\tilde{\nabla}_i\phi - \tilde{\nabla}_j\tilde{\nabla}_i\psi + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j$$
$$+\tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j + (k - \dot{\Omega}^2\Omega^{-2} + 2\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_i E_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i$$
$$-\tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i + (k - \dot{\Omega}^2\Omega^{-2} + 2\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_j E_i - \ddot{E}_{ij} - 2\dot{\Omega}\Omega^{-1}\dot{E}_{ij}$$
$$+(-2\dot{\Omega}^2\Omega^{-2} + 4\ddot{\Omega}\Omega^{-1})E_{ij} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij} \tag{3.10}$$

$$g^{\mu\nu}\delta G_{\mu\nu} = -6\Omega^{-2}\ddot{\psi} - 6\dot{\Omega}\Omega^{-3}\dot{\phi} - 18\dot{\Omega}\Omega^{-3}\dot{\psi} + (6\dot{\Omega}^2\Omega^{-4} - 12\ddot{\Omega}\Omega^{-3} + 6k\Omega^{-2})\phi$$
$$+(6\dot{\Omega}^2\Omega^{-4} - 12\ddot{\Omega}\Omega^{-3} + 6k\Omega^{-2})\psi - 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a \dot{B} + 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a \ddot{E}$$
$$+6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a \dot{E} + (-2\dot{\Omega}^2\Omega^{-4} + 4\ddot{\Omega}\Omega^{-3} + 2k\Omega^{-2})\tilde{\nabla}_a\tilde{\nabla}^a E - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi \tag{3.11}$$

4 Field Equations

$$\Delta_{\mu\nu} \equiv \delta G_{\mu\nu} + \delta T_{\mu\nu} \tag{4.1}$$

$$\Delta_{00} = \Omega^2\delta\rho + 6\dot{\Omega}\Omega^{-1}\dot{\psi} + 6\dot{\Omega}^2\Omega^{-2}\phi - 6k\psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a \dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi \tag{4.2}$$

$$\Delta_{0i} = 2k\tilde{\nabla}_i B - 2k\tilde{\nabla}_i\dot{E} - 2\tilde{\nabla}_i\dot{\psi} + (-4\dot{\Omega}^2\Omega^{-3} + 2\ddot{\Omega}\Omega^{-2} - 2k\Omega^{-1})\tilde{\nabla}_i V - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + kB_i - k\dot{E}_i$$
$$+(-4\dot{\Omega}^2\Omega^{-3} + 2\ddot{\Omega}\Omega^{-2} - 2k\Omega^{-1})V_i + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a \dot{E}_i \tag{4.3}$$

$$\Delta_{ij} = -2\ddot{\psi}\tilde{g}_{ij} + \Omega^2\delta p\tilde{g}_{ij} - 2\dot{\Omega}\Omega^{-1}\dot{\phi}\tilde{g}_{ij} - 4\dot{\Omega}\Omega^{-1}\dot{\psi}\tilde{g}_{ij} + (2\dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} - 4\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij})\phi + 2k\tilde{g}_{ij}\psi$$
$$-2\dot{\Omega}\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a \dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a \ddot{E} + 2\dot{\Omega}\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a \dot{E} - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi$$
$$+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi + (-2k + 2\dot{\Omega}^2\Omega^{-2} - 4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_i\tilde{\nabla}_j E + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E}$$

$$-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} + (2k - 2\dot{\Omega}^2\Omega^{-2} + 4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_j\tilde{\nabla}_iE + \tilde{\nabla}_j\tilde{\nabla}_i\phi - \tilde{\nabla}_j\tilde{\nabla}_i\psi + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_iB_j$$
$$+\tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_jB_i + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i - \ddot{E}_{ij}$$
$$-2\dot{\Omega}\Omega^{-1}\dot{E}_{ij} - 2kE_{ij} + \tilde{\nabla}_a\tilde{\nabla}^aE_{ij} \tag{4.4}$$

$$g^{\mu\nu}\Delta_{\mu\nu} = -6\Omega^{-2}\ddot{\psi} + 3\delta p - \delta\rho - 6\dot{\Omega}\Omega^{-3}\dot{\phi} - 18\dot{\Omega}\Omega^{-3}\dot{\psi} - 12\ddot{\Omega}\Omega^{-3}\phi + 12k\Omega^{-2}\psi - 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aB$$
$$-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi \tag{4.5}$$

5 Field Equations (G.I. Form)

$$\alpha = \phi + \psi + \dot{B} - \ddot{E}, \qquad \gamma = -\dot{\Omega}^{-1}\Omega\psi + B - \dot{E}, \qquad B_i - \dot{E}_i, \qquad E_{ij}, \qquad V_i \tag{5.1}$$

$$V^{GI} = V - \Omega^2\dot{\Omega}^{-1}\psi \tag{5.2}$$

$$\delta\rho^{GI} = \delta\rho - 12\dot{\Omega}^2\psi\Omega^{-4} + 6\ddot{\Omega}\psi\Omega^{-3} - 6k\psi\Omega^{-2} \tag{5.3}$$

$$\delta p^{GI} = \delta p - 4\dot{\Omega}^2\psi\Omega^{-4} + 8\ddot{\Omega}\psi\Omega^{-3} + 2k\psi\Omega^{-2} - 2\dddot{\Omega}\dot{\Omega}^{-1}\psi\Omega^{-2} \tag{5.4}$$

$$\Delta_{00} = \Omega^2\delta\rho^{GI} - 6\dot{\Omega}^2\Omega^{-2}\dot{\gamma} + 6\dot{\Omega}^2\Omega^{-2}\alpha + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\gamma \tag{5.5}$$

$$\Delta_{0i} = 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{\gamma} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\alpha + 2k\tilde{\nabla}_i\gamma + kQ_i + (-4\dot{\Omega}^2\Omega^{-3} + 2\ddot{\Omega}\Omega^{-2} - 2k\Omega^{-1})V_i + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^aQ_i$$
$$+(-4\dot{\Omega}^2\Omega^{-3} + 2\ddot{\Omega}\Omega^{-2} - 2k\Omega^{-1})\tilde{\nabla}_iV^{GI} \tag{5.6}$$

$$\Delta_{ij} = 2\dot{\Omega}\Omega^{-1}\ddot{\gamma}\tilde{g}_{ij} + \Omega^2\delta p^{GI}\tilde{g}_{ij} - 2\dot{\Omega}\Omega^{-1}\dot{\alpha}\tilde{g}_{ij} + \dot{\gamma}(-2\dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} + 4\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij})$$
$$+(2\dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} - 4\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij})\alpha - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\alpha - 2\dot{\Omega}\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\gamma + \tilde{\nabla}_j\tilde{\nabla}_i\alpha$$
$$+2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\gamma + \tfrac{1}{2}\tilde{\nabla}_i\dot{Q}_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_iQ_j + \tfrac{1}{2}\tilde{\nabla}_j\dot{Q}_i + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_jQ_i - \ddot{E}_{ij} - 2\dot{\Omega}\Omega^{-1}\dot{E}_{ij}$$
$$-2kE_{ij} + \tilde{\nabla}_a\tilde{\nabla}^aE_{ij} \tag{5.7}$$

$$g^{\mu\nu}\Delta_{\mu\nu} = 6\dot{\Omega}\Omega^{-3}\ddot{\gamma} + 3\delta p^{GI} - \delta\rho^{GI} - 6\dot{\Omega}\Omega^{-3}\dot{\alpha} + 12\ddot{\Omega}\Omega^{-3}\dot{\gamma} - 12\ddot{\Omega}\Omega^{-3}\alpha - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\alpha$$
$$-6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\gamma \tag{5.8}$$

6 Conservation

Variations are with respect to background (2.8).

$$\delta(\nabla_\mu A^{\mu\nu}) = \Omega^{-4}\tilde{\nabla}_\alpha\delta A^{\nu\alpha} + \tfrac{1}{2}A^{(0)\nu}{}_\alpha\Omega^{-4}\tilde{\nabla}^\alpha f + 2\delta A^\nu{}_\alpha\Omega^{-5}\tilde{\nabla}^\alpha\Omega - 2A^{(0)\nu\beta}f_{\alpha\beta}\Omega^{-5}\tilde{\nabla}^\alpha\Omega$$
$$+A^{(0)\beta}{}_\beta f^\nu{}_\alpha\Omega^{-5}\tilde{\nabla}^\alpha\Omega - 2A^{(0)}{}_\alpha{}^\beta f^\nu{}_\beta\Omega^{-5}\tilde{\nabla}^\alpha\Omega - f^{\nu\alpha}\Omega^{-4}\tilde{\nabla}_\beta A^{(0)}{}_\alpha{}^\beta - f^{\alpha\beta}\Omega^{-4}\tilde{\nabla}_\beta A^{(0)\nu}{}_\alpha$$
$$-A^{(0)\nu\alpha}\Omega^{-4}\tilde{\nabla}_\beta f_\alpha{}^\beta - \tfrac{1}{2}A^{(0)\alpha\beta}\Omega^{-4}\tilde{\nabla}^\nu f_{\alpha\beta} - \delta A^\alpha{}_\alpha\Omega^{-5}\tilde{\nabla}^\nu\Omega + A^{(0)\alpha\beta}f_{\alpha\beta}\Omega^{-5}\tilde{\nabla}^\nu\Omega \tag{6.1}$$

$$\delta(\nabla_\mu T^{\mu 0}) = \delta T^a{}_a\dot{\Omega}\Omega^{-5} + \delta T_{00}\dot{\Omega}\Omega^{-5} - T^{ab}\dot{\Omega}f_{ab}\Omega^{-5} + T^a{}_a\dot{\Omega}f_{00}\Omega^{-5} + 2T_{00}\dot{\Omega}f_{00}\Omega^{-5} - 2T_0{}^a\dot{\Omega}f_{0a}\Omega^{-5}$$
$$+\delta\dot{T}_{00}\Omega^{-4} + \tfrac{1}{2}T^{ab}\dot{f}_{ab}\Omega^{-4} + \tfrac{3}{2}T_{00}\dot{f}_{00}\Omega^{-4} - 2T_0{}^a\dot{f}_{0a}\Omega^{-4} + \tfrac{1}{2}T_{00}\dot{f}\Omega^{-4} + 2\dot{T}_{00}f_{00}\Omega^{-4}$$
$$-2\dot{T}_0{}^af_{0a}\Omega^{-4} - \Omega^{-4}\tilde{\nabla}_a\delta T_0{}^a - f_0{}^a\Omega^{-4}\tilde{\nabla}_aT_{00} - f_{00}\Omega^{-4}\tilde{\nabla}_aT_0{}^a - T_{00}\Omega^{-4}\tilde{\nabla}_af_0{}^a$$
$$-\tfrac{1}{2}T_0{}^a\Omega^{-4}\tilde{\nabla}_af + f_0{}^a\Omega^{-4}\tilde{\nabla}_bT_a{}^b + T_0{}^a\Omega^{-4}\tilde{\nabla}_bf_a{}^b + f_{ab}\Omega^{-4}\tilde{\nabla}^bT_0{}^a \tag{6.2}$$

4

$$
\begin{aligned}
=\ & \Omega^{-2}\dot{\delta\rho} + 3\dot{\Omega}\Omega^{-3}\delta p + 3\dot{\Omega}\Omega^{-3}\delta\rho + (-12\dot{\Omega}^2\Omega^{-6} + 6\ddot{\Omega}\Omega^{-5} - 6k\Omega^{-4})\dot{\psi} \\
& + (-4\dot{\Omega}^2\Omega^{-6} + 2\ddot{\Omega}\Omega^{-5} - 2k\Omega^{-4})\tilde{\nabla}_a\tilde{\nabla}^a B + (4\dot{\Omega}^2\Omega^{-6} - 2\ddot{\Omega}\Omega^{-5} + 2k\Omega^{-4})\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
& + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\tilde{\nabla}_a\tilde{\nabla}^a V
\end{aligned}
\tag{6.3}
$$

$$
\begin{aligned}
=\ & \Omega^{-2}\dot{\delta\rho}^{GI} + 3\dot{\Omega}\Omega^{-3}\delta p^{GI} + 3\dot{\Omega}\Omega^{-3}\delta\rho^{GI} \\
& + (-4\dot{\Omega}^2\Omega^{-6} + 2\ddot{\Omega}\Omega^{-5} - 2k\Omega^{-4})\tilde{\nabla}_a\tilde{\nabla}^a\gamma + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\tilde{\nabla}_a\tilde{\nabla}^a V^{GI}
\end{aligned}
\tag{6.4}
$$

$$
\begin{aligned}
\delta(\nabla_\mu T^{\mu i}) =\ & -2\delta T_0{}^i\dot{\Omega}\Omega^{-5} + 2T_0{}^a\dot{\Omega}f^i{}_a\Omega^{-5} - 2T_0{}^i\dot{\Omega}f_{00}\Omega^{-5} + 2T^i{}_a\dot{\Omega}f_0{}^a\Omega^{-5} - T^a{}_a\dot{\Omega}f_0{}^i\Omega^{-5} \\
& - T_{00}\dot{\Omega}f_0{}^i\Omega^{-5} - \dot{\delta T_0{}^i}\Omega^{-4} - T_0{}^i\dot{f}_{00}\Omega^{-4} + T^i{}_a\dot{f}_0{}^a\Omega^{-4} - \tfrac{1}{2}T_0{}^i\dot{f}\Omega^{-4} + \dot{T}_0{}^a f^i{}_a\Omega^{-4} \\
& - \dot{T}_0{}^i f_{00}\Omega^{-4} + \dot{T}^i{}_a f_0{}^a\Omega^{-4} - \dot{T}_{00}f_0{}^i\Omega^{-4} + \Omega^{-4}\tilde{\nabla}_a\delta T^{ia} + f_0{}^i\Omega^{-4}\tilde{\nabla}_a T_0{}^a + f_0{}^a\Omega^{-4}\tilde{\nabla}_a T_0{}^i \\
& + T_0{}^i\Omega^{-4}\tilde{\nabla}_a f_0{}^a + \tfrac{1}{2}T^i{}_a\Omega^{-4}\tilde{\nabla}^a f - f^{ia}\Omega^{-4}\tilde{\nabla}_b T_a{}^b - f^{ab}\Omega^{-4}\tilde{\nabla}_b T^i{}_a - T^{ia}\Omega^{-4}\tilde{\nabla}_b f_a{}^b \\
& - \tfrac{1}{2}T^{ab}\Omega^{-4}\tilde{\nabla}^i f_{ab} - \tfrac{1}{2}T_{00}\Omega^{-4}\tilde{\nabla}^i f_{00} + T_0{}^a\Omega^{-4}\tilde{\nabla}^i f_{0a}
\end{aligned}
\tag{6.5}
$$

$$
\begin{aligned}
=\ & \Omega^{-2}\tilde{\nabla}^i\delta p + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\tilde{\nabla}^i\dot{V} \\
& + (-4\dot{\Omega}^3\Omega^{-8} + 8\ddot{\Omega}\dot{\Omega}\Omega^{-7} - 2\dddot{\Omega}\Omega^{-6} + 2\dot{\Omega}k\Omega^{-6})\tilde{\nabla}^i V \\
& + (4\dot{\Omega}^2\Omega^{-6} - 2\ddot{\Omega}\Omega^{-5} + 2k\Omega^{-4})\tilde{\nabla}^i\phi + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\dot{V}^i \\
& + (-4\dot{\Omega}^3\Omega^{-8} + 8\ddot{\Omega}\dot{\Omega}\Omega^{-7} - 2\dddot{\Omega}\Omega^{-6} + 2\dot{\Omega}k\Omega^{-6})V^i
\end{aligned}
\tag{6.6}
$$

$$
\begin{aligned}
=\ & \Omega^{-2}\tilde{\nabla}^i\delta p^{GI} + (-4\dot{\Omega}^2\Omega^{-6} + 2\ddot{\Omega}\Omega^{-5} - 2k\Omega^{-4})\tilde{\nabla}^i\dot{\gamma} \\
& + (4\dot{\Omega}^2\Omega^{-6} - 2\ddot{\Omega}\Omega^{-5} + 2k\Omega^{-4})\tilde{\nabla}^i\alpha + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\dot{V}^i \\
& + (-4\dot{\Omega}^3\Omega^{-8} + 8\ddot{\Omega}\dot{\Omega}\Omega^{-7} - 2\dddot{\Omega}\Omega^{-6} + 2\dot{\Omega}k\Omega^{-6})V^i + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\tilde{\nabla}^i\dot{V}^{GI} \\
& + (-4\dot{\Omega}^3\Omega^{-8} + 8\ddot{\Omega}\dot{\Omega}\Omega^{-7} - 2\dddot{\Omega}\Omega^{-6} + 2\dot{\Omega}k\Omega^{-6})\tilde{\nabla}^i V^{GI}
\end{aligned}
\tag{6.7}
$$

$$
\begin{aligned}
\delta(\nabla_\mu G^{\mu 0}) =\ & \delta G^a{}_a\dot{\Omega}\Omega^{-5} + \delta G_{00}\dot{\Omega}\Omega^{-5} - 2B^a G_{0a}\dot{\Omega}\Omega^{-5} - 2G^{ab}\dot{\Omega}E_{ab}\Omega^{-5} - 2G^a{}_a\dot{\Omega}\phi\Omega^{-5} - 4G_{00}\dot{\Omega}\phi\Omega^{-5} \\
& + 2G^a{}_a\dot{\Omega}\psi\Omega^{-5} + \dot{\delta G_{00}}\Omega^{-4} - 2B^a\dot{G}_{0a}\Omega^{-4} - 2\dot{B}^a G_{0a}\Omega^{-4} + G^{ab}\dot{E}_{ab}\Omega^{-4} - 2G_{00}\dot{\phi}\Omega^{-4} \\
& - G^a{}_a\dot{\psi}\Omega^{-4} - 3G_{00}\dot{\psi}\Omega^{-4} + 2kG_0{}^a E_a\Omega^{-4} - 4\dot{G}_{00}\phi\Omega^{-4} - 2G_0{}^a\dot{\Omega}\Omega^{-5}\tilde{\nabla}_a B - 2\dot{G}_0{}^a\Omega^{-4}\tilde{\nabla}_a B \\
& - 2G_0{}^a\Omega^{-4}\tilde{\nabla}_a\dot{B} - \Omega^{-4}\tilde{\nabla}_a\delta G_0{}^a - B^a\Omega^{-4}\tilde{\nabla}_a G_{00} + 2\phi\Omega^{-4}\tilde{\nabla}_a G_0{}^a - 2\psi\Omega^{-4}\tilde{\nabla}_a G_0{}^a \\
& + 2kG_0{}^a\Omega^{-4}\tilde{\nabla}_a E - G_0{}^a\Omega^{-4}\tilde{\nabla}_a\phi + G_0{}^a\Omega^{-4}\tilde{\nabla}_a\psi - G_{00}\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^a B + G_{00}\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
& - \Omega^{-4}\tilde{\nabla}_a G_{00}\tilde{\nabla}^a B + B^a\Omega^{-4}\tilde{\nabla}_b G_a{}^b + \Omega^{-4}\tilde{\nabla}^a B\tilde{\nabla}_b G_a{}^b + G_0{}^a\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}^b E_a \\
& + G_0{}^a\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E + 2E_{ab}\Omega^{-4}\tilde{\nabla}^b G_0{}^a + \Omega^{-4}\tilde{\nabla}_a E_b\tilde{\nabla}^b G_0{}^a + \Omega^{-4}\tilde{\nabla}_b E_a\tilde{\nabla}^b G_0{}^a \\
& + 2\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b G_0{}^a + G_{ab}\Omega^{-4}\tilde{\nabla}^b\dot{E}^a - 2G_{ab}\dot{\Omega}\Omega^{-5}\tilde{\nabla}^b E^a + G_{ab}\Omega^{-4}\tilde{\nabla}^b\tilde{\nabla}^a\dot{E} \\
& - 2G_{ab}\dot{\Omega}\Omega^{-5}\tilde{\nabla}^b\tilde{\nabla}^a E
\end{aligned}
\tag{6.8}
$$

$$
=\ 0
\tag{6.9}
$$

$$
\begin{aligned}
\delta(\nabla_\mu G^{\mu i}) =\ & -2\delta G_0{}^i\dot{\Omega}\Omega^{-5} - B^i G^a{}_a\dot{\Omega}\Omega^{-5} + 2B^a G^i{}_a\dot{\Omega}\Omega^{-5} - B^i G_{00}\dot{\Omega}\Omega^{-5} + 4G_0{}^a\dot{\Omega}E^i{}_a\Omega^{-5} \\
& + 4G_0{}^i\dot{\Omega}\phi\Omega^{-5} - 4G_0{}^i\dot{\Omega}\psi\Omega^{-5} - \dot{\delta G_0{}^i}\Omega^{-4} + B^a\dot{G}^i{}_a\Omega^{-4} - B^i\dot{G}_{00}\Omega^{-4} + \dot{B}^a G^i{}_a\Omega^{-4} \\
& + G_0{}^i\dot{\phi}\Omega^{-4} + 3G_0{}^i\dot{\psi}\Omega^{-4} + 2\dot{G}_0{}^a E^i{}_a\Omega^{-4} - 2kG^i{}_a E^a\Omega^{-4} + 2\dot{G}_0{}^i\phi\Omega^{-4} - 2\dot{G}_0{}^i\psi\Omega^{-4} \\
& + \Omega^{-4}\tilde{\nabla}_a\delta G^{ia} + 4\psi\Omega^{-4}\tilde{\nabla}_a G^{ia} + B^i\Omega^{-4}\tilde{\nabla}_a G_0{}^a + B^a\Omega^{-4}\tilde{\nabla}_a G_0{}^i + 2G_0{}^a\dot{\Omega}\Omega^{-5}\tilde{\nabla}_a E^i \\
& + \dot{G}_0{}^a\Omega^{-4}\tilde{\nabla}_a E^i + G_0{}^i\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^a B - G_0{}^i\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} + 2G^i{}_a\dot{\Omega}\Omega^{-5}\tilde{\nabla}^a B + \dot{G}^i{}_a\Omega^{-4}\tilde{\nabla}^a B
\end{aligned}
$$

$$+\Omega^{-4}\tilde{\nabla}_a G_0{}^i \tilde{\nabla}^a B + G^i{}_a \Omega^{-4}\tilde{\nabla}^a \dot{B} - 2k G^i{}_a \Omega^{-4}\tilde{\nabla}^a E + G^i{}_a \Omega^{-4}\tilde{\nabla}^a \phi - G^i{}_a \Omega^{-4}\tilde{\nabla}^a \psi$$
$$-2E^{ia}\Omega^{-4}\tilde{\nabla}_b G_a{}^b - \Omega^{-4}\tilde{\nabla}^a E^i \tilde{\nabla}_b G_a{}^b - 2E^{ab}\Omega^{-4}\tilde{\nabla}_b G^i{}_a - G^i{}_a \Omega^{-4}\tilde{\nabla}_b \tilde{\nabla}^b E^a$$
$$-G^i{}_a \Omega^{-4}\tilde{\nabla}_b \tilde{\nabla}^b \tilde{\nabla}^a E - \Omega^{-4}\tilde{\nabla}_a G^i{}_b \tilde{\nabla}^b E^a - \Omega^{-4}\tilde{\nabla}_b G^i{}_a \tilde{\nabla}^b E^a - 2\Omega^{-4}\tilde{\nabla}_b G^i{}_a \tilde{\nabla}^b \tilde{\nabla}^a E$$
$$-G^a{}_a \dot{\Omega}\Omega^{-5}\tilde{\nabla}^i B - G_{00}\dot{\Omega}\Omega^{-5}\tilde{\nabla}^i B - \dot{G}_{00}\Omega^{-4}\tilde{\nabla}^i B + \Omega^{-4}\tilde{\nabla}_a G_0{}^a \tilde{\nabla}^i B + G_0{}^a \Omega^{-4}\tilde{\nabla}^i B_a$$
$$-G^{ab}\Omega^{-4}\tilde{\nabla}^i E_{ab} + 2G_0{}^a \dot{\Omega}\Omega^{-5}\tilde{\nabla}^i E_a + \dot{G}_0{}^a \Omega^{-4}\tilde{\nabla}^i E_a - \Omega^{-4}\tilde{\nabla}_b G_a{}^b \tilde{\nabla}^i E^a + G_{00}\Omega^{-4}\tilde{\nabla}^i \phi$$
$$+G^a{}_a \Omega^{-4}\tilde{\nabla}^i \psi + G_0{}^a \Omega^{-4}\tilde{\nabla}^i \tilde{\nabla}_a B + 4G_0{}^a \dot{\Omega}\Omega^{-5}\tilde{\nabla}^i \tilde{\nabla}_a E + 2\dot{G}_0{}^a \Omega^{-4}\tilde{\nabla}^i \tilde{\nabla}_a E$$
$$-2\Omega^{-4}\tilde{\nabla}_b G_a{}^b \tilde{\nabla}^i \tilde{\nabla}^a E - G_{ab}\Omega^{-4}\tilde{\nabla}^i \tilde{\nabla}^b E^a - G_{ab}\Omega^{-4}\tilde{\nabla}^i \tilde{\nabla}^b \tilde{\nabla}^a E \tag{6.10}$$

$$= \quad 0 \tag{6.11}$$

$$\delta(\nabla_\mu \Delta^{\mu 0}) = \Omega^{-2}\dot{\delta\rho}^{GI} + 3\dot{\Omega}\Omega^{-3}\delta p^{GI} + 3\dot{\Omega}\Omega^{-3}\delta\rho^{GI}$$
$$+(-4\dot{\Omega}^2\Omega^{-6} + 2\ddot{\Omega}\Omega^{-5} - 2k\Omega^{-4})\tilde{\nabla}_a\tilde{\nabla}^a\gamma + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\tilde{\nabla}_a\tilde{\nabla}^a V^{GI} \tag{6.12}$$

$$\delta(\nabla_\mu \Delta^{\mu i}) = \Omega^{-2}\tilde{\nabla}^i \delta p^{GI} + (-4\dot{\Omega}^2\Omega^{-6} + 2\ddot{\Omega}\Omega^{-5} - 2k\Omega^{-4})\tilde{\nabla}^i\dot{\gamma}$$
$$+(4\dot{\Omega}^2\Omega^{-6} - 2\ddot{\Omega}\Omega^{-5} + 2k\Omega^{-4})\tilde{\nabla}^i\alpha + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\dot{V}^i$$
$$+(-4\dot{\Omega}^3\Omega^{-8} + 8\ddot{\Omega}\dot{\Omega}\Omega^{-7} - 2\dddot{\Omega}\Omega^{-6} + 2\dot{\Omega}k\Omega^{-6})V^i + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\tilde{\nabla}^i\dot{V}^{GI}$$
$$+(-4\dot{\Omega}^3\Omega^{-8} + 8\ddot{\Omega}\dot{\Omega}\Omega^{-7} - 2\dddot{\Omega}\Omega^{-6} + 2\dot{\Omega}k\Omega^{-6})\tilde{\nabla}^i V^{GI} \tag{6.13}$$

$$\nabla_i\delta(\nabla_\mu \Delta^{\mu i}) = \Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a \delta p^{GI} + (-4\dot{\Omega}^2\Omega^{-6} + 2\ddot{\Omega}\Omega^{-5} - 2k\Omega^{-4})\tilde{\nabla}_a\tilde{\nabla}^a\dot{\gamma}$$
$$+(4\dot{\Omega}^2\Omega^{-6} - 2\ddot{\Omega}\Omega^{-5} + 2k\Omega^{-4})\tilde{\nabla}_a\tilde{\nabla}^a\alpha + (4\dot{\Omega}^2\Omega^{-7} - 2\ddot{\Omega}\Omega^{-6} + 2k\Omega^{-5})\tilde{\nabla}_a\tilde{\nabla}^a\dot{V}^{GI}$$
$$+(-4\dot{\Omega}^3\Omega^{-8} + 8\ddot{\Omega}\dot{\Omega}\Omega^{-7} - 2\dddot{\Omega}\Omega^{-6} + 2\dot{\Omega}k\Omega^{-6})\tilde{\nabla}_a\tilde{\nabla}^a V^{GI} \tag{6.14}$$

Computationally, we find that $\delta(\nabla_\mu G^{\mu\nu})$ evaluates to zero as expected from the Bianchi identity and that $\delta(\nabla_\mu \Delta^{\mu\nu}) = \delta(\nabla_\mu T^{\mu\nu})$. This is the perturbed covariant conservation condition for a RW perfect fluid in analogy to (2.20).

Appendix A   Possibly Useful Relations

$$\tilde{\nabla}^i\Delta_{0i} = 2\dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot\gamma - 2\dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\alpha + 2k\tilde{\nabla}_a\tilde{\nabla}^a\gamma + (-4\dot\Omega^2\Omega^{-3} + 2\ddot\Omega\Omega^{-2} - 2k\Omega^{-1})\tilde{\nabla}_a\tilde{\nabla}^a V^G \tag{A.1}$$

$$
\begin{aligned}
\tilde{g}^{ij}\Delta_{ij} = {}& 6\dot\Omega\Omega^{-1}\ddot\gamma + 3\Omega^2\delta p^{GI} - 6\dot\Omega\Omega^{-1}\dot\alpha + (-6\dot\Omega^2\Omega^{-2} + 12\ddot\Omega\Omega^{-1})\dot\gamma + (6\dot\Omega^2\Omega^{-2} - 12\ddot\Omega\Omega^{-1})\alpha \\
& -2\tilde{\nabla}_a\tilde{\nabla}^a\alpha - 4\dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\gamma
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
\tilde{\nabla}^i\Delta_{ij} = {}& 2\dot\Omega\Omega^{-1}\tilde{\nabla}_j\ddot\gamma + \Omega^2\tilde{\nabla}_j\delta p^{GI} - 2\dot\Omega\Omega^{-1}\tilde{\nabla}_j\dot\alpha + (-2\dot\Omega^2\Omega^{-2} + 4\ddot\Omega\Omega^{-1})\tilde{\nabla}_j\dot\gamma \\
& + (2k + 2\dot\Omega^2\Omega^{-2} - 4\ddot\Omega\Omega^{-1})\tilde{\nabla}_j\alpha + 4\dot\Omega k\Omega^{-1}\tilde{\nabla}_j\gamma + k\dot Q_j + 2\dot\Omega k\Omega^{-1}Q_j + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot Q_j \\
& + \dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a Q_j
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
\tilde{\nabla}^i\tilde{\nabla}^j\Delta_{ij} = {}& 2\dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\ddot\gamma + \Omega^2\tilde{\nabla}_a\tilde{\nabla}^a\delta p^{GI} - 2\dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot\alpha + (-2\dot\Omega^2\Omega^{-2} + 4\ddot\Omega\Omega^{-1})\tilde{\nabla}_a\tilde{\nabla}^a\dot\gamma \\
& + (2k + 2\dot\Omega^2\Omega^{-2} - 4\ddot\Omega\Omega^{-1})\tilde{\nabla}_a\tilde{\nabla}^a\alpha + 4\dot\Omega k\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\gamma
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
\tilde{\nabla}_a\tilde{\nabla}^a\Delta_{ij} = {}& 2\dot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot\gamma + \Omega^2\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\delta p^{GI} - 2\dot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot\alpha \\
& + (-2\dot\Omega^2\Omega^{-2}\tilde{g}_{ij} + 4\ddot\Omega\Omega^{-1}\tilde{g}_{ij})\tilde{\nabla}_a\tilde{\nabla}^a\dot\gamma + (2\dot\Omega^2\Omega^{-2}\tilde{g}_{ij} - 4\ddot\Omega\Omega^{-1}\tilde{g}_{ij})\tilde{\nabla}_a\tilde{\nabla}^a\alpha + \tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j\tilde{\nabla}_i\alpha \\
& + 2\dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j\tilde{\nabla}_i\gamma - \tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\alpha - 2\dot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\gamma + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_i\dot Q_j \\
& + \dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_i Q_j + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j\dot Q_i + \dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j Q_i - \tilde{\nabla}_a\tilde{\nabla}^a\ddot E_{ij} - 2\dot\Omega\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot E_{ij} \\
& - 2k\tilde{\nabla}_a\tilde{\nabla}^a E_{ij} + \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a E_{ij}
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b\Delta_{ij} = {}& 2\dot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot\gamma + \Omega^2\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\delta p^{GI} - 2\dot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot\alpha \\
& - 2\dot\Omega^2\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot\gamma + 4\ddot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot\gamma + 2\dot\Omega^2\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\alpha \\
& - 4\ddot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\alpha + \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j\tilde{\nabla}_i\alpha + 2\dot\Omega\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j\tilde{\nabla}_i\gamma \\
& - \tilde{g}_{ij}\tilde{\nabla}_c\tilde{\nabla}^c\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\alpha - 2\dot\Omega\Omega^{-1}\tilde{g}_{ij}\tilde{\nabla}_c\tilde{\nabla}^c\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\gamma + \tfrac{1}{2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_i\dot Q_j \\
& + \dot\Omega\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_i Q_j + \tfrac{1}{2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j\dot Q_i + \dot\Omega\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_j Q_i - \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot E_{ij} \\
& - 2\dot\Omega\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot E_{ij} - 2k\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a E_{ij} + \tilde{\nabla}_c\tilde{\nabla}^c\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a E_{ij}
\end{aligned}
\tag{A.6}
$$