Astrophysics & Cosmology
HW 6

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8.1 The leakage rate of photons from the interior of the sun is

$$L = \frac{(4\pi R^3/3)(aT^4)}{3R^2/lc}.$$

Since in this problem we are only interested in proportionalities, this is

$$L \propto RT^4 l \tag{1}$$

where l is the mean-free path length. This path length is

$$l \propto T^{3.5}/\rho^2 \text{ low to medium mass} \tag{2}$$

$$l \propto 1/\rho \text{ high to very high mass.} \tag{3}$$

We also have, from definition of average density,

$$\rho = M/V = M/(4/3\pi R^3) \propto M/R^3.$$

From 5.9, we showed that the central pressure at the sun is given as

$$P_c = 19GM^2/R^4 \propto M^2/R^4.$$

For a stable star, this central pressure must be equal to the total pressure arising from gas and radiation. For stars with low to high mass, gas pressure dominates thus

$$P = nkT = \frac{N}{V}kT = \frac{M}{mV}kT = \frac{\rho}{m_i}kT \propto \rho T$$

where m is the mass of an effective gas molecule (individial species can be added seperately in which $\rho = \sum_i \rho_i$). For stars with very high mass, radiation pressure dominates, so

$$P = \frac{1}{3}n\langle vp\rangle = \frac{1}{3}n\langle E\rangle = \frac{1}{3}\mathcal{E} = \frac{1}{3}aT^4 \propto T^4.$$

Equating the gravitational pressure to the total pressure for each scenario, we find

$$T \propto M^2/(\rho R^4) = M/R \text{ low to high mass}$$

$$T^4 = M^2/R^4 \text{ very high mass.}$$

Now we substitute this into the luminosity, to get the leakage rate dependence upon the mass,

$$L \propto RT^{7.5}(R^6/M^2) = M^{5.5}/R^{0.5} \text{ low to medium mass}$$

$$L \propto RT^4/\rho = R^4T^4/M = R^4M^4/R^4M = M^3 \text{ high mass}$$

$$L \propto RT^4 = R^4T^4/M = R^4(M^2/R^4)/M = M \text{ very high mass.}$$

For low to medium mass stars, $R \propto M$ and thus $L \propto M^5$. Since the masses of stars on the main sequences vary from low to high (very massive stars are quite rare), the range of luminosity varies from $L \propto M^3 - M^5$. Including the fact that very low mass stars are convective and do not follow the same luminosity relationship, is it reasonable to take a representative power of magnitude for stars on the main sequence as $L \propto M^4$, as this gives a good average compromise over the ranges of low to high mass stars.