RW SVT4 $k \neq 0$ v4

1 Background

$$ds^2 = \Omega^2 \tilde{g}_{\mu\nu} dx^\mu dx^\nu \tag{1.1}$$

$$P_{\mu\nu} = \tilde{g}_{\mu\nu} + U_\mu U_\nu \tag{1.2}$$

$$\tilde{R}_{\lambda\mu\nu\kappa} = k(P_{\lambda\nu}P_{\mu\kappa} - P_{\mu\nu}P_{\lambda\kappa}) \qquad \tilde{R}_{\mu\nu} = -2kP_{\mu\nu}, \qquad \tilde{R} = -6k \tag{1.3}$$

$$\begin{aligned} G_{\mu\nu} &= -\tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} + \tilde{R}_{\mu\nu} - 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega \\ &= k\tilde{g}_{\mu\nu} - 2kU_\mu U_\nu - 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega \end{aligned} \tag{1.4}$$

$$g^{\mu\nu}G_{\mu\nu} = 6k\Omega^{-2} - 6\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \tag{1.5}$$

$$T_{\mu\nu} = \Omega^2(\rho+p)U_\mu U_\nu + \Omega^2 p\tilde{g}_{\mu\nu}, \qquad U_\mu = -\delta_\mu^0, \qquad U^\mu = \delta_0^\mu \tag{1.6}$$

$$g^{\mu\nu}T_{\mu\nu} = 3p - \rho \tag{1.7}$$

$$\begin{aligned} \Delta_{\mu\nu}^{(0)} &= k\tilde{g}_{\mu\nu} - 2kU_\mu U_\nu + \tilde{g}_{\mu\nu}p\Omega^2 + pU_\mu U_\nu\Omega^2 + U_\mu U_\nu\rho\Omega^2 - 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega \\ &\quad -4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega \end{aligned} \tag{1.8}$$

$$g^{\mu\nu}\Delta_{\mu\nu}^{(0)} = 3p - \rho + 6k\Omega^{-2} - 6\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \tag{1.9}$$

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= U^\alpha U^\nu\Omega^{-2}\tilde{\nabla}_\alpha p + U^\alpha U^\nu\Omega^{-2}\tilde{\nabla}_\alpha\rho + 4pU^\alpha U^\nu\Omega^{-3}\tilde{\nabla}_\alpha\Omega + 4U^\alpha U^\nu\rho\Omega^{-3}\tilde{\nabla}_\alpha\Omega + \Omega^{-2}\tilde{\nabla}^\nu p \\ &\quad + p\Omega^{-3}\tilde{\nabla}^\nu\Omega + \rho\Omega^{-3}\tilde{\nabla}^\nu\Omega \end{aligned} \tag{1.10}$$

Solving for ρ within (1.9), substituting into (1.8), and projecting $U^\mu U^\nu\Delta_{\mu\nu}$ we can solve for p as

$$\begin{aligned} p &= -k\Omega^{-2} + \tfrac{4}{3}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tfrac{1}{3}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + \tfrac{4}{3}U^\alpha U^\beta\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega - \tfrac{2}{3}U^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega \\ &= -k\Omega^{-2} - \Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + 2\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega. \end{aligned} \tag{1.11}$$

From (1.9) we can then determine ρ as

$$\begin{aligned} \rho &= 3k\Omega^{-2} - 2\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + 4U^\alpha U^\beta\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega - 2U^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega \\ &= 3k\Omega^{-2} - 3\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega. \end{aligned} \tag{1.12}$$

1.1 $\Omega(\tau)$ Identities

$$U^\alpha U^\beta\tilde{\nabla}_\alpha\tilde{\nabla}_\beta\Omega = -\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \tag{1.13}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega = -\tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha \Omega \tag{1.14}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha F \tilde{\nabla}_\beta \Omega = -\tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha F \tag{1.15}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\beta F (\tilde{\nabla}^\gamma \tilde{\nabla}_\gamma \Omega) = -\tilde{\nabla}_\alpha \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\alpha \tilde{\nabla}^\beta F \tag{1.16}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\beta F (\tilde{\nabla}^\gamma \Omega \tilde{\nabla}_\gamma \Omega) = -\tilde{\nabla}^\beta \Omega \tilde{\nabla}^\alpha \Omega \tilde{\nabla}_\alpha \tilde{\nabla}_\beta F \tag{1.17}$$

$$\tilde{\nabla}^\alpha F \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\beta \Omega = \tilde{\nabla}^\alpha F \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega \tag{1.18}$$

$$F^\alpha U_\alpha U^\beta \tilde{\nabla}_\beta \Omega = -F^\alpha \tilde{\nabla}_\alpha \Omega \tag{1.19}$$

$$U^\alpha U^\beta \tilde{\nabla}_\beta F_\alpha (\tilde{\nabla}_\gamma \Omega \tilde{\nabla}^\gamma \Omega) = -\tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta F^\alpha \tag{1.20}$$

$$U^\alpha U^\beta \tilde{\nabla}_\beta F_\alpha (\tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \Omega) = -\tilde{\nabla}_\alpha \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta F^\alpha \tag{1.21}$$

$$F^\alpha \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\beta \Omega = F^\alpha \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega \tag{1.22}$$

$$U^\alpha U^\beta F_{\alpha\beta} \tilde{\nabla}_\gamma \Omega \tilde{\nabla}^\gamma \Omega = -F_{\alpha\beta} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega \tag{1.23}$$

$$U^\alpha U^\beta F_{\alpha\beta} \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \Omega = -F_{\alpha\beta} \tilde{\nabla}^\alpha \tilde{\nabla}^\beta \Omega \tag{1.24}$$

2   Fluctuations

$$ds^2 = \Omega^2(\tau)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu, \quad \text{with } \tilde{g}_{\mu\nu} \text{ obeying (1.3)} \tag{2.1}$$

$$f_{\mu\nu} = -2\tilde{g}_{\mu\nu}\chi + 2\tilde{\nabla}_\mu \tilde{\nabla}_\nu F + \tilde{\nabla}_\mu F_\nu + \tilde{\nabla}_\nu F_\mu + 2F_{\mu\nu} \tag{2.2}$$

$$\tilde{g}^{\mu\nu} F_{\mu\nu} = 0, \qquad \tilde{\nabla}^\mu F_{\mu\nu} = 0, \qquad \tilde{\nabla}^\mu F_\mu = 0 \tag{2.3}$$

$$U^\mu \delta U_\mu = \tfrac{1}{2} U^\mu U^\nu f_{\mu\nu} \tag{2.4}$$

$$\delta U_\mu = (V_\mu + \tilde{\nabla}_\mu V) + U_\mu U^\alpha (V_\alpha + \tilde{\nabla}_\alpha V) - U_\mu \left(\tfrac{1}{2} U^\alpha U^\beta f_{\alpha\beta} \right) \tag{2.5}$$

$$\begin{aligned} \delta T_{\mu\nu} = {}& \delta p \tilde{g}_{\mu\nu} \Omega^2 + \delta p U_\mu U_\nu \Omega^2 + \delta\rho U_\mu U_\nu \Omega^2 - 2\tilde{g}_{\mu\nu} p\chi \Omega^2 + 2p\Omega^2 \tilde{\nabla}_\mu \tilde{\nabla}_\nu F + \delta U_\nu p U_\mu \Omega^2 + \delta U_\mu p U_\nu \Omega^2 \\ & + \delta U_\nu U_\mu \rho \Omega^2 + \delta U_\mu U_\nu \rho \Omega^2 + p\Omega^2 \tilde{\nabla}_\mu F_\nu + p\Omega^2 \tilde{\nabla}_\nu F_\mu + 2F_{\mu\nu} p\Omega^2 \end{aligned} \tag{2.6}$$

$$\begin{aligned} g^{\mu\nu} \delta T_{\mu\nu} = {}& 3\delta p - \delta\rho - 6p\chi + 2\rho\chi + 2p\tilde{\nabla}_\alpha \tilde{\nabla}^\alpha F + 2pU^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha F + 2U^\alpha U^\beta \rho \tilde{\nabla}_\beta \tilde{\nabla}_\alpha F + 2pU^\alpha U^\beta \tilde{\nabla}_\beta F_\alpha \\ & + 2U^\alpha U^\beta \rho \tilde{\nabla}_\beta F_\alpha + 2F_{\alpha\beta} pU^\alpha U^\beta + 2F_{\alpha\beta} U^\alpha U^\beta \rho \end{aligned} \tag{2.7}$$

$$\delta G_{\mu\nu} = 2\tilde{g}_{\mu\nu} \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha \chi + 2k\tilde{g}_{\mu\nu} \Omega^{-1} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha F - 2kU_\mu U_\nu \Omega^{-1} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha F + 2\tilde{g}_{\mu\nu} \Omega^{-1} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha \chi$$

$$+2k\tilde{g}_{\mu\nu}U^\alpha U^\beta \Omega^{-1}\tilde{\nabla}_\alpha F\tilde{\nabla}_\beta\Omega + 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha F - 2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}_\alpha F\tilde{\nabla}^\beta\Omega$$
$$4\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta\tilde{\nabla}^\alpha F + 2kU^\alpha U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\mu F - 2kU^\alpha U_\nu\tilde{\nabla}_\mu\tilde{\nabla}_\alpha F + 2\Omega^{-1}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\chi$$
$$+2kU^\alpha U_\mu\Omega^{-1}\tilde{\nabla}_\alpha F\tilde{\nabla}_\nu\Omega + 2k\Omega^{-1}\tilde{\nabla}_\mu F\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\mu\chi\tilde{\nabla}_\nu\Omega - 2kU^\alpha U_\mu\tilde{\nabla}_\nu\tilde{\nabla}_\alpha F$$
$$+2k\tilde{\nabla}_\nu\tilde{\nabla}_\mu F - 4\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu F + 2\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu F - 2\tilde{\nabla}_\nu\tilde{\nabla}_\mu\chi$$
$$-2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{\nabla}_\alpha F + 2kF^\alpha\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\Omega + 2kF_\mu U^\alpha U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega - 2kF^\alpha U_\mu U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega$$
$$+2kF^\alpha\tilde{g}_{\mu\nu}U_\alpha U^\beta\Omega^{-1}\tilde{\nabla}_\beta\Omega + 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta F_\alpha - 2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta F^\alpha$$
$$+4\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta F^\alpha - 2kU^\alpha U_\nu\tilde{\nabla}_\mu F_\alpha + k\tilde{\nabla}_\mu F_\nu - 2\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu F_\nu$$
$$+\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu F_\nu - 2kU^\alpha U_\mu\tilde{\nabla}_\nu F_\alpha + k\tilde{\nabla}_\nu F_\mu - 2\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu F_\mu$$
$$+\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu F_\mu + 2kF_\mu\Omega^{-1}\tilde{\nabla}_\nu\Omega + 2kF^\alpha U_\alpha U_\mu\Omega^{-1}\tilde{\nabla}_\nu\Omega$$
$$-2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu F_\alpha - 4kF_{\nu\alpha}U^\alpha U_\mu - 4kF_{\mu\alpha}U^\alpha U_\nu + \tilde{\nabla}_\alpha\tilde{\nabla}^\alpha F_{\mu\nu} - 4F_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega$$
$$+2\Omega^{-1}\tilde{\nabla}_\alpha F_{\mu\nu}\tilde{\nabla}^\alpha\Omega + 2F_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 2F_{\alpha\beta}\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega + 4F_{\alpha\beta}\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}^\beta\tilde{\nabla}^\alpha\Omega$$
$$-2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu F_{\nu\alpha} - 2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu F_{\mu\alpha} \tag{2.8}$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} =\ & 2k\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha F + 6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\chi + 12k\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F + 12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\chi \\
& +12kU^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\alpha F\tilde{\nabla}_\beta\Omega - 4kU^\alpha U^\beta\Omega^{-2}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha F + 2\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta F \\
& -4\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha F\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega + 6\Omega^{-3}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha F - 8\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}_\alpha F\tilde{\nabla}^\beta\Omega \\
& +16\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta\tilde{\nabla}^\alpha F + 12kF^\alpha\Omega^{-3}\tilde{\nabla}_\alpha\Omega - 4kU^\alpha U^\beta\Omega^{-2}\tilde{\nabla}_\beta F_\alpha + 12kF^\alpha U_\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta\Omega \\
& +6\Omega^{-3}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta F_\alpha - 8\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta F^\alpha + 16\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta F^\alpha - 8kF_{\alpha\beta}U^\alpha U^\beta\Omega^{-2} \\
& -8F_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega + 16F_{\alpha\beta}\Omega^{-3}\tilde{\nabla}^\beta\tilde{\nabla}^\alpha\Omega \tag{2.9}
\end{aligned}
$$

3 Field Equations

$$\Delta_{\mu\nu} = \delta G_{\mu\nu} + \delta T_{\mu\nu} \tag{3.1}$$

$$
\begin{aligned}
\Delta_{\mu\nu} =\ & 2k\tilde{g}_{\mu\nu}\chi + \delta p\tilde{g}_{\mu\nu}\Omega^2 + \delta p U_\mu U_\nu\Omega^2 + \delta\rho U_\mu U_\nu\Omega^2 + 2\tilde{g}_{\mu\nu}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\chi - 4\tilde{g}_{\mu\nu}\chi\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \\
& +2k\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F - 2kU_\mu U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F + 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\chi + 2\tilde{g}_{\mu\nu}\chi\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega \\
& +2k\tilde{g}_{\mu\nu}U^\alpha U^\beta\Omega^{-1}\tilde{\nabla}_\alpha F\tilde{\nabla}_\beta\Omega + 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha F - 2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}_\alpha F\tilde{\nabla}^\beta\Omega \\
& +4\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta\tilde{\nabla}^\alpha F + 2kU^\alpha U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\mu F + 2kU_\nu\tilde{\nabla}_\mu V + 2U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu V \\
& -4U_\nu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu V - 2kU^\alpha U_\nu\tilde{\nabla}_\mu\tilde{\nabla}_\alpha F - 2k\tilde{\nabla}_\mu\tilde{\nabla}_\nu F + 4\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu\tilde{\nabla}_\nu F \\
& -2\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu\tilde{\nabla}_\nu F + 2kU_\mu\tilde{\nabla}_\nu V + 2U_\mu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu V - 4U_\mu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu V \\
& +2\Omega^{-1}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\chi + 2kU^\alpha U_\mu\Omega^{-1}\tilde{\nabla}_\alpha F\tilde{\nabla}_\nu\Omega + 2k\Omega^{-1}\tilde{\nabla}_\mu F\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\mu\chi\tilde{\nabla}_\nu\Omega \\
& -2kU^\alpha U_\mu\tilde{\nabla}_\nu\tilde{\nabla}_\alpha F + 2k\tilde{\nabla}_\nu\tilde{\nabla}_\mu F - 4\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu F + 2\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu F \\
& -2\tilde{\nabla}_\nu\tilde{\nabla}_\mu\chi - 2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{\nabla}_\alpha F + 2kU_\nu V_\mu + 2kU_\mu V_\nu + 2kF^\alpha\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\Omega \\
& +2kF_\mu U^\alpha U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega - 2kF^\alpha U_\mu U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega + 2U_\nu V_\mu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + 2U_\mu V_\nu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \\
& -4U_\nu V_\mu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 4U_\mu V_\nu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + 2kF^\alpha\tilde{g}_{\mu\nu}U_\alpha U^\beta\Omega^{-1}\tilde{\nabla}_\beta\Omega \\
& +2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta F_\alpha - 2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta F^\alpha + 4\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta F^\alpha \\
& -2kU^\alpha U_\nu\tilde{\nabla}_\mu F_\alpha - 2kU^\alpha U_\mu\tilde{\nabla}_\nu F_\alpha + 2kF_\mu\Omega^{-1}\tilde{\nabla}_\nu\Omega + 2kF^\alpha U_\alpha U_\mu\Omega^{-1}\tilde{\nabla}_\nu\Omega \\
& -2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu F_\alpha - 2kF_{\mu\nu} - 4kF_{\nu\alpha}U^\alpha U_\mu - 4kF_{\mu\alpha}U^\alpha U_\nu + \tilde{\nabla}_\alpha\tilde{\nabla}^\alpha F_{\mu\nu} \\
& +2\Omega^{-1}\tilde{\nabla}_\alpha F_{\mu\nu}\tilde{\nabla}^\alpha\Omega - 2F_{\alpha\beta}\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega + 4F_{\alpha\beta}\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}^\beta\tilde{\nabla}^\alpha\Omega - 2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu F_{\nu\alpha} \\
& -2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu F_{\mu\alpha} \tag{3.2}
\end{aligned}
$$

3

$$g^{\mu\nu}\Delta_{\mu\nu} = 3\delta p - \delta\rho + 12k\chi\Omega^{-2} + 6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\chi - 12\chi\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + 12k\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F$$
$$+12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\chi + 12kU^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\alpha F\tilde{\nabla}_\beta\Omega + 6\Omega^{-3}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha F$$
$$-8\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}_\alpha F\tilde{\nabla}^\beta\Omega + 16\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta\tilde{\nabla}^\alpha F + 4U^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha F\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma\Omega$$
$$-8U^\alpha U^\beta\Omega^{-4}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha F\tilde{\nabla}_\gamma\Omega\tilde{\nabla}^\gamma\Omega + 12kF^\alpha\Omega^{-3}\tilde{\nabla}_\alpha\Omega + 12kF^\alpha U_\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta\Omega$$
$$+6\Omega^{-3}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta F_\alpha - 8\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta F^\alpha + 16\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta F^\alpha$$
$$+4U^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta F_\alpha\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma\Omega - 8U^\alpha U^\beta\Omega^{-4}\tilde{\nabla}_\beta F_\alpha\tilde{\nabla}_\gamma\Omega\tilde{\nabla}^\gamma\Omega - 4kF_{\alpha\beta}U^\alpha U^\beta\Omega^{-2}$$
$$-8F_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega + 16F_{\alpha\beta}\Omega^{-3}\tilde{\nabla}^\beta\tilde{\nabla}^\alpha\Omega + 4F_{\alpha\beta}U^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma\Omega$$
$$-8F_{\alpha\beta}U^\alpha U^\beta\Omega^{-4}\tilde{\nabla}_\gamma\Omega\tilde{\nabla}^\gamma\Omega \tag{3.3}$$

4 Field Equations (G.I. Form)

$$\alpha = \chi + \Omega^{-1}\tilde{\nabla}_\alpha\Omega(F^\alpha + \tilde{\nabla}^\alpha F) = \chi - \Omega^{-1}U^\alpha U^\beta\tilde{\nabla}_\alpha\Omega(F_\beta + \tilde{\nabla}_\beta F) \tag{4.1}$$

$$V^{GI} = V - U^\alpha\tilde{\nabla}_\alpha F - U^\alpha F_\alpha \tag{4.2}$$

$$\delta\rho^{GI} = \delta\rho + 12\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\chi - 6k\Omega^{-2}\chi - 6\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\chi \tag{4.3}$$

$$\delta p^{GI} = \delta p + 4\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\chi - 2\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega(\tilde{\nabla}^\alpha F + F^\alpha) + 2\Omega^{-2}k\chi - 8\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\chi \tag{4.4}$$

$$\Delta_{\mu\nu} = U_\mu U_\nu\delta\rho^{GI}\Omega^2 + \delta p^{GI}(\tilde{g}_{\mu\nu}\Omega^2 + U_\mu U_\nu\Omega^2) + 4kU^\alpha U_\mu U_\nu\tilde{\nabla}_\alpha V^{GI} + 2\tilde{g}_{\mu\nu}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\alpha$$
$$+2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\alpha$$
$$+\alpha\big(10U_\mu U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 8U_\mu U_\nu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}(4\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 2\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega)\big)$$
$$+4U^\alpha U_\mu U_\nu\Omega^{-1}\tilde{\nabla}_\alpha V^{GI}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega - 8U^\alpha U_\mu U_\nu\Omega^{-2}\tilde{\nabla}_\alpha V^{GI}\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta\Omega$$
$$+(2kU_\nu + 2U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 4U_\nu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega)\tilde{\nabla}_\mu V^{GI}$$
$$+(2kU_\mu + 2U_\mu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 4U_\mu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega)\tilde{\nabla}_\nu V^{GI} + 2\Omega^{-1}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\alpha + 2\Omega^{-1}\tilde{\nabla}_\mu\alpha\tilde{\nabla}_\nu\Omega$$
$$-2\tilde{\nabla}_\nu\tilde{\nabla}_\mu\alpha + 4kU^\alpha U_\mu U_\nu V_\alpha + V_\nu(2kU_\mu + 2U_\mu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 4U_\mu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega)$$
$$+V_\mu(2kU_\nu + 2U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 4U_\nu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega) + 4U^\alpha U_\mu U_\nu V_\alpha\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega$$
$$-8U^\alpha U_\mu U_\nu V_\alpha\Omega^{-2}\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta\Omega - 4kU^\alpha U^\beta U_\mu U_\nu F_{\alpha\beta} - 4kU^\alpha U_\nu F_{\mu\alpha} - 2kF_{\mu\nu} - 4kU^\alpha U_\mu F_{\nu\alpha}$$
$$+\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha F_{\mu\nu} + 2\Omega^{-1}\tilde{\nabla}_\alpha F_{\mu\nu}\tilde{\nabla}^\alpha\Omega + 4\tilde{g}_{\mu\nu}F^{\alpha\beta}\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega + 4U_\mu U_\nu F^{\alpha\beta}\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega$$
$$-2\tilde{g}_{\mu\nu}F_{\alpha\beta}\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega - 8U_\mu U_\nu F_{\alpha\beta}\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega - 2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu F_{\nu\alpha}$$
$$-2\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu F_{\mu\alpha} \tag{4.5}$$

$$g^{\mu\nu}\Delta_{\mu\nu} = 3\delta p^{GI} - \delta\rho^{GI} + 6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\alpha + 6\alpha\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + 12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\alpha - 4kU^\alpha U^\beta F_{\alpha\beta}\Omega^{-2}$$
$$+12F^{\alpha\beta}\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega \tag{4.6}$$

In evaluating (4.5), we are initially left over with non-gauge-invariant quantities, collectively denoted as $Z_{\mu\nu}$,

$$Z_{\mu\nu} = -2kU_\mu U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F - 2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega - 10U_\mu U_\nu\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega$$
$$+2U_\mu U_\nu\Omega^{-1}\tilde{\nabla}^\alpha F\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega + 8U_\mu U_\nu\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta\Omega$$
$$+2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}^\alpha F\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\beta\Omega + 2U^\alpha U_\nu\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega\tilde{\nabla}_\mu\tilde{\nabla}_\alpha F - 4U^\alpha U_\nu\Omega^{-2}\tilde{\nabla}_\beta\Omega\tilde{\nabla}^\beta\Omega\tilde{\nabla}_\mu\tilde{\nabla}_\alpha F$$
$$+2\Omega^{-1}\tilde{\nabla}^\alpha\tilde{\nabla}_\nu F\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\Omega + 2kU^\alpha U_\mu\Omega^{-1}\tilde{\nabla}_\alpha F\tilde{\nabla}_\nu\Omega + 8\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha F\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega$$
$$-4\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu\tilde{\nabla}_\alpha F\tilde{\nabla}_\nu\Omega - 4\Omega^{-2}\tilde{\nabla}^\alpha F\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\nu\Omega$$
$$+\tilde{\nabla}_\mu F(2kU^\alpha U_\nu\Omega^{-1}\tilde{\nabla}_\alpha\Omega + 2k\Omega^{-1}\tilde{\nabla}_\nu\Omega) + 2U^\alpha U_\mu\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\alpha F$$

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$$-4U^\alpha U_\mu \Omega^{-2} \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta \Omega \tilde{\nabla}_\nu \tilde{\nabla}_\alpha F - 4\Omega^{-2} \tilde{\nabla}^\alpha \Omega \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu \tilde{\nabla}_\alpha F + 2\Omega^{-1} \tilde{\nabla}^\alpha \tilde{\nabla}_\mu F \tilde{\nabla}_\nu \tilde{\nabla}_\alpha \Omega$$

$$-4\Omega^{-2} \tilde{\nabla}^\alpha F \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu \tilde{\nabla}_\alpha \Omega - 2\Omega^{-2} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha F \tilde{\nabla}_\nu \tilde{\nabla}_\mu \Omega$$

$$+2\Omega^{-1} \tilde{\nabla}^\alpha F \tilde{\nabla}_\nu \tilde{\nabla}_\mu \tilde{\nabla}_\alpha \Omega - 2kF^\alpha U_\mu U_\nu \Omega^{-1} \tilde{\nabla}_\alpha \Omega - 2F^\alpha \tilde{g}_{\mu\nu} \Omega^{-2} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega$$

$$-10F^\alpha U_\mu U_\nu \Omega^{-2} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega + 2F^\alpha U_\mu U_\nu \Omega^{-1} \tilde{\nabla}_\beta \tilde{\nabla}^\beta \tilde{\nabla}_\alpha \Omega + 8F^\alpha U_\mu U_\nu \Omega^{-3} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta \Omega$$

$$+2F^\alpha \tilde{g}_{\mu\nu} \Omega^{-2} \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\beta \Omega + 2U^\alpha U_\nu \Omega^{-1} \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega \tilde{\nabla}_\mu F_\alpha - 4U^\alpha U_\nu \Omega^{-2} \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta \Omega \tilde{\nabla}_\mu F_\alpha$$

$$+2U^\alpha U_\mu \Omega^{-1} \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega \tilde{\nabla}_\nu F_\alpha - 4U^\alpha U_\mu \Omega^{-2} \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta \Omega \tilde{\nabla}_\nu F_\alpha - 4\Omega^{-2} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu F^\alpha$$

$$+2\Omega^{-1} \tilde{\nabla}_\mu \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\nu F^\alpha + 2kF^\alpha U_\alpha U_\mu \Omega^{-1} \tilde{\nabla}_\nu \Omega - 4\Omega^{-2} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\mu F^\alpha \tilde{\nabla}_\nu \Omega$$

$$+8F^\alpha \Omega^{-3} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu \Omega - 4F^\alpha \Omega^{-2} \tilde{\nabla}_\mu \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\nu \Omega + F_\mu (2kU^\alpha U_\nu \Omega^{-1} \tilde{\nabla}_\alpha \Omega + 2k\Omega^{-1} \tilde{\nabla}_\nu \Omega)$$

$$+2\Omega^{-1} \tilde{\nabla}_\mu F^\alpha \tilde{\nabla}_\nu \tilde{\nabla}_\alpha \Omega - 4F^\alpha \Omega^{-2} \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu \tilde{\nabla}_\alpha \Omega - 2F^\alpha \Omega^{-2} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\nu \tilde{\nabla}_\mu \Omega$$

$$+2F^\alpha \Omega^{-1} \tilde{\nabla}_\nu \tilde{\nabla}_\mu \tilde{\nabla}_\alpha \Omega \tag{4.7}$$

Performing a $3+1$ splitting on $Z_{\mu\nu}$ results in $Z_{\mu\nu}$ vanishing for all components, thereby permitting (4.5) to be expressed gauge invariantly.

Appendix A $\Omega = 1$

$$
\begin{aligned}
\Delta_{\mu\nu} &= (g_{\mu\nu} + U_\mu U_\nu)\delta p^{GI} + U_\mu U_\nu \delta\rho^{GI} + 4kU^\alpha U_\mu U_\nu \nabla_\alpha V^{GI} + 2g_{\mu\nu}\nabla_\alpha\nabla^\alpha\alpha + 2kU_\nu\nabla_\mu V^{GI} \\
&\quad + 2kU_\mu\nabla_\nu V^{GI} - 2\nabla_\nu\nabla_\mu\alpha + 4kU^\alpha U_\mu U_\nu V_\alpha + 2kU_\nu V_\mu + 2kU_\mu V_\nu - 4kU^\alpha U^\beta U_\mu U_\nu F_{\alpha\beta} \\
&\quad - 4kU^\alpha U_\nu F_{\mu\alpha} - 2kF_{\mu\nu} - 4kU^\alpha U_\mu F_{\nu\alpha} + \nabla_\alpha\nabla^\alpha F_{\mu\nu}
\end{aligned}
\tag{A.1}
$$

$$
g^{\mu\nu}\Delta_{\mu\nu} = 3\delta p^{GI} - \delta\rho^{GI} + 6\nabla_\alpha\nabla^\alpha\alpha - 4kU^\alpha U^\beta F_{\alpha\beta}
\tag{A.2}
$$

Appendix B Conformal Transformations

Given the geometry $g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}$, the relations between curvature tensors are

$$
R_{\mu\nu} = \tilde{R}_{\mu\nu} + \tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega
\tag{B.1}
$$

$$
R = \tilde{R}\Omega^{-2} + 6\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega
\tag{B.2}
$$

If we instead evaluate within geometry $\tilde{g}_{\mu\nu} = \Omega^{-2}g_{\mu\nu}$, we find

$$
\tilde{R}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}\Omega^{-1}\nabla_\alpha\nabla^\alpha\Omega + 3g_{\mu\nu}\Omega^{-2}\nabla_\alpha\Omega\nabla^\alpha\Omega - 2\Omega^{-1}\nabla_\nu\nabla_\mu\Omega.
\tag{B.3}
$$

In a maximal 3-space geometry, it follows that

$$
\tilde{R}_{\mu\nu} = -2k\tilde{P}_{\mu\nu} = -2k(\tilde{g}_{\mu\nu} + \tilde{U}_\mu\tilde{U}_\nu).
\tag{B.4}
$$

The projector $\tilde{P}_{\mu\nu}$ may be constructed in relation to $g_{\mu\nu}$ as

$$
\tilde{P}_{\mu\nu} = \Omega^{-2}(g_{\mu\nu} + U_\mu U_\nu),
\tag{B.5}
$$

where $U_\mu = \Omega\tilde{U}_\mu$. (This holds given $\Omega(\tau)$ and $\tilde{U}_\mu = -\delta^0_\mu$. For the more general $\Omega(x)$, there will be non-zero spatial components of U_μ, and thus $U_\mu \neq \Omega\tilde{U}_\mu$).

With (B.3) and (B.4), we may form the relation for the RW Ricci tensor

$$
R_{\mu\nu} = -2k\Omega^{-2}(g_{\mu\nu} + U_\mu U_\nu) + g_{\mu\nu}\Omega^{-1}\nabla_\alpha\nabla^\alpha\Omega - 3g_{\mu\nu}\Omega^{-2}\nabla_\alpha\Omega\nabla^\alpha\Omega + 2\Omega^{-1}\nabla_\nu\nabla_\mu\Omega,
\tag{B.6}
$$

again, given that $U_\mu = \Omega\tilde{U}_\mu$.

One may check when evaluated within geometry $g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}$, (B.6) becomes

$$
R_{\mu\nu} = -2k(\tilde{g}_{\mu\nu} + \tilde{U}_\mu\tilde{U}_\nu) + \tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega,
\tag{B.7}
$$

which is equivalent to (B.1) in a maximal 3-space.