Weyl Tensor Simplifications v8 Matthew

Direct Output, No Simplification

$$W_{\mu\nu} = -\tfrac{1}{6}g_{\mu\nu}R^2 + \tfrac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \tfrac{2}{3}RR_{\mu\nu} - R^{\alpha\beta}R_{\beta\mu\alpha\nu} - R^{\alpha\beta}R_{\beta\nu\alpha\mu} - \tfrac{1}{6}g_{\mu\nu}\nabla_\alpha\nabla^\alpha R + \nabla_\alpha\nabla^\alpha R_{\mu\nu} - \tfrac{1}{3}\nabla_\nu\nabla_\mu R$$

62 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(h_{\mu\nu}) = & -\tfrac{1}{6}h_{\mu\nu}R^2 + \tfrac{1}{3}g_{\mu\nu}h^{\alpha\beta}RR_{\alpha\beta} + \tfrac{1}{2}h_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}h^{\alpha\beta}R_\alpha{}^\gamma R_{\beta\gamma} - \tfrac{2}{3}h^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\
& + h^{\alpha\beta}R_\alpha{}^\gamma R_{\mu\beta\nu\gamma} + h^{\alpha\beta}R_\alpha{}^\gamma R_{\mu\gamma\nu\beta} - \tfrac{1}{6}h_{\mu\nu}\nabla_\alpha\nabla^\alpha R + \tfrac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_\beta\nabla_\alpha R \\
& - h^{\alpha\beta}\nabla_\beta\nabla_\alpha R_{\mu\nu} + \tfrac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_\gamma\nabla^\gamma R_{\alpha\beta} + \tfrac{1}{3}h^{\alpha\beta}\nabla_\nu\nabla_\mu R_{\alpha\beta} \\
\\
& + \tfrac{1}{3}R\nabla_\alpha\nabla^\alpha h_{\mu\nu} + R_{\mu\beta\nu\gamma}\nabla_\alpha\nabla^\gamma h^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_\alpha\nabla^\gamma h^{\alpha\beta} - \tfrac{1}{3}R\nabla_\alpha\nabla_\mu h_\nu{}^\alpha - \tfrac{1}{3}R\nabla_\alpha\nabla_\nu h_\mu{}^\alpha \\
& - \tfrac{1}{6}\nabla_\alpha h_{\mu\nu}\nabla^\alpha R + \tfrac{1}{6}g_{\mu\nu}\nabla^\alpha R\nabla_\beta h_\alpha{}^\beta - \nabla_\alpha h^{\alpha\beta}\nabla_\beta R_{\mu\nu} + \tfrac{1}{3}g_{\mu\nu}R\nabla_\beta\nabla_\alpha h^{\alpha\beta} \\
& - \tfrac{2}{3}R_{\mu\nu}\nabla_\beta\nabla_\alpha h^{\alpha\beta} + \tfrac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla_\alpha h_\mu{}^\beta - R^{\alpha\beta}\nabla_\beta\nabla_\alpha h_{\mu\nu} + \tfrac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla_\alpha h_\nu{}^\beta \\
& - \tfrac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla^\beta h_{\mu\alpha} - \tfrac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla^\beta h_{\nu\alpha} + \tfrac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla^\alpha h_{\mu\nu} - \tfrac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla_\mu h_\nu{}^\alpha \\
& - \tfrac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla_\nu h_\mu{}^\alpha - \tfrac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla_\mu h_\alpha{}^\beta + R^{\alpha\beta}\nabla_\beta\nabla_\mu h_{\nu\alpha} - \tfrac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla_\nu h_\alpha{}^\beta \\
& + R^{\alpha\beta}\nabla_\beta\nabla_\nu h_{\mu\alpha} + \nabla_\alpha R_{\nu\beta}\nabla^\beta h_\mu{}^\alpha - \nabla_\beta R_{\nu\alpha}\nabla^\beta h_\mu{}^\alpha + \nabla_\alpha R_{\mu\beta}\nabla^\beta h_\nu{}^\alpha - \nabla_\beta R_{\mu\alpha}\nabla^\beta h_\nu{}^\alpha \\
& - g_{\mu\nu}R^{\alpha\beta}\nabla_\gamma\nabla_\beta h_\alpha{}^\gamma + \tfrac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_\gamma\nabla^\gamma h_{\alpha\beta} - R_{\mu\alpha\nu\beta}\nabla_\gamma\nabla^\gamma h^{\alpha\beta} + \tfrac{1}{6}g_{\mu\nu}\nabla_\gamma\nabla^\gamma\nabla_\beta\nabla_\alpha h^{\alpha\beta} \\
& + \tfrac{1}{3}g_{\mu\nu}\nabla_\gamma R_{\alpha\beta}\nabla^\gamma h^{\alpha\beta} - \nabla_\beta R_{\nu\alpha}\nabla_\mu h^{\alpha\beta} + \tfrac{1}{6}\nabla^\alpha R\nabla_\mu h_{\nu\alpha} - \tfrac{1}{2}R^{\alpha\beta}\nabla_\mu\nabla_\nu h_{\alpha\beta} \\
& - \nabla_\beta R_{\mu\alpha}\nabla_\nu h^{\alpha\beta} + \tfrac{1}{3}\nabla_\mu R_{\alpha\beta}\nabla_\nu h^{\alpha\beta} + \tfrac{1}{6}\nabla^\alpha R\nabla_\nu h_{\mu\alpha} + \tfrac{1}{3}\nabla_\mu h^{\alpha\beta}\nabla_\nu R_{\alpha\beta} \\
& - \tfrac{1}{6}R^{\alpha\beta}\nabla_\nu\nabla_\mu h_{\alpha\beta} + \tfrac{1}{3}\nabla_\nu\nabla_\mu\nabla_\beta\nabla_\alpha h^{\alpha\beta} \\
\\
& - \tfrac{1}{3}g_{\mu\nu}R\nabla_\alpha\nabla^\alpha h + \tfrac{2}{3}R_{\mu\nu}\nabla_\alpha\nabla^\alpha h + \tfrac{1}{2}\nabla_\alpha\nabla^\alpha\nabla_\nu\nabla_\mu h - \tfrac{1}{12}g_{\mu\nu}\nabla_\alpha h\nabla^\alpha R \\
& + \tfrac{1}{2}\nabla_\alpha R_{\mu\nu}\nabla^\alpha h + \tfrac{1}{2}g_{\mu\nu}R^{\alpha\beta}\nabla_\beta\nabla_\alpha h - \tfrac{1}{6}g_{\mu\nu}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla^\alpha h - R_{\mu\alpha\nu\beta}\nabla^\beta\nabla^\alpha h \\
& + \tfrac{1}{3}R\nabla_\nu\nabla_\mu h - \tfrac{1}{3}\nabla_\nu\nabla_\mu\nabla_\alpha\nabla^\alpha h. \qquad (1)
\end{aligned}$$

Substituting $h_{\mu\nu} = K_{\mu\nu} + \frac{h}{4}g^{(0)}_{\mu\nu}$

71 Terms

$$\begin{aligned}\delta W_{\mu\nu}(K_{\mu\nu}) = & -\tfrac{1}{6}K_{\mu\nu}R^2 + \tfrac{1}{3}g_{\mu\nu}K_{\alpha\beta}RR^{\alpha\beta} + \tfrac{1}{2}K_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}K_{\alpha\beta}R^{\alpha}{}_{\gamma}R^{\beta\gamma} - \tfrac{2}{3}K_{\alpha\beta}R^{\alpha\beta}R_{\mu\nu} \\ & + K_{\alpha\beta}R^{\alpha}{}_{\gamma}R_{\mu}{}^{\beta}{}_{\nu}{}^{\gamma} + K_{\alpha\beta}R^{\alpha}{}_{\gamma}R_{\mu}{}^{\gamma}{}_{\nu}{}^{\beta} - \tfrac{1}{6}K_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \tfrac{1}{6}g_{\mu\nu}K_{\alpha\beta}\nabla^{\beta}\nabla^{\alpha}R \\ & - K_{\alpha\beta}\nabla^{\beta}\nabla^{\alpha}R_{\mu\nu} + \tfrac{1}{6}g_{\mu\nu}K_{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R^{\alpha\beta} + \tfrac{1}{3}K_{\alpha\beta}\nabla_{\nu}\nabla_{\mu}R^{\alpha\beta} \\ \\ & + \tfrac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} + R_{\mu\beta\nu\gamma}\nabla_{\alpha}\nabla^{\gamma}K^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_{\alpha}\nabla^{\gamma}K^{\alpha\beta} - \tfrac{1}{3}R\nabla_{\alpha}\nabla_{\mu}K_{\nu}{}^{\alpha} \\ & - \tfrac{1}{3}R\nabla_{\alpha}\nabla_{\nu}K_{\mu}{}^{\alpha} - \tfrac{1}{6}\nabla_{\alpha}R\nabla^{\alpha}K_{\mu\nu} + \tfrac{1}{6}g_{\mu\nu}\nabla_{\alpha}R\nabla_{\beta}K^{\alpha\beta} + \nabla_{\alpha}R_{\nu}{}^{\beta}\nabla_{\beta}K_{\mu}{}^{\alpha} \\ & + \nabla_{\alpha}R_{\mu}{}^{\beta}\nabla_{\beta}K_{\nu}{}^{\alpha} - \nabla_{\alpha}K^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} + \tfrac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} - \tfrac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} \\ & + \tfrac{1}{2}R_{\nu\alpha}\nabla_{\beta}\nabla^{\alpha}K_{\mu}{}^{\beta} + \tfrac{1}{2}R_{\mu\alpha}\nabla_{\beta}\nabla^{\alpha}K_{\nu}{}^{\beta} - \tfrac{1}{2}R_{\nu\alpha}\nabla_{\beta}\nabla^{\beta}K_{\mu}{}^{\alpha} - \tfrac{1}{2}R_{\mu\alpha}\nabla_{\beta}\nabla^{\beta}K_{\nu}{}^{\alpha} \\ & + \tfrac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} - \tfrac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\mu}K_{\nu}{}^{\alpha} - \tfrac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\nu}K_{\mu}{}^{\alpha} - \tfrac{1}{2}R_{\nu\alpha}\nabla_{\beta}\nabla_{\mu}K^{\alpha\beta} \\ & - \tfrac{1}{2}R_{\mu\alpha}\nabla_{\beta}\nabla_{\nu}K^{\alpha\beta} - \nabla_{\beta}K_{\nu}{}^{\alpha}\nabla^{\beta}R_{\mu\alpha} - \nabla_{\beta}K_{\mu}{}^{\alpha}\nabla^{\beta}R_{\nu\alpha} - R_{\alpha\beta}\nabla^{\beta}\nabla^{\alpha}K_{\mu\nu} \\ & + R_{\alpha\beta}\nabla^{\beta}\nabla_{\mu}K_{\nu}{}^{\alpha} + R_{\alpha\beta}\nabla^{\beta}\nabla_{\nu}K_{\mu}{}^{\alpha} - g_{\mu\nu}R_{\alpha\beta}\nabla_{\gamma}\nabla^{\beta}K^{\alpha\gamma} + \tfrac{2}{3}g_{\mu\nu}R_{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}K^{\alpha\beta} \\ & - R_{\mu\alpha\nu\beta}\nabla_{\gamma}\nabla^{\gamma}K^{\alpha\beta} + \tfrac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} + \tfrac{1}{3}g_{\mu\nu}\nabla_{\gamma}K^{\alpha\beta}\nabla^{\gamma}R_{\alpha\beta} - \nabla_{\beta}R_{\nu\alpha}\nabla_{\mu}K^{\alpha\beta} \\ & + \tfrac{1}{6}\nabla_{\alpha}R\nabla_{\mu}K_{\nu}{}^{\alpha} - \tfrac{1}{2}R_{\alpha\beta}\nabla_{\mu}\nabla_{\nu}K^{\alpha\beta} + \tfrac{1}{3}\nabla_{\mu}R^{\alpha\beta}\nabla_{\nu}K_{\alpha\beta} - \nabla_{\beta}R_{\mu\alpha}\nabla_{\nu}K^{\alpha\beta} \\ & + \tfrac{1}{6}\nabla_{\alpha}R\nabla_{\nu}K_{\mu}{}^{\alpha} + \tfrac{1}{3}\nabla_{\mu}K^{\alpha\beta}\nabla_{\nu}R_{\alpha\beta} - \tfrac{1}{6}R_{\alpha\beta}\nabla_{\nu}\nabla_{\mu}K^{\alpha\beta} + \tfrac{1}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} \\ \\ & + \tfrac{1}{24}g_{\mu\nu}R^2h - \tfrac{1}{8}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}h - \tfrac{1}{6}RR_{\mu\nu}h + \tfrac{1}{2}R_{\alpha\beta}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}h + \tfrac{1}{24}g_{\mu\nu}h\nabla_{\alpha}\nabla^{\alpha}R \\ & - \tfrac{1}{4}h\nabla_{\alpha}\nabla^{\alpha}R_{\mu\nu} + \tfrac{1}{4}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}\nabla_{\mu}h - \tfrac{1}{4}\nabla_{\alpha}h\nabla^{\alpha}R_{\mu\nu} - \tfrac{1}{2}R_{\mu\alpha\nu}{}^{\beta}\nabla_{\beta}\nabla^{\alpha}h + \tfrac{1}{4}\nabla_{\alpha}h\nabla_{\mu}R_{\nu}{}^{\alpha} \\ & - \tfrac{1}{4}\nabla_{\alpha}R_{\nu}{}^{\alpha}\nabla_{\mu}h + \tfrac{1}{4}R_{\nu\alpha}\nabla_{\mu}\nabla^{\alpha}h + \tfrac{1}{8}\nabla_{\mu}h\nabla_{\nu}R + \tfrac{1}{4}\nabla_{\alpha}h\nabla_{\nu}R_{\mu}{}^{\alpha} - \tfrac{1}{4}\nabla_{\alpha}R_{\mu}{}^{\alpha}\nabla_{\nu}h \\ & + \tfrac{1}{8}\nabla_{\mu}R\nabla_{\nu}h + \tfrac{1}{4}R_{\mu\alpha}\nabla_{\nu}\nabla^{\alpha}h + \tfrac{1}{12}h\nabla_{\nu}\nabla_{\mu}R - \tfrac{1}{4}\nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}h. \end{aligned} \tag{2}$$

Weyl Tensor (General)

$$C_{\lambda\mu\nu\kappa} = \tfrac{1}{6}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu})R + \tfrac{1}{2}(g_{\mu\nu}R_{\lambda\kappa} - g_{\mu\kappa}R_{\lambda\nu} - g_{\lambda\nu}R_{\mu\kappa} + g_{\lambda\kappa}R_{\mu\nu}) + R_{\lambda\mu\nu\kappa} \tag{3}$$

$$\begin{aligned}\delta C_{\lambda\mu\nu\kappa}(h_{\mu\nu}) = & -\tfrac{1}{6}g_{\mu\nu}h_{\kappa\lambda}R + \tfrac{1}{6}g_{\lambda\nu}h_{\kappa\mu}R + \tfrac{1}{6}g_{\kappa\mu}h_{\lambda\nu}R - \tfrac{1}{6}g_{\kappa\lambda}h_{\mu\nu}R - \tfrac{1}{6}g_{\kappa\mu}g_{\lambda\nu}h^{\alpha\beta}R_{\alpha\beta} \\ & + \tfrac{1}{6}g_{\kappa\lambda}g_{\mu\nu}h^{\alpha\beta}R_{\alpha\beta} + \tfrac{1}{2}h_{\mu\nu}R_{\kappa\lambda} - \tfrac{1}{2}h_{\lambda\nu}R_{\kappa\mu} - \tfrac{1}{2}h_{\kappa\mu}R_{\lambda\nu} + \tfrac{1}{2}h_{\kappa\lambda}R_{\mu\nu} + h_{\lambda}{}^{\alpha}R_{\kappa\nu\mu\alpha} \\ & + \tfrac{1}{4}g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}h_{\kappa\lambda} - \tfrac{1}{4}g_{\lambda\nu}\nabla_{\alpha}\nabla^{\alpha}h_{\kappa\mu} - \tfrac{1}{4}g_{\kappa\mu}\nabla_{\alpha}\nabla^{\alpha}h_{\lambda\nu} + \tfrac{1}{4}g_{\kappa\lambda}\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} \\ & - \tfrac{1}{4}g_{\mu\nu}\nabla_{\alpha}\nabla_{\kappa}h_{\lambda}{}^{\alpha} + \tfrac{1}{4}g_{\lambda\nu}\nabla_{\alpha}\nabla_{\kappa}h_{\mu}{}^{\alpha} - \tfrac{1}{4}g_{\mu\nu}\nabla_{\alpha}\nabla_{\lambda}h_{\kappa}{}^{\alpha} + \tfrac{1}{4}g_{\kappa\mu}\nabla_{\alpha}\nabla_{\lambda}h_{\nu}{}^{\alpha} \\ & + \tfrac{1}{4}g_{\lambda\nu}\nabla_{\alpha}\nabla_{\mu}h_{\kappa}{}^{\alpha} - \tfrac{1}{4}g_{\kappa\lambda}\nabla_{\alpha}\nabla_{\mu}h_{\nu}{}^{\alpha} + \tfrac{1}{4}g_{\kappa\mu}\nabla_{\alpha}\nabla_{\nu}h_{\lambda}{}^{\alpha} - \tfrac{1}{4}g_{\kappa\lambda}\nabla_{\alpha}\nabla_{\nu}h_{\mu}{}^{\alpha} \\ & - \tfrac{1}{6}g_{\kappa\mu}g_{\lambda\nu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} + \tfrac{1}{6}g_{\kappa\lambda}g_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} - \tfrac{1}{2}\nabla_{\kappa}\nabla_{\lambda}h_{\mu\nu} + \tfrac{1}{2}\nabla_{\kappa}\nabla_{\mu}h_{\lambda\nu} + \tfrac{1}{2}\nabla_{\kappa}\nabla_{\nu}h_{\lambda\mu} \\ & - \tfrac{1}{2}\nabla_{\nu}\nabla_{\kappa}h_{\lambda\mu} + \tfrac{1}{2}\nabla_{\nu}\nabla_{\lambda}h_{\kappa\mu} - \tfrac{1}{2}\nabla_{\nu}\nabla_{\mu}h_{\kappa\lambda} \\ \\ & + \tfrac{1}{6}g_{\kappa\mu}g_{\lambda\nu}\nabla_{\alpha}\nabla^{\alpha}h - \tfrac{1}{6}g_{\kappa\lambda}g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}h + \tfrac{1}{4}g_{\mu\nu}\nabla_{\lambda}\nabla_{\kappa}h - \tfrac{1}{4}g_{\lambda\nu}\nabla_{\mu}\nabla_{\kappa}h - \tfrac{1}{4}g_{\kappa\mu}\nabla_{\nu}\nabla_{\lambda}h \\ & + \tfrac{1}{4}g_{\kappa\lambda}\nabla_{\nu}\nabla_{\mu}h. \end{aligned} \tag{4}$$

$$
\begin{aligned}
\delta C_{\lambda\mu\nu\kappa}(K_{\mu\nu}+\tfrac{h}{4}g_{\mu\nu}) = & -\tfrac{1}{6}g_{\mu\nu}K_{\kappa\lambda}R+\tfrac{1}{6}g_{\lambda\nu}K_{\kappa\mu}R+\tfrac{1}{6}g_{\kappa\mu}K_{\lambda\nu}R-\tfrac{1}{6}g_{\kappa\lambda}K_{\mu\nu}R-\tfrac{1}{6}g_{\kappa\mu}g_{\lambda\nu}K^{\alpha\beta}R_{\alpha\beta} \\
& +\tfrac{1}{6}g_{\kappa\lambda}g_{\mu\nu}K^{\alpha\beta}R_{\alpha\beta}+\tfrac{1}{2}K_{\mu\nu}R_{\kappa\lambda}-\tfrac{1}{2}K_{\lambda\nu}R_{\kappa\mu}-\tfrac{1}{2}K_{\kappa\mu}R_{\lambda\nu}+\tfrac{1}{2}K_{\kappa\lambda}R_{\mu\nu}+K_{\lambda}{}^{\alpha}R_{\kappa\nu\mu\alpha} \\
& \\
& +\tfrac{1}{4}g_{\mu\nu}\nabla_\alpha\nabla^\alpha K_{\kappa\lambda}-\tfrac{1}{4}g_{\lambda\nu}\nabla_\alpha\nabla^\alpha K_{\kappa\mu}-\tfrac{1}{4}g_{\kappa\mu}\nabla_\alpha\nabla^\alpha K_{\lambda\nu}+\tfrac{1}{4}g_{\kappa\lambda}\nabla_\alpha\nabla^\alpha K_{\mu\nu} \\
& -\tfrac{1}{4}g_{\mu\nu}\nabla_\alpha\nabla_\kappa K_\lambda{}^\alpha+\tfrac{1}{4}g_{\lambda\nu}\nabla_\alpha\nabla_\kappa K_\mu{}^\alpha-\tfrac{1}{4}g_{\mu\nu}\nabla_\alpha\nabla_\lambda K_\kappa{}^\alpha+\tfrac{1}{4}g_{\kappa\mu}\nabla_\alpha\nabla_\lambda K_\nu{}^\alpha \\
& +\tfrac{1}{4}g_{\lambda\nu}\nabla_\alpha\nabla_\mu K_\kappa{}^\alpha-\tfrac{1}{4}g_{\kappa\lambda}\nabla_\alpha\nabla_\mu K_\nu{}^\alpha+\tfrac{1}{4}g_{\kappa\mu}\nabla_\alpha\nabla_\nu K_\lambda{}^\alpha-\tfrac{1}{4}g_{\kappa\lambda}\nabla_\alpha\nabla_\nu K_\mu{}^\alpha \\
& -\tfrac{1}{6}g_{\kappa\mu}g_{\lambda\nu}\nabla_\beta\nabla_\alpha K^{\alpha\beta}+\tfrac{1}{6}g_{\kappa\lambda}g_{\mu\nu}\nabla_\beta\nabla_\alpha K^{\alpha\beta}-\tfrac{1}{2}\nabla_\kappa\nabla_\lambda K_{\mu\nu}+\tfrac{1}{2}\nabla_\kappa\nabla_\mu K_{\lambda\nu} \\
& +\tfrac{1}{2}\nabla_\kappa\nabla_\nu K_{\lambda\mu}-\tfrac{1}{2}\nabla_\nu\nabla_\kappa K_{\lambda\mu}+\tfrac{1}{2}\nabla_\nu\nabla_\lambda K_{\kappa\mu}-\tfrac{1}{2}\nabla_\nu\nabla_\mu K_{\kappa\lambda} \\
& \\
& +\tfrac{1}{24}g_{\kappa\mu}g_{\lambda\nu}Rh-\tfrac{1}{24}g_{\kappa\lambda}g_{\mu\nu}Rh+\tfrac{1}{8}g_{\mu\nu}R_{\kappa\lambda}h-\tfrac{1}{8}g_{\lambda\nu}R_{\kappa\mu}h-\tfrac{1}{8}g_{\kappa\mu}R_{\lambda\nu}h+\tfrac{1}{8}g_{\kappa\lambda}R_{\mu\nu}h \\
& -\tfrac{1}{4}R_{\kappa\nu\lambda\mu}h. \qquad (5)
\end{aligned}
$$

$$
\delta C_{\lambda\mu\nu\kappa}(\tfrac{h}{4}g_{\mu\nu}) = \frac{1}{4}hC_{\lambda\mu\nu\kappa} \qquad (6)
$$