adS SVT3 $\Omega(z)$

1 Background

$$ds^2 = \Omega^2(z)\left[-dt^2+dx^2+dy^2+dz^2\right] \tag{1.1}$$

$$\Omega(z) = \frac{1}{Kz} \tag{1.2}$$

$$R_{\lambda\mu\nu\kappa} = -K^2(g_{\mu\nu}g_{\lambda\kappa}-g_{\lambda\nu}g_{\mu\kappa}), \qquad R_{\mu\nu}=3K^2g_{\mu\nu}, \qquad R=12K^2 \tag{1.3}$$

$$G_{\mu\nu} = -3K^2g_{\mu\nu}, \qquad T_{\mu\nu}=3K^2g_{\mu\nu} \tag{1.4}$$

2 Perturbations

$$ds^2 = \Omega^2(z)\left[-dt^2+\tilde{g}_{ij}dx^idx^j+f_{\mu\nu}dx^\mu dx^\nu\right], \qquad \tilde{g}_{ij}=\delta_{ij} \tag{2.1}$$

$$f_{00} = -2\phi, \qquad f_{0i}=\tilde{\nabla}_iB+B_i \tag{2.2}$$

$$f_{ij} = -2\psi\tilde{g}_{ij}+2\tilde{\nabla}_i\tilde{\nabla}_jE+\tilde{\nabla}_iE_j+\tilde{\nabla}_iE_j+2E_{ij} \tag{2.3}$$

$$\delta T_{\mu\nu} = 3\Omega^2K^2f_{\mu\nu} \tag{2.4}$$

$$\begin{aligned}\delta G_{00} = {} & 6\dot{\psi}\dot{\Omega}\Omega^{-1}+2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^aB-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}-2\tilde{\nabla}_a\tilde{\nabla}^a\psi+4\phi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\\ & +4\psi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega+4\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^aB-2\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB-2\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi\\ & -2\phi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega-2\psi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE\\ & +2\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega-4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE\\ & +4B^a\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}-2B^a\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a+2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a\\ & -4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a-4E^{ab}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega+2E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega\end{aligned} \tag{2.5}$$

$$\begin{aligned}\delta G_{0i} = {} & -\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_iB+2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_iB-2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_iB+\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_iB-2\tilde{\nabla}_i\dot{\psi}\\ & -2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi+2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i\Omega-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i\tilde{\nabla}_a\dot{E}-B_i\dot{\Omega}^2\Omega^{-2}+2B_i\ddot{\Omega}\Omega^{-1}\\ & +\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^aB_i-\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i-2B_i\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega+\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB_i-\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{E}_i\\ & +B_i\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega-\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_iB^a-\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i\dot{E}^a-2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a\Omega\end{aligned} \tag{2.6}$$

$$\begin{aligned}\delta G_{ij} = {} & -2\ddot{\psi}\tilde{g}_{ij}+2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2}+2\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2}-2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}-4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}-4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1}\\ & -4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1}-2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^aB-\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}+2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}\end{aligned}$$

$$
\begin{aligned}
&-\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi-4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^aB+2\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB\\
&-2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B}-2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi+2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE\\
&-2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega+4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE+2\Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\psi\\
&+2\Omega^{-1}\tilde{\nabla}_i\psi\tilde{\nabla}_j\Omega+2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_iB+\tilde{\nabla}_j\tilde{\nabla}_i\dot{B}-\tilde{\nabla}_j\tilde{\nabla}_i\ddot{E}-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E}\\
&-2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_iE+4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_iE-4\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_iE+2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_iE\\
&+\tilde{\nabla}_j\tilde{\nabla}_i\phi-\tilde{\nabla}_j\tilde{\nabla}_i\psi-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_aE\\
&-4B^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}+2B^a\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega-2\dot{B}^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega+2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a\\
&-2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a+4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_iB_j+\tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j-\tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j\\
&-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j-\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_iE_j+2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_iE_j-2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_iE_j\\
&+\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_iE_j+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_jB_i+\tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i-\tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i-\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_jE_i\\
&+2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_jE_i-2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_jE_i+\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_jE_i-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_iE_a\\
&-\ddot{E}_{ij}-2\dot{\Omega}^2E_{ij}\Omega^{-2}-2\dot{E}_{ij}\dot{\Omega}\Omega^{-1}+4\ddot{\Omega}E_{ij}\Omega^{-1}+\tilde{\nabla}_a\tilde{\nabla}^aE_{ij}-4E_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\\
&+2\Omega^{-1}\tilde{\nabla}_aE_{ij}\tilde{\nabla}^a\Omega+2E_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega+4E^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\\
&-2E_{ab}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_iE_{ja}-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_jE_{ia} \qquad (2.7)
\end{aligned}
$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} &= \Omega^{-2}(-\delta G_{00}+\tilde{g}^{ab}\delta G_{ab})\\
&= 6\dot{\Omega}^2\phi\Omega^{-4}+6\dot{\Omega}^2\psi\Omega^{-4}-6\dot{\phi}\dot{\Omega}\Omega^{-3}-18\dot{\psi}\dot{\Omega}\Omega^{-3}-12\ddot{\Omega}\phi\Omega^{-3}-12\ddot{\Omega}\psi\Omega^{-3}-6\ddot{\psi}\Omega^{-2}\\
&\quad-6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aB-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}+6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}\\
&\quad-2\dot{\Omega}^2\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^aE+4\ddot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi+4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi-4\phi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\\
&\quad-4\psi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega-16\Omega^{-3}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^aB+8\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB-6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B}\\
&\quad-6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi+6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi+2\phi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega+2\psi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\\
&\quad+2\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE-4\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE\tilde{\nabla}_b\tilde{\nabla}^b\Omega+6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE\\
&\quad-8\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega+16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE.-16B^a\Omega^{-3}\tilde{\nabla}_a\dot{\Omega}\\
&\quad+8B^a\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega-6\dot{B}^a\Omega^{-3}\tilde{\nabla}_a\Omega+6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a-8\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a\\
&\quad+16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a+16E^{ab}\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega-8E_{ab}\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega \qquad (2.8)
\end{aligned}
$$

3 Field Equations

$$
\begin{aligned}
\Delta_{00} &= 6z^{-2}\psi-2\tilde{\nabla}_1\tilde{\nabla}_1\psi+2z^{-1}\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_3E-2\tilde{\nabla}_2\tilde{\nabla}_2\psi+2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_3E+2z^{-1}\tilde{\nabla}_3\psi\\
&\quad-6z^{-2}\tilde{\nabla}_3\tilde{\nabla}_3E-2\tilde{\nabla}_3\tilde{\nabla}_3\psi+2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3E+2z^{-1}\tilde{\nabla}_1\tilde{\nabla}_1E_3+2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_2E_3-6z^{-2}\tilde{\nabla}_3E_3\\
&\quad+2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_3E_3-6E_{33}z^{-2} \qquad (3.1)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{01} &= -2\tilde{\nabla}_1\dot{\psi}+2z^{-1}\tilde{\nabla}_1\tilde{\nabla}_3\dot{E}+z^{-1}\tilde{\nabla}_1B_3+z^{-1}\tilde{\nabla}_1\dot{E}_3+\tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1B_1-\tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}_1+\tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2B_1\\
&\quad-\tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E}_1-z^{-1}\tilde{\nabla}_3B_1+z^{-1}\tilde{\nabla}_3\dot{E}_1+\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3B_1-\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}_1+2\dot{E}_{13}z^{-1} \qquad (3.2)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{02} &= -2\tilde{\nabla}_2\dot{\psi}+2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_3\dot{E}+\tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1B_2-\tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}_2+z^{-1}\tilde{\nabla}_2B_3+z^{-1}\tilde{\nabla}_2\dot{E}_3+\tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2B_2\\
&\quad-\tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E}_2-z^{-1}\tilde{\nabla}_3B_2+z^{-1}\tilde{\nabla}_3\dot{E}_2+\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3B_2-\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}_2+2\dot{E}_{23}z^{-1} \qquad (3.3)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{03} &= -2\dot{\psi}z^{-1}-2\tilde{\nabla}_3\dot{\psi}+2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}+\tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1B_3-\tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}_3+\tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2B_3-\tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E}_3\\
&\quad+2z^{-1}\tilde{\nabla}_3\dot{E}_3+\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3B_3-\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}_3+2\dot{E}_{33}z^{-1} \qquad (3.4)
\end{aligned}
$$

$$
\Delta_{11} = -2\ddot{\psi}-6z^{-2}\psi-\tilde{\nabla}_2\tilde{\nabla}_2\dot{B}+\tilde{\nabla}_2\tilde{\nabla}_2\ddot{E}-\tilde{\nabla}_2\tilde{\nabla}_2\phi+\tilde{\nabla}_2\tilde{\nabla}_2\psi-2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_3E+2z^{-1}\tilde{\nabla}_3\dot{B}
$$

$$
\begin{aligned}
&+2z^{-1}\tilde{\nabla}_3\phi - \tilde{\nabla}_3\tilde{\nabla}_3\dot{B} + \tilde{\nabla}_3\tilde{\nabla}_3\ddot{E} + 6z^{-2}\tilde{\nabla}_3\tilde{\nabla}_3 E - \tilde{\nabla}_3\tilde{\nabla}_3\phi + \tilde{\nabla}_3\tilde{\nabla}_3\psi \\
&-2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3 E + 2\dot{B}_3 z^{-1} + \tilde{\nabla}_1\dot{B}_1 - \tilde{\nabla}_1\ddot{E}_1 - 2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_2 E_3 + 6z^{-2}\tilde{\nabla}_3 E_3 \\
&-2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_3 E_3 - \ddot{E}_{11} + 6E_{33}z^{-2} + 4z^{-1}\tilde{\nabla}_1 E_{13} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{11} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{11} - 2z^{-1}\tilde{\nabla}_3 E_{11} \\
&+\tilde{\nabla}_3\tilde{\nabla}_3 E_{11}
\end{aligned} \tag{3.5}
$$

$$
\begin{aligned}
\Delta_{22} \;=\; & -2\ddot{\psi} - 6z^{-2}\psi - \tilde{\nabla}_1\tilde{\nabla}_1\dot{B} + \tilde{\nabla}_1\tilde{\nabla}_1\ddot{E} - \tilde{\nabla}_1\tilde{\nabla}_1\phi + \tilde{\nabla}_1\tilde{\nabla}_1\psi - 2z^{-1}\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_3 E + 2z^{-1}\tilde{\nabla}_3\dot{B} \\
&+2z^{-1}\tilde{\nabla}_3\phi - \tilde{\nabla}_3\tilde{\nabla}_3\dot{B} + \tilde{\nabla}_3\tilde{\nabla}_3\ddot{E} + 6z^{-2}\tilde{\nabla}_3\tilde{\nabla}_3 E - \tilde{\nabla}_3\tilde{\nabla}_3\phi + \tilde{\nabla}_3\tilde{\nabla}_3\psi \\
&-2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3 E + 2\dot{B}_3 z^{-1} - 2z^{-1}\tilde{\nabla}_1\tilde{\nabla}_1 E_3 + \tilde{\nabla}_2\dot{B}_2 - \tilde{\nabla}_2\ddot{E}_2 + 6z^{-2}\tilde{\nabla}_3 E_3 \\
&-2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_3 E_3 - \ddot{E}_{22} + 6E_{33}z^{-2} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{22} + 4z^{-1}\tilde{\nabla}_2 E_{23} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{22} - 2z^{-1}\tilde{\nabla}_3 E_{22} \\
&+\tilde{\nabla}_3\tilde{\nabla}_3 E_{22}
\end{aligned} \tag{3.6}
$$

$$
\begin{aligned}
\Delta_{33} \;=\; & -2\ddot{\psi} - 6z^{-2}\psi - \tilde{\nabla}_1\tilde{\nabla}_1\dot{B} + \tilde{\nabla}_1\tilde{\nabla}_1\ddot{E} - \tilde{\nabla}_1\tilde{\nabla}_1\phi + \tilde{\nabla}_1\tilde{\nabla}_1\psi - 2z^{-1}\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_3 E - \tilde{\nabla}_2\tilde{\nabla}_2\dot{B} \\
&+\tilde{\nabla}_2\tilde{\nabla}_2\ddot{E} - \tilde{\nabla}_2\tilde{\nabla}_2\phi + \tilde{\nabla}_2\tilde{\nabla}_2\psi - 2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_3 E + 2z^{-1}\tilde{\nabla}_3\dot{B} + 2z^{-1}\tilde{\nabla}_3\phi - 4z^{-1}\tilde{\nabla}_3\psi \\
&+6z^{-2}\tilde{\nabla}_3\tilde{\nabla}_3 E + 2\dot{B}_3 z^{-1} - 2z^{-1}\tilde{\nabla}_1\tilde{\nabla}_1 E_3 - 2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_2 E_3 + \tilde{\nabla}_3\dot{B}_3 - \tilde{\nabla}_3\ddot{E}_3 \\
&+6z^{-2}\tilde{\nabla}_3 E_3 - \ddot{E}_{33} + 6E_{33}z^{-2} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{33} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{33} + 2z^{-1}\tilde{\nabla}_3 E_{33} + \tilde{\nabla}_3\tilde{\nabla}_3 E_{33}
\end{aligned} \tag{3.7}
$$

$$
\begin{aligned}
\Delta_{12} \;=\; & 6z^{-2}\tilde{\nabla}_1\tilde{\nabla}_2 E + \tilde{\nabla}_2\tilde{\nabla}_1\dot{B} - \tilde{\nabla}_2\tilde{\nabla}_1\ddot{E} - 6z^{-2}\tilde{\nabla}_2\tilde{\nabla}_1 E + \tilde{\nabla}_2\tilde{\nabla}_1\phi - \tilde{\nabla}_2\tilde{\nabla}_1\psi \\
&+2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_1\tilde{\nabla}_3 E + \tfrac{1}{2}\tilde{\nabla}_1\dot{B}_2 - \tfrac{1}{2}\tilde{\nabla}_1\ddot{E}_2 + \tfrac{1}{2}\tilde{\nabla}_2\dot{B}_1 - \tfrac{1}{2}\tilde{\nabla}_2\ddot{E}_1 + 2z^{-1}\tilde{\nabla}_2\tilde{\nabla}_1 E_3 - \ddot{E}_{12} \\
&+2z^{-1}\tilde{\nabla}_1 E_{23} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{12} + 2z^{-1}\tilde{\nabla}_2 E_{13} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{12} - 2z^{-1}\tilde{\nabla}_3 E_{12} + \tilde{\nabla}_3\tilde{\nabla}_3 E_{12}
\end{aligned} \tag{3.8}
$$

$$
\begin{aligned}
\Delta_{13} \;=\; & -2z^{-1}\tilde{\nabla}_1\psi + 6z^{-2}\tilde{\nabla}_1\tilde{\nabla}_3 E + \tilde{\nabla}_3\tilde{\nabla}_1\dot{B} - \tilde{\nabla}_3\tilde{\nabla}_1\ddot{E} - 6z^{-2}\tilde{\nabla}_3\tilde{\nabla}_1 E + \tilde{\nabla}_3\tilde{\nabla}_1\phi - \tilde{\nabla}_3\tilde{\nabla}_1\psi \\
&+2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_1\tilde{\nabla}_3 E + \tfrac{1}{2}\tilde{\nabla}_1\dot{B}_3 - \tfrac{1}{2}\tilde{\nabla}_1\ddot{E}_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{B}_1 - \tfrac{1}{2}\tilde{\nabla}_3\ddot{E}_1 + 2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_1 E_3 - \ddot{E}_{13} \\
&+2z^{-1}\tilde{\nabla}_1 E_{33} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{13} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{13} + \tilde{\nabla}_3\tilde{\nabla}_3 E_{13}
\end{aligned} \tag{3.9}
$$

$$
\begin{aligned}
\Delta_{23} \;=\; & -2z^{-1}\tilde{\nabla}_2\psi + 6z^{-2}\tilde{\nabla}_2\tilde{\nabla}_3 E + \tilde{\nabla}_3\tilde{\nabla}_2\dot{B} - \tilde{\nabla}_3\tilde{\nabla}_2\ddot{E} - 6z^{-2}\tilde{\nabla}_3\tilde{\nabla}_2 E + \tilde{\nabla}_3\tilde{\nabla}_2\phi - \tilde{\nabla}_3\tilde{\nabla}_2\psi \\
&+2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_3 E + \tfrac{1}{2}\tilde{\nabla}_2\dot{B}_3 - \tfrac{1}{2}\tilde{\nabla}_2\ddot{E}_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{B}_2 - \tfrac{1}{2}\tilde{\nabla}_3\ddot{E}_2 + 2z^{-1}\tilde{\nabla}_3\tilde{\nabla}_2 E_3 - \ddot{E}_{23} \\
&+\tilde{\nabla}_1\tilde{\nabla}_1 E_{23} + 2z^{-1}\tilde{\nabla}_2 E_{33} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{23} + \tilde{\nabla}_3\tilde{\nabla}_3 E_{23}
\end{aligned} \tag{3.10}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} \;=\; & -6K^2\ddot{\psi}z^2 - 24K^2\psi - 2K^2z^2\tilde{\nabla}_1\tilde{\nabla}_1\dot{B} + 2K^2z^2\tilde{\nabla}_1\tilde{\nabla}_1\ddot{E} - 2K^2z^2\tilde{\nabla}_1\tilde{\nabla}_1\phi + 4K^2z^2\tilde{\nabla}_1\tilde{\nabla}_1\psi \\
&-6K^2z\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_3 E - 2K^2z^2\tilde{\nabla}_2\tilde{\nabla}_2\dot{B} + 2K^2z^2\tilde{\nabla}_2\tilde{\nabla}_2\ddot{E} - 2K^2z^2\tilde{\nabla}_2\tilde{\nabla}_2\phi + 4K^2z^2\tilde{\nabla}_2\tilde{\nabla}_2\psi \\
&-6K^2z\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_3 E + 6K^2z\tilde{\nabla}_3\dot{B} + 6K^2z\tilde{\nabla}_3\phi - 6K^2z\tilde{\nabla}_3\psi - 2K^2z^2\tilde{\nabla}_3\tilde{\nabla}_3\dot{B} + 2K^2z^2\tilde{\nabla}_3\tilde{\nabla}_3\ddot{E} \\
&+24K^2\tilde{\nabla}_3\tilde{\nabla}_3 E - 2K^2z^2\tilde{\nabla}_3\tilde{\nabla}_3\phi + 4K^2z^2\tilde{\nabla}_3\tilde{\nabla}_3\psi - 6K^2z\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3 E + 6K^2\dot{B}_3 z \\
&-6K^2z\tilde{\nabla}_1\tilde{\nabla}_1 E_3 - 6K^2z\tilde{\nabla}_2\tilde{\nabla}_2 E_3 + 24K^2\tilde{\nabla}_3 E_3 - 6K^2z\tilde{\nabla}_3\tilde{\nabla}_3 E_3 + 24K^2 E_{33}
\end{aligned} \tag{3.11}
$$

4 Field Equations (G.I. Form)

Gauge invariant quantities may be separated in terms of ψ and ϕ by taking 1/2[(70a)+(70d)] and 1/2[(70a)-(70d)]

$$\begin{aligned}\alpha &= \phi + \dot{B} - \ddot{E} + B\dot{\Omega}\Omega^{-1} - \dot{E}\dot{\Omega}\Omega^{-1} - E^a\Omega^{-1}\tilde{\nabla}_a\Omega - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a E \\ &= \phi + \dot{B} - \ddot{E} + z^{-1}E_3 + z^{-1}\tilde{\nabla}_3 E \end{aligned} \tag{4.1}$$

$$\begin{aligned}\gamma &= \psi - B\dot{\Omega}\Omega^{-1} + \dot{E}\dot{\Omega}\Omega^{-1} + E^a\Omega^{-1}\tilde{\nabla}_a\Omega + \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a E \\ &= \psi - z^{-1}E_3 - z^{-1}\tilde{\nabla}_3 E \end{aligned} \tag{4.2}$$

$$Q_i = B_i - \dot{E}_i \tag{4.3}$$

In the following, $\tilde{\nabla}^2 = \tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b$.

$$\Delta_{00} = -2\nabla^2\gamma + 6z^{-2}\gamma + 2z^{-1}\tilde{\nabla}_3\gamma - 6z^{-2}E_{33} \tag{4.4}$$

$$\Delta_{01} = -2\tilde{\nabla}_1\dot{\gamma} + \tfrac{1}{2}\nabla^2 Q_1 + z^{-1}\tilde{\nabla}_1 Q_3 - z^{-1}\tilde{\nabla}_3 Q_1 + 2z^{-1}\dot{E}_{13} \tag{4.5}$$

$$\Delta_{02} = -2\tilde{\nabla}_2\dot{\gamma} + \tfrac{1}{2}\nabla^2 Q_2 + z^{-1}\tilde{\nabla}_2 Q_3 - z^{-1}\tilde{\nabla}_3 Q_2 + 2z^{-1}\dot{E}_{23} \tag{4.6}$$

$$\Delta_{03} = -2z^{-1}\dot{\gamma} - 2\tilde{\nabla}_3\dot{\gamma} + \tfrac{1}{2}\nabla^2 Q_3 + 2z^{-1}\dot{E}_{33} \tag{4.7}$$

$$\begin{aligned}\Delta_{11} &= -2\ddot{\gamma} - \nabla^2\alpha + \nabla^2\gamma - 6z^{-2}\gamma + \tilde{\nabla}_1\tilde{\nabla}_1\alpha - \tilde{\nabla}_1\tilde{\nabla}_1\gamma + 2z^{-1}\tilde{\nabla}_3\alpha + 2z^{-1}\dot{Q}_3 + \tilde{\nabla}_1\dot{Q}_1 - \ddot{E}_{11} \\ &\quad +6z^{-2}E_{33} + \nabla^2 E_{11} + 4z^{-1}\tilde{\nabla}_1 E_{13} - 2z^{-1}\tilde{\nabla}_3 E_{11} \end{aligned} \tag{4.8}$$

$$\begin{aligned}\Delta_{22} &= -2\ddot{\gamma} - \nabla^2\alpha + \nabla^2\gamma - 6z^{-2}\gamma + \tilde{\nabla}_2\tilde{\nabla}_2\alpha - \tilde{\nabla}_2\tilde{\nabla}_2\gamma + 2z^{-1}\tilde{\nabla}_3\alpha + 2z^{-1}\dot{Q}_3 + \tilde{\nabla}_2\dot{Q}_2 - \ddot{E}_{22} \\ &\quad +6z^{-2}E_{33} + \nabla^2 E_{22} + 4z^{-1}\tilde{\nabla}_2 E_{23} - 2z^{-1}\tilde{\nabla}_3 E_{22} \end{aligned} \tag{4.9}$$

$$\begin{aligned}\Delta_{33} &= -2\ddot{\gamma} - \nabla^2\alpha + \nabla^2\gamma - 6z^{-2}\gamma + 2z^{-1}\tilde{\nabla}_3\alpha - 4z^{-1}\tilde{\nabla}_3\gamma + \tilde{\nabla}_3\tilde{\nabla}_3\alpha - \tilde{\nabla}_3\tilde{\nabla}_3\gamma + 2z^{-1}\dot{Q}_3 \\ &\quad +\tilde{\nabla}_3\dot{Q}_3 - \ddot{E}_{33} + 6z^{-2}E_{33} + \nabla^2 E_{33} + 2z^{-1}\tilde{\nabla}_3 E_{33} \end{aligned} \tag{4.10}$$

$$\begin{aligned}\Delta_{12} &= \tilde{\nabla}_2\tilde{\nabla}_1\alpha - \tilde{\nabla}_2\tilde{\nabla}_1\gamma + \tfrac{1}{2}\tilde{\nabla}_1\dot{Q}_2 + \tfrac{1}{2}\tilde{\nabla}_2\dot{Q}_1 - \ddot{E}_{12} + \nabla^2 E_{12} + 2z^{-1}\tilde{\nabla}_1 E_{23} + 2z^{-1}\tilde{\nabla}_2 E_{13} \\ &\quad -2z^{-1}\tilde{\nabla}_3 E_{12} \end{aligned} \tag{4.11}$$

$$\Delta_{13} = -2z^{-1}\tilde{\nabla}_1\gamma + \tilde{\nabla}_3\tilde{\nabla}_1\alpha - \tilde{\nabla}_3\tilde{\nabla}_1\gamma + \tfrac{1}{2}\tilde{\nabla}_1\dot{Q}_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{Q}_1 - \ddot{E}_{13} + \nabla^2 E_{13} + 2z^{-1}\tilde{\nabla}_1 E_{33} \tag{4.12}$$

$$\Delta_{23} = -2z^{-1}\tilde{\nabla}_2\gamma + \tilde{\nabla}_3\tilde{\nabla}_2\alpha - \tilde{\nabla}_3\tilde{\nabla}_2\gamma + \tfrac{1}{2}\tilde{\nabla}_2\dot{Q}_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{Q}_2 - \ddot{E}_{23} + \nabla^2 E_{23} + 2z^{-1}\tilde{\nabla}_2 E_{33} \tag{4.13}$$

$$\begin{aligned}g^{\mu\nu}\Delta_{\mu\nu} &= -6K^2z^2\ddot{\gamma} - 2K^2z^2\nabla^2\alpha + 4K^2z^2\nabla^2\gamma - 24K^2\gamma + 6K^2z\tilde{\nabla}_3\alpha \\ &\quad -6K^2z\tilde{\nabla}_3\gamma + 6K^2z\dot{Q}_3 + 24K^2E_{33} \end{aligned} \tag{4.14}$$