General Gauge:

$$\eta^{\alpha\beta}\ \partial_\alpha h_{\beta\nu} == \frac{J\ \eta^{\alpha\beta}\ h_{\nu\alpha}\ \partial_\beta\Omega}{\Omega} + P\,\Omega^2\,\partial_\nu h + R\,h\,\Omega\,\partial_\nu\Omega$$

Perturbed Einstein Tensor in RW

J = 0, P = 1, R = -1

$$\frac{\eta^{\mu\nu}}{\Omega^2}\delta G_{\mu\nu} =$$

$$-\frac{10\ h_{00}\ \Omega'[t]^2}{\Omega[t]^6} + \frac{2\,h\,\Omega'[t]^2}{\Omega[t]^4} + \frac{6\ h_{00}\ \Omega''[t]}{\Omega[t]^5} + \frac{3\,h\,\Omega''[t]}{\Omega[t]^3}$$

00	$-\frac{\partial_0\partial_0 h}{2} + \frac{\partial_0 h_{00}\ \Omega'[t]}{\Omega[t]^3} + \frac{\partial_0 h\,\Omega'[t]}{2\,\Omega[t]} + \frac{2\,h_{00}\ \Omega'[t]^2}{\Omega[t]^4} - \frac{3\,h\,\Omega'[t]^2}{2\,\Omega[t]^2} + \frac{h\,\Omega''[t]}{2\,\Omega[t]} + \frac{1}{2\,\Omega[t]^2}\ \Box h_{00}$
11	$-\frac{\partial_1\partial_1 h}{2} + \frac{\partial_0 h_{11}\ \Omega'[t]}{\Omega[t]^3} + \frac{\partial_0 h\,\Omega'[t]}{2\,\Omega[t]} - \frac{2\,h_{00}\ \Omega'[t]^2}{\Omega[t]^4} - \frac{2\,h_{11}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{h\,\Omega'[t]^2}{2\,\Omega[t]^2} + \frac{h_{00}\ \Omega''[t]}{\Omega[t]^3} + \frac{3\,h_{11}\ \Omega''[t]}{\Omega[t]^3} + \frac{h\,\Omega''[t]}{2\,\Omega[t]} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{11}$
22	$-\frac{\partial_2\partial_2 h}{2} + \frac{\partial_0 h_{22}\ \Omega'[t]}{\Omega[t]^3} + \frac{\partial_0 h\,\Omega'[t]}{2\,\Omega[t]} - \frac{2\,h_{00}\ \Omega'[t]^2}{\Omega[t]^4} - \frac{2\,h_{22}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{h\,\Omega'[t]^2}{2\,\Omega[t]^2} + \frac{h_{00}\ \Omega''[t]}{\Omega[t]^3} + \frac{3\,h_{22}\ \Omega''[t]}{\Omega[t]^3} + \frac{h\,\Omega''[t]}{2\,\Omega[t]} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{22}$
33	$-\frac{\partial_3\partial_3 h}{2} + \frac{\partial_0 h_{33}\ \Omega'[t]}{\Omega[t]^3} + \frac{\partial_0 h\,\Omega'[t]}{2\,\Omega[t]} - \frac{2\,h_{00}\ \Omega'[t]^2}{\Omega[t]^4} - \frac{2\,h_{33}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{h\,\Omega'[t]^2}{2\,\Omega[t]^2} + \frac{h_{00}\ \Omega''[t]}{\Omega[t]^3} + \frac{3\,h_{33}\ \Omega''[t]}{\Omega[t]^3} + \frac{h\,\Omega''[t]}{2\,\Omega[t]} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{33}$
01	$-\frac{\partial_0\partial_1 h}{2} + \frac{\partial_0 h_{01}\ \Omega'[t]}{\Omega[t]^3} + \frac{\partial_1 h\,\Omega'[t]}{2\,\Omega[t]} - \frac{h_{01}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{2\,h_{01}\ \Omega''[t]}{\Omega[t]^3} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{01}$
02	$-\frac{\partial_0\partial_2 h}{2} + \frac{\partial_0 h_{02}\ \Omega'[t]}{\Omega[t]^3} + \frac{\partial_2 h\,\Omega'[t]}{2\,\Omega[t]} - \frac{h_{02}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{2\,h_{02}\ \Omega''[t]}{\Omega[t]^3} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{02}$
03	$-\frac{\partial_0\partial_3 h}{2} + \frac{\partial_0 h_{03}\ \Omega'[t]}{\Omega[t]^3} + \frac{\partial_3 h\,\Omega'[t]}{2\,\Omega[t]} - \frac{h_{03}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{2\,h_{03}\ \Omega''[t]}{\Omega[t]^3} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{03}$
12	$-\frac{\partial_1\partial_2 h}{2} + \frac{\partial_0 h_{12}\ \Omega'[t]}{\Omega[t]^3} - \frac{2\,h_{12}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{3\,h_{12}\ \Omega''[t]}{\Omega[t]^3} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{12}$
13	$-\frac{\partial_1\partial_3 h}{2} + \frac{\partial_0 h_{13}\ \Omega'[t]}{\Omega[t]^3} - \frac{2\,h_{13}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{3\,h_{13}\ \Omega''[t]}{\Omega[t]^3} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{13}$
23	$-\frac{\partial_2\partial_3 h}{2} + \frac{\partial_0 h_{23}\ \Omega'[t]}{\Omega[t]^3} - \frac{2\,h_{23}\ \Omega'[t]^2}{\Omega[t]^4} + \frac{3\,h_{23}\ \Omega''[t]}{\Omega[t]^3} + \frac{1}{2\,\Omega[t]^2}\ \Box\ h_{23}$

SVT

As given in Bertschinger (Structure Formation 2000)

$$h_{\mu\nu} = 2\ S_{\mu\nu} - u_\nu\ w_\mu - u_\mu\ w_\nu - 2\ u_\mu\ u_\nu\ \phi - 2\ (\eta_{\mu\nu} + u_\mu\ u_\nu)\ \psi$$

where

$u_\mu = (-1,\ 0,\ 0,\ 0)$

$u^\mu u^\nu S_{\mu\nu} = 0,\ g^{\mu\nu} S_{\mu\nu} = 0,\ u^\mu w_\mu = 0$

$$h = \frac{2\,\phi - 6\,\psi}{\Omega^2}$$

If $\delta G_{\mu\nu}$ decomposes in RW, it must also in flat space.

No gauge, $\delta G_{\mu\nu}$ flat:

00	$\frac{1}{2}\partial_1\partial_1 h_{00} - \frac{1}{2}\partial_1\partial_1 h_{11} + \frac{\partial_1\partial_1 h}{2} - \partial_2\partial_1 h_{12} + \frac{1}{2}\partial_2\partial_2 h_{00} - \frac{1}{2}\partial_2\partial_2 h_{22} + \frac{\partial_2\partial_2 h}{2} - \partial_3\partial_1 h_{13} - \partial_3\partial_2 h_{23} + \frac{1}{2}\partial_3\partial_3 h_{00} - \frac{1}{2}\partial_3\partial_3 h_{33} + \frac{\partial_3\partial_3 h}{2}$
11	$\frac{1}{2}\partial_0\partial_0 h_{00} - \frac{1}{2}\partial_0\partial_0 h_{11} + \frac{\partial_0\partial_0 h}{2} - \partial_1\partial_0 h_{01} + h_{10}\ \partial_1\partial_0 h_{01} - \partial_2\partial_0 h_{02} + \frac{1}{2}\partial_2\partial_2 h_{11} + \frac{1}{2}\partial_2\partial_2 h_{22} - \frac{\partial_2\partial_2 h}{2} - \partial_3\partial_0 h_{03} + \partial_3\partial_2 h_{23} + \frac{1}{2}\partial_3\partial_3 h_{11} + \frac{1}{2}\partial_3\partial_3 h_{33} - \frac{\partial_3\partial_3 h}{2}$
22	$\frac{1}{2}\partial_0\partial_0 h_{00} - \frac{1}{2}\partial_0\partial_0 h_{22} + \frac{\partial_0\partial_0 h}{2} - \partial_1\partial_0 h_{01} + \frac{1}{2}\partial_1\partial_1 h_{11} + \frac{1}{2}\partial_1\partial_1 h_{22} - \frac{\partial_1\partial_1 h}{2} - \partial_2\partial_0 h_{02} + h_{20}\ \partial_2\partial_0 h_{02} - \partial_3\partial_0 h_{03} + \partial_3\partial_1 h_{13} + \frac{1}{2}\partial_3\partial_3 h_{22} + \frac{1}{2}\partial_3\partial_3 h_{33} - \frac{\partial_3\partial_3 h}{2}$
33	$\frac{1}{2}\partial_0\partial_0 h_{00} - \frac{1}{2}\partial_0\partial_0 h_{33} + \frac{\partial_0\partial_0 h}{2} - \partial_1\partial_0 h_{01} + \frac{1}{2}\partial_1\partial_1 h_{11} + \frac{1}{2}\partial_1\partial_1 h_{33} - \frac{\partial_1\partial_1 h}{2} - \partial_2\partial_0 h_{02} + \partial_2\partial_1 h_{12} + \frac{1}{2}\partial_2\partial_2 h_{22} + \frac{1}{2}\partial_2\partial_2 h_{33} - \frac{\partial_2\partial_2 h}{2} - \partial_3\partial_0 h_{03} + h_{30}\ \partial_3\partial_0 h_{03}$
01	$\frac{1}{2}\partial_1\partial_0 h_{00} - \frac{1}{2}\partial_1\partial_0 h_{11} + \frac{\partial_1\partial_0 h}{2} - \frac{1}{2}\partial_2\partial_0 h_{12} - \frac{1}{2}\partial_2\partial_1 h_{02} + \frac{1}{2}\partial_2\partial_2 h_{01} - \frac{1}{2}\partial_3\partial_0 h_{13} - \frac{1}{2}\partial_3\partial_1 h_{03} + \frac{1}{2}\partial_3\partial_3 h_{01}$
02	$-\frac{1}{2}\partial_1\partial_0 h_{12} + \frac{1}{2}\partial_1\partial_1 h_{02} + \frac{1}{2}\partial_2\partial_0 h_{00} - \frac{1}{2}\partial_2\partial_0 h_{22} + \frac{\partial_2\partial_0 h}{2} - \frac{1}{2}\partial_2\partial_1 h_{01} - \frac{1}{2}\partial_3\partial_0 h_{23} - \frac{1}{2}\partial_3\partial_2 h_{03} + \frac{1}{2}\partial_3\partial_3 h_{02}$
03	$-\frac{1}{2}\partial_1\partial_0 h_{13} + \frac{1}{2}\partial_1\partial_1 h_{03} - \frac{1}{2}\partial_2\partial_0 h_{23} + \frac{1}{2}\partial_2\partial_2 h_{03} + \frac{1}{2}\partial_3\partial_0 h_{00} - \frac{1}{2}\partial_3\partial_0 h_{33} + \frac{\partial_3\partial_0 h}{2} - \frac{1}{2}\partial_3\partial_1 h_{01} - \frac{1}{2}\partial_3\partial_2 h_{02}$
12	$-\frac{1}{2}\partial_0\partial_0 h_{12} + \frac{1}{2}\partial_1\partial_0 h_{02} + \frac{1}{2}\partial_2\partial_0 h_{01} - \frac{1}{2}\partial_2\partial_1 h_{11} - \frac{1}{2}\partial_2\partial_1 h_{22} + \frac{\partial_2\partial_1 h}{2} - \frac{1}{2}\partial_3\partial_1 h_{23} - \frac{1}{2}\partial_3\partial_2 h_{13} + \frac{1}{2}\partial_3\partial_3 h_{12}$
13	$-\frac{1}{2}\partial_0\partial_0 h_{13} + \frac{1}{2}\partial_1\partial_0 h_{03} - \frac{1}{2}\partial_2\partial_1 h_{23} + \frac{1}{2}\partial_2\partial_2 h_{13} + \frac{1}{2}\partial_3\partial_0 h_{01} - \frac{1}{2}\partial_3\partial_1 h_{11} - \frac{1}{2}\partial_3\partial_1 h_{33} + \frac{\partial_3\partial_1 h}{2} - \frac{1}{2}\partial_3\partial_2 h_{12}$
23	$-\frac{1}{2}\partial_0\partial_0 h_{23} + \frac{1}{2}\partial_1\partial_1 h_{23} + \frac{1}{2}\partial_2\partial_0 h_{03} - \frac{1}{2}\partial_2\partial_1 h_{13} + \frac{1}{2}\partial_3\partial_0 h_{02} - \frac{1}{2}\partial_3\partial_1 h_{12} - \frac{1}{2}\partial_3\partial_2 h_{22} - \frac{1}{2}\partial_3\partial_2 h_{33} + \frac{\partial_3\partial_2 h}{2}$

Compare to SVT $\delta G_{\mu\nu}$ flat, where

$h_{00} = -2\phi,\ h_{0i} = w_i,\ h_{ij} = 2S_{ij},\ h = 2\phi - 6\psi$

00	$-\partial_1\partial_1 S_{11} - 2\,\partial_1\partial_1\psi - 2\,\partial_2\partial_1 S_{12} - \partial_2\partial_2 S_{22} - 2\,\partial_2\partial_2\psi - 2\,\partial_3\partial_1 S_{13} - 2\,\partial_3\partial_2 S_{23} - \partial_3\partial_3 S_{33} - 2\,\partial_3\partial_3\psi$
11	$-\partial_0\partial_0 S_{11} - 2\,\partial_0\partial_0\psi - \partial_2\partial_0 w_2 + \partial_2\partial_2 S_{11} + \partial_2\partial_2 S_{22} - \partial_2\partial_2\phi + \partial_2\partial_2\psi - \partial_3\partial_0 w_3 + 2\,\partial_3\partial_2 S_{23} + \partial_3\partial_3 S_{11} + \partial_3\partial_3 S_{33} - \partial_3\partial_3\phi + \partial_3\partial_3\psi$
22	$-\partial_0\partial_0 S_{22} - 2\,\partial_0\partial_0\psi - \partial_1\partial_0 w_1 + \partial_1\partial_1 S_{11} + \partial_1\partial_1 S_{22} - \partial_1\partial_1\phi + \partial_1\partial_1\psi - \partial_3\partial_0 w_3 + 2\,\partial_3\partial_1 S_{13} + \partial_3\partial_3 S_{22} + \partial_3\partial_3 S_{33} - \partial_3\partial_3\phi + \partial_3\partial_3\psi$
33	$-\partial_0\partial_0 S_{33} - 2\,\partial_0\partial_0\psi - \partial_1\partial_0 w_1 + \partial_1\partial_1 S_{11} + \partial_1\partial_1 S_{33} - \partial_1\partial_1\phi + \partial_1\partial_1\psi - \partial_2\partial_0 w_2 + 2\,\partial_2\partial_1 S_{12} + \partial_2\partial_2 S_{22} + \partial_2\partial_2 S_{33} - \partial_2\partial_2\phi + \partial_2\partial_2\psi$
01	$-\partial_1\partial_0 S_{11} - 2\,\partial_1\partial_0\psi - \partial_2\partial_0 S_{12} - \frac{1}{2}\partial_2\partial_1 w_2 + \frac{1}{2}\partial_2\partial_2 w_1 - \partial_3\partial_0 S_{13} - \frac{1}{2}\partial_3\partial_1 w_3 + \frac{1}{2}\partial_3\partial_3 w_1$
02	$-\partial_1\partial_0 S_{12} + \frac{1}{2}\partial_1\partial_1 w_2 - \partial_2\partial_0 S_{22} - 2\,\partial_2\partial_0\psi - \frac{1}{2}\partial_2\partial_1 w_1 - \partial_3\partial_0 S_{23} - \frac{1}{2}\partial_3\partial_2 w_3 + \frac{1}{2}\partial_3\partial_3 w_2$
03	$-\partial_1\partial_0 S_{13} + \frac{1}{2}\partial_1\partial_1 w_3 - \partial_2\partial_0 S_{23} + \frac{1}{2}\partial_2\partial_2 w_3 - \partial_3\partial_0 S_{33} - 2\,\partial_3\partial_0\psi - \frac{1}{2}\partial_3\partial_1 w_1 - \frac{1}{2}\partial_3\partial_2 w_2$
12	$-\partial_0\partial_0 S_{12} + \frac{1}{2}\partial_1\partial_0 w_2 + \frac{1}{2}\partial_2\partial_0 w_1 - \partial_2\partial_1 S_{11} - \partial_2\partial_1 S_{22} + \partial_2\partial_1\phi - \partial_2\partial_1\psi - \partial_3\partial_1 S_{23} - \partial_3\partial_2 S_{13} + \partial_3\partial_3 S_{12}$
13	$-\partial_0\partial_0 S_{13} + \frac{1}{2}\partial_1\partial_0 w_3 - \partial_2\partial_1 S_{23} + \partial_2\partial_2 S_{13} + \frac{1}{2}\partial_3\partial_0 w_1 - \partial_3\partial_1 S_{11} - \partial_3\partial_1 S_{33} + \partial_3\partial_1\phi - \partial_3\partial_1\psi - \partial_3\partial_2 S_{12}$
23	$-\partial_0\partial_0 S_{23} + \partial_1\partial_1 S_{23} + \frac{1}{2}\partial_2\partial_0 w_3 - \partial_2\partial_1 S_{13} + \frac{1}{2}\partial_3\partial_0 w_2 - \partial_3\partial_1 S_{12} - \partial_3\partial_2 S_{22} - \partial_3\partial_2 S_{33} + \partial_3\partial_2\phi - \partial_3\partial_2\psi$

We then further decompose S_{ij} and w_i as

$S_{ij} = (\nabla_i \nabla_j - 1/3\ \delta_{ij} \nabla^2) S + (\nabla_i S_j + \nabla_j S_i) + S_{ij}{}^T$

$w_i = \nabla_i E + E_i$

with conditions

$\nabla^i S_i = 0,\ \nabla^i S_{ij}{}^T = 0,\ \nabla^i E_i = 0$

Taking δG_{01} as an example, we see that it will consist of scalars, vectors, and tensor components. According to SVT, we equate each spin component to each spin component of δT_{01}, i.e.

$\delta G_{01}{}^{(S)} = \delta T_{01}{}^{(S)},\ \delta G_{01}{}^{(V)} = \delta T_{01}{}^{(V)},\ \delta G_{01}{}^{(T)} = \delta T_{01}{}^{(T)}$