SVT4 dS$_4$ Conformal Einstein

1 Background and Fluctuations

$$G^{(0)}_{\mu\nu} = 3H^2 g_{\mu\nu} \tag{1.1}$$

$$R^{(0)}_{\lambda\mu\nu\kappa} = H^2(g_{\mu\nu}g_{\lambda\kappa} - g_{\lambda\nu}g_{\mu\kappa}), \qquad R^{(0)}_{\mu\kappa} = -3H^2 g_{\mu\kappa}, \qquad R^{(0)} = -12H^2, \tag{1.2}$$

$$ds^2 = \Omega^2(\tau)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu, \qquad \Omega(\tau) = \frac{1}{H\tau} = e^{Ht} \tag{1.3}$$

$$\tilde{g}_{\mu\nu} = \mathrm{diag}(-1,1,1,1) \quad \text{or} \quad \mathrm{diag}(-1,1,r^2,r^2\sin^2\theta) \tag{1.4}$$

$$f_{\mu\nu} = -2\tilde{g}_{\mu\nu}\chi + 2\tilde{\nabla}_\mu\tilde{\nabla}_\nu F + \tilde{\nabla}_\mu F_\nu + \tilde{\nabla}_\nu F_\mu + 2F_{\mu\nu} \tag{1.5}$$

$$\tilde{\nabla}_\mu\Omega = \dot{\Omega}\delta^0_\mu, \qquad \Omega = \frac{1}{H\tau}, \qquad \dot{\Omega} = -\frac{1}{H\tau^2}, \qquad \ddot{\Omega} = \frac{2}{H\tau^3} \tag{1.6}$$

$$\delta G_{00} = -6\dot{\chi}\tau^{-1} - 2\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{F} - 2\tilde{\nabla}_a\tilde{\nabla}^a\chi - 2\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a F_0 - \ddot{F}_{00} - 2\dot{F}_{00}\tau^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a F_{00} \tag{1.7}$$

$$\begin{aligned}\delta G_{0i} = {} & -2\tau^{-1}\tilde{\nabla}_i\ddot{F} + 6\tau^{-2}\tilde{\nabla}_i\dot{F} - 2\tilde{\nabla}_i\dot{\chi} - 2\tau^{-1}\tilde{\nabla}_i\chi + 3\dot{F}_i\tau^{-2} - 2\tau^{-1}\tilde{\nabla}_i\dot{F}_0 + 3\tau^{-2}\tilde{\nabla}_i F_0 - \ddot{F}_{0i} \\ & +6F_{0i}\tau^{-2} + \tilde{\nabla}_a\tilde{\nabla}^a F_{0i} - 2\tau^{-1}\tilde{\nabla}_i F_{00}\end{aligned} \tag{1.8}$$

$$\begin{aligned}\delta G_{ij} = {} & -2\ddot{\chi}\tilde{g}_{ij} + 6\ddot{F}\tilde{g}_{ij}\tau^{-2} - 2\dddot{F}\tilde{g}_{ij}\tau^{-1} + 2\dot{\chi}\tilde{g}_{ij}\tau^{-1} + 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{F} + 2\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\chi - 2\tau^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{F} \\ & +6\tau^{-2}\tilde{\nabla}_j\tilde{\nabla}_i F - 2\tilde{\nabla}_j\tilde{\nabla}_i\chi + 6\dot{F}_0\tilde{g}_{ij}\tau^{-2} - 2\ddot{F}_0\tilde{g}_{ij}\tau^{-1} + 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a F_0 + 3\tau^{-2}\tilde{\nabla}_i F_j \\ & +3\tau^{-2}\tilde{\nabla}_j F_i - 2\tau^{-1}\tilde{\nabla}_j\tilde{\nabla}_i F_0 - \ddot{F}_{ij} + 6F_{ij}\tau^{-2} + 6F_{00}\tilde{g}_{ij}\tau^{-2} + 2\dot{F}_{ij}\tau^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a F_{ij} \\ & -2\tau^{-1}\tilde{\nabla}_i F_{0j} - 2\tau^{-1}\tilde{\nabla}_j F_{0i}\end{aligned} \tag{1.9}$$

$$\begin{aligned}g^{\mu\nu}\delta G_{\mu\nu} = {} & 18H^2\ddot{F} - 6H^2\dddot{F}\tau + 12H^2\dot{\chi}\tau - 6H^2\ddot{\chi}\tau^2 + 6H^2\tau\tilde{\nabla}_a\tilde{\nabla}^a\dot{F} + 6H^2\tilde{\nabla}_a\tilde{\nabla}^a F \\ & +6H^2\tau^2\tilde{\nabla}_a\tilde{\nabla}^a\chi + 24H^2\dot{F}_0 - 6H^2\ddot{F}_0\tau + 6H^2\tau\tilde{\nabla}_a\tilde{\nabla}^a F_0 + 24H^2 F_{00}\end{aligned} \tag{1.10}$$

$$-\kappa_4^2\delta T_{00} = 6\ddot{F}\tau^{-2} + 6\tau^{-2}\chi + 6\dot{F}_0\tau^{-2} + 6F_{00}\tau^{-2} \tag{1.11}$$

$$-\kappa_4^2\delta T_{0i} = 6\tau^{-2}\tilde{\nabla}_i\dot{F} + 3\dot{F}_i\tau^{-2} + 3\tau^{-2}\tilde{\nabla}_i F_0 + 6F_{0i}\tau^{-2} \tag{1.12}$$

$$-\kappa_4^2\delta T_{ij} = -6\tilde{g}_{ij}\tau^{-2}\chi + 6\tau^{-2}\tilde{\nabla}_j\tilde{\nabla}_i F + 3\tau^{-2}\tilde{\nabla}_i F_j + 3\tau^{-2}\tilde{\nabla}_j F_i + 6F_{ij}\tau^{-2} \tag{1.13}$$

$$-\kappa_4^2 g^{\mu\nu}\delta T_{\mu\nu} = -6H^2\ddot{F} - 24H^2\chi + 6H^2\tilde{\nabla}_a\tilde{\nabla}^a F \tag{1.14}$$

2 Field Equations

$$\Delta_{\mu\nu} \equiv \delta G_{\mu\nu} + \kappa_4^2 \delta T_{\mu\nu} = 0 \tag{2.1}$$

$$\begin{aligned}\Delta_{00} &= -6\ddot{F}\tau^{-2} - 6\dot{\chi}\tau^{-1} - 6\tau^{-2}\chi - 2\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{F} - 2\tilde{\nabla}_a\tilde{\nabla}^a\chi - 6\dot{F}_0\tau^{-2} - 2\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a F_0 - \ddot{F}_{00} \\ &\quad -6F_{00}\tau^{-2} - 2\dot{F}_{00}\tau^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a F_{00}\end{aligned} \tag{2.2}$$

$$\Delta_{0i} = -2\tau^{-1}\tilde{\nabla}_i\ddot{F} - 2\tilde{\nabla}_i\dot{\chi} - 2\tau^{-1}\tilde{\nabla}_i\chi - 2\tau^{-1}\tilde{\nabla}_i\dot{F}_0 - \ddot{F}_{0i} + \tilde{\nabla}_a\tilde{\nabla}^a F_{0i} - 2\tau^{-1}\tilde{\nabla}_i F_{00} \tag{2.3}$$

$$\begin{aligned}\Delta_{ij} &= -2\ddot{\chi}\tilde{g}_{ij} + 6\ddot{F}\tilde{g}_{ij}\tau^{-2} - 2\dddot{F}\tilde{g}_{ij}\tau^{-1} + 2\dot{\chi}\tilde{g}_{ij}\tau^{-1} + 6\tilde{g}_{ij}\tau^{-2}\chi + 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{F} + 2\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\chi \\ &\quad -2\tau^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{F} - 2\tilde{\nabla}_j\tilde{\nabla}_i\chi + 6\dot{F}_0\tilde{g}_{ij}\tau^{-2} - 2\ddot{F}_0\tilde{g}_{ij}\tau^{-1} + 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a F_0 \\ &\quad -2\tau^{-1}\tilde{\nabla}_j\tilde{\nabla}_i F_0 - \ddot{F}_{ij} + 6F_{00}\tilde{g}_{ij}\tau^{-2} + 2\dot{F}_{ij}\tau^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a F_{ij} - 2\tau^{-1}\tilde{\nabla}_i F_{0j} - 2\tau^{-1}\tilde{\nabla}_j F_{0i}\end{aligned} \tag{2.4}$$

$$\begin{aligned}g^{\mu\nu}\Delta_{\mu\nu} &= 24H^2\ddot{F} - 6H^2\dddot{F}\tau + 12H^2\dot{\chi}\tau - 6H^2\ddot{\chi}\tau^2 + 24H^2\chi + 6H^2\tau\tilde{\nabla}_a\tilde{\nabla}^a\dot{F} \\ &\quad +6H^2\tau^2\tilde{\nabla}_a\tilde{\nabla}^a\chi + 24H^2\dot{F}_0 - 6H^2\ddot{F}_0\tau + 6H^2\tau\tilde{\nabla}_a\tilde{\nabla}^a F_0 + 24H^2 F_{00}\end{aligned} \tag{2.5}$$

3 Field Equations (G.I. Form)

$$\alpha = \dot{F} + F_0 - \dot{\Omega}^{-1}\chi\Omega = \dot{F} + F_0 + \tau\chi \tag{3.1}$$

$$\Delta_{00} = -6\dot{\alpha}\tau^{-2} - 2\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\alpha - \ddot{F}_{00} - 6F_{00}\tau^{-2} - 2\dot{F}_{00}\tau^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a F_{00} \tag{3.2}$$

$$\Delta_{0i} = -2\tau^{-1}\tilde{\nabla}_i\dot{\alpha} - \ddot{F}_{0i} + \tilde{\nabla}_a\tilde{\nabla}^a F_{0i} - 2\tau^{-1}\tilde{\nabla}_i F_{00} \tag{3.3}$$

$$\begin{aligned}\Delta_{ij} &= 6\dot{\alpha}\tilde{g}_{ij}\tau^{-2} - 2\ddot{\alpha}\tilde{g}_{ij}\tau^{-1} + 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\alpha - 2\tau^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\alpha - \ddot{F}_{ij} + 6F_{00}\tilde{g}_{ij}\tau^{-2} + 2\dot{F}_{ij}\tau^{-1} \\ &\quad +\tilde{\nabla}_a\tilde{\nabla}^a F_{ij} - 2\tau^{-1}\tilde{\nabla}_i F_{0j} - 2\tau^{-1}\tilde{\nabla}_j F_{0i}\end{aligned} \tag{3.4}$$

$$g^{\mu\nu}\Delta_{\mu\nu} = 24H^2\dot{\alpha} - 6H^2\ddot{\alpha}\tau + 6H^2\tau\tilde{\nabla}_a\tilde{\nabla}^a\alpha + 24H^2F_{00} \tag{3.5}$$

4 Covariant Conservation

(4.1)

$$\begin{aligned}\nabla_\mu\Delta^{\mu\nu} &= \tilde{g}^{\alpha\gamma}\tilde{g}^{\nu\beta}\Omega^{-4}\tilde{\nabla}_\alpha\Delta_{\beta\gamma} + 2\tilde{g}^{\alpha\gamma}\tilde{g}^{\nu\beta}\Delta_{\alpha\beta}\Omega^{-5}\tilde{\nabla}_\gamma\Omega - \tilde{g}^{\nu\gamma}\tilde{g}^{\alpha\beta}\Delta_{\alpha\beta}\Omega^{-5}\tilde{\nabla}_\gamma\Omega \\ &= \Omega^{-4}(\tilde{g}^{\nu\beta}\tilde{\nabla}^\alpha\Delta_{\alpha\beta} - 2\Omega^{-1}\dot{\Omega}\tilde{g}^{\nu\beta}\Delta_{0\beta} - \Omega^{-1}\dot{\Omega}\tilde{g}^{\nu0}\tilde{g}^{\alpha\beta}\Delta_{\alpha\beta})\end{aligned} \tag{4.2}$$

$$
\begin{aligned}
\dot{F}_{00} &= \tilde{\nabla}^a F_{0a}, \qquad \dot{F}_{0i} = \tilde{\nabla}^j F_{ij} && (4.3)
\end{aligned}
$$

$$
\nu = 0 \tag{4.4}
$$

$$
\begin{aligned}
\Longrightarrow\ 0 &\overset{!}{=} -\tilde{\nabla}^\alpha \Delta_{\alpha 0} + 2\Omega^{-1}\dot{\Omega}\Delta_{00} + \Omega\dot{\Omega}\tilde{g}^{\alpha\beta}\Delta_{\alpha\beta} \\
&= \dot{\Delta}_{00} - \tilde{\nabla}^i \Delta_{0i} - \frac{2}{\tau}\Delta_{00} - \frac{1}{H^2\tau^3} g^{\alpha\beta}\Delta_{\alpha\beta} \\
&= 0 && (4.5)
\end{aligned}
$$

$$
\nu = i \tag{4.6}
$$

$$
\begin{aligned}
\Longrightarrow\ 0 &\overset{!}{=} -\dot{\Delta}_{0i} + \tilde{\nabla}^j \Delta_{ij} - 2\Omega^{-1}\dot{\Omega}\Delta_{0i} \\
&= -\dot{\Delta}_{0i} + \tilde{\nabla}^j \Delta_{ij} + \frac{2}{\tau}\Delta_{0i} \\
&= 0 && (4.7)
\end{aligned}
$$

Appendix A Gauge Invariants

A.1 dS_4

$$x'^{\mu} = x^{\mu} - \epsilon^{\mu}(x) \tag{A.1}$$

$$h'_{\mu\nu} = h_{\mu\nu} + \nabla_{\mu}\epsilon_{\nu} + \nabla_{\nu}\epsilon_{\mu} \tag{A.2}$$

$$\begin{aligned} \delta R'_{\mu\nu} &= \delta R_{\mu\nu} + R^{(0)}_{\lambda\nu}\nabla_{\mu}\epsilon^{\lambda} + R^{(0)}_{\lambda\mu}\nabla_{\nu}\epsilon^{\lambda} + \nabla_{\lambda}R^{(0)}_{\mu\nu}\epsilon^{\lambda} \\ &= \delta R_{\mu\nu} - 3H^2(\nabla_{\mu}\epsilon_{\nu} + \nabla_{\nu}\epsilon_{\mu}) \end{aligned} \tag{A.3}$$

$$\delta R'_{\mu\nu} + 3H^2 h'_{\mu\nu} = \delta R_{\mu\nu} + 3H^2 h_{\mu\nu} \tag{A.4}$$

$$g^{\mu\nu}\delta R'_{\mu\nu} + 3H^2 h' = g^{\mu\nu}\delta R_{\mu\nu} + 3H^2 h \tag{A.5}$$

Evaluating (A.4) and (A.5) in terms of $f_{\mu\nu}$,

$$\begin{aligned} (\delta R_{\mu\nu} + 3H^2 h_{\mu\nu}) = {} & -6H^2\ddot{F}g_{\mu\nu}(1-H\tau)^{-2} + 4H\dot{\chi}g_{\mu\nu}(1-H\tau)^{-1} - 6H^2 g_{\mu\nu}(1-H\tau)^{-2}\chi \\ & + Hg_{\mu\nu}(1-H\tau)^{-1}\nabla_{\alpha}\nabla^{\alpha}\dot{F} - g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}\chi - 2H(1-H\tau)^{-1}U_{\nu}\nabla_{\mu}\chi \\ & - 2H(1-H\tau)^{-1}U_{\mu}\nabla_{\nu}\chi + 2H(1-H\tau)^{-1}\nabla_{\nu}\nabla_{\mu}\dot{F} - 2\nabla_{\nu}\nabla_{\mu}\chi \\ & - 6H^2\dot{F}^{\alpha}g_{\mu\nu}(1-H\tau)^{-2}U_{\alpha} + Hg_{\mu\nu}(1-H\tau)^{-1}U^{\alpha}\nabla_{\beta}\nabla^{\beta}F_{\alpha} \\ & + 2H(1-H\tau)^{-1}U^{\alpha}\nabla_{\nu}\nabla_{\mu}F_{\alpha} - 2H\dot{F}_{\mu\nu}(1-H\tau)^{-1} - 6H^2F_{\alpha\beta}g_{\mu\nu}(1-H\tau)^{-2}U^{\alpha}U^{\beta} \\ & + \nabla_{\alpha}\nabla^{\alpha}F_{\mu\nu} + 2H(1-H\tau)^{-1}U^{\alpha}\nabla_{\mu}F_{\nu\alpha} + 2H(1-H\tau)^{-1}U^{\alpha}\nabla_{\nu}F_{\mu\alpha} \end{aligned} \tag{A.6}$$

$$\begin{aligned} (g^{\mu\nu}\delta R_{\mu\nu} + 3H^2 h) &= -\Delta \\ &= -24H^2\ddot{F} + 12H\dot{\chi} - 12H^2\dot{\chi}\tau - 24H^2(1-H\tau)^{-2}\chi + 48H^3\tau(1-H\tau)^{-2}\chi \\ & \quad - 24H^4\tau^2(1-H\tau)^{-2}\chi + 6H\nabla_{\alpha}\nabla^{\alpha}\dot{F} - 6H^2\tau\nabla_{\alpha}\nabla^{\alpha}\dot{F} - 6H^2\nabla_{\alpha}\nabla^{\alpha}F \\ & \quad + 6H^2(1-H\tau)^{-2}\nabla_{\alpha}\nabla^{\alpha}F - 12H^3\tau(1-H\tau)^{-2}\nabla_{\alpha}\nabla^{\alpha}F + 6H^4\tau^2(1-H\tau)^{-2}\nabla_{\alpha}\nabla^{\alpha}F \\ & \quad - 6\nabla_{\alpha}\nabla^{\alpha}\chi + 12H\tau\nabla_{\alpha}\nabla^{\alpha}\chi - 6H^2\tau^2\nabla_{\alpha}\nabla^{\alpha}\chi - 24H^2\dot{F}^{\alpha}U_{\alpha} \\ & \quad + 6HU^{\alpha}\nabla_{\beta}\nabla^{\beta}F_{\alpha} - 6H^2\tau U^{\alpha}\nabla_{\beta}\nabla^{\beta}F_{\alpha} - 24H^2F_{\alpha\beta}U^{\alpha}U^{\beta} \end{aligned} \tag{A.7}$$

Here $(g^{\mu\nu}\delta R_{\mu\nu} + 3H^2 h)$ is the fundamental scalar gauge invariant, which includes vector and tensor contributions. Unlike the flat space case shown below, we cannot reduce Δ into combinations of the pure scalars F and χ that are separate from F_{α} and $F_{\mu\nu}$.

A.2 $R^{(0)}_{\mu\nu} = 0$ (Flat)

$$x'^{\mu} = x^{\mu} - \epsilon^{\mu}(x) \tag{A.8}$$

$$h'_{\mu\nu} = h_{\mu\nu} + \nabla_{\mu}\epsilon_{\nu} + \nabla_{\nu}\epsilon_{\mu} \tag{A.9}$$

$$\delta R'_{\mu\nu} = \delta R_{\mu\nu}, \qquad g^{\mu\nu}\delta R'_{\mu\nu} = g^{\mu\nu}\delta R_{\mu\nu} \tag{A.10}$$

$$\delta R_{\mu\nu} = \nabla_{\alpha}\nabla^{\alpha}F_{\mu\nu} - g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}\chi - 2\nabla_{\nu}\nabla_{\mu}\chi \tag{A.11}$$

$$g^{\mu\nu}\delta R_{\mu\nu} = -6\nabla_{\alpha}\nabla^{\alpha}\chi \tag{A.12}$$

$$\begin{aligned}\delta G_{\mu\nu} &= \delta R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(g^{\alpha\beta}\delta R_{\alpha\beta}) \\ &= \nabla_\alpha \nabla^\alpha F_{\mu\nu} + 2 g_{\mu\nu} \nabla_\alpha \nabla^\alpha \chi - 2\nabla_\nu \nabla_\mu \chi \end{aligned} \tag{A.13}$$

For a χ that vanishes asymptotically, we can define a gauge invariant χ as

$$\chi = -6 \int D g^{\mu\nu} \delta R_{\mu\nu}. \tag{A.14}$$

Since δG is gauge invariant, all SVT scalars arising within δG can be entirely expressed in terms of derivatives onto a single χ.

Appendix B SVT Gauge Transformations (Incomplete)

$$x'^\mu = x^\mu - \epsilon^\mu(x), \qquad \epsilon_\mu = \Omega^2 \ell_\mu, \qquad \ell_\mu = L_\mu + \tilde{\nabla}_\mu L \tag{B.1}$$

$$\nabla_\mu \epsilon_\nu = \Omega^2 \tilde{\nabla}_\mu \ell_\nu - \frac{1}{2}\left(\ell_\mu \tilde{\nabla}_\nu - \ell_\nu \tilde{\nabla}_\mu - \tilde{g}_{\mu\nu} \ell^\rho \tilde{\nabla}_\rho\right)\Omega^2$$

$$\begin{aligned} f'_{\mu\nu} &= f_{\mu\nu} + \tilde{\nabla}_\mu \ell_\nu + \tilde{\nabla}_\nu \ell_\mu + 2\Omega^{-1} \tilde{g}_{\mu\nu} \ell^\rho \tilde{\nabla}_\rho \Omega \\ &= f_{\mu\nu} + \tilde{\nabla}_\mu \ell_\nu + \tilde{\nabla}_\nu \ell_\mu + 2\tau^{-1} \tilde{g}_{\mu\nu} U^\rho \ell_\rho \end{aligned} \tag{B.2}$$

$$\chi = \frac{1}{6}\left(\tilde{\nabla}^\sigma W_\sigma - f\right) \tag{B.3}$$

$$F = \frac{1}{6}\int D(4\tilde{\nabla}^\sigma W_\sigma - f) \tag{B.4}$$

$$F_\mu = W_\mu - \tilde{\nabla}_\mu \int D \tilde{\nabla}^\sigma W_\sigma \tag{B.5}$$

$$2F_{\mu\nu} = h_{\mu\nu} - \tilde{\nabla}_\mu W_\nu - \tilde{\nabla}_\nu W_\mu + \frac{2}{3}\tilde{\nabla}_\mu \tilde{\nabla}_\nu \int D\tilde{\nabla}^\sigma W_\sigma + \frac{g_{\mu\nu}}{3}(\tilde{\nabla}^\sigma W_\sigma - h) + \frac{\tilde{\nabla}_\mu \tilde{\nabla}_\nu}{3}\int Df \tag{B.6}$$

$$\begin{aligned}\tilde{\nabla}^\mu f'_{\mu\nu} &= \tilde{\nabla}^\mu f_{\mu\nu} + \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha L_\nu + 2\tilde{\nabla}_\alpha \tilde{\nabla}^\alpha \tilde{\nabla}_\nu L + 2\tau^{-2} U_\nu U^\alpha L_\alpha + 2\tau^{-1} U^\alpha \nabla_\nu L_\alpha \\ &\quad + 2\tau^{-2} U_\nu \dot{L} + 2\tau^{-1} \nabla_\nu \dot{L}\end{aligned} \tag{B.7}$$

$$W'_\nu = W_\nu + L_\nu + 2\tilde{\nabla}_\nu L + \int D(x - x')(2\tau^{-2} U_\nu U^\alpha L_\alpha + 2\tau^{-1} U^\alpha \nabla_\nu L_\alpha + 2\tau^{-2} U_\nu \dot{L} + 2\tau^{-1} \nabla_\nu \dot{L}) \tag{B.8}$$