S.V.T. Decomposition in Conformal to Flat Space

$$\begin{array}{llll} h_{\theta\theta} &=& -2\phi \\ \\ h_{\theta\,\,\mathbf{i}} &=& w_{\mathbf{i}} &=& \partial_{\mathbf{i}} B \; - \; B_{\mathbf{i}} \\ \\ h_{\mathbf{i}\,\mathbf{j}} &=& -2\psi \; + \; S_{\mathbf{i}\,\mathbf{j}} \; = \; -2\psi \; + \; 2\partial_{\mathbf{i}}\partial_{\mathbf{j}} E \; + \; \partial_{\mathbf{i}} E_{\mathbf{j}} \; + \; \partial_{\mathbf{j}} E_{\mathbf{i}} \; + \; 2E_{\mathbf{i}\,\mathbf{j}} \\ \\ \text{where} \\ \\ \partial_{\mathbf{i}} B^{\mathbf{i}} &=& \partial_{\mathbf{i}} E^{\mathbf{i}} \; = \; \emptyset \\ \\ \partial_{\mathbf{i}} E^{\mathbf{i}\,\mathbf{j}} &=& \emptyset \\ \\ \delta_{\mathbf{i}\,\mathbf{j}} E^{\mathbf{i}\,\mathbf{j}} &=& \emptyset \end{array}$$

(Same as Ellis Relativistic Cosmology, apart from $E_{\rm ij}$)

Ω =1 Gauge invariant quantities:

$$\begin{array}{llll} \psi &=& \psi \\ \\ \mathcal{P} &=& \phi &+& \partial_{\theta} B &-& \partial_{\theta} \partial_{\theta} E \\ \\ \mathcal{F}_{\mathbf{i}} &=& \partial_{\theta} E_{\mathbf{i}} &+& B_{\mathbf{i}} \\ \\ E_{\mathbf{i}\mathbf{j}} &=& E_{\mathbf{i}\mathbf{j}} \end{array}$$

Ω = $\Omega(t)$ Gauge invariant quantities are given in Ellis



$\Omega = 1$

00	$-2 abla^2\psi$
11	$ (-2\partial_{\theta}\partial_{\theta}-\partial_{1}\partial_{1}+\nabla^{2})\psi - (\nabla^{2}-\partial_{1}\partial_{1})(\phi+\partial_{\theta}B-\partial_{\theta}\partial_{\theta}E) - \partial_{1}\partial_{\theta}(B_{1}+\partial_{\theta}E_{1}) + \Box E_{11} $
22	$ (-2\partial_{\theta}\partial_{\theta} - \partial_{2}\partial_{2} + \nabla^{2}) \psi - (\nabla^{2} - \partial_{2}\partial_{2}) (\phi + \partial_{\theta}B - \partial_{\theta}\partial_{\theta}E) - \partial_{2}\partial_{\theta} (B_{2} + \partial_{\theta}E_{2}) + \Box E_{22} $
33	$ (-2\partial_{\theta}\partial_{\theta} - \partial_{3}\partial_{3} + \nabla^{2}) \psi - (\nabla^{2} - \partial_{3}\partial_{3}) (\phi + \partial_{\theta}B - \partial_{\theta}\partial_{\theta}E) - \partial_{3}\partial_{\theta} (B_{3} + \partial_{\theta}E_{3}) + \Box E_{33} $
01	$-2 \partial_{1} \partial_{0} \psi + \frac{1}{2} \nabla^{2} (\mathbf{B_{1}} + \partial_{0} \mathbf{E_{1}})$
02	$-2\partial_2\partial_\theta\psi + \frac{1}{2}\nabla^2(B_2 + \partial_\theta E_2)$
03	$-2\partial_3\partial_{\theta}\psi\ +\ \frac{1}{2}\nabla^2\left(B_3+\partial_{\theta}E_3\right)$
12	$-\partial_2\partial_1\psi + \partial_2\partial_1(\phi + \partial_0B - \partial_0\partial_0E) - \frac{1}{2}\partial_1\partial_0(\partial_0E_2 + B_2) - \frac{1}{2}\partial_2\partial_0(\partial_0E_1 + B_1) + \Box E_{12}$
13	$-\partial_3\partial_1\psi + \partial_3\partial_1(\phi + \partial_0B - \partial_0\partial_0E) - \frac{1}{2}\partial_1\partial_0(\partial_0E_3 + B_3) - \frac{1}{2}\partial_3\partial_0(\partial_0E_1 + B_1) + \Box E_{13}$
23	$-\partial_3\partial_2\psi + \partial_3\partial_2\left(\phi + \partial_0B - \partial_0\partial_0E\right) - \frac{1}{2}\partial_2\partial_0\left(\partial_0E_3 + B_3\right) - \frac{1}{2}\partial_3\partial_0\left(\partial_0E_2 + B_2\right) + \Box E_{23}$

$\Omega = \Omega(t)$

00	$6 \frac{\Omega'}{\Omega} \partial_{\theta} \psi - 2 \nabla^{2} \psi + 2 \frac{\Omega'}{\Omega} \nabla^{2} (B - \partial_{\theta} E)$
11	$-2\partial_{\boldsymbol{\theta}}\partial_{\boldsymbol{\theta}}\psi \ - \ 2\frac{\Omega'}{\Omega}\partial_{\boldsymbol{\theta}}\left(\phi + 2\psi + E_{11}\right) \ + \ 2\left[\left(\frac{\Omega'}{\Omega}\right)^2 - 2\frac{\Omega''}{\Omega}\right]\left(\phi + \psi - \partial_{1}\partial_{1}E - \partial_{1}E_{1} - E_{11}\right) \ -$
	$ (\nabla^2 - \partial_1 \partial_1) \ (\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E) \ - \ 2 \frac{\Omega'}{\Omega} \ (\nabla^2 - \partial_1 \partial_1) \ (B - \partial_\theta E) \ - \ (\partial_1 \partial_\theta + 2 \frac{\Omega'}{\Omega} \partial_1) \ (B_1 + \partial_\theta E_1) \ + \ \Box E_{11} $
22	$-2\partial_{\theta}\partial_{\theta}\psi - 2\frac{\Omega'}{\Omega}\partial_{\theta}\left(\phi + 2\psi + E_{22}\right) + 2\left[\left(\frac{\Omega'}{\Omega}\right)^{2} - 2\frac{\Omega''}{\Omega}\right]\left(\phi + \psi - \partial_{2}\partial_{2}E - \partial_{2}E_{2} - E_{22}\right) -$
	$ (\nabla^2 - \partial_2 \partial_2) \ (\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E) \ - \ 2 \frac{\Omega'}{\Omega} \ (\nabla^2 - \partial_2 \partial_2) \ (B - \partial_\theta E) \ - \ (\partial_2 \partial_\theta + 2 \frac{\Omega'}{\Omega} \partial_2) \ (B_2 + \partial_\theta E_2) \ + \ \Box E_{22} $
33	$-2\partial_{\theta}\partial_{\theta}\psi - 2\frac{\Omega'}{\Omega}\partial_{\theta}\left(\phi + 2\psi + E_{33}\right) + 2\left[\left(\frac{\Omega'}{\Omega}\right)^{2} - 2\frac{\Omega''}{\Omega}\right]\left(\phi + \psi - \partial_{3}\partial_{3}E - \partial_{3}E_{3} - E_{33}\right) -$
	$ (\nabla^2 - \partial_3 \partial_3) \ (\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E) \ - \ 2 \frac{\Omega'}{\Omega} \ (\nabla^2 - \partial_3 \partial_3) \ (B - \partial_\theta E) \ - \ (\partial_3 \partial_\theta + 2 \frac{\Omega'}{\Omega} \partial_3) \ (B_3 + \partial_\theta E_3) \ + \ \Box E_{33} $
01	$-2\partial_{1}\partial_{0}\psi - 2\frac{\Omega'}{\Omega}\partial_{1}\phi - \left[\left(\frac{\Omega'}{\Omega}\right)^{2} - 2\frac{\Omega''}{\Omega}\right]\left(\partial_{1}B - B_{1}\right) - \frac{1}{2}\nabla^{2}\left(B_{1} + \partial_{0}E_{1}\right)$
02	$-2\partial_2\partial_\theta\psi - 2\frac{\Omega'}{\Omega}\partial_2\phi - \left[\left(\frac{\Omega'}{\Omega}\right)^2 - 2\frac{\Omega''}{\Omega}\right]\left(\partial_2B - B_2\right) - \frac{1}{2}\nabla^2\left(B_2 + \partial_\theta E_2\right)$
03	$-2\partial_{3}\partial_{\theta}\psi - 2\frac{\Omega'}{\Omega}\partial_{3}\phi - \left[\left(\frac{\Omega'}{\Omega}\right)^{2} - 2\frac{\Omega''}{\Omega}\right]\left(\partial_{3}B - B_{3}\right) - \frac{1}{2}\nabla^{2}\left(B_{3} + \partial_{\theta}E_{3}\right)$
12	$\partial_1 \partial_2 \left(\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E \right) -$
	$ (\frac{1}{2}\partial_{\theta} + \frac{\Omega'}{\Omega}) \ (\partial_{1}B_{2} + \partial_{1}\partial_{\theta}E_{2} + \partial_{2}B_{1} + \partial_{2}\partial_{\theta}E_{1}) \ - \ [\ (\frac{\Omega'}{\Omega})^{2} - 2\frac{\Omega''}{\Omega}\] \ (\partial_{1}E_{2} + \partial_{2}E_{1} - 2\partial_{1}\partial_{2}E) \ + \ \BoxE_{12} $
13	$\partial_1 \partial_3 \left(\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E \right) -$
	$(\frac{1}{2}\partial_{\theta} + \frac{\Omega'}{\Omega}) \ (\partial_{1}B_{3} + \partial_{1}\partial_{\theta}E_{3} + \partial_{3}B_{1} + \partial_{3}\partial_{\theta}E_{1}) \ - \ [\ (\frac{\Omega'}{\Omega})^{2} - 2\frac{\Omega''}{\Omega}\] \ (\partial_{1}E_{3} + \partial_{3}E_{1} - 2\partial_{1}\partial_{3}E) \ + \ \Box E_{13}$
23	$\partial_2\partial_3\left(\phi-\psi+\partial_\thetaB-\partial_\theta\partial_\thetaE\right)$ –
	$(\frac{1}{2}\partial_{\theta} + \frac{\Omega'}{\Omega}) \ (\partial_{2}B_{3} + \partial_{2}\partial_{\theta}E_{3} + \partial_{3}B_{2} + \partial_{3}\partial_{\theta}E_{2}) \ - \ [\ (\frac{\Omega'}{\Omega})^{2} - 2\frac{\Omega''}{\Omega}\] \ (\partial_{2}E_{3} + \partial_{3}E_{2} - 2\partial_{2}\partial_{3}E) \ + \ \Box E_{23}$

$\delta(G^{\mu}_{v})$

$\Omega = \Omega(t)$

00	$-6\frac{\Omega'}{\Omega}\partial_{\theta}\psi + 2\nabla^{2}\psi - 6\left(\frac{\Omega'}{\Omega}\right)^{2}\phi - 2\frac{\Omega'}{\Omega}\nabla^{2}\left(B - \partial_{\theta}E\right)$
11	$-2\partial_{\boldsymbol{\theta}}\partial_{\boldsymbol{\theta}}\psi \ -\ 2\frac{\Omega'}{\Omega}\partial_{\boldsymbol{\theta}}\left(\phi+2\psi+E_{11}\right) \ +\ 2\left[\left(\frac{\Omega'}{\Omega}\right)^{2}-2\frac{\Omega''}{\Omega}\right]\phi \ -$
	$ (\nabla^2 - \partial_1 \partial_1) \ (\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E) \ - \ 2 \frac{\Omega'}{\Omega} \ (\nabla^2 - \partial_1 \partial_1) \ (B - \partial_\theta E) \ - \ (\partial_1 \partial_\theta + 2 \frac{\Omega'}{\Omega} \partial_1) \ (B_1 + \partial_\theta E_1) \ + \ \Box E_{11} $
22	$-2\partial_{\boldsymbol{\theta}}\partial_{\boldsymbol{\theta}}\psi - 2\frac{\Omega'}{\Omega}\partial_{\boldsymbol{\theta}}\left(\phi + 2\psi + E_{22}\right) + 2\left[\left(\frac{\Omega'}{\Omega}\right)^{2} - 2\frac{\Omega''}{\Omega}\right]\phi -$
	$ (\nabla^2 - \partial_2 \partial_2) \ (\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E) \ - \ 2 \frac{\Omega'}{\Omega} \ (\nabla^2 - \partial_2 \partial_2) \ (B - \partial_\theta E) \ - \ (\partial_2 \partial_\theta + 2 \frac{\Omega'}{\Omega} \partial_2) \ (B_2 + \partial_\theta E_2) \ + \ \Box E_{22} $
33	$-2\partial_{\boldsymbol{\theta}}\partial_{\boldsymbol{\theta}}\psi - 2\frac{\Omega'}{\Omega}\partial_{\boldsymbol{\theta}}\left(\phi + 2\psi + E_{33}\right) + 2\left[\left(\frac{\Omega'}{\Omega}\right)^{2} - 2\frac{\Omega''}{\Omega}\right]\phi -$
	$ (\nabla^2 - \partial_3 \partial_3) \ (\phi - \psi + \partial_\theta \mathbf{B} - \partial_\theta \partial_\theta \mathbf{E}) \ - \ 2 \frac{\Omega'}{\Omega} \ (\nabla^2 - \partial_3 \partial_3) \ (\mathbf{B} - \partial_\theta \mathbf{E}) \ - \ (\partial_3 \partial_\theta + 2 \frac{\Omega'}{\Omega} \partial_3) \ (\mathbf{B}_3 + \partial_\theta \mathbf{E}_3) \ + \ \Box \mathbf{E}_{33} $
01	$2\partial_{1}\partial_{\theta}\psi + 2\frac{\Omega'}{\Omega}\partial_{1}\phi + \frac{1}{2}\nabla^{2}(B_{1}+\partial_{\theta}E_{1})$
02	$2\partial_2\partial_\theta\psi + 2\frac{\Omega'}{\Omega}\partial_2\phi + \frac{1}{2}\nabla^2(B_2 + \partial_\thetaE_2)$
03	$2\partial_3\partial_\theta\psi + 2\frac{\Omega'}{\Omega}\partial_3\phi + \frac{1}{2}\nabla^2(B_3 + \partial_\thetaE_3)$
12	$ \boxed{ \partial_1 \partial_2 \left(\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E \right) - \left(\frac{1}{2} \partial_\theta + \frac{\Omega'}{\Omega} \right) \left(\partial_1 B_2 + \partial_1 \partial_\theta E_2 + \partial_2 B_1 - \partial_2 \partial_\theta E_1 \right) + 2 \frac{\Omega'}{\Omega} \left(\partial_1 \partial_2 B - \partial_1 \partial_2 \partial_\theta E - \partial_\theta E_{12} \right) + \Box E_{12} } $
13	$ \boxed{ \partial_1 \partial_3 \left(\phi - \psi + \partial_\theta B - \partial_\theta \partial_\theta E \right) - \left(\frac{1}{2} \partial_\theta + \frac{\Omega'}{\Omega} \right) \left(\partial_1 B_3 + \partial_1 \partial_\theta E_3 + \partial_3 B_1 - \partial_3 \partial_\theta E_1 \right) + 2 \frac{\Omega'}{\Omega} \left(\partial_1 \partial_3 B - \partial_1 \partial_3 \partial_\theta E - \partial_\theta E_{13} \right) + \Box E_{13} } $
23	$ \boxed{ \partial_2\partial_3\left(\phi - \psi + \partial_\theta B - \partial_\theta\partial_\theta E\right) \ - \ \left(\frac{1}{2}\partial_\theta + \frac{\Omega'}{\Omega}\right)\left(\partial_2 B_3 + \partial_2\partial_\theta E_3 + \partial_3 B_2 - \partial_3\partial_\theta E_2\right) \ + \ 2\frac{\Omega'}{\Omega}\left(\partial_2\partial_3 B - \partial_2\partial_3\partial_\theta E - \partial_\theta E_{23}\right) \ + \ \square E_{23} } $

Compare to Ellis eqs. 10.62-10.65, 10.89-10.91