Weyl SVT v1 Matthew

The Weyl tensor is perturbed according to

$$ds^2 = \Omega^2(x)\big\{-(1+2\phi)d\tau^2 + 2(\tilde{\nabla}_i B + B_i)d\tau dx^i + [(1-2\psi)\gamma_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij}]dx^i dx^j\big\} \quad (1)$$

where

$$\gamma^{ij}\tilde{\nabla}_i B_j = 0, \gamma^{ij}\tilde{\nabla}_i E_j = 0,\ \gamma^{ij}\tilde{\nabla}_i E_{kj} = 0,\ \gamma^{ij}E_{ij} = 0. \quad (2)$$

Covariant derivatives are defined with respect to the 3-space background γ_{ij} and are indicated as $\tilde{\nabla}_i$.

$\Omega(x)$

$$\delta C_{0000} = 0 \quad (3)$$

$$\delta C_{0i00} = 0 \quad (4)$$

$$\delta C^{(S)}_{0i0j} = \Omega^2\bigg[\tfrac{1}{6}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} - \tfrac{1}{6}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + \tfrac{1}{6}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tfrac{1}{6}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi - \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_i\dot{B} + \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} - \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_i\phi - \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_i\psi\bigg]. \quad (5)$$

$$\delta C^{(V)}_{0i0j} = \Omega^2\bigg[-\tfrac{1}{4}\tilde{\nabla}_i\dot{B}_j + \tfrac{1}{4}\tilde{\nabla}_i\ddot{E}_j - \tfrac{1}{4}\tilde{\nabla}_j\dot{B}_i + \tfrac{1}{4}\tilde{\nabla}_j\ddot{E}_i\bigg]. \quad (6)$$

$$\delta C^{(T)}_{0i0j} = \Omega^2\bigg[\tfrac{1}{2}\ddot{E}_{ij} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a E_{ij}\bigg]. \quad (7)$$

$$\delta C^{(S)}_{0ijk} = 0 \quad (8)$$

$$\delta C^{(V)}_{0ijk} = \Omega^2\bigg[\tfrac{1}{4}\gamma_{jk}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{4}\gamma_{ik}\tilde{\nabla}_a\tilde{\nabla}^a B_j - \tfrac{1}{4}\gamma_{jk}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i + \tfrac{1}{4}\gamma_{ik}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_j + \tfrac{1}{2}\tilde{\nabla}_k\tilde{\nabla}_i B_j - \tfrac{1}{2}\tilde{\nabla}_k\tilde{\nabla}_i\dot{E}_j - \tfrac{1}{2}\tilde{\nabla}_k\tilde{\nabla}_j B_i + \tfrac{1}{2}\tilde{\nabla}_k\tilde{\nabla}_j\dot{E}_i\bigg]. \quad (9)$$

$$\delta C^{(T)}_{0ijk} = \Omega^2\bigg[-\tilde{\nabla}_i\dot{E}_{jk} + \tilde{\nabla}_j\dot{E}_{ik}\bigg]. \quad (10)$$

$$\delta C^{(S)}_{ijkl} = \Omega^2 \Big[\tfrac{1}{3}\gamma_{ik}\gamma_{jl}\tilde{\nabla}_a\tilde{\nabla}^a \dot{B} - \tfrac{1}{3}\gamma_{ij}\gamma_{kl}\tilde{\nabla}_a\tilde{\nabla}^a \dot{B} - \tfrac{1}{3}\gamma_{ik}\gamma_{jl}\tilde{\nabla}_a\tilde{\nabla}^a \ddot{E} + \tfrac{1}{3}\gamma_{ij}\gamma_{kl}\tilde{\nabla}_a\tilde{\nabla}^a \ddot{E}$$
$$+ \tfrac{1}{3}\gamma_{ik}\gamma_{jl}\tilde{\nabla}_a\tilde{\nabla}^a \phi - \tfrac{1}{3}\gamma_{ij}\gamma_{kl}\tilde{\nabla}_a\tilde{\nabla}^a \phi + \tfrac{1}{3}\gamma_{ik}\gamma_{jl}\tilde{\nabla}_a\tilde{\nabla}^a \psi - \tfrac{1}{3}\gamma_{ij}\gamma_{kl}\tilde{\nabla}_a\tilde{\nabla}^a \psi + \tfrac{1}{2}\gamma_{kl}\tilde{\nabla}_j\tilde{\nabla}_i \dot{B}$$
$$- \tfrac{1}{2}\gamma_{kl}\tilde{\nabla}_j\tilde{\nabla}_i \ddot{E} + \tfrac{1}{2}\gamma_{kl}\tilde{\nabla}_j\tilde{\nabla}_i \phi + \tfrac{1}{2}\gamma_{kl}\tilde{\nabla}_j\tilde{\nabla}_i \psi - \tfrac{1}{2}\gamma_{jl}\tilde{\nabla}_k\tilde{\nabla}_i \dot{B} + \tfrac{1}{2}\gamma_{jl}\tilde{\nabla}_k\tilde{\nabla}_i \ddot{E}$$
$$- \tfrac{1}{2}\gamma_{jl}\tilde{\nabla}_k\tilde{\nabla}_i \phi - \tfrac{1}{2}\gamma_{jl}\tilde{\nabla}_k\tilde{\nabla}_i \psi - \tfrac{1}{2}\gamma_{ik}\tilde{\nabla}_l\tilde{\nabla}_j \dot{B} + \tfrac{1}{2}\gamma_{ik}\tilde{\nabla}_l\tilde{\nabla}_j \ddot{E} - \tfrac{1}{2}\gamma_{ik}\tilde{\nabla}_l\tilde{\nabla}_j \phi$$
$$- \tfrac{1}{2}\gamma_{ik}\tilde{\nabla}_l\tilde{\nabla}_j \psi + \tfrac{1}{2}\gamma_{ij}\tilde{\nabla}_l\tilde{\nabla}_k \dot{B} - \tfrac{1}{2}\gamma_{ij}\tilde{\nabla}_l\tilde{\nabla}_k \ddot{E} + \tfrac{1}{2}\gamma_{ij}\tilde{\nabla}_l\tilde{\nabla}_k \phi + \tfrac{1}{2}\gamma_{ij}\tilde{\nabla}_l\tilde{\nabla}_k \psi \Big]. \tag{11}$$

$$\delta C^{(V)}_{ijkl} = \Omega^2 \Big[\tfrac{1}{4}\gamma_{kl}\tilde{\nabla}_i \dot{B}_j - \tfrac{1}{4}\gamma_{jl}\tilde{\nabla}_i \dot{B}_k - \tfrac{1}{4}\gamma_{kl}\tilde{\nabla}_i \ddot{E}_j + \tfrac{1}{4}\gamma_{jl}\tilde{\nabla}_i \ddot{E}_k + \tfrac{1}{4}\gamma_{kl}\tilde{\nabla}_j \dot{B}_i - \tfrac{1}{4}\gamma_{ik}\tilde{\nabla}_j \dot{B}_l - \tfrac{1}{4}\gamma_{kl}\tilde{\nabla}_j \ddot{E}_i$$
$$+ \tfrac{1}{4}\gamma_{ik}\tilde{\nabla}_j \ddot{E}_l - \tfrac{1}{4}\gamma_{jl}\tilde{\nabla}_k \dot{B}_i + \tfrac{1}{4}\gamma_{ij}\tilde{\nabla}_k \dot{B}_l + \tfrac{1}{4}\gamma_{jl}\tilde{\nabla}_k \ddot{E}_i - \tfrac{1}{4}\gamma_{ij}\tilde{\nabla}_k \ddot{E}_l - \tfrac{1}{4}\gamma_{ik}\tilde{\nabla}_l \dot{B}_j$$
$$+ \tfrac{1}{4}\gamma_{ij}\tilde{\nabla}_l \dot{B}_k + \tfrac{1}{4}\gamma_{ik}\tilde{\nabla}_l \ddot{E}_j - \tfrac{1}{4}\gamma_{ij}\tilde{\nabla}_l \ddot{E}_k \Big]. \tag{12}$$

$$\delta C^{(T)}_{ijkl} = \Omega^2 \Big[- \tfrac{1}{2}\ddot{E}_{kl}\gamma_{ij} + \tfrac{1}{2}\ddot{E}_{jl}\gamma_{ik} + \tfrac{1}{2}\ddot{E}_{ik}\gamma_{jl} - \tfrac{1}{2}\ddot{E}_{ij}\gamma_{kl} + \tfrac{1}{2}\gamma_{kl}\tilde{\nabla}_a\tilde{\nabla}^a E_{ij} - \tfrac{1}{2}\gamma_{jl}\tilde{\nabla}_a\tilde{\nabla}^a E_{ik}$$
$$- \tfrac{1}{2}\gamma_{ik}\tilde{\nabla}_a\tilde{\nabla}^a E_{jl} + \tfrac{1}{2}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a E_{kl} - \tilde{\nabla}_j\tilde{\nabla}_i E_{kl} + \tilde{\nabla}_k\tilde{\nabla}_i E_{jl} + \tilde{\nabla}_l\tilde{\nabla}_j E_{ik} - \tilde{\nabla}_l\tilde{\nabla}_k E_{ij} \Big]. \tag{13}$$