Barred Quantities in $\delta W_{\mu\nu}$

Under conformal transformation the metric decomposes as

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} = \bar{g}^{(0)}_{\mu\nu} + \bar{h}_{\mu\nu} \tag{1}$$

and the perturbed Weyl tensor transforms as

$$\delta W_{\mu\nu}(h_{\mu\nu}) \to \delta \bar{W}_{\mu\nu}(\bar{h}_{\mu\nu}). \tag{2}$$

We seek to obtain solutions to the fluctuation equation

$$\delta \bar{W}_{\mu\nu}(\bar{h}_{\mu\nu}) = \delta \bar{T}_{\mu\nu}(\bar{h}_{\mu\nu})/4\alpha_g \tag{3}$$

as evaluated in a conformal to flat background (given above eq (12) in paper being written).

The perturbed tensor after conformal transformation takes the form (where $\bar{g}_{\mu\nu} \equiv \bar{g}^{(0)}_{\mu\nu}$)

$$\begin{aligned}
\delta \bar{W}_{\mu\nu}(\bar{h}_{\mu\nu}) = & -\tfrac{1}{6}\bar{h}_{\mu\nu}\bar{R}^2 + \tfrac{1}{3}\bar{g}_{\mu\nu}\bar{h}^{\alpha\beta}\bar{R}\bar{R}_{\alpha\beta} + \tfrac{1}{2}\bar{h}_{\mu\nu}\bar{R}_{\alpha\beta}\bar{R}^{\alpha\beta} + \tfrac{1}{3}\bar{h}_\nu{}^\alpha\bar{R}\bar{R}_{\mu\alpha} - \tfrac{1}{2}\bar{h}_\nu{}^\alpha\bar{R}_{\alpha\beta}\bar{R}_\mu{}^\beta \\
& - \tfrac{2}{3}\bar{h}^{\alpha\beta}\bar{R}_{\alpha\beta}\bar{R}_{\mu\nu} + \tfrac{1}{3}\bar{h}_\mu{}^\alpha\bar{R}\bar{R}_{\nu\alpha} + \bar{h}^{\alpha\beta}\bar{R}_{\mu\alpha}\bar{R}_{\nu\beta} - \tfrac{1}{2}\bar{h}_\mu{}^\alpha\bar{R}_{\alpha\beta}\bar{R}_\nu{}^\beta - \bar{g}_{\mu\nu}\bar{h}^{\alpha\beta}\bar{R}^{\gamma\eta}\bar{R}_{\alpha\gamma\beta\eta} \\
& - \tfrac{2}{3}\bar{h}^{\alpha\beta}\bar{R}\bar{R}_{\mu\alpha\nu\beta} - \bar{h}_\nu{}^\alpha\bar{R}^{\beta\gamma}\bar{R}_{\mu\beta\alpha\gamma} + 2\bar{h}^{\alpha\beta}\bar{R}_\alpha{}^\gamma\bar{R}_{\mu\gamma\nu\beta} + 2\bar{h}^{\alpha\beta}\bar{R}_{\alpha\gamma\beta\eta}\bar{R}_\mu{}^\gamma{}_\nu{}^\eta \\
& - \bar{h}_\mu{}^\alpha\bar{R}^{\beta\gamma}\bar{R}_{\nu\beta\alpha\gamma} + \tfrac{1}{3}\bar{R}\bar{\nabla}_\alpha\bar{\nabla}^\alpha\bar{h}_{\mu\nu} - \tfrac{1}{6}\bar{h}_{\mu\nu}\bar{\nabla}_\alpha\bar{\nabla}^\alpha\bar{R} + \tfrac{1}{2}\bar{R}_\nu{}^\alpha\bar{\nabla}_\alpha\bar{\nabla}_\beta\bar{h}_\mu{}^\beta \\
& + \tfrac{1}{2}\bar{R}_\mu{}^\alpha\bar{\nabla}_\alpha\bar{\nabla}_\beta\bar{h}_\nu{}^\beta - \tfrac{1}{2}\bar{R}_\nu{}^\alpha\bar{\nabla}_\alpha\bar{\nabla}_\mu\bar{h}^\beta{}_\beta - \tfrac{1}{2}\bar{R}_\mu{}^\alpha\bar{\nabla}_\alpha\bar{\nabla}_\nu\bar{h}^\beta{}_\beta - \tfrac{1}{12}\bar{g}_{\mu\nu}\bar{\nabla}_\alpha\bar{h}^\beta{}_\beta\bar{\nabla}^\alpha\bar{R} \\
& - \tfrac{1}{6}\bar{\nabla}_\alpha\bar{h}_{\mu\nu}\bar{\nabla}^\alpha\bar{R} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{\nabla}^\alpha\bar{R}\bar{\nabla}_\beta\bar{h}_\alpha{}^\beta - \bar{\nabla}_\alpha\bar{h}^{\alpha\beta}\bar{\nabla}_\beta\bar{R}_{\mu\nu} + \tfrac{1}{3}\bar{g}_{\mu\nu}\bar{R}\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{h}^{\alpha\beta} \\
& - \tfrac{2}{3}\bar{R}_{\mu\nu}\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{h}^{\alpha\beta} + \tfrac{1}{2}\bar{g}_{\mu\nu}\bar{R}^{\alpha\beta}\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{h}^\gamma{}_\gamma - \bar{R}^{\alpha\beta}\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{h}_{\mu\nu} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{h}^{\alpha\beta}\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{R} \\
& - \bar{h}^{\alpha\beta}\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{R}_{\mu\nu} - \tfrac{1}{3}\bar{g}_{\mu\nu}\bar{R}\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{h}^\alpha{}_\alpha + \tfrac{2}{3}\bar{R}_{\mu\nu}\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{h}^\alpha{}_\alpha + \tfrac{1}{2}\bar{h}_\nu{}^\alpha\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{R}_{\mu\alpha} \\
& + \tfrac{1}{2}\bar{h}_\mu{}^\alpha\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{R}_{\nu\alpha} + \tfrac{1}{2}\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{\nabla}_\alpha\bar{\nabla}^\alpha\bar{h}_{\mu\nu} - \tfrac{1}{2}\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{\nabla}_\mu\bar{\nabla}_\alpha\bar{h}_\nu{}^\alpha - \tfrac{1}{2}\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{\nabla}_\nu\bar{\nabla}_\alpha\bar{h}_\mu{}^\alpha \\
& - \bar{g}_{\mu\nu}\bar{R}^{\alpha\beta}\bar{\nabla}_\beta\bar{\nabla}_\gamma\bar{h}_\alpha{}^\gamma + \tfrac{1}{2}\bar{\nabla}_\beta\bar{R}_{\mu\nu}\bar{\nabla}^\beta\bar{h}^\alpha{}_\alpha + \bar{\nabla}_\alpha\bar{R}_{\nu\beta}\bar{\nabla}^\beta\bar{h}_\mu{}^\alpha + \bar{\nabla}_\alpha\bar{R}_{\mu\beta}\bar{\nabla}^\beta\bar{h}_\nu{}^\alpha \\
& + \tfrac{1}{2}\bar{\nabla}^\beta\bar{h}^\alpha{}_\alpha\bar{\nabla}_\gamma\bar{R}_\mu{}^\gamma{}_{\nu\beta} + \tfrac{2}{3}\bar{g}_{\mu\nu}\bar{R}^{\alpha\beta}\bar{\nabla}_\gamma\bar{\nabla}^\gamma\bar{h}_{\alpha\beta} - 2\bar{R}_{\mu\alpha\nu\beta}\bar{\nabla}_\gamma\bar{\nabla}^\gamma\bar{h}^{\alpha\beta} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{h}^{\alpha\beta}\bar{\nabla}_\gamma\bar{\nabla}^\gamma\bar{R}_{\alpha\beta} \\
& - \bar{h}^{\alpha\beta}\bar{\nabla}_\gamma\bar{\nabla}^\gamma\bar{R}_{\mu\alpha\nu\beta} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{\nabla}_\gamma\bar{\nabla}^\gamma\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{h}^{\alpha\beta} - \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{\nabla}_\gamma\bar{\nabla}^\gamma\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{h}^\alpha{}_\alpha \\
& + \tfrac{1}{3}g_{\mu\nu}\bar{\nabla}_\gamma\bar{R}_{\alpha\beta}\bar{\nabla}^\gamma\bar{h}^{\alpha\beta} - 2\bar{\nabla}_\gamma\bar{R}_{\mu\alpha\nu\beta}\bar{\nabla}^\gamma\bar{h}^{\alpha\beta} + \bar{R}_{\mu\beta\nu\gamma}\bar{\nabla}^\gamma\bar{\nabla}_\alpha\bar{h}^{\alpha\beta} + \bar{R}_{\mu\gamma\nu\beta}\bar{\nabla}^\gamma\bar{\nabla}_\alpha\bar{h}^{\alpha\beta} \\
& - \bar{\nabla}_\beta\bar{R}_{\nu\alpha}\bar{\nabla}_\mu\bar{h}^{\alpha\beta} + \tfrac{1}{6}\bar{\nabla}^\alpha\bar{R}\bar{\nabla}_\mu\bar{h}_{\nu\alpha} - \tfrac{1}{2}\bar{\nabla}^\beta\bar{h}^\alpha{}_\alpha\bar{\nabla}_\mu\bar{R}_{\nu\beta} - \tfrac{1}{3}\bar{R}\bar{\nabla}_\mu\bar{\nabla}_\alpha\bar{h}_\nu{}^\alpha \\
& - \tfrac{1}{2}\bar{R}_\nu{}^\alpha\bar{\nabla}_\mu\bar{\nabla}_\beta\bar{h}_\alpha{}^\beta + \bar{R}^{\alpha\beta}\bar{\nabla}_\mu\bar{\nabla}_\beta\bar{h}_{\nu\alpha} - \bar{\nabla}_\beta\bar{R}_{\mu\alpha}\bar{\nabla}_\nu\bar{h}^{\alpha\beta} + \tfrac{1}{3}\bar{\nabla}_\mu\bar{R}_{\alpha\beta}\bar{\nabla}_\nu\bar{h}^{\alpha\beta} \\
& + \tfrac{1}{6}\bar{\nabla}^\alpha\bar{R}\bar{\nabla}_\nu\bar{h}_{\mu\alpha} + \tfrac{1}{3}\bar{\nabla}_\mu\bar{h}^{\alpha\beta}\bar{\nabla}_\nu\bar{R}_{\alpha\beta} - \tfrac{1}{3}\bar{R}\bar{\nabla}_\nu\bar{\nabla}_\alpha\bar{h}_\mu{}^\alpha - \tfrac{1}{2}\bar{R}_\mu{}^\alpha\bar{\nabla}_\nu\bar{\nabla}_\beta\bar{h}_\alpha{}^\beta \\
& + \bar{R}^{\alpha\beta}\bar{\nabla}_\nu\bar{\nabla}_\beta\bar{h}_{\mu\alpha} - \tfrac{2}{3}\bar{R}^{\alpha\beta}\bar{\nabla}_\nu\bar{\nabla}_\mu\bar{h}_{\alpha\beta} + \tfrac{1}{3}\bar{R}\bar{\nabla}_\nu\bar{\nabla}_\mu\bar{h}^\alpha{}_\alpha + \tfrac{1}{3}\bar{h}^{\alpha\beta}\bar{\nabla}_\nu\bar{\nabla}_\mu\bar{R}_{\alpha\beta} \\
& + \tfrac{1}{3}\bar{\nabla}_\nu\bar{\nabla}_\mu\bar{\nabla}_\beta\bar{\nabla}_\alpha\bar{h}^{\alpha\beta} + \tfrac{1}{6}\bar{\nabla}_\nu\bar{\nabla}_\mu\bar{\nabla}_\beta\bar{\nabla}^\beta\bar{h}^\alpha{}_\alpha.
\end{aligned} \tag{4}$$

Now we make the substitution

$$\bar{h}_{\mu\nu} = \bar{K}_{\mu\nu} + \tfrac{1}{4}\bar{g}_{\mu\nu}h \tag{5}$$

and we have

$$\delta \bar{W}_{\mu\nu}(\bar{h}_{\mu\nu}) = -\tfrac{1}{6}\bar{K}_{\mu\nu}\bar{R}^2 + \tfrac{1}{3}\bar{g}_{\mu\nu}\bar{K}^{\alpha\beta}\bar{R}\bar{R}_{\alpha\beta} + \tfrac{1}{2}\bar{K}_{\mu\nu}\bar{R}_{\alpha\beta}\bar{R}^{\alpha\beta} + \tfrac{1}{3}\bar{K}_\nu{}^\alpha\bar{R}\bar{R}_{\mu\alpha} - \tfrac{1}{2}\bar{K}_\nu{}^\alpha\bar{R}_{\alpha\beta}\bar{R}_\mu{}^\beta$$

$$
\begin{aligned}
&- \tfrac{2}{3}\bar{K}^{\alpha\beta}\bar{R}_{\alpha\beta}\bar{R}_{\mu\nu} + \tfrac{1}{3}\bar{K}_{\mu}{}^{\alpha}\bar{R}\bar{R}_{\nu\alpha} + \bar{K}^{\alpha\beta}\bar{R}_{\mu\alpha}\bar{R}_{\nu\beta} - \tfrac{1}{2}\bar{K}_{\mu}{}^{\alpha}\bar{R}_{\alpha\beta}\bar{R}_{\nu}{}^{\beta} - \bar{g}_{\mu\nu}\bar{K}^{\alpha\beta}\bar{R}^{\gamma\eta}\bar{R}_{\alpha\gamma\beta\eta} \\
&- \tfrac{2}{3}\bar{K}^{\alpha\beta}\bar{R}\bar{R}_{\mu\alpha\nu\beta} - \bar{K}_{\nu}{}^{\alpha}\bar{R}^{\beta\gamma}\bar{R}_{\mu\beta\alpha\gamma} + 2\bar{K}^{\alpha\beta}\bar{R}_{\alpha}{}^{\gamma}\bar{R}_{\mu\gamma\nu\beta} + 2\bar{K}^{\alpha\beta}\bar{R}_{\alpha\gamma\beta\eta}\bar{R}_{\mu}{}^{\gamma}{}_{\nu}{}^{\eta} \\
&- \bar{K}_{\mu}{}^{\alpha}\bar{R}^{\beta\gamma}\bar{R}_{\nu\beta\alpha\gamma} + \tfrac{1}{3}\bar{R}\bar{\nabla}_{\alpha}\bar{\nabla}^{\alpha}\bar{K}_{\mu\nu} - \tfrac{1}{6}\bar{K}_{\mu\nu}\bar{\nabla}_{\alpha}\bar{\nabla}^{\alpha}\bar{R} + \tfrac{1}{2}\bar{R}_{\nu}{}^{\alpha}\bar{\nabla}_{\alpha}\bar{\nabla}_{\beta}\bar{K}_{\mu}{}^{\beta} \\
&+ \tfrac{1}{2}\bar{R}_{\mu}{}^{\alpha}\bar{\nabla}_{\alpha}\bar{\nabla}_{\beta}\bar{K}_{\nu}{}^{\beta} - \tfrac{1}{6}\bar{\nabla}_{\alpha}\bar{K}_{\mu\nu}\bar{\nabla}^{\alpha}\bar{R} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{\nabla}^{\alpha}\bar{R}\bar{\nabla}_{\beta}\bar{K}_{\alpha}{}^{\beta} - \bar{\nabla}_{\alpha}\bar{K}^{\alpha\beta}\bar{\nabla}_{\beta}\bar{R}_{\mu\nu} \\
&+ \tfrac{1}{3}\bar{g}_{\mu\nu}\bar{R}\bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}\bar{K}^{\alpha\beta} - \tfrac{2}{3}\bar{R}_{\mu\nu}\bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}\bar{K}^{\alpha\beta} - \bar{R}^{\alpha\beta}\bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}\bar{K}_{\mu\nu} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{K}^{\alpha\beta}\bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}\bar{R} \\
&- \bar{K}^{\alpha\beta}\bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}\bar{R}_{\mu\nu} + \tfrac{1}{2}\bar{K}_{\nu}{}^{\alpha}\bar{\nabla}_{\beta}\bar{\nabla}^{\beta}\bar{R}_{\mu\alpha} + \tfrac{1}{2}\bar{K}_{\mu}{}^{\alpha}\bar{\nabla}_{\beta}\bar{\nabla}^{\beta}\bar{R}_{\nu\alpha} + \tfrac{1}{2}\bar{\nabla}_{\beta}\bar{\nabla}^{\beta}\bar{\nabla}_{\alpha}\bar{\nabla}^{\alpha}\bar{K}_{\mu\nu} \\
&- \tfrac{1}{2}\bar{\nabla}_{\beta}\bar{\nabla}^{\beta}\bar{\nabla}_{\mu}\bar{\nabla}_{\alpha}\bar{K}_{\nu}{}^{\alpha} - \tfrac{1}{2}\bar{\nabla}_{\beta}\bar{\nabla}^{\beta}\bar{\nabla}_{\nu}\bar{\nabla}_{\alpha}\bar{K}_{\mu}{}^{\alpha} - \bar{g}_{\mu\nu}\bar{R}^{\alpha\beta}\bar{\nabla}_{\beta}\bar{\nabla}_{\gamma}\bar{K}_{\alpha}{}^{\gamma} + \bar{\nabla}_{\alpha}\bar{R}_{\nu\beta}\bar{\nabla}^{\beta}\bar{K}_{\mu}{}^{\alpha} \\
&+ \bar{\nabla}_{\alpha}\bar{R}_{\mu\beta}\bar{\nabla}^{\beta}\bar{K}_{\nu}{}^{\alpha} + \tfrac{2}{3}\bar{g}_{\mu\nu}\bar{R}^{\alpha\beta}\bar{\nabla}_{\gamma}\bar{\nabla}^{\gamma}\bar{K}_{\alpha\beta} - 2\bar{R}_{\mu\alpha\nu\beta}\bar{\nabla}_{\gamma}\bar{\nabla}^{\gamma}\bar{K}^{\alpha\beta} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{K}^{\alpha\beta}\bar{\nabla}_{\gamma}\bar{\nabla}^{\gamma}\bar{R}_{\alpha\beta} \\
&- \bar{K}^{\alpha\beta}\bar{\nabla}_{\gamma}\bar{\nabla}^{\gamma}\bar{R}_{\mu\alpha\nu\beta} + \tfrac{1}{6}\bar{g}_{\mu\nu}\bar{\nabla}_{\gamma}\bar{\nabla}^{\gamma}\bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}\bar{K}^{\alpha\beta} + \tfrac{1}{3}\bar{g}_{\mu\nu}\bar{\nabla}_{\gamma}\bar{R}_{\alpha\beta}\bar{\nabla}^{\gamma}\bar{K}^{\alpha\beta} \\
&- 2\bar{\nabla}_{\gamma}\bar{R}_{\mu\alpha\nu\beta}\bar{\nabla}^{\gamma}\bar{K}^{\alpha\beta} + \bar{R}_{\mu\beta\nu\gamma}\bar{\nabla}^{\gamma}\bar{\nabla}_{\alpha}\bar{K}^{\alpha\beta} + \bar{R}_{\mu\gamma\nu\beta}\bar{\nabla}^{\gamma}\bar{\nabla}_{\alpha}\bar{K}^{\alpha\beta} - \bar{\nabla}_{\beta}\bar{R}_{\nu\alpha}\bar{\nabla}_{\mu}\bar{K}^{\alpha\beta} \\
&+ \tfrac{1}{6}\bar{\nabla}^{\alpha}\bar{R}\bar{\nabla}_{\mu}\bar{K}_{\nu\alpha} - \tfrac{1}{3}\bar{R}\bar{\nabla}_{\mu}\bar{\nabla}_{\alpha}\bar{K}_{\nu}{}^{\alpha} - \tfrac{1}{2}\bar{R}_{\nu}{}^{\alpha}\bar{\nabla}_{\mu}\bar{\nabla}_{\beta}\bar{K}_{\alpha}{}^{\beta} + \bar{R}^{\alpha\beta}\bar{\nabla}_{\mu}\bar{\nabla}_{\beta}\bar{K}_{\nu\alpha} \\
&- \bar{\nabla}_{\beta}\bar{R}_{\mu\alpha}\bar{\nabla}_{\nu}\bar{K}^{\alpha\beta} + \tfrac{1}{3}\bar{\nabla}_{\mu}\bar{R}_{\alpha\beta}\bar{\nabla}_{\nu}\bar{K}^{\alpha\beta} + \tfrac{1}{6}\bar{\nabla}^{\alpha}\bar{R}\bar{\nabla}_{\nu}\bar{K}_{\mu\alpha} + \tfrac{1}{3}\bar{\nabla}_{\mu}\bar{K}^{\alpha\beta}\bar{\nabla}_{\nu}\bar{R}_{\alpha\beta} \\
&- \tfrac{1}{3}\bar{R}\bar{\nabla}_{\nu}\bar{\nabla}_{\alpha}\bar{K}_{\mu}{}^{\alpha} - \tfrac{1}{2}\bar{R}_{\mu}{}^{\alpha}\bar{\nabla}_{\nu}\bar{\nabla}_{\beta}\bar{K}_{\alpha}{}^{\beta} + \bar{R}^{\alpha\beta}\bar{\nabla}_{\nu}\bar{\nabla}_{\beta}\bar{K}_{\mu\alpha} - \tfrac{2}{3}\bar{R}^{\alpha\beta}\bar{\nabla}_{\nu}\bar{\nabla}_{\mu}\bar{K}_{\alpha\beta} \\
&+ \tfrac{1}{3}\bar{K}^{\alpha\beta}\bar{\nabla}_{\nu}\bar{\nabla}_{\mu}\bar{R}_{\alpha\beta} + \tfrac{1}{3}\bar{\nabla}_{\nu}\bar{\nabla}_{\mu}\bar{\nabla}_{\beta}\bar{\nabla}_{\alpha}\bar{K}^{\alpha\beta} - \tfrac{1}{4}h\bar{W}_{\mu\nu}(\bar{g}_{\mu\nu}). \qquad (6)
\end{aligned}
$$

Now if we expand all curvature tensors and covariant derivatives in terms of connections evaluated in a conformal to flat background, viz.

$$\Gamma^{\lambda}_{\mu\nu} = \Omega^{-1}(x)[\delta^{\lambda}_{\mu}\partial_{\nu} + \delta^{\lambda}_{\mu}\partial_{\mu} - \eta^{\lambda\rho}\eta_{\mu\nu}\partial_{\rho}]\Omega(x) \qquad (7)$$

the dependence upon h drops out and (6) becomes

$$
\begin{aligned}
\delta\bar{W}_{\mu\nu}(\bar{K}_{\mu\nu}) = &-\frac{48\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\gamma}\Omega\partial_{\eta}\bar{K}_{\mu\nu}}{\Omega^{7}} + \frac{24\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\gamma}\partial_{\beta}\Omega\partial_{\eta}\bar{K}_{\mu\nu}}{\Omega^{6}} + \frac{20\eta^{\alpha\beta}\eta^{\gamma\kappa}\eta^{\eta\lambda}\eta_{\mu\nu}\bar{K}_{\kappa\lambda}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\gamma}\Omega\partial_{\eta}\Omega}{\Omega^{8}} \\
&+ \frac{60\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\nu}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\gamma}\Omega\partial_{\eta}\Omega}{\Omega^{8}} - \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\gamma}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\beta}\bar{K}_{\mu\nu}}{\Omega^{5}} + \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\nu}\partial_{\gamma}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\beta}\Omega}{\Omega^{6}} \\
&+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\beta}\partial_{\mu}\bar{K}_{\nu\gamma}}{\Omega^{5}} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\beta}\partial_{\nu}\bar{K}_{\mu\gamma}}{\Omega^{5}} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\eta}\partial_{\beta}\partial_{\nu}\partial_{\mu}\bar{K}_{\alpha\gamma}}{3\Omega^{4}} \\
&- \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\alpha\gamma}\partial_{\eta}\partial_{\beta}\partial_{\nu}\partial_{\mu}\Omega}{3\Omega^{5}} + \frac{12\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\gamma}\bar{K}_{\mu\nu}}{\Omega^{6}} + \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\gamma}\bar{K}_{\mu\nu}}{\Omega^{6}} \\
&- \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\beta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\gamma}\bar{K}_{\mu\nu}}{\Omega^{5}} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\beta}\bar{K}_{\mu\nu}\partial_{\eta}\partial_{\gamma}\Omega}{\Omega^{6}} - \frac{48\eta^{\alpha\gamma}\eta^{\beta\eta}\bar{K}_{\mu\nu}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\gamma}\Omega}{\Omega^{7}} \\
&- \frac{24\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\nu}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\gamma}\Omega}{\Omega^{7}} + \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\nu}\partial_{\beta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\gamma}\Omega}{\Omega^{6}} - \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\gamma}\partial_{\beta}\bar{K}_{\mu\nu}}{\Omega^{5}} \\
&- \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\bar{K}_{\mu\nu}\partial_{\eta}\partial_{\gamma}\partial_{\beta}\Omega}{\Omega^{5}} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\nu}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\gamma}\partial_{\beta}\Omega}{\Omega^{6}} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\eta}\partial_{\gamma}\partial_{\beta}\partial_{\alpha}\bar{K}_{\mu\nu}}{2\Omega^{4}} \\
&- \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\nu}\partial_{\eta}\partial_{\gamma}\partial_{\beta}\partial_{\alpha}\Omega}{\Omega^{5}} - \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\eta}\partial_{\gamma}\partial_{\beta}\partial_{\mu}\bar{K}_{\nu\alpha}}{2\Omega^{4}} - \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\eta}\partial_{\gamma}\partial_{\beta}\partial_{\nu}\bar{K}_{\mu\alpha}}{2\Omega^{4}} \\
&+ \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\gamma}\partial_{\mu}\bar{K}_{\nu\beta}}{\Omega^{5}} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\alpha}\partial_{\eta}\partial_{\gamma}\partial_{\mu}\partial_{\beta}\Omega}{\Omega^{5}} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\gamma}\partial_{\nu}\bar{K}_{\mu\beta}}{\Omega^{5}} \\
&+ \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\alpha}\partial_{\eta}\partial_{\gamma}\partial_{\nu}\partial_{\beta}\Omega}{\Omega^{5}} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\gamma}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\mu}\bar{K}_{\nu\beta}}{\Omega^{5}} - \frac{6\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\mu}\bar{K}_{\nu\gamma}}{\Omega^{6}} \\
&- \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\mu}\bar{K}_{\nu\gamma}}{\Omega^{6}} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\beta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\mu}\bar{K}_{\nu\gamma}}{\Omega^{5}} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\gamma}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\mu}\partial_{\beta}\Omega}{\Omega^{6}} \\
&+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\gamma}\bar{K}_{\nu\alpha}\partial_{\eta}\partial_{\mu}\partial_{\beta}\Omega}{\Omega^{5}} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\beta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\mu}\partial_{\gamma}\Omega}{\Omega^{6}} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\beta}\bar{K}_{\nu\alpha}\partial_{\eta}\partial_{\mu}\partial_{\gamma}\Omega}{\Omega^{5}} \\
&+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\gamma}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\nu}\bar{K}_{\mu\beta}}{\Omega^{5}} - \frac{6\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\nu}\bar{K}_{\mu\gamma}}{\Omega^{6}} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\alpha}\Omega\partial_{\beta}\Omega\partial_{\eta}\partial_{\nu}\bar{K}_{\mu\gamma}}{\Omega^{6}} \\
&+ \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\beta}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\nu}\bar{K}_{\mu\gamma}}{\Omega^{5}} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\gamma}\partial_{\alpha}\Omega\partial_{\eta}\partial_{\nu}\partial_{\beta}\Omega}{\Omega^{6}} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_{\gamma}\bar{K}_{\mu\alpha}\partial_{\eta}\partial_{\nu}\partial_{\beta}\Omega}{\Omega^{5}}
\end{aligned}
$$

$$
- \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\beta}\partial_\alpha\Omega\partial_\eta\partial_\nu\partial_\gamma\Omega}{\Omega^6} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\beta\bar{K}_{\mu\alpha}\partial_\eta\partial_\nu\partial_\gamma\Omega}{\Omega^5} - \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\nu\partial_\mu\bar{K}_{\beta\gamma}}{3\Omega^5}
$$
$$
+ \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\beta\gamma}\partial_\alpha\Omega\partial_\eta\partial_\nu\partial_\mu\Omega}{\Omega^6} - \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\beta\bar{K}_{\alpha\gamma}\partial_\eta\partial_\nu\partial_\mu\Omega}{3\Omega^5} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\beta\bar{K}_{\eta\lambda}\partial_\kappa\partial_\gamma\Omega}{\Omega^6}
$$
$$
+ \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\kappa\partial_\gamma\Omega\partial_\lambda\bar{K}_{\beta\eta}}{\Omega^6} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\eta\partial_\gamma\Omega\partial_\lambda\bar{K}_{\beta\kappa}}{\Omega^6} - \frac{8\eta^{\alpha\eta}\eta^{\beta\kappa}\eta^{\gamma\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\gamma\Omega\partial_\lambda\bar{K}_{\eta\kappa}}{\Omega^7}
$$
$$
- \frac{8\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\gamma\Omega\partial_\lambda\bar{K}_{\eta\kappa}}{\Omega^7} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\gamma\partial_\beta\Omega\partial_\lambda\bar{K}_{\eta\kappa}}{\Omega^6} -
$$
$$
\frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\gamma\partial_\alpha\Omega\partial_\lambda\partial_\eta\bar{K}_{\beta\kappa}}{3\Omega^5} + \frac{4\eta^{\alpha\gamma}\eta^{\beta\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\lambda\partial_\eta\bar{K}_{\gamma\kappa}}{\Omega^6} +
$$
$$
\frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\lambda\partial_\eta\bar{K}_{\gamma\kappa}}{\Omega^6} - \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\beta\partial_\alpha\Omega\partial_\lambda\partial_\eta\bar{K}_{\gamma\kappa}}{3\Omega^5} -
$$
$$
\frac{16\eta^{\alpha\gamma}\eta^{\beta\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\gamma\kappa}\partial_\alpha\Omega\partial_\beta\Omega\partial_\lambda\partial_\eta\Omega}{\Omega^7} - \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\gamma\kappa}\partial_\alpha\Omega\partial_\beta\Omega\partial_\lambda\partial_\eta\Omega}{\Omega^7}
$$
$$
+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\alpha\gamma}\partial_\kappa\partial_\beta\Omega\partial_\lambda\partial_\eta\Omega}{\Omega^6} - \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\lambda\partial_\eta\partial_\beta\bar{K}_{\gamma\kappa}}{3\Omega^5}
$$
$$
+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\gamma\kappa}\partial_\alpha\Omega\partial_\lambda\partial_\eta\partial_\beta\Omega}{\Omega^6} - \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\kappa\bar{K}_{\alpha\gamma}\partial_\lambda\partial_\eta\partial_\beta\Omega}{3\Omega^5}
$$
$$
- \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\gamma\partial_\alpha\Omega\partial_\lambda\partial_\kappa\bar{K}_{\beta\eta}}{3\Omega^5} + \frac{\eta^{\alpha\gamma}\eta^{\beta\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\lambda\partial_\kappa\bar{K}_{\gamma\eta}}{\Omega^6}
$$
$$
- \frac{4\eta^{\alpha\gamma}\eta^{\beta\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\gamma\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\lambda\partial_\kappa\Omega}{\Omega^7} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\alpha\gamma}\partial_\eta\partial_\beta\Omega\partial_\lambda\partial_\kappa\Omega}{\Omega^6}
$$
$$
- \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\alpha\Omega\partial_\lambda\partial_\kappa\partial_\eta\bar{K}_{\beta\gamma}}{3\Omega^5} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\beta\gamma}\partial_\alpha\Omega\partial_\lambda\partial_\kappa\partial_\eta\Omega}{\Omega^6}
$$
$$
- \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\beta\bar{K}_{\alpha\gamma}\partial_\lambda\partial_\kappa\partial_\eta\Omega}{3\Omega^5} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\partial_\lambda\partial_\kappa\partial_\eta\partial_\beta\bar{K}_{\alpha\gamma}}{6\Omega^4} - \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\eta^{\kappa\lambda}\eta_{\mu\nu}\bar{K}_{\alpha\gamma}\partial_\lambda\partial_\kappa\partial_\eta\partial_\beta\Omega}{3\Omega^5}
$$
$$
- \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\nu\partial_\beta\Omega\partial_\mu\bar{K}_{\alpha\gamma}}{3\Omega^5} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\gamma\partial_\beta\Omega\partial_\mu\bar{K}_{\nu\alpha}}{\Omega^5} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\gamma\Omega\partial_\mu\bar{K}_{\nu\beta}}{\Omega^6}
$$
$$
+ \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\gamma\Omega\partial_\mu\bar{K}_{\nu\eta}}{\Omega^7} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\gamma\partial_\beta\Omega\partial_\mu\bar{K}_{\nu\eta}}{\Omega^6} - \frac{60\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\gamma\Omega\partial_\mu\Omega}{\Omega^8}
$$
$$
- \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\gamma\partial_\alpha\Omega\partial_\eta\bar{K}_{\nu\beta}\partial_\mu\Omega}{\Omega^6} + \frac{24\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\eta\bar{K}_{\nu\gamma}\partial_\mu\Omega}{\Omega^7} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\eta\bar{K}_{\nu\gamma}\partial_\mu\Omega}{\Omega^7}
$$
$$
- \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\beta\partial_\alpha\Omega\partial_\eta\bar{K}_{\nu\gamma}\partial_\mu\Omega}{\Omega^6} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\beta\bar{K}_{\nu\gamma}\partial_\mu\Omega}{\Omega^6} + \frac{24\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\gamma}\partial_\alpha\Omega\partial_\eta\partial_\beta\Omega\partial_\mu\Omega}{\Omega^7}
$$
$$
- \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\beta\partial_\nu\bar{K}_{\alpha\gamma}\partial_\mu\Omega}{3\Omega^5} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\gamma\bar{K}_{\nu\beta}\partial_\mu\Omega}{\Omega^6} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\beta}\partial_\alpha\Omega\partial_\eta\partial_\gamma\Omega\partial_\mu\Omega}{\Omega^7}
$$
$$
+ \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\gamma\partial_\beta\bar{K}_{\nu\alpha}\partial_\mu\Omega}{\Omega^5} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\alpha}\partial_\eta\partial_\gamma\partial_\beta\Omega\partial_\mu\Omega}{\Omega^6} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\nu\bar{K}_{\beta\gamma}\partial_\mu\Omega}{\Omega^6}
$$
$$
+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\alpha\gamma}\partial_\eta\partial_\nu\partial_\beta\Omega\partial_\mu\Omega}{\Omega^6} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\beta\bar{K}_{\nu\gamma}\partial_\mu\partial_\alpha\Omega}{\Omega^5} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\gamma\bar{K}_{\nu\beta}\partial_\mu\partial_\alpha\Omega}{\Omega^5}
$$
$$
- \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\nu\bar{K}_{\beta\gamma}\partial_\mu\partial_\alpha\Omega}{3\Omega^5} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\bar{K}_{\nu\gamma}\partial_\mu\partial_\beta\Omega}{\Omega^6} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\alpha}\partial_\eta\partial_\gamma\Omega\partial_\mu\partial_\beta\Omega}{\Omega^6}
$$
$$
- \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\beta\bar{K}_{\nu\eta}\partial_\mu\partial_\gamma\Omega}{\Omega^6} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\bar{K}_{\nu\beta}\partial_\mu\partial_\gamma\Omega}{\Omega^6} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\alpha}\partial_\eta\partial_\beta\Omega\partial_\mu\partial_\gamma\Omega}{\Omega^6}
$$
$$
+ \frac{24\eta^{\alpha\gamma}\eta^{\beta\eta}\bar{K}_{\nu\gamma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\mu\partial_\eta\Omega}{\Omega^7} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\nu\gamma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\mu\partial_\eta\Omega}{\Omega^7} - \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\mu\partial_\beta\Omega\partial_\nu\bar{K}_{\alpha\gamma}}{3\Omega^5}
$$
$$
+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\gamma\partial_\alpha\Omega\partial_\mu\Omega\partial_\nu\bar{K}_{\beta\eta}}{\Omega^6} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\mu\partial_\gamma\Omega\partial_\nu\bar{K}_{\beta\eta}}{\Omega^6} - \frac{8\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\mu\Omega\partial_\nu\bar{K}_{\gamma\eta}}{\Omega^7}
$$
$$
+ \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\gamma\partial_\beta\Omega\partial_\nu\bar{K}_{\mu\alpha}}{\Omega^5} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\gamma\Omega\partial_\nu\bar{K}_{\mu\beta}}{\Omega^6} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\gamma\Omega\partial_\nu\bar{K}_{\mu\eta}}{\Omega^7}
$$

$$
\begin{aligned}
&- \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\gamma\partial_\beta\Omega\partial_\nu\bar{K}_{\mu\eta}}{\Omega^6} - \frac{60\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\gamma\Omega\partial_\nu\Omega}{\Omega^8} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\gamma\partial_\alpha\Omega\partial_\eta\bar{K}_{\mu\beta}\partial_\nu\Omega}{\Omega^6} \\
&+ \frac{24\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\eta\bar{K}_{\mu\gamma}\partial_\nu\Omega}{\Omega^7} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\eta\bar{K}_{\mu\gamma}\partial_\nu\Omega}{\Omega^7} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\beta\partial_\alpha\Omega\partial_\eta\bar{K}_{\mu\gamma}\partial_\nu\Omega}{\Omega^6} \\
&- \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\beta\bar{K}_{\mu\gamma}\partial_\nu\Omega}{\Omega^6} + \frac{24\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\gamma}\partial_\alpha\Omega\partial_\eta\partial_\beta\Omega\partial_\nu\Omega}{\Omega^7} - \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\beta\partial_\mu\bar{K}_{\alpha\gamma}\partial_\nu\Omega}{3\Omega^5} \\
&- \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\gamma\bar{K}_{\mu\beta}\partial_\nu\Omega}{\Omega^6} + \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\beta}\partial_\alpha\Omega\partial_\eta\partial_\gamma\Omega\partial_\nu\Omega}{\Omega^7} + \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\gamma\partial_\beta\bar{K}_{\mu\alpha}\partial_\nu\Omega}{\Omega^5} \\
&- \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\alpha}\partial_\eta\partial_\gamma\partial_\beta\Omega\partial_\nu\Omega}{\Omega^6} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\partial_\mu\bar{K}_{\beta\gamma}\partial_\nu\Omega}{\Omega^6} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\alpha\gamma}\partial_\eta\partial_\mu\partial_\beta\Omega\partial_\nu\Omega}{\Omega^6} \\
&+ \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\gamma\partial_\alpha\Omega\partial_\mu\bar{K}_{\beta\eta}\partial_\nu\Omega}{\Omega^6} - \frac{8\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\mu\bar{K}_{\gamma\eta}\partial_\nu\Omega}{\Omega^7} + \frac{40\eta^{\alpha\gamma}\eta^{\beta\eta}\bar{K}_{\gamma\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\mu\Omega\partial_\nu\Omega}{\Omega^8} \\
&- \frac{16\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\bar{K}_{\beta\gamma}\partial_\mu\Omega\partial_\nu\Omega}{\Omega^7} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\beta\bar{K}_{\alpha\gamma}\partial_\mu\Omega\partial_\nu\Omega}{\Omega^6} - \frac{8\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\alpha\gamma}\partial_\eta\partial_\beta\Omega\partial_\mu\Omega\partial_\nu\Omega}{\Omega^7} \\
&+ \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\bar{K}_{\beta\gamma}\partial_\mu\partial_\alpha\Omega\partial_\nu\Omega}{\Omega^6} - \frac{16\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\beta\gamma}\partial_\alpha\Omega\partial_\mu\partial_\eta\Omega\partial_\nu\Omega}{\Omega^7} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\beta\bar{K}_{\mu\gamma}\partial_\nu\partial_\alpha\Omega}{\Omega^5} \\
&+ \frac{\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\gamma\bar{K}_{\mu\beta}\partial_\nu\partial_\alpha\Omega}{\Omega^5} - \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\mu\bar{K}_{\beta\gamma}\partial_\nu\partial_\alpha\Omega}{3\Omega^5} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\bar{K}_{\beta\gamma}\partial_\mu\Omega\partial_\nu\partial_\alpha\Omega}{\Omega^6} \\
&- \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\bar{K}_{\mu\gamma}\partial_\nu\partial_\beta\Omega}{\Omega^6} - \frac{3\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\alpha}\partial_\eta\partial_\gamma\Omega\partial_\nu\partial_\beta\Omega}{\Omega^6} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\beta\bar{K}_{\mu\eta}\partial_\nu\partial_\gamma\Omega}{\Omega^6} \\
&- \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\bar{K}_{\mu\beta}\partial_\nu\partial_\gamma\Omega}{\Omega^6} - \frac{6\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\alpha}\partial_\eta\partial_\beta\Omega\partial_\nu\partial_\gamma\Omega}{\Omega^6} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\mu\bar{K}_{\beta\eta}\partial_\nu\partial_\gamma\Omega}{\Omega^6} \\
&+ \frac{24\eta^{\alpha\gamma}\eta^{\beta\eta}\bar{K}_{\mu\gamma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\nu\partial_\eta\Omega}{\Omega^7} \\
&+ \frac{12\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\mu\gamma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\nu\partial_\eta\Omega}{\Omega^7} \\
&- \frac{16\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\beta\gamma}\partial_\alpha\Omega\partial_\mu\Omega\partial_\nu\partial_\eta\Omega}{\Omega^7} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\alpha\gamma}\partial_\mu\partial_\beta\Omega\partial_\nu\partial_\eta\Omega}{\Omega^6} - \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\gamma\partial_\alpha\Omega\partial_\nu\partial_\mu\bar{K}_{\beta\eta}}{3\Omega^5} \\
&+ \frac{2\eta^{\alpha\gamma}\eta^{\beta\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\nu\partial_\mu\bar{K}_{\gamma\eta}}{\Omega^6} - \frac{8\eta^{\alpha\gamma}\eta^{\beta\eta}\bar{K}_{\gamma\eta}\partial_\alpha\Omega\partial_\beta\Omega\partial_\nu\partial_\mu\Omega}{\Omega^7} + \frac{4\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\alpha\Omega\partial_\eta\bar{K}_{\beta\gamma}\partial_\nu\partial_\mu\Omega}{\Omega^6} \\
&- \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\partial_\eta\partial_\beta\bar{K}_{\alpha\gamma}\partial_\nu\partial_\mu\Omega}{3\Omega^5} + \frac{2\eta^{\alpha\beta}\eta^{\gamma\eta}\bar{K}_{\alpha\gamma}\partial_\eta\partial_\beta\Omega\partial_\nu\partial_\mu\Omega}{\Omega^6}. \qquad (8)
\end{aligned}
$$

Now we apply the gauge condition

$$\bar{\nabla}_\nu\bar{K}^{\mu\nu} = 4\Omega^{-1}\bar{K}^{\mu\nu}\partial_\nu\Omega \qquad (9)$$

or the equivalent gauge covariant in $K_{\mu\nu}$

$$\eta^{\alpha\beta}\partial_\alpha\bar{K}_{\mu\beta} = 2\Omega^{-1}\eta^{\alpha\beta}\bar{K}_{\mu\beta}\partial_\alpha\Omega. \qquad (10)$$

and $\delta\bar{W}_{\mu\nu}$ reduces to

$$
\begin{aligned}
\delta\bar{W}_{\mu\nu}(\bar{K}_{\mu\nu}) = &-48\Omega^{-7}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\rho\Omega\partial_\sigma\bar{K}_{\mu\nu} + 24\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\rho\partial_\beta\Omega\partial_\sigma\bar{K}_{\mu\nu} \\
&+ 60\Omega^{-8}\eta^{\alpha\beta}\eta^{\rho\sigma}\bar{K}_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\rho\Omega\partial_\sigma\Omega - 4\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\rho\partial_\alpha\Omega\partial_\sigma\partial_\beta\bar{K}_{\mu\nu} \\
&+ 6\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\bar{K}_{\mu\nu}\partial_\rho\partial_\alpha\Omega\partial_\sigma\partial_\beta\Omega + 12\Omega^{-6}\eta^{\alpha\rho}\eta^{\beta\sigma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho\bar{K}_{\mu\nu} \\
&+ 6\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho\bar{K}_{\mu\nu} - 2\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\beta\partial_\alpha\Omega\partial_\sigma\partial_\rho\bar{K}_{\mu\nu} \\
&+ 12\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\beta\bar{K}_{\mu\nu}\partial_\sigma\partial_\rho\Omega - 48\Omega^{-7}\eta^{\alpha\rho}\eta^{\beta\sigma}\bar{K}_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho\Omega \\
&- 24\Omega^{-7}\eta^{\alpha\beta}\eta^{\rho\sigma}\bar{K}_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho\Omega + 3\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\bar{K}_{\mu\nu}\partial_\beta\partial_\alpha\Omega\partial_\sigma\partial_\rho\Omega \\
&- 4\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\sigma\partial_\rho\partial_\beta\bar{K}_{\mu\nu} - 4\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\bar{K}_{\mu\nu}\partial_\sigma\partial_\rho\partial_\beta\Omega \\
&+ 12\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\bar{K}_{\mu\nu}\partial_\alpha\Omega\partial_\sigma\partial_\rho\partial_\beta\Omega + \tfrac{1}{2}\Omega^{-4}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\sigma\partial_\rho\partial_\beta\partial_\alpha\bar{K}_{\mu\nu} \\
&- \Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\bar{K}_{\mu\nu}\partial_\sigma\partial_\rho\partial_\beta\partial_\alpha\Omega
\end{aligned}
$$

$$= \frac{1}{2}\Omega^{-2}\eta^{\sigma\rho}\eta^{\alpha\beta}\partial_\sigma\partial_\rho\partial_\alpha\partial_\beta(\Omega^{-2}\bar{K}_{\mu\nu}) \tag{11}$$

From this last result we see that

$$\delta\bar{W}_{\mu\nu}(\bar{h}_{\mu\nu}) = \Omega^{-2}\delta W_{\mu\nu}(h_{\mu\nu}) \tag{12}$$

since

$$\bar{h}_{\mu\nu} = \Omega^2 h_{\mu\nu}$$

$$\nabla_\kappa\nabla_\nu T_{\lambda\mu} = \nabla_\nu\nabla_\kappa T_{\lambda\mu} + R_{\lambda\sigma\nu\kappa}T^\sigma{}_\mu - R_{\sigma\mu\nu\kappa}T_\lambda{}^\sigma \tag{13}$$

obeyed by any rank two tensor, so that we can write $W^{\mu\nu}$ as

$$\begin{aligned} W^{\mu\nu} &= -\frac{1}{6}g^{\mu\nu}\nabla_\beta\nabla^\beta R^\alpha{}_\alpha + \nabla_\beta\nabla^\beta R^{\mu\nu} - \frac{1}{3}\nabla_\mu\nabla_\nu R^\alpha{}_\alpha - R^\beta{}_\sigma R^\sigma{}_{\mu\beta\nu} \\ &- R^\beta{}_\sigma R^\sigma{}_{\nu\beta\mu} + \frac{1}{2}g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \frac{2}{3}R^\alpha{}_\alpha R^{\mu\nu} - \frac{1}{6}g^{\mu\nu}(R^\alpha{}_\alpha)^2 \end{aligned} \tag{14}$$

Perturbing $W^{\mu\nu}$ about metric $g_{\mu\nu} + h_{\mu\nu}$ with background metric $g_{\mu\nu}$ and fluctuation $h_{\mu\nu}$ gives (following a machine calculation)