Physics 6300 Astrophysics and Modern Cosmology

Spring 2017 Take Home Exam due 12:30pm Thursday April 27, 2017.

Answer all five questions.

(1) In the standard hot big bang cosmology the key evolution equation for the universe is given by

$$\dot{R}^2(t) + kc^2 = \left(\frac{8\pi G}{3c^2}\right) R^2(t)\rho(t)$$

where $\rho(t)$ is the energy density of the matter content of the universe. Derive this equation in both Newtonian and general relativistic cosmology where each case requires its own identification of the quantity $R(t)$. Discuss the expansion of the universe from the point of view of the quantities k and $\Omega(t) = \rho(t)/\rho_c(t)$ where $\rho_c(t) = 3c^2H^2(t)/8\pi G$ and $H(t) = \dot{R}(t)/R(t)$, to establish the behavior of $R(t)$ in the open ($k = -1$), flat ($k = 0$) and closed ($k = +1$) cases. Make a numerical estimate for $\Omega(t)$ in the present epoch. What does its value tell you about the ultimate fate of the universe in cosmologies described by the above evolution equation.

(2) Typical stars put out Newtonian potentials of the form $V(r) = -\beta c^2/r$ where β is the Schwarzschild radius of the sun. An infinitesimally thin disk of such stars with surface mass distribution $\Sigma(R) = \Sigma_0 \exp(-R/R_0)$ (where R is the radial distance from the center of the disk, and R_0 is the disk scale length) is regarded as a good first approximation to the mass distribution of disk dominated Sc galaxies. For such a matter distribution derive a closed form expression for the dependence $v(R)$ of the rotational velocities of circular orbits in the plane of the disk on the radial coordinate R. Similarly, consider a spherically symmetric 3-dimensional distribution of such stars with radial matter density $\sigma(r)$, and derive a closed form expression for its contribution to the rotational velocity $v(r)$ at distance r. Then obtain a closed form expression for the isothermal sphere halo contribution to $v(r)$ associated with the halo $\sigma(r) = \sigma_0/(r^2 + r_0^2)$.

(3) Identify all the nuclear channels which produce solar neutrinos, and give the threshold neutrino energy associated with each relevant reaction. Explain briefly what the solar neutrino problem is, and describe briefly how neutrino oscillations could possibly solve the solar neutrino problem. Discuss the experimental evidence which has confirmed this neutrino oscillation hypothesis.

(4) Using the Euler hydrostatic equilibrium equation in the presence of a gravitational field and the quantum statistical mechanics of a degenerate electron gas, show that there is a maximum mass (the Chandrasehkar mass) for white dwarf stars and derive its magnitude in a closed form.

(5) The galaxy NGC3198 possesses one of the most strikingly flat HI rotation curves ever obtained (see e.g. Begeman, K.G., Astronomy and Astrophysics, 223, 47 (1989)). The galaxy lies at a distance of 9.36 Mpc from us, has a total luminosity of $L = 9.00 \times 10^9\ L_\odot$ in solar luminosity units, and is found to have the exponential surface luminosity brightness characteristic of Sc galaxies. With radial distances in the disk of NGC3198 being measured via their angular positions on the sky, the surface luminosity is found to have a scale length $R_0 = 1'$, i.e. exactly one minute of arc. For the galaxy the rotation data extend to the radial distance $11'$ which is way beyond the region where the luminosity is concentrated. Reported rotational velocity data are obtained in steps of $0.25'$ in the region from $0.25'$ to $2.75'$, and are found to be (in units of km/sec)

$$[55, 92, 110, 123, 134, 142, 145, 147, 148, 152, 155]$$

with error of plus/minus

$$[8, 8, 6, 5, 4, 4, 3, 3, 3, 2, 2]$$

(again in km/sec). Reported rotational velocity data are also obtained in steps of $0.50'$ in the region from $3.0'$ to $11.0'$, and are found to be (in units of km/sec)

$$[156, 157, 153, 153, 154, 153, 150, 149, 148, 146, 147, 148, 148, 149, 150, 150, 149]$$

with error (again in km/sec) of plus/minus

$$[2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3]$$

Make a least squares fit to these data using a numerical fitting program such as Mathematica in two cases: one using the luminous disk alone, and the other using a $\sigma(r) = \sigma_0/(r^2 + r_0^2)$ halo as well as the disk. In each case present your fits graphically, and report out the fitted values of the disk mass to light ratio M/L in units of $M_\odot/L_\odot$ obtained in each case as well as the fitted halo parameters. In the disk plus halo fit calculate the ratio of the total disk mass to total halo mass contained in the observed $0'$ to $11'$ region.