Dissertation Talking Points

1 Title Page

- Thank you everyone for coming. Welcome to my dissertation defense on the topic Cosmological Perturbations as applied to both standard Einstein Gravity and Conformal gravity

2 Overview

- Alright so, to give an overview of what will be covered,
- we will first form the necessary background of cosmological perturbation theory. Talking about the geometry of the universe, introduce einstein gravity, perturbations, and coordinate invariance
- As an approach to simplify and solve the equations arising in cosmology, we're going to analyze something called the SVT decomp. in 3 dimensions and then perform some generalizations to four dimensions, demonstrating specific applications of both within a de Sitter background geometry
- Then we'll cover a discussion of cosmological perturbations in conformal gravity, and demonstrate a calculation of the fluctuations in the early universe radiation era
- Finally we'll wrap it up with overall conclusions and I'll demonstrate some of the computations involved in order to do the calculations that we'll see through this presentation

3 Cosmological Geometries

- Let's first discuss the geometry that is relevant to cosmology. If we take a look at the distribution of matter in the universe, we will see something like this image taken from the hubble telescope (which I'll add in served as my desktop background for quite a number of years).
- What we observe is that no matter where you are in the universe and no matter what direction you look, the universe is the same on large scales. More specifically, while the structure of universe may be in-homogeneous on small scales, on a large scale the universe is statistically homogeneous and isotropic. These two features are embodied in what is called the cosmological principle.
- Based solely on arguments of homogeneity and isotropy, the large scale geometry of such a universe is described through the RW metric and de Sitter metric (of which one can show that de Sitter space is actually a subset of the RW geometry).
- It of interest to note that with a proper choice of coordinates, the roberston walker geometries can all be expressed in a conformal to flat form
- with this equation here showing the spacetime line element being expressed as a minkowski spacetime multiplied by an overall conformal factor

4 Cosmological Geometries - Robertson Walker

- To see what the RW geometry entails, we have an expression for the spacetime line element in eq. (1), here in comoving coordinates. The geometry describes the expansion of space over time as characterized by the functional form of the scale factor $a(t)$
- The comoving coordinates are at rest with respect to the hubble flow, and if we look at the figure here we can see that while the comoving distance between these two galaxies remains constant, the proper distance increases as space expands according to $a(t)$
- The space itself that is expanding, referred to as the 3-space, is a space of uniform curvature, which can be represented by the curvature constant k, taking the values of $-1, 0, 1$. These values correspond to hyperbolic, flat, or spherical space respectively.
- Spaces of constant curvature are called maximally symmetric, where the curvature tensors take the specific form in eq(2). So here we have the Riemann tensor, which we might recall is the unique tensor composed of second order derivatives of the metric which measures the local curvature. We also have its contractions the ricci tensor, and the ricci scalar and we see that the ricci scalar is a constant in a max. symm. 3-space.
- As mentioned before, with a proper choice of coordinates, the RW metric can be cast into a conformal to flat form
- The simplest case is for $k = 0$, in which if we define the conformal time τ and set $k = 0$, then the line element take the form of eq (4), which we can recognize as conformal to a spherical polar flat metric
- For $k = \pm 1$, we have to perform additional coord. transformations, and we just note that for these the conformal factor is a function of both space and time.
- So this describes the geometry of the large scale universe, but in order to discuss the interaction of gravity and matter, one needs to introduce Einstein field equations, which we do now

5 Einstein Gravity

- One starts with the Einstein Hilbert action defined as the coordinate invariant integral over the Ricci scalar
- Functional variation w.r.t. the metric yields the Einstein tensor $G_{\mu\nu}$, and likewise upon specification of a matter action, one obtains the energy momentum tensor
- In requiring the sum of both the E.H. action and the matter action to be stationary with respect to arbitrary variations in the metric, we obtain the EFE's.
- With (10) showing an identity that relates the derivative of the ricci tensor to its contraction, we can see that the Einstein tensor is conserved
- In the EFE's the interaction of gravity and matter can be seen via the LHS being a pure function of the metric representing the curvature of space while the RHS defines the source of matter and energy
- We'll now look at the linearization of the Einstein field equations according to cosmological perturbation theory

6 Cosmological Perturbation Theory

- So as discussed prior, on a large scale the universe is homogeneous and isotropic which we can think of as the smooth surface of this sphere.
- Now in order to capture the departures from homogeneity and isotropy, things that are necessary in order to form localized structures in spacetime, we introduce small fluctuations on top of the otherwise smooth background. Thus we define the metric according to a background contribution and first order perturbation $h_{\mu\nu}$

- If we then substitute the metric into $G_{\mu\nu}$, it then can be split into a background piece and fluctuation tensor
- Upon similarly perturbing $T_{\mu\nu}$, we can then form the entire background field equations and first order fluctuation equations, where here we've combined them into the tensor $\Delta_{\mu\nu}$
- The background equations serve to define the rate of expansion of space given the source, whereas the fluctuation equations describe the evolution of metric perturbations due to things like over densities arising from source

7 Coordinate Transformations

- So upon perturbing the EFE's, we will need to consider the effect of coordinate transformations
- The field equations are covariant w.r.t. general coordinate transformations, with the metric transforming as in (18)
- If we now consider an infinitesimal coordinate transformation with the vector field ϵ small in the same sense that h is small, then it is conveinent to attribute the whole change in $g_{\mu\nu}$ to a change in the perturbation $h_{\mu\nu}$
- The fluctuation eqns are then to be invariant under the so called gauge transformation of eq (20), where $\Delta h_{\mu\nu}$ is given by this symmetric sum of derivatives onto epsilon
- Thus if $h_{\mu\nu}$ serves as a solution to the EFE's, then and $h'_{\mu\nu}$ defined by (20) will also serve as a solution
- Now since $h_{\mu\nu}$ is a 4x4 symmetric rank 2 tensor it has 10 components, and with the four coordinate functions that define the vector field ϵ, one can then use the coordinate freedom to reduce $h_{\mu\nu}$ to six independent components
- Its also quite instructive to look at the transformation of the fluctuation tensors themselves
- Here we note that if the background tensor vanishes, then the fluctuation tensors themselves are separately gauge invariant
- However, if the background does not vanish, then it only the entire sum of $\delta G_{\mu\nu} + \delta T_{\mu\nu}$ that is gauge invariant. In Einstein gravity the background only vanishes in spaces where the Ricci tensor itself vanishes, and thus for non-flat cosmological geomtries, its only $\Delta_{\mu\nu}$ that is gauge invariant

8 Solution Methods

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9 SVT3 Decomposition

- a

10 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 1/3

- a

11 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 2/3

- a

12 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 3/3

- a

13 SVT3 Integral Formulation 1/4

- Show identical vanishing of surface term upon application of $\partial_i \partial^i$
- Wording: Harmonic function, generalized Laplacian. divergence of gradient.
- Think about conditions requires to vanish

14 SVT3 Integral Formulation 2/4

- a

15 SVT3 Integral Formulation 3/4

- a

16 SVT3 Integral Formulation 4/4

- a

17 SVT4 Setup

- a

18 SVTD Integral Formulation - Maximally Symmetric Space 1/2

- a

19 SVTD Integral Formulation - Maximally Symmetric Space 2/2

- a

20 SVT4 $\delta G_{\mu\nu}$ in a de Sitter Background 1/2

- a

21 SVT4 $\delta G_{\mu\nu}$ in a de Sitter Background 2/2

- a

22 Conformal Gravity Introduction

- a

23 Conformal Invariance in Conformal Gravity

- a

24 Trace Properties in Conformal Gravity

- a

25 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 1/3

- a

26 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 2/3

- a

27 SVT4 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 3/3

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28 Conformal Gravity Robertson-Walker Radiation Era

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29 Conclusions

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30 Computational Methods

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31 References

- a

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33 The End