Astrophysics & Cosmology
HW 8

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11.1 Since the mean free path between collision is l, the volume swept out by a dust particle with cross sectional area πR^2 is

$$V = l\pi R^2.$$

Since only one collision is expected to occur within such a volume, we have

$$n/N = n = l\pi R^2$$

or

$$l = \frac{1}{n\pi R^2}.$$

Given $l = 3000$ ly and $R = 10^{-5}$ cm, we have a density

$$n = \frac{1}{l\pi R^2} = 33.6 \text{ stars cm}^{-3}.$$

Given a football stadium volume of 10^{12} cm^3 this given the number of stars as

$$N = 33.65(10^{12}) = 3.36 \times 10^{12} \text{ stars}.$$

12.1 The number of stars with $f > f_0$ is all stars with $r < r_0$. Thus the total number of stars will lie in a volume spanned by the limit of r_0, i.e.

$$V = \frac{4}{3}\pi r^3.$$

Multiplying by the density

$$N = n(L)\frac{4}{3}\pi r^3.$$

From the luminosity relation

$$L = 4\pi r^2 f_0$$

we have

$$\frac{4}{3}\pi r_0^3 = \frac{L^{3/2} f_0^{-3/2}}{3(4\pi)^{1/2}}$$

and thus the total number of stars is

$$N_L(f > f_0) = \frac{n(L) L^{3/2} f_0^{-3/2}}{3(4\pi)^{1/2}}.$$

12.3 Equating the pressure due to gravity to the pressure from kinetic energy

$$\begin{aligned} P &= F/A = \rho_k \bar{g}_z H \\ &= P_k = \rho_k v_z^2 \end{aligned}$$

this leads to

$$\bar{g}_z = \frac{H}{v_z^2}.$$

Poisson eq:

$$\nabla \cdot \mathbf{g} = -4\pi G\rho.$$

Divergence theorem:

$$\int \nabla \cdot \mathbf{g}\, d^3x = \oint \mathbf{g} \cdot \hat{\mathbf{n}}\, dA = -4\pi G \int d^3x\ \rho$$

$$\oint \mathbf{g} \cdot \hat{\mathbf{n}}\, dA = -4\pi G M.$$

Taking a surface as a rectange of height $2H$ centered at $z = 0$, the only flux of the field through the suface is that at $z = \pm H$. In this case, the gravitational field is in opposite direction to the normal of the surface, thus

$$\oint \mathbf{g} \cdot \hat{\mathbf{n}}\, dA = -2Ag_z.$$

$$\begin{aligned} -2Ag_z &= -4\pi G M \\ \mu = \frac{M}{A} &= \frac{g_z}{2\pi G}. \end{aligned}$$

Given that $g_z = 2\bar{g}_z$ and from the above $\bar{g}_z = H/v_z^2$, this yields

$$\mu = \frac{H}{v^2 \pi G}.$$

12.4

$$\begin{aligned} A - B &= -\frac{r}{2}\frac{d\Omega}{dr} + \frac{1}{2r}\frac{d}{dr}(r^2\Omega) \\ &= -\frac{r}{2}\frac{d\Omega}{dr} + \frac{1}{2r}\left(2r\Omega + r^2\frac{d\Omega}{dr}\right) \\ &= \Omega \end{aligned}$$

If $A = 0$, it follows that $\Omega = -B$.

With

$$(1 - A/B)^{1/2} = 1.6$$

and

$$A = 0.005 \text{ km/sec/lyr}$$

it follows that

$$B = -A/1.56 = -.0032 \text{ km/sec/lyr}.$$

Then the period is

$$T = 2\pi/\Omega = 2\pi/(A - B) = 765.8 \text{ s}^{-1}$$

At a radius of $r = 3 \times 10^4$ lyr, this implies a linear velocity of

$$v = r\Omega = 3 \times 10^4 (0.0082) = 246.15 \ \text{ km s}^{-1}.$$

From the mass galaxy formula

$$M_G = \frac{rv^2}{G}$$

we then have

$$M_G = 2 \times 10^12 \text{ g} = 10^{-21} M_{sun}$$

I must have made an error somewhere within the period $T = 2\pi/\Omega$?

12.5 From Gauss's law, let us take our surface to enclose the mass at radius $r = 30000$ lyr. Then

$$\oint \mathbf{g} \cdot \hat{\mathbf{n}}\, dA = -4\pi GM$$

$$-4\pi r^2 g(r) - 4\pi GM$$

$$M = \frac{r^2 g(r)}{G}.$$

At the radius r, if in circular orbit, the centripetal accleration is that due to gravtiation

$$mg(r) = m\frac{v^2}{r}$$

or

$$g(r) = \frac{v^2}{r}.$$

Thus we have

$$M = \frac{v^2 r}{G}.$$

For $v = 300$ km s^{-1} and $r = 6 \times 10^4$ lyr this yields

$$M_G = 5.1 \times 10^{37}\text{g} = 10^5 M_{sun}.$$