

# $\delta G_{\mu\nu}$ SVT3 in Cosmological Geometries

## 1 Flat $R_{\mu\nu} = 0$

$$ds^2 = (-dt^2 + g_{ij}dx^i dx^j + h_{\mu\nu}dx^\mu dx^\nu) \quad (1.1)$$

$$g_{ij} = \text{diag}(1, 1, 1) \quad \text{or} \quad \text{diag}(1, r^2, r^2 \sin^2 \theta) \quad (1.2)$$

$$h_{00} = -2\phi, \quad h_{0i} = \nabla_i B + B_i, \quad h_{ij} = -2g_{ij}\psi + 2\nabla_i \nabla_j E + \nabla_i E_j + \nabla_j E_i + 2E_{ij} \quad (1.3)$$

$$\delta G_{00} = -2g^{ab}\nabla_a \nabla_b \psi \quad (1.4)$$

$$\delta G_{0i} = \frac{1}{2}g^{ab}\nabla_a \nabla_b B_i - \frac{1}{2}g^{ab}\nabla_a \nabla_b \dot{E}_i - 2\nabla_i \dot{\psi} \quad (1.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\ddot{\psi}g_{ij} - g^{ab}g_{ij}\nabla_b \nabla_a \dot{B} + g^{ab}g_{ij}\nabla_b \nabla_a \ddot{E} - g^{ab}g_{ij}\nabla_b \nabla_a \phi + g^{ab}g_{ij}\nabla_b \nabla_a \psi + \nabla_j \nabla_i \dot{B} \\ & - \nabla_j \nabla_i \ddot{E} + \nabla_j \nabla_i \phi - \nabla_j \nabla_i \psi + \frac{1}{2}\nabla_i \dot{B}_j - \frac{1}{2}\nabla_i \ddot{E}_j + \frac{1}{2}\nabla_j \dot{B}_i - \frac{1}{2}\nabla_j \ddot{E}_i \\ & - \ddot{E}_{ij} + g^{ab}\nabla_a \nabla_b E_{ij} \end{aligned} \quad (1.6)$$

$$\begin{aligned} g^{\mu\nu}\delta G_{\mu\nu} = & -\delta G_{00} + g^{ab}\delta G_{ab} \\ = & -6\ddot{\psi} - 2g^{ab}\nabla_b \nabla_a \dot{B} + 2g^{ab}\nabla_b \nabla_a \ddot{E} - 2g^{ab}\nabla_b \nabla_a \phi + 4g^{ab}\nabla_b \nabla_a \psi \end{aligned} \quad (1.7)$$

## 2 RW

$$ds^2 = \Omega^2(\tau)(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \quad (2.1)$$

$$\tilde{g}_{ij} = \text{diag}\left(\frac{1}{1-kr^2}, r^2, r^2 \sin^2 \theta\right) \quad (2.2)$$

$$f_{00} = -2\phi, \quad f_{0i} = \tilde{\nabla}_i B + B_i, \quad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i \tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \quad (2.3)$$

$$\delta G_{00} = -6k\phi - 6k\psi + 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a B - 2\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} - 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \psi \quad (2.4)$$

$$\begin{aligned} \delta G_{0i} = & 3k\tilde{\nabla}_i B - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2k\tilde{\nabla}_i \dot{E} - 2\tilde{\nabla}_i \dot{\psi} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \phi \\ & + 2kB_i - k\dot{E}_i - B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} + \frac{1}{2}\tilde{g}^{ab}\tilde{\nabla}_a \tilde{\nabla}_b B_i - \frac{1}{2}\tilde{g}^{ab}\tilde{\nabla}_a \tilde{\nabla}_b \dot{E}_i \end{aligned} \quad (2.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\ddot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\ & - 4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a B - \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \dot{B} + \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \ddot{E} \\ & + 2\dot{\Omega}\tilde{g}^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} - \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \phi + \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i B + \tilde{\nabla}_j \tilde{\nabla}_i \dot{B} \\ & - \tilde{\nabla}_j \tilde{\nabla}_i \ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{E} + 2k\tilde{\nabla}_j \tilde{\nabla}_i E - 2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j \tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i E + \tilde{\nabla}_j \tilde{\nabla}_i \phi \\ & - \tilde{\nabla}_j \tilde{\nabla}_i \psi + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \frac{1}{2}\tilde{\nabla}_i \dot{B}_j - \frac{1}{2}\tilde{\nabla}_i \ddot{E}_j - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \dot{E}_j + k\tilde{\nabla}_i E_j - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i E_j \\ & + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \frac{1}{2}\tilde{\nabla}_j \dot{B}_i - \frac{1}{2}\tilde{\nabla}_j \ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \dot{E}_i + k\tilde{\nabla}_j E_i - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j E_i \\ & + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i - \ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{\Omega}E_{ij}\Omega^{-1} + \tilde{g}^{ab}\tilde{\nabla}_a \tilde{\nabla}_b E_{ij} \end{aligned} \quad (2.6)$$

$$\begin{aligned} g^{\mu\nu}\delta G_{\mu\nu} = & \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\ = & 6\dot{\Omega}^2\phi\Omega^{-4} + 6\dot{\Omega}^2\psi\Omega^{-4} - 6\dot{\phi}\dot{\Omega}\Omega^{-3} - 18\dot{\psi}\dot{\Omega}\Omega^{-3} - 12\ddot{\Omega}\phi\Omega^{-3} - 12\ddot{\Omega}\psi\Omega^{-3} - 6\ddot{\psi}\Omega^{-2} \\ & + 6k\phi\Omega^{-2} + 6k\psi\Omega^{-2} - 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-3}\tilde{\nabla}_b \tilde{\nabla}_a B - 2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \dot{B} + 2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \ddot{E} \\ & + 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-3}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} - 2\dot{\Omega}^2\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b \tilde{\nabla}_a E + 4\ddot{\Omega}\tilde{g}^{ab}\Omega^{-3}\tilde{\nabla}_b \tilde{\nabla}_a E + 2k\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a E \\ & - 2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \phi + 4\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \psi \end{aligned} \quad (2.7)$$

$$\begin{aligned} \tilde{g}^{\mu\nu}\delta G_{\mu\nu} = & -\delta G_{00} + \tilde{g}^{ab}\delta G_{ab} \\ = & -6\ddot{\psi} + 6k\phi + 6k\psi + 6\dot{\Omega}^2\phi\Omega^{-2} + 6\dot{\Omega}^2\psi\Omega^{-2} - 6\dot{\phi}\dot{\Omega}\Omega^{-1} - 18\dot{\psi}\dot{\Omega}\Omega^{-1} - 12\ddot{\Omega}\phi\Omega^{-1} \\ & - 12\ddot{\Omega}\psi\Omega^{-1} - 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a B - 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \dot{B} + 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \ddot{E} + 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} \\ & + 2k\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a E - 2\dot{\Omega}^2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a E + 4\ddot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a E - 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \phi \\ & + 4\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \psi \end{aligned} \quad (2.8)$$

### 3 Conformal Flat $\Omega(\tau)$

$$ds^2 = \Omega^2(\tau)(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \quad (3.1)$$

$$\tilde{g}_{ij} = \text{diag}(1, 1, 1) \quad \text{or} \quad \text{diag}(1, r^2, r^2 \sin^2 \theta) \quad (3.2)$$

$$f_{00} = -2\phi, \quad f_{0i} = \tilde{\nabla}_i B + B_i, \quad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i \tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \quad (3.3)$$

$$\delta G_{00} = 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a B - 2\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} - 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \psi \quad (3.4)$$

$$\begin{aligned} \delta G_{0i} = & -\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2\tilde{\nabla}_i \dot{\psi} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \phi \\ & -B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} + \frac{1}{2}\tilde{g}^{ab}\tilde{\nabla}_a \tilde{\nabla}_b B_i - \frac{1}{2}\tilde{g}^{ab}\tilde{\nabla}_a \tilde{\nabla}_b \dot{E}_i \end{aligned} \quad (3.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\ddot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\ & -4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a B - \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \dot{B} + \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \ddot{E} \\ & + 2\dot{\Omega}\tilde{g}^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} - \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \phi + \tilde{g}^{ab}\tilde{g}_{ij}\tilde{\nabla}_b \tilde{\nabla}_a \psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i B + \tilde{\nabla}_j \tilde{\nabla}_i \dot{B} \\ & - \tilde{\nabla}_j \tilde{\nabla}_i \dot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{E} - 2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j \tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i E + \tilde{\nabla}_j \tilde{\nabla}_i \phi - \tilde{\nabla}_j \tilde{\nabla}_i \psi \\ & + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \frac{1}{2}\tilde{\nabla}_i \dot{B}_j - \frac{1}{2}\tilde{\nabla}_i \dot{E}_j - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \dot{E}_j - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i E_j + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j \\ & + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \frac{1}{2}\tilde{\nabla}_j \dot{B}_i - \frac{1}{2}\tilde{\nabla}_j \dot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \dot{E}_i - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j E_i + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i \\ & - \ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{\Omega}E_{ij}\Omega^{-1} + \tilde{g}^{ab}\tilde{\nabla}_a \tilde{\nabla}_b E_{ij} \end{aligned} \quad (3.6)$$

$$\begin{aligned} g^{\mu\nu}\delta G_{\mu\nu} = & \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\ = & 6\dot{\Omega}^2\phi\Omega^{-4} + 6\dot{\Omega}^2\psi\Omega^{-4} - 6\dot{\phi}\dot{\Omega}\Omega^{-3} - 18\dot{\psi}\dot{\Omega}\Omega^{-3} - 12\ddot{\Omega}\phi\Omega^{-3} - 12\ddot{\Omega}\psi\Omega^{-3} - 6\ddot{\psi}\Omega^{-2} \\ & - 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-3}\tilde{\nabla}_b \tilde{\nabla}_a B - 2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \dot{B} + 2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \ddot{E} + 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-3}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} \\ & - 2\dot{\Omega}^2\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b \tilde{\nabla}_a E + 4\ddot{\Omega}\tilde{g}^{ab}\Omega^{-3}\tilde{\nabla}_b \tilde{\nabla}_a E - 2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \phi + 4\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a \psi \end{aligned} \quad (3.7)$$

$$\begin{aligned} \tilde{g}^{\mu\nu}\delta G_{\mu\nu} = & -\delta G_{00} + \tilde{g}^{ab}\delta G_{ab} \\ = & -6\ddot{\psi} + 6\dot{\Omega}^2\phi\Omega^{-2} + 6\dot{\Omega}^2\psi\Omega^{-2} - 6\dot{\phi}\dot{\Omega}\Omega^{-1} - 18\dot{\psi}\dot{\Omega}\Omega^{-1} - 12\ddot{\Omega}\phi\Omega^{-1} - 12\ddot{\Omega}\psi\Omega^{-1} \\ & - 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a B - 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \dot{B} + 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \ddot{E} + 6\dot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \dot{E} \\ & - 2\dot{\Omega}^2\tilde{g}^{ab}\Omega^{-2}\tilde{\nabla}_b \tilde{\nabla}_a E + 4\ddot{\Omega}\tilde{g}^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a E - 2\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \phi + 4\tilde{g}^{ab}\tilde{\nabla}_b \tilde{\nabla}_a \psi \end{aligned} \quad (3.8)$$

## 4 Conformal Flat $\Omega(\tau) = \tau$

$$ds^2 = \tau^2(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \quad (4.1)$$

$$\tilde{g}_{ij} = \text{diag}(1, 1, 1) \quad \text{or} \quad \text{diag}(1, r^2, r^2 \sin^2 \theta) \quad (4.2)$$

$$f_{00} = -2\phi, \quad f_{0i} = \tilde{\nabla}_i B + B_i, \quad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i \tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \quad (4.3)$$

$$\delta G_{00} = 6\dot{\psi}\tau^{-1} + 2\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tilde{\nabla}_a \tilde{\nabla}^a \psi \quad (4.4)$$

$$\delta G_{0i} = -\tau^{-2}\tilde{\nabla}_i B - 2\tilde{\nabla}_i \dot{\psi} - 2\tau^{-1}\tilde{\nabla}_i \phi - B_i \tau^{-2} + \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a B_i - \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E}_i \quad (4.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\ddot{\psi}\tilde{g}_{ij} - 2\dot{\phi}\tilde{g}_{ij}\tau^{-1} - 4\dot{\psi}\tilde{g}_{ij}\tau^{-1} + 2\tilde{g}_{ij}\tau^{-2}\phi + 2\tilde{g}_{ij}\tau^{-2}\psi - 4\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B \\ & - 2\tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + 2\tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} + 4\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \phi + 2\tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \psi \\ & + 2\tau^{-1}\tilde{\nabla}_j \tilde{\nabla}_i B + \tilde{\nabla}_j \tilde{\nabla}_i \dot{B} - \tilde{\nabla}_j \tilde{\nabla}_i \ddot{E} - 2\tau^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{E} - 2\tau^{-2}\tilde{\nabla}_j \tilde{\nabla}_i E + \tilde{\nabla}_j \tilde{\nabla}_i \phi - \tilde{\nabla}_j \tilde{\nabla}_i \psi \\ & + \tau^{-1}\tilde{\nabla}_i B_j + \frac{1}{2}\tilde{\nabla}_i \dot{B}_j - \frac{1}{2}\tilde{\nabla}_i \ddot{E}_j - \tau^{-1}\tilde{\nabla}_i \dot{E}_j - \tau^{-2}\tilde{\nabla}_i E_j + \tau^{-1}\tilde{\nabla}_j B_i + \frac{1}{2}\tilde{\nabla}_j \dot{B}_i - \frac{1}{2}\tilde{\nabla}_j \ddot{E}_i \\ & - \tau^{-1}\tilde{\nabla}_j \dot{E}_i - \tau^{-2}\tilde{\nabla}_j E_i - \ddot{E}_{ij} - 2E_{ij}\tau^{-2} - 2\dot{E}_{ij}\tau^{-1} + 2\tilde{\nabla}_a \tilde{\nabla}^a E_{ij} \end{aligned} \quad (4.6)$$

$$\begin{aligned} g^{\mu\nu}\delta G_{\mu\nu} = & \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\ = & -6\dot{\phi}\tau^{-3} - 18\dot{\psi}\tau^{-3} - 6\ddot{\psi}\tau^{-2} + 6\tau^{-4}\phi + 6\tau^{-4}\psi - 6\tau^{-3}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} \\ & + 2\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} + 6\tau^{-3}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tau^{-4}\tilde{\nabla}_a \tilde{\nabla}^a E - 2\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \phi + 4\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \psi \end{aligned} \quad (4.7)$$

$$\begin{aligned} \tilde{g}^{\mu\nu}\delta G_{\mu\nu} = & -\delta G_{00} + \tilde{g}^{ab}\delta G_{ab} \\ = & -6\ddot{\psi} - 6\dot{\phi}\tau^{-1} - 18\dot{\psi}\tau^{-1} + 6\tau^{-2}\phi + 6\tau^{-2}\psi - 6\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + 2\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} \\ & + 6\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a E - 2\tilde{\nabla}_a \tilde{\nabla}^a \phi + 4\tilde{\nabla}_a \tilde{\nabla}^a \psi \end{aligned} \quad (4.8)$$

## 5 Conformal Flat $\Omega(\tau) = (H\tau)^{-1}$

$$ds^2 = \frac{1}{(H\tau)^2}(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \quad (5.1)$$

$$\tilde{g}_{ij} = \text{diag}(1, 1, 1) \quad \text{or} \quad \text{diag}(1, r^2, r^2 \sin^2 \theta) \quad (5.2)$$

$$f_{00} = -2\phi, \quad f_{0i} = \tilde{\nabla}_i B + B_i, \quad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i \tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \quad (5.3)$$

$$\delta G_{00} = -6\dot{\psi}\tau^{-1} - 2\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B + 2\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tilde{\nabla}_a \tilde{\nabla}^a \psi \quad (5.4)$$

$$\delta G_{0i} = 3\tau^{-2}\tilde{\nabla}_i B - 2\tilde{\nabla}_i \dot{\psi} + 2\tau^{-1}\tilde{\nabla}_i \phi + 3B_i\tau^{-2} + \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a B_i - \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E}_i \quad (5.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\ddot{\psi}\tilde{g}_{ij} + 2\dot{\phi}\tilde{g}_{ij}\tau^{-1} + 4\dot{\psi}\tilde{g}_{ij}\tau^{-1} - 6\tilde{g}_{ij}\tau^{-2}\phi - 6\tilde{g}_{ij}\tau^{-2}\psi + 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B \\ & - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} - 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \phi + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \psi \\ & - 2\tau^{-1}\tilde{\nabla}_j \tilde{\nabla}_i B + \tilde{\nabla}_j \tilde{\nabla}_i \dot{B} - \tilde{\nabla}_j \tilde{\nabla}_i \ddot{E} + 2\tau^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{E} + 6\tau^{-2}\tilde{\nabla}_j \tilde{\nabla}_i E + \tilde{\nabla}_j \tilde{\nabla}_i \phi - \tilde{\nabla}_j \tilde{\nabla}_i \psi \\ & - \tau^{-1}\tilde{\nabla}_i B_j + \frac{1}{2}\tilde{\nabla}_i \dot{B}_j - \frac{1}{2}\tilde{\nabla}_i \ddot{E}_j + \tau^{-1}\tilde{\nabla}_i \dot{E}_j + 3\tau^{-2}\tilde{\nabla}_i E_j - \tau^{-1}\tilde{\nabla}_j B_i + \frac{1}{2}\tilde{\nabla}_j \dot{B}_i \\ & - \frac{1}{2}\tilde{\nabla}_j \ddot{E}_i + \tau^{-1}\tilde{\nabla}_j \dot{E}_i + 3\tau^{-2}\tilde{\nabla}_j E_i - \ddot{E}_{ij} + 6E_{ij}\tau^{-2} + 2\dot{E}_{ij}\tau^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{ij} \end{aligned} \quad (5.6)$$

$$\begin{aligned} g^{\mu\nu}\delta G_{\mu\nu} &= \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\ &= H^2 \left( 6\dot{\phi}\tau + 18\dot{\psi}\tau - 6\ddot{\psi}\tau^2 - 18\phi - 18\psi + 6\tau\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + 2\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} \right. \\ &\quad \left. - 6\tau\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} + 6\tilde{\nabla}_a \tilde{\nabla}^a E - 2\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \phi + 4\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \psi \right) \end{aligned} \quad (5.7)$$

$$\begin{aligned} \tilde{g}^{\mu\nu}\delta G_{\mu\nu} &= -\delta G_{00} + \tilde{g}^{ab}\delta G_{ab} \\ &= -6\ddot{\psi} + 6\dot{\phi}\tau^{-1} + 18\dot{\psi}\tau^{-1} - 18\tau^{-2}\phi - 18\tau^{-2}\psi + 6\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + 2\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} \\ &\quad - 6\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} + 6\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a E - 2\tilde{\nabla}_a \tilde{\nabla}^a \phi + 4\tilde{\nabla}_a \tilde{\nabla}^a \psi \end{aligned} \quad (5.8)$$

$$\begin{aligned} \Omega^{-2}\tilde{g}^{ab}\delta G_{ab} &= H^2 \left( 6\dot{\phi}\tau + 12\dot{\psi}\tau - 6\ddot{\psi}\tau^2 - 18\phi - 18\psi + 4\tau\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + 2\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} \right. \\ &\quad \left. - 4\tau\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} + 6\tilde{\nabla}_a \tilde{\nabla}^a E - 2\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \phi + 2\tau^2\tilde{\nabla}_a \tilde{\nabla}^a \psi \right) \end{aligned} \quad (5.9)$$

## 6 Conformal Flat $\Omega(x)$

$$ds^2 = \Omega^2(x)(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \quad (6.1)$$

$$\tilde{g}_{ij} = \text{diag}(1, 1, 1) \quad \text{or} \quad \text{diag}(1, r^2, r^2 \sin^2 \theta) \quad (6.2)$$

$$f_{00} = -2\phi, \quad f_{0i} = \tilde{\nabla}_i B + B_i, \quad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i \tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \quad (6.3)$$

$$\begin{aligned} \delta G_{00} = & 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tilde{\nabla}_a \tilde{\nabla}^a \psi + 4\phi\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega \\ & + 4\psi\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega + 4\Omega^{-1}\tilde{\nabla}_a \dot{\Omega}\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a B - 2\Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \psi \\ & - 2\phi\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega - 2\psi\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b \tilde{\nabla}_a E \\ & + 2\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}_a E\tilde{\nabla}^b \Omega - 4\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b \tilde{\nabla}^a E \\ & + 4B^a \Omega^{-1}\tilde{\nabla}_a \dot{\Omega} - 2B^a \dot{\Omega}\Omega^{-2}\tilde{\nabla}_a \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b E_a + 2\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}_b \Omega\tilde{\nabla}^b E^a \\ & - 4\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b E^a - 4E^{ab}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega + 2E_{ab}\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}^b \Omega \end{aligned} \quad (6.4)$$

$$\begin{aligned} \delta G_{0i} = & -\dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_i B + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_i B + \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_i B - 2\tilde{\nabla}_i \dot{\psi} \\ & - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \phi + 2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_i \tilde{\nabla}_a \dot{E} - B_i \dot{\Omega}^2 \Omega^{-2} + 2B_i \dot{\Omega}\Omega^{-1} \\ & + \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a B_i - \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E}_i - 2B_i \Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega + \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a B_i - \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \dot{E}_i \\ & + B_i \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega - \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}_i B^a - \Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}_i \dot{E}^a - 2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a \Omega \end{aligned} \quad (6.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\dot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2 \tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2 \tilde{g}_{ij}\psi\Omega^{-2} - 2\phi\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\psi\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\ & - 4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} \\ & - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \phi + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \psi - 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \dot{\Omega}\tilde{\nabla}^a B + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a B \\ & - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \dot{B} - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \phi + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b \tilde{\nabla}_a E \\ & - 2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}_a E\tilde{\nabla}^b \Omega + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b \tilde{\nabla}^a E + 2\Omega^{-1}\tilde{\nabla}_i \Omega\tilde{\nabla}_j \psi \\ & + 2\Omega^{-1}\tilde{\nabla}_i \psi\tilde{\nabla}_j \Omega + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i B + \tilde{\nabla}_j \tilde{\nabla}_i \dot{B} - \tilde{\nabla}_j \tilde{\nabla}_i \ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{E} \\ & - 2\dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_j \tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j \tilde{\nabla}_i E - 4\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i E + 2\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i E \\ & + \tilde{\nabla}_j \tilde{\nabla}_i \phi - \tilde{\nabla}_j \tilde{\nabla}_i \psi - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i \tilde{\nabla}_a E \\ & - 4B^a \tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \dot{\Omega} + 2B^a \dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega - 2\dot{B}^a \tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a \Omega + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_b \tilde{\nabla}^b E_a \\ & - 2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}_b \Omega\tilde{\nabla}^b E^a + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega\tilde{\nabla}^b E^a + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \frac{1}{2}\tilde{\nabla}_i \dot{B}_j - \frac{1}{2}\tilde{\nabla}_i \ddot{E}_j \\ & - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i \dot{E}_j - \dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_i E_j + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j - 2\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_i E_j \\ & + \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_i E_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \frac{1}{2}\tilde{\nabla}_j \dot{B}_i - \frac{1}{2}\tilde{\nabla}_j \ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j \dot{E}_i - \dot{\Omega}^2 \Omega^{-2}\tilde{\nabla}_j E_i \\ & + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega\tilde{\nabla}_j E_i + \Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_j \tilde{\nabla}_i E_a \\ & - \ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{E}_{ij}\Omega^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{ij} - 4E_{ij}\Omega^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \Omega \\ & + 2\Omega^{-1}\tilde{\nabla}_a E_{ij}\tilde{\nabla}^a \Omega + 2E_{ij}\Omega^{-2}\tilde{\nabla}_a \Omega\tilde{\nabla}^a \Omega + 4E^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b \tilde{\nabla}_a \Omega \\ & - 2E_{ab}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a \Omega\tilde{\nabla}^b \Omega - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_i E_{ja} - 2\Omega^{-1}\tilde{\nabla}^a \Omega\tilde{\nabla}_j E_{ia} \end{aligned} \quad (6.6)$$

$$\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} &= \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\
&= 6\dot{\Omega}^2\phi\Omega^{-4} + 6\dot{\Omega}^2\psi\Omega^{-4} - 6\dot{\phi}\dot{\Omega}\Omega^{-3} - 18\dot{\psi}\dot{\Omega}\Omega^{-3} - 12\ddot{\Omega}\phi\Omega^{-3} - 12\ddot{\Omega}\psi\Omega^{-3} - 6\ddot{\psi}\Omega^{-2} \\
&\quad - 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aB - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
&\quad - 2\dot{\Omega}^2\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^aE + 4\ddot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 4\phi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\
&\quad - 4\psi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega - 16\Omega^{-3}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^aB + 8\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB - 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} \\
&\quad - 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi + 2\phi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega + 2\psi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega \\
&\quad + 2\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE - 4\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE\tilde{\nabla}_b\tilde{\nabla}^b\Omega + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE \\
&\quad - 8\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega + 16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE - 16B^a\Omega^{-3}\tilde{\nabla}_a\dot{\Omega} \\
&\quad + 8B^a\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega - 6\dot{B}^a\Omega^{-3}\tilde{\nabla}_a\Omega + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a - 8\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a \\
&\quad + 16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a + 16E^{ab}\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega - 8E_{ab}\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{6.7}$$

$$\begin{aligned}
\tilde{g}^{\mu\nu}\delta G_{\mu\nu} &= -\delta G_{00} + \tilde{g}^{ab}\delta G_{ab} \\
&= -6\ddot{\psi} + 6\dot{\Omega}^2\phi\Omega^{-2} + 6\dot{\Omega}^2\psi\Omega^{-2} - 6\dot{\phi}\dot{\Omega}\Omega^{-1} - 18\dot{\psi}\dot{\Omega}\Omega^{-1} - 12\ddot{\Omega}\phi\Omega^{-1} - 12\ddot{\Omega}\psi\Omega^{-1} \\
&\quad - 6\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^aB - 2\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 6\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^aE \\
&\quad + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^aE - 2\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\tilde{\nabla}_a\tilde{\nabla}^a\psi - 4\phi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega - 4\psi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\
&\quad - 16\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^aB + 8\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB - 6\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} - 6\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi \\
&\quad + 6\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi + 2\phi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega + 2\psi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega + 2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE \\
&\quad - 4\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^aE\tilde{\nabla}_b\tilde{\nabla}^b\Omega + 6\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE - 8\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega \\
&\quad + 16\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE - 16B^a\Omega^{-1}\tilde{\nabla}_a\dot{\Omega} + 8B^a\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega - 6\dot{B}^a\Omega^{-1}\tilde{\nabla}_a\Omega \\
&\quad + 6\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a - 8\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a + 16\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a \\
&\quad + 16E^{ab}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega - 8E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{6.8}$$

## 7 Conformal Flat $\Omega(\tau) = \tau/2$

$$ds^2 = \frac{\tau^2}{4} (-d\tau^2 + \tilde{g}_{ij} dx^i dx^j + f_{\mu\nu} dx^\mu dx^\nu) \quad (7.1)$$

$$\tilde{g}_{ij} = \text{diag}(1, 1, 1) \quad \text{or} \quad \text{diag}(1, r^2, r^2 \sin^2 \theta) \quad (7.2)$$

$$f_{00} = -2\phi, \quad f_{0i} = \tilde{\nabla}_i B + B_i, \quad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i \tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \quad (7.3)$$

$$\delta G_{00} = 6\dot{\psi}\tau^{-1} + 2\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tilde{\nabla}_a \tilde{\nabla}^a \psi \quad (7.4)$$

$$\delta G_{0i} = -\tau^{-2}\tilde{\nabla}_i B - 2\tilde{\nabla}_i \dot{\psi} - 2\tau^{-1}\tilde{\nabla}_i \phi - B_i \tau^{-2} + \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a B_i - \frac{1}{2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E}_i \quad (7.5)$$

$$\begin{aligned} \delta G_{ij} = & -2\ddot{\psi}\tilde{g}_{ij} - 2\dot{\phi}\tilde{g}_{ij}\tau^{-1} - 4\dot{\psi}\tilde{g}_{ij}\tau^{-1} + 2\tilde{g}_{ij}\tau^{-2}\phi + 2\tilde{g}_{ij}\tau^{-2}\psi - 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B \\ & - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} + 2\tilde{g}_{ij}\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \phi + \tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \psi \\ & + 2\tau^{-1}\tilde{\nabla}_j \tilde{\nabla}_i B + \tilde{\nabla}_j \tilde{\nabla}_i \dot{B} - \tilde{\nabla}_j \tilde{\nabla}_i \ddot{E} - 2\tau^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{E} - 2\tau^{-2}\tilde{\nabla}_j \tilde{\nabla}_i E + \tilde{\nabla}_j \tilde{\nabla}_i \phi - \tilde{\nabla}_j \tilde{\nabla}_i \psi \\ & + \tau^{-1}\tilde{\nabla}_i B_j + \frac{1}{2}\tilde{\nabla}_i \dot{B}_j - \frac{1}{2}\tilde{\nabla}_i \ddot{E}_j - \tau^{-1}\tilde{\nabla}_i \dot{E}_j - \tau^{-2}\tilde{\nabla}_i E_j + \tau^{-1}\tilde{\nabla}_j B_i + \frac{1}{2}\tilde{\nabla}_j \dot{B}_i - \frac{1}{2}\tilde{\nabla}_j \ddot{E}_i \\ & - \tau^{-1}\tilde{\nabla}_j \dot{E}_i - \tau^{-2}\tilde{\nabla}_j E_i - \ddot{E}_{ij} - 2E_{ij}\tau^{-2} - 2\dot{E}_{ij}\tau^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{ij} \end{aligned} \quad (7.6)$$

$$\begin{aligned} g^{\mu\nu}\delta G_{\mu\nu} = & \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\ = & -24\dot{\phi}\tau^{-3} - 72\dot{\psi}\tau^{-3} - 24\ddot{\psi}\tau^{-2} + 24\tau^{-4}\phi + 24\tau^{-4}\psi - 24\tau^{-3}\tilde{\nabla}_a \tilde{\nabla}^a B - 8\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} \\ & + 8\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} + 24\tau^{-3}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 8\tau^{-4}\tilde{\nabla}_a \tilde{\nabla}^a E - 8\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \phi + 16\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \psi \end{aligned} \quad (7.7)$$

$$\begin{aligned} \tilde{g}^{\mu\nu}\delta G_{\mu\nu} = & -\delta G_{00} + \tilde{g}^{ab}\delta G_{ab} \\ = & -6\ddot{\psi} - 6\dot{\phi}\tau^{-1} - 18\dot{\psi}\tau^{-1} + 6\tau^{-2}\phi + 6\tau^{-2}\psi - 6\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a B - 2\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} + 2\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} \\ & + 6\tau^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 2\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a E - 2\tilde{\nabla}_a \tilde{\nabla}^a \phi + 4\tilde{\nabla}_a \tilde{\nabla}^a \psi \end{aligned} \quad (7.8)$$

$$\begin{aligned} \Omega^{-2}\tilde{g}^{ab}\delta G_{ab} = & -24\dot{\phi}\tau^{-3} - 48\dot{\psi}\tau^{-3} - 24\ddot{\psi}\tau^{-2} + 24\tau^{-4}\phi + 24\tau^{-4}\psi - 16\tau^{-3}\tilde{\nabla}_a \tilde{\nabla}^a B - 8\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \dot{B} \\ & + 8\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \ddot{E} + 16\tau^{-3}\tilde{\nabla}_a \tilde{\nabla}^a \dot{E} - 8\tau^{-4}\tilde{\nabla}_a \tilde{\nabla}^a E - 8\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \phi + 8\tau^{-2}\tilde{\nabla}_a \tilde{\nabla}^a \psi \end{aligned} \quad (7.9)$$