TT Projection Curved Space v1

1 $h_{\mu\nu}$ General Decomposition

At present, I'm unable to express the general curved space decomposition for $h_{\mu\nu}$ in terms of scalar propagators.

1.1 Maximally Symmetric Space

$$\begin{aligned} h_{\mu\nu} &= h^{T\theta}_{\mu\nu} + \nabla_\mu W_\nu + \nabla_\nu W_\mu - \frac{g_{\mu\nu}}{D-1}(\nabla^\sigma W_\sigma - h) \\ &+ \frac{2-D}{D-1}\left(\nabla_\mu\nabla_\nu - \frac{g_{\mu\nu}R}{D(D-1)}\right)\int D(x,x')\nabla^\sigma W_\sigma - \frac{1}{D-1}\left(\nabla_\mu\nabla_\nu - \frac{g_{\mu\nu}R}{D(D-1)}\right)\int D(x,x')h \end{aligned} \tag{1.1}$$

$$\begin{aligned} \left(\nabla_\alpha\nabla^\alpha - \frac{R}{D-1}\right)D(x,x') &= g^{-1/2}\delta^4(x-x') \\ \nabla^\mu h_{\mu\nu} &= \left(\nabla_\alpha\nabla^\alpha - \frac{R}{D}\right)W_\nu \end{aligned} \tag{1.2}$$

1.2 Curved Space

Below is a generalization of the decomposition above. When the space is maximally symmetric, we expect to be able to bring the decomposition to the form of (1.1).

$$\begin{aligned} h_{\mu\nu} &= h^{T\theta}_{\mu\nu} + \nabla_\mu W_\nu + \nabla_\nu W_\mu - \frac{g_{\mu\nu}}{D-1}A_1(\nabla^\sigma W_\sigma - h) \\ &+ \frac{2-D}{D-1}A_2\left(B_3\nabla_\mu\nabla_\nu - B_1\frac{g_{\mu\nu}R}{D(D-1)} + B_2R_{\mu\nu}\right)\int D(x,x')\nabla^\sigma W_\sigma \\ &- \frac{1}{D-1}A_3\left(C_3\nabla_\mu\nabla_\nu - C_1\frac{g_{\mu\nu}R}{D(D-1)} + C_2R_{\mu\nu}\right)\int D(x,x')h \end{aligned} \tag{1.3}$$

We take $D = 4$ and define

$$\begin{aligned} J(x) &= \int D(x,x')\nabla^\sigma W_\sigma \\ K(x) &= \int D(x,x')h. \end{aligned} \tag{1.4}$$

The transverse and trace conditions are

$$\begin{aligned} \nabla_\alpha h_\nu{}^\alpha &= -R_{\nu\alpha}W^\alpha + \nabla_\alpha\nabla^\alpha W_\nu - \tfrac{2}{3}A_2B_2R_{\nu\alpha}\nabla^\alpha J(x) + \tfrac{2}{3}A_2B_3R_{\nu\alpha}\nabla^\alpha J(x) \\ &-\tfrac{1}{3}A_3C_2R_{\nu\alpha}\nabla^\alpha K(x) + \tfrac{1}{3}A_3C_3R_{\nu\alpha}\nabla^\alpha K(x) + \tfrac{1}{18}A_2B_1R\nabla_\nu J(x) + \tfrac{1}{36}A_3C_1R\nabla_\nu K(x) \\ &+\tfrac{1}{3}A_1\nabla_\nu h + \tfrac{1}{18}A_2B_1J(x)\nabla_\nu R - \tfrac{1}{3}A_2B_2J(x)\nabla_\nu R + \tfrac{1}{36}A_3C_1K(x)\nabla_\nu R - \tfrac{1}{6}A_3C_2K(x)\nabla_\nu R \\ &+\nabla_\nu\nabla_\alpha W^\alpha - \tfrac{1}{3}A_1\nabla_\nu\nabla_\alpha W^\alpha - \tfrac{2}{3}A_2B_3\nabla_\nu\nabla_\alpha\nabla^\alpha J(x) - \tfrac{1}{3}A_3C_3\nabla_\nu\nabla_\alpha\nabla^\alpha K(x) \end{aligned} \tag{1.5}$$

$$
\begin{aligned}
h &= \tfrac{4}{3}A_1 h + \tfrac{2}{9}A_2B_1J(x)R - \tfrac{2}{3}A_2B_2J(x)R + \tfrac{1}{9}A_3C_1K(x)R - \tfrac{1}{3}A_3C_2K(x)R + 2\nabla_\alpha W^\alpha \\
&\quad -\tfrac{4}{3}A_1\nabla_\alpha W^\alpha - \tfrac{2}{3}A_2B_3\nabla_\alpha\nabla^\alpha J(x) - \tfrac{1}{3}A_3C_3\nabla_\alpha\nabla^\alpha K(x)
\end{aligned} \tag{1.6}
$$

(1.1) corresponds to $A_1 = A_2 = A_3 = 1$, $B_3 = B_1 = C_3 = C_1 = 1$, $B_2 = C_2 = 0$:

$$
\begin{aligned}
\nabla_\alpha h_\nu{}^\alpha &= -R_{\nu\alpha}W^\alpha + \nabla_\alpha\nabla^\alpha W_\nu + \tfrac{2}{3}R_{\nu\alpha}\nabla^\alpha J(x) + \tfrac{1}{3}R_{\nu\alpha}\nabla^\alpha K(x) + \tfrac{1}{18}R\nabla_\nu J(x) \\
&\quad + \tfrac{1}{36}R\nabla_\nu K(x) + \tfrac{1}{3}\nabla_\nu h + \tfrac{1}{18}J(x)\nabla_\nu R + \tfrac{1}{36}K(x)\nabla_\nu R + \tfrac{2}{3}\nabla_\nu\nabla_\alpha W^\alpha - \tfrac{2}{3}\nabla_\nu\nabla_\alpha\nabla^\alpha J(x) \\
&\quad -\tfrac{1}{3}\nabla_\nu\nabla_\alpha\nabla^\alpha K(x)
\end{aligned} \tag{1.7}
$$

$$
h = -\tfrac{2}{3}J(x)R - \tfrac{1}{3}K(x)R - 2\nabla_\alpha W^\alpha + 2\nabla_\alpha\nabla^\alpha J(x) + \nabla_\alpha\nabla^\alpha K(x) \tag{1.8}
$$

If we straight forwardly covariantize the flat space decomposition this corresponds to $A_1 = A_2 = A_3 = 1$, $B_3 = C_3 = 1$, $B_2 = C_2 = B_1 = C_1 = 0$:

$$
\begin{aligned}
\nabla_\alpha h_\nu{}^\alpha &= -R_{\nu\alpha}W^\alpha + \nabla_\alpha\nabla^\alpha W_\nu + \tfrac{2}{3}R_{\nu\alpha}\nabla^\alpha J(x) + \tfrac{1}{3}R_{\nu\alpha}\nabla^\alpha K(x) + \tfrac{1}{3}\nabla_\nu h + \tfrac{2}{3}\nabla_\nu\nabla_\alpha W^\alpha \\
&\quad -\tfrac{2}{3}\nabla_\nu\nabla_\alpha\nabla^\alpha J(x) - \tfrac{1}{3}\nabla_\nu\nabla_\alpha\nabla^\alpha K(x)
\end{aligned} \tag{1.9}
$$

$$
h = -2\nabla_\alpha W^\alpha + 2\nabla_\alpha\nabla^\alpha J(x) + \nabla_\alpha\nabla^\alpha K(x) \tag{1.10}
$$