Dissertation Notes

Cosmological Geometries

- $k \in \{-1, 0, 1\}$ topological?
- Statistically homeogeneous and isoptropic on scales large enough to includes clusters of galaxies (10^8 or 10^9 lightyears)
- General form derived from homogeneity and isotropy, Einstein equations only serve to define $a(t)$.
- 3D space of uniform curvature. Almost homogeneous and isotropic when averaged over a large scale

Einstein Gravity

- Total action I is stationary with respect to arbitrary variation in $g_{\mu\nu}$ if and only if $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$ (Weinberg pg. 364)
- Can show conservation of $\nabla^\mu T_{\mu\nu}$ by imposing field equation (only holds for stationary paths), or by imposing (scalar) field equations of motion to $T_{\mu\nu}$ directly (holds for stationary paths beyond those satisfying field equations)

Perturbations

- Small compared to what? "We are helped in this task by the fact that we expect such inhomogeneities to be of very small amplitude early on so we can adopt a kind of perturbative approach, at least for the early stages of the problem. If the length scale of the perturbations is smaller than the effective cosmological horizon $d_H = c/H_0$, a Newtonian treatment of the subject is expected to be valid."
- "Cosmological perturbation theory applies to largescales, up to and beyond the particle horizon of the observable Universe. Such length scales are, by definition,comparable to the characteristic wavelength, $\lambda_c \sim L_C$, where L_C the typical length scale associated with the regime of cosmological perturbation theory. "
- "The most natural explanation for the large-scale structures seen in galaxy surveys (e.g. superclusters, walls, and filaments) is that they are the result of gravitational amplification of small primordial fluctuations due to the gravitational interaction of collisionless cold dark matter (CDM) particles in an expanding universe"
- Fluctuations capture departure from homogeneity and isotropy
- "As an essential feature of the analysis presented here, we assume that during most of the history of the universe all departures from homogeneity and isotropy have been small, so that they can be treated as first-order perturbations."
- First given by Lifshitz 1946, created notation $\delta g_{\mu\nu} = h_{\mu\nu}$
- As discussed prior, on a large scale the universe is homogeneous and isoptropic which we may think of as the smooth surface of a sphere. To capture the departures from homogenenity and isotropy necessary in forming localized structures in spacetime, we introduce the fluctuation $h_{\mu\nu}$ and define the metric according to a background contribution and first order perturbation.

Gauge Transformations

- EFE covariant w.r.t. general coordinate transformations
- Weinberg G&C 10.9, 15.10 580, Cosmology 235
- Maggiore, $x^\mu \to x'^\mu$. x'^μ must be invertible, differentiable, and with a differential inverse (i.e. an arbitrary diffeomorphism)
- Brane Localized Mannheim pg. 82 (shows change of entire $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$ and collects with $h_{\mu\nu}$)
- Attribute the whole change in $g_{\mu\nu}$ to change in perturbation $h_{\mu\nu}$

SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background

- How to address gauge invariants?

SVT3 Integral Formulation

- Show identical vanishing of surface term upon application of $\partial_i \partial^i$
- Wording: Harmonic function, generalized Laplacian. divergence of gradient.
- Delta functions only take support at parameter value zero