Coordinate Transformation SVT3 RW v3

1 Background and Fluctuations

$$ds^2 \;=\; \Omega^2(\tau)(\tilde{g}_{\mu\nu} + f_{\mu\nu})dx^\mu dx^\nu = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \tag{1.1}$$

$$\tilde{g}_{\mu\nu} \;=\; \mathrm{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right), \qquad \tilde{\Gamma}^\lambda_{\alpha\beta} = \delta^\lambda_i \delta^j_\alpha \delta^k_\beta \tilde{\Gamma}^i_{jk} \tag{1.2}$$

2 SVT3

2.1 $f_{\mu\nu}(SVT3)$

$$
\begin{aligned}
f_{00} &= -2\phi \\
f_{0i} &= B_i + \tilde{\nabla}_i B \\
f_{ij} &= -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \\
\tilde{g}^{ij} f_{ij} &= -6\psi + 2\tilde{\nabla}^k\tilde{\nabla}_k E \\
\tilde{g}^{\mu\nu} f_{\mu\nu} &= 2\phi - 6\psi + 2\tilde{\nabla}^k\tilde{\nabla}_k E
\end{aligned}
\tag{2.1}
$$

2.2 $SVT3(f_{\mu\nu})$

$$\phi \;=\; -\tfrac{1}{2}f_{00} \tag{2.2}$$

$$\tilde{\nabla}_a\tilde{\nabla}^a B \;=\; \tilde{\nabla}^a f_{0a} \tag{2.3}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)B_i \;=\; (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)f_{0i} - \tilde{\nabla}_i\tilde{\nabla}^a f_{0a} \tag{2.4}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\psi \;=\; \frac{1}{4}\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} - (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{g}^{bc} f_{bc})\right] \tag{2.5}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b E \;=\; \frac{3}{4}\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} - \frac{1}{3}\tilde{\nabla}_a\tilde{\nabla}^a(\tilde{g}^{bc} f_{bc})\right] \tag{2.6}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)E_i \;=\; (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\tilde{\nabla}^b f_{ib} - \tilde{\nabla}_i\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} \tag{2.7}$$

$$
\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)(2E_{ij}) \;=\;\; & (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)f_{ij} + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} + (\tilde{\nabla}_a\tilde{\nabla}^a + 4k)(\tilde{g}^{bc} f_{bc})\right] \\
& + \tfrac{1}{2}\tilde{g}_{ij}\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 4k)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - (\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b - 2k\tilde{\nabla}_a\tilde{\nabla}^a + 4k^2)(\tilde{g}^{bc} f_{bc})\right] \\
& - (\tilde{\nabla}_a\tilde{\nabla}^a - 3k)(\tilde{\nabla}_i\tilde{\nabla}^b f_{jb} + \tilde{\nabla}_j\tilde{\nabla}^b f_{ib})
\end{aligned}
\tag{2.8}
$$

2.3 Gauge Invariants

We mix time derivative notation a bit, using ∂_0 upon $f_{\mu\nu}$ and dot upon Ω and SVT3 quantities.

$$
\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\phi + \psi + \dot{B} - \ddot{E}] &= (\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}^b(\partial_0 f_{0b}) - \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k - \partial_0^2)\tilde{\nabla}_b\tilde{\nabla}^b(\tilde{g}^{cd}f_{cd}) \\
&\quad + \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a - 3\partial_0^2)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - \tfrac{1}{2}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b f_{00}
\end{aligned}
\tag{2.9}
$$

$$
\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\psi - \dot{\Omega}\Omega^{-1}(B - \dot{E})] &= -\dot{\Omega}\Omega^{-1}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}^b f_{0b} + \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 3\dot{\Omega}\Omega^{-1}\partial_0)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} \\
&\quad - \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k + \dot{\Omega}\Omega^{-1}\partial_0)\tilde{\nabla}_b\tilde{\nabla}^b(\tilde{g}^{cd}f_{cd})
\end{aligned}
\tag{2.10}
$$

$$
\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)[B_i - \dot{E}_i] &= (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)f_{0i} - (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\tilde{\nabla}^b(\partial_0 f_{ib}) \\
&\quad - \tilde{\nabla}_i(\tilde{\nabla}_a\tilde{\nabla}^a + 4k)\tilde{\nabla}^b f_{0b} + \tilde{\nabla}_i\tilde{\nabla}^a\tilde{\nabla}^b(\partial_0 f_{ab})
\end{aligned}
\tag{2.11}
$$

$$
\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)[2E_{ij}] &= (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)f_{ij} + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\big[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} + (\tilde{\nabla}_a\tilde{\nabla}^a + 4k)(\tilde{g}^{bc}f_{bc})\big] \\
&\quad + \tfrac{1}{2}\tilde{g}_{ij}\big[(\tilde{\nabla}_a\tilde{\nabla}^a - 4k)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - (\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b - 2k\tilde{\nabla}_a\tilde{\nabla}^a + 4k^2)(\tilde{g}^{bc}f_{bc})\big] \\
&\quad - (\tilde{\nabla}_a\tilde{\nabla}^a - 3k)(\tilde{\nabla}_i\tilde{\nabla}^b f_{jb} + \tilde{\nabla}_j\tilde{\nabla}^b f_{ib})
\end{aligned}
\tag{2.12}
$$

2.4 $f_{(1,3)} \to f_4$

We use $U = \delta_0^\mu$, $U_\mu = -\delta_\mu^0$ and $P_{\mu\nu} = (g_{\mu\nu} + U_\mu U_\nu)$ to transform from $3+1$ to a covariant 4 index.

$$
\begin{aligned}
&(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\phi + \psi + \dot{B} - \ddot{E}] = \\
&-\tfrac{1}{2}k\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha f - \tfrac{1}{2}kU^\alpha U^\beta\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f - \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma f + \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma f^{\gamma\zeta} \\
&-\tfrac{1}{4}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha f + 3kU^\alpha U^\beta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f_\alpha{}^\gamma - 2kU^\alpha U^\beta\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma f_{\alpha\beta} + \tfrac{1}{4}\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f^{\alpha\beta} \\
&+kU^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma f_{\alpha\beta} - \tfrac{3}{4}U^\alpha U^\beta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f^{\gamma\zeta} + \tfrac{1}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta\tilde{\nabla}_\eta f_\alpha{}^\eta \\
&+\tfrac{5}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\eta\tilde{\nabla}_\beta f_\alpha{}^\eta - \tfrac{3}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\eta\tilde{\nabla}^\eta f_{\alpha\beta} + \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\zeta\tilde{\nabla}^\zeta\tilde{\nabla}_\beta\tilde{\nabla}_\gamma f_\alpha{}^\gamma \\
&+\tfrac{5}{4}U^\alpha U^\beta\tilde{\nabla}_\zeta\tilde{\nabla}^\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f_\alpha{}^\gamma - \tfrac{3}{4}U^\alpha U^\beta\tilde{\nabla}_\zeta\tilde{\nabla}^\zeta\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma f_{\alpha\beta} - \tfrac{3}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\eta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f_\alpha{}^\eta \\
&-\tfrac{3}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\eta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f_\alpha{}^\eta + \tfrac{3}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\eta\tilde{\nabla}^\eta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma f_{\alpha\beta}
\end{aligned}
\tag{2.13}
$$

$$
\begin{aligned}
&(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\psi - \dot{\Omega}\Omega^{-1}(B - \dot{E})] = \\
&-\tfrac{1}{2}k\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha f - \tfrac{1}{2}kU^\alpha U^\beta\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f - \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma f + \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma f^{\gamma\zeta} \\
&-\tfrac{1}{4}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha f - 3kU^\alpha U^\beta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\gamma f_\beta{}^\gamma - \tfrac{1}{2}kU^\alpha U^\beta\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma f_{\alpha\beta} \\
&-\tfrac{1}{4}U^\alpha U^\beta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma\tilde{\nabla}_\beta f + \tfrac{1}{4}\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f^{\alpha\beta} - \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f \\
&-3kU^\alpha U^\beta U^\gamma U^\zeta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\zeta f_{\beta\gamma} - \tfrac{1}{2}kU^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma f_{\alpha\beta} \\
&+\tfrac{3}{4}U^\alpha U^\beta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f^{\gamma\zeta} - \tfrac{1}{4}U^\alpha U^\beta U^\gamma U^\zeta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f \\
&-\tfrac{1}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f + \tfrac{1}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta\tilde{\nabla}_\eta f_\alpha{}^\eta \\
&-U^\alpha U^\beta U^\gamma U^\zeta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\eta f_\beta{}^\eta + \tfrac{1}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\eta\tilde{\nabla}_\beta f_\alpha{}^\eta \\
&-\tfrac{1}{4}U^\alpha U^\beta U^\gamma U^\zeta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\eta\tilde{\nabla}^\eta f_{\alpha\beta} + \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\zeta\tilde{\nabla}^\zeta\tilde{\nabla}_\beta\tilde{\nabla}_\gamma f_\alpha{}^\gamma - U^\alpha U^\beta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\zeta\tilde{\nabla}^\zeta\tilde{\nabla}_\gamma f_\beta{}^\gamma \\
&+\tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\zeta\tilde{\nabla}^\zeta\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f_\alpha{}^\gamma - \tfrac{1}{4}U^\alpha U^\beta\tilde{\nabla}_\zeta\tilde{\nabla}^\zeta\tilde{\nabla}_\gamma\tilde{\nabla}^\gamma f_{\alpha\beta} + \tfrac{3}{4}U^\alpha U^\beta U^\gamma U^\zeta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\zeta\tilde{\nabla}_\eta\tilde{\nabla}_\gamma f_\beta{}^\eta \\
&+\tfrac{3}{4}U^\alpha U^\beta U^\gamma U^\zeta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\eta\tilde{\nabla}_\zeta\tilde{\nabla}_\gamma f_\beta{}^\eta - \tfrac{5}{4}U^\alpha U^\beta U^\gamma U^\zeta\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\eta\tilde{\nabla}^\eta\tilde{\nabla}_\zeta f_{\beta\gamma} \\
&-\tfrac{1}{2}U^\alpha U^\beta U^\gamma U^\zeta U^\eta U^\kappa\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\kappa\tilde{\nabla}_\eta\tilde{\nabla}_\zeta f_{\beta\gamma}
\end{aligned}
\tag{2.14}
$$

$$
\begin{aligned}
&(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)[B_i - \dot{E}_i] = \\
&-4k^2 f_{\mu\alpha}U^\alpha - 4k^2 f_{\alpha\beta}U^\alpha U^\beta U_\mu + 2kU^\alpha\tilde{\nabla}_\beta\tilde{\nabla}_\alpha f_\mu{}^\beta + 2kU^\alpha U^\beta U_\mu\tilde{\nabla}_\gamma\tilde{\nabla}_\beta f_\alpha{}^\gamma
\end{aligned}
$$

$$+2kU^\alpha U^\beta U^\gamma \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_{\mu\alpha} - U^\alpha U^\beta U^\gamma \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\zeta \tilde{\nabla}_\alpha f_\mu{}^\zeta + U^\alpha U^\beta U^\gamma \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta f_{\mu\alpha}$$
$$-U^\alpha U^\beta U^\gamma \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\mu \tilde{\nabla}_\zeta f_\alpha{}^\zeta - U^\alpha \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f_\mu{}^\beta + U^\alpha \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}^\beta f_{\mu\alpha}$$
$$-U^\alpha \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\mu \tilde{\nabla}_\beta f_\alpha{}^\beta + 2kU^\alpha U^\beta U^\gamma U^\zeta U_\mu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f_{\alpha\beta} - U^\alpha U^\beta U^\gamma U^\zeta U_\mu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\eta \tilde{\nabla}_\beta f_\alpha{}^\eta$$
$$+U^\alpha U^\beta U^\gamma U^\zeta U_\mu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\eta \tilde{\nabla}^\eta f_{\alpha\beta} - U^\alpha U^\beta U_\mu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_\alpha{}^\gamma + U^\alpha U^\beta U_\mu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma f_{\alpha\beta}$$
$$-U^\alpha U^\beta U^\gamma \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\mu \tilde{\nabla}_\gamma f_{\alpha\beta} - U^\alpha U^\beta U^\gamma U^\zeta U^\eta \tilde{\nabla}_\eta \tilde{\nabla}_\zeta \tilde{\nabla}_\mu \tilde{\nabla}_\gamma f_{\alpha\beta} - 2kU^\alpha \tilde{\nabla}_\mu \tilde{\nabla}_\beta f_\alpha{}^\beta$$
$$-2kU^\alpha U^\beta U^\gamma \tilde{\nabla}_\mu \tilde{\nabla}_\gamma f_{\alpha\beta} + U^\alpha \tilde{\nabla}_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f^{\beta\gamma} + U^\alpha U^\beta U^\gamma \tilde{\nabla}_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\zeta \tilde{\nabla}_\beta f_\alpha{}^\zeta$$
$$+U^\alpha U^\beta U^\gamma \tilde{\nabla}_\mu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_\alpha{}^\zeta + U^\alpha U^\beta U^\gamma U^\zeta U^\eta \tilde{\nabla}_\mu \tilde{\nabla}_\eta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f_{\alpha\beta} \qquad (2.15)$$

$$(\tilde{\nabla}_a \tilde{\nabla}^a - 2k)(\tilde{\nabla}_b \tilde{\nabla}^b - 3k)[2E_{ij}] =$$
$$6k^2 f_{\mu\nu} - 2k^2 f \tilde{g}_{\mu\nu} - 2k^2 f_{\alpha\beta} \tilde{g}_{\mu\nu} U^\alpha U^\beta + 6k^2 f_{\nu\alpha} U^\alpha U_\mu + 6k^2 f_{\mu\alpha} U^\alpha U_\nu - 2k^2 f U_\mu U_\nu$$
$$+4k^2 f_{\alpha\beta} U^\alpha U^\beta U_\mu U_\nu - 5k \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha f_{\mu\nu} + k \tilde{g}_{\mu\nu} \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha f + k U_\mu U_\nu \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha f + 3k U^\alpha U_\nu \tilde{\nabla}_\alpha \tilde{\nabla}_\beta f_\mu{}^\beta$$
$$+3k U^\alpha U_\mu \tilde{\nabla}_\alpha \tilde{\nabla}_\beta f_\nu{}^\beta + \tfrac{1}{2} U^\alpha U_\mu \tilde{\nabla}_\alpha \tilde{\nabla}_\nu \tilde{\nabla}_\beta \tilde{\nabla}^\beta f + \tfrac{1}{2} U^\alpha U_\mu \tilde{\nabla}_\alpha \tilde{\nabla}_\nu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f^{\beta\gamma} - 2k \tilde{g}_{\mu\nu} \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f^{\alpha\beta}$$
$$-2k U_\mu U_\nu \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f^{\alpha\beta} - 5k U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f_{\mu\nu} + k \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f + 3k U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f$$
$$+U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma f_{\mu\nu} - \tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma f + \tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f^{\gamma\zeta}$$
$$+U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f^{\gamma\zeta} - U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\mu \tilde{\nabla}_\gamma f_\nu{}^\gamma - U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\nu \tilde{\nabla}_\gamma f_\mu{}^\gamma$$
$$-5k U^\alpha U_\nu \tilde{\nabla}_\beta \tilde{\nabla}^\beta f_{\mu\alpha} - 5k U^\alpha U_\mu \tilde{\nabla}_\beta \tilde{\nabla}^\beta f_{\nu\alpha} + \tilde{\nabla}_\beta \tilde{\nabla}^\beta \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha f_{\mu\nu} - \tfrac{1}{2} \tilde{g}_{\mu\nu} \tilde{\nabla}_\beta \tilde{\nabla}^\beta \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha f$$
$$-\tfrac{1}{2} U_\mu U_\nu \tilde{\nabla}_\beta \tilde{\nabla}^\beta \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha f - \tilde{\nabla}_\beta \tilde{\nabla}^\beta \tilde{\nabla}_\mu \tilde{\nabla}_\alpha f_\nu{}^\alpha - \tilde{\nabla}_\beta \tilde{\nabla}^\beta \tilde{\nabla}_\nu \tilde{\nabla}_\alpha f_\mu{}^\alpha - 2k \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\gamma f_\alpha{}^\gamma$$
$$+4k U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\beta \tilde{\nabla}_\gamma f_\alpha{}^\gamma - 2k \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_\alpha{}^\gamma - 2k U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_\alpha{}^\gamma$$
$$-2k U^\alpha U^\beta U^\gamma U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_{\mu\alpha} - 2k U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_{\nu\alpha} - U^\alpha U^\beta U^\gamma U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\zeta f_\mu{}^\zeta$$
$$-U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\zeta f_\nu{}^\zeta + U^\alpha U^\beta U^\gamma U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta f_{\mu\alpha} + U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta f_{\nu\alpha}$$
$$-U^\alpha U^\beta U^\gamma U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\mu \tilde{\nabla}_\zeta f_\alpha{}^\zeta - U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\nu \tilde{\nabla}_\zeta f_\alpha{}^\zeta + k \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma f_{\alpha\beta}$$
$$-4k U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma f_{\alpha\beta} - U^\alpha U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\alpha \tilde{\nabla}_\beta f_\mu{}^\beta - U^\alpha U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\alpha \tilde{\nabla}_\beta f_\nu{}^\beta$$
$$+\tfrac{1}{2} \tilde{g}_{\mu\nu} \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f^{\alpha\beta} + \tfrac{1}{2} U_\mu U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f^{\alpha\beta} + U^\alpha U^\beta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f_{\mu\nu}$$
$$-\tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f - \tfrac{1}{2} U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f + U^\alpha U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}^\beta f_{\mu\alpha}$$
$$+U^\alpha U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\beta \tilde{\nabla}^\beta f_{\nu\alpha} - U^\alpha U_\nu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\mu \tilde{\nabla}_\beta f_\alpha{}^\beta - U^\alpha U^\beta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\mu \tilde{\nabla}_\beta f_{\nu\alpha}$$
$$-U^\alpha U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\nu \tilde{\nabla}_\beta f_\alpha{}^\beta - U^\alpha U^\beta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \tilde{\nabla}_\nu \tilde{\nabla}_\beta f_{\mu\alpha} + 2k U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\nu f_{\alpha\beta}$$
$$+\tfrac{1}{2} U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\nu \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f + \tfrac{1}{2} U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\nu \tilde{\nabla}_\beta \tilde{\nabla}_\zeta f_\alpha{}^\zeta$$
$$+\tfrac{1}{2} U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\nu \tilde{\nabla}_\zeta \tilde{\nabla}_\beta f_\alpha{}^\zeta + \tfrac{1}{2} U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\gamma \tilde{\nabla}_\nu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta f_{\alpha\beta}$$
$$-k \tilde{g}_{\mu\nu} U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f_{\alpha\beta} + 2k U^\alpha U^\beta U^\gamma U^\zeta U_\mu U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f_{\alpha\beta}$$
$$+U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f_{\mu\nu} - \tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\alpha f$$
$$+\tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\eta f_\alpha{}^\eta - U^\alpha U^\beta U^\gamma U^\zeta U_\mu U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta \tilde{\nabla}_\eta f_\alpha{}^\eta$$
$$+\tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\eta \tilde{\nabla}_\beta f_\alpha{}^\eta + U^\alpha U^\beta U^\gamma U^\zeta U_\mu U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\eta \tilde{\nabla}_\beta f_\alpha{}^\eta$$
$$-\tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\eta \tilde{\nabla}^\eta f_{\alpha\beta} + U^\alpha U^\beta U^\gamma U^\zeta U_\mu U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\eta \tilde{\nabla}^\eta f_{\alpha\beta}$$
$$-U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\mu \tilde{\nabla}_\beta f_{\nu\alpha} - U^\alpha U^\beta U^\gamma U^\zeta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\nu \tilde{\nabla}_\beta f_{\mu\alpha} + \tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\beta \tilde{\nabla}_\gamma f_\alpha{}^\gamma$$
$$-\tfrac{3}{2} U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\beta \tilde{\nabla}_\gamma f_\alpha{}^\gamma + \tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_\alpha{}^\gamma$$
$$+\tfrac{1}{2} U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\gamma \tilde{\nabla}_\beta f_\alpha{}^\gamma - \tfrac{1}{2} \tilde{g}_{\mu\nu} U^\alpha U^\beta \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma f_{\alpha\beta}$$
$$+\tfrac{1}{2} U^\alpha U^\beta U_\mu U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma f_{\alpha\beta} - U^\alpha U^\beta U^\gamma U_\nu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\mu \tilde{\nabla}_\gamma f_{\alpha\beta}$$
$$-U^\alpha U^\beta U^\gamma U_\mu \tilde{\nabla}_\zeta \tilde{\nabla}^\zeta \tilde{\nabla}_\nu \tilde{\nabla}_\gamma f_{\alpha\beta} - U^\alpha U^\beta U^\gamma U^\zeta U^\eta U_\nu \tilde{\nabla}_\eta \tilde{\nabla}_\zeta \tilde{\nabla}_\mu \tilde{\nabla}_\gamma f_{\alpha\beta}$$
$$-U^\alpha U^\beta U^\gamma U^\zeta U^\eta U_\mu \tilde{\nabla}_\eta \tilde{\nabla}_\zeta \tilde{\nabla}_\nu \tilde{\nabla}_\gamma f_{\alpha\beta} - U^\alpha U^\beta U^\gamma U^\zeta U_\mu U_\nu \tilde{\nabla}_\eta \tilde{\nabla}^\eta \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f_{\alpha\beta}$$
$$+U^\alpha U^\beta U^\gamma U^\zeta U^\eta U_\mu \tilde{\nabla}_\eta \tilde{\nabla}_\nu \tilde{\nabla}_\zeta \tilde{\nabla}_\gamma f_{\alpha\beta} + 3k \tilde{\nabla}_\mu \tilde{\nabla}_\alpha f_\nu{}^\alpha + 2k U^\alpha U_\nu \tilde{\nabla}_\mu \tilde{\nabla}_\alpha f$$

$$+\tfrac{1}{2}U^{\alpha}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\alpha}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}f + \tfrac{1}{2}U^{\alpha}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\alpha}\tilde{\nabla}_{\gamma}\tilde{\nabla}_{\beta}f^{\beta\gamma} + 3kU^{\alpha}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\beta}f_{\alpha}{}^{\beta}$$

$$+3kU^{\alpha}U^{\beta}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\beta}f_{\nu\alpha} + 5kU^{\alpha}U^{\beta}U^{\gamma}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\gamma}f_{\alpha\beta} + \tfrac{1}{2}U^{\alpha}U^{\beta}U^{\gamma}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\gamma}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}f$$

$$+\tfrac{1}{2}U^{\alpha}U^{\beta}U^{\gamma}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\gamma}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\zeta}f_{\alpha}{}^{\zeta} + \tfrac{1}{2}U^{\alpha}U^{\beta}U^{\gamma}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\gamma}\tilde{\nabla}_{\zeta}\tilde{\nabla}_{\beta}f_{\alpha}{}^{\zeta}$$

$$+\tfrac{1}{2}U^{\alpha}U^{\beta}U^{\gamma}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\gamma}\tilde{\nabla}_{\zeta}\tilde{\nabla}^{\zeta}f_{\alpha\beta} + U^{\alpha}U^{\beta}U^{\gamma}U^{\zeta}U^{\eta}U_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\eta}\tilde{\nabla}_{\zeta}\tilde{\nabla}_{\gamma}f_{\alpha\beta} + 2kU^{\alpha}U^{\beta}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}f_{\alpha\beta}$$

$$+\tfrac{1}{2}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}f + \tfrac{1}{2}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}f^{\alpha\beta} + \tfrac{1}{2}U^{\alpha}U^{\beta}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}f + \tfrac{1}{2}U^{\alpha}U^{\beta}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\gamma}f_{\alpha}{}^{\gamma}$$

$$+\tfrac{1}{2}U^{\alpha}U^{\beta}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\gamma}\tilde{\nabla}_{\beta}f_{\alpha}{}^{\gamma} + \tfrac{1}{2}U^{\alpha}U^{\beta}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\gamma}\tilde{\nabla}^{\gamma}f_{\alpha\beta} + U^{\alpha}U^{\beta}U^{\gamma}U^{\zeta}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\zeta}\tilde{\nabla}_{\gamma}f_{\alpha\beta}$$

$$+3k\tilde{\nabla}_{\nu}\tilde{\nabla}_{\alpha}f_{\mu}{}^{\alpha} + 2kU^{\alpha}U_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\alpha}f + 3kU^{\alpha}U_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\beta}f_{\alpha}{}^{\beta} + 3kU^{\alpha}U^{\beta}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\beta}f_{\mu\alpha}$$

$$+3kU^{\alpha}U^{\beta}U^{\gamma}U_{\mu}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\gamma}f_{\alpha\beta} + 2k\tilde{\nabla}_{\nu}\tilde{\nabla}_{\mu}f \qquad (2.16)$$

Appendix A γ Alternative

$$
\begin{aligned}
(\tilde{\nabla}_a \tilde{\nabla}^a + 3k)\tilde{\nabla}_b \tilde{\nabla}^b[-\dot{\Omega}^{-1}\Omega\psi + B - \dot{E}] \;=\; & (\tilde{\nabla}_a \tilde{\nabla}^a + 3k)\tilde{\nabla}^b f_{0b} - \tfrac{1}{4}(\dot{\Omega}^{-1}\Omega\tilde{\nabla}_a \tilde{\nabla}^a + 3\partial_0)\tilde{\nabla}^b \tilde{\nabla}^c f_{bc} \\
& + \tfrac{1}{4}\left[\dot{\Omega}^{-1}\Omega(\tilde{\nabla}_a \tilde{\nabla}^a + 2k) + \partial_0\right]\tilde{\nabla}_b \tilde{\nabla}^b (\tilde{g}^{cd} f_{cd})
\end{aligned}
\tag{A.1}
$$

Appendix B SVTD in Max. Sym. Space

$$
\left(\nabla_\alpha \nabla^\alpha - \frac{R}{D-1}\right)\chi \;=\; \frac{1}{2(D-1)}\left[\nabla^\alpha \nabla^\beta h_{\alpha\beta} - \left(\nabla_\alpha \nabla^\alpha - \frac{R}{D}\right)h\right]
\tag{B.1}
$$

$$
\left(\nabla_\alpha \nabla^\alpha - \frac{R}{D-1}\right)\nabla_\beta \nabla^\beta F \;=\; \frac{D}{2(D-1)}\left(\nabla^\alpha \nabla^\beta h_{\alpha\beta} - \frac{1}{D}\nabla_\alpha \nabla^\alpha h\right)
\tag{B.2}
$$

$$
\left(\nabla_\alpha \nabla^\alpha - \frac{R}{D}\right)\left(\nabla_\beta \nabla^\beta + \frac{R}{D}\right)F_\mu \;=\; \left(\nabla_\alpha \nabla^\alpha + \frac{R}{D}\right)\nabla^\sigma h_{\sigma\mu} - \nabla_\mu \nabla^\alpha \nabla^\beta h_{\alpha\beta},
\tag{B.3}
$$