$$W_{int} = k\sum_{i=1}^{n}\sum_{j<i}\frac{q_i q_j}{|\mathbf{x}_i-\mathbf{x}_j|} = \frac{k}{2}\sum_{i}^{n}\sum_{\substack{j\\ j\neq i}}\frac{q_i q_j}{|\mathbf{x}_i-\mathbf{x}_j|} = \frac{1}{2}\sum_{i=1}^{n}q_i\sum_{\substack{j\\ j\neq i}}k\frac{q_j}{|\mathbf{x}_i-\mathbf{x}_j|}$$

$$= \frac{1}{2}\sum_{i=1}^{n}q_i\Phi(\mathbf{x}_i) = \frac{1}{2}\int_\rho \rho(\mathbf{x})\Phi(\mathbf{x})\,d^3x = \frac{\epsilon_0}{2}\int_{all}|\mathbf{E}|^2\,d^3x = -\int_\infty^r \mathbf{F}\cdot d\mathbf{l} = \frac{1}{2}q\Phi_*(\mathbf{x})$$

$$W_{int\text{-}dip} = \mathbf{p}\cdot\mathbf{E};\quad W_{12} = \frac{\mathbf{p}_1\cdot\mathbf{p}_2 - 3(\mathbf{n}\cdot\mathbf{p}_1)(\mathbf{n}\cdot\mathbf{p}_2)}{4\pi\epsilon_0|\mathbf{x}_1-\mathbf{x}_2|^3}\quad \text{where}\quad \mathbf{x}_1\neq\mathbf{x}_2\quad \mathbf{n} = \frac{\mathbf{x}_1-\mathbf{x}_2}{|\mathbf{x}_1-\mathbf{x}_2|}$$

$$\Phi_{dipole\text{-}o}(\mathbf{r}) = k\frac{\mathbf{p}\cdot\mathbf{r}}{r^3};\quad \mathbf{F}_{dip} = (\mathbf{p}\cdot\nabla)\mathbf{E};\quad W_{intqsp} = \frac{q^2}{2C};\quad -kq^2/2z\quad -k/2q^2R/(r^2-R^2)$$

$$\mathbf{E}_{dip}(\mathbf{x}) = k\left(\frac{3\mathbf{n}(\mathbf{p}\cdot\mathbf{n})-\mathbf{p}}{|\mathbf{x}-\mathbf{x}_0|^3} - \frac{4\pi}{3}\mathbf{p}\delta(\mathbf{x}-\mathbf{x}_0)\right)\quad \text{where}\quad \mathbf{n} = \frac{\mathbf{x}-\mathbf{x}_0}{|\mathbf{x}-\mathbf{x}_0|}$$

$$\Phi(\mathbf{x}) \approx k\left(\frac{1}{r}\int\rho(\mathbf{x}')\,d^3x' + \frac{\mathbf{p}\cdot\mathbf{x}}{r^3}\right)\quad \text{for}\quad r>r';\quad \mathbf{p} = \int\mathbf{x}'\rho(\mathbf{x}')\,d^3x'$$

$$\nabla = \frac{\partial}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi};\quad -\nabla_x\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right) = \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\phi^2};\quad \nabla^2\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right) = -4\pi\delta(\mathbf{x}-\mathbf{x}')$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2};\quad R(\rho) = AJ_\nu(k\rho)+BN_\nu(k\rho);\quad Z(z) = e^{\pm kz};\quad Q(\phi) = e^{\pm i\nu\phi}$$

$$\Phi(\mathbf{x}) = k\int_V\rho(\mathbf{x}')G(\mathbf{x},\mathbf{x}')\,d^3x' + \frac{1}{4\pi}\oint_{\partial V}\left[G(\mathbf{x},\mathbf{x}')\frac{\partial\Phi}{\partial n'} - \Phi(\mathbf{x}')\frac{\partial G(\mathbf{x},\mathbf{x}')}{\partial n'}\right]da';\quad G_D(\mathbf{x},\mathbf{x}')|_{x'\in\partial V} = 0$$

$$\Phi = \frac{U(r)}{r}P(\theta)Q(\phi);\quad Q = e^{\pm im\phi};\quad U = Ar^{l+1} + Br^{-l};\quad P(\theta) = P_l^m(\cos\theta)$$

$$\Phi(r,\theta) = \sum_{l=0}^{\infty}[A_l r^l + B_l r^{-(l+1)}]P_l(\cos\theta);\quad A_l = \frac{2l+1}{2}\int_0^\pi\Phi(r,\theta)|_{BC}P_l(\cos\theta)$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi};\quad Y_{l-m} = (-1)^m Y*_{lm}$$

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}^*(\theta',\phi')Y_{lm}(\theta,\phi)$$

$$q' = -q\frac{R}{r_0};\quad r_0' = \frac{R^2}{r_0};\quad \sigma = -\frac{q}{4\pi R^2}\frac{R}{r_0}\frac{1-R^2/r_0^2}{(1+R^2/r_0^2-2R/r_0\cos\gamma)^{3/2}}$$

$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l;\quad P_l^m(x) = (-1)^m(1-x^2)^{m/2}\frac{d^m}{dx^m}P_l(x);$$

$$\Phi(\mathbf{x}) = -k\int_{\partial V}D(\mathbf{x}')\,d\Omega;\quad \Delta\Phi = \frac{D}{\epsilon_0};\quad \mathbf{p} = \mathbf{n}D\,da'$$

$$\delta(r) = \begin{cases}\frac{1}{\frac{4\pi}{3}\gamma^3} & r\leq\gamma\\ 0 & r>\gamma\end{cases};\quad \frac{1}{|\mathbf{x}+\mathbf{a}|} = \frac{1}{x} + \mathbf{a}\cdot\nabla\left(\frac{1}{x}\right) + \ldots$$