Astrophysics & Cosmology
HW 10

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14.5 From the virial theorem

$$E_{kinetic} = -\frac{1}{2}E_{grav}.$$

For N bodies of mass m with average separation between bodies R, the graviational potential of the total $N(N-1)/2$ pairings is

$$E_{grav} = -\frac{GN(N-1)m^2}{2R}$$

and the total kinetic energy is

$$E_{kinetic} = \frac{1}{2}mv^2.$$

Now using the virial theorem

$$\frac{Nmv^2}{2} = \frac{GN(N-1)m^2}{4R}.$$

This rearranges to

$$(N-1)m = \frac{2Rv^2}{G}.$$

For $N \gg 1$ and $M = Nm$, the above reduces to

$$M = \frac{2Rv^2}{G}.$$

For a galaxy cluster, on average $v^2 = 3V^2$, where V is the mean line-of-sight velocity displacement of a galaxy with respect to the center. In this case,

$$M = \frac{6RV^2}{G}.$$

Contrast this with the approximate mass of a single galaxy, as a function of radius r from the center

$$M(r) = \frac{rv^2}{G}$$

and we see the mass of a spherical cluster of galaxies scales by a numerical factor of 6 for mean line-of-sight velocity and a factor of 2 for mean random velocity.

For $V = 10^8$ cm s^{-1} and $R = 2.8 \times 10^{24}$ cm, the total mass is

$$M = \frac{6RV^2}{G} = 2.55 \times 10^{48}\ g = 1.3 \times 10^{15} M_{sun}.$$

Problem 14.4 assumes a stellar disk of μ corresponding to $10^{11} M_{sun}$, with radius $R_{disk} = 10^{-2}R$. At the same radius then, the mass of the galaxy cluster would be $M \approx 10^{13} M_{sun}$. Thus a spherical galaxy distribution is only about 100 times more massive than the stellar gas disk mass.

14.10 From the Hubble law

$$v = H_0 r$$

it follows that

$$z = \frac{H_0 r}{c}.$$

If the hubble constant is

$$H_0 = 20 \text{ km/sec/million lyr}$$

and we spot a galaxy moving at redshift $z = 0.02$, then its distance would be

$$r = \frac{zc}{H_0} = 300 \text{ million lyr} = 2.8 \times 10^{26} \text{ cm}.$$

Small redshifts can be caused by gravitational interactions between objects and not that due to the expansion of space and therefore may not always be a reliable metric for computing distances.