RW SVT3 $\delta G_{\mu\nu} = -\kappa_4^2 \delta T_{\mu\nu}$ (v2)

1 Background

$$ds^2 = \Omega^2(\tau)\tilde{g}_{\mu\nu}dx^\mu dx^\nu, \qquad \tilde{g}_{\mu\nu} = \text{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right) \tag{1.1}$$

$$G_{00}^{(0)} = -3k - 3\dot{\Omega}^2\Omega^{-2} \qquad G_{0i}^{(0)} = 0 \qquad G_{ij}^{(0)} = k\tilde{g}_{ij} - \dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} + 2\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij} \tag{1.2}$$

$$\kappa_4^2 T_{\mu\nu}^{(0)} = (\rho+p)U_\mu U_\nu + p\Omega^2\tilde{g}_{\mu\nu}, \qquad U_\mu = -\Omega\delta_\mu^0 \qquad \text{[Evaluated in (1.1)]} \tag{1.3}$$

$$\Delta_{\mu\nu}^{(0)} = G_{\mu\nu}^{(0)} + \kappa_4^2 T_{\mu\nu}^{(0)} = 0 \tag{1.4}$$

$$\Delta_{00}^{(0)} = -3k - 3\dot{\Omega}^2\Omega^{-2} + \Omega^2\rho \tag{1.5}$$

$$\rightarrow \boxed{\rho = 3k\Omega^{-2} + 3\dot{\Omega}^2\Omega^{-4}} \tag{1.6}$$

$$\Delta_{ij}^{(0)} = k\tilde{g}_{ij} - \dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} + 2\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij} + \Omega^2 p\tilde{g}_{ij} \tag{1.7}$$

$$\rightarrow \boxed{p = -k\Omega^{-2} + \dot{\Omega}^2\Omega^{-4} - 2\ddot{\Omega}\Omega^{-3}} \tag{1.8}$$

$$\kappa_4^2\nabla_\mu T^{\mu\nu}_{(0)} = 0$$

$$\rightarrow \boxed{p = -\rho - \frac{1}{3}\frac{\Omega}{\dot{\Omega}}\dot{\rho}} \tag{1.9}$$

$$p + \rho + \tfrac{1}{3}\dot{\Omega}^{-1}\Omega\dot{\rho} = \tfrac{1}{3}\Omega^{-2}\tilde{g}^{ij}\Delta_{ij}^{(0)} + \tfrac{1}{3}\Omega^{-2}\Delta_{00}^{(0)} + \tfrac{1}{3}\dot{\Omega}^{-1}\Omega^{-1}\dot{\Delta}_{00}^{(0)} \tag{1.10}$$

2 Fluctuations

$$ds^2 = \Omega^2(\tau)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu \tag{2.1}$$

$$\tilde{g}_{\mu\nu} = \text{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right) \tag{2.2}$$

$$f_{00} = -2\phi, \qquad f_{0i} = \tilde{\nabla}_i B + B_i, \qquad f_{ij} = -2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{2.3}$$

$$\delta G_{00} = -6k\phi - 6k\psi + 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi \tag{2.4}$$

$$
\begin{aligned}
\delta G_{0i} &= 3k\tilde{\nabla}_i B - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2k\tilde{\nabla}_i \dot{E} - 2\tilde{\nabla}_i\dot{\psi} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + 2kB_i - k\dot{E}_i \\
&\quad -B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i
\end{aligned}
\tag{2.5}
$$

$$
\begin{aligned}
\delta G_{ij} &= -2\ddot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\
&\quad -4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
&\quad -\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} \\
&\quad +2k\tilde{\nabla}_j\tilde{\nabla}_i E - 2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i E + \tilde{\nabla}_j\tilde{\nabla}_i\phi - \tilde{\nabla}_j\tilde{\nabla}_i\psi + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j \\
&\quad -\tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j + k\tilde{\nabla}_i E_j - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i E_j + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i \\
&\quad -\tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i + k\tilde{\nabla}_j E_i - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j E_i + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i - \ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} \\
&\quad -2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{\Omega}E_{ij}\Omega^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij}
\end{aligned}
\tag{2.6}
$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} &= 6\dot{\Omega}^2\phi\Omega^{-4} + 6\dot{\Omega}^2\psi\Omega^{-4} - 6\dot{\phi}\dot{\Omega}\Omega^{-3} - 18\dot{\psi}\dot{\Omega}\Omega^{-3} - 12\ddot{\Omega}\phi\Omega^{-3} - 12\ddot{\Omega}\psi\Omega^{-3} - 6\ddot{\psi}\Omega^{-2} + 6k\phi\Omega^{-2} \\
&\quad +6k\psi\Omega^{-2} - 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
&\quad -2\dot{\Omega}^2\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^a E + 4\ddot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a E + 2k\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a E - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi
\end{aligned}
\tag{2.7}
$$

$$
\kappa_4^2\delta T_{\mu\nu} = (\delta\rho + \delta p)U_\mu U_\nu + (\rho + p)(\delta U_\mu U_\nu + U_\mu \delta U_\nu) + \Omega^2\delta p\tilde{g}_{\mu\nu} + \Omega^2 p f_{\mu\nu}
\tag{2.8}
$$

$$
\delta U_0 = -\Omega\phi, \qquad \delta U_i = \tilde{\nabla}_i V + V_i
\tag{2.9}
$$

$$
\kappa_4^2\delta T_{00} = \Omega^2\delta\rho + 2\Omega^2\rho\phi, \qquad \text{[Substituting (2.9)]}
\tag{2.10}
$$

$$
\kappa_4^2\delta T_{0i} = -\Omega(\rho + p)(\tilde{\nabla}_i V + V_i) + \Omega^2 p(\tilde{\nabla}_i B + B_i) \qquad \text{[Substituting (2.9)]}
\tag{2.11}
$$

$$
\kappa_4^2\delta T_{ij} = \Omega^2\delta p\tilde{g}_{ij} + \Omega^2 p(-2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij})
\tag{2.12}
$$

$$
\kappa_4^2 g^{\mu\nu}\delta T_{\mu\nu} = -\delta\rho + 3\delta p - 2\rho\phi + p(-6\psi + 2\tilde{\nabla}_a\tilde{\nabla}^a E) \qquad \text{[Substituting (2.9)]}
\tag{2.13}
$$

3 Field Equations

We express the background EM quantities ρ and p in terms of Ω via substitution (1.6) and (1.8).

$$
\Delta_{\mu\nu} \equiv \delta G_{\mu\nu} + \kappa_4^2\delta T_{\mu\nu} = 0
\tag{3.1}
$$

$$
\Delta_{00} = -6k\psi + 6\dot{\Omega}^2\phi\Omega^{-2} + 6\dot{\psi}\dot{\Omega}\Omega^{-1} + \delta\rho\Omega^2 + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi
\tag{3.2}
$$

$$
\begin{aligned}
\Delta_{0i} &= 2k\tilde{\nabla}_i B - 2k\tilde{\nabla}_i\dot{E} - 2\tilde{\nabla}_i\dot{\psi} - 4\dot{\Omega}^2\Omega^{-3}\tilde{\nabla}_i V + 2\ddot{\Omega}\Omega^{-2}\tilde{\nabla}_i V - 2k\Omega^{-1}\tilde{\nabla}_i V - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + kB_i \\
&\quad -k\dot{E}_i - 4\dot{\Omega}^2 V_i\Omega^{-3} + 2\ddot{\Omega}V_i\Omega^{-2} - 2kV_i\Omega^{-1} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
\Delta_{ij} &= -2\ddot{\psi}\tilde{g}_{ij} + 2k\tilde{g}_{ij}\psi + 2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} + \delta p\tilde{g}_{ij}\Omega^2 \\
&\quad -2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi \\
&\quad +\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} + \tilde{\nabla}_j\tilde{\nabla}_i\phi
\end{aligned}
$$

$$
\begin{aligned}
& -\tilde{\nabla}_j\tilde{\nabla}_i\psi+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_iB_j+\tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j-\tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_jB_i+\tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i-\tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i \\
& -\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i-\ddot{E}_{ij}-2kE_{ij}-2\dot{E}_{ij}\dot{\Omega}\Omega^{-1}+\tilde{\nabla}_a\tilde{\nabla}^aE_{ij} \qquad (3.4)
\end{aligned}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} \;=\;& 3\delta p-\delta\rho-6\dot{\phi}\dot{\Omega}\Omega^{-3}-18\dot{\psi}\dot{\Omega}\Omega^{-3}-12\ddot{\Omega}\phi\Omega^{-3}-6\ddot{\psi}\Omega^{-2}+12k\psi\Omega^{-2}-6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aB \\
& -2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}+6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi+4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi \qquad (3.5)
\end{aligned}
$$

4 Field Equations (G.I. Form)

$$
\alpha = \phi+\psi+\dot{B}-\ddot{E}, \qquad \gamma=-\dot{\Omega}^{-1}\Omega\psi+B-\dot{E}, \qquad B_i-\dot{E}_i, \qquad E_{ij}, \qquad V_i \qquad (4.1)
$$

$$
V^{GI} = V-\Omega^2\dot{\Omega}^{-1}\psi \qquad (4.2)
$$

$$
\delta\rho^{GI} = \delta\rho-12\dot{\Omega}^2\psi\Omega^{-4}+6\ddot{\Omega}\psi\Omega^{-3}-6k\psi\Omega^{-2} \qquad (4.3)
$$

$$
\delta p^{GI} = \delta p-4\dot{\Omega}^2\psi\Omega^{-4}+8\ddot{\Omega}\psi\Omega^{-3}+2k\psi\Omega^{-2}-2\dddot{\Omega}\dot{\Omega}^{-1}\psi\Omega^{-2} \qquad (4.4)
$$

$$
\Delta_{00} = 6\dot{\Omega}^2\Omega^{-2}(\alpha-\dot{\gamma})+\delta\rho^{GI}\Omega^2+2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\gamma \qquad (4.5)
$$

$$
\begin{aligned}
\Delta_{0i} \;=\;& -2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i(\alpha-\dot{\gamma})+2k\tilde{\nabla}_i\gamma+(-4\dot{\Omega}^2\Omega^{-3}+2\ddot{\Omega}\Omega^{-2}-2k\Omega^{-1})\tilde{\nabla}_iV^{GI} \\
& +k(B_i-\dot{E}_i)+\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a(B_i-\dot{E}_i)+(-4\dot{\Omega}^2\Omega^{-3}+2\ddot{\Omega}\Omega^{-2}-2k\Omega^{-1})V_i \qquad (4.6)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{ij} \;=\;& \tilde{g}_{ij}\big[2\dot{\Omega}^2\Omega^{-2}(\alpha-\dot{\gamma})-2\dot{\Omega}\Omega^{-1}(\dot{\alpha}-\ddot{\gamma})-4\ddot{\Omega}\Omega^{-1}(\alpha-\dot{\gamma})+\Omega^2\delta p^{GI}-\tilde{\nabla}_a\tilde{\nabla}^a(\alpha+2\dot{\Omega}\Omega^{-1}\gamma)\big] \\
& +\tilde{\nabla}_i\tilde{\nabla}_j(\alpha+2\dot{\Omega}\Omega^{-1}\gamma) \\
& +\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i(B_j-\dot{E}_j)+\tfrac{1}{2}\tilde{\nabla}_i(\dot{B}_j-\ddot{E}_j)+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j(B_i-\dot{E}_i)+\tfrac{1}{2}\tilde{\nabla}_j(\dot{B}_i-\ddot{E}_i) \\
& -\ddot{E}_{ij}-2kE_{ij}-2\dot{E}_{ij}\dot{\Omega}\Omega^{-1}+\tilde{\nabla}_a\tilde{\nabla}^aE_{ij} \qquad (4.7)
\end{aligned}
$$

$$
g^{\mu\nu}\Delta_{\mu\nu} = 3\delta p^{GI}-\delta\rho^{GI}-12\ddot{\Omega}\Omega^{-3}(\alpha-\dot{\gamma})-6\dot{\Omega}\Omega^{-3}(\dot{\alpha}-\ddot{\gamma})-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a(\alpha+3\dot{\Omega}\Omega^{-1}\gamma) \qquad (4.8)
$$

$$
\tilde{\nabla}^i\Delta_{0i} = \tilde{\nabla}_a\tilde{\nabla}^a\big[-2\dot{\Omega}\Omega^{-1}(\alpha-\dot{\gamma})+2k\gamma+(-4\dot{\Omega}^2\Omega^{-3}+2\ddot{\Omega}\Omega^{-2}-2k\Omega^{-1})V^{GI}\big] \qquad (4.9)
$$

$$
\tilde{g}^{ij}\Delta_{ij} = 6\dot{\Omega}^2\Omega^{-2}(\alpha-\dot{\gamma})-6\dot{\Omega}\Omega^{-1}(\dot{\alpha}-\ddot{\gamma})-12\ddot{\Omega}\Omega^{-1}(\alpha-\dot{\gamma})+3\Omega^2\delta p^{GI}-2\tilde{\nabla}_a\tilde{\nabla}^a(\alpha+2\dot{\Omega}\Omega^{-1}\gamma) \qquad (4.10)
$$

5 Conservation Equations

$$\rho = 3k\Omega^{-2} + 3\dot{\Omega}^2\Omega^{-4} \tag{5.1}$$

$$p = -k\Omega^{-2} + \dot{\Omega}^2\Omega^{-4} - 2\ddot{\Omega}\Omega^{-3} \tag{5.2}$$

$$\delta\rho^{GI} = \delta\rho - 12\dot{\Omega}^2\psi\Omega^{-4} + 6\ddot{\Omega}\psi\Omega^{-3} - 6k\psi\Omega^{-2} \tag{5.3}$$

$$\begin{aligned}\delta\dot{\rho}^{GI} &= \delta\dot{\rho} + 48\dot{\Omega}^3\psi\Omega^{-5} - 12\dot{\psi}\dot{\Omega}^2\Omega^{-4} - 42\ddot{\Omega}\dot{\Omega}\psi\Omega^{-4} + 6\ddot{\Omega}\dot{\psi}\Omega^{-3} \\ &\quad + 6\dddot{\Omega}\psi\Omega^{-3} + 12k\dot{\Omega}\psi\Omega^{-3} - 6k\dot{\psi}\Omega^{-2}\end{aligned} \tag{5.4}$$

$$\delta p^{GI} = \delta p - 4\dot{\Omega}^2\psi\Omega^{-4} + 8\ddot{\Omega}\psi\Omega^{-3} + 2k\psi\Omega^{-2} - 2\dddot{\Omega}\dot{\Omega}^{-1}\psi\Omega^{-2} \tag{5.5}$$

$$\tag{5.6}$$

$$p + \rho + \tfrac{1}{3}\dot{\rho}\dot{\Omega}^{-1}\Omega = 0 \tag{5.7}$$

$$\begin{aligned}\delta p + \delta\rho + \tfrac{1}{3}\delta\dot{\rho}\dot{\Omega}^{-1}\Omega &= \delta p^{GI} + \delta\rho^{GI} + \tfrac{1}{3}\delta\dot{\rho}^{GI}\dot{\Omega}^{-1}\Omega + (4\dot{\Omega}\Omega^{-3} - 2\dddot{\Omega}\dot{\Omega}^{-1}\Omega^{-2} + 2k\dot{\Omega}^{-1}\Omega^{-1})\dot{\psi} \\ &= \delta p^{GI} + \delta\rho^{GI} + \tfrac{1}{3}\delta\dot{\rho}^{GI} - \tfrac{1}{3}\Omega^2\dot{\Omega}^{-2}\dot{\rho}\dot{\psi}\end{aligned} \tag{5.8}$$

$$\tag{5.9}$$

$$p + \rho + \tfrac{1}{3}\dot{\Omega}^{-1}\Omega\dot{\rho} = \tfrac{1}{3}\Omega^{-2}\tilde{g}^{ij}\Delta_{ij}^{(0)} + \tfrac{1}{3}\Omega^{-2}\Delta_{00}^{(0)} + \tfrac{1}{3}\dot{\Omega}^{-1}\Omega^{-1}\dot{\Delta}_{00}^{(0)} \tag{5.10}$$

$$\begin{aligned}\tfrac{1}{3}\Omega^{-2}\tilde{g}^{ij}\Delta_{ij} + \tfrac{1}{3}\Omega^{-2}\Delta_{00} + \tfrac{1}{3}\dot{\Omega}^{-1}\Omega^{-1}\dot{\Delta}_{00} &= \delta p^{GI} + \delta\rho^{GI} + \tfrac{1}{3}\dot{\Omega}^{-1}\Omega\delta\dot{\rho}^{GI} - \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\alpha \\ &\quad - \tfrac{2}{3}(2\Omega^{-3}\dot{\Omega} - \ddot{\Omega}\dot{\Omega}^{-1}\Omega^{-2})\tilde{\nabla}_a\tilde{\nabla}^a\gamma + \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{\gamma}\end{aligned} \tag{5.11}$$