

SVT

$$h_{\theta\theta} = -2\phi$$

$$h_{\theta i} = w_i = \partial_i B + B_i$$

$$h_{ij} = -2\psi + S_{ij} = -2\psi + 2\partial_i \partial_j E + \partial_i E_j + \partial_j E_i + 2E_{ij}$$

where

$$\partial_i B^i = \partial_i E^i = 0$$

$$\partial_i E^{ij} = 0$$

$$\delta_{ij} E^{ij} = 0$$

No gauge, $\delta G_{\mu\nu}$ flat:

00	$\frac{1}{2} \partial_1 \partial_1 h_{\theta\theta} - \frac{1}{2} \partial_1 \partial_1 h_{11} + \frac{\partial_1 \partial_1 h}{2} - \partial_2 \partial_1 h_{12} + \frac{1}{2} \partial_2 \partial_2 h_{\theta\theta} - \frac{1}{2} \partial_2 \partial_2 h_{22} + \frac{\partial_2 \partial_2 h}{2} - \partial_3 \partial_1 h_{13} - \partial_3 \partial_2 h_{23} + \frac{1}{2} \partial_3 \partial_3 h_{\theta\theta} - \frac{1}{2} \partial_3 \partial_3 h_{33} + \frac{\partial_3 \partial_3 h}{2}$
11	$\frac{1}{2} \partial_\theta \partial_\theta h_{\theta\theta} - \frac{1}{2} \partial_\theta \partial_\theta h_{11} + \frac{\partial_\theta \partial_\theta h}{2} - \partial_1 \partial_\theta h_{\theta 1} + h_{1\theta} \partial_1 \partial_\theta h_{\theta 1} - \partial_2 \partial_\theta h_{\theta 2} + \frac{1}{2} \partial_2 \partial_2 h_{11} + \frac{1}{2} \partial_2 \partial_2 h_{22} - \frac{\partial_2 \partial_2 h}{2} - \partial_3 \partial_\theta h_{\theta 3} + \partial_3 \partial_2 h_{23} + \frac{1}{2} \partial_3 \partial_3 h_{11} + \frac{1}{2} \partial_3 \partial_3 h_{33} - \frac{\partial_3 \partial_3 h}{2}$
22	$\frac{1}{2} \partial_\theta \partial_\theta h_{\theta\theta} - \frac{1}{2} \partial_\theta \partial_\theta h_{22} + \frac{\partial_\theta \partial_\theta h}{2} - \partial_1 \partial_\theta h_{\theta 1} + \frac{1}{2} \partial_1 \partial_1 h_{11} + \frac{1}{2} \partial_1 \partial_1 h_{22} - \frac{\partial_1 \partial_1 h}{2} - \partial_2 \partial_\theta h_{\theta 2} + h_{2\theta} \partial_2 \partial_\theta h_{\theta 2} - \partial_3 \partial_\theta h_{\theta 3} + \partial_3 \partial_1 h_{13} + \frac{1}{2} \partial_3 \partial_3 h_{22} + \frac{1}{2} \partial_3 \partial_3 h_{33} - \frac{\partial_3 \partial_3 h}{2}$
33	$\frac{1}{2} \partial_\theta \partial_\theta h_{\theta\theta} - \frac{1}{2} \partial_\theta \partial_\theta h_{33} + \frac{\partial_\theta \partial_\theta h}{2} - \partial_1 \partial_\theta h_{\theta 1} + \frac{1}{2} \partial_1 \partial_1 h_{11} + \frac{1}{2} \partial_1 \partial_1 h_{33} - \frac{\partial_1 \partial_1 h}{2} - \partial_2 \partial_\theta h_{\theta 2} + \partial_2 \partial_1 h_{12} + \frac{1}{2} \partial_2 \partial_2 h_{22} + \frac{1}{2} \partial_2 \partial_2 h_{33} - \frac{\partial_2 \partial_2 h}{2} - \partial_3 \partial_\theta h_{\theta 3} + h_{3\theta} \partial_3 \partial_\theta h_{\theta 3}$
01	$\frac{1}{2} \partial_1 \partial_\theta h_{\theta\theta} - \frac{1}{2} \partial_1 \partial_\theta h_{11} + \frac{\partial_1 \partial_\theta h}{2} - \frac{1}{2} \partial_2 \partial_\theta h_{12} - \frac{1}{2} \partial_2 \partial_1 h_{\theta 2} + \frac{1}{2} \partial_2 \partial_2 h_{\theta 1} - \frac{1}{2} \partial_3 \partial_\theta h_{13} - \frac{1}{2} \partial_3 \partial_1 h_{\theta 3} + \frac{1}{2} \partial_3 \partial_3 h_{\theta 1}$
02	$-\frac{1}{2} \partial_1 \partial_\theta h_{12} + \frac{1}{2} \partial_1 \partial_1 h_{\theta 2} + \frac{1}{2} \partial_2 \partial_\theta h_{\theta\theta} - \frac{1}{2} \partial_2 \partial_\theta h_{22} + \frac{\partial_2 \partial_\theta h}{2} - \frac{1}{2} \partial_2 \partial_1 h_{\theta 1} - \frac{1}{2} \partial_3 \partial_\theta h_{23} - \frac{1}{2} \partial_3 \partial_2 h_{\theta 3} + \frac{1}{2} \partial_3 \partial_3 h_{\theta 2}$
03	$-\frac{1}{2} \partial_1 \partial_\theta h_{13} + \frac{1}{2} \partial_1 \partial_1 h_{\theta 3} - \frac{1}{2} \partial_2 \partial_\theta h_{23} + \frac{1}{2} \partial_2 \partial_2 h_{\theta 3} + \frac{1}{2} \partial_3 \partial_\theta h_{\theta\theta} - \frac{1}{2} \partial_3 \partial_\theta h_{33} + \frac{\partial_3 \partial_\theta h}{2} - \frac{1}{2} \partial_3 \partial_1 h_{\theta 1} - \frac{1}{2} \partial_3 \partial_2 h_{\theta 2}$
12	$-\frac{1}{2} \partial_\theta \partial_\theta h_{12} + \frac{1}{2} \partial_1 \partial_\theta h_{\theta 2} + \frac{1}{2} \partial_2 \partial_\theta h_{\theta 1} - \frac{1}{2} \partial_2 \partial_1 h_{11} - \frac{1}{2} \partial_2 \partial_1 h_{22} + \frac{\partial_2 \partial_1 h}{2} - \frac{1}{2} \partial_3 \partial_1 h_{23} - \frac{1}{2} \partial_3 \partial_2 h_{13} + \frac{1}{2} \partial_3 \partial_3 h_{12}$
13	$-\frac{1}{2} \partial_\theta \partial_\theta h_{13} + \frac{1}{2} \partial_1 \partial_\theta h_{\theta 3} - \frac{1}{2} \partial_2 \partial_1 h_{23} + \frac{1}{2} \partial_2 \partial_2 h_{13} + \frac{1}{2} \partial_3 \partial_\theta h_{\theta 1} - \frac{1}{2} \partial_3 \partial_1 h_{11} - \frac{1}{2} \partial_3 \partial_1 h_{33} + \frac{\partial_3 \partial_1 h}{2} - \frac{1}{2} \partial_3 \partial_2 h_{12}$
23	$-\frac{1}{2} \partial_\theta \partial_\theta h_{23} + \frac{1}{2} \partial_1 \partial_1 h_{23} + \frac{1}{2} \partial_2 \partial_\theta h_{\theta 3} - \frac{1}{2} \partial_2 \partial_1 h_{13} + \frac{1}{2} \partial_3 \partial_\theta h_{\theta 2} - \frac{1}{2} \partial_3 \partial_1 h_{12} - \frac{1}{2} \partial_3 \partial_2 h_{22} - \frac{1}{2} \partial_3 \partial_2 h_{33} + \frac{\partial_3 \partial_2 h}{2}$

Now decompose into the form with ϕ , ψ , w_i , and S_{ij} .

Note that the trace condition is

```
"h = 2!\!(\*
InterpretationBox[
StyleBox["\phi",
ShowAutoStyles->False,
AutoSpacing->False],
$CellContext`\phi[],
Editable->False]\) - 6!\!(\*
InterpretationBox[
StyleBox["\psi",
ShowAutoStyles->False,
AutoSpacing->False],
$CellContext`\psi[],
Editable->False]\)
```

00	$-2 \partial^i \partial_i \psi - \frac{1}{2} \partial^i \partial^j S_{ij}$
11	$-2 \partial_\theta \partial_\theta \psi - \partial_2 \partial_2 \phi + \partial_2 \partial_2 \psi - \partial_3 \partial_3 \phi + \partial_3 \partial_3 \psi - \frac{1}{2} \partial_2 \partial_\theta w_2 - \frac{1}{2} \partial_3 \partial_\theta w_3 -$ $\frac{1}{2} \partial_\theta \partial_\theta S_{11} + \frac{1}{2} \partial_2 \partial_2 S_{11} + \frac{1}{2} \partial_2 \partial_2 S_{22} + \frac{1}{2} \partial_3 \partial_3 S_{11} + \partial_3 \partial_2 S_{23} + \frac{1}{2} \partial_3 \partial_3 S_{33}$
22	$-2 \partial_\theta \partial_\theta \psi - \partial_1 \partial_1 \phi + \partial_1 \partial_1 \psi - \partial_3 \partial_3 \phi + \partial_3 \partial_3 \psi - \frac{1}{2} \partial_1 \partial_\theta w_1 -$ $\frac{1}{2} \partial_3 \partial_\theta w_3 - \partial_\theta \partial_\theta S_{22} + \partial_1 \partial_1 S_{11} + \partial_1 \partial_1 S_{22} + 2 \partial_3 \partial_1 S_{13} + \partial_3 \partial_3 S_{22} + \partial_3 \partial_3 S_{33}$
33	$-2 \partial_\theta \partial_\theta \psi - \partial_1 \partial_1 \phi + \partial_1 \partial_1 \psi - \partial_2 \partial_2 \phi + \partial_2 \partial_2 \psi - \frac{1}{2} \partial_1 \partial_\theta w_1 - \frac{1}{2} \partial_2 \partial_\theta w_2 -$ $\frac{1}{2} \partial_\theta \partial_\theta S_{33} + \frac{1}{2} \partial_1 \partial_1 S_{11} + \frac{1}{2} \partial_1 \partial_1 S_{33} + \partial_2 \partial_1 S_{12} + \frac{1}{2} \partial_2 \partial_2 S_{22} + \frac{1}{2} \partial_2 \partial_2 S_{33}$
01	$-2 \partial_1 \partial_\theta \psi - \frac{1}{2} \partial_\theta \partial^i S_{i1} - \frac{1}{4} \partial_2 \partial_1 w_2 + \frac{1}{4} \partial_2 \partial_2 w_1 - \frac{1}{4} \partial_3 \partial_1 w_3 + \frac{1}{4} \partial_3 \partial_3 w_1$
02	$-2 \partial_2 \partial_\theta \psi - \frac{1}{2} \partial_\theta \partial^i S_{i2} + \frac{1}{4} \partial_1 \partial_1 w_2 - \frac{1}{4} \partial_2 \partial_1 w_1 - \frac{1}{4} \partial_3 \partial_2 w_3 + \frac{1}{4} \partial_3 \partial_3 w_2$
03	$-2 \partial_3 \partial_\theta \psi - \frac{1}{2} \partial_\theta \partial^i S_{i3} + \frac{1}{4} \partial_1 \partial_1 w_3 + \frac{1}{4} \partial_2 \partial_2 w_3 - \frac{1}{4} \partial_3 \partial_1 w_1 - \frac{1}{4} \partial_3 \partial_2 w_2$
12	$\partial_2 \partial_1 \phi - \partial_2 \partial_1 \psi + \frac{1}{4} \partial_1 \partial_\theta w_2 + \frac{1}{4} \partial_2 \partial_\theta w_1 - \frac{1}{2} \partial_\theta \partial_\theta S_{12} -$ $\frac{1}{2} \partial_2 \partial_1 S_{11} - \frac{1}{2} \partial_2 \partial_1 S_{22} - \frac{1}{2} \partial_3 \partial_1 S_{23} - \frac{1}{2} \partial_3 \partial_2 S_{13} + \frac{1}{2} \partial_3 \partial_3 S_{12}$
13	$\partial_3 \partial_1 \phi - \partial_3 \partial_1 \psi + \frac{1}{4} \partial_1 \partial_\theta w_3 + \frac{1}{4} \partial_3 \partial_\theta w_1 - \frac{1}{2} \partial_\theta \partial_\theta S_{13} -$ $\frac{1}{2} \partial_2 \partial_1 S_{23} + \frac{1}{2} \partial_2 \partial_2 S_{13} - \frac{1}{2} \partial_3 \partial_1 S_{11} - \frac{1}{2} \partial_3 \partial_1 S_{33} - \frac{1}{2} \partial_3 \partial_2 S_{12}$
23	$\partial_3 \partial_2 \phi - \partial_3 \partial_2 \psi + \frac{1}{4} \partial_2 \partial_\theta w_3 + \frac{1}{4} \partial_3 \partial_\theta w_2 - \frac{1}{2} \partial_\theta \partial_\theta S_{23} +$ $\frac{1}{2} \partial_1 \partial_1 S_{23} - \frac{1}{2} \partial_2 \partial_1 S_{13} - \frac{1}{2} \partial_3 \partial_1 S_{12} - \frac{1}{2} \partial_3 \partial_2 S_{22} - \frac{1}{2} \partial_3 \partial_2 S_{33}$

Now further decompose S_{ij} and w_i as given in the first equations.

(The following still needs to be compared to literature)

00	$-2 \partial_1 \partial_1 \psi - \partial_1 \partial_1 \partial_1 E - 2 \partial_2 \partial_2 \psi - 2 \partial_2 \partial_2 \partial_1 E - \partial_2 \partial_2 \partial_2 E - 2 \partial_3 \partial_3 \psi - 2 \partial_3 \partial_3 \partial_1 E - 2 \partial_3 \partial_3 \partial_2 E - \partial_3 \partial_3 \partial_3 E$
11	$-\partial_0 \partial_0 E_{11} - 2 \partial_0 \partial_0 \psi + \frac{1}{2} \partial_1 \partial_0 B_1 + \partial_1 \partial_1 E_{11} - \partial_1 \partial_1 \partial_0 E + \partial_2 \partial_2 E_{11} -$ $\partial_2 \partial_2 \phi + \partial_2 \partial_2 \psi - \frac{1}{2} \partial_2 \partial_2 \partial_0 B - \partial_2 \partial_2 \partial_0 \partial_0 E + \partial_2 \partial_2 \partial_1 \partial_1 E + \partial_2 \partial_2 \partial_2 \partial_2 E + \partial_3 \partial_3 E_{11} -$ $\partial_3 \partial_3 \phi + \partial_3 \partial_3 \psi - \frac{1}{2} \partial_3 \partial_3 \partial_0 B - \partial_3 \partial_3 \partial_0 \partial_0 E + \partial_3 \partial_3 \partial_1 \partial_1 E + 2 \partial_3 \partial_3 \partial_2 \partial_2 E + \partial_3 \partial_3 \partial_3 \partial_3 E$
22	$-\partial_0 \partial_0 E_{22} - 2 \partial_0 \partial_0 \psi + \partial_1 \partial_1 E_{22} - \partial_1 \partial_1 \phi + \partial_1 \partial_1 \psi - \frac{1}{2} \partial_1 \partial_1 \partial_0 B -$ $\partial_1 \partial_1 \partial_0 \partial_0 E + \partial_1 \partial_1 \partial_1 \partial_1 E + \frac{1}{2} \partial_2 \partial_0 B_2 + \partial_2 \partial_2 E_{22} - \partial_2 \partial_2 \partial_0 \partial_0 E + \partial_2 \partial_2 \partial_1 \partial_1 E + \partial_3 \partial_3 E_{22} -$ $\partial_3 \partial_3 \phi + \partial_3 \partial_3 \psi - \frac{1}{2} \partial_3 \partial_3 \partial_0 B - \partial_3 \partial_3 \partial_0 \partial_0 E + 2 \partial_3 \partial_3 \partial_1 \partial_1 E + \partial_3 \partial_3 \partial_2 \partial_2 E + \partial_3 \partial_3 \partial_3 \partial_3 E$
33	$-\partial_0 \partial_0 E_{33} - 2 \partial_0 \partial_0 \psi + \partial_1 \partial_1 E_{33} - \partial_1 \partial_1 \phi + \partial_1 \partial_1 \psi - \frac{1}{2} \partial_1 \partial_1 \partial_0 B -$ $\partial_1 \partial_1 \partial_0 \partial_0 E + \partial_1 \partial_1 \partial_1 \partial_1 E + \partial_2 \partial_2 E_{33} - \partial_2 \partial_2 \phi + \partial_2 \partial_2 \psi - \frac{1}{2} \partial_2 \partial_2 \partial_0 B - \partial_2 \partial_2 \partial_0 \partial_0 E +$ $2 \partial_2 \partial_2 \partial_1 \partial_1 E + \partial_2 \partial_2 \partial_2 \partial_2 E + \frac{1}{2} \partial_3 \partial_0 B_3 + \partial_3 \partial_3 E_{33} - \partial_3 \partial_3 \partial_0 \partial_0 E + \partial_3 \partial_3 \partial_1 \partial_1 E + \partial_3 \partial_3 \partial_2 \partial_2 E$
01	$-2 \partial_1 \partial_0 \psi + \frac{1}{4} \partial_1 \partial_1 B_1 - \partial_1 \partial_1 \partial_1 \partial_0 E + \frac{1}{4} \partial_2 \partial_2 B_1 - \partial_2 \partial_2 \partial_1 \partial_0 E + \frac{1}{4} \partial_3 \partial_3 B_1 - \partial_3 \partial_3 \partial_1 \partial_0 E$
02	$\frac{1}{4} \partial_1 \partial_1 B_2 - 2 \partial_2 \partial_0 \psi - \partial_2 \partial_1 \partial_1 \partial_0 E + \frac{1}{4} \partial_2 \partial_2 B_2 - \partial_2 \partial_2 \partial_2 \partial_0 E + \frac{1}{4} \partial_3 \partial_3 B_2 - \partial_3 \partial_3 \partial_2 \partial_0 E$
03	$\frac{1}{4} \partial_1 \partial_1 B_3 + \frac{1}{4} \partial_2 \partial_2 B_3 - 2 \partial_3 \partial_0 \psi - \partial_3 \partial_1 \partial_1 \partial_0 E - \partial_3 \partial_2 \partial_2 \partial_0 E + \frac{1}{4} \partial_3 \partial_3 B_3 - \partial_3 \partial_3 \partial_3 \partial_0 E$
12	$-\partial_0 \partial_0 E_{12} + \frac{1}{4} \partial_1 \partial_0 B_2 + \partial_1 \partial_1 E_{12} + \frac{1}{4} \partial_2 \partial_0 B_1 + \partial_2 \partial_1 \phi - \partial_2 \partial_1 \psi +$ $\frac{1}{2} \partial_2 \partial_1 \partial_0 B - \partial_2 \partial_1 \partial_1 \partial_1 E + \partial_2 \partial_2 E_{12} - \partial_2 \partial_2 \partial_2 \partial_1 E + \partial_3 \partial_3 E_{12} - \partial_3 \partial_3 \partial_2 \partial_1 E$
13	$-\partial_0 \partial_0 E_{13} + \frac{1}{4} \partial_1 \partial_0 B_3 + \partial_1 \partial_1 E_{13} + \partial_2 \partial_2 E_{13} + \frac{1}{4} \partial_3 \partial_0 B_1 +$ $\partial_3 \partial_1 \phi - \partial_3 \partial_1 \psi + \frac{1}{2} \partial_3 \partial_1 \partial_0 B - \partial_3 \partial_1 \partial_1 \partial_1 E - \partial_3 \partial_2 \partial_2 \partial_1 E + \partial_3 \partial_3 E_{13} - \partial_3 \partial_3 \partial_3 \partial_1 E$
23	$-\partial_0 \partial_0 E_{23} + \partial_1 \partial_1 E_{23} + \frac{1}{4} \partial_2 \partial_0 B_3 + \partial_2 \partial_2 E_{23} + \frac{1}{4} \partial_3 \partial_0 B_2 +$ $\partial_3 \partial_2 \phi - \partial_3 \partial_2 \psi + \frac{1}{2} \partial_3 \partial_2 \partial_0 B - \partial_3 \partial_2 \partial_1 \partial_1 E - \partial_3 \partial_2 \partial_2 \partial_2 E + \partial_3 \partial_3 E_{23} - \partial_3 \partial_3 \partial_3 \partial_2 E$