Einstein SVT $\Omega(x)$ Matthew

The Einstein tensor is perturbed according to

$$ds^2 = \Omega^2(x)\big\{-(1+2\phi)d\tau^2 + 2(\tilde{\nabla}_i B + B_i)d\tau dx^i + [(1-2\psi)\gamma_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij}]dx^i dx^j\big\} \tag{1}$$

where

$$\gamma^{ij}\tilde{\nabla}_i B_j = 0, \gamma^{ij}\tilde{\nabla}_i E_j = 0,\ \gamma^{ij}\tilde{\nabla}_i E_{kj} = 0,\ \gamma^{ij}E_{ij} = 0. \tag{2}$$

Covariant derivatives are defined with respect to the 3-space background γ_{ij} and are indicated as $\tilde{\nabla}_i$. The perturbed Einstein tensor is calculated as:

$$\begin{aligned}
\delta G_{00}^{(S)} = &-2\gamma^{ij}\tilde{\nabla}_i\tilde{\nabla}_j(\psi - \Omega^{-1}\tilde{\nabla}_0\Omega B + \Omega^{-1}\tilde{\nabla}_0\Omega\tilde{\nabla}_0 E + \Omega^{-1}\gamma^{kl}\tilde{\nabla}_k\Omega\tilde{\nabla}_l E) + 6\Omega^{-1}\tilde{\nabla}_0\Omega\tilde{\nabla}_0\psi \\
&+ 2\Omega^{-2}\gamma^{ij}\gamma^{kl}\tilde{\nabla}_i\Omega\tilde{\nabla}_k\Omega\tilde{\nabla}_j\tilde{\nabla}_l E - 4\Omega^{-1}\gamma^{ij}\gamma^{kl}\tilde{\nabla}_i\tilde{\nabla}_k\Omega\tilde{\nabla}_j\tilde{\nabla}_l E - 2\Omega^{-1}\gamma^{ij}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\psi \\
&- 2\Omega^{-2}\gamma^{ij}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\Omega(\psi+\phi) + 4\Omega^{-1}\gamma^{ij}\tilde{\nabla}_i\tilde{\nabla}_j\Omega(\psi+\phi) - 2\Omega^{-2}\gamma^{ij}\tilde{\nabla}_0\Omega\tilde{\nabla}_i\Omega\tilde{\nabla}_j B \\
&+ 4\Omega^{-1}\gamma^{ij}\tilde{\nabla}_i\tilde{\nabla}_0\Omega\tilde{\nabla}_j B
\end{aligned} \tag{3}$$

$$\begin{aligned}
\delta G_{00}^{(V)} = &\, 2\Omega^{-1}\gamma^{ij}\gamma^{kl}\tilde{\nabla}_i\Omega\tilde{\nabla}_k\tilde{\nabla}_l E_j + 2\Omega^{-2}\gamma^{ij}\gamma^{kl}\tilde{\nabla}_i\Omega\tilde{\nabla}_k\Omega\tilde{\nabla}_j E_l - 4\Omega^{-1}\gamma^{ij}\gamma^{kl}\tilde{\nabla}_i\tilde{\nabla}_k\Omega\tilde{\nabla}_j E_l \\
&- 2\Omega^{-2}\gamma^{ij}\tilde{\nabla}_0\Omega\tilde{\nabla}_i\Omega B_j + 4\Omega^{-1}\gamma^{ij}\tilde{\nabla}_0\tilde{\nabla}_i\Omega B_j
\end{aligned} \tag{4}$$

$$\delta G_{00}^{(T)} = 2\Omega^{-2}\gamma^{ij}\gamma^{kl}\tilde{\nabla}_i\Omega\tilde{\nabla}_k\Omega E_{jl} - 4\Omega^{-1}\gamma^{ij}\gamma^{kl}\tilde{\nabla}_i\tilde{\nabla}_k\Omega E_{jl} \tag{5}$$