Coordinate Transformation RW SVT3 v2

1 RW $\Omega(\tau)$

$$ds^2 = (g_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu = \Omega^2(\tau)(\tilde{g}_{\mu\nu}+f_{\mu\nu})dx^\mu dx^\nu \tag{1.1}$$

$$\tilde{g}_{\mu\nu} = \text{diag}\left(-1,\frac{1}{1-kr^2},r^2,r^2\sin^2\theta\right) \qquad \tilde{\Gamma}^\lambda_{\alpha\beta}=\delta^\lambda_i\delta^j_\alpha\delta^k_\beta\tilde{\Gamma}^i_{jk} \tag{1.2}$$

1.1 W_1

As evaluated in the background geometry of $\Omega^2(x)\tilde{g}_{\mu\nu}dx^\mu dx^\nu$.

$$\begin{aligned}
W^{(1)}_{\mu\nu} = {} & \tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}^2\Omega^{-2}-2\tilde{R}\tilde{R}_{\mu\nu}\Omega^{-2}+2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R}-12\tilde{R}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-6\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R}\\
&+6\tilde{g}_{\mu\nu}\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega-30\tilde{g}_{\mu\nu}\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega+96\tilde{g}_{\mu\nu}\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega\\
&-60\tilde{g}_{\mu\nu}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega+12\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-60\tilde{g}_{\mu\nu}\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega\\
&+6\Omega^{-3}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\tilde{R}+6\Omega^{-3}\tilde{\nabla}_\mu\tilde{R}\tilde{\nabla}_\nu\Omega-12\tilde{R}\Omega^{-4}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega-168\Omega^{-5}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega\\
&+48\Omega^{-4}\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\Omega+48\Omega^{-4}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-2\Omega^{-2}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{R}+12\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega\\
&-12\Omega^{-3}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega+\tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}^2\Omega^{-2}-2\tilde{R}\tilde{R}_{\mu\nu}\Omega^{-2}+2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R}\\
&-12\tilde{R}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-6\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R}+6\tilde{g}_{\mu\nu}\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\\
&-30\tilde{g}_{\mu\nu}\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega+96\tilde{g}_{\mu\nu}\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega-60\tilde{g}_{\mu\nu}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega\\
&+12\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-60\tilde{g}_{\mu\nu}\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega+6\Omega^{-3}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\tilde{R}+6\Omega^{-3}\tilde{\nabla}_\mu\tilde{R}\tilde{\nabla}_\nu\Omega\\
&-12\tilde{R}\Omega^{-4}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega-168\Omega^{-5}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega+48\Omega^{-4}\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\Omega\\
&+48\Omega^{-4}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-2\Omega^{-2}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{R}+12\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega\\
&-12\Omega^{-3}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega+\tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}^2\Omega^{-2}-2\tilde{R}\tilde{R}_{\mu\nu}\Omega^{-2}+2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R}\\
&-12\tilde{R}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-6\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R}+6\tilde{g}_{\mu\nu}\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\\
&-30\tilde{g}_{\mu\nu}\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega+96\tilde{g}_{\mu\nu}\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega-60\tilde{g}_{\mu\nu}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega\\
&+12\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-60\tilde{g}_{\mu\nu}\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega+6\Omega^{-3}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\tilde{R}+6\Omega^{-3}\tilde{\nabla}_\mu\tilde{R}\tilde{\nabla}_\nu\Omega\\
&-12\tilde{R}\Omega^{-4}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega-168\Omega^{-5}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega+48\Omega^{-4}\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\Omega\\
&+48\Omega^{-4}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-2\Omega^{-2}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{R}+12\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega-12\Omega^{-3}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega
\end{aligned} \tag{1.3}$$

$$\begin{aligned}
g^{\mu\nu}W^{(1)}_{\mu\nu} = {} & 6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R}-12\tilde{R}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R}+12\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\\
&-108\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega+216\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega-144\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega\\
&+36\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-144\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega+6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R}-12\tilde{R}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\\
&-12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R}+12\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega-108\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega\\
&+216\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega-144\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega+36\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\\
&-144\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega+6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R}-12\tilde{R}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega-12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R}\\
&+12\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega-108\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega+216\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega
\end{aligned}$$

$$-144\Omega^{-4}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}\Omega+36\Omega^{-3}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega-144\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}^{\beta}\Omega \tag{1.4}$$

$$= 36\Omega^{-7}\Big[6\ddot{\Omega}\dot{\Omega}^2-3\ddot{\Omega}^2\Omega-4\dddot{\Omega}\dot{\Omega}\Omega+2k\dot{\Omega}^2\Omega+\ddddot{\Omega}\Omega^2-2k\ddot{\Omega}\Omega^2\Big] \tag{1.5}$$

1.2 $g^{\mu\nu}\delta W^{(1)}_{\mu\nu}$

$$\begin{aligned}
g^{\mu\nu}\delta W^{(1)}_{\mu\nu} = 12\Omega^{-4}\Big[& -3\ddddot{\psi}+\psi(3k^2-48\ddot{\Omega}\dot{\Omega}^2\Omega^{-3}+15\ddot{\Omega}^2\Omega^{-2}+30\dddot{\Omega}\dot{\Omega}\Omega^{-2}-6k\dot{\Omega}^2\Omega^{-2}-6\ddddot{\Omega}\Omega^{-1}) \\
& +\ddot{\phi}(12\dot{\Omega}^2\Omega^{-2}-12\ddot{\Omega}\Omega^{-1})+\ddot{\psi}(6k+30\dot{\Omega}^2\Omega^{-2}-12\ddot{\Omega}\Omega^{-1}) \\
& +\phi(3k^2-84\ddot{\Omega}\dot{\Omega}^2\Omega^{-3}+33\ddot{\Omega}^2\Omega^{-2}+54\dddot{\Omega}\dot{\Omega}\Omega^{-2}-18k\dot{\Omega}^2\Omega^{-2}-12\ddddot{\Omega}\Omega^{-1}+12k\ddot{\Omega}\Omega^{-1}) \\
& +\dot{\phi}(-18\dot{\Omega}^3\Omega^{-3}+54\ddot{\Omega}\dot{\Omega}\Omega^{-2}-18\dddot{\Omega}\Omega^{-1}+6k\dot{\Omega}\Omega^{-1}) \\
& +\dot{\psi}(-54\dot{\Omega}^3\Omega^{-3}+90\ddot{\Omega}\dot{\Omega}\Omega^{-2}-18\dddot{\Omega}\Omega^{-1}+6k\dot{\Omega}\Omega^{-1})-3\dddot{\phi}\dot{\Omega}\Omega^{-1}-3\dddot{\psi}\dot{\Omega}\Omega^{-1} \\
& +(-18\dot{\Omega}^3\Omega^{-3}+30\ddot{\Omega}\dot{\Omega}\Omega^{-2}-6\dddot{\Omega}\Omega^{-1}+6k\dot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b B+(10\dot{\Omega}^2\Omega^{-2}-4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\dot{B} \\
& -\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\ddot{B}-\tilde{\nabla}_b\tilde{\nabla}^b\dddot{B}+\tilde{\nabla}_b\tilde{\nabla}^b\dddot{E}+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\ddot{E}+(-10\dot{\Omega}^2\Omega^{-2}+4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\ddot{E} \\
& -\tilde{\nabla}_b\tilde{\nabla}^b\ddot{\phi}+5\tilde{\nabla}_b\tilde{\nabla}^b\ddot{\psi}+(18\dot{\Omega}^3\Omega^{-3}-30\ddot{\Omega}\dot{\Omega}\Omega^{-2}+6\dddot{\Omega}\Omega^{-1}-6k\dot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\dot{E} \\
& +5\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\dot{\phi}+5\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\dot{\psi} \\
& +(-k^2+16\ddot{\Omega}\dot{\Omega}^2\Omega^{-3}-5\ddot{\Omega}^2\Omega^{-2}-10\dddot{\Omega}\dot{\Omega}\Omega^{-2}+6k\dot{\Omega}^2\Omega^{-2}+2\ddddot{\Omega}\Omega^{-1}-4k\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b E \\
& +(-2\dot{\Omega}^2\Omega^{-2}+8\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\phi+(-6k+4\dot{\Omega}^2\Omega^{-2}-4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\psi+3\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a B \\
& +\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}-\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}-3\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}+\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi-2\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi\Big]
\end{aligned} \tag{1.6}$$