

# General Gauge:

$$\eta^{\alpha\beta} \partial_\alpha h_{\beta\gamma} = \frac{J \eta^{\alpha\beta} h_{\gamma\alpha} \partial_\beta \Omega}{\Omega} + P \Omega^2 \partial_\gamma h + R h \Omega \partial_\gamma \Omega$$

## Ricci Tensor

In General Gauge:

$$\begin{aligned} \frac{\eta^{\mu\nu}}{\Omega^2} \delta R_{\mu\nu} = & \frac{3 \eta^{\alpha\mu} \partial_\alpha h \partial_\mu \Omega}{\Omega^3} - \frac{4 P \eta^{\alpha\mu} \partial_\alpha h \partial_\mu \Omega}{\Omega^3} - \frac{J P \eta^{\alpha\mu} \partial_\alpha h \partial_\mu \Omega}{\Omega^3} - \frac{R \eta^{\alpha\mu} \partial_\alpha h \partial_\mu \Omega}{\Omega^3} + \frac{2 \eta^{\alpha\nu} \eta^{\mu\beta} h_{\nu\beta} \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^6} - \\ & \frac{J \eta^{\alpha\nu} \eta^{\mu\beta} h_{\nu\beta} \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^6} - \frac{J^2 \eta^{\alpha\nu} \eta^{\mu\beta} h_{\nu\beta} \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^6} + \frac{\eta^{\alpha\mu} h \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^4} - \frac{3 R \eta^{\alpha\mu} h \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^4} - \\ & \frac{J R \eta^{\alpha\mu} h \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^4} + \frac{\eta^{\alpha\mu} \partial_\mu \partial_\alpha h}{\Omega^2} - \frac{P \eta^{\alpha\mu} \partial_\mu \partial_\alpha h}{\Omega^2} + \frac{\eta^{\alpha\mu} h \partial_\mu \partial_\alpha \Omega}{\Omega^3} - \frac{R \eta^{\alpha\mu} h \partial_\mu \partial_\alpha \Omega}{\Omega^3} - \frac{2 \eta^{\alpha\mu} \eta^{\nu\beta} h_{\alpha\nu} \partial_\mu \partial_\beta \Omega}{\Omega^5} - \\ & \frac{J \eta^{\alpha\mu} \eta^{\nu\beta} h_{\alpha\nu} \partial_\mu \partial_\beta \Omega}{\Omega^5} \end{aligned}$$

Non trace terms from above:

$$\begin{aligned} \frac{\eta^{\mu\nu}}{\Omega^2} \delta R_{\mu\nu} = & \frac{2 \eta^{\alpha\nu} \eta^{\mu\beta} h_{\nu\beta} \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^6} - \frac{J \eta^{\alpha\nu} \eta^{\mu\beta} h_{\nu\beta} \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^6} - \frac{J^2 \eta^{\alpha\nu} \eta^{\mu\beta} h_{\nu\beta} \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^6} - \\ & \frac{2 \eta^{\alpha\mu} \eta^{\nu\beta} h_{\alpha\nu} \partial_\mu \partial_\beta \Omega}{\Omega^5} - \frac{J \eta^{\alpha\mu} \eta^{\nu\beta} h_{\alpha\nu} \partial_\mu \partial_\beta \Omega}{\Omega^5} \end{aligned}$$

We see that we must have J = -2, in which all non-trace terms vanish. Thus for J = -2:

$$\begin{aligned} \frac{\eta^{\mu\nu}}{\Omega^2} \delta R_{\mu\nu} = & \frac{3 \eta^{\alpha\mu} \partial_\alpha h \partial_\mu \Omega}{\Omega^3} - \frac{2 P \eta^{\alpha\mu} \partial_\alpha h \partial_\mu \Omega}{\Omega^3} - \frac{R \eta^{\alpha\mu} \partial_\alpha h \partial_\mu \Omega}{\Omega^3} + \frac{\eta^{\alpha\mu} h \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^4} - \frac{R \eta^{\alpha\mu} h \partial_\alpha \Omega \partial_\mu \Omega}{\Omega^4} + \frac{\eta^{\alpha\mu} \partial_\mu \partial_\alpha h}{\Omega^2} - \\ & \frac{P \eta^{\alpha\mu} \partial_\mu \partial_\alpha h}{\Omega^2} + \frac{\eta^{\alpha\mu} h \partial_\mu \partial_\alpha \Omega}{\Omega^3} - \frac{R \eta^{\alpha\mu} h \partial_\mu \partial_\alpha \Omega}{\Omega^3} \end{aligned}$$

## Perturbed Einstein Tensor

$$\frac{\eta^{\mu\nu}}{\Omega^2} \delta G_{\mu\nu} =$$

$$- \frac{3 \eta^{\alpha\beta} \partial_\alpha h \partial_\beta \Omega}{\Omega^3} + \frac{4 P \eta^{\alpha\beta} \partial_\alpha h \partial_\beta \Omega}{\Omega^3} + \frac{J P \eta^{\alpha\beta} \partial_\alpha h \partial_\beta \Omega}{\Omega^3} + \frac{R \eta^{\alpha\beta} \partial_\alpha h \partial_\beta \Omega}{\Omega^3} - \frac{10 \eta^{\alpha\mu} \eta^{\beta\nu} h_{\mu\nu} \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^6} +$$

$$\frac{J \eta^{\alpha\mu} \eta^{\beta\nu} h_{\mu\nu} \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^6} + \frac{J^2 \eta^{\alpha\mu} \eta^{\beta\nu} h_{\mu\nu} \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^6} + \frac{\eta^{\alpha\beta} h \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^4} + \frac{3 R \eta^{\alpha\beta} h \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^4} +$$

$$\frac{J R \eta^{\alpha\beta} h \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^4} - \frac{\eta^{\alpha\beta} \partial_\beta \partial_\alpha h}{\Omega^2} + \frac{P \eta^{\alpha\beta} \partial_\beta \partial_\alpha h}{\Omega^2} - \frac{2 \eta^{\alpha\beta} h \partial_\beta \partial_\alpha \Omega}{\Omega^3} + \frac{R \eta^{\alpha\beta} h \partial_\beta \partial_\alpha \Omega}{\Omega^3} +$$

$$\frac{6 \eta^{\alpha\beta} \eta^{\mu\nu} h_{\alpha\mu} \partial_\nu \partial_\beta \Omega}{\Omega^5} + \frac{J \eta^{\alpha\beta} \eta^{\mu\nu} h_{\alpha\mu} \partial_\nu \partial_\beta \Omega}{\Omega^5}$$

Looking at non-trace terms:

$$\frac{\eta^{\mu\nu}}{\Omega^2} \delta G_{\mu\nu} =$$

$$- \frac{10 \eta^{\alpha\mu} \eta^{\beta\nu} h_{\mu\nu} \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^6} + \frac{J \eta^{\alpha\mu} \eta^{\beta\nu} h_{\mu\nu} \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^6} + \frac{J^2 \eta^{\alpha\mu} \eta^{\beta\nu} h_{\mu\nu} \partial_\alpha \Omega \partial_\beta \Omega}{\Omega^6} +$$

$$\frac{6 \eta^{\alpha\beta} \eta^{\mu\nu} h_{\alpha\mu} \partial_\nu \partial_\beta \Omega}{\Omega^5} + \frac{J \eta^{\alpha\beta} \eta^{\mu\nu} h_{\alpha\mu} \partial_\nu \partial_\beta \Omega}{\Omega^5}$$

For  $\Omega = 1/Ht$ , this becomes

$$= 2 H^4 h_{\theta\theta} t^2 + 3 H^4 J h_{\theta\theta} t^2 + H^4 J^2 h_{\theta\theta} t^2$$

For  $J = -1$  or  $J = -2$ , the above vanishes.

Full Einstein perturbation

$$\delta G_{\mu\nu} =$$

00	$2 H^2 h_{00} - \frac{3}{2} H^2 J h_{00} - \frac{1}{2} H^2 J^2 h_{00} + \frac{\frac{R h}{2} - \frac{J R h}{2}}{t^2} + t \left( -H^2 \partial_0 h_{00} - H^2 J \partial_0 h_{00} \right) +$ $\frac{\partial_0 h - P \partial_0 h - \frac{1}{2} J P \partial_0 h + \frac{R \partial_0 h}{2}}{t} - \frac{1}{2} P \partial_0 \partial_0 h + \frac{\partial_1 \partial_1 h}{2} - \frac{1}{2} P \partial_1 \partial_1 h + \frac{\partial_2 \partial_2 h}{2} - \frac{1}{2} P \partial_2 \partial_2 h +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{00} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{00} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{00} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{00} \right) + \frac{\partial_3 \partial_3 h}{2} - \frac{1}{2} P \partial_3 \partial_3 h$
11	$\frac{1}{2} H^2 J h_{00} + \frac{1}{2} H^2 J^2 h_{00} + 4 H^2 h_{11} + \frac{\frac{3 R h}{2} - \frac{J R h}{2}}{t^2} + \frac{-\partial_0 h + P \partial_0 h + \frac{1}{2} J P \partial_0 h + \frac{R \partial_0 h}{2}}{t} +$ $\frac{\partial_0 \partial_0 h}{2} - \frac{1}{2} P \partial_0 \partial_0 h + t \left( -H^2 \partial_0 h_{11} - H^2 J \partial_1 h_{01} \right) - \frac{1}{2} P \partial_1 \partial_1 h - \frac{\partial_2 \partial_2 h}{2} + \frac{1}{2} P \partial_2 \partial_2 h +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{11} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{11} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{11} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{11} \right) - \frac{\partial_3 \partial_3 h}{2} + \frac{1}{2} P \partial_3 \partial_3 h$
22	$\frac{1}{2} H^2 J h_{00} + \frac{1}{2} H^2 J^2 h_{00} + 4 H^2 h_{22} + \frac{\frac{3 R h}{2} - \frac{J R h}{2}}{t^2} + \frac{-\partial_0 h + P \partial_0 h + \frac{1}{2} J P \partial_0 h + \frac{R \partial_0 h}{2}}{t} +$ $\frac{\partial_0 \partial_0 h}{2} - \frac{1}{2} P \partial_0 \partial_0 h - \frac{\partial_1 \partial_1 h}{2} + \frac{1}{2} P \partial_1 \partial_1 h + t \left( -H^2 \partial_0 h_{22} - H^2 J \partial_2 h_{02} \right) - \frac{1}{2} P \partial_2 \partial_2 h +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{22} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{22} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{22} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{22} \right) - \frac{\partial_3 \partial_3 h}{2} + \frac{1}{2} P \partial_3 \partial_3 h$
33	$\frac{1}{2} H^2 J h_{00} + \frac{1}{2} H^2 J^2 h_{00} + 4 H^2 h_{33} + \frac{\frac{3 R h}{2} - \frac{J R h}{2}}{t^2} + \frac{-\partial_0 h + P \partial_0 h + \frac{1}{2} J P \partial_0 h + \frac{R \partial_0 h}{2}}{t} +$ $\frac{\partial_0 \partial_0 h}{2} - \frac{1}{2} P \partial_0 \partial_0 h - \frac{\partial_1 \partial_1 h}{2} + \frac{1}{2} P \partial_1 \partial_1 h - \frac{\partial_2 \partial_2 h}{2} + \frac{1}{2} P \partial_2 \partial_2 h + t \left( -H^2 \partial_0 h_{33} - H^2 J \partial_3 h_{03} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{33} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{33} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{33} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{33} \right) - \frac{1}{2} P \partial_3 \partial_3 h$
01	$3 H^2 h_{01} - \frac{1}{2} H^2 J h_{01} + \frac{\partial_0 \partial_1 h}{2} - P \partial_0 \partial_1 h + t \left( -H^2 \partial_0 h_{01} - \frac{1}{2} H^2 J \partial_0 h_{01} - \frac{1}{2} H^2 J \partial_1 h_{00} \right) +$ $\frac{R \partial_1 h}{2 t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{01} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{01} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{01} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{01} \right)$
02	$3 H^2 h_{02} - \frac{1}{2} H^2 J h_{02} + \frac{\partial_0 \partial_2 h}{2} - P \partial_0 \partial_2 h + t \left( -H^2 \partial_0 h_{02} - \frac{1}{2} H^2 J \partial_0 h_{02} - \frac{1}{2} H^2 J \partial_2 h_{00} \right) +$ $\frac{R \partial_2 h}{2 t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{02} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{02} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{02} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{02} \right)$
03	$3 H^2 h_{03} - \frac{1}{2} H^2 J h_{03} + \frac{\partial_0 \partial_3 h}{2} - P \partial_0 \partial_3 h + t \left( -H^2 \partial_0 h_{03} - \frac{1}{2} H^2 J \partial_0 h_{03} - \frac{1}{2} H^2 J \partial_3 h_{00} \right) +$ $\frac{R \partial_3 h}{2 t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{03} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{03} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{03} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{03} \right)$
12	$4 H^2 h_{12} + \frac{\partial_1 \partial_2 h}{2} - P \partial_1 \partial_2 h + t \left( -H^2 \partial_0 h_{12} - \frac{1}{2} H^2 J \partial_1 h_{02} - \frac{1}{2} H^2 J \partial_2 h_{01} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{12} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{12} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{12} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{12} \right)$
13	$4 H^2 h_{13} + \frac{\partial_1 \partial_3 h}{2} - P \partial_1 \partial_3 h + t \left( -H^2 \partial_0 h_{13} - \frac{1}{2} H^2 J \partial_1 h_{03} - \frac{1}{2} H^2 J \partial_3 h_{01} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{13} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{13} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{13} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{13} \right)$
23	$4 H^2 h_{23} + \frac{\partial_2 \partial_3 h}{2} - P \partial_2 \partial_3 h + t \left( -H^2 \partial_0 h_{23} - \frac{1}{2} H^2 J \partial_2 h_{03} - \frac{1}{2} H^2 J \partial_3 h_{02} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{23} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{23} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{23} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{23} \right)$

In covariant Harmonic gauge: J = -2, P = 1/2, R = 1

$$\eta^{\alpha\beta} \partial_\beta h_{\nu\alpha} = -\frac{2 \eta^{\alpha\beta} h_{\nu\beta} \partial_\alpha \Omega}{\Omega} + \frac{1}{2} \Omega^2 \partial_\nu h + h_{\Omega} \partial_\nu \Omega$$

00	$3 H^2 h_{00} - \frac{h}{2t^2} + H^2 t \partial_0 h_{00} + \frac{3 \partial_0 h}{2t} - \frac{\partial_0 \partial_0 h}{4} + \frac{\partial_1 \partial_1 h}{4} + \frac{\partial_2 \partial_2 h}{4} +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{00} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{00} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{00} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{00} \right) + \frac{\partial_3 \partial_3 h}{4}$
11	$H^2 h_{00} + 4 H^2 h_{11} - \frac{h}{2t^2} - \frac{\partial_0 h}{2t} + \frac{\partial_0 \partial_0 h}{4} + t \left( -H^2 \partial_0 h_{11} + 2 H^2 \partial_1 h_{01} \right) - \frac{\partial_1 \partial_1 h}{4} -$ $\frac{\partial_2 \partial_2 h}{4} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{11} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{11} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{11} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{11} \right) - \frac{\partial_3 \partial_3 h}{4}$
22	$H^2 h_{00} + 4 H^2 h_{22} - \frac{h}{2t^2} - \frac{\partial_0 h}{2t} + \frac{\partial_0 \partial_0 h}{4} - \frac{\partial_1 \partial_1 h}{4} + t \left( -H^2 \partial_0 h_{22} + 2 H^2 \partial_2 h_{02} \right) -$ $\frac{\partial_2 \partial_2 h}{4} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{22} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{22} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{22} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{22} \right) - \frac{\partial_3 \partial_3 h}{4}$
33	$H^2 h_{00} + 4 H^2 h_{33} - \frac{h}{2t^2} - \frac{\partial_0 h}{2t} + \frac{\partial_0 \partial_0 h}{4} - \frac{\partial_1 \partial_1 h}{4} - \frac{\partial_2 \partial_2 h}{4} + t \left( -H^2 \partial_0 h_{33} + 2 H^2 \partial_3 h_{03} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{33} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{33} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{33} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{33} \right) - \frac{\partial_3 \partial_3 h}{4}$
01	$4 H^2 h_{01} + H^2 t \partial_1 h_{00} + \frac{\partial_1 h}{2t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{01} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{01} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{01} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{01} \right)$
02	$4 H^2 h_{02} + H^2 t \partial_2 h_{00} + \frac{\partial_2 h}{2t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{02} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{02} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{02} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{02} \right)$
03	$4 H^2 h_{03} + H^2 t \partial_3 h_{00} + \frac{\partial_3 h}{2t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{03} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{03} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{03} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{03} \right)$
12	$4 H^2 h_{12} + t \left( -H^2 \partial_0 h_{12} + H^2 \partial_1 h_{02} + H^2 \partial_2 h_{01} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{12} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{12} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{12} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{12} \right)$
13	$4 H^2 h_{13} + t \left( -H^2 \partial_0 h_{13} + H^2 \partial_1 h_{03} + H^2 \partial_3 h_{01} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{13} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{13} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{13} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{13} \right)$
23	$4 H^2 h_{23} + t \left( -H^2 \partial_0 h_{23} + H^2 \partial_2 h_{03} + H^2 \partial_3 h_{02} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{23} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{23} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{23} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{23} \right)$

In covariant transverse gauge: J = -2, P = 0, R = 1

$$\eta^{\alpha\beta} \partial_\beta h_{\nu\alpha} = -\frac{2 \eta^{\alpha\beta} h_{\nu\beta} \partial_\alpha \Omega}{\Omega} + h_{\nu\Omega} \partial_\nu \Omega$$

00	$3 H^2 h_{00} - \frac{h}{2t^2} + H^2 t \partial_0 h_{00} + \frac{3 \partial_0 h}{2t} + \frac{\partial_1 \partial_1 h}{2} + \frac{\partial_2 \partial_2 h}{2} +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{00} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{00} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{00} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{00} \right) + \frac{\partial_3 \partial_3 h}{2}$
11	$H^2 h_{00} + 4 H^2 h_{11} - \frac{h}{2t^2} - \frac{\partial_0 h}{2t} + \frac{\partial_0 \partial_0 h}{2} + t \left( -H^2 \partial_0 h_{11} + 2 H^2 \partial_1 h_{01} \right) -$ $\frac{\partial_2 \partial_2 h}{2} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{11} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{11} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{11} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{11} \right) - \frac{\partial_3 \partial_3 h}{2}$
22	$H^2 h_{00} + 4 H^2 h_{22} - \frac{h}{2t^2} - \frac{\partial_0 h}{2t} + \frac{\partial_0 \partial_0 h}{2} - \frac{\partial_1 \partial_1 h}{2} + t \left( -H^2 \partial_0 h_{22} + 2 H^2 \partial_2 h_{02} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{22} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{22} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{22} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{22} \right) - \frac{\partial_3 \partial_3 h}{2}$
33	$H^2 h_{00} + 4 H^2 h_{33} - \frac{h}{2t^2} - \frac{\partial_0 h}{2t} + \frac{\partial_0 \partial_0 h}{2} - \frac{\partial_1 \partial_1 h}{2} - \frac{\partial_2 \partial_2 h}{2} + t \left( -H^2 \partial_0 h_{33} + 2 H^2 \partial_3 h_{03} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{33} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{33} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{33} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{33} \right)$
01	$4 H^2 h_{01} + \frac{\partial_0 \partial_1 h}{2} + H^2 t \partial_1 h_{00} + \frac{\partial_1 h}{2t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{01} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{01} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{01} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{01} \right)$
02	$4 H^2 h_{02} + \frac{\partial_0 \partial_2 h}{2} + H^2 t \partial_2 h_{00} + \frac{\partial_2 h}{2t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{02} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{02} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{02} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{02} \right)$
03	$4 H^2 h_{03} + \frac{\partial_0 \partial_3 h}{2} + H^2 t \partial_3 h_{00} + \frac{\partial_3 h}{2t} + t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{03} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{03} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{03} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{03} \right)$
12	$4 H^2 h_{12} + \frac{\partial_1 \partial_2 h}{2} + t \left( -H^2 \partial_0 h_{12} + H^2 \partial_1 h_{02} + H^2 \partial_2 h_{01} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{12} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{12} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{12} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{12} \right)$
13	$4 H^2 h_{13} + \frac{\partial_1 \partial_3 h}{2} + t \left( -H^2 \partial_0 h_{13} + H^2 \partial_1 h_{03} + H^2 \partial_3 h_{01} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{13} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{13} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{13} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{13} \right)$
23	$4 H^2 h_{23} + \frac{\partial_2 \partial_3 h}{2} + t \left( -H^2 \partial_0 h_{23} + H^2 \partial_2 h_{03} + H^2 \partial_3 h_{02} \right) +$ $t^2 \left( -\frac{1}{2} H^2 \partial_0 \partial_0 h_{23} + \frac{1}{2} H^2 \partial_1 \partial_1 h_{23} + \frac{1}{2} H^2 \partial_2 \partial_2 h_{23} + \frac{1}{2} H^2 \partial_3 \partial_3 h_{23} \right)$

We noted from previous work that  $h_{0i}$  terms will be present in  $h_{ii}$  components unless  $J = 0$ . Seems that to have the trace of  $\delta G_{\mu\nu}$  be expressed in terms of  $h$  only, we lose some of the diagonalizability.

\*Note: When taking trace of a gauged perturbation, gauge condition must be used again.