Radiation zone: $d \ll \lambda \ll r$

$$\mathbf{B} = \nabla \times \mathbf{A} = i\mathbf{k} \times \mathbf{A}, \qquad \mathbf{E} = c(\mathbf{B} \times \hat{\mathbf{e}}_k)$$

$$\mathbf{S} = \frac{cB^2}{\mu_0}\hat{\mathbf{e}}_k, \qquad I = \frac{dP}{d\Omega} = Sr^2$$

$$t' = t - \frac{r}{c}$$

Electric Dipole:

$$\mathbf{p} = \int d^3r' \; \mathbf{r}' \rho(\mathbf{r}')$$

$$\begin{aligned}\mathbf{A} &= \frac{\mu_0}{4\pi r}\dot{\mathbf{p}}(t') \\ &= -\frac{i\omega\mu_0}{4\pi}\frac{e^{i(kr-\omega t)}}{r}\mathbf{p}_\omega \quad \text{(Harmonic)}\end{aligned}$$

$$\mathbf{B} = \frac{\mu_0 ck^2}{4\pi}\frac{e^{i(kr-\omega t)}}{r}(\hat{\mathbf{e}}_k \times \mathbf{p}_\omega) \quad \text{(Harmonic)}$$

$$\mathbf{S} = \frac{\mu_0 c^3 k^4}{16\pi^2 r^2} p_\omega^2 \sin^2\theta \cos^2(kr - \omega t)\hat{\mathbf{e}}_k \quad \text{(Harmonic)}$$

$$I = \frac{\ddot{\mathbf{p}}^2}{4\pi c^3}\sin^2\theta$$

$$P_{total} = \frac{2}{3c^3}\ddot{\mathbf{p}}^2$$

Magnetic Dipole:

$$\mathbf{m} = \frac{1}{2}\int d^3r' \; \mathbf{r}' \times \mathbf{j}(\mathbf{r}')$$

$$\begin{aligned}\mathbf{A} &= \frac{\mu_0}{4\pi rc}\dot{\mathbf{m}}(t') \times \hat{\mathbf{e}}_k \\ &= -\frac{i\omega\mu_0}{4\pi c}\frac{e^{i(kr-\omega t)}}{r}(\mathbf{m}_\omega \times \hat{\mathbf{e}}_k) \quad \text{(Harmonic)}\end{aligned}$$

$$\mathbf{B} = \frac{\mu_0 k^2}{4\pi}\frac{e^{i(kr-\omega t)}}{r}(\hat{\mathbf{e}}_k \times \mathbf{m}_\omega) \times \hat{\mathbf{e}}_k \quad \text{(Harmonic)}$$

$$\mathbf{S} = \frac{c\mu_0 k^4}{16\pi^2} m_\omega^2 \sin^2\theta \frac{\cos^2(kr - \omega t)}{r^2} \quad \text{(Harmonic)}$$

$$P_{total} = \frac{2}{3c^3}\ddot{\mathbf{m}}^2$$

Quadrupole:

$$Q_\alpha(\hat{\mathbf{e}}_k) = \sum_\beta Q_{\alpha\beta}(\hat{\mathbf{e}}_k)_\beta$$

$$Q_{\alpha\beta} = \int d^3x \; (3x_\alpha x_\beta - \delta_{\alpha\beta} r^2)\rho(\mathbf{x})$$

$$\begin{aligned}\mathbf{A} &= \frac{\mu_0}{24\pi rc}\ddot{\mathbf{Q}}_k(t') \\ &= -\frac{\mu\omega^2}{24\pi c}\frac{e^{i(kr-\omega t)}}{r}\mathbf{Q}_\omega \quad \text{(Harmonic)}\end{aligned}$$

$$\mathbf{B} = -\frac{i\mu_0 ck^3}{24\pi}\frac{e^{i(kr-\omega t)}}{r}(\hat{\mathbf{e}}_k \times \mathbf{Q}_k) \quad \text{(Harmonic)}$$

$$\langle I \rangle_t = \frac{\mu_0\omega^6}{1152\pi^2 c^3}|(\hat{\mathbf{e}}_k \times \mathbf{Q}_k) \times \hat{\mathbf{e}}_k|^2 \quad \text{(Harmonic)}$$

$$P_{total} = \frac{1}{180c^3}\dddot{\mathbf{Q}}_k^2$$

Radiation by accelerated charge:

$$P = \frac{2q^2}{3c^3}|\ddot{\mathbf{r}}|^2$$

$$I(\theta) = \frac{q^2}{4\pi c^3}|\ddot{\mathbf{r}}|^2 \sin^2\theta$$

Scattering:

$$d\sigma(\hat{\mathbf{e}}_k) = \frac{dP}{dS_0} = I(\theta)\frac{d\Omega}{dS_0}$$

$$\frac{d\sigma}{d\Omega} = \frac{I(\theta)}{S_0}$$

$$\sigma_{total} = \frac{P}{S_0}$$

$$\mathbf{S}_0 = \frac{c}{4\pi}E_0^2\cos^2(kr - \omega t)\hat{\mathbf{e}}_k \quad \text{(Plane Waves)}$$

$$|\mathbf{S}_0| = \frac{c}{4\pi}E_0^2 \quad \text{(Constant Field)}$$

$$\sigma_{total} = \frac{8\pi}{3}\left(\frac{e^2}{m_3c^2}\right)^2 = \frac{8\pi}{3}r_0^2 \quad \text{(Free electron, incoming plane wave)}$$

$$\sigma_{total} = \frac{8\pi}{3}r_0^2\left(\frac{\omega^2}{\omega^2 - \omega_0^2}\right)^2 \quad \text{(Harmonically bound electron, incoming plane wave)}$$