Einstein SVT Matthew

The Einstein tensor is perturbed according to

$$ds^2 = \Omega^2(x)\big\{ -(1+2\phi)d\tau^2 + 2(\tilde{\nabla}_i B + B_i)d\tau dx^i + [(1-2\psi)\gamma_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij}]dx^i dx^j\big\} \quad (1)$$

where

$$\gamma^{ij}\tilde{\nabla}_i B_j = 0, \gamma^{ij}\tilde{\nabla}_i E_j = 0,\ \gamma^{ij}\tilde{\nabla}_i E_{kj} = 0,\ \gamma^{ij}E_{ij} = 0. \quad (2)$$

Covariant derivatives are defined with respect to the 3-space background γ_{ij} and are indicated as $\tilde{\nabla}_i$.

$\Omega(x)$

$$\begin{aligned}\delta G^{(S)}_{00} = {} & 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi + 4\phi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\ & + 4\psi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 4\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B - 2\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi \\ & - 2\phi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - 2\psi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\ & + 2\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega - 4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E. \end{aligned} \quad (3)$$

$$\begin{aligned}\delta G^{(V)}_{00} = {} & 4B^a\Omega^{-1}\tilde{\nabla}_a\dot{\Omega} - 2B^a\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a + 2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^b E^a \\ & - 4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b E^a. \end{aligned} \quad (4)$$

$$\delta G^{(T)}_{00} = -4E^{ab}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega + 2E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega. \quad (5)$$

$$\begin{aligned}\delta G^{(S)}_{0i} = {} & -\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_i B + \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_i B - 2\tilde{\nabla}_i\dot{\psi} \\ & - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + 2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i\tilde{\nabla}_a\dot{E}. \end{aligned} \quad (6)$$

$$\begin{aligned}\delta G^{(V)}_{0i} = = {} & -B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i - 2B_i\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B_i \\ & - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{E}_i + B_i\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i B^a - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i\dot{E}^a. \end{aligned} \quad (7)$$

$$\delta G^{(T)}_{0i} = -2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a\Omega. \quad (8)$$

$$\begin{aligned}\delta G^{(S)}_{ij} = {} & -2\ddot{\psi}\gamma_{ij} + 2\dot{\Omega}^2\gamma_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\gamma_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\gamma_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\gamma_{ij}\Omega^{-1} - 4\ddot{\Omega}\gamma_{ij}\phi\Omega^{-1} \\ & - 4\ddot{\Omega}\gamma_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 2\dot{\Omega}\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\ & - \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 4\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^a B + 2\dot{\Omega}\gamma_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B \\ & - 2\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} - 2\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 2\gamma_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\ & - 2\gamma_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega + 4\gamma_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E + 2\Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\psi \\ & + 2\Omega^{-1}\tilde{\nabla}_i\psi\tilde{\nabla}_j\Omega + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} \\ & - 2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i E - 4\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E + 2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E \\ & + \tilde{\nabla}_j\tilde{\nabla}_i\phi - \tilde{\nabla}_j\tilde{\nabla}_i\psi - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a E. \end{aligned} \quad (9)$$

$$\begin{aligned}\delta G^{(V)}_{ij} = & -4B^a\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega} + 2B^a\dot{\Omega}\gamma_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega - 2\dot{B}^a\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega + 2\gamma_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a \\ & - 2\gamma_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^b E^a + 4\gamma_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b E^a + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j \\ & - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i E_j + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j - 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_j \\ & + \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j E_i \\ & + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_i + \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E_a. \end{aligned} \tag{10}$$

$$\begin{aligned}\delta G^{(T)}_{ij} = & -\ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{\Omega}E_{ij}\Omega^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij} - 4E_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\ & + 2\Omega^{-1}\tilde{\nabla}_a E_{ij}\tilde{\nabla}^a\Omega + 2E_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega + 4E^{ab}\gamma_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega \\ & - 2E_{ab}\gamma_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_{ja} - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_{ia}. \end{aligned} \tag{11}$$

$\Omega(t)$

$$\delta G^{(S)}_{00} = 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi. \tag{12}$$

$$\delta G^{(V)}_{00} = \delta G^{(T)}_{00} = 0. \tag{13}$$

$$\delta G^{(S)}_{0i} = -\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2\tilde{\nabla}_i\dot{\psi} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi. \tag{14}$$

$$\delta G^{(V)}_{0i} = -B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i. \tag{15}$$

$$\delta G^{(T)}_{0i} = 0. \tag{16}$$

$$\begin{aligned}\delta G^{(S)}_{ij} = & -2\ddot{\psi}\gamma_{ij} + 2\dot{\Omega}^2\gamma_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\gamma_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\gamma_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\gamma_{ij}\Omega^{-1} - 4\ddot{\Omega}\gamma_{ij}\phi\Omega^{-1} \\ & - 4\ddot{\Omega}\gamma_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 2\dot{\Omega}\gamma_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\ & - \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} \\ & - 2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i E + \tilde{\nabla}_j\tilde{\nabla}_i\phi - \tilde{\nabla}_j\tilde{\nabla}_i\psi. \end{aligned} \tag{17}$$

$$\begin{aligned}\delta G^{(V)}_{ij} = & \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i E_j + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j \\ & + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j E_i + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i. \end{aligned} \tag{18}$$

$$\delta G^{(T)}_{ij} = -\ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{\Omega}E_{ij}\Omega^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij}. \tag{19}$$