RW SVT3 $\delta G_{\mu\nu} = -\kappa_4^2 \delta T_{\mu\nu}$

1 Background

$$ds^2 = \Omega^2(\tau)\tilde{g}_{\mu\nu}dx^\mu dx^\nu, \qquad \tilde{g}_{\mu\nu} = \mathrm{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right) \tag{1.1}$$

$$G^{(0)}_{00} = -3k - 3\dot{\Omega}^2\Omega^{-2} \qquad G^{(0)}_{0i} = 0 \qquad G^{(0)}_{ij} = k\tilde{g}_{ij} - \dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} + 2\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij} \tag{1.2}$$

$$\kappa_4^2 T^{(0)}_{\mu\nu} = (\rho+p)U_\mu U_\nu + p\Omega^2\tilde{g}_{\mu\nu}, \qquad U_\mu = -\Omega\delta^0_\mu \qquad \text{[Evaluated in (1.1)]} \tag{1.3}$$

$$\Delta^{(0)}_{\mu\nu} = G^{(0)}_{\mu\nu} + \kappa_4^2 T^{(0)}_{\mu\nu} = 0 \tag{1.4}$$

$$\Delta^{(0)}_{00} = -3k - 3\dot{\Omega}^2\Omega^{-2} + \Omega^2\rho \tag{1.5}$$

$$\rightarrow \boxed{\rho = 3k\Omega^{-2} + 3\dot{\Omega}^2\Omega^{-4}} \tag{1.6}$$

$$\Delta^{(0)}_{ij} = k\tilde{g}_{ij} - \dot{\Omega}^2\Omega^{-2}\tilde{g}_{ij} + 2\ddot{\Omega}\Omega^{-1}\tilde{g}_{ij} + \Omega^2 p\tilde{g}_{ij} \tag{1.7}$$

$$\rightarrow \boxed{p = -k\Omega^{-2} + \dot{\Omega}^2\Omega^{-4} - 2\ddot{\Omega}\Omega^{-3}} \tag{1.8}$$

2 Fluctuations

$$ds^2 = \Omega^2(\tau)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu \tag{2.1}$$

$$\tilde{g}_{\mu\nu} = \mathrm{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right) \tag{2.2}$$

$$f_{00} = -2\phi, \qquad f_{0i} = \tilde{\nabla}_i B + B_i, \qquad f_{ij} = -2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{2.3}$$

$$\delta G_{00} = -6k\phi - 6k\psi + 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi \tag{2.4}$$

$$\begin{aligned}\delta G_{0i} = {} & 3k\tilde{\nabla}_i B - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2k\tilde{\nabla}_i\dot{E} - 2\tilde{\nabla}_i\dot{\psi} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + 2kB_i - k\dot{E}_i \\ & - B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i\end{aligned} \tag{2.5}$$

$$\begin{aligned}\delta G_{ij} = {} & -2\ddot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\ & -4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}\end{aligned}$$

$$
\begin{aligned}
&-\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi+2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_iB+\tilde{\nabla}_j\tilde{\nabla}_i\dot{B}-\tilde{\nabla}_j\tilde{\nabla}_i\ddot{E}-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E}\\
&+2k\tilde{\nabla}_j\tilde{\nabla}_iE-2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_iE+4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_iE+\tilde{\nabla}_j\tilde{\nabla}_i\phi-\tilde{\nabla}_j\tilde{\nabla}_i\psi+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_iB_j+\tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j\\
&-\tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j+k\tilde{\nabla}_iE_j-\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_iE_j+2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_iE_j+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_jB_i+\tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i\\
&-\tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i+k\tilde{\nabla}_jE_i-\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_jE_i+2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_jE_i-\ddot{E}_{ij}-2\dot{\Omega}^2E_{ij}\Omega^{-2}\\
&-2\dot{E}_{ij}\dot{\Omega}\Omega^{-1}+4\ddot{\Omega}E_{ij}\Omega^{-1}+\tilde{\nabla}_a\tilde{\nabla}^aE_{ij} \qquad (2.6)
\end{aligned}
$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} &= 6\dot{\Omega}^2\phi\Omega^{-4}+6\dot{\Omega}^2\psi\Omega^{-4}-6\dot{\phi}\dot{\Omega}\Omega^{-3}-18\dot{\psi}\dot{\Omega}\Omega^{-3}-12\ddot{\Omega}\phi\Omega^{-3}-12\ddot{\Omega}\psi\Omega^{-3}-6\ddot{\psi}\Omega^{-2}+6k\phi\Omega^{-2}\\
&\quad+6k\psi\Omega^{-2}-6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aB-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}+6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}\\
&\quad-2\dot{\Omega}^2\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^aE+4\ddot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE+2k\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^aE-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi+4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi \qquad (2.7)
\end{aligned}
$$

$$
\kappa_4^2\delta T_{\mu\nu} = (\delta\rho+\delta p)U_\mu U_\nu+(\rho+p)(\delta U_\mu U_\nu+U_\mu\delta U_\nu)+\Omega^2\delta p\tilde{g}_{\mu\nu}+\Omega^2 pf_{\mu\nu} \qquad (2.8)
$$

$$
\delta U_0 = -\Omega\phi, \qquad \delta U_i=\tilde{\nabla}_iV+V_i \qquad (2.9)
$$

$$
\kappa_4^2\delta T_{00} = \Omega^2\delta\rho+2\Omega^2\rho\phi, \qquad \text{[Substituting (2.9)]} \qquad (2.10)
$$

$$
\kappa_4^2\delta T_{0i} = -\Omega(\rho+p)(\tilde{\nabla}_iV+V_i)+\Omega^2p(\tilde{\nabla}_iB+B_i) \qquad \text{[Substituting (2.9)]} \qquad (2.11)
$$

$$
\kappa_4^2\delta T_{ij} = \Omega^2\delta p\tilde{g}_{ij}+\Omega^2p(-2\psi\tilde{g}_{ij}+2\tilde{\nabla}_i\tilde{\nabla}_jE+\tilde{\nabla}_iE_j+\tilde{\nabla}_jE_i+2E_{ij}) \qquad (2.12)
$$

$$
\kappa_4^2g^{\mu\nu}\delta T_{\mu\nu} = -\delta\rho+3\delta p-2\rho\phi+p(-6\psi+2\tilde{\nabla}_a\tilde{\nabla}^aE) \qquad \text{[Substituting (2.9)]} \qquad (2.13)
$$

3 Field Equations

We express the background EM quantities ρ and p in terms of Ω via substitution (1.6) and (1.8).

$$
\Delta_{\mu\nu} \equiv \delta G_{\mu\nu}+\kappa_4^2\delta T_{\mu\nu}=0 \qquad (3.1)
$$

$$
\Delta_{00} = -6k\psi+6\dot{\Omega}^2\phi\Omega^{-2}+6\dot{\psi}\dot{\Omega}\Omega^{-1}+\delta\rho\Omega^2+2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^aB-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}-2\tilde{\nabla}_a\tilde{\nabla}^a\psi \qquad (3.2)
$$

$$
\begin{aligned}
\Delta_{0i} &= 2k\tilde{\nabla}_iB-2k\tilde{\nabla}_i\dot{E}-2\tilde{\nabla}_i\dot{\psi}-4\dot{\Omega}^2\Omega^{-3}\tilde{\nabla}_iV+2\ddot{\Omega}\Omega^{-2}\tilde{\nabla}_iV-2k\Omega^{-1}\tilde{\nabla}_iV-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi+kB_i\\
&\quad-k\dot{E}_i-4\dot{\Omega}^2V_i\Omega^{-3}+2\ddot{\Omega}V_i\Omega^{-2}-2kV_i\Omega^{-1}+\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^aB_i-\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i \qquad (3.3)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{ij} &= -2\ddot{\psi}\tilde{g}_{ij}+2k\tilde{g}_{ij}\psi+2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2}-2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}-4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}-4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1}+\delta p\tilde{g}_{ij}\Omega^2\\
&\quad-2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^aB-\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}+2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}-\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi\\
&\quad+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi+2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_iB+\tilde{\nabla}_j\tilde{\nabla}_i\dot{B}-\tilde{\nabla}_j\tilde{\nabla}_i\ddot{E}-2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E}+\tilde{\nabla}_j\tilde{\nabla}_i\phi\\
&\quad-\tilde{\nabla}_j\tilde{\nabla}_i\psi+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_iB_j+\tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j-\tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_jB_i+\tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i-\tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i\\
&\quad-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i-\ddot{E}_{ij}-2kE_{ij}-2\dot{E}_{ij}\dot{\Omega}\Omega^{-1}+\tilde{\nabla}_a\tilde{\nabla}^aE_{ij} \qquad (3.4)
\end{aligned}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} &= 3\delta p-\delta\rho-6\dot{\phi}\dot{\Omega}\Omega^{-3}-18\dot{\psi}\dot{\Omega}\Omega^{-3}-12\ddot{\Omega}\phi\Omega^{-3}-6\ddot{\psi}\Omega^{-2}+12k\psi\Omega^{-2}-6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aB\\
&\quad-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}+6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi+4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi \qquad (3.5)
\end{aligned}
$$

4 Field Equations (G.I. Form)

$$\alpha = \phi + \psi + \dot{B} - \ddot{E}, \qquad \gamma = -\dot{\Omega}^{-1}\Omega\psi + B - \dot{E} \tag{4.1}$$

$$\Delta_{00} = -6k\psi - 6\dot{\gamma}\dot{\Omega}^2\Omega^{-2} + 6\dot{\Omega}^2\alpha\Omega^{-2} - 12\dot{\Omega}^2\psi\Omega^{-2} + 6\ddot{\Omega}\psi\Omega^{-1} + \delta\rho\Omega^2 + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\gamma \tag{4.2}$$

$$\begin{aligned}\Delta_{0i} = {} & 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{\gamma} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\alpha + 2k\tilde{\nabla}_i\gamma - 4\dot{\Omega}^2\Omega^{-3}\tilde{\nabla}_iV + 2\ddot{\Omega}\Omega^{-2}\tilde{\nabla}_iV - 2k\Omega^{-1}\tilde{\nabla}_iV - 2\dddot{\Omega}\dot{\Omega}^{-1}\tilde{\nabla}_i\psi \\ & +4\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\psi + 2k\dot{\Omega}^{-1}\Omega\tilde{\nabla}_i\psi + kB_i - k\dot{E}_i - 4\dot{\Omega}^2V_i\Omega^{-3} + 2\ddot{\Omega}V_i\Omega^{-2} - 2kV_i\Omega^{-1} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^aB_i \\ & -\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i\end{aligned} \tag{4.3}$$

$$\begin{aligned}\Delta_{ij} = {} & 2k\tilde{g}_{ij}\psi - 2\dddot{\Omega}\dot{\Omega}^{-1}\tilde{g}_{ij}\psi - 2\dot{\gamma}\dot{\Omega}^2\tilde{g}_{ij}\Omega^{-2} + 2\dot{\Omega}^2\tilde{g}_{ij}\alpha\Omega^{-2} - 4\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2} + 4\ddot{\Omega}\dot{\gamma}\tilde{g}_{ij}\Omega^{-1} \\ & +2\ddot{\gamma}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 2\dot{\alpha}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\alpha\Omega^{-1} + 8\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} + \delta p\tilde{g}_{ij}\Omega^2 - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\alpha \\ & -2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\gamma + \tilde{\nabla}_j\tilde{\nabla}_i\alpha + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\gamma + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_iB_j + \tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j \\ & -\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_jB_i + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i - \ddot{E}_{ij} - 2kE_{ij} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} \\ & +\tilde{\nabla}_a\tilde{\nabla}^aE_{ij}\end{aligned} \tag{4.4}$$

$$\begin{aligned}g^{\mu\nu}\Delta_{\mu\nu} = {} & 3\delta p - \delta\rho + 12\ddot{\Omega}\dot{\gamma}\Omega^{-3} + 6\ddot{\gamma}\dot{\Omega}\Omega^{-3} - 6\dot{\alpha}\dot{\Omega}\Omega^{-3} - 12\ddot{\Omega}\alpha\Omega^{-3} + 18\ddot{\Omega}\psi\Omega^{-3} + 12k\psi\Omega^{-2} \\ & -6\dddot{\Omega}\dot{\Omega}^{-1}\psi\Omega^{-2} - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\alpha - 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\gamma\end{aligned} \tag{4.5}$$

From Δ_{00} and $g^{\mu\nu}\Delta_{\mu\nu}$, the two EM gauge invariant combinations are:

$$\boxed{\delta\rho^{GI} = \delta\rho - 12\dot{\Omega}^2\psi\Omega^{-4} + 6\ddot{\Omega}\psi\Omega^{-3} - 6k\psi\Omega^{-2}} \tag{4.6}$$

$$\boxed{\delta p^{GI} = \delta p - 4\dot{\Omega}^2\psi\Omega^{-4} + 8\ddot{\Omega}\psi\Omega^{-3} + 2k\psi\Omega^{-2} - 2\dddot{\Omega}\dot{\Omega}^{-1}\psi\Omega^{-2}}. \tag{4.7}$$

Given $\Omega(\tau) = \tau/2$, we find the combinations are of the form

$$\delta\rho^{GI} = \delta\rho - 48\tau^{-4}\psi - 24k\tau^{-2}\psi \tag{4.8}$$

$$\delta p^{GI} = \delta p - 16\tau^{-4}\psi + 8k\tau^{-2}\psi. \tag{4.9}$$

With $k = 0$ and $\delta\rho^{GI} - 3\delta p^{GI} = \delta\rho - 3\delta p$, we find the result matches previous work in RW radiation. Using identity

$$[\tilde{\nabla}_a\tilde{\nabla}^a, \tilde{\nabla}_i]E_j = 2k(\tilde{\nabla}_iE_j + \tilde{\nabla}_jE_i) \tag{4.10}$$

we find from $\tilde{\nabla}^i\Delta_{0i}$ a combination

$$\tilde{\nabla}_a\tilde{\nabla}^a\left(-2\dddot{\Omega}\dot{\Omega}^{-1}\psi - 4\dot{\Omega}^2V\Omega^{-3} + 2\ddot{\Omega}V\Omega^{-2} - 2kV\Omega^{-1} + 4\dot{\Omega}\psi\Omega^{-1} + 2k\dot{\Omega}^{-1}\psi\Omega\right). \tag{4.11}$$

This leads to the gauge invariant combination (also agreeing with RW radiation)

$$\boxed{V - \Omega^2\dot{\Omega}^{-1}\psi}. \tag{4.12}$$

With $B_i - \dot{E}_i$ being gauge invariant, it follows that the remaining V_i is itself gauge invariant.

5 Equation of State $w = p/\rho$

$$p = -k\Omega^{-2} + \dot{\Omega}^2\Omega^{-4} - 2\ddot{\Omega}\Omega^{-3} \tag{5.1}$$

$$\rho = 3k\Omega^{-2} + 3\dot{\Omega}^2\Omega^{-4} \tag{5.2}$$

$$\rightarrow \quad 0 = (1-3w)\dot{\Omega}^2 - 2\ddot{\Omega}\Omega - k(1+3w)\Omega^2 \tag{5.3}$$

$$\Omega(\tau) = \cos\left[\sqrt{k}\tau(1+3w)\right]^{\frac{2}{1+3w}} \tag{5.4}$$

$$\Omega(\tau) = \begin{cases} [\tau(1+3w)]^{\frac{2}{1+3w}} & k=0 \\ \cos\left[\frac{1}{2}\tau(1+3w)\right]^{\frac{2}{1+3w}} & k=1 \\ \cosh\left[\frac{1}{2}\tau(1+3w)\right]^{\frac{2}{1+3w}} & k=-1 \end{cases} \tag{5.5}$$

$$\delta\rho^{GI} = \delta\rho - 9k\Big(\cos\big(\tfrac{1}{2}k^{1/2}(\tau+3w\tau)\big)\Big)^{-6(1+w)(1+3w)^{-1}}(1+w)\psi \tag{5.6}$$

$$\delta p^{GI} = \delta p - 9k\Big(\cos\big(\tfrac{1}{2}k^{1/2}(\tau+3w\tau)\big)\Big)^{-6(1+w)(1+3w)^{-1}}w(1+w)\psi \tag{5.7}$$

$$w\delta\rho^{GI} - \delta p^{GI} = w\delta\rho - \delta p \tag{5.8}$$