

First order Weyl fluctuations in dS₄

$$W_{\mu\nu}^{(1)} = \frac{1}{2} g_{\mu\nu} R^2 - 2 R R_{\mu\nu} + 2 g_{\mu\nu} \nabla_\alpha \nabla^\alpha R - 2 \nabla_\nu \nabla_\mu R$$

$$W_{\mu\nu}^{(2)} = \frac{1}{2} g_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} + 2 R^{\alpha\beta} R_{\mu\alpha\beta\nu} + \frac{1}{2} g_{\mu\nu} \nabla_\alpha \nabla^\alpha R + \nabla_\alpha \nabla^\alpha R_{\mu\nu} - \nabla_\nu \nabla_\mu R^\alpha{}_\alpha$$

Eq. 38

$$\delta W_{\mu\nu}^{(1)} = 6 H^4 g_{\mu\nu} h^\alpha{}_\alpha - 24 H^4 h_{\mu\nu} + 12 H^2 \nabla_\alpha \nabla^\alpha h_{\mu\nu} + 6 H^2 g_{\mu\nu} \nabla_\beta \nabla_\alpha h^{\alpha\beta} - 2 g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla_\alpha h^{\alpha\beta} +$$

$$2 g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - 12 H^2 \nabla_\mu \nabla_\alpha h_\nu{}^\alpha - 12 H^2 \nabla_\nu \nabla_\alpha h_\mu{}^\alpha + 6 H^2 \nabla_\nu \nabla_\mu h^\alpha{}_\alpha + 2 \nabla_\nu \nabla_\mu \nabla_\beta \nabla_\alpha h^{\alpha\beta} - 2 \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha$$

Eq. 39

$$\delta W_{\mu\nu}^{(1)} = 6 H^4 g_{\mu\nu} K^\alpha{}_\alpha - 24 H^4 K_{\mu\nu} + 12 H^2 \nabla_\alpha \nabla^\alpha K_{\mu\nu} + 6 H^2 g_{\mu\nu} \nabla_\beta \nabla_\alpha K^{\alpha\beta} + \frac{9}{2} H^2 g_{\mu\nu} \nabla_\beta \nabla^\beta h^\alpha{}_\alpha -$$

$$2 g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla_\alpha K^{\alpha\beta} + \frac{3}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha + 2 g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta K^\alpha{}_\alpha - 12 H^2 \nabla_\mu \nabla_\alpha K_\nu{}^\alpha -$$

$$12 H^2 \nabla_\nu \nabla_\alpha K_\mu{}^\alpha + 6 H^2 \nabla_\nu \nabla_\mu K^\alpha{}_\alpha + 2 \nabla_\nu \nabla_\mu \nabla_\beta \nabla_\alpha K^{\alpha\beta} - \frac{3}{2} \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - 2 \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta K^\alpha{}_\alpha$$

Eq. 45

$$\delta W_{\mu\nu}^{(2)} = H^4 g_{\mu\nu} h^\alpha{}_\alpha - 4 H^4 h_{\mu\nu} + H^2 \nabla_\alpha \nabla^\alpha h_{\mu\nu} + H^2 g_{\mu\nu} \nabla_\beta \nabla_\alpha h^{\alpha\beta} + \frac{3}{2} H^2 g_{\mu\nu} \nabla_\beta \nabla^\beta h^\alpha{}_\alpha +$$

$$\frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\alpha \nabla^\alpha h_{\mu\nu} - \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\mu \nabla_\alpha h_\nu{}^\alpha - \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\nu \nabla_\alpha h_\mu{}^\alpha + \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\nu \nabla_\mu h^\alpha{}_\alpha - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla_\alpha h^{\alpha\beta} +$$

$$\frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - 2 H^2 \nabla_\mu \nabla_\alpha h_\nu{}^\alpha - 2 H^2 \nabla_\nu \nabla_\alpha h_\mu{}^\alpha - H^2 \nabla_\nu \nabla_\mu h^\alpha{}_\alpha + \nabla_\nu \nabla_\mu \nabla_\beta \nabla_\alpha h^{\alpha\beta} - \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha$$

Eq. 48

$$\delta W_{\mu\nu}^{(2)} = -4 H^4 K_{\mu\nu} + H^2 \nabla_\alpha \nabla^\alpha K_{\mu\nu} + \frac{1}{2} H^2 g^{\alpha\beta} g_{\mu\nu} \nabla_\beta \nabla_\alpha h^\gamma{}_\gamma + H^2 g_{\mu\nu} \nabla_\beta \nabla_\alpha K^{\alpha\beta} + H^2 g_{\mu\nu} \nabla_\beta \nabla^\beta h^\alpha{}_\alpha +$$

$$\frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\alpha \nabla^\alpha K_{\mu\nu} - \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\mu \nabla_\alpha K_\nu{}^\alpha - \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\nu \nabla_\alpha K_\mu{}^\alpha - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla_\alpha K^{\alpha\beta} + \frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha -$$

$$2 H^2 \nabla_\mu \nabla_\alpha K_\nu{}^\alpha - \frac{3}{4} H^2 \nabla_\mu \nabla_\nu h^\alpha{}_\alpha - 2 H^2 \nabla_\nu \nabla_\alpha K_\mu{}^\alpha + \frac{3}{4} H^2 \nabla_\nu \nabla_\mu h^\alpha{}_\alpha + \nabla_\nu \nabla_\mu \nabla_\beta \nabla_\alpha K^{\alpha\beta} - \frac{1}{2} \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha$$

-1/3 Trace[$\delta W_{\mu\nu}^{(1)}$]

$$\text{Tr} \left[-1/3 \delta W_{\mu\nu}^{(1)} \right] = -\frac{3}{2} H^2 g_{\mu\nu} \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha + \frac{1}{2} \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha$$

Trace[$\delta W_{\mu\nu}^{(2)}$]

$$\text{Tr} \left[\delta W_{\mu\nu}^{(2)} \right] = \frac{3}{2} H^2 g_{\mu\nu} \nabla_\beta \nabla^\beta h^\alpha{}_\alpha + \frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - \frac{1}{2} \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha$$