Special Gauge Matthew v11

Setup

Metric decomposed to first order:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} = \Omega^2(\tau)(\eta_{\mu\nu} + f_{\mu\nu}). \tag{1}$$

We then split $f_{\mu\nu}$ into its traceless and trace components, i.e.

$$f_{\mu\nu} = k_{\mu\nu} + \frac{1}{4}\eta_{\mu\nu} f \tag{2}$$

where $f = \eta^{\mu\nu} f_{\mu\nu}$. We impose a generalized gauge of the form

$$\eta^{\alpha\beta}\partial_\alpha k_{\beta\nu} = \Omega^{-1} J \eta^{\alpha\beta} k_{\nu\alpha}\partial_\beta\Omega + P\partial_\nu f + R\Omega^{-1} f\partial_\nu\Omega. \tag{3}$$

and take

$$J = -4, \qquad R = 2P - \frac{3}{2}. \tag{4}$$

$\Omega(\tau)$: $k_{\mu\nu}$, f

$$\begin{aligned}\eta^{\mu\nu}\delta G_{\mu\nu} = {} & (-8\Omega^{-2}\dot\Omega^2 + 4\Omega^{-1}\ddot\Omega)k_{00} + (\tfrac{3}{2}\Omega^{-2}\dot\Omega^2 - 2P\Omega^{-2}\dot\Omega^2 + \tfrac{3}{2}\Omega^{-1}\ddot\Omega - 2P\Omega^{-1}\ddot\Omega - \tfrac{3}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & + P\eta^{\mu\nu}\partial_\mu\partial_\nu + 3\Omega^{-1}\dot\Omega\partial_0 - 4P\Omega^{-1}\dot\Omega\partial_0)f. \\ = {} & (-8\Omega^{-2}\dot\Omega^2 + 4\Omega^{-1}\ddot\Omega)k_{00} + (P - \tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta(\Omega^2 f)\end{aligned} \tag{5}$$

$$\begin{aligned}\delta G_{00} = {} & (2\Omega^{-2}\dot\Omega^2 - 2\Omega^{-1}\ddot\Omega + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - 3\Omega^{-1}\dot\Omega\partial_0)k_{00} + (-\tfrac{3}{4}\Omega^{-2}\dot\Omega^2 + P\Omega^{-2}\dot\Omega^2 + \tfrac{3}{4}\Omega^{-1}\ddot\Omega \\ & - P\Omega^{-1}\ddot\Omega + \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu - P\Omega^{-1}\dot\Omega\partial_0 + \tfrac{1}{4}\partial_0\partial_0 - P\partial_0\partial_0)f.\end{aligned} \tag{6}$$

$$\begin{aligned}\delta G_{0i} = {} & -\Omega^{-1}\dot\Omega\partial_i k_{00} + (\Omega^{-2}\dot\Omega^2 + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - 2\Omega^{-1}\dot\Omega\partial_0)k_{0i} + (\tfrac{1}{2}\Omega^{-1}\dot\Omega\partial_i - P\Omega^{-1}\dot\Omega\partial_i \\ & + \tfrac{1}{4}\partial_i\partial_0 - P\partial_i\partial_0)f.\end{aligned} \tag{7}$$

$$\begin{aligned}\delta G_{ij} = {} & \delta_{ij}\Omega^{-2}\dot\Omega^2 k_{00} - \Omega^{-1}\dot\Omega\partial_j k_{0i} - \Omega^{-1}\dot\Omega\partial_i k_{0j} + (-\Omega^{-2}\dot\Omega^2 + 2\Omega^{-1}\ddot\Omega + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-1}\dot\Omega\partial_0)k_{ij} \\ & + \delta_{ij}(-\tfrac{3}{4}\Omega^{-2}\dot\Omega^2 + P\Omega^{-2}\dot\Omega^2 + \tfrac{3}{4}\Omega^{-1}\ddot\Omega - P\Omega^{-1}\ddot\Omega - \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu + \Omega^{-1}\dot\Omega\partial_0 \\ & - P\Omega^{-1}\dot\Omega\partial_0)f + (\tfrac{1}{4}\partial_i\partial_j - P\partial_i\partial_j)f.\end{aligned} \tag{8}$$

$\Omega(\tau) = \frac{1}{H\tau}$: $k_{\mu\nu}$, f

Now set $\Omega(\tau) = \frac{1}{H\tau}$, with the fluctuations being evaluated as

$$\eta^{\mu\nu}\delta G_{\mu\nu} = (P - \tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta(\Omega^2 f) \tag{9}$$

$$\begin{aligned}\delta G_{00} = &(-2\tau^{-2} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 3\tau^{-1}\partial_0)k_{00} + (\tfrac{3}{4}\tau^{-2} - P\tau^{-2} + \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu \\ &+ P\tau^{-1}\partial_0 + \tfrac{1}{4}\partial_0\partial_0 - P\partial_0\partial_0)f.\end{aligned} \tag{10}$$

$$\begin{aligned}\delta G_{0i} = &\tau^{-1}\partial_i k_{00} + (\tau^{-2} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 2\tau^{-1}\partial_0)k_{0i} + (-\tfrac{1}{2}\tau^{-1}\partial_i + P\tau^{-1}\partial_i + \tfrac{1}{4}\partial_i\partial_0 \\ &- P\partial_i\partial_0)f.\end{aligned} \tag{11}$$

$$\begin{aligned}\delta G_{ij} = &\delta_{ij}\tau^{-2}k_{00} + \tau^{-1}\partial_j k_{0i} + \tau^{-1}\partial_i k_{0j} + (3\tau^{-2} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)k_{ij} + \delta_{ij}(\tfrac{3}{4}\tau^{-2} - P\tau^{-2} \\ &- \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu - \tau^{-1}\partial_0 + P\tau^{-1}\partial_0)f + (\tfrac{1}{4}\partial_i\partial_j - P\partial_i\partial_j)f.\end{aligned} \tag{12}$$

Now we will further express our results in terms of

$$K_{\mu\nu} = \Omega^2 k_{\mu\nu} \qquad \text{and} \qquad h = \Omega^2 f \tag{13}$$

$\Omega(\tau)$: $K_{\mu\nu}$, h

Working with a time dependent conformal factor, $\Omega(\tau)$, the fluctuations are evaluated as

$$\begin{aligned}\eta^{\mu\nu}\delta G_{\mu\nu} &= (-8\Omega^{-4}\dot{\Omega}^2 + 4\Omega^{-3}\ddot{\Omega})K_{00} + (-\tfrac{3}{4}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + P\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu)h \\ &= \Omega^{-2}(-8\Omega^{-2}\dot{\Omega}^2 + 4\Omega^{-1}\ddot{\Omega})K_{00} + (P - \tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta h\end{aligned} \tag{14}$$

$$\begin{aligned}\delta G_{00} = &(5\Omega^{-4}\dot{\Omega}^2 - \Omega^{-3}\ddot{\Omega} + \tfrac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-3}\dot{\Omega}\partial_0)K_{00} + (-\tfrac{3}{4}\Omega^{-4}\dot{\Omega}^2 + \tfrac{3}{4}\Omega^{-3}\ddot{\Omega} \\ &+ \tfrac{1}{4}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}P\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + P\Omega^{-3}\dot{\Omega}\partial_0 + \tfrac{1}{4}\Omega^{-2}\partial_0\partial_0 - P\Omega^{-2}\partial_0\partial_0)h.\end{aligned} \tag{15}$$

$$\begin{aligned}\delta G_{0i} = &-\Omega^{-3}\dot{\Omega}\partial_i K_{00} + (2\Omega^{-4}\dot{\Omega}^2 + \Omega^{-3}\ddot{\Omega} + \tfrac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu)K_{0i} + (P\Omega^{-3}\dot{\Omega}\partial_i + \tfrac{1}{4}\Omega^{-2}\partial_i\partial_0 \\ &- P\Omega^{-2}\partial_i\partial_0)h.\end{aligned} \tag{16}$$

$$\begin{aligned}\delta G_{ij} = &\delta_{ij}\Omega^{-4}\dot{\Omega}^2 K_{00} - \Omega^{-3}\dot{\Omega}\partial_j K_{0i} - \Omega^{-3}\dot{\Omega}\partial_i K_{0j} + (-2\Omega^{-4}\dot{\Omega}^2 + 3\Omega^{-3}\ddot{\Omega} + \tfrac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \Omega^{-3}\dot{\Omega}\partial_0)K_{ij} \\ &+ \delta_{ij}(-\tfrac{5}{4}\Omega^{-4}\dot{\Omega}^2 + \tfrac{1}{4}\Omega^{-3}\ddot{\Omega} - \tfrac{1}{4}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}P\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + P\Omega^{-3}\dot{\Omega}\partial_0)h + (\tfrac{1}{4}\Omega^{-2}\partial_i\partial_j \\ &- P\Omega^{-2}\partial_i\partial_j)h.\end{aligned} \tag{17}$$

$\Omega(\tau) = \frac{1}{H\tau}$: $K_{\mu\nu}$, h

Now set $\Omega(\tau) = \frac{1}{H\tau}$, with the fluctuations being evaluated as

$$\begin{aligned}\eta^{\mu\nu}\delta G_{\mu\nu} =&(-\tfrac{3}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu)h \\ =&(P - \tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta h\end{aligned} \tag{18}$$

$$\delta G_{00} = (3H^2 + \tfrac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + H^2\tau\partial_0)K_{00} + (\tfrac{3}{4}H^2 + \tfrac{1}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu$$
$$- H^2P\tau\partial_0 + \tfrac{1}{4}H^2\tau^2\partial_0\partial_0 - H^2P\tau^2\partial_0\partial_0)h. \tag{19}$$

$$\delta G_{0i} = H^2\tau\partial_i K_{00} + (4H^2 + \tfrac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu)K_{0i} + (-H^2P\tau\partial_i + \tfrac{1}{4}H^2\tau^2\partial_i\partial_0$$
$$- H^2P\tau^2\partial_i\partial_0)h. \tag{20}$$

$$\delta G_{ij} = \delta_{ij}H^2K_{00} + H^2\tau\partial_j K_{0i} + H^2\tau\partial_i K_{0j} + (4H^2 + \tfrac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - H^2\tau\partial_0)K_{ij}$$
$$+ \delta_{ij}(-\tfrac{3}{4}H^2 - \tfrac{1}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - H^2P\tau\partial_0)h + (\tfrac{1}{4}H^2\tau^2\partial_i\partial_j - H^2P\tau^2\partial_i\partial_j)h. \tag{21}$$