$\delta W_{\mu\nu}$ SVT4 Conformal Flat

$$ds^{2} = \Omega^{2}(x)(\tilde{g}_{\mu\nu} + f_{\mu\nu})dx^{\mu}dx^{\nu} = \Omega^{2}(x)(\tilde{g}_{\mu\nu} + \Omega^{-2}h_{\mu\nu})dx^{\mu}dx^{\nu}$$
(0.1)

$$\tilde{g}_{\mu\nu} = \operatorname{diag}(-1, 1, 1, 1)$$
 (up to a coordinate transformation) (0.2)

$$W_{\mu\nu} = 0 \tag{0.3}$$

$$f_{\mu\nu} = -2\tilde{g}_{\mu\nu}\chi + 2\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}F + \tilde{\nabla}_{\mu}F_{\nu} + \tilde{\nabla}_{\nu}F_{\mu} + 2F_{\mu\nu} \tag{0.4}$$

$$\begin{split} \delta W_{\mu\nu}(h_{\mu\nu}) &= -\frac{1}{6}h_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}h^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}h_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}h^{\alpha\beta}R_{\alpha}^{\gamma}R_{\beta\gamma} - \frac{2}{3}h^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\ &+ 2h^{\alpha\beta}R_{\alpha}^{\gamma}R_{\mu\beta\nu\gamma} + \frac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} - \frac{1}{3}g_{\mu\nu}R\nabla_{\alpha}\nabla^{\alpha}h + \frac{2}{3}R_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}h - \frac{1}{6}h_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R \\ &+ \frac{1}{2}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}\nabla_{\mu}h + R_{\mu\beta\nu\gamma}\nabla_{\alpha}\nabla^{\gamma}h^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_{\alpha}\nabla^{\gamma}h^{\alpha\beta} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\mu}h_{\nu}^{\alpha} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\nu}h_{\mu}^{\alpha} \\ &- \frac{1}{12}g_{\mu\nu}\nabla_{\alpha}R\nabla^{\alpha}h + \frac{1}{2}\nabla_{\alpha}R_{\mu\nu}\nabla^{\alpha}h - \frac{1}{6}\nabla_{\alpha}h_{\mu\nu}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}\nabla^{\alpha}R\nabla_{\beta}h_{\alpha}^{\beta} - \nabla_{\alpha}h^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} \\ &+ \frac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} - \frac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} + \frac{1}{2}R_{\nu}^{\alpha}\nabla_{\beta}\nabla_{\alpha}h_{\mu}^{\beta} - R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h_{\mu\nu} \\ &+ \frac{1}{2}R_{\mu}^{\alpha}\nabla_{\beta}\nabla_{\alpha}h_{\nu}^{\beta} + \frac{1}{2}g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} \\ &- \frac{1}{2}R_{\nu}^{\alpha}\nabla_{\beta}\nabla_{\beta}h_{\mu\alpha} - \frac{1}{2}R_{\mu}^{\alpha}\nabla_{\beta}\nabla_{\beta}\nabla_{\alpha}h + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} \\ &- \frac{1}{2}R_{\nu}^{\alpha}\nabla_{\beta}\nabla_{\beta}h_{\mu\alpha} - \frac{1}{2}R_{\mu}^{\alpha}\nabla_{\beta}\nabla_{\beta}\nabla_{\alpha}h^{\alpha} - \frac{1}{2}R_{\nu}^{\alpha}\nabla_{\beta}\nabla_{\mu}h_{\alpha}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}\nabla^{\alpha}h \\ &- \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\mu}h_{\nu}^{\alpha} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\nu}h_{\mu}^{\alpha} - \frac{1}{2}R_{\nu}^{\alpha}\nabla_{\beta}\nabla_{\mu}h_{\alpha}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\mu}h_{\nu\alpha} \\ &- \frac{1}{2}R_{\mu}^{\alpha}\nabla_{\beta}\nabla_{\nu}h_{\alpha}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\nu}h_{\mu\alpha} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}h_{\mu}^{\alpha} - \nabla_{\beta}R_{\nu\alpha}\nabla^{\beta}h_{\nu}^{\alpha} - \nabla_{\beta}R_{\mu\alpha}\nabla^{\beta}h_{\nu}^{\alpha} \\ &- \frac{1}{2}R_{\mu}^{\alpha}\nabla_{\beta}\nabla_{\nu}h_{\alpha}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\nu}h_{\mu\alpha} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}h_{\mu}^{\alpha} - \nabla_{\beta}R_{\nu\alpha}\nabla^{\beta}h_{\mu}^{\alpha} + \nabla_{\alpha}R_{\mu\beta}\nabla^{\beta}h_{\nu}^{\alpha} \\ &- \nabla_{\beta}R_{\mu\alpha}\nabla^{\beta}h_{\nu}^{\alpha} - R_{\mu\alpha\nu}\beta\nabla^{\beta}\nabla_{\nu}h_{\mu\alpha} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}h_{\mu}^{\alpha} - \nabla_{\beta}R_{\nu\alpha}\nabla^{\gamma}\nabla_{\alpha}h_{\alpha\beta} \\ &- R_{\mu\alpha\nu}\beta\nabla_{\gamma}\nabla^{\gamma}h_{\alpha\beta} + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}R_{\alpha\beta} + \frac{1}{3}g_{\mu\nu}\nabla_{\gamma}R_{\alpha\beta}\nabla^{\gamma}h_{\alpha\beta} \\ &+ \frac{1}{6}\nabla^{\alpha}R\nabla_{\mu}h_{\nu\alpha} - \frac{1}{2}\nabla^{\alpha}h\nabla_{\mu}R_{\mu\alpha} + \nabla_{\alpha}R_{\mu}\nabla^{\beta}\nabla_{\nu}R_{\mu\alpha} + \frac{1}{2}\nabla_{\mu}\nabla_{\alpha}\nabla^{\gamma}h_{\alpha\beta} \\ &+ \frac{1}{6}\nabla^{\alpha}R_{\mu}\nabla^{\alpha}\nabla_{\beta}\nabla^{\beta}h_{\mu}\nabla^{\alpha}\nabla_{\alpha}\nabla^{\beta}\nabla_{\alpha}h_{\mu}\nabla^{\alpha}\nabla_{\alpha}\nabla^{\beta}h_{\mu}\nabla^{\alpha}\nabla^{\beta}\nabla_{\alpha$$

$$\delta W_{\mu\nu}(f_{\mu\nu}) = \frac{1}{2}\Omega^{-2}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}f_{\mu\nu} - \frac{1}{6}\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}f + \frac{1}{6}\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_{\gamma}\tilde{\nabla}^{\gamma}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}f^{\alpha\beta}
- \frac{1}{2}\Omega^{-2}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}f_{\nu}{}^{\alpha} - \frac{1}{2}\Omega^{-2}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}f_{\mu}{}^{\alpha} + \frac{1}{6}\Omega^{-2}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}f
+ \frac{1}{2}\Omega^{-2}\tilde{\nabla}_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}f^{\alpha\beta}$$
(0.6)

$$= \Omega^{-2} \tilde{\nabla}_{\beta} \tilde{\nabla}^{\beta} \tilde{\nabla}_{\alpha} \tilde{\nabla}^{\alpha} F_{\mu\nu} \tag{0.7}$$