Phys. 6430 - THEORY OF RELATIVITY - HOMEWORK 2

Due in class on Thursday, September 29.

1. Show that the equations
$$\partial_\alpha F^{\alpha\beta} = J^\beta$$
$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\beta\alpha} = 0$$
are equivalent to Maxwell's equations written in terms of three component vectors $\vec{E}$ and $\vec{B}$.

2. Consider a transformation from Cartesian to polar coordinates on a plane, $x = r\cos\phi$; $y = r\sin\phi$. Write down the length element dl^2 in polar coordinates and thus determine the metric $g_{\mu\nu}$. Calculate the Christoffel symbol in polar coordinates. Write down the free equation of motion in polar coordinates (time coordinate is an additional one, and the transformation to the polar coordinates does not affect it).

3. Calculate the elements of the transformation matrix $\partial x'^\mu/\partial x^\nu$ for the transformation from Cartesian to polar coordinates.

4. For the vector field V^μ whose Cartesian components are $(x^2 + 3y, y^2 + 3x)$

   a). Compute $V^\mu_{;\nu}$ in Cartesian coordinates.

   b). Transform $V^\mu_{;\nu}$ to polar coordinates using the tranformation matrix calculated in the previous problem.

   c). Compute $V^\mu_{;\nu}$ directly in polar coordinates using the Christoffel symbols.

5. Consider the $x - t$ plane of an inertial observer. A certain uniformly accelerated observer wishes to set up an orthonormal coordinate system. The worldline of this observer is
$$t(\lambda) = a\sinh\lambda; \quad x(\lambda) = a\cosh\lambda$$

   a). Show that $a\lambda$ is the proper time of the accelerated observer (clock time on his wrist watch).

b). Show that the spacelike line described by this equation with a as the variable parameter and λ fixed is orthogonal to his world line where they intersect. Changing λ then generates a family of such lines.

c). Show that this equation defines a transformation from coordinates (t, x) to coordinates (λ, a) which form an orthogonal coordinate system. Draw these coordinates and show that they cover only one half of the original $x - t$ plane. Show that the coordinates are bad on the lines $|x| = |t|$, so they really cover two disjoint quadrants. The right hand quadrant in these coordinates is called Rindler space.

d). Find the metric tensor and all the Christoffel symbols in this coordinate system.

6. Calculate the metric on a sphere by considering a radius r sphere $x^2 + y^2 + z^2 = r^2$ embedded in three dimensional flat space $ds^2 = dx^2 + dy^2 + dz^2$. To do this change to polar coordinates

$$x = r\cos\phi\sin\theta;\ \ y = r\cos\phi\cos\theta;\ \ z = r\cos\theta$$

and then set $dr = 0$ to restrict to surface of the sphere.