

Weyl Tensor Simplifications

General, no Bianchi, no explicit covariant derivative commutation

$$W_{\mu\nu}^{(1)} = \frac{1}{2}g_{\mu\nu}R^2 - 2RR_{\mu\nu} + 2g_{\mu\nu}\nabla_\alpha\nabla^\alpha R - 2\nabla_\nu\nabla_\mu R. \quad (1)$$

Here we apply the Bianchi only to $W_{\mu\nu}^{(2)}$

$$W_{\mu\nu}^{(2)} = \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - 2R^{\alpha\beta}R_{\alpha\mu\beta\nu} + \frac{1}{2}g_{\mu\nu}\nabla_\alpha\nabla^\alpha R + \nabla_\alpha\nabla^\alpha R_{\mu\nu} - \nabla_\nu\nabla_\mu R. \quad (2)$$

$$\begin{aligned} W_{\mu\nu} = & -\frac{1}{6}g_{\mu\nu}R^2 + \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \frac{2}{3}RR_{\mu\nu} - 2R^{\alpha\beta}R_{\alpha\mu\beta\nu} - \frac{1}{6}g_{\mu\nu}\nabla_\alpha\nabla^\alpha R + \nabla_\alpha\nabla^\alpha R_{\mu\nu} \\ & - \frac{1}{3}\nabla_\nu\nabla_\mu R. \end{aligned} \quad (3)$$

where we use the perturbed quantities

$$\begin{aligned} \delta R_{\lambda\mu\nu\kappa} = & h^\alpha{}_\lambda R_{\alpha\mu\nu\kappa} - \frac{1}{2}\nabla_\kappa\nabla_\lambda h_{\mu\nu} + \frac{1}{2}\nabla_\kappa\nabla_\mu h_{\nu\lambda} + \frac{1}{2}\nabla_\kappa\nabla_\nu h_{\mu\lambda} - \frac{1}{2}\nabla_\nu\nabla_\kappa h_{\mu\lambda} + \frac{1}{2}\nabla_\nu\nabla_\lambda h_{\kappa\mu} \\ & - \frac{1}{2}\nabla_\nu\nabla_\mu h_{\kappa\lambda} \end{aligned} \quad (4)$$

$$\delta R_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\nabla_\alpha\nabla_\beta h_{\mu\nu} - \nabla_\alpha\nabla_\mu h_{\beta\nu} - \nabla_\alpha\nabla_\nu h_{\beta\mu} + \nabla_\nu\nabla_\mu h_{\alpha\beta}). \quad (5)$$

60 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(h_{\mu\nu}) = & -\frac{1}{6}h_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}h^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}h_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}h^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\beta\gamma} - \frac{2}{3}h^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\
& + 2h^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\mu\gamma\nu\beta} - \frac{1}{6}h_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} \\
& + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\alpha\beta} + \frac{1}{3}h^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}R_{\alpha\beta} \\
& + \frac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} + R_{\mu\beta\nu\gamma}\nabla_{\alpha}\nabla^{\gamma}h^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_{\alpha}\nabla^{\gamma}h^{\alpha\beta} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\mu}h_{\nu}{}^{\alpha} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\nu}h_{\mu}{}^{\alpha} \\
& - \frac{1}{6}\nabla_{\alpha}h_{\mu\nu}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}\nabla^{\alpha}R\nabla_{\beta}h_{\alpha}{}^{\beta} - \nabla_{\alpha}h^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} + \frac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} \\
& - \frac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} + \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}h_{\mu}{}^{\beta} - R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h_{\mu\nu} + \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}h_{\nu}{}^{\beta} \\
& - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}h_{\mu\alpha} - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}h_{\nu\alpha} + \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\mu}h_{\nu}{}^{\alpha} \\
& - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\nu}h_{\mu}{}^{\alpha} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\mu}h_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\mu}h_{\nu\alpha} - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\nu}h_{\alpha}{}^{\beta} \\
& + R^{\alpha\beta}\nabla_{\beta}\nabla_{\nu}h_{\mu\alpha} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}h_{\mu}{}^{\alpha} - \nabla_{\beta}R_{\nu\alpha}\nabla^{\beta}h_{\mu}{}^{\alpha} + \nabla_{\alpha}R_{\mu\beta}\nabla^{\beta}h_{\nu}{}^{\alpha} - \nabla_{\beta}R_{\mu\alpha}\nabla^{\beta}h_{\nu}{}^{\alpha} \\
& - g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla_{\beta}h_{\alpha}{}^{\gamma} + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}h_{\alpha\beta} - R_{\mu\alpha\nu\beta}\nabla_{\gamma}\nabla^{\gamma}h^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} \\
& + \frac{1}{3}g_{\mu\nu}\nabla_{\gamma}R_{\alpha\beta}\nabla^{\gamma}h^{\alpha\beta} - \nabla_{\beta}R_{\nu\alpha}\nabla_{\mu}h^{\alpha\beta} + \frac{1}{6}\nabla^{\alpha}R\nabla_{\mu}h_{\nu\alpha} - \nabla_{\beta}R_{\mu\alpha}\nabla_{\nu}h^{\alpha\beta} \\
& + \frac{1}{3}\nabla_{\mu}R_{\alpha\beta}\nabla_{\nu}h^{\alpha\beta} + \frac{1}{6}\nabla^{\alpha}R\nabla_{\nu}h_{\mu\alpha} + \frac{1}{3}\nabla_{\mu}h^{\alpha\beta}\nabla_{\nu}R_{\alpha\beta} - \frac{2}{3}R^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}h_{\alpha\beta} \\
& + \frac{1}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} \\
& - \frac{1}{3}g_{\mu\nu}R\nabla_{\alpha}\nabla^{\alpha}h + \frac{2}{3}R_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}h + \frac{1}{2}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}\nabla_{\mu}h - \frac{1}{12}g_{\mu\nu}\nabla_{\alpha}h\nabla^{\alpha}R \\
& + \frac{1}{2}\nabla_{\alpha}R_{\mu\nu}\nabla^{\alpha}h + \frac{1}{2}g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h - \frac{1}{6}g_{\mu\nu}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}h - R_{\mu\alpha\nu\beta}\nabla^{\beta}\nabla^{\alpha}h \\
& + \frac{1}{3}R\nabla_{\nu}\nabla_{\mu}h - \frac{1}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}h.
\end{aligned} \tag{6}$$

General, no Bianchi, no explicit covariant derivative commutation

$$W_{\mu\nu}^{(1)} = \frac{1}{2}g_{\mu\nu}R^2 - 2RR_{\mu\nu} + 2g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R - 2\nabla_{\nu}\nabla_{\mu}R. \tag{7}$$

No Bianchi applied to $W_{\mu\nu}^2$

$$W_{\mu\nu}^{(2)} = \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - 2R^{\alpha\beta}R_{\alpha\mu\beta\nu} + \frac{1}{2}g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \nabla_{\alpha}\nabla^{\alpha}R_{\mu\nu} - \nabla_{\mu}\nabla^{\alpha}R_{\nu\alpha} - \nabla_{\nu}\nabla^{\alpha}R_{\mu\alpha}. \tag{8}$$

$$\begin{aligned}
W_{\mu\nu} = & -\frac{1}{6}g_{\mu\nu}R^2 + \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \frac{2}{3}RR_{\mu\nu} - 2R^{\alpha\beta}R_{\alpha\mu\beta\nu} - \frac{1}{6}g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \nabla_{\alpha}\nabla^{\alpha}R_{\mu\nu} \\
& - \nabla_{\mu}\nabla^{\alpha}R_{\nu\alpha} - \nabla_{\nu}\nabla^{\alpha}R_{\mu\alpha} + \frac{2}{3}\nabla_{\nu}\nabla_{\mu}R.
\end{aligned} \tag{9}$$

General, no Bianchi, no explicit covariant derivative commutation

80 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(h_{\mu\nu}) = & -\frac{1}{6}h_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}h^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}h_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}h^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\beta\gamma} - \frac{2}{3}h^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\
& + 2h^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\mu\gamma\nu\beta} - \frac{1}{6}h_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} \\
& + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\alpha\beta} + h^{\alpha\beta}\nabla_{\mu}\nabla_{\beta}R_{\nu\alpha} + h^{\alpha\beta}\nabla_{\nu}\nabla_{\beta}R_{\mu\alpha} - \frac{2}{3}h^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}R_{\alpha\beta} \\
& + \frac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} + R_{\mu\beta\nu\gamma}\nabla_{\alpha}\nabla^{\gamma}h^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_{\alpha}\nabla^{\gamma}h^{\alpha\beta} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\mu}h_{\nu}{}^{\alpha} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\nu}h_{\mu}{}^{\alpha} \\
& + \frac{1}{3}\nabla_{\alpha}h_{\mu\nu}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}\nabla^{\alpha}R\nabla_{\beta}h_{\alpha}{}^{\beta} - \nabla^{\alpha}h_{\mu\nu}\nabla_{\beta}R_{\alpha}{}^{\beta} - \nabla_{\alpha}h^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} \\
& + \frac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} - \frac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} + \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}h_{\mu}{}^{\beta} - R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h_{\mu\nu} \\
& + \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}h_{\nu}{}^{\beta} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}h_{\mu\alpha} - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}h_{\nu\alpha} + \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} \\
& - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\mu}h_{\nu}{}^{\alpha} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\nu}h_{\mu}{}^{\alpha} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\mu}h_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\mu}h_{\nu\alpha} \\
& - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\nu}h_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\nu}h_{\mu\alpha} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}h_{\mu}{}^{\alpha} - \nabla_{\beta}R_{\nu\alpha}\nabla^{\beta}h_{\mu}{}^{\alpha} \\
& + \nabla_{\alpha}R_{\mu\beta}\nabla^{\beta}h_{\nu}{}^{\alpha} - \nabla_{\beta}R_{\mu\alpha}\nabla^{\beta}h_{\nu}{}^{\alpha} - g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla_{\beta}h_{\alpha}{}^{\gamma} + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}h_{\alpha\beta} \\
& - R_{\mu\alpha\nu\beta}\nabla_{\gamma}\nabla^{\gamma}h^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} + \frac{1}{3}g_{\mu\nu}\nabla_{\gamma}R_{\alpha\beta}\nabla^{\gamma}h^{\alpha\beta} - \frac{1}{3}\nabla^{\alpha}R\nabla_{\mu}h_{\nu\alpha} \\
& + \nabla_{\beta}R_{\alpha}{}^{\beta}\nabla_{\mu}h_{\nu}{}^{\alpha} + \nabla_{\alpha}h^{\alpha\beta}\nabla_{\mu}R_{\nu\beta} - \frac{1}{2}\nabla_{\mu}\nabla_{\alpha}\nabla_{\beta}\nabla^{\beta}h_{\nu}{}^{\alpha} + R_{\nu}{}^{\alpha}\nabla_{\mu}\nabla_{\beta}h_{\alpha}{}^{\beta} \\
& + \frac{1}{2}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}\nabla^{\beta}h_{\nu}{}^{\alpha} + \frac{1}{2}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}\nabla_{\nu}h^{\alpha\beta} + \frac{1}{2}R^{\alpha\beta}\nabla_{\mu}\nabla_{\nu}h_{\alpha\beta} - \frac{1}{6}\nabla_{\mu}R_{\alpha\beta}\nabla_{\nu}h^{\alpha\beta} \\
& - \frac{1}{3}\nabla^{\alpha}R\nabla_{\nu}h_{\mu\alpha} + \nabla_{\beta}R_{\alpha}{}^{\beta}\nabla_{\nu}h_{\mu}{}^{\alpha} - \frac{1}{6}\nabla_{\mu}h^{\alpha\beta}\nabla_{\nu}R_{\alpha\beta} + \nabla_{\alpha}h^{\alpha\beta}\nabla_{\nu}R_{\mu\beta} \\
& - \frac{1}{2}\nabla_{\nu}\nabla_{\alpha}\nabla_{\beta}\nabla^{\beta}h_{\mu}{}^{\alpha} + R_{\mu}{}^{\alpha}\nabla_{\nu}\nabla_{\beta}h_{\alpha}{}^{\beta} + \frac{1}{2}\nabla_{\nu}\nabla_{\beta}\nabla_{\alpha}\nabla^{\beta}h_{\mu}{}^{\alpha} + \frac{1}{2}\nabla_{\nu}\nabla_{\beta}\nabla_{\alpha}\nabla_{\mu}h^{\alpha\beta} \\
& - \frac{7}{6}R^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}h_{\alpha\beta} - \frac{2}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} \\
& - \frac{1}{3}g_{\mu\nu}R\nabla_{\alpha}\nabla^{\alpha}h + \frac{2}{3}R_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}h + \frac{1}{2}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}\nabla_{\mu}h - \frac{1}{12}g_{\mu\nu}\nabla_{\alpha}h\nabla^{\alpha}R \\
& + \frac{1}{2}\nabla_{\alpha}R_{\mu\nu}\nabla^{\alpha}h + \frac{1}{2}g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h - \frac{1}{6}g_{\mu\nu}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}h - R_{\mu\alpha\nu\beta}\nabla^{\beta}\nabla^{\alpha}h \\
& - \frac{1}{2}\nabla^{\alpha}h\nabla_{\mu}R_{\nu\alpha} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\mu}\nabla_{\alpha}h - \frac{1}{2}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}h - \frac{1}{2}\nabla^{\alpha}h\nabla_{\nu}R_{\mu\alpha} - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\nu}\nabla_{\alpha}h \\
& - \frac{1}{2}\nabla_{\nu}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\mu}h + \frac{1}{3}R\nabla_{\nu}\nabla_{\mu}h + \frac{2}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}h.
\end{aligned} \tag{10}$$

Now make substitution

$$h_{\mu\nu} = K_{\mu\nu} + \frac{1}{4}hg_{\mu\nu}^{(0)} \tag{11}$$

in which it follows

$$\delta W_{\mu\nu}(h_{\mu\nu}) = \delta W_{\mu\nu}(K_{\mu\nu} + \frac{1}{4}hg_{\mu\nu}^{(0)}) = \delta W_{\mu\nu}(K_{\mu\nu}) + \delta W_{\mu\nu}(\frac{1}{4}hg_{\mu\nu}^{(0)}). \tag{12}$$

64 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(K_{\mu\nu}) = & -\frac{1}{6}K_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}K^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}K_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}K^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\beta\gamma} - \frac{2}{3}K^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\
& + 2K^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\mu\gamma\nu\beta} + \frac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} - \frac{1}{6}K_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + R_{\mu\beta\nu\gamma}\nabla_{\alpha}\nabla^{\gamma}K^{\alpha\beta} \\
& + R_{\mu\gamma\nu\beta}\nabla_{\alpha}\nabla^{\gamma}K^{\alpha\beta} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\mu}K_{\nu}{}^{\alpha} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\nu}K_{\mu}{}^{\alpha} + \frac{1}{3}\nabla_{\alpha}K_{\mu\nu}\nabla^{\alpha}R \\
& + \frac{1}{6}g_{\mu\nu}\nabla^{\alpha}R\nabla_{\beta}K_{\alpha}{}^{\beta} - \nabla^{\alpha}K_{\mu\nu}\nabla_{\beta}R_{\alpha}{}^{\beta} - \nabla_{\alpha}K^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} + \frac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} \\
& - \frac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} + \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}K_{\mu}{}^{\beta} - R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}K_{\mu\nu} + \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}K_{\nu}{}^{\beta} \\
& + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - K^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}K_{\mu\alpha} - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}K_{\nu\alpha} \\
& + \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\mu}K_{\nu}{}^{\alpha} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\nu}K_{\mu}{}^{\alpha} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\mu}K_{\alpha}{}^{\beta} \\
& + R^{\alpha\beta}\nabla_{\beta}\nabla_{\mu}K_{\nu\alpha} - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\nu}K_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\nu}K_{\mu\alpha} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}K_{\mu}{}^{\alpha} \\
& - \nabla_{\beta}R_{\nu\alpha}\nabla^{\beta}K_{\mu}{}^{\alpha} + \nabla_{\alpha}R_{\mu\beta}\nabla^{\beta}K_{\nu}{}^{\alpha} - \nabla_{\beta}R_{\mu\alpha}\nabla^{\beta}K_{\nu}{}^{\alpha} - g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla_{\beta}K_{\alpha}{}^{\gamma} \\
& + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}K_{\alpha\beta} - R_{\mu\alpha\nu\beta}\nabla_{\gamma}\nabla^{\gamma}K^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\alpha\beta} \\
& + \frac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} + \frac{1}{3}g_{\mu\nu}\nabla_{\gamma}R_{\alpha\beta}\nabla^{\gamma}K^{\alpha\beta} - \frac{1}{3}\nabla^{\alpha}R\nabla_{\mu}K_{\nu\alpha} + \nabla_{\beta}R_{\alpha}{}^{\beta}\nabla_{\mu}K_{\nu}{}^{\alpha} \\
& + \nabla_{\alpha}K^{\alpha\beta}\nabla_{\mu}R_{\nu\beta} - \frac{1}{2}\nabla_{\mu}\nabla_{\alpha}\nabla_{\beta}\nabla^{\beta}K_{\nu}{}^{\alpha} + R_{\nu}{}^{\alpha}\nabla_{\mu}\nabla_{\beta}K_{\alpha}{}^{\beta} + K^{\alpha\beta}\nabla_{\mu}\nabla_{\beta}R_{\nu\alpha} \\
& + \frac{1}{2}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}\nabla^{\beta}K_{\nu}{}^{\alpha} + \frac{1}{2}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}\nabla_{\nu}K^{\alpha\beta} + \frac{1}{2}R^{\alpha\beta}\nabla_{\mu}\nabla_{\nu}K_{\alpha\beta} - \frac{1}{6}\nabla_{\mu}R_{\alpha\beta}\nabla_{\nu}K^{\alpha\beta} \\
& - \frac{1}{3}\nabla^{\alpha}R\nabla_{\nu}K_{\mu\alpha} + \nabla_{\beta}R_{\alpha}{}^{\beta}\nabla_{\nu}K_{\mu}{}^{\alpha} - \frac{1}{6}\nabla_{\mu}K^{\alpha\beta}\nabla_{\nu}R_{\alpha\beta} + \nabla_{\alpha}K^{\alpha\beta}\nabla_{\nu}R_{\mu\beta} \\
& - \frac{1}{2}\nabla_{\nu}\nabla_{\alpha}\nabla_{\beta}\nabla^{\beta}K_{\mu}{}^{\alpha} + R_{\mu}{}^{\alpha}\nabla_{\nu}\nabla_{\beta}K_{\alpha}{}^{\beta} + K^{\alpha\beta}\nabla_{\nu}\nabla_{\beta}R_{\mu\alpha} + \frac{1}{2}\nabla_{\nu}\nabla_{\beta}\nabla_{\alpha}\nabla^{\beta}K_{\mu}{}^{\alpha} \\
& + \frac{1}{2}\nabla_{\nu}\nabla_{\beta}\nabla_{\alpha}\nabla_{\mu}K^{\alpha\beta} - \frac{7}{6}R^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}K_{\alpha\beta} - \frac{2}{3}K^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}R_{\alpha\beta} - \frac{2}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta}. \tag{13}
\end{aligned}$$

21 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(\frac{h}{4}g_{\mu\nu}^{(0)}) = & \frac{1}{24}g_{\mu\nu}R^2h - \frac{1}{8}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}h - \frac{1}{6}RR_{\mu\nu}h + \frac{1}{2}R^{\alpha\beta}R_{\mu\alpha\nu\beta}h + \frac{1}{24}g_{\mu\nu}h\nabla_{\alpha}\nabla^{\alpha}R \\
& - \frac{1}{4}h\nabla_{\alpha}\nabla^{\alpha}R_{\mu\nu} + \frac{1}{4}h\nabla_{\mu}\nabla_{\alpha}R_{\nu}{}^{\alpha} + \frac{1}{4}h\nabla_{\nu}\nabla_{\alpha}R_{\mu}{}^{\alpha} - \frac{1}{6}h\nabla_{\nu}\nabla_{\mu}R \\
& + \frac{1}{4}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}\nabla_{\mu}h + \frac{1}{8}g_{\mu\nu}\nabla_{\alpha}h\nabla^{\alpha}R - \frac{1}{4}\nabla_{\alpha}R_{\mu\nu}\nabla^{\alpha}h - \frac{1}{4}g_{\mu\nu}\nabla^{\alpha}h\nabla_{\beta}R_{\alpha}{}^{\beta} \\
& - \frac{1}{2}R_{\mu\alpha\nu\beta}\nabla^{\beta}\nabla^{\alpha}h + \frac{1}{4}\nabla_{\alpha}R_{\nu}{}^{\alpha}\nabla_{\mu}h - \frac{1}{4}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}h - \frac{1}{8}\nabla_{\mu}h\nabla_{\nu}R + \frac{1}{4}\nabla_{\alpha}R_{\mu}{}^{\alpha}\nabla_{\nu}h \\
& - \frac{1}{8}\nabla_{\mu}R\nabla_{\nu}h - \frac{1}{4}\nabla_{\nu}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\mu}h + \frac{1}{4}\nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}h. \tag{14}
\end{aligned}$$

General, with Bianchi, no explicit covariant derivative commutation

52 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(K_{\mu\nu}) = & -\frac{1}{6}K_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}K^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}K_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}K^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\beta\gamma} - \frac{2}{3}K^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\
& + 2K^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\mu\gamma\nu\beta} - \frac{1}{6}K_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - K^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} \\
& + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\alpha\beta} + \frac{1}{2}K^{\alpha\beta}\nabla_{\mu}\nabla_{\nu}R_{\alpha\beta} - \frac{1}{6}K^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}R_{\alpha\beta} \\
& + \frac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} + R_{\mu\beta\nu\gamma}\nabla_{\alpha}\nabla^{\gamma}K^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_{\alpha}\nabla^{\gamma}K^{\alpha\beta} - \frac{1}{3}R\nabla_{\alpha}\nabla_{\mu}K_{\nu}{}^{\alpha} \\
& - \frac{1}{3}R\nabla_{\alpha}\nabla_{\nu}K_{\mu}{}^{\alpha} - \frac{1}{6}\nabla_{\alpha}K_{\mu\nu}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}\nabla^{\alpha}R\nabla_{\beta}K_{\alpha}{}^{\beta} - \nabla_{\alpha}K^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} \\
& + \frac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} - \frac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} + \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}K_{\mu}{}^{\beta} - R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}K_{\mu\nu} \\
& + \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\alpha}K_{\nu}{}^{\beta} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}K_{\mu\alpha} - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}K_{\nu\alpha} + \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} \\
& - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\mu}K_{\nu}{}^{\alpha} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla_{\nu}K_{\mu}{}^{\alpha} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\beta}\nabla_{\mu}K_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\mu}K_{\nu\alpha} \\
& - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\beta}\nabla_{\nu}K_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\beta}\nabla_{\nu}K_{\mu\alpha} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}K_{\mu}{}^{\alpha} - \nabla_{\beta}R_{\nu\alpha}\nabla^{\beta}K_{\mu}{}^{\alpha} \\
& + \nabla_{\alpha}R_{\mu\beta}\nabla^{\beta}K_{\nu}{}^{\alpha} - \nabla_{\beta}R_{\mu\alpha}\nabla^{\beta}K_{\nu}{}^{\alpha} - g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla_{\beta}K_{\alpha}{}^{\gamma} + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}K_{\alpha\beta} \\
& - R_{\mu\alpha\nu\beta}\nabla_{\gamma}\nabla^{\gamma}K^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} + \frac{1}{3}g_{\mu\nu}\nabla_{\gamma}R_{\alpha\beta}\nabla^{\gamma}K^{\alpha\beta} - \nabla_{\beta}R_{\nu\alpha}\nabla_{\mu}K^{\alpha\beta} \\
& + \frac{1}{6}\nabla^{\alpha}R\nabla_{\mu}K_{\nu\alpha} + \frac{1}{2}R^{\alpha\beta}\nabla_{\mu}\nabla_{\nu}K_{\alpha\beta} - \nabla_{\beta}R_{\mu\alpha}\nabla_{\nu}K^{\alpha\beta} + \frac{1}{3}\nabla_{\mu}R_{\alpha\beta}\nabla_{\nu}K^{\alpha\beta} \\
& + \frac{1}{6}\nabla^{\alpha}R\nabla_{\nu}K_{\mu\alpha} + \frac{1}{3}\nabla_{\mu}K^{\alpha\beta}\nabla_{\nu}R_{\alpha\beta} - \frac{7}{6}R^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}K_{\alpha\beta} + \frac{1}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta}. \tag{15}
\end{aligned}$$

15 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(\frac{h}{4}g_{\mu\nu}^{(0)}) = & \frac{1}{24}g_{\mu\nu}R^2h - \frac{1}{8}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}h - \frac{1}{6}RR_{\mu\nu}h + \frac{1}{2}R^{\alpha\beta}R_{\mu\alpha\nu\beta}h + \frac{1}{24}g_{\mu\nu}h\nabla_{\alpha}\nabla^{\alpha}R \\
& - \frac{1}{4}h\nabla_{\alpha}\nabla^{\alpha}R_{\mu\nu} + \frac{1}{12}h\nabla_{\nu}\nabla_{\mu}R + \frac{1}{4}\nabla_{\alpha}\nabla^{\alpha}\nabla_{\nu}\nabla_{\mu}h - \frac{1}{4}\nabla_{\alpha}R_{\mu\nu}\nabla^{\alpha}h - \frac{1}{2}R_{\mu\alpha\nu\beta}\nabla^{\beta}\nabla^{\alpha}h \\
& + \frac{1}{4}\nabla^{\alpha}h\nabla_{\mu}R_{\nu\alpha} + \frac{1}{4}R_{\nu}{}^{\alpha}\nabla_{\mu}\nabla_{\alpha}h + \frac{1}{4}\nabla^{\alpha}h\nabla_{\nu}R_{\mu\alpha} + \frac{1}{4}R_{\mu}{}^{\alpha}\nabla_{\nu}\nabla_{\alpha}h - \frac{1}{4}\nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}h. \tag{16}
\end{aligned}$$

With Covariant Derivative Commutation and Bianchi

71 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(h_{\mu\nu}) = & -\frac{1}{6}h_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}h^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}h_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{3}h_{\nu}{}^{\alpha}RR_{\mu\alpha} - \frac{1}{2}h_{\nu}{}^{\alpha}R_{\alpha\beta}R_{\mu}{}^{\beta} \\
& -\frac{2}{3}h^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} + \frac{1}{3}h_{\mu}{}^{\alpha}RR_{\nu\alpha} + h^{\alpha\beta}R_{\mu\alpha}R_{\nu\beta} - \frac{1}{2}h_{\mu}{}^{\alpha}R_{\alpha\beta}R_{\nu}{}^{\beta} - g_{\mu\nu}h^{\alpha\beta}R^{\gamma\eta}R_{\alpha\gamma\beta\eta} \\
& -\frac{2}{3}h^{\alpha\beta}RR_{\mu\alpha\nu\beta} - h_{\nu}{}^{\alpha}R^{\beta\gamma}R_{\mu\beta\alpha\gamma} + 2h^{\alpha\beta}R_{\alpha}{}^{\gamma}R_{\mu\gamma\nu\beta} + 2h^{\alpha\beta}R_{\alpha\gamma\beta\eta}R_{\mu}{}^{\gamma}{}_{\nu}{}^{\eta} \\
& -h_{\mu}{}^{\alpha}R^{\beta\gamma}R_{\nu\beta\alpha\gamma} - \frac{1}{6}h_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - h^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} \\
& + \frac{1}{2}h_{\nu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}R_{\mu\alpha} + \frac{1}{2}h_{\mu}{}^{\alpha}\nabla_{\beta}\nabla^{\beta}R_{\nu\alpha} + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\alpha\beta} - h^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\mu\alpha\nu\beta} \\
& + \frac{1}{3}h^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}R_{\alpha\beta} \\
& + \frac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} + \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\alpha}\nabla_{\beta}h_{\mu}{}^{\beta} + \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\alpha}\nabla_{\beta}h_{\nu}{}^{\beta} - \frac{1}{6}\nabla_{\alpha}h_{\mu\nu}\nabla^{\alpha}R \\
& + \frac{1}{6}g_{\mu\nu}\nabla^{\alpha}R\nabla_{\beta}h_{\alpha}{}^{\beta} - \nabla_{\alpha}h^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} + \frac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} - \frac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} \\
& - R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h_{\mu\nu} + \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}h_{\mu\nu} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\mu}\nabla_{\alpha}h_{\nu}{}^{\alpha} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\nu}\nabla_{\alpha}h_{\mu}{}^{\alpha} \\
& - g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\gamma}h_{\alpha}{}^{\gamma} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}h_{\mu}{}^{\alpha} + \nabla_{\alpha}R_{\mu\beta}\nabla^{\beta}h_{\nu}{}^{\alpha} + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}h_{\alpha\beta} \\
& - 2R_{\mu\alpha\nu\beta}\nabla_{\gamma}\nabla^{\gamma}h^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} + \frac{1}{3}g_{\mu\nu}\nabla_{\gamma}R_{\alpha\beta}\nabla^{\gamma}h^{\alpha\beta} \\
& - 2\nabla_{\gamma}R_{\mu\alpha\nu\beta}\nabla^{\gamma}h^{\alpha\beta} + R_{\mu\beta\nu\gamma}\nabla^{\gamma}\nabla_{\alpha}h^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla^{\gamma}\nabla_{\alpha}h^{\alpha\beta} - \nabla_{\beta}R_{\nu\alpha}\nabla_{\mu}h^{\alpha\beta} \\
& + \frac{1}{6}\nabla^{\alpha}R\nabla_{\mu}h_{\nu\alpha} - \frac{1}{3}R\nabla_{\mu}\nabla_{\alpha}h_{\nu}{}^{\alpha} - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\mu}\nabla_{\beta}h_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\mu}\nabla_{\beta}h_{\nu\alpha} - \nabla_{\beta}R_{\mu\alpha}\nabla_{\nu}h^{\alpha\beta} \\
& + \frac{1}{3}\nabla_{\mu}R_{\alpha\beta}\nabla_{\nu}h^{\alpha\beta} + \frac{1}{6}\nabla^{\alpha}R\nabla_{\nu}h_{\mu\alpha} + \frac{1}{3}\nabla_{\mu}h^{\alpha\beta}\nabla_{\nu}R_{\alpha\beta} - \frac{1}{3}R\nabla_{\nu}\nabla_{\alpha}h_{\mu}{}^{\alpha} \\
& - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\nu}\nabla_{\beta}h_{\alpha}{}^{\beta} + R^{\alpha\beta}\nabla_{\nu}\nabla_{\beta}h_{\mu\alpha} - \frac{2}{3}R^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}h_{\alpha\beta} + \frac{1}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}h^{\alpha\beta} \\
& - \frac{1}{3}g_{\mu\nu}R\nabla_{\alpha}\nabla^{\alpha}h + \frac{2}{3}R_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}h - \frac{1}{12}g_{\mu\nu}\nabla_{\alpha}h\nabla^{\alpha}R + \frac{1}{2}\nabla_{\alpha}R_{\mu\nu}\nabla^{\alpha}h \\
& + \frac{1}{2}\nabla^{\alpha}h\nabla_{\beta}R_{\mu}{}^{\beta}{}_{\nu\alpha} + \frac{1}{2}g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}h - \frac{1}{6}g_{\mu\nu}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}h - \frac{1}{2}\nabla^{\alpha}h\nabla_{\mu}R_{\nu\alpha} \\
& - \frac{1}{2}R_{\nu}{}^{\alpha}\nabla_{\mu}\nabla_{\alpha}h - \frac{1}{2}R_{\mu}{}^{\alpha}\nabla_{\nu}\nabla_{\alpha}h + \frac{1}{3}R\nabla_{\nu}\nabla_{\mu}h + \frac{1}{6}\nabla_{\nu}\nabla_{\mu}\nabla_{\alpha}\nabla^{\alpha}h.
\end{aligned} \tag{17}$$

59 Terms

$$\begin{aligned}
\delta W_{\mu\nu}(K_{\mu\nu}) = & -\frac{1}{6}K_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}K^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}K_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{3}K_{\nu}^{\alpha}RR_{\mu\alpha} - \frac{1}{2}K_{\nu}^{\alpha}R_{\alpha\beta}R_{\mu}^{\beta} \\
& -\frac{2}{3}K^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} + \frac{1}{3}K_{\mu}^{\alpha}RR_{\nu\alpha} + K^{\alpha\beta}R_{\mu\alpha}R_{\nu\beta} - \frac{1}{2}K_{\mu}^{\alpha}R_{\alpha\beta}R_{\nu}^{\beta} - g_{\mu\nu}K^{\alpha\beta}R^{\gamma\eta}R_{\alpha\gamma\beta\eta} \\
& -\frac{2}{3}K^{\alpha\beta}RR_{\mu\alpha\nu\beta} - K_{\nu}^{\alpha}R^{\beta\gamma}R_{\mu\beta\alpha\gamma} + 2K^{\alpha\beta}R_{\alpha}^{\gamma}R_{\mu\gamma\nu\beta} + 2K^{\alpha\beta}R_{\alpha\gamma\beta\eta}R_{\mu}^{\gamma}{}_{\nu}{}^{\eta} \\
& -K_{\mu}^{\alpha}R^{\beta\gamma}R_{\nu\beta\alpha\gamma} - \frac{1}{6}K_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}R + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R - K^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}R_{\mu\nu} \\
& + \frac{1}{2}K_{\nu}^{\alpha}\nabla_{\beta}\nabla^{\beta}R_{\mu\alpha} + \frac{1}{2}K_{\mu}^{\alpha}\nabla_{\beta}\nabla^{\beta}R_{\nu\alpha} + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\alpha\beta} - K^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}R_{\mu\alpha\nu\beta} \\
& + \frac{1}{3}K^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}R_{\alpha\beta} \\
& + \frac{1}{3}R\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} + \frac{1}{2}R_{\nu}^{\alpha}\nabla_{\alpha}\nabla_{\beta}K_{\mu}^{\beta} + \frac{1}{2}R_{\mu}^{\alpha}\nabla_{\alpha}\nabla_{\beta}K_{\nu}^{\beta} - \frac{1}{6}\nabla_{\alpha}K_{\mu\nu}\nabla^{\alpha}R \\
& + \frac{1}{6}g_{\mu\nu}\nabla^{\alpha}R\nabla_{\beta}K_{\alpha}^{\beta} - \nabla_{\alpha}K^{\alpha\beta}\nabla_{\beta}R_{\mu\nu} + \frac{1}{3}g_{\mu\nu}R\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} - \frac{2}{3}R_{\mu\nu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} \\
& - R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha}K_{\mu\nu} + \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\alpha}\nabla^{\alpha}K_{\mu\nu} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\mu}\nabla_{\alpha}K_{\nu}^{\alpha} - \frac{1}{2}\nabla_{\beta}\nabla^{\beta}\nabla_{\nu}\nabla_{\alpha}K_{\mu}^{\alpha} \\
& - g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\gamma}K_{\alpha}^{\gamma} + \nabla_{\alpha}R_{\nu\beta}\nabla^{\beta}K_{\mu}^{\alpha} + \nabla_{\alpha}R_{\mu\beta}\nabla^{\beta}K_{\nu}^{\alpha} + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_{\gamma}\nabla^{\gamma}K_{\alpha\beta} \\
& - 2R_{\mu\alpha\nu\beta}\nabla_{\gamma}\nabla^{\gamma}K^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_{\gamma}\nabla^{\gamma}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta} + \frac{1}{3}g_{\mu\nu}\nabla_{\gamma}R_{\alpha\beta}\nabla^{\gamma}K^{\alpha\beta} \\
& - 2\nabla_{\gamma}R_{\mu\alpha\nu\beta}\nabla^{\gamma}K^{\alpha\beta} + R_{\mu\beta\nu\gamma}\nabla^{\gamma}\nabla_{\alpha}K^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla^{\gamma}\nabla_{\alpha}K^{\alpha\beta} - \nabla_{\beta}R_{\nu\alpha}\nabla_{\mu}K^{\alpha\beta} \\
& + \frac{1}{6}\nabla^{\alpha}R\nabla_{\mu}K_{\nu\alpha} - \frac{1}{3}R\nabla_{\mu}\nabla_{\alpha}K_{\nu}^{\alpha} - \frac{1}{2}R_{\nu}^{\alpha}\nabla_{\mu}\nabla_{\beta}K_{\alpha}^{\beta} + R^{\alpha\beta}\nabla_{\mu}\nabla_{\beta}K_{\nu\alpha} \\
& - \nabla_{\beta}R_{\mu\alpha}\nabla_{\nu}K^{\alpha\beta} + \frac{1}{3}\nabla_{\mu}R_{\alpha\beta}\nabla_{\nu}K^{\alpha\beta} + \frac{1}{6}\nabla^{\alpha}R\nabla_{\nu}K_{\mu\alpha} + \frac{1}{3}\nabla_{\mu}K^{\alpha\beta}\nabla_{\nu}R_{\alpha\beta} \\
& - \frac{1}{3}R\nabla_{\nu}\nabla_{\alpha}K_{\mu}^{\alpha} - \frac{1}{2}R_{\mu}^{\alpha}\nabla_{\nu}\nabla_{\beta}K_{\alpha}^{\beta} + R^{\alpha\beta}\nabla_{\nu}\nabla_{\beta}K_{\mu\alpha} - \frac{2}{3}R^{\alpha\beta}\nabla_{\nu}\nabla_{\mu}K_{\alpha\beta} \\
& + \frac{1}{3}\nabla_{\nu}\nabla_{\mu}\nabla_{\beta}\nabla_{\alpha}K^{\alpha\beta}.
\end{aligned} \tag{18}$$

$$\delta W_{\mu\nu}(\frac{\hbar}{4}g_{\mu\nu}^{(0)}) = -\frac{1}{4}\hbar W_{\mu\nu}(g_{\mu\nu}^{(0)}) \tag{19}$$

Weyl Tensor Flat

$$\begin{aligned}
\delta C_{\lambda\mu\nu\kappa} = & \frac{1}{4}\eta_{\mu\nu}\partial_{\alpha}\partial^{\alpha}K_{\kappa\lambda} - \frac{1}{4}\eta_{\lambda\nu}\partial_{\alpha}\partial^{\alpha}K_{\kappa\mu} - \frac{1}{4}\eta_{\kappa\mu}\partial_{\alpha}\partial^{\alpha}K_{\lambda\nu} + \frac{1}{4}\eta_{\kappa\lambda}\partial_{\alpha}\partial^{\alpha}K_{\mu\nu} \\
& - \frac{1}{6}\eta_{\kappa\mu}\eta_{\lambda\nu}\partial_{\beta}\partial_{\alpha}K^{\alpha\beta} + \frac{1}{6}\eta_{\kappa\lambda}\eta_{\mu\nu}\partial_{\beta}\partial_{\alpha}K^{\alpha\beta} - \frac{1}{4}\eta_{\mu\nu}\partial_{\kappa}\partial_{\alpha}K_{\lambda}^{\alpha} + \frac{1}{4}\eta_{\lambda\nu}\partial_{\kappa}\partial_{\alpha}K_{\mu}^{\alpha} \\
& + \frac{1}{2}\partial_{\kappa}\partial_{\mu}K_{\lambda\nu} - \frac{1}{4}\eta_{\mu\nu}\partial_{\lambda}\partial_{\alpha}K_{\kappa}^{\alpha} + \frac{1}{4}\eta_{\kappa\mu}\partial_{\lambda}\partial_{\alpha}K_{\nu}^{\alpha} - \frac{1}{2}\partial_{\lambda}\partial_{\kappa}K_{\mu\nu} + \frac{1}{2}\partial_{\lambda}\partial_{\nu}K_{\kappa\mu} \\
& + \frac{1}{4}\eta_{\lambda\nu}\partial_{\mu}\partial_{\alpha}K_{\kappa}^{\alpha} - \frac{1}{4}\eta_{\kappa\lambda}\partial_{\mu}\partial_{\alpha}K_{\nu}^{\alpha} - \frac{1}{2}\partial_{\mu}\partial_{\nu}K_{\kappa\lambda} + \frac{1}{4}\eta_{\kappa\mu}\partial_{\nu}\partial_{\alpha}K_{\lambda}^{\alpha} - \frac{1}{4}\eta_{\kappa\lambda}\partial_{\nu}\partial_{\alpha}K_{\mu}^{\alpha}.
\end{aligned} \tag{20}$$

Applying Gauge

Now we apply the gauge condition

$$\nabla_{\nu}K^{\mu\nu} = 4\Omega^{-1}K^{\mu\nu}\partial_{\nu}\Omega \tag{21}$$

or the equivalent gauge covariant in $K_{\mu\nu}$

$$\eta^{\alpha\beta}\partial_{\alpha}K_{\mu\beta} = 2\Omega^{-1}\eta^{\alpha\beta}K_{\mu\beta}\partial_{\alpha}\Omega. \tag{22}$$

and $\delta W_{\mu\nu}$ reduces to

$$\begin{aligned}
\delta W_{\mu\nu}(K_{\mu\nu}) = & -48\Omega^{-7}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\rho\Omega\partial_\sigma K_{\mu\nu} + 24\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\rho\partial_\beta\Omega\partial_\sigma K_{\mu\nu} \\
& + 60\Omega^{-8}\eta^{\alpha\beta}\eta^{\rho\sigma}K_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\rho\Omega\partial_\sigma\Omega - 4\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\rho\partial_\alpha\Omega\partial_\sigma\partial_\beta K_{\mu\nu} \\
& + 6\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}K_{\mu\nu}\partial_\rho\partial_\alpha\Omega\partial_\sigma\partial_\beta\Omega + 12\Omega^{-6}\eta^{\alpha\rho}\eta^{\beta\sigma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho K_{\mu\nu} \\
& + 6\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho K_{\mu\nu} - 2\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\beta\partial_\alpha\Omega\partial_\sigma\partial_\rho K_{\mu\nu} \\
& + 12\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\beta K_{\mu\nu}\partial_\sigma\partial_\rho\Omega - 48\Omega^{-7}\eta^{\alpha\rho}\eta^{\beta\sigma}K_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho\Omega \\
& - 24\Omega^{-7}\eta^{\alpha\beta}\eta^{\rho\sigma}K_{\mu\nu}\partial_\alpha\Omega\partial_\beta\Omega\partial_\sigma\partial_\rho\Omega + 3\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}K_{\mu\nu}\partial_\beta\partial_\alpha\Omega\partial_\sigma\partial_\rho\Omega \\
& - 4\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha\Omega\partial_\sigma\partial_\rho\partial_\beta K_{\mu\nu} - 4\Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\alpha K_{\mu\nu}\partial_\sigma\partial_\rho\partial_\beta\Omega \\
& + 12\Omega^{-6}\eta^{\alpha\beta}\eta^{\rho\sigma}K_{\mu\nu}\partial_\alpha\Omega\partial_\sigma\partial_\rho\partial_\beta\Omega + \frac{1}{2}\Omega^{-4}\eta^{\alpha\beta}\eta^{\rho\sigma}\partial_\sigma\partial_\rho\partial_\beta\partial_\alpha K_{\mu\nu} \\
& - \Omega^{-5}\eta^{\alpha\beta}\eta^{\rho\sigma}K_{\mu\nu}\partial_\sigma\partial_\rho\partial_\beta\partial_\alpha\Omega \\
& = \frac{1}{2}\Omega^{-2}\eta^{\sigma\rho}\eta^{\alpha\beta}\partial_\sigma\partial_\rho\partial_\alpha\partial_\beta(\Omega^{-2}K_{\mu\nu})
\end{aligned} \tag{23}$$

$$\begin{aligned}
W^{\mu\nu} = & -\frac{1}{6}g^{\mu\nu}\nabla_\beta\nabla^\beta R^\alpha{}_\alpha + \nabla_\beta\nabla^\beta R^{\mu\nu} - \frac{1}{3}\nabla_\mu\nabla_\nu R^\alpha{}_\alpha - R^{\beta\sigma}R_{\sigma\mu\beta\nu} - R^{\beta\sigma}R_{\sigma\nu\beta\mu} + \frac{1}{2}g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \frac{2}{3}R^\alpha{}_\alpha R^{\mu\nu} - \frac{1}{6}g^{\mu\nu}(R^\alpha{}_\alpha)^2 \\
& - \frac{1}{6}g_{\mu\nu}R^2 + \frac{1}{2}g_{\mu\nu}R_{\beta\alpha}R^{\beta\alpha} + \frac{2}{3}RR_{\mu\nu} - R^{\alpha\beta}R_{\beta\mu\alpha\nu} - R^{\alpha\beta}R_{\beta\nu\alpha\mu} - \frac{1}{6}g_{\mu\nu}\nabla_\alpha\nabla^\alpha R + \nabla_\alpha\nabla^\alpha R_{\mu\nu} - \frac{1}{3}\nabla_\nu\nabla_\mu R
\end{aligned} \tag{24}$$

$$\begin{aligned}
\delta W_{\mu\nu}(h_{\mu\nu}) = & -\frac{1}{6}h_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}h^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}h_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\beta\gamma} - \frac{2}{3}h^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\
& + h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\mu\beta\nu\gamma} + h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\mu\gamma\nu\beta} - \frac{1}{6}h_{\mu\nu}\nabla_\alpha\nabla^\alpha R + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_\beta\nabla_\alpha R \\
& - h^{\alpha\beta}\nabla_\beta\nabla_\alpha R_{\mu\nu} + \frac{1}{6}g_{\mu\nu}h^{\alpha\beta}\nabla_\gamma\nabla^\gamma R_{\alpha\beta} + \frac{1}{3}h^{\alpha\beta}\nabla_\nu\nabla_\mu R_{\alpha\beta}
\end{aligned} \tag{25}$$

$$\begin{aligned}
& + \frac{1}{3}R\nabla_\alpha\nabla^\alpha h_{\mu\nu} + R_{\mu\beta\nu\gamma}\nabla_\alpha\nabla^\gamma h^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_\alpha\nabla^\gamma h^{\alpha\beta} - \frac{1}{3}R\nabla_\alpha\nabla_\mu h_\nu{}^\alpha - \frac{1}{3}R\nabla_\alpha\nabla_\nu h_\mu{}^\alpha \\
& - \frac{1}{6}\nabla_\alpha h_{\mu\nu}\nabla^\alpha R + \frac{1}{6}g_{\mu\nu}\nabla^\alpha R\nabla_\beta h_\alpha{}^\beta - \nabla_\alpha h^{\alpha\beta}\nabla_\beta R_{\mu\nu} + \frac{1}{3}g_{\mu\nu}R\nabla_\beta\nabla_\alpha h^{\alpha\beta} \\
& - \frac{2}{3}R_{\mu\nu}\nabla_\beta\nabla_\alpha h^{\alpha\beta} + \frac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla_\alpha h_\mu{}^\beta - R^{\alpha\beta}\nabla_\beta\nabla_\alpha h_{\mu\nu} + \frac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla_\alpha h_\nu{}^\beta \\
& - \frac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla^\beta h_{\mu\alpha} - \frac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla^\beta h_{\nu\alpha} + \frac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla^\alpha h_{\mu\nu} - \frac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla_\mu h_\nu{}^\alpha \\
& - \frac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla_\nu h_\mu{}^\alpha - \frac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla_\mu h_\alpha{}^\beta + R^{\alpha\beta}\nabla_\beta\nabla_\mu h_{\nu\alpha} - \frac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla_\nu h_\alpha{}^\beta \\
& + R^{\alpha\beta}\nabla_\beta\nabla_\nu h_{\mu\alpha} + \nabla_\alpha R_{\nu\beta}\nabla^\beta h_\mu{}^\alpha - \nabla_\beta R_{\nu\alpha}\nabla^\beta h_\mu{}^\alpha + \nabla_\alpha R_{\mu\beta}\nabla^\beta h_\nu{}^\alpha - \nabla_\beta R_{\mu\alpha}\nabla^\beta h_\nu{}^\alpha \\
& - g_{\mu\nu}R^{\alpha\beta}\nabla_\gamma\nabla_\beta h_\alpha{}^\gamma + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_\gamma\nabla^\gamma h_{\alpha\beta} - R_{\mu\alpha\nu\beta}\nabla_\gamma\nabla^\gamma h^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_\gamma\nabla^\gamma\nabla_\beta\nabla_\alpha h^{\alpha\beta} \\
& + \frac{1}{3}g_{\mu\nu}\nabla_\gamma R_{\alpha\beta}\nabla^\gamma h^{\alpha\beta} - \nabla_\beta R_{\nu\alpha}\nabla_\mu h^{\alpha\beta} + \frac{1}{6}\nabla^\alpha R\nabla_\mu h_{\nu\alpha} - \frac{1}{2}R^{\alpha\beta}\nabla_\mu\nabla_\nu h_{\alpha\beta} \\
& - \nabla_\beta R_{\mu\alpha}\nabla_\nu h^{\alpha\beta} + \frac{1}{3}\nabla_\mu R_{\alpha\beta}\nabla_\nu h^{\alpha\beta} + \frac{1}{6}\nabla^\alpha R\nabla_\nu h_{\mu\alpha} + \frac{1}{3}\nabla_\mu h^{\alpha\beta}\nabla_\nu R_{\alpha\beta} \\
& - \frac{1}{6}R^{\alpha\beta}\nabla_\nu\nabla_\mu h_{\alpha\beta} + \frac{1}{3}\nabla_\nu\nabla_\mu\nabla_\beta\nabla_\alpha h^{\alpha\beta}
\end{aligned} \tag{26}$$

$$\begin{aligned}
& - \frac{1}{3}g_{\mu\nu}R\nabla_\alpha\nabla^\alpha h + \frac{2}{3}R_{\mu\nu}\nabla_\alpha\nabla^\alpha h + \frac{1}{2}\nabla_\alpha\nabla^\alpha\nabla_\nu\nabla_\mu h - \frac{1}{12}g_{\mu\nu}\nabla_\alpha h\nabla^\alpha R \\
& + \frac{1}{2}\nabla_\alpha R_{\mu\nu}\nabla^\alpha h + \frac{1}{2}g_{\mu\nu}R^{\alpha\beta}\nabla_\beta\nabla_\alpha h - \frac{1}{6}g_{\mu\nu}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla^\alpha h - R_{\mu\alpha\nu\beta}\nabla^\beta\nabla^\alpha h \\
& + \frac{1}{3}R\nabla_\nu\nabla_\mu h - \frac{1}{3}\nabla_\nu\nabla_\mu\nabla_\alpha\nabla^\alpha h.
\end{aligned} \tag{27}$$

Asanka:

$$2h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\mu\beta\nu\gamma} + 2h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\mu\gamma\nu\beta} - 2h_\mu{}^\alpha R^{\beta\gamma}R_{\nu\beta\alpha\gamma} - R^{\alpha\beta}\nabla_\beta\nabla_\nu h_{\mu\alpha} + 2R^{\alpha\beta}\nabla_\nu\nabla_\beta h_{\mu\alpha} - \frac{2}{3}R^{\alpha\beta}\nabla_\nu\nabla_\mu h_{\alpha\beta} \tag{28}$$

Matthew:

$$h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\mu\beta\nu\gamma} + h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\mu\gamma\nu\beta} + R^{\alpha\beta}\nabla_\beta\nabla_\nu h_{\mu\alpha} - \frac{1}{2}R^{\alpha\beta}\nabla_\mu\nabla_\nu h_{\alpha\beta} - \frac{1}{6}R^{\alpha\beta}\nabla_\nu\nabla_\mu h_{\alpha\beta} \tag{29}$$

Commutations:

$$2R^{\alpha\beta}\nabla_\nu\nabla_\beta h_{\mu\alpha} = -2h^{\alpha\beta}R_{\alpha}{}^\gamma R_{\mu\beta\nu\gamma} + 2h_\mu{}^\alpha R^{\beta\gamma}R_{\nu\beta\alpha\gamma} + 2R^{\alpha\beta}\nabla_\beta\nabla_\nu h_{\mu\alpha} \tag{30}$$

$$-\frac{1}{2}R^{\alpha\beta}\nabla_\mu\nabla_\nu h_{\alpha\beta} = -h^{\alpha\beta}R_\alpha{}^\gamma R_{\mu\nu\beta\gamma} - \frac{1}{2}R^{\alpha\beta}\nabla_\nu\nabla_\mu h_{\alpha\beta} \quad (31)$$

Cyclicity:

$$-R_{\mu\nu\beta\gamma} = R_{\mu\gamma\nu\beta} + R_{\mu\beta\gamma\nu} \quad (32)$$

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$$\begin{aligned} \delta W_{\mu\nu}(K_{\mu\nu}) = & -\frac{1}{6}K_{\mu\nu}R^2 + \frac{1}{3}g_{\mu\nu}K^{\alpha\beta}RR_{\alpha\beta} + \frac{1}{2}K_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - g_{\mu\nu}K^{\alpha\beta}R_\alpha{}^\gamma R_{\beta\gamma} - \frac{2}{3}K^{\alpha\beta}R_{\alpha\beta}R_{\mu\nu} \\ & + K^{\alpha\beta}R_\alpha{}^\gamma R_{\mu\beta\nu\gamma} + K^{\alpha\beta}R_\alpha{}^\gamma R_{\mu\gamma\nu\beta} - \frac{1}{6}K_{\mu\nu}\nabla_\alpha\nabla^\alpha R + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_\beta\nabla_\alpha R \\ & - K^{\alpha\beta}\nabla_\beta\nabla_\alpha R_{\mu\nu} + \frac{1}{6}g_{\mu\nu}K^{\alpha\beta}\nabla_\gamma\nabla^\gamma R_{\alpha\beta} + \frac{1}{3}K^{\alpha\beta}\nabla_\nu\nabla_\mu R_{\alpha\beta} \\ & + \frac{1}{3}R\nabla_\alpha\nabla^\alpha K_{\mu\nu} + R_{\mu\beta\nu\gamma}\nabla_\alpha\nabla^\gamma K^{\alpha\beta} + R_{\mu\gamma\nu\beta}\nabla_\alpha\nabla^\gamma K^{\alpha\beta} - \frac{1}{3}R\nabla_\alpha\nabla_\mu K_\nu{}^\alpha \\ & - \frac{1}{3}R\nabla_\alpha\nabla_\nu K_\mu{}^\alpha - \frac{1}{6}\nabla_\alpha K_{\mu\nu}\nabla^\alpha R + \frac{1}{6}g_{\mu\nu}\nabla^\alpha R\nabla_\beta K_\alpha{}^\beta - \nabla_\alpha K^{\alpha\beta}\nabla_\beta R_{\mu\nu} \\ & + \frac{1}{3}g_{\mu\nu}R\nabla_\beta\nabla_\alpha K^{\alpha\beta} - \frac{2}{3}R_{\mu\nu}\nabla_\beta\nabla_\alpha K^{\alpha\beta} + \frac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla_\alpha K_\mu{}^\beta - R^{\alpha\beta}\nabla_\beta\nabla_\alpha K_{\mu\nu} \\ & + \frac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla_\alpha K_\nu{}^\beta - \frac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla^\beta K_{\mu\alpha} - \frac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla^\beta K_{\nu\alpha} + \frac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla^\alpha K_{\mu\nu} \\ & - \frac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla_\mu K_\nu{}^\alpha - \frac{1}{2}\nabla_\beta\nabla^\beta\nabla_\alpha\nabla_\nu K_\mu{}^\alpha - \frac{1}{2}R_\nu{}^\alpha\nabla_\beta\nabla_\mu K_\alpha{}^\beta + R^{\alpha\beta}\nabla_\beta\nabla_\mu K_{\nu\alpha} \\ & - \frac{1}{2}R_\mu{}^\alpha\nabla_\beta\nabla_\nu K_\alpha{}^\beta + R^{\alpha\beta}\nabla_\beta\nabla_\nu K_{\mu\alpha} + \nabla_\alpha R_{\nu\beta}\nabla^\beta K_\mu{}^\alpha - \nabla_\beta R_{\nu\alpha}\nabla^\beta K_\mu{}^\alpha \\ & + \nabla_\alpha R_{\mu\beta}\nabla^\beta K_\nu{}^\alpha - \nabla_\beta R_{\mu\alpha}\nabla^\beta K_\nu{}^\alpha - g_{\mu\nu}R^{\alpha\beta}\nabla_\gamma\nabla_\beta K_\alpha{}^\gamma + \frac{2}{3}g_{\mu\nu}R^{\alpha\beta}\nabla_\gamma\nabla^\gamma K_{\alpha\beta} \\ & - R_{\mu\alpha\nu\beta}\nabla_\gamma\nabla^\gamma K^{\alpha\beta} + \frac{1}{6}g_{\mu\nu}\nabla_\gamma\nabla^\gamma\nabla_\beta\nabla_\alpha K^{\alpha\beta} + \frac{1}{3}g_{\mu\nu}\nabla_\gamma R_{\alpha\beta}\nabla^\gamma K^{\alpha\beta} - \nabla_\beta R_{\nu\alpha}\nabla_\mu K^{\alpha\beta} \\ & + \frac{1}{6}\nabla^\alpha R\nabla_\mu K_{\nu\alpha} - \frac{1}{2}R^{\alpha\beta}\nabla_\mu\nabla_\nu K_{\alpha\beta} - \nabla_\beta R_{\mu\alpha}\nabla_\nu K^{\alpha\beta} + \frac{1}{3}\nabla_\mu R_{\alpha\beta}\nabla_\nu K^{\alpha\beta} \\ & + \frac{1}{6}\nabla^\alpha R\nabla_\nu K_{\mu\alpha} + \frac{1}{3}\nabla_\mu K^{\alpha\beta}\nabla_\nu R_{\alpha\beta} - \frac{1}{6}R^{\alpha\beta}\nabla_\nu\nabla_\mu K_{\alpha\beta} + \frac{1}{3}\nabla_\nu\nabla_\mu\nabla_\beta\nabla_\alpha K^{\alpha\beta} \\ & + \frac{1}{24}g_{\mu\nu}R^2h - \frac{1}{4}g_{\mu\nu}R_\alpha{}^\beta R^\alpha{}_\beta h + \frac{1}{8}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}h - \frac{1}{6}RR_{\mu\nu}h + \frac{1}{4}R^\alpha{}_\beta R_{\mu\alpha\nu}{}^\beta h \\ & + \frac{1}{4}R^\alpha{}_\beta R_\mu{}^\beta{}_\nu{}_\alpha h + \frac{1}{24}g_{\mu\nu}h\nabla_\alpha\nabla^\alpha R - \frac{1}{4}h\nabla_\alpha\nabla^\alpha R_{\mu\nu} + \frac{1}{4}\nabla_\alpha\nabla^\alpha\nabla_\nu\nabla_\mu h \\ & - \frac{1}{12}g_{\mu\nu}\nabla_\alpha h\nabla^\alpha R - \frac{1}{4}\nabla_\alpha h\nabla^\alpha R_{\mu\nu} + \frac{1}{12}g_{\mu\nu}\nabla_\alpha R\nabla^\alpha h - \frac{1}{2}R_{\mu\alpha\nu\beta}\nabla^\beta\nabla^\alpha h \\ & + \frac{1}{4}\nabla^\alpha h\nabla_\mu R_{\nu\alpha} - \frac{1}{4}\nabla_\alpha R_\nu{}^\alpha\nabla_\mu h + \frac{1}{4}R_\nu{}^\alpha\nabla_\mu\nabla_\alpha h + \frac{1}{8}\nabla_\mu h\nabla_\nu R + \frac{1}{4}\nabla^\alpha h\nabla_\nu R_{\mu\alpha} \\ & - \frac{1}{4}\nabla_\alpha R_\mu{}^\alpha\nabla_\nu h + \frac{1}{8}\nabla_\mu R\nabla_\nu h + \frac{1}{4}R_\mu{}^\alpha\nabla_\nu\nabla_\alpha h + \frac{1}{12}h\nabla_\nu\nabla_\mu R - \frac{1}{4}\nabla_\nu\nabla_\mu\nabla_\alpha\nabla^\alpha h. \end{aligned} \quad (33)$$