Special Gauge Matthew v9

Setup

Metric decomposed to first order:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} = \Omega^2(\tau)(\eta_{\mu\nu} + f_{\mu\nu}). \tag{1}$$

We then split $f_{\mu\nu}$ into its traceless and trace components, i.e.

$$f_{\mu\nu} = k_{\mu\nu} + \frac{1}{4}\eta_{\mu\nu} f \tag{2}$$

where $f = \eta^{\mu\nu} f_{\mu\nu}$. We impose a generalized gauge of the form

$$\eta^{\alpha\beta}\partial_\alpha k_{\beta\nu} = \Omega^{-1} J \eta^{\alpha\beta} k_{\nu\alpha}\partial_\beta \Omega + P\partial_\nu f + R\Omega^{-1} f \partial_\nu \Omega. \tag{3}$$

and take

$$J = -4, \qquad R = 2P - \frac{3}{2}. \tag{4}$$

$\Omega(\tau)$

Working with a time dependent conformal factor, $\Omega(\tau)$, the fluctuations are evaluated as

$$\begin{aligned}\eta^{\mu\nu}\delta G_{\mu\nu} &= (-8\Omega^{-2}\dot{\Omega}^2 + 4\Omega^{-1}\ddot{\Omega})k_{00} + (\tfrac{3}{2}\Omega^{-2}\dot{\Omega}^2 - 2P\Omega^{-2}\dot{\Omega}^2 + \tfrac{3}{2}\Omega^{-1}\ddot{\Omega} - 2P\Omega^{-1}\ddot{\Omega} - \tfrac{3}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu \\ &\qquad + P\eta^{\mu\nu}\partial_\mu\partial_\nu + 3\Omega^{-1}\dot{\Omega}\partial_0 - 4P\Omega^{-1}\dot{\Omega}\partial_0)f. \\ &= (-8\Omega^{-2}\dot{\Omega}^2 + 4\Omega^{-1}\ddot{\Omega})k_{00} + (P - \tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta(\Omega^2 f)\end{aligned} \tag{5}$$

$$\begin{aligned}\delta G_{00} = &(2\Omega^{-2}\dot{\Omega}^2 - 2\Omega^{-1}\ddot{\Omega} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - 3\Omega^{-1}\dot{\Omega}\partial_0)k_{00} + (-\tfrac{3}{4}\Omega^{-2}\dot{\Omega}^2 + P\Omega^{-2}\dot{\Omega}^2 + \tfrac{3}{4}\Omega^{-1}\ddot{\Omega} \\ &- P\Omega^{-1}\ddot{\Omega} + \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu - P\Omega^{-1}\dot{\Omega}\partial_0 + \tfrac{1}{4}\partial_0\partial_0 - P\partial_0\partial_0)f.\end{aligned} \tag{6}$$

$$\begin{aligned}\delta G_{01} = &-\Omega^{-1}\dot{\Omega}\partial_1 k_{00} + (\Omega^{-2}\dot{\Omega}^2 + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - 2\Omega^{-1}\dot{\Omega}\partial_0)k_{01} + (\tfrac{1}{2}\Omega^{-1}\dot{\Omega}\partial_1 - P\Omega^{-1}\dot{\Omega}\partial_1 \\ &+ \tfrac{1}{4}\partial_1\partial_0 - P\partial_1\partial_0)f.\end{aligned} \tag{7}$$

$$\begin{aligned}\delta G_{11} = &\,\Omega^{-2}\dot{\Omega}^2 k_{00} - 2\Omega^{-1}\dot{\Omega}\partial_1 k_{01} + (-\Omega^{-2}\dot{\Omega}^2 + 2\Omega^{-1}\ddot{\Omega} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-1}\dot{\Omega}\partial_0)k_{11} \\ &+ (-\tfrac{3}{4}\Omega^{-2}\dot{\Omega}^2 + P\Omega^{-2}\dot{\Omega}^2 + \tfrac{3}{4}\Omega^{-1}\ddot{\Omega} - P\Omega^{-1}\ddot{\Omega} - \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu + \Omega^{-1}\dot{\Omega}\partial_0 \\ &- P\Omega^{-1}\dot{\Omega}\partial_0 + \tfrac{1}{4}\partial_1\partial_1 - P\partial_1\partial_1)f.\end{aligned} \tag{8}$$

$$\begin{aligned}\delta G_{12} = &-\Omega^{-1}\dot{\Omega}\partial_2 k_{01} - \Omega^{-1}\dot{\Omega}\partial_1 k_{02} + (-\Omega^{-2}\dot{\Omega}^2 + 2\Omega^{-1}\ddot{\Omega} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-1}\dot{\Omega}\partial_0)k_{12} \\ &+ (\tfrac{1}{4}\partial_2\partial_1 - P\partial_2\partial_1)f.\end{aligned} \tag{9}$$

$\Omega(\tau) = \frac{1}{H\tau}$

Now set $\Omega(\tau) = \frac{1}{H\tau}$, with the fluctuations being evaluated as

$$\eta^{\mu\nu}\delta G_{\mu\nu} = (P - \tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta(\Omega^2 f) \tag{10}$$

$$\begin{aligned}\delta G_{00} = {} & (-2\tau^{-2} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 3\tau^{-1}\partial_0)k_{00} + (\tfrac{3}{4}\tau^{-2} - P\tau^{-2} + \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & + P\tau^{-1}\partial_0 + \tfrac{1}{4}\partial_0\partial_0 - P\partial_0\partial_0)f.\end{aligned} \tag{11}$$

$$\begin{aligned}\delta G_{01} = {} & \tau^{-1}\partial_1 k_{00} + (\tau^{-2} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 2\tau^{-1}\partial_0)k_{01} + (-\tfrac{1}{2}\tau^{-1}\partial_1 + P\tau^{-1}\partial_1 + \tfrac{1}{4}\partial_1\partial_0 \\ & - P\partial_1\partial_0)f.\end{aligned} \tag{12}$$

$$\begin{aligned}\delta G_{11} = {} & \tau^{-2}k_{00} + 2\tau^{-1}\partial_1 k_{01} + (3\tau^{-2} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)k_{11} + (\tfrac{3}{4}\tau^{-2} - P\tau^{-2} \\ & - \tfrac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu - \tau^{-1}\partial_0 + P\tau^{-1}\partial_0 + \tfrac{1}{4}\partial_1\partial_1 - P\partial_1\partial_1)f.\end{aligned} \tag{13}$$

$$\delta G_{12} = \tau^{-1}\partial_2 k_{01} + \tau^{-1}\partial_1 k_{02} + (3\tau^{-2} + \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)k_{12} + (\tfrac{1}{4}\partial_2\partial_1 - P\partial_2\partial_1)f. \tag{14}$$