Gauged Perturbed Einstein Factorization

For $\Omega(t)$, and in gauge $P = 1/2$, $J = 0$, $R = -1$

$$\eta^{\alpha\beta}\partial_\alpha h_{\nu\beta} = \frac{1}{2}\Omega^2\partial_\nu h - \Omega h\partial_\nu\Omega$$

$$\begin{aligned}
\delta G_{00} &= \frac{1}{2}\Box(\Omega^{-2}h_{00}) - \frac{1}{2}\Omega^{-4}\partial_0(h_{00}\partial_0(\Omega^2)) + 6\Omega^{-4}h_{00}(\partial_0\Omega)^2 - \frac{1}{4}\Box h + \frac{1}{4}\Omega^2 h\Box(\Omega^{-2}) \\
\delta G_{ij} &= \frac{1}{2}\Box(\Omega^{-2}h_{ij}) - \frac{1}{2}\Omega^{-4}\partial_0(h_{ij}\partial_0(\Omega^2)) - 3\Omega^{-3}h_{ij}\Box\Omega + 2\Omega^{-4}h_{ij}(\partial_0\Omega)^2 - \frac{1}{4}\Omega^{-2}\Box(\Omega^2 h) - \delta_{ij}\Omega^{-1}h_{00}\Box(\Omega^{-1}) \\
\delta G_{0i} &= \frac{1}{2}\Box(\Omega^{-2}h_{0i}) - \frac{1}{2}\Omega^{-4}\partial_0(h_{0i}\partial_0(\Omega^2)) - 2\Omega^{-3}h_{0i}\Box\Omega + 3\Omega^{-4}h_{0i}(\partial_0\Omega)^2 - \frac{1}{4}h\Omega^2\Box(\Omega^{-2})
\end{aligned}$$

In gauge $P = 1$, $J = R = 0$:

$$\eta^{\alpha\beta}\partial_\alpha h_{\nu\beta} = \Omega^2\partial_\nu h$$

$$\begin{aligned}
\delta G_{00} &= \left[\frac{1}{2}\Omega^{-2}\Box + \Omega^{-3}\partial_0\Omega\partial_0 + 2\Omega^{-4}(\partial_0\Omega)^2\right]h_{00} - \frac{1}{2}\partial_0\partial_0 h \\
\delta G_{ij} &= \left[\frac{1}{2}\Omega^{-2}\Box + \Omega^{-3}\partial_0\Omega\partial_0 - 2\Omega^{-4}(\partial_0\Omega)^2 + 3\Omega^{-3}\partial_0\partial_0\Omega\right]h_{ij} + \delta_{ij}\left[\Omega^{-3}\partial_0\partial_0\Omega - 2\Omega^{-4}(\partial_0\Omega)^2\right]h_{00} - \frac{1}{2}\partial_i\partial_j h \\
\delta G_{0i} &= \left[\frac{1}{2}\Omega^{-2}\Box + \Omega^{-3}\partial_0\Omega\partial_0 - \Omega^{-4}(\partial_0\Omega)^2 + 2\Omega^{-3}\partial_0\partial_0\Omega\right]h_{0i} - \frac{1}{2}\partial_0\partial_i h
\end{aligned}$$