RW SVT4 $k \neq 0$ v5

For a RW geometry of the form

$$ds^2 = -dt^2 + a(t)^2 \tilde{g}_{ij} dx^i dx^j, \qquad R_{ij} = -2k\tilde{g}_{ij} \tag{0.1}$$

or

$$ds^2 = \Omega^2(\tau)\left(-d\tau^2 + \tilde{g}_{ij} dx^i dx^j\right), \qquad R_{ij} = -2k\tilde{g}_{ij}, \tag{0.2}$$

the Ricci tensor may be $3+1$ decomposed into a perfect fluid form as in (1.2). For $k \neq 0$, following a $3+1$ decomposition the conformal to flat geometry will necessarily include terms proportional to q_μ and $\pi_{\mu\nu}$. (If $R_{0i} = 0$ and $R_{ij} \propto g_{ij}$ then q_μ and $\pi_{\mu\nu}$ will vanish).

1 Background

$$R_{\lambda\mu\nu\kappa} = -\tfrac{1}{6} g_{\lambda\nu} g_{\mu\kappa} R + \tfrac{1}{6} g_{\lambda\kappa} g_{\mu\nu} R - \tfrac{1}{2} g_{\mu\nu} R_{\lambda\kappa} + \tfrac{1}{2} g_{\mu\kappa} R_{\lambda\nu} + \tfrac{1}{2} g_{\lambda\nu} R_{\mu\kappa} - \tfrac{1}{2} g_{\lambda\kappa} R_{\mu\nu} \tag{1.1}$$

$$R_{\mu\nu} = (A+B) U_\mu U_\nu + g_{\mu\nu} B, \qquad R = 3B - A \tag{1.2}$$

$$G_{\mu\nu} = \tfrac{1}{2} A g_{\mu\nu} - \tfrac{1}{2} B g_{\mu\nu} + A U_\mu U_\nu + B U_\mu U_\nu \tag{1.3}$$

$$g^{\mu\nu} G_{\mu\nu} = A - 3B \tag{1.4}$$

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} \tag{1.5}$$

$$g^{\mu\nu} T_{\mu\nu} = 3p - \rho \tag{1.6}$$

$$\Delta^{(0)}_{\mu\nu} = \tfrac{1}{2} A g_{\mu\nu} - \tfrac{1}{2} B g_{\mu\nu} + g_{\mu\nu} p + A U_\mu U_\nu + B U_\mu U_\nu + p U_\mu U_\nu + U_\mu U_\nu \rho \tag{1.7}$$

$$g^{\mu\nu} \Delta^{(0)}_{\mu\nu} = A - 3B + 3p - \rho \tag{1.8}$$

Solving for ρ within (1.8), substituting into (1.7), and projecting $U^\mu U^\nu \Delta^{(0)}_{\mu\nu}$ we can solve for p as

$$p = \tfrac{1}{2}(-A + B) \tag{1.9}$$

From (1.8) we can then determine ρ as

$$\rho = \tfrac{1}{2}(-A - 3B). \tag{1.10}$$

1.1 Identities

A and B are functions only of coordinate x^0.

$$U^\alpha U^\beta \nabla_\alpha F \nabla_\beta A = -\nabla^\alpha F \nabla_\alpha A \tag{1.11}$$

$$F^\alpha U_\alpha U^\beta \nabla_\beta A = -F^\alpha \nabla_\alpha A \tag{1.12}$$

2 Fluctuations

$$ds^2 = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \tag{2.1}$$

$$h_{\mu\nu} = -2g_{\mu\nu}\chi + 2\nabla_\mu\nabla_\nu F + \nabla_\mu F_\nu + \nabla_\nu F_\mu + 2F_{\mu\nu} \tag{2.2}$$

$$g^{\mu\nu}F_{\mu\nu} = 0, \qquad \nabla^\mu F_{\mu\nu} = 0, \qquad \nabla^\mu F_\mu = 0 \tag{2.3}$$

$$U^\mu \delta U_\mu = \tfrac{1}{2}U^\mu U^\nu h_{\mu\nu}, \qquad U^\mu U_\mu = -1, \qquad U^\alpha \nabla_\alpha U^\mu = 0 \tag{2.4}$$

$$\delta U_\mu = (V_\mu + \nabla_\mu V) + U_\mu U^\alpha (V_\alpha + \nabla_\alpha V) - U_\mu \left(\tfrac{1}{2}U^\alpha U^\beta h_{\alpha\beta}\right) \tag{2.5}$$

$$\begin{aligned}\delta T_{\mu\nu} = {} & \delta p g_{\mu\nu} + \delta p U_\mu U_\nu + \delta\rho U_\mu U_\nu - 2g_{\mu\nu}p\chi + 2p\nabla_\nu\nabla_\mu F + \delta U_\nu p U_\mu + \delta U_\mu p U_\nu + \delta U_\nu U_\mu \rho \\ & + \delta U_\mu U_\nu \rho + p\nabla_\mu F_\nu + p\nabla_\nu F_\mu + 2F_{\mu\nu}p\end{aligned} \tag{2.6}$$

$$\begin{aligned}g^{\mu\nu}\delta T_{\mu\nu} = {} & 3\delta p - \delta\rho - 6p\chi + 2\rho\chi + 2p\nabla_\alpha\nabla^\alpha F + 2pU^\alpha U^\beta \nabla_\beta\nabla_\alpha F + 2U^\alpha U^\beta \rho\nabla_\beta\nabla_\alpha F + 2pU^\alpha U^\beta \nabla_\beta F_\alpha \\ & + 2U^\alpha U^\beta \rho\nabla_\beta F_\alpha + 2F_{\alpha\beta}pU^\alpha U^\beta + 2F_{\alpha\beta}U^\alpha U^\beta \rho\end{aligned} \tag{2.7}$$

$$\begin{aligned}\delta G_{\mu\nu} = {} & 2g_{\mu\nu}\nabla_\alpha\nabla^\alpha\chi + \tfrac{2}{3}g_{\mu\nu}\nabla_\alpha F\nabla^\alpha A + \tfrac{1}{2}U_\mu U_\nu \nabla_\alpha F\nabla^\alpha A + \tfrac{1}{2}U_\mu U_\nu \nabla_\alpha F\nabla^\alpha B + \tfrac{1}{2}AU_\nu \nabla_\alpha U_\mu \nabla^\alpha F \\ & + \tfrac{1}{2}BU_\nu \nabla_\alpha U_\mu \nabla^\alpha F + \tfrac{1}{2}AU_\mu \nabla_\alpha U_\nu \nabla^\alpha F + \tfrac{1}{2}BU_\mu \nabla_\alpha U_\nu \nabla^\alpha F - \tfrac{1}{2}Ag_{\mu\nu}U^\alpha \nabla_\alpha F\nabla_\beta U^\beta \\ & - \tfrac{1}{2}Bg_{\mu\nu}U^\alpha \nabla_\alpha F\nabla_\beta U^\beta + \tfrac{1}{2}U^\alpha U_\nu \nabla_\alpha F\nabla_\mu A + \tfrac{1}{2}U^\alpha U_\nu \nabla_\alpha F\nabla_\mu B + \tfrac{1}{2}AU_\nu \nabla^\alpha F\nabla_\mu U_\alpha \\ & + \tfrac{1}{2}BU_\nu \nabla^\alpha F\nabla_\mu U_\alpha + \tfrac{1}{2}AU^\alpha \nabla_\alpha F\nabla_\mu U_\nu + \tfrac{1}{2}BU^\alpha \nabla_\alpha F\nabla_\mu U_\nu + AU^\alpha U_\nu \nabla_\mu\nabla_\alpha F \\ & + BU^\alpha U_\nu \nabla_\mu\nabla_\alpha F - \tfrac{1}{2}U^\alpha U_\mu \nabla_\alpha A\nabla_\nu F - \tfrac{1}{2}U^\alpha U_\mu \nabla_\alpha B\nabla_\nu F - \tfrac{1}{2}AU_\mu \nabla_\alpha U^\alpha \nabla_\nu F \\ & - \tfrac{1}{2}BU_\mu \nabla_\alpha U^\alpha \nabla_\nu F - \tfrac{1}{6}\nabla_\mu A\nabla_\nu F + \tfrac{1}{2}\nabla_\mu B\nabla_\nu F + AU^\alpha U_\mu \nabla_\nu\nabla_\alpha F + BU^\alpha U_\mu \nabla_\nu\nabla_\alpha F \\ & + A\nabla_\nu\nabla_\mu F - B\nabla_\nu\nabla_\mu F - 2\nabla_\nu\nabla_\mu\chi + \tfrac{2}{3}F^\alpha g_{\mu\nu}\nabla_\alpha A - \tfrac{1}{4}F_\nu U^\alpha U_\mu \nabla_\alpha A - \tfrac{1}{4}F_\mu U^\alpha U_\nu \nabla_\alpha A \\ & + \tfrac{1}{2}F^\alpha U_\mu U_\nu \nabla_\alpha A - \tfrac{1}{4}F_\nu U^\alpha U_\mu \nabla_\alpha B - \tfrac{1}{4}F_\mu U^\alpha U_\nu \nabla_\alpha B + \tfrac{1}{2}F^\alpha U_\mu U_\nu \nabla_\alpha B - \tfrac{1}{4}AF_\nu U_\mu \nabla_\alpha U^\alpha \\ & - \tfrac{1}{4}BF_\nu U_\mu \nabla_\alpha U^\alpha - \tfrac{1}{4}AF_\mu U_\nu \nabla_\alpha U^\alpha - \tfrac{1}{4}BF_\mu U_\nu \nabla_\alpha U^\alpha + \tfrac{1}{2}AF^\alpha U_\nu \nabla_\alpha U_\mu + \tfrac{1}{2}BF^\alpha U_\nu \nabla_\alpha U_\mu \\ & + \tfrac{1}{2}AF^\alpha U_\mu \nabla_\alpha U_\nu + \tfrac{1}{2}BF^\alpha U_\mu \nabla_\alpha U_\nu - \tfrac{1}{2}AF^\alpha g_{\mu\nu}U_\alpha \nabla_\beta U^\beta - \tfrac{1}{2}BF^\alpha g_{\mu\nu}U_\alpha \nabla_\beta U^\beta - \tfrac{1}{12}F_\nu \nabla_\mu A \\ & + \tfrac{1}{4}F^\alpha U_\alpha U_\nu \nabla_\mu A + \tfrac{1}{4}F_\nu \nabla_\mu B + \tfrac{1}{4}F^\alpha U_\alpha U_\nu \nabla_\mu B + AU^\alpha U_\nu \nabla_\mu F_\alpha + BU^\alpha U_\nu \nabla_\mu F_\alpha \\ & + \tfrac{1}{2}A\nabla_\mu F_\nu - \tfrac{1}{2}B\nabla_\mu F_\nu + \tfrac{1}{4}AF^\alpha U_\nu \nabla_\mu U_\alpha + \tfrac{1}{4}BF^\alpha U_\nu \nabla_\mu U_\alpha + \tfrac{1}{4}AF^\alpha U_\alpha \nabla_\mu U_\nu \\ & + \tfrac{1}{4}BF^\alpha U_\alpha \nabla_\mu U_\nu - \tfrac{1}{12}F_\mu \nabla_\nu A + \tfrac{1}{4}F^\alpha U_\alpha U_\mu \nabla_\nu A + \tfrac{1}{4}F_\mu \nabla_\nu B + \tfrac{1}{4}F^\alpha U_\alpha U_\mu \nabla_\nu B \\ & + AU^\alpha U_\mu \nabla_\nu F_\alpha + BU^\alpha U_\mu \nabla_\nu F_\alpha + \tfrac{1}{2}A\nabla_\nu F_\mu - \tfrac{1}{2}B\nabla_\nu F_\mu + \tfrac{1}{4}AF^\alpha U_\mu \nabla_\nu U_\alpha \\ & + \tfrac{1}{4}BF^\alpha U_\mu \nabla_\nu U_\alpha + \tfrac{1}{4}AF^\alpha U_\alpha \nabla_\nu U_\mu + \tfrac{1}{4}BF^\alpha U_\alpha \nabla_\nu U_\mu + \tfrac{4}{3}AF_{\mu\nu} + 2AF_{\nu\alpha}U^\alpha U_\mu \\ & + 2BF_{\nu\alpha}U^\alpha U_\mu + 2AF_{\mu\alpha}U^\alpha U_\nu + 2BF_{\mu\alpha}U^\alpha U_\nu + \nabla_\alpha\nabla^\alpha F_{\mu\nu}\end{aligned} \tag{2.8}$$

$$\begin{aligned}g^{\mu\nu}\delta G_{\mu\nu} = {} & A\nabla_\alpha\nabla^\alpha F - B\nabla_\alpha\nabla^\alpha F + 6\nabla_\alpha\nabla^\alpha\chi + 2\nabla_\alpha F\nabla^\alpha A - 2AU^\alpha \nabla_\alpha F\nabla_\beta U^\beta - 2BU^\alpha \nabla_\alpha F\nabla_\beta U^\beta \\ & + 2AU^\alpha U^\beta \nabla_\beta\nabla_\alpha F + 2BU^\alpha U^\beta \nabla_\beta\nabla_\alpha F + AU^\alpha \nabla_\beta U_\alpha \nabla^\beta F + BU^\alpha \nabla_\beta U_\alpha \nabla^\beta F + 2F^\alpha \nabla_\alpha A \\ & + AF^\alpha U^\beta \nabla_\alpha U_\beta + BF^\alpha U^\beta \nabla_\alpha U_\beta + 2AU^\alpha U^\beta \nabla_\beta F_\alpha + 2BU^\alpha U^\beta \nabla_\beta F_\alpha - 2AF^\alpha U_\alpha \nabla_\beta U^\beta \\ & - 2BF^\alpha U_\alpha \nabla_\beta U^\beta + 4AF_{\alpha\beta}U^\alpha U^\beta + 4BF_{\alpha\beta}U^\alpha U^\beta\end{aligned} \tag{2.9}$$

3 Field Equations

$$
\begin{aligned}
\Delta_{\mu\nu} &= \delta p g_{\mu\nu} + \delta p U_\mu U_\nu + \delta\rho U_\mu U_\nu + A g_{\mu\nu}\chi - B g_{\mu\nu}\chi + 2A U_\mu U_\nu \chi + 2B U_\mu U_\nu \chi - 2A U^\alpha U_\mu U_\nu \nabla_\alpha V \\
&\quad -2B U^\alpha U_\mu U_\nu \nabla_\alpha V + 2 g_{\mu\nu}\nabla_\alpha\nabla^\alpha\chi + \tfrac{2}{3} g_{\mu\nu}\nabla_\alpha F \nabla^\alpha A + \tfrac{1}{2} U_\mu U_\nu \nabla_\alpha F \nabla^\alpha A \\
&\quad +\tfrac{1}{2} U_\mu U_\nu \nabla_\alpha F \nabla^\alpha B + \tfrac{1}{2} A U_\nu \nabla_\alpha U_\mu \nabla^\alpha F + \tfrac{1}{2} B U_\nu \nabla_\alpha U_\mu \nabla^\alpha F + \tfrac{1}{2} A U_\mu \nabla_\alpha U_\nu \nabla^\alpha F \\
&\quad +\tfrac{1}{2} B U_\mu \nabla_\alpha U_\nu \nabla^\alpha F - \tfrac{1}{2} A g_{\mu\nu} U^\alpha \nabla_\alpha F \nabla_\beta U^\beta - \tfrac{1}{2} B g_{\mu\nu} U^\alpha \nabla_\alpha F \nabla_\beta U^\beta \\
&\quad +2A U^\alpha U^\beta U_\mu U_\nu \nabla_\beta \nabla_\alpha F + 2B U^\alpha U^\beta U_\mu U_\nu \nabla_\beta \nabla_\alpha F + \tfrac{1}{2} U^\alpha U_\nu \nabla_\alpha F \nabla_\mu A + \tfrac{1}{2} U^\alpha U_\nu \nabla_\alpha F \nabla_\mu B \\
&\quad +\tfrac{1}{2} A U_\nu \nabla^\alpha F \nabla_\mu U_\alpha + \tfrac{1}{2} B U_\nu \nabla^\alpha F \nabla_\mu U_\alpha + \tfrac{1}{2} A U^\alpha \nabla_\alpha F \nabla_\mu U_\nu + \tfrac{1}{2} B U^\alpha \nabla_\alpha F \nabla_\mu U_\nu \\
&\quad -A U_\nu \nabla_\mu V - B U_\nu \nabla_\mu V + A U^\alpha U_\nu \nabla_\mu \nabla_\alpha F + B U^\alpha U_\nu \nabla_\mu \nabla_\alpha F - \tfrac{1}{2} U^\alpha U_\mu \nabla_\alpha A \nabla_\nu F \\
&\quad -\tfrac{1}{2} U^\alpha U_\mu \nabla_\alpha B \nabla_\nu F - \tfrac{1}{2} A U_\mu \nabla_\alpha U^\alpha \nabla_\nu F - \tfrac{1}{2} B U_\mu \nabla_\alpha U^\alpha \nabla_\nu F - \tfrac{1}{6} \nabla_\mu A \nabla_\nu F + \tfrac{1}{2} \nabla_\mu B \nabla_\nu F \\
&\quad -A U_\mu \nabla_\nu V - B U_\mu \nabla_\nu V + A U^\alpha U_\mu \nabla_\nu \nabla_\alpha F + B U^\alpha U_\mu \nabla_\nu \nabla_\alpha F - 2\nabla_\nu\nabla_\mu\chi - 2A U^\alpha U_\mu U_\nu V_\alpha \\
&\quad -2B U^\alpha U_\mu U_\nu V_\alpha - A U_\nu V_\mu - B U_\nu V_\mu - A U_\mu V_\nu - B U_\mu V_\nu + \tfrac{2}{3} F^\alpha g_{\mu\nu} \nabla_\alpha A - \tfrac{1}{4} F_\nu U^\alpha U_\mu \nabla_\alpha A \\
&\quad -\tfrac{1}{4} F_\mu U^\alpha U_\nu \nabla_\alpha A + \tfrac{1}{2} F^\alpha U_\mu U_\nu \nabla_\alpha A - \tfrac{1}{4} F_\nu U^\alpha U_\mu \nabla_\alpha B - \tfrac{1}{4} F_\mu U^\alpha U_\nu \nabla_\alpha B + \tfrac{1}{2} F^\alpha U_\mu U_\nu \nabla_\alpha B \\
&\quad -\tfrac{1}{4} A F_\nu U_\mu \nabla_\alpha U^\alpha - \tfrac{1}{4} B F_\nu U_\mu \nabla_\alpha U^\alpha - \tfrac{1}{4} A F_\mu U_\nu \nabla_\alpha U^\alpha - \tfrac{1}{4} B F_\mu U_\nu \nabla_\alpha U^\alpha + \tfrac{1}{2} A F^\alpha U_\nu \nabla_\alpha U_\mu \\
&\quad +\tfrac{1}{2} B F^\alpha U_\nu \nabla_\alpha U_\mu + \tfrac{1}{2} A F^\alpha U_\mu \nabla_\alpha U_\nu + \tfrac{1}{2} B F^\alpha U_\mu \nabla_\alpha U_\nu + 2A U^\alpha U^\beta U_\mu U_\nu \nabla_\beta F_\alpha \\
&\quad +2B U^\alpha U^\beta U_\mu U_\nu \nabla_\beta F_\alpha - \tfrac{1}{2} A F^\alpha g_{\mu\nu} U_\alpha \nabla_\beta U^\beta - \tfrac{1}{2} B F^\alpha g_{\mu\nu} U_\alpha \nabla_\beta U^\beta - \tfrac{1}{12} F_\nu \nabla_\mu A \\
&\quad +\tfrac{1}{4} F^\alpha U_\alpha U_\nu \nabla_\mu A + \tfrac{1}{4} F_\nu \nabla_\mu B + \tfrac{1}{4} F^\alpha U_\alpha U_\nu \nabla_\mu B + A U^\alpha U_\nu \nabla_\mu F_\alpha + B U^\alpha U_\nu \nabla_\mu F_\alpha \\
&\quad +\tfrac{1}{4} A F^\alpha U_\nu \nabla_\mu U_\alpha + \tfrac{1}{4} B F^\alpha U_\nu \nabla_\mu U_\alpha + \tfrac{1}{4} A F^\alpha U_\alpha \nabla_\mu U_\nu + \tfrac{1}{4} B F^\alpha U_\alpha \nabla_\mu U_\nu - \tfrac{1}{12} F_\mu \nabla_\nu A \\
&\quad +\tfrac{1}{4} F^\alpha U_\alpha U_\mu \nabla_\nu A + \tfrac{1}{4} F_\mu \nabla_\nu B + \tfrac{1}{4} F^\alpha U_\alpha U_\mu \nabla_\nu B + A U^\alpha U_\mu \nabla_\nu F_\alpha + B U^\alpha U_\mu \nabla_\nu F_\alpha \\
&\quad +\tfrac{1}{4} A F^\alpha U_\mu \nabla_\nu U_\alpha + \tfrac{1}{4} B F^\alpha U_\mu \nabla_\nu U_\alpha + \tfrac{1}{4} A F^\alpha U_\alpha \nabla_\nu U_\mu + \tfrac{1}{4} B F^\alpha U_\alpha \nabla_\nu U_\mu + \tfrac{1}{3} A F_{\mu\nu} + B F_{\mu\nu} \\
&\quad +2A F_{\nu\alpha} U^\alpha U_\mu + 2B F_{\nu\alpha} U^\alpha U_\mu + 2A F_{\mu\alpha} U^\alpha U_\nu + 2B F_{\mu\alpha} U^\alpha U_\nu + 2A F_{\alpha\beta} U^\alpha U^\beta U_\mu U_\nu \\
&\quad +2B F_{\alpha\beta} U^\alpha U^\beta U_\mu U_\nu + \nabla_\alpha \nabla^\alpha F_{\mu\nu} \qquad (3.1)
\end{aligned}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} &= 3\delta p - \delta\rho + 2A\chi - 6B\chi + 6\nabla_\alpha\nabla^\alpha\chi + 2\nabla_\alpha F \nabla^\alpha A - 2A U^\alpha \nabla_\alpha F \nabla_\beta U^\beta - 2B U^\alpha \nabla_\alpha F \nabla_\beta U^\beta \\
&\quad +A U^\alpha \nabla_\beta U_\alpha \nabla^\beta F + B U^\alpha \nabla_\beta U_\alpha \nabla^\beta F + 2F^\alpha \nabla_\alpha A + A F^\alpha U^\beta \nabla_\alpha U_\beta + B F^\alpha U^\beta \nabla_\alpha U_\beta \\
&\quad -2A F^\alpha U_\alpha \nabla_\beta U^\beta - 2B F^\alpha U_\alpha \nabla_\beta U^\beta + 2A F_{\alpha\beta} U^\alpha U^\beta + 2B F_{\alpha\beta} U^\alpha U^\beta \qquad (3.2)
\end{aligned}
$$

$$
\begin{aligned}
U^\mu U^\nu \Delta_{\mu\nu} &= \delta\rho + A\chi + 3B\chi - 2\nabla_\alpha\nabla^\alpha\chi + A U^\alpha \nabla_\alpha F \nabla_\beta U^\beta + B U^\alpha \nabla_\alpha F \nabla_\beta U^\beta - 2U^\alpha U^\beta \nabla_\beta \nabla_\alpha \chi \\
&\quad -A U^\alpha \nabla_\beta U_\alpha \nabla^\beta F - B U^\alpha \nabla_\beta U_\alpha \nabla^\beta F - A F^\alpha U^\beta \nabla_\alpha U_\beta - B F^\alpha U^\beta \nabla_\alpha U_\beta + A F^\alpha U_\alpha \nabla_\beta U^\beta \\
&\quad +B F^\alpha U_\alpha \nabla_\beta U^\beta - \tfrac{5}{3} A F_{\alpha\beta} U^\alpha U^\beta - B F_{\alpha\beta} U^\alpha U^\beta + U^\alpha U^\beta \nabla_\gamma \nabla^\gamma F_{\alpha\beta} \qquad (3.3)
\end{aligned}
$$

$$
\begin{aligned}
(U^\mu U^\nu + g^{\mu\nu})\Delta_{\mu\nu} &= 3\delta p + 3A\chi - 3B\chi + 4\nabla_\alpha\nabla^\alpha\chi + 2\nabla_\alpha F \nabla^\alpha A - A U^\alpha \nabla_\alpha F \nabla_\beta U^\beta - B U^\alpha \nabla_\alpha F \nabla_\beta U^\beta \\
&\quad -2U^\alpha U^\beta \nabla_\beta \nabla_\alpha \chi + 2F^\alpha \nabla_\alpha A - A F^\alpha U_\alpha \nabla_\beta U^\beta - B F^\alpha U_\alpha \nabla_\beta U^\beta + \tfrac{1}{3} A F_{\alpha\beta} U^\alpha U^\beta \\
&\quad +B F_{\alpha\beta} U^\alpha U^\beta + U^\alpha U^\beta \nabla_\gamma \nabla^\gamma F_{\alpha\beta} \qquad (3.4)
\end{aligned}
$$

4 Field Equations (G.I. Form)

The results below hold within the geometry of (4.9).

$$(Q_\mu \equiv F_\mu + \nabla_\mu F) \tag{4.1}$$

$$\delta\rho^{GI} = \delta\rho - AQ^\alpha U^\beta \nabla_\alpha U_\beta - BQ^\alpha U^\beta \nabla_\alpha U_\beta + AQ^\alpha U_\alpha \nabla_\beta U^\beta + BQ^\alpha U_\alpha \nabla_\beta U^\beta \tag{4.2}$$

$$\delta p^{GI} = 3\delta p + 2Q^\alpha \nabla_\alpha A - AQ^\alpha U_\alpha \nabla_\beta U^\beta - BQ^\alpha U_\alpha \nabla_\beta U^\beta \tag{4.3}$$

$$V^{GI} = V - U^\alpha Q_\alpha, \qquad \chi, \quad F_{\mu\nu} \quad V \tag{4.4}$$

$$\begin{aligned}\Delta_{\mu\nu} = {} & (\tfrac{1}{3}g_{\mu\nu} + \tfrac{1}{3}U_\mu U_\nu)\delta p^{GI} + U_\mu U_\nu \delta\rho^{GI} + \big((A-B)g_{\mu\nu} + 2AU_\mu U_\nu + 2BU_\mu U_\nu\big)\chi \\ & -2AU^\alpha U_\mu U_\nu \nabla_\alpha V^{GI} - 2BU^\alpha U_\mu U_\nu \nabla_\alpha V^{GI} + 2g_{\mu\nu}\nabla_\alpha\nabla^\alpha\chi + (-AU_\nu - BU_\nu)\nabla_\mu V^{GI} \\ & +(-AU_\mu - BU_\mu)\nabla_\nu V^{GI} - 2\nabla_\nu\nabla_\mu\chi - 2AU^\alpha U_\mu U_\nu V_\alpha - 2BU^\alpha U_\mu U_\nu V_\alpha \\ & +(-AU_\nu - BU_\nu)V_\mu + (-AU_\mu - BU_\mu)V_\nu + 2AU^\alpha U^\beta U_\mu U_\nu F_{\alpha\beta} + 2BU^\alpha U^\beta U_\mu U_\nu F_{\alpha\beta} \\ & +2AU^\alpha U_\nu F_{\mu\alpha} + 2BU^\alpha U_\nu F_{\mu\alpha} + (\tfrac{1}{3}A + B)F_{\mu\nu} + 2AU^\alpha U_\mu F_{\nu\alpha} + 2BU^\alpha U_\mu F_{\nu\alpha} + \nabla_\alpha\nabla^\alpha F_{\mu\nu}\end{aligned} \tag{4.5}$$

$$g^{\mu\nu}\Delta_{\mu\nu} = \delta p^{GI} - \delta\rho^{GI} + (2A-6B)\chi + 6\nabla_\alpha\nabla^\alpha\chi + 2AU^\alpha U^\beta F_{\alpha\beta} + 2BU^\alpha U^\beta F_{\alpha\beta} \tag{4.6}$$

$$\begin{aligned}U^\mu U^\nu \Delta_{\mu\nu} = {} & \delta\rho^{GI} + (A+3B)\chi - 2\nabla_\alpha\nabla^\alpha\chi - 2U^\alpha U^\beta \nabla_\beta\nabla_\alpha\chi - \tfrac{5}{3}AU^\alpha U^\beta F_{\alpha\beta} - BU^\alpha U^\beta F_{\alpha\beta} \\ & +U^\alpha U^\beta \nabla_\gamma\nabla^\gamma F_{\alpha\beta}\end{aligned} \tag{4.7}$$

$$\begin{aligned}(U^\mu U^\nu + g^{\mu\nu})\Delta_{\mu\nu} = {} & \delta p^{GI} + (3A-3B)\chi + 4\nabla_\alpha\nabla^\alpha\chi - 2U^\alpha U^\beta \nabla_\beta\nabla_\alpha\chi + \tfrac{1}{3}AU^\alpha U^\beta F_{\alpha\beta} + BU^\alpha U^\beta F_{\alpha\beta} \\ & +U^\alpha U^\beta \nabla_\gamma\nabla^\gamma F_{\alpha\beta}\end{aligned} \tag{4.8}$$

Evaluating in geometry

$$ds^2 = \Omega^2(\tau)\left(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j\right), \qquad R_{ij} = -2k\tilde{g}_{ij}, \tag{4.9}$$

we find that the quantity $Z_{\mu\nu}(F, F_\mu)$ appearing within $\Delta_{\mu\nu}$ vanishes:

$$\begin{aligned}Z_{\mu\nu} = {} & -\tfrac{1}{6}U_\mu U_\nu \nabla_\alpha F\nabla^\alpha A + \tfrac{1}{2}U_\mu U_\nu \nabla_\alpha F\nabla^\alpha B + \tfrac{1}{2}AU_\nu \nabla_\alpha U_\mu \nabla^\alpha F + \tfrac{1}{2}BU_\nu \nabla_\alpha U_\mu \nabla^\alpha F \\ & +\tfrac{1}{2}AU_\mu \nabla_\alpha U_\nu \nabla^\alpha F + \tfrac{1}{2}BU_\mu \nabla_\alpha U_\nu \nabla^\alpha F - \tfrac{1}{6}Ag_{\mu\nu}U^\alpha \nabla_\alpha F\nabla_\beta U^\beta - \tfrac{1}{6}Bg_{\mu\nu}U^\alpha \nabla_\alpha F\nabla_\beta U^\beta \\ & -\tfrac{2}{3}AU^\alpha U_\mu U_\nu \nabla_\alpha F\nabla_\beta U^\beta - \tfrac{2}{3}BU^\alpha U_\mu U_\nu \nabla_\alpha F\nabla_\beta U^\beta + AU^\alpha U_\mu U_\nu \nabla_\beta U_\alpha \nabla^\beta F \\ & +BU^\alpha U_\mu U_\nu \nabla_\beta U_\alpha \nabla^\beta F + \tfrac{1}{2}U^\alpha U_\nu \nabla_\alpha F\nabla_\mu A + \tfrac{1}{2}U^\alpha U_\nu \nabla_\alpha F\nabla_\mu B - \tfrac{1}{2}AU_\nu \nabla^\alpha F\nabla_\mu U_\alpha \\ & -\tfrac{1}{2}BU_\nu \nabla^\alpha F\nabla_\mu U_\alpha + \tfrac{1}{2}AU^\alpha \nabla_\alpha F\nabla_\mu U_\nu + \tfrac{1}{2}BU^\alpha \nabla_\alpha F\nabla_\mu U_\nu - \tfrac{1}{2}U^\alpha U_\mu \nabla_\alpha A\nabla_\nu F \\ & -\tfrac{1}{2}U^\alpha U_\mu \nabla_\alpha B\nabla_\nu F - \tfrac{1}{2}AU_\mu \nabla_\alpha U^\alpha \nabla_\nu F - \tfrac{1}{2}BU_\mu \nabla_\alpha U^\alpha \nabla_\nu F - \tfrac{1}{6}\nabla_\mu A\nabla_\nu F + \tfrac{1}{2}\nabla_\mu B\nabla_\nu F \\ & -AU_\mu \nabla^\alpha F\nabla_\nu U_\alpha - BU_\mu \nabla^\alpha F\nabla_\nu U_\alpha - \tfrac{1}{4}F_\nu U^\alpha U_\mu \nabla_\alpha A - \tfrac{1}{4}F_\mu U^\alpha U_\nu \nabla_\alpha A - \tfrac{1}{6}F^\alpha U_\mu U_\nu \nabla_\alpha A \\ & -\tfrac{1}{4}F_\nu U^\alpha U_\mu \nabla_\alpha B - \tfrac{1}{4}F_\mu U^\alpha U_\nu \nabla_\alpha B + \tfrac{1}{2}F^\alpha U_\mu U_\nu \nabla_\alpha B - \tfrac{1}{4}AF_\nu U_\mu \nabla_\alpha U^\alpha - \tfrac{1}{4}BF_\nu U_\mu \nabla_\alpha U^\alpha \\ & -\tfrac{1}{4}AF_\mu U_\nu \nabla_\alpha U^\alpha - \tfrac{1}{4}BF_\mu U_\nu \nabla_\alpha U^\alpha + AF^\alpha U^\beta U_\mu U_\nu \nabla_\alpha U_\beta + BF^\alpha U^\beta U_\mu U_\nu \nabla_\alpha U_\beta \\ & +\tfrac{1}{2}AF^\alpha U_\nu \nabla_\alpha U_\mu + \tfrac{1}{2}BF^\alpha U_\nu \nabla_\alpha U_\mu + \tfrac{1}{2}AF^\alpha U_\mu \nabla_\alpha U_\nu + \tfrac{1}{2}BF^\alpha U_\mu \nabla_\alpha U_\nu \\ & -\tfrac{1}{6}AF^\alpha g_{\mu\nu}U_\alpha \nabla_\beta U^\beta - \tfrac{1}{6}BF^\alpha g_{\mu\nu}U_\alpha \nabla_\beta U^\beta - \tfrac{2}{3}AF^\alpha U_\alpha U_\mu U_\nu \nabla_\beta U^\beta - \tfrac{2}{3}BF^\alpha U_\alpha U_\mu U_\nu \nabla_\beta U^\beta \\ & -\tfrac{1}{12}F_\nu \nabla_\mu A + \tfrac{1}{4}F^\alpha U_\alpha U_\nu \nabla_\mu A + \tfrac{1}{4}F_\nu \nabla_\mu B + \tfrac{1}{4}F^\alpha U_\alpha U_\nu \nabla_\mu B - \tfrac{3}{4}AF^\alpha U_\nu \nabla_\mu U_\alpha \\ & -\tfrac{3}{4}BF^\alpha U_\nu \nabla_\mu U_\alpha + \tfrac{1}{4}AF^\alpha U_\alpha \nabla_\mu U_\nu + \tfrac{1}{4}BF^\alpha U_\alpha \nabla_\mu U_\nu - \tfrac{1}{12}F_\mu \nabla_\nu A + \tfrac{1}{4}F^\alpha U_\alpha U_\mu \nabla_\nu A\end{aligned}$$

$$
\begin{aligned}
& + \tfrac{1}{4} F_\mu \nabla_\nu B + \tfrac{1}{4} F^\alpha U_\alpha U_\mu \nabla_\nu B - \tfrac{3}{4} A F^\alpha U_\mu \nabla_\nu U_\alpha - \tfrac{3}{4} B F^\alpha U_\mu \nabla_\nu U_\alpha + \tfrac{1}{4} A F^\alpha U_\alpha \nabla_\nu U_\mu \\
& + \tfrac{1}{4} B F^\alpha U_\alpha \nabla_\nu U_\mu \\
= \quad & 0 \qquad (4.10)
\end{aligned}
$$