

RW SVT4 Decomposition v1

1 Background

$$ds^2 = \Omega^2(\tau) (-d\tau^2 + \tilde{g}_{ij} dx^i dx^j), \quad R_{ij} = -2k\tilde{g}_{ij} \quad (1.1)$$

$$R_{\lambda\mu\nu\kappa} = -\frac{1}{6}g_{\lambda\nu}g_{\mu\kappa}R + \frac{1}{6}g_{\lambda\kappa}g_{\mu\nu}R - \frac{1}{2}g_{\mu\nu}R_{\lambda\kappa} + \frac{1}{2}g_{\mu\kappa}R_{\lambda\nu} + \frac{1}{2}g_{\lambda\nu}R_{\mu\kappa} - \frac{1}{2}g_{\lambda\kappa}R_{\mu\nu} \quad (1.2)$$

$$R_{\mu\nu} = (A+B)U_\mu U_\nu + g_{\mu\nu}B, \quad R = 3B - A \quad (1.3)$$

$$\begin{aligned} A &= -\frac{1}{2}(3p + \rho) \\ &= -3\dot{\Omega}^2\Omega^{-4} + 3\ddot{\Omega}\Omega^{-3} \end{aligned} \quad (1.4)$$

$$\begin{aligned} B &= \frac{1}{2}(p - \rho) \\ &= -\dot{\Omega}^2\Omega^{-4} - \ddot{\Omega}\Omega^{-3} - 2k\Omega^{-2} \end{aligned} \quad (1.5)$$

$$\begin{aligned} \rho &= \frac{1}{2}(-A - 3B) \\ &= 3\dot{\Omega}^2\Omega^{-4} + 3k\Omega^{-2} \end{aligned} \quad (1.6)$$

$$\begin{aligned} p &= \frac{1}{2}(-A + B) \\ &= \dot{\Omega}^2\Omega^{-4} - 2\ddot{\Omega}\Omega^{-3} - k\Omega^{-2} \end{aligned} \quad (1.7)$$

1.1 Identities

A, B, ρ, p are functions only of coordinate x^0 .

$$U^\alpha U^\beta \nabla_\alpha F \nabla_\beta A = -\nabla^\alpha F \nabla_\alpha A \quad (1.8)$$

$$F^\alpha U_\alpha U^\beta \nabla_\beta A = -F^\alpha \nabla_\alpha A \quad (1.9)$$

$$U^\alpha \nabla_\alpha U^\mu = 0 \quad (1.10)$$

$$\nabla_\mu U_\nu = \dot{\Omega}\Omega^{-2}(g_{\mu\nu} + U_\mu U_\nu) \quad (1.11)$$

2 Fluctuations

$$ds^2 = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \quad (2.1)$$

$$g^{\mu\nu}F_{\mu\nu} = 0, \quad \nabla^\mu F_{\mu\nu} = 0, \quad \nabla^\mu F_\mu = 0 \quad (2.2)$$

$$h_{\mu\nu} = -2g_{\mu\nu}\chi + 2\nabla_\mu \nabla_\nu F + \nabla_\mu F_\nu + \nabla_\nu F_\mu + 2F_{\mu\nu} \quad (2.3)$$

$$g^{\alpha\beta}h_{\alpha\beta} = -8\chi + 2\nabla_\alpha \nabla^\alpha F \quad (2.4)$$

$$\begin{aligned} \nabla^\mu h_{\mu\nu} &= 2U^\alpha U_\nu(p + \rho)\nabla_\alpha F + (-p + \rho)\nabla_\nu F - 2\nabla_\nu \chi + 2\nabla_\nu \nabla_\alpha \nabla^\alpha F + \frac{1}{2}F_\nu(-p + \rho) \\ &\quad + F^\alpha U_\alpha U_\nu(p + \rho) + \nabla_\alpha \nabla^\alpha F_\nu \end{aligned} \quad (2.5)$$

$$\begin{aligned} \nabla^\mu \nabla^\nu h_{\mu\nu} &= (-p + \rho)\nabla_\alpha \nabla^\alpha F - 2\nabla_\alpha \nabla^\alpha \chi + (-3\nabla_\alpha p - \nabla_\alpha \rho)\nabla^\alpha F + 2U^\alpha(p + \rho)\nabla_\alpha F \nabla_\beta U^\beta \\ &\quad + 2U^\alpha U^\beta(p + \rho)\nabla_\beta \nabla_\alpha F + 2\nabla_\beta \nabla^\beta \nabla_\alpha \nabla^\alpha F + 2U^\alpha U^\beta(p + \rho)\nabla_\beta F_\alpha \\ &\quad + F^\alpha(-3\nabla_\alpha p - \nabla_\alpha \rho + 2U_\alpha(p + \rho)\nabla_\beta U^\beta) \end{aligned} \quad (2.6)$$

$$U^\mu U^\nu h_{\mu\nu} = 2\chi + 2U^\alpha U^\beta \nabla_\beta \nabla_\alpha F + 2U^\alpha U^\beta \nabla_\beta F_\alpha + 2F_{\alpha\beta} U^\alpha U^\beta \quad (2.7)$$

$$(U^\mu U^\nu + g^{\mu\nu})h_{\mu\nu} = -6\chi + 2\nabla_\alpha \nabla^\alpha F + 2U^\alpha U^\beta \nabla_\beta \nabla_\alpha F + 2U^\alpha U^\beta \nabla_\beta F_\alpha + 2F_{\alpha\beta} U^\alpha U^\beta \quad (2.8)$$

$$(U^\mu \nabla^\nu + U^\nu \nabla^\mu)h_{\mu\nu} = -4U^\alpha \nabla_\alpha \chi + 4U^\alpha \nabla_\beta \nabla^\beta \nabla_\alpha F - F^\alpha U_\alpha(3p + \rho) + 2U^\alpha \nabla_\beta \nabla^\beta F_\alpha \quad (2.9)$$

$$\begin{aligned} \Delta'_{\mu\nu} &= 2(p + \rho)U_\mu U_\nu U^\alpha \nabla_\alpha V^{GI} + (\frac{2}{3}g_{\mu\nu} + \frac{2}{3}U_\mu U_\nu)\nabla_\alpha \nabla^\alpha \chi + (\frac{2}{3}g_{\mu\nu}U^\alpha U^\beta + \frac{8}{3}U_\mu U_\nu U^\alpha U^\beta)\nabla_\beta \nabla_\alpha \chi \\ &\quad + (p + \rho)U_\nu \nabla_\mu V^{GI} + (p + \rho)U_\mu \nabla_\nu V^{GI} - 2\nabla_\nu \nabla_\mu \chi + 2(p + \rho)U_\mu U_\nu U^\alpha V_\alpha + (p + \rho)U_\nu V_\mu \\ &\quad + (p + \rho)U_\mu V_\nu + (\frac{2}{9}\rho g_{\mu\nu}U^\alpha U^\beta - \frac{4}{9}(9p + 7\rho)U_\mu U_\nu U^\alpha U^\beta)F_{\alpha\beta} - 2(p + \rho)U_\nu U^\alpha F_{\mu\alpha} - \frac{2}{3}\rho F_{\mu\nu} \\ &\quad - 2(p + \rho)U_\mu U^\alpha F_{\nu\alpha} + \nabla_\alpha \nabla^\alpha F_{\mu\nu} + (-\frac{1}{3}g_{\mu\nu}U^\alpha U^\beta - \frac{4}{3}U_\mu U_\nu U^\alpha U^\beta)\nabla_\gamma \nabla^\gamma F_{\alpha\beta} \end{aligned} \quad (2.10)$$