

Coordinate Transformation RW SVT3 v2

1 RW $\Omega(\tau)$

$$ds^2 = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = \Omega^2(\tau)(\tilde{g}_{\mu\nu} + f_{\mu\nu})dx^\mu dx^\nu \quad (1.1)$$

$$\tilde{g}_{\mu\nu} = \text{diag}\left(-1, \frac{1}{1 - kr^2}, r^2, r^2 \sin^2 \theta\right) \quad \tilde{\Gamma}_{\alpha\beta}^\lambda = \delta_i^\lambda \delta_\alpha^j \delta_\beta^k \tilde{\Gamma}_{jk}^i \quad (1.2)$$

1.1 W_1

As evaluated in the background geometry of $\Omega^2(x)\tilde{g}_{\mu\nu}dx^\mu dx^\nu$.

[illegible]

$$\begin{aligned}
g^{\mu\nu}W_{\mu\nu}^{(1)} = & 6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R} - 12\tilde{R}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R} + 12\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega \\
& - 108\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega + 216\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega - 144\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega \\
& + 36\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 144\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega + 6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R} - 12\tilde{R}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \\
& - 12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R} + 12\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 108\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega \\
& + 216\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega - 144\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega + 36\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \\
& - 144\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega + 6\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\tilde{R} - 12\tilde{R}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 12\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\tilde{R} \\
& + 12\tilde{R}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 108\Omega^{-4}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega + 216\Omega^{-5}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\Omega
\end{aligned}$$

$$-144\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\Omega + 36\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}^\beta\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 144\tilde{R}_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega \quad (1.4)$$

$$= 36\Omega^{-7}\left[6\ddot{\Omega}\dot{\Omega}^2 - 3\ddot{\Omega}^2\Omega - 4\ddot{\Omega}\dot{\Omega}\Omega + 2k\dot{\Omega}^2\Omega + \ddot{\Omega}\Omega^2 - 2k\ddot{\Omega}\Omega^2\right] \quad (1.5)$$

1.2 $g^{\mu\nu}\delta W_{\mu\nu}^{(1)}$

$$\begin{aligned} g^{\mu\nu}\delta W_{\mu\nu}^{(1)} = & 12\Omega^{-4}\left[-3\ddot{\psi} + \psi(3k^2 - 48\ddot{\Omega}\dot{\Omega}^2\Omega^{-3} + 15\ddot{\Omega}^2\Omega^{-2} + 30\ddot{\Omega}\dot{\Omega}\Omega^{-2} - 6k\dot{\Omega}^2\Omega^{-2} - 6\ddot{\Omega}\Omega^{-1}) \right. \\ & + \ddot{\phi}(12\dot{\Omega}^2\Omega^{-2} - 12\ddot{\Omega}\Omega^{-1}) + \ddot{\psi}(6k + 30\dot{\Omega}^2\Omega^{-2} - 12\ddot{\Omega}\Omega^{-1}) \\ & + \phi(3k^2 - 84\ddot{\Omega}\dot{\Omega}^2\Omega^{-3} + 33\ddot{\Omega}^2\Omega^{-2} + 54\ddot{\Omega}\dot{\Omega}\Omega^{-2} - 18k\dot{\Omega}^2\Omega^{-2} - 12\ddot{\Omega}\Omega^{-1} + 12k\ddot{\Omega}\Omega^{-1}) \\ & + \dot{\phi}(-18\dot{\Omega}^3\Omega^{-3} + 54\ddot{\Omega}\dot{\Omega}\Omega^{-2} - 18\ddot{\Omega}\Omega^{-1} + 6k\dot{\Omega}\Omega^{-1}) \\ & + \dot{\psi}(-54\dot{\Omega}^3\Omega^{-3} + 90\ddot{\Omega}\dot{\Omega}\Omega^{-2} - 18\ddot{\Omega}\Omega^{-1} + 6k\dot{\Omega}\Omega^{-1}) - 3\ddot{\phi}\dot{\Omega}\Omega^{-1} - 3\ddot{\psi}\dot{\Omega}\Omega^{-1} \\ & + (-18\dot{\Omega}^3\Omega^{-3} + 30\ddot{\Omega}\dot{\Omega}\Omega^{-2} - 6\ddot{\Omega}\Omega^{-1} + 6k\dot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^bB + (10\dot{\Omega}^2\Omega^{-2} - 4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\dot{B} \\ & - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\ddot{B} - \tilde{\nabla}_b\tilde{\nabla}^b\ddot{B} + \tilde{\nabla}_b\tilde{\nabla}^b\ddot{E} + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\ddot{E} + (-10\dot{\Omega}^2\Omega^{-2} + 4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\dot{E} \\ & - \tilde{\nabla}_b\tilde{\nabla}^b\ddot{\phi} + 5\tilde{\nabla}_b\tilde{\nabla}^b\ddot{\psi} + (18\dot{\Omega}^3\Omega^{-3} - 30\ddot{\Omega}\dot{\Omega}\Omega^{-2} + 6\ddot{\Omega}\Omega^{-1} - 6k\dot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\dot{E} \\ & + 5\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\dot{\phi} + 5\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\dot{\psi} \\ & + (-k^2 + 16\ddot{\Omega}\dot{\Omega}^2\Omega^{-3} - 5\ddot{\Omega}^2\Omega^{-2} - 10\ddot{\Omega}\dot{\Omega}\Omega^{-2} + 6k\dot{\Omega}^2\Omega^{-2} + 2\ddot{\Omega}\Omega^{-1} - 4k\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^bE \\ & + (-2\dot{\Omega}^2\Omega^{-2} + 8\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\phi + (-6k + 4\dot{\Omega}^2\Omega^{-2} - 4\ddot{\Omega}\Omega^{-1})\tilde{\nabla}_b\tilde{\nabla}^b\psi + 3\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^aB \\ & \left. + \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} - \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - 3\dot{\Omega}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} + \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi - 2\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi\right] \quad (1.6) \end{aligned}$$