

SVT4 dS₄ Conformal(2) Einstein

1 Background and Fluctuations

$$G_{\mu\nu}^{(0)} = 3H^2 g_{\mu\nu} \quad (1.1)$$

$$R_{\lambda\mu\nu\kappa}^{(0)} = H^2(g_{\mu\nu}g_{\lambda\kappa} - g_{\lambda\nu}g_{\mu\kappa}), \quad R_{\mu\kappa}^{(0)} = -3H^2 g_{\mu\kappa}, \quad R^{(0)} = -12H^2, \quad (1.2)$$

$$ds^2 = \Omega^2(\tau)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu, \quad \Omega(\tau) = \frac{1}{1-H\tau} = e^{Ht}, \quad \tau(t=0) = 0 \quad (1.3)$$

$$\tilde{g}_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \quad \text{or} \quad \text{diag}(-1, 1, r^2, r^2 \sin^2 \theta) \quad (1.4)$$

$$f_{\mu\nu} = -2\tilde{g}_{\mu\nu}\chi + 2\tilde{\nabla}_\mu \tilde{\nabla}_\nu F + \tilde{\nabla}_\mu E_\nu + \tilde{\nabla}_\nu E_\mu + 2E_{\mu\nu} \quad (1.5)$$

$$\tilde{\nabla}_\mu \Omega = \dot{\Omega} \delta_\mu^0, \quad \Omega = \frac{1}{1-H\tau}, \quad \dot{\Omega} = \frac{H}{(1-H\tau)^2}, \quad \ddot{\Omega} = \frac{2H^2}{(1-H\tau)^3} \quad (1.6)$$

$$\delta G_{00} = 6H\dot{\chi}(1-H\tau)^{-1} + 2H(1-H\tau)^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{F} - 2\tilde{\nabla}_a \tilde{\nabla}^a \chi + 2H(1-H\tau)^{-1}\tilde{\nabla}_a \tilde{\nabla}^a E_0 - \ddot{E}_{00} + 2H\dot{E}_{00}(1-H\tau)^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{00} \quad (1.7)$$

$$\begin{aligned} \delta G_{0i} = & 2H(1-H\tau)^{-1}\tilde{\nabla}_i \ddot{F} + 6H^2(1-H\tau)^{-2}\tilde{\nabla}_i \dot{F} - 2\tilde{\nabla}_i \dot{\chi} \\ & + 2H(1-H\tau)^{-1}\tilde{\nabla}_i \chi + 3H^2 \dot{E}_i(1-H\tau)^{-2} + 2H(1-H\tau)^{-1}\tilde{\nabla}_i \dot{E}_0 \\ & + 3H^2(1-H\tau)^{-2}\tilde{\nabla}_i E_0 - \ddot{E}_{0i} + 6H^2 E_{0i}(1-H\tau)^{-2} - 4H\dot{E}_{0i}(1-H\tau)^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{0i} \\ & + 2H(1-H\tau)^{-1}\tilde{\nabla}_i E_{00} \end{aligned} \quad (1.8)$$

$$\begin{aligned} \delta G_{ij} = & -2\dot{\chi}\tilde{g}_{ij} + 6H^2 \ddot{F}\tilde{g}_{ij}(1-H\tau)^{-2} + 2H\ddot{F}\tilde{g}_{ij}(1-H\tau)^{-1} - 2H\dot{\chi}\tilde{g}_{ij}(1-H\tau)^{-1} \\ & - 2H\tilde{g}_{ij}(1-H\tau)^{-1}\tilde{\nabla}_a \tilde{\nabla}^a \dot{F} + 2\tilde{g}_{ij}\tilde{\nabla}_a \tilde{\nabla}^a \chi + 2H(1-H\tau)^{-1}\tilde{\nabla}_j \tilde{\nabla}_i \dot{F} \\ & + 6H^2(1-H\tau)^{-2}\tilde{\nabla}_j \tilde{\nabla}_i F - 2\tilde{\nabla}_j \tilde{\nabla}_i \chi + 6H^2 \dot{E}_{00}\tilde{g}_{ij}(1-H\tau)^{-2} + 2H\ddot{E}_{00}\tilde{g}_{ij}(1-H\tau)^{-1} \\ & - 2H\tilde{g}_{ij}(1-H\tau)^{-1}\tilde{\nabla}_a \tilde{\nabla}^a E_0 + 3H^2(1-H\tau)^{-2}\tilde{\nabla}_i E_j + 3H^2(1-H\tau)^{-2}\tilde{\nabla}_j E_i \\ & + 2H(1-H\tau)^{-1}\tilde{\nabla}_j \tilde{\nabla}_i E_0 - \ddot{E}_{ij} + 6H^2 E_{ij}(1-H\tau)^{-2} + 6H^2 E_{00}\tilde{g}_{ij}(1-H\tau)^{-2} \\ & - 2H\dot{E}_{ij}(1-H\tau)^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{ij} - 2H(1-H\tau)^{-1}\tilde{\nabla}_i E_{0j} - 2H(1-H\tau)^{-1}\tilde{\nabla}_j E_{0i} \end{aligned} \quad (1.9)$$

$$\begin{aligned} g^{\mu\nu} \delta G_{\mu\nu} = & 6H\ddot{F} + 18H^2 \ddot{F} - 6\ddot{\chi} - 12H\dot{\chi} - 6H^2 \ddot{F}\tau + 12H\ddot{\chi}\tau + 12H^2 \dot{\chi}\tau - 6H^2 \dot{\chi}\tau^2 - 6H\tilde{\nabla}_a \tilde{\nabla}^a \dot{F} \\ & + 6H^2 \tau \tilde{\nabla}_a \tilde{\nabla}^a \dot{F} + 6H^2 \tilde{\nabla}_a \tilde{\nabla}^a F + 6\tilde{\nabla}_a \tilde{\nabla}^a \chi - 12H\tau \tilde{\nabla}_a \tilde{\nabla}^a \chi + 6H^2 \tau^2 \tilde{\nabla}_a \tilde{\nabla}^a \chi + 6H\ddot{E}_0 \\ & + 24H^2 \dot{E}_0 - 6H^2 \ddot{E}_0 \tau - 6H\tilde{\nabla}_a \tilde{\nabla}^a E_0 + 6H^2 \tau \tilde{\nabla}_a \tilde{\nabla}^a E_0 + 24H^2 E_{00} \end{aligned} \quad (1.10)$$

$$-\kappa_4^2 \delta T_{00} = 6H^2 \ddot{F}(1-H\tau)^{-2} + 6H^2(1-H\tau)^{-2}\chi + 6H^2 \dot{E}_0(1-H\tau)^{-2} + 6H^2 E_{00}(1-H\tau)^{-2} \quad (1.11)$$

$$-\kappa_4^2 \delta T_{0i} = 6H^2(1-H\tau)^{-2} \tilde{\nabla}_i \dot{F} + 3H^2 \dot{E}_i (1-H\tau)^{-2} + 3H^2(1-H\tau)^{-2} \tilde{\nabla}_i E_0 + 6H^2 E_{0i} (1-H\tau)^{-2} \quad (1.12)$$

$$\begin{aligned} -\kappa_4^2 \delta T_{ij} = & -6H^2 \tilde{g}_{ij} (1-H\tau)^{-2} \chi + 6H^2 (1-H\tau)^{-2} \tilde{\nabla}_j \tilde{\nabla}_i F + 3H^2 (1-H\tau)^{-2} \tilde{\nabla}_i E_j \\ & + 3H^2 (1-H\tau)^{-2} \tilde{\nabla}_j E_i + 6H^2 E_{ij} (1-H\tau)^{-2} \end{aligned} \quad (1.13)$$

$$-\kappa_4^2 g^{\mu\nu} \delta T_{\mu\nu} = -6H^2 \ddot{F} - 24H^2 \chi + 6H^2 \tilde{\nabla}_a \tilde{\nabla}^a F \quad (1.14)$$

2 Field Equations

$$\Delta_{\mu\nu} \equiv \delta G_{\mu\nu} + \kappa_4^2 \delta T_{\mu\nu} = 0 \quad (2.1)$$

$$\begin{aligned} \Delta_{00} = & -6H^2 \ddot{F} (1-H\tau)^{-2} + 6H \dot{\chi} (1-H\tau)^{-1} - 6H^2 (1-H\tau)^{-2} \chi + 2H (1-H\tau)^{-1} \tilde{\nabla}_a \tilde{\nabla}^a \dot{F} \\ & - 2\tilde{\nabla}_a \tilde{\nabla}^a \chi - 6H^2 \dot{E}_0 (1-H\tau)^{-2} + 2H (1-H\tau)^{-1} \tilde{\nabla}_a \tilde{\nabla}^a E_0 - \ddot{E}_{00} - 6H^2 E_{00} (1-H\tau)^{-2} \\ & + 2H \dot{E}_{00} (1-H\tau)^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{00} \end{aligned} \quad (2.2)$$

$$\begin{aligned} \Delta_{0i} = & 2H (1-H\tau)^{-1} \tilde{\nabla}_i \ddot{F} - 2\tilde{\nabla}_i \dot{\chi} + 2H (1-H\tau)^{-1} \tilde{\nabla}_i \chi + 2H (1-H\tau)^{-1} \tilde{\nabla}_i \dot{E}_0 - \ddot{E}_{0i} \\ & - 4H \dot{E}_{0i} (1-H\tau)^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{0i} + 2H (1-H\tau)^{-1} \tilde{\nabla}_i E_{00} \end{aligned} \quad (2.3)$$

$$\begin{aligned} \Delta_{ij} = & -2\ddot{\chi} \tilde{g}_{ij} + 6H^2 \ddot{F} \tilde{g}_{ij} (1-H\tau)^{-2} + 2H \ddot{F} \tilde{g}_{ij} (1-H\tau)^{-1} - 2H \dot{\chi} \tilde{g}_{ij} (1-H\tau)^{-1} \\ & + 6H^2 \tilde{g}_{ij} (1-H\tau)^{-2} \chi - 2H \tilde{g}_{ij} (1-H\tau)^{-1} \tilde{\nabla}_a \tilde{\nabla}^a \dot{F} + 2\tilde{g}_{ij} \tilde{\nabla}_a \tilde{\nabla}^a \chi + 2H (1-H\tau)^{-1} \tilde{\nabla}_j \tilde{\nabla}_i \dot{F} \\ & - 2\tilde{\nabla}_j \tilde{\nabla}_i \chi + 6H^2 \dot{E}_0 \tilde{g}_{ij} (1-H\tau)^{-2} + 2H \ddot{E}_0 \tilde{g}_{ij} (1-H\tau)^{-1} - 2H \tilde{g}_{ij} (1-H\tau)^{-1} \tilde{\nabla}_a \tilde{\nabla}^a E_0 \\ & + 2H (1-H\tau)^{-1} \tilde{\nabla}_j \tilde{\nabla}_i E_0 - \ddot{E}_{ij} + 6H^2 E_{00} \tilde{g}_{ij} (1-H\tau)^{-2} - 2H \dot{E}_{ij} (1-H\tau)^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{ij} \\ & - 2H (1-H\tau)^{-1} \tilde{\nabla}_i E_{0j} - 2H (1-H\tau)^{-1} \tilde{\nabla}_j E_{0i} \end{aligned} \quad (2.4)$$

$$\begin{aligned} g^{\mu\nu} \Delta_{\mu\nu} = & 6H \ddot{F} + 24H^2 \ddot{F} - 6\ddot{\chi} - 12H \dot{\chi} - 6H^2 \ddot{F} \tau + 12H \dot{\chi} \tau + 12H^2 \dot{\chi} \tau - 6H^2 \ddot{\chi} \tau^2 + 24H^2 \chi \\ & - 6H \tilde{\nabla}_a \tilde{\nabla}^a \dot{F} + 6H^2 \tau \tilde{\nabla}_a \tilde{\nabla}^a \dot{F} + 6\tilde{\nabla}_a \tilde{\nabla}^a \chi - 12H \tau \tilde{\nabla}_a \tilde{\nabla}^a \chi + 6H^2 \tau^2 \tilde{\nabla}_a \tilde{\nabla}^a \chi + 6H \ddot{E}_0 \\ & + 24H^2 \dot{E}_0 - 6H^2 \ddot{E}_0 \tau - 6H \tilde{\nabla}_a \tilde{\nabla}^a E_0 + 6H^2 \tau \tilde{\nabla}_a \tilde{\nabla}^a E_0 + 24H^2 E_{00} \end{aligned} \quad (2.5)$$

3 Field Equations (G.I. Form)

$$\alpha = \dot{\Omega} \Omega^{-1} \dot{F} + \dot{\Omega} \Omega^{-1} E_0 - \chi = H \dot{F} (1-H\tau)^{-1} + H E_0 (1-H\tau)^{-1} - \chi \quad (3.1)$$

$$\begin{aligned} \Delta_{00} = & 6H^2 \alpha (1-H\tau)^{-2} - 6H \dot{\alpha} (1-H\tau)^{-1} + 2\tilde{\nabla}_a \tilde{\nabla}^a \alpha - \ddot{E}_{00} - 6H^2 E_{00} (1-H\tau)^{-2} \\ & + 2H \dot{E}_{00} (1-H\tau)^{-1} + \tilde{\nabla}_a \tilde{\nabla}^a E_{00} \end{aligned} \quad (3.2)$$

$$\Delta_{0i} = 2\tilde{\nabla}_i\dot{\alpha} - 2H(1-H\tau)^{-1}\tilde{\nabla}_i\alpha - \ddot{E}_{0i} - 4H\dot{E}_{0i}(1-H\tau)^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a E_{0i} + 2H(1-H\tau)^{-1}\tilde{\nabla}_i E_{00} \quad (3.3)$$

$$\begin{aligned} \Delta_{ij} = & 2\ddot{\alpha}\tilde{g}_{ij} - 6H^2\tilde{g}_{ij}\alpha(1-H\tau)^{-2} + 2H\dot{\alpha}\tilde{g}_{ij}(1-H\tau)^{-1} - 2\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\alpha + 2\tilde{\nabla}_j\tilde{\nabla}_i\alpha - \ddot{E}_{ij} \\ & + 6H^2E_{00}\tilde{g}_{ij}(1-H\tau)^{-2} - 2H\dot{E}_{ij}(1-H\tau)^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij} - 2H(1-H\tau)^{-1}\tilde{\nabla}_i E_{0j} \\ & - 2H(1-H\tau)^{-1}\tilde{\nabla}_j E_{0i} \end{aligned} \quad (3.4)$$

$$\begin{aligned} g^{\mu\nu}\Delta_{\mu\nu} = & 6\ddot{\alpha} + 12H\dot{\alpha} - 24H^2\alpha - 12H\ddot{\alpha}\tau - 12H^2\dot{\alpha}\tau + 6H^2\ddot{\alpha}\tau^2 - 6\tilde{\nabla}_a\tilde{\nabla}^a\alpha + 12H\tau\tilde{\nabla}_a\tilde{\nabla}^a\alpha \\ & - 6H^2\tau^2\tilde{\nabla}_a\tilde{\nabla}^a\alpha + 24H^2E_{00} \end{aligned} \quad (3.5)$$