SVT3 RW Radiation $\Omega(x)$ $k<0$ Cartesian v2

1 Background

1.1 Comoving $a(t)$

First, determine the form of $a(t)$ for $\rho = 3p$ radiation in comoving coordinates

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right) = -dt^2 + a(t)^2\tilde{g}_{ij}dx^i dx^j \tag{1.1}$$

$$T_{\mu\nu} = p(4U_\mu U_\nu + g_{\mu\nu}), \qquad U_\mu = -\delta^0_\mu \tag{1.2}$$

$$T_{00} = 3p, \qquad T_{ij} = a^2(t)p\tilde{g}_{ij} \tag{1.3}$$

$$G_{00} = -3ka^{-2} - 3\dot{a}^2a^{-2}, \qquad G_{ij} = \tilde{g}_{ij}(k + \dot{a}^2 + 2a\ddot{a}) \tag{1.4}$$

$$\Delta_{\mu\nu} = G_{\mu\nu} + T_{\mu\nu} = 0 \tag{1.5}$$

$$\Delta_{00} = 3(p - ka^{-2} - \dot{a}^2a^{-2}), \qquad \Delta_{ij} = \tilde{g}_{ij}(a^2p + k + \dot{a}^2 + 2a\ddot{a}) \tag{1.6}$$

$$\rightarrow \qquad \boxed{p = ka^{-2} + \dot{a}^2a^{-2}} \qquad \boxed{0 = k + \dot{a}^2 + a\ddot{a}} \tag{1.7}$$

With $k = -1/L^2$, we will follow APM (B1) and take

$$a^2(t) = \frac{1}{L^2}(d^2 + t^2) \tag{1.8}$$

$$p = -\frac{d^2}{(d^2+t^2)^2} = -\frac{d^2}{L^4a^4} \tag{1.9}$$

1.2 Conformal T, R Coordinates

Given $a(t)$ in the form (1.8), we may transform the metric from

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1+r^2/L^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right), \tag{1.10}$$

to the conformal flat form

$$ds^2 = \Omega^2(X)(-dT^2 + dR^2 + R^2 d\theta^2 + R^2\sin^2\theta d\phi^2). \tag{1.11}$$

Making definitions $u \equiv t/d$, $v \equiv r/L$, the coordinate transformations go as

$$T = \left(u + \sqrt{u^2+1}\right)\left(1+v^2\right)^{1/2}, \qquad R = \left(u + \sqrt{u^2+1}\right)v, \qquad X^2 = T^2 - R^2 = \left(u + \sqrt{u^2+1}\right)^2$$

$$u = \frac{X^2-1}{2X}, \qquad v = \frac{R}{X} \tag{1.12}$$

$$a^2(X) = \frac{d^2}{L^2}\frac{(1+X^2)^2}{4X^2} \tag{1.13}$$

$$\Omega^2(X) = L^2\frac{a^2(X)}{X^2} = \frac{d^2}{4}\frac{(1+X^2)^2}{X^4} = d^2\frac{[1+u^2+u(u^2+1)^{1/2}]^2}{[u+(u^2+1)^{1/2}]^4} \tag{1.14}$$

Transformation matrix components:

$$x'^{\mu} = (T,R,\theta,\phi), \qquad x^{\mu} = (t,r,\theta,\phi) \tag{1.15}$$

$$\frac{\partial t}{\partial T} = d\frac{\partial u}{\partial X}\frac{\partial X}{\partial T} = \Omega\frac{\partial X}{\partial T} = \Omega\frac{T}{X} \tag{1.16}$$

$$\frac{\partial r}{\partial T} = L\frac{\partial v}{\partial X}\frac{\partial X}{\partial T} = -L\frac{R}{X^2}\frac{\partial X}{\partial T} = -L\frac{TR}{X^3} \tag{1.17}$$

$$\frac{\partial t}{\partial R} = d\frac{\partial u}{\partial X}\frac{\partial X}{\partial R} = \Omega\frac{\partial X}{\partial R} = -\Omega\frac{R}{X} \tag{1.18}$$

$$\frac{\partial r}{\partial R} = L\frac{\partial v}{\partial X}\frac{\partial X}{\partial R} = -L\frac{R}{X^2}\frac{\partial X}{\partial R} = L\frac{R^2}{X^3} \tag{1.19}$$

1.3 $T'_{\mu\nu}(T,R)$

$$T'_{\mu\nu} = p(4U'_\mu U'_\nu + g'_{\mu\nu}) \tag{1.20}$$

$$p = -d^2\Omega^{-4}X^{-4} \tag{1.21}$$

$$\begin{aligned} U'_\mu &= \frac{\partial x^\alpha}{\partial x'^\mu}U_\alpha = -\frac{\partial t}{\partial x'^\mu} \\ &= \Omega\left(-\frac{T}{X},\frac{R}{X},0,0\right) \end{aligned} \tag{1.22}$$

$$g'_{\mu\nu} = \Omega^2\mathrm{diag}\left(-1,1,R^2,R^2\sin^2\theta\right) \tag{1.23}$$

1.4 $T'_{\mu\nu}(T,x,y,z)$

$$T'_{\mu\nu} = p(4U'_\mu U'_\nu + g'_{\mu\nu}) \tag{1.24}$$

$$p = -d^2\Omega^{-4}X^{-4} \tag{1.25}$$

$$\begin{aligned} U'_i &= \frac{\partial R}{\partial x'^i}U_R = \frac{x_i}{R}U_R \\ U'_\mu &= \Omega\left(-\frac{T}{X},\frac{x}{X},\frac{y}{X},\frac{z}{X}\right) \end{aligned} \tag{1.26}$$

$$U'^{\mu} = \Omega^{-1}\left(\frac{T}{X},\frac{x}{X},\frac{y}{X},\frac{z}{X}\right) \tag{1.27}$$

$$g'_{\mu\nu} = \Omega^2 \mathrm{diag}\,(-1,1,1,1) \tag{1.28}$$

2 Fluctuations

$$ds^2 = \Omega^2(x)(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \tag{2.1}$$

$$\tilde{g}_{ij} = \delta_{ij} \tag{2.2}$$

$$f_{00} = -2\phi, \qquad f_{0i} = \tilde{\nabla}_i B + B_i, \qquad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{2.3}$$

2.1 $\delta G_{\mu\nu}$

$$\begin{aligned}
\delta G_{00} = {} & 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi + 4\phi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\
& + 4\psi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 4\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B - 2\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi \\
& - 2\phi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - 2\psi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\
& + 2\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega - 4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E \\
& + 4B^a\Omega^{-1}\tilde{\nabla}_a\dot{\Omega} - 2B^a\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a + 2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^b E^a \\
& - 4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b E^a - 4E^{ab}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega + 2E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
\delta G_{0i} = {} & -\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_i B + \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_i B - 2\tilde{\nabla}_i\dot{\psi} \\
& - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + 2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i\tilde{\nabla}_a\dot{E} - B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} \\
& + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i - 2B_i\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B_i - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{E}_i \\
& + B_i\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i B^a - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i\dot{E}^a - 2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a\Omega
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
\delta G_{ij} = {} & -2\ddot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\
& - 4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
& - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^a B + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B \\
& - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\
& - 2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E + 2\Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\psi \\
& + 2\Omega^{-1}\tilde{\nabla}_i\psi\tilde{\nabla}_j\Omega + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} \\
& - 2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i E - 4\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E + 2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E \\
& + \tilde{\nabla}_j\tilde{\nabla}_i\phi - \tilde{\nabla}_j\tilde{\nabla}_i\psi - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a E \\
& - 4B^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega} + 2B^a\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega - 2\dot{B}^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a \\
& - 2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^b E^a + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b E^a + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i B_j + \tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j \\
& - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i E_j + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i E_j - 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_j \\
& + \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_j + \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j B_i + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i - \dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i - \dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j E_i \\
& + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_i + \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E_a \\
& - \ddot{E}_{ij} - 2\dot{\Omega}^2 E_{ij}\Omega^{-2} - 2\dot{E}_{ij}\dot{\Omega}\Omega^{-1} + 4\ddot{\Omega}E_{ij}\Omega^{-1} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij} - 4E_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\
& + 2\Omega^{-1}\tilde{\nabla}_a E_{ij}\tilde{\nabla}^a\Omega + 2E_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega + 4E^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega \\
& - 2E_{ab}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_{ja} - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_{ia}
\end{aligned} \tag{2.6}$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} &= \Omega^{-2}(-\delta G_{00} + \tilde{g}^{ab}\delta G_{ab}) \\
&= 6\dot{\Omega}^2\phi\Omega^{-4} + 6\dot{\Omega}^2\psi\Omega^{-4} - 6\dot{\phi}\dot{\Omega}\Omega^{-3} - 18\dot{\psi}\dot{\Omega}\Omega^{-3} - 12\ddot{\Omega}\phi\Omega^{-3} - 12\ddot{\Omega}\psi\Omega^{-3} - 6\ddot{\psi}\Omega^{-2} \\
&\quad -6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
&\quad -2\dot{\Omega}^2\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^a E + 4\ddot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a E - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 4\phi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\
&\quad -4\psi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega - 16\Omega^{-3}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^a B + 8\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B - 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} \\
&\quad -6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi + 2\phi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega + 2\psi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega \\
&\quad +2\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E - 4\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a E\tilde{\nabla}_b\tilde{\nabla}^b\Omega + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\
&\quad -8\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega + 16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E. - 16B^a\Omega^{-3}\tilde{\nabla}_a\dot{\Omega} \\
&\quad +8B^a\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega - 6\dot{B}^a\Omega^{-3}\tilde{\nabla}_a\Omega + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a - 8\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^b E^a \\
&\quad +16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b E^a + 16E^{ab}\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega - 8E_{ab}\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{2.7}
$$

2.2 $\delta T_{\mu\nu}$

From $\delta(g^{\mu\nu}U_\mu U_\nu) = 0$ we find

$$
\begin{aligned}
U^\mu\delta U_\mu &= \frac{1}{2}\left(h_{\mu\nu}U^\mu U^\nu\right) \\
\rightarrow \Omega^{-1}\left(TX^{-1}\delta U_0 + x^i X^{-1}\delta U_i\right) &= \frac{1}{2}\left(T^2X^{-2}f_{00} + 2Tx^iX^{-2}f_{0i} + x^ix^jX^{-2}f_{ij}\right) \\
\rightarrow T\delta U_0 + x^i\delta U_i &= \Omega X^{-1}\left[-\phi T^2 + Tx^i(\tilde{\nabla}_i B + B_i) + x^ix^j(-\psi g_{ij} + \tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + E_{ij})\right]
\end{aligned} \tag{2.8}
$$

$$
p = -d^2\Omega^{-4}X^{-4}, \qquad U_\mu = \Omega\left(-\frac{T}{X}, \frac{x}{X}, \frac{y}{X}, \frac{z}{X}\right), \quad U^\mu = \Omega^{-1}\left(\frac{T}{X}, \frac{x}{X}, \frac{y}{X}, \frac{z}{X}\right) \tag{2.9}
$$

$$
\delta T_{\mu\nu} = \delta p(4U_\mu U_\nu + \Omega^2\tilde{g}_{\mu\nu}) + p\left(4\delta U_\mu U_\nu + 4U_\mu\delta U_\nu + \Omega^2 f_{\mu\nu}\right) \tag{2.10}
$$

$$
\delta T_{00} = \Omega^2\delta p(4T^2X^{-2} - 1) - 8\Omega TX^{-1}p\delta U_0 - 2\Omega^2 p\phi \tag{2.11}
$$

$$
\delta T_{0i} = -4\Omega TX^{-1}U_i\delta p + 4pU_i\delta U_0 - 4\Omega TX^{-1}p\delta U_i + \Omega^2 p(\tilde{\nabla}_i B + B_i) \tag{2.12}
$$

$$
\begin{aligned}
\delta T_{ij} &= 4U_iU_j\delta p + \Omega^2\tilde{g}_{ij}\delta p + 4pU_i\delta U_j + 4pU_j\delta U_i \\
&\quad +\Omega^2 p(-2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij})
\end{aligned} \tag{2.13}
$$

$$
g^{\mu\nu}\delta T_{\mu\nu} = 8p\Omega^{-1}\left(TX^{-1}\delta U_0 + x^iX^{-1}\delta U_i\right) + p(2\phi - 6\psi + 2\tilde{\nabla}_a\tilde{\nabla}^a E) \tag{2.14}
$$

3 Field Equations (G.I. Form)

In the following, $\tilde{\nabla}^2 = \delta^{ij}\tilde{\nabla}_i\tilde{\nabla}_j$.

$$\alpha = \phi + \dot{B} - \ddot{E} + B\dot{\Omega}\Omega^{-1} - \dot{E}\dot{\Omega}\Omega^{-1} - E^a\Omega^{-1}\tilde{\nabla}_a\Omega - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a E \tag{3.1}$$

$$\gamma = \psi - B\dot{\Omega}\Omega^{-1} + \dot{E}\dot{\Omega}\Omega^{-1} + E^a\Omega^{-1}\tilde{\nabla}_a\Omega + \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a E \tag{3.2}$$

$$\delta p^{GI} = \delta p - 32d^{-2}X^4(-1+X^2)(1+X^2)^{-4}\psi \tag{3.3}$$

$$V^{GI} = V + \tfrac{1}{4}dX^{-1}(1+X^2)^2\psi \tag{3.4}$$

$$Q_i = B_i - \dot{E}_i, \qquad V_i, \qquad E_{ij} \tag{3.5}$$

$$\Delta_{\mu\nu} = \delta G_{\mu\nu} + \delta T_{\mu\nu} \tag{3.6}$$

$$\begin{aligned}
\Delta_{00} = {} & -\tfrac{1}{4}d^2X^{-6}(1+X^2)^2(-4T^2+X^2)\delta p^{GI} - 12T(X^2+X^4)^{-1}\dot{\gamma} - 2\tilde{\nabla}^2\gamma \\
& +24T^2X^{-4}(1+X^2)^{-2}\alpha + 8X^{-4}(1+X^2)^{-2}\Big(X^4(2+5X^2) + T^2\big(3-2(X^2+X^4)\big)\Big)\gamma \\
& -64d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_1V^{GI} + 4(-1+4X^2)(X^2+X^4)^{-1}x\tilde{\nabla}_1\gamma \\
& -64d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_2V^{GI} + 4(-1+4X^2)(X^2+X^4)^{-1}y\tilde{\nabla}_2\gamma \\
& -64d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_3V^{GI} \\
& +4(-1+4X^2)(X^2+X^4)^{-1}z\tilde{\nabla}_3\gamma - 24TX^{-4}(1+X^2)^{-2}xQ_1 - 24TX^{-4}(1+X^2)^{-2}yQ_2 \\
& -24TX^{-4}(1+X^2)^{-2}zQ_3 - 64d^{-1}X(1+X^2)^{-3}xV_1 - 64d^{-1}X(1+X^2)^{-3}yV_2 \\
& -64d^{-1}X(1+X^2)^{-3}zV_3 - 8X^{-4}(1+X^2)^{-2}(X^2+X^4+3x^2)E_{11} \\
& -48X^{-4}(1+X^2)^{-2}xyE_{12} - 48X^{-4}(1+X^2)^{-2}xzE_{13} \\
& -8X^{-4}(1+X^2)^{-2}(X^2+X^4+3y^2)E_{22} - 48X^{-4}(1+X^2)^{-2}yzE_{23} \\
& -8X^{-4}(1+X^2)^{-2}(X^2+X^4+3z^2)E_{33}
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
\Delta_{11} = {} & -2\ddot{\gamma} + \tfrac{1}{4}d^2X^{-6}(1+X^2)^2(X^2+4x^2)\delta p^{GI} + 4T(X^2+X^4)^{-1}\dot{\alpha} + 8T(X^2+X^4)^{-1}\dot{\gamma} - \tilde{\nabla}^2\alpha \\
& +\tilde{\nabla}^2\gamma + 8X^{-4}(1+X^2)^{-2}\big(X^2+X^4-T^2(3+4X^2)\big)\alpha \\
& +8X^{-4}(1+X^2)^{-2}\big(X^2-T^2(3+4X^2)+X^2(1+X^2)(X^2-2x^2)\big)\gamma \\
& -64d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_1V^{GI} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_1\alpha + 8(1+2X^2)(X^2+X^4)^{-1}x\tilde{\nabla}_1\gamma \\
& +\tilde{\nabla}_1\tilde{\nabla}_1\alpha - \tilde{\nabla}_1\tilde{\nabla}_1\gamma - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\alpha - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\alpha - 4(X^2+X^4)^{-1}x\dot{Q}_1 \\
& -4(X^2+X^4)^{-1}y\dot{Q}_2 - 4(X^2+X^4)^{-1}z\dot{Q}_3 + 8TX^{-4}(1+X^2)^{-2}(3+4X^2)xQ_1 \\
& +8TX^{-4}(1+X^2)^{-2}(3+4X^2)yQ_2 + 8TX^{-4}(1+X^2)^{-2}(3+4X^2)zQ_3 \\
& -64d^{-1}X(1+X^2)^{-3}xV_1 + \tilde{\nabla}_1\dot{Q}_1 - 4T(X^2+X^4)^{-1}\tilde{\nabla}_1Q_1 - \ddot{E}_{11} + 4T(X^2+X^4)^{-1}\dot{E}_{11} \\
& +8X^{-4}(1+X^2)^{-2}(3+4X^2)x^2E_{11} + 16X^{-4}(1+X^2)^{-2}(3+4X^2)xyE_{12} \\
& +16X^{-4}(1+X^2)^{-2}(3+4X^2)xzE_{13} + 8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)E_{22} \\
& +16X^{-4}(1+X^2)^{-2}(3+4X^2)yzE_{23} + 8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)E_{33} \\
& +\tilde{\nabla}^2E_{11} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_1E_{11} - 8(X^2+X^4)^{-1}y\tilde{\nabla}_1E_{12} - 8(X^2+X^4)^{-1}z\tilde{\nabla}_1E_{13} \\
& +4(X^2+X^4)^{-1}y\tilde{\nabla}_2E_{11} + 4(X^2+X^4)^{-1}z\tilde{\nabla}_3E_{11}
\end{aligned} \tag{3.8}$$

$$\Delta_{22} = -2\ddot{\gamma} + \tfrac{1}{4}d^2X^{-6}(1+X^2)^2(X^2+4y^2)\delta p^{GI} + 4T(X^2+X^4)^{-1}\dot{\alpha} + 8T(X^2+X^4)^{-1}\dot{\gamma} - \tilde{\nabla}^2\alpha$$

$$
\begin{aligned}
&+\tilde{\nabla}^2\gamma+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4-T^2(3+4X^2)\big)\alpha\\
&-8X^{-4}(1+X^2)^{-2}\Big(T^2(3-4X^2-6X^4)+X^2\big(-1+(8+6X^2)x^2+10y^2+8z^2+X^2(7+5X^2+8y^2+6z^2)\big)\Big)\gamma\\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\alpha-64d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_2V^{GI}-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\alpha\\
&+8(1+2X^2)(X^2+X^4)^{-1}y\tilde{\nabla}_2\gamma+\tilde{\nabla}_2\tilde{\nabla}_2\alpha-\tilde{\nabla}_2\tilde{\nabla}_2\gamma\\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\alpha-4(X^2+X^4)^{-1}x\dot{Q}_1-4(X^2+X^4)^{-1}y\dot{Q}_2-4(X^2+X^4)^{-1}z\dot{Q}_3\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)xQ_1+8TX^{-4}(1+X^2)^{-2}(3+4X^2)yQ_2\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)zQ_3-64d^{-1}X(1+X^2)^{-3}yV_2+\tilde{\nabla}_2\dot{Q}_2\\
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_2Q_2-\ddot{E}_{22}+4T(X^2+X^4)^{-1}\dot{E}_{22}\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)E_{11}+16X^{-4}(1+X^2)^{-2}(3+4X^2)xyE_{12}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xzE_{13}+8X^{-4}(1+X^2)^{-2}(3+4X^2)y^2E_{22}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)yzE_{23}+8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)E_{33}\\
&+\tilde{\nabla}^2E_{22}+4(X^2+X^4)^{-1}x\tilde{\nabla}_1E_{22}-8(X^2+X^4)^{-1}x\tilde{\nabla}_2E_{12}-4(X^2+X^4)^{-1}y\tilde{\nabla}_2E_{22}\\
&-8(X^2+X^4)^{-1}z\tilde{\nabla}_2E_{23}+4(X^2+X^4)^{-1}z\tilde{\nabla}_3E_{22}
\end{aligned}
\tag{3.9}
$$

$$
\begin{aligned}
\Delta_{33} \;=\;& -2\ddot{\gamma}+\tfrac{1}{4}d^2X^{-6}(1+X^2)^2(X^2+4z^2)\delta p^{GI}+4T(X^2+X^4)^{-1}\dot{\alpha}+8T(X^2+X^4)^{-1}\dot{\gamma}-\tilde{\nabla}^2\alpha\\
&+\tilde{\nabla}^2\gamma+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4-T^2(3+4X^2)\big)\alpha\\
&-8X^{-4}(1+X^2)^{-2}\Big(T^2(3-4X^2-6X^4)+X^2\big(-1+(8+6X^2)x^2+8y^2+10z^2+X^2(7+5X^2+6y^2+8z^2)\big)\Big)\gamma\\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\alpha-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\alpha-64d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_3V^{GI}\\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\alpha+8(1+2X^2)(X^2+X^4)^{-1}z\tilde{\nabla}_3\gamma+\tilde{\nabla}_3\tilde{\nabla}_3\alpha\\
&-\tilde{\nabla}_3\tilde{\nabla}_3\gamma-4(X^2+X^4)^{-1}x\dot{Q}_1-4(X^2+X^4)^{-1}y\dot{Q}_2-4(X^2+X^4)^{-1}z\dot{Q}_3\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)xQ_1+8TX^{-4}(1+X^2)^{-2}(3+4X^2)yQ_2\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)zQ_3-64d^{-1}X(1+X^2)^{-3}zV_3+\tilde{\nabla}_3\dot{Q}_3\\
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_3Q_3-\ddot{E}_{33}+4T(X^2+X^4)^{-1}\dot{E}_{33}\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)E_{11}+16X^{-4}(1+X^2)^{-2}(3+4X^2)xyE_{12}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xzE_{13}+8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)E_{22}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)yzE_{23}+8X^{-4}(1+X^2)^{-2}(3+4X^2)z^2E_{33}+\tilde{\nabla}^2E_{33}\\
&+4(X^2+X^4)^{-1}x\tilde{\nabla}_1E_{33}+4(X^2+X^4)^{-1}y\tilde{\nabla}_2E_{33}-8(X^2+X^4)^{-1}x\tilde{\nabla}_3E_{13}\\
&-8(X^2+X^4)^{-1}y\tilde{\nabla}_3E_{23}-4(X^2+X^4)^{-1}z\tilde{\nabla}_3E_{33}
\end{aligned}
\tag{3.10}
$$

$$
\begin{aligned}
\Delta_{01} \;=\;& -d^2TX^{-6}x\delta p^{GI}-2d^2TX^{-4}x\delta p^{GI}-d^2TX^{-2}x\delta p^{GI}+4(X^2+X^4)^{-1}x\dot{\gamma}\\
&+16T(X+X^3)^{-2}x\alpha+32T(X+X^3)^{-2}x\gamma-8T^{-1}X^2(X+X^3)^{-2}x\gamma\\
&+16TX^2(X+X^3)^{-2}x\gamma-24T^{-1}X^4(X+X^3)^{-2}x\gamma-2\tilde{\nabla}_1\dot{\gamma}+32d^{-1}TX(1+X^2)^{-3}\tilde{\nabla}_1V^{GI}\\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}x^2\tilde{\nabla}_1V^{GI}+4T(X^2+X^4)^{-1}\tilde{\nabla}_1\alpha-8T(1+X^2)^{-1}\tilde{\nabla}_1\gamma\\
&-8T^{-1}(1+X^2)^{-1}x^2\tilde{\nabla}_1\gamma+32d^{-1}T^{-1}X(1+X^2)^{-3}xy\tilde{\nabla}_2V^{GI}-8(T+TX^2)^{-1}xy\tilde{\nabla}_2\gamma\\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}xz\tilde{\nabla}_3V^{GI}-8(T+TX^2)^{-1}xz\tilde{\nabla}_3\gamma+\tfrac{1}{2}\tilde{\nabla}^2Q_1-4(X+X^3)^{-2}Q_1\\
&-4X^2(X+X^3)^{-2}Q_1-16(X+X^3)^{-2}x^2Q_1-16(X+X^3)^{-2}xyQ_2-16(X+X^3)^{-2}xzQ_3\\
&+32d^{-1}TX(1+X^2)^{-3}V_1+32d^{-1}T^{-1}X(1+X^2)^{-3}x^2V_1+32d^{-1}T^{-1}X(1+X^2)^{-3}xyV_2\\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}xzV_3-2(X^2+X^4)^{-1}y\tilde{\nabla}_1Q_2-2(X^2+X^4)^{-1}z\tilde{\nabla}_1Q_3\\
&+2(X^2+X^4)^{-1}y\tilde{\nabla}_2Q_1+2(X^2+X^4)^{-1}z\tilde{\nabla}_3Q_1-4(X^2+X^4)^{-1}x\dot{E}_{11}\\
&-4(X^2+X^4)^{-1}y\dot{E}_{12}-4(X^2+X^4)^{-1}z\dot{E}_{13}-16T^{-1}(X+X^3)^{-2}x^3E_{11}\\
&-32T^{-1}(X+X^3)^{-2}x^2yE_{12}-32T^{-1}(X+X^3)^{-2}x^2zE_{13}-16T^{-1}(X+X^3)^{-2}xy^2E_{22}
\end{aligned}
$$

$$-32T^{-1}(X+X^3)^{-2}xyzE_{23}-16T^{-1}(X+X^3)^{-2}xz^2E_{33} \tag{3.11}$$

$$\begin{aligned}
\Delta_{02} &= -d^2TX^{-6}y\delta p^{GI}-2d^2TX^{-4}y\delta p^{GI}-d^2TX^{-2}y\delta p^{GI}+4(X^2+X^4)^{-1}y\dot{\gamma}\\
&+16T(X+X^3)^{-2}y\alpha+32T(X+X^3)^{-2}y\gamma-8T^{-1}X^2(X+X^3)^{-2}y\gamma\\
&+16TX^2(X+X^3)^{-2}y\gamma-24T^{-1}X^4(X+X^3)^{-2}y\gamma+32d^{-1}T^{-1}X(1+X^2)^{-3}xy\tilde{\nabla}_1V^{GI}\\
&-8(T+TX^2)^{-1}xy\tilde{\nabla}_1\gamma-2\tilde{\nabla}_2\dot{\gamma}+32d^{-1}TX(1+X^2)^{-3}\tilde{\nabla}_2V^{GI}\\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}y^2\tilde{\nabla}_2V^{GI}+4T(X^2+X^4)^{-1}\tilde{\nabla}_2\alpha-8T(1+X^2)^{-1}\tilde{\nabla}_2\gamma\\
&-8T^{-1}(1+X^2)^{-1}y^2\tilde{\nabla}_2\gamma+32d^{-1}T^{-1}X(1+X^2)^{-3}yz\tilde{\nabla}_3V^{GI}\\
&-8(T+TX^2)^{-1}yz\tilde{\nabla}_3\gamma+\tfrac{1}{2}\tilde{\nabla}^2Q_2-16(X+X^3)^{-2}xyQ_1-4(X+X^3)^{-2}Q_2\\
&-4X^2(X+X^3)^{-2}Q_2-16(X+X^3)^{-2}y^2Q_2-16(X+X^3)^{-2}yzQ_3\\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}xyV_1+32d^{-1}TX(1+X^2)^{-3}V_2+32d^{-1}T^{-1}X(1+X^2)^{-3}y^2V_2\\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}yzV_3+2(X^2+X^4)^{-1}x\tilde{\nabla}_1Q_2-2(X^2+X^4)^{-1}x\tilde{\nabla}_2Q_1\\
&-2(X^2+X^4)^{-1}z\tilde{\nabla}_2Q_3+2(X^2+X^4)^{-1}z\tilde{\nabla}_3Q_2-4(X^2+X^4)^{-1}x\dot{E}_{12}\\
&-4(X^2+X^4)^{-1}y\dot{E}_{22}-4(X^2+X^4)^{-1}z\dot{E}_{23}-16T^{-1}(X+X^3)^{-2}x^2yE_{11}\\
&-32T^{-1}(X+X^3)^{-2}xy^2E_{12}-32T^{-1}(X+X^3)^{-2}xyzE_{13}-16T^{-1}(X+X^3)^{-2}y^3E_{22}\\
&-32T^{-1}(X+X^3)^{-2}y^2zE_{23}-16T^{-1}(X+X^3)^{-2}yz^2E_{33}
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\Delta_{03} &= -d^2TX^{-6}z\delta p^{GI}-2d^2TX^{-4}z\delta p^{GI}-d^2TX^{-2}z\delta p^{GI}+4(X^2+X^4)^{-1}z\dot{\gamma}\\
&+16T(X+X^3)^{-2}z\alpha+32T(X+X^3)^{-2}z\gamma-8T^{-1}X^2(X+X^3)^{-2}z\gamma\\
&+16TX^2(X+X^3)^{-2}z\gamma-24T^{-1}X^4(X+X^3)^{-2}z\gamma+32d^{-1}T^{-1}X(1+X^2)^{-3}xz\tilde{\nabla}_1V^{GI}\\
&-8(T+TX^2)^{-1}xz\tilde{\nabla}_1\gamma+32d^{-1}T^{-1}X(1+X^2)^{-3}yz\tilde{\nabla}_2V^{GI}-8(T+TX^2)^{-1}yz\tilde{\nabla}_2\gamma\\
&-2\tilde{\nabla}_3\dot{\gamma}+32d^{-1}TX(1+X^2)^{-3}\tilde{\nabla}_3V^{GI}+32d^{-1}T^{-1}X(1+X^2)^{-3}z^2\tilde{\nabla}_3V^{GI}\\
&+4T(X^2+X^4)^{-1}\tilde{\nabla}_3\alpha-8T(1+X^2)^{-1}\tilde{\nabla}_3\gamma-8T^{-1}(1+X^2)^{-1}z^2\tilde{\nabla}_3\gamma+\tfrac{1}{2}\tilde{\nabla}^2Q_3\\
&-16(X+X^3)^{-2}xzQ_1-16(X+X^3)^{-2}yzQ_2-4(X+X^3)^{-2}Q_3-4X^2(X+X^3)^{-2}Q_3\\
&-16(X+X^3)^{-2}z^2Q_3+32d^{-1}T^{-1}X(1+X^2)^{-3}xzV_1+32d^{-1}T^{-1}X(1+X^2)^{-3}yzV_2\\
&+32d^{-1}TX(1+X^2)^{-3}V_3+32d^{-1}T^{-1}X(1+X^2)^{-3}z^2V_3+2(X^2+X^4)^{-1}x\tilde{\nabla}_1Q_3\\
&+2(X^2+X^4)^{-1}y\tilde{\nabla}_2Q_3-2(X^2+X^4)^{-1}x\tilde{\nabla}_3Q_1\\
&-2(X^2+X^4)^{-1}y\tilde{\nabla}_3Q_2-4(X^2+X^4)^{-1}x\dot{E}_{13}-4(X^2+X^4)^{-1}y\dot{E}_{23}-4(X^2+X^4)^{-1}z\dot{E}_{33}\\
&-16T^{-1}(X+X^3)^{-2}x^2zE_{11}-32T^{-1}(X+X^3)^{-2}xyzE_{12}-32T^{-1}(X+X^3)^{-2}xz^2E_{13}\\
&-16T^{-1}(X+X^3)^{-2}y^2zE_{22}-32T^{-1}(X+X^3)^{-2}yz^2E_{23}-16T^{-1}(X+X^3)^{-2}z^3E_{33}
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\Delta_{12} &= d^2X^{-6}xy\delta p^{GI}+2d^2X^{-4}xy\delta p^{GI}+d^2X^{-2}xy\delta p^{GI}-16(X^2+X^4)^{-1}xy\gamma\\
&-32d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_1V^{GI}+4(X^2+X^4)^{-1}y\tilde{\nabla}_1\gamma+8X^2(X^2+X^4)^{-1}y\tilde{\nabla}_1\gamma\\
&-32d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_2V^{GI}+4(X^2+X^4)^{-1}x\tilde{\nabla}_2\gamma+8X^2(X^2+X^4)^{-1}x\tilde{\nabla}_2\gamma+\tilde{\nabla}_2\tilde{\nabla}_1\alpha\\
&-\tilde{\nabla}_2\tilde{\nabla}_1\gamma-32d^{-1}X(1+X^2)^{-3}yV_1-32d^{-1}X(1+X^2)^{-3}xV_2+\tfrac{1}{2}\tilde{\nabla}_1\dot{Q}_2\\
&-2T(X^2+X^4)^{-1}\tilde{\nabla}_1Q_2+\tfrac{1}{2}\tilde{\nabla}_2\dot{Q}_1-2T(X^2+X^4)^{-1}\tilde{\nabla}_2Q_1-\ddot{E}_{12}+4T(X^2+X^4)^{-1}\dot{E}_{12}\\
&-8(X^2+X^4)^{-1}E_{12}+\tilde{\nabla}^2E_{12}-4(X^2+X^4)^{-1}y\tilde{\nabla}_1E_{22}-4(X^2+X^4)^{-1}z\tilde{\nabla}_1E_{23}\\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_2E_{11}-4(X^2+X^4)^{-1}z\tilde{\nabla}_2E_{13}+4(X^2+X^4)^{-1}z\tilde{\nabla}_3E_{12}
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\Delta_{13} &= d^2X^{-6}xz\delta p^{GI}+2d^2X^{-4}xz\delta p^{GI}+d^2X^{-2}xz\delta p^{GI}-16(X^2+X^4)^{-1}xz\gamma\\
&-32d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_1V^{GI}+4(X^2+X^4)^{-1}z\tilde{\nabla}_1\gamma+8X^2(X^2+X^4)^{-1}z\tilde{\nabla}_1\gamma\\
&-32d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_3V^{GI}+4(X^2+X^4)^{-1}x\tilde{\nabla}_3\gamma+8X^2(X^2+X^4)^{-1}x\tilde{\nabla}_3\gamma+\tilde{\nabla}_3\tilde{\nabla}_1\alpha\\
&-\tilde{\nabla}_3\tilde{\nabla}_1\gamma-32d^{-1}X(1+X^2)^{-3}zV_1-32d^{-1}X(1+X^2)^{-3}xV_3+\tfrac{1}{2}\tilde{\nabla}_1\dot{Q}_3
\end{aligned}$$

$$
\begin{aligned}
&-2T(X^2+X^4)^{-1}\tilde{\nabla}_1 Q_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{Q}_1 - 2T(X^2+X^4)^{-1}\tilde{\nabla}_3 Q_1 - \ddot{E}_{13} + 4T(X^2+X^4)^{-1}\dot{E}_{13}\\
&-8(X^2+X^4)^{-1}E_{13} + \tilde{\nabla}^2 E_{13} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_1 E_{23} - 4(X^2+X^4)^{-1}z\tilde{\nabla}_1 E_{33}\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_2 E_{13} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3 E_{11} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_3 E_{12}
\end{aligned} \tag{3.15}
$$

$$
\begin{aligned}
\Delta_{23} \;=\;& d^2X^{-6}yz\delta p^{GI} + 2d^2X^{-4}yz\delta p^{GI} + d^2X^{-2}yz\delta p^{GI} - 16(X^2+X^4)^{-1}yz\gamma\\
&-32d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_2 V^{GI} + 4(X^2+X^4)^{-1}z\tilde{\nabla}_2\gamma + 8X^2(X^2+X^4)^{-1}z\tilde{\nabla}_2\gamma\\
&-32d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_3 V^{GI} + 4(X^2+X^4)^{-1}y\tilde{\nabla}_3\gamma + 8X^2(X^2+X^4)^{-1}y\tilde{\nabla}_3\gamma + \tilde{\nabla}_3\tilde{\nabla}_2\alpha\\
&-\tilde{\nabla}_3\tilde{\nabla}_2\gamma - 32d^{-1}X(1+X^2)^{-3}zV_2 - 32d^{-1}X(1+X^2)^{-3}yV_3 + \tfrac{1}{2}\tilde{\nabla}_2\dot{Q}_3\\
&-2T(X^2+X^4)^{-1}\tilde{\nabla}_2 Q_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{Q}_2 - 2T(X^2+X^4)^{-1}\tilde{\nabla}_3 Q_2 - \ddot{E}_{23} + 4T(X^2+X^4)^{-1}\dot{E}_{23}\\
&-8(X^2+X^4)^{-1}E_{23} + \tilde{\nabla}^2 E_{23} + 4(X^2+X^4)^{-1}x\tilde{\nabla}_1 E_{23} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_2 E_{13}\\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_2 E_{33} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3 E_{12} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_3 E_{22}
\end{aligned} \tag{3.16}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} \;=\;& -24d^{-2}X^4(1+X^2)^{-2}\ddot{\gamma} + 48d^{-2}TX^2(1+X^2)^{-3}\dot{\alpha} + 144d^{-2}TX^2(1+X^2)^{-3}\dot{\gamma}\\
&-8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}^2\alpha + 16d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}^2\gamma + 96d^{-2}(1+X^2)^{-3}(-4T^2+X^2)\alpha\\
&+96d^{-2}(1+X^2)^{-3}(-4T^2+X^2)\gamma - 48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_1\alpha\\
&+48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_1\gamma - 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\alpha + 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\gamma\\
&-48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\alpha + 48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\gamma - 48d^{-2}X^2(1+X^2)^{-3}x\dot{Q}_1\\
&-48d^{-2}X^2(1+X^2)^{-3}y\dot{Q}_2 - 48d^{-2}X^2(1+X^2)^{-3}z\dot{Q}_3 + 384d^{-2}T(1+X^2)^{-3}xQ_1\\
&+384d^{-2}T(1+X^2)^{-3}yQ_2 + 384d^{-2}T(1+X^2)^{-3}zQ_3 + 384d^{-2}(1+X^2)^{-3}x^2E_{11}\\
&+768d^{-2}(1+X^2)^{-3}xyE_{12} + 768d^{-2}(1+X^2)^{-3}xzE_{13} + 384d^{-2}(1+X^2)^{-3}y^2E_{22}\\
&+768d^{-2}(1+X^2)^{-3}yzE_{23} + 384d^{-2}(1+X^2)^{-3}z^2E_{33}
\end{aligned} \tag{3.17}
$$

4 Covariant Conservation (Incomplete)

4.1 EM Tensor

$$
\begin{aligned}
\delta(\nabla_\mu T^{\mu\nu}) \;=\;& \tfrac{1}{2}T^\nu{}_\alpha\Omega^{-4}\tilde{\nabla}^\alpha f + 2\delta T_{\alpha\beta}\tilde{g}^{\nu\beta}\Omega^{-5}\tilde{\nabla}^\alpha\Omega - 2T^{\nu\beta}f_{\alpha\beta}\Omega^{-5}\tilde{\nabla}^\alpha\Omega + T^\beta{}_\beta f^\nu{}_\alpha\Omega^{-5}\tilde{\nabla}^\alpha\Omega\\
&-2T_\alpha{}^\beta f^\nu{}_\beta\Omega^{-5}\tilde{\nabla}^\alpha\Omega - f^{\nu\alpha}\Omega^{-4}\tilde{\nabla}_\beta T_\alpha{}^\beta - f^{\alpha\beta}\Omega^{-4}\tilde{\nabla}_\beta T^\nu{}_\alpha - T^{\nu\alpha}\Omega^{-4}\tilde{\nabla}_\beta f_\alpha{}^\beta\\
&+\tilde{g}^{\beta\gamma}\tilde{g}^{\nu\alpha}\Omega^{-4}\tilde{\nabla}_\gamma\delta T_{\alpha\beta} - \tfrac{1}{2}T^{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\nu f_{\alpha\beta} - \delta T_{\alpha\beta}\tilde{g}^{\alpha\beta}\Omega^{-5}\tilde{\nabla}^\nu\Omega + T^{\alpha\beta}f_{\alpha\beta}\Omega^{-5}\tilde{\nabla}^\nu\Omega
\end{aligned} \tag{4.1}
$$

$$
\begin{aligned}
\delta(\nabla_\mu T^{\mu 0}) \;=\;& \delta T_{00}\dot{\Omega}\Omega^{-5} + \delta T_{ab}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5} - 4T_{00}\dot{\Omega}\phi\Omega^{-5} - 2T_{ab}\dot{\Omega}\tilde{g}^{ab}\phi\Omega^{-5} + 2T_{ab}\dot{\Omega}\tilde{g}^{ab}\psi\Omega^{-5} + \delta\dot{T}_{00}\Omega^{-4}\\
&-2T_{00}\dot{\phi}\Omega^{-4} - 3T_{00}\dot{\psi}\Omega^{-4} - T_{ab}\dot{\psi}\tilde{g}^{ab}\Omega^{-4} - 4\dot{T}_{00}\phi\Omega^{-4} - T_{0d}\tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}_c\tilde{\nabla}_b E\\
&-2T_{0a}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b B - 2\dot{T}_{0a}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b B - 2T_{0a}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b\dot{B} - \tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_a B\tilde{\nabla}_b T_{00}\\
&+2\tilde{g}^{ab}\phi\Omega^{-4}\tilde{\nabla}_b T_{0a} - 2\tilde{g}^{ab}\psi\Omega^{-4}\tilde{\nabla}_b T_{0a} - \tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b\delta T_{0a} - T_{0a}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b\phi + T_{0a}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b\psi\\
&-2\delta T_{0a}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega + 4T_{0a}\tilde{g}^{ab}\phi\Omega^{-5}\tilde{\nabla}_b\Omega - 4T_{0a}\tilde{g}^{ab}\psi\Omega^{-5}\tilde{\nabla}_b\Omega - T_{00}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a B\tilde{\nabla}_b\Omega\\
&+2T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_a B\tilde{\nabla}_b\Omega - T_{00}\tilde{g}^{ba}\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}_a B + T_{00}\tilde{g}^{ba}\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}_a\dot{E}\\
&+T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}_a\dot{E} - 2T_{cd}\dot{\Omega}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b\tilde{\nabla}_a E + 2T_{0d}\tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_c\tilde{\nabla}_b\tilde{\nabla}_a E\\
&+\tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_c T_{ab}\tilde{\nabla}_d B + 2\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}_d T_{0c} - T_{bc}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_a B\tilde{\nabla}_d\Omega\\
&+4T_{0c}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}_d\Omega + \delta T_{00}\dot{\Omega}\Omega^{-5} - 2B_a T_{0b}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5} + \delta T_{ab}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5} + \delta\dot{T}_{00}\Omega^{-4}\\
&-2B_a\dot{T}_{0b}\tilde{g}^{ab}\Omega^{-4} - 2\dot{B}_a T_{0b}\tilde{g}^{ab}\Omega^{-4} - B_a\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b T_{00} - \tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b\delta T_{0a}\\
&+T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_b\dot{E}_a - 2T_{cd}\dot{\Omega}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b E_a - B_a T_{00}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega - 2\delta T_{0a}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega\\
&+2B_a T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b\Omega + B_d\tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_c T_{ab} + \tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_a T_{0b}\tilde{\nabla}_d E_c\\
&+\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_b T_{0a}\tilde{\nabla}_d E_c - B_a T_{bc}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_d\Omega + 2T_{0a}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b E_c\tilde{\nabla}_d\Omega
\end{aligned}
$$

$$
\begin{aligned}
&+2T_{0a}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_c E_b\tilde{\nabla}_d\Omega + T_{0a}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_d\tilde{\nabla}_b E_c + \delta T_{00}\dot{\Omega}\Omega^{-5} + \delta T_{ab}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5}\\
&-2T_{ab}\dot{\Omega}E_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5} + \delta\dot{T}_{00}\Omega^{-4} + T_{ab}\dot{E}_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4} + 2E_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_b T_{0a}\\
&-\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b\delta T_{0a} - 2\delta T_{0a}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega + 4T_{0a}E_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b\Omega
\end{aligned}
\tag{4.2}
$$

$$
\begin{aligned}
\delta(\nabla_\mu T^{\mu i}) \;=\;& -2\delta T_{0i}\dot{\Omega}\Omega^{-5} + 4T_{0i}\dot{\Omega}\phi\Omega^{-5} - 4T_{0i}\dot{\Omega}\psi\Omega^{-5} - \delta\dot{T}_{0i}\Omega^{-4} + T_{0i}\dot{\phi}\Omega^{-4} + 3T_{0i}\dot{\psi}\Omega^{-4} + 2\dot{T}_{0i}\phi\Omega^{-4}\\
&-2\dot{T}_{0i}\psi\Omega^{-4} + 2T_{ib}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a B + \dot{T}_{ib}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_a B + T_{ib}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_a\dot{B} + T_{ib}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_a\phi\\
&-T_{ib}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_a\psi + 2\delta T_{ib}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a\Omega + 8T_{ib}\tilde{g}^{ab}\psi\Omega^{-5}\tilde{\nabla}_a\Omega + T_{id}\tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}_c\tilde{\nabla}_b E\\
&+\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_a B\tilde{\nabla}_b T_{0i} + 4\tilde{g}^{ba}\psi\Omega^{-4}\tilde{\nabla}_b T_{ia} + \tilde{g}^{ba}\Omega^{-4}\tilde{\nabla}_b\delta T_{ia} + 2T_{0i}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a B\tilde{\nabla}_b\Omega\\
&+T_{0i}\tilde{g}^{ba}\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}_a B - T_{0i}\tilde{g}^{ba}\Omega^{-4}\tilde{\nabla}_b\tilde{\nabla}_a\dot{E} - 2T_{id}\tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_c\tilde{\nabla}_b\tilde{\nabla}_a E\\
&-2\tilde{g}^{ba}\tilde{g}^{dc}\Omega^{-4}\tilde{\nabla}_c\tilde{\nabla}_a E\tilde{\nabla}_d T_{ib} - 4T_{ic}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}_d\Omega - T_{00}\dot{\Omega}\Omega^{-5}\tilde{\nabla}_i B\\
&-T_{ab}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_i B - \dot{T}_{00}\Omega^{-4}\tilde{\nabla}_i B + \tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b T_{0a}\tilde{\nabla}_i B + 2T_{0a}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega\tilde{\nabla}_i B\\
&+T_{00}\Omega^{-4}\tilde{\nabla}_i\phi + T_{ab}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_i\psi + \delta T_{00}\Omega^{-5}\tilde{\nabla}_i\Omega - \delta T_{ab}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_i\Omega - 2T_{00}\phi\Omega^{-5}\tilde{\nabla}_i\Omega\\
&+2T_{00}\psi\Omega^{-5}\tilde{\nabla}_i\Omega - 4T_{ab}\tilde{g}^{ab}\psi\Omega^{-5}\tilde{\nabla}_i\Omega - 2T_{0a}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b B\tilde{\nabla}_i\Omega + 2T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}_i\Omega\\
&+T_{0b}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_i\tilde{\nabla}_a B + 4T_{0b}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_i\tilde{\nabla}_a E + 2\dot{T}_{0b}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_i\tilde{\nabla}_a E\\
&-2T_{00}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega\tilde{\nabla}_i\tilde{\nabla}_a E - 2\tilde{g}^{ba}\tilde{g}^{dc}\Omega^{-4}\tilde{\nabla}_d T_{bc}\tilde{\nabla}_i\tilde{\nabla}_a E + 2T_{bc}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_d\Omega\tilde{\nabla}_i\tilde{\nabla}_a E\\
&-4T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_a\Omega\tilde{\nabla}_i\tilde{\nabla}_b E - T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_i\tilde{\nabla}_b\tilde{\nabla}_a E - B_i T_{00}\dot{\Omega}\Omega^{-5} - 2\delta T_{0i}\dot{\Omega}\Omega^{-5}\\
&-B_i T_{ab}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5} + 2B_a T_{ib}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5} - B_i\dot{T}_{00}\Omega^{-4} - \delta\dot{T}_{0i}\Omega^{-4} + B_a\dot{T}_{ib}\tilde{g}^{ab}\Omega^{-4} + \dot{B}_a T_{ib}\tilde{g}^{ab}\Omega^{-4}\\
&+2\delta T_{ib}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a\Omega + B_i\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b T_{0a} + B_a\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b T_{0i} + \tilde{g}^{ba}\Omega^{-4}\tilde{\nabla}_b\delta T_{ia}\\
&+2T_{0a}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b E_i + \dot{T}_{0a}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_b E_i + 2B_i T_{0a}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega + 2B_a T_{0i}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega\\
&-T_{00}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a E_i\tilde{\nabla}_b\Omega - 2T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_a E_i\tilde{\nabla}_b\Omega - \tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_a T_{ib}\tilde{\nabla}_d E_c\\
&-\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_b T_{ia}\tilde{\nabla}_d E_c - \tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_c T_{ab}\tilde{\nabla}_d E_i - 2T_{ic}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_a E_b\tilde{\nabla}_d\Omega\\
&+T_{bc}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_a E_i\tilde{\nabla}_d\Omega - 2T_{ic}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_b E_a\tilde{\nabla}_d\Omega - T_{ic}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-4}\tilde{\nabla}_d\tilde{\nabla}_a E_b\\
&+T_{0a}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_i B_b - T_{00}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_b\Omega\tilde{\nabla}_i E_a - 2T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b\Omega\tilde{\nabla}_i E_a\\
&+T_{bc}\tilde{g}^{ad}\tilde{g}^{bc}\Omega^{-5}\tilde{\nabla}_d\Omega\tilde{\nabla}_i E_a + 2T_{0a}\dot{\Omega}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_i E_b + \dot{T}_{0a}\tilde{g}^{ab}\Omega^{-4}\tilde{\nabla}_i E_b - \tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_c T_{ab}\tilde{\nabla}_i E_d\\
&+\delta T_{00}\Omega^{-5}\tilde{\nabla}_i\Omega - 2B_a T_{0b}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_i\Omega - \delta T_{ab}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_i\Omega + 2T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b E_a\tilde{\nabla}_i\Omega\\
&-T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_i\tilde{\nabla}_b E_a - 2\delta T_{0i}\dot{\Omega}\Omega^{-5} + 4T_{0a}\dot{\Omega}E_{ib}\tilde{g}^{ab}\Omega^{-5} - \delta\dot{T}_{0i}\Omega^{-4} + 2\dot{T}_{0a}E_{ib}\tilde{g}^{ab}\Omega^{-4}\\
&+2\delta T_{ib}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a\Omega - 2T_{00}E_{ib}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_a\Omega - 4T_{ib}E_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_a\Omega\\
&-4T_{cb}E_{id}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_a\Omega - 2E_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_b T_{ia} + \tilde{g}^{ba}\Omega^{-4}\tilde{\nabla}_b\delta T_{ia} + 2T_{ac}E_{id}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_b\Omega\\
&-2E_{id}\tilde{g}^{ad}\tilde{g}^{cb}\Omega^{-4}\tilde{\nabla}_c T_{ab} - T_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-4}\tilde{\nabla}_i E_{ab} + \delta T_{00}\Omega^{-5}\tilde{\nabla}_i\Omega - \delta T_{ab}\tilde{g}^{ab}\Omega^{-5}\tilde{\nabla}_i\Omega\\
&+2T_{ab}E_{cd}\tilde{g}^{ac}\tilde{g}^{bd}\Omega^{-5}\tilde{\nabla}_i\Omega
\end{aligned}
\tag{4.3}
$$

4.2 $\Delta_{\mu\nu}$

(4.4)

$$
\nabla_\mu\Delta^{\mu\nu} \;=\; \tilde{g}^{\alpha\gamma}\tilde{g}^{\nu\beta}\Omega^{-4}\tilde{\nabla}_\alpha\Delta_{\beta\gamma} + 2\tilde{g}^{\alpha\gamma}\tilde{g}^{\nu\beta}\Delta_{\alpha\beta}\Omega^{-5}\tilde{\nabla}_\gamma\Omega - \tilde{g}^{\nu\gamma}\tilde{g}^{\alpha\beta}\Delta_{\alpha\beta}\Omega^{-5}\tilde{\nabla}_\gamma\Omega \tag{4.5}
$$

$$
\nu \;=\; 0 \tag{4.6}
$$

$$
\begin{aligned}
0 \;\stackrel{!}{=}\;& \Delta_{00}\dot{\Omega}\Omega^{-5} + \Delta_{11}\dot{\Omega}\Omega^{-5} + \Delta_{22}\dot{\Omega}\Omega^{-5} + \Delta_{33}\dot{\Omega}\Omega^{-5} + \dot{\Delta}_{00}\Omega^{-4} - \Omega^{-4}\tilde{\nabla}_1\Delta_{01}\\
&-2\Delta_{01}\Omega^{-5}\tilde{\nabla}_1\Omega - \Omega^{-4}\tilde{\nabla}_2\Delta_{02} - 2\Delta_{02}\Omega^{-5}\tilde{\nabla}_2\Omega - \Omega^{-4}\tilde{\nabla}_3\Delta_{03} - 2\Delta_{03}\Omega^{-5}\tilde{\nabla}_3\Omega
\end{aligned}
\tag{4.7}
$$

$$
= \tag{4.8}
$$

$$\nu = 1 \tag{4.9}$$

$$\begin{aligned} 0 \stackrel{!}{=} & -2\Delta_{01}\dot{\Omega}\Omega^{-5} - \dot{\Delta}_{01}\Omega^{-4} + \Omega^{-4}\tilde{\nabla}_1\Delta_{11} + \Delta_{00}\Omega^{-5}\tilde{\nabla}_1\Omega + \Delta_{11}\Omega^{-5}\tilde{\nabla}_1\Omega - \Delta_{22}\Omega^{-5}\tilde{\nabla}_1\Omega \\ & -\Delta_{33}\Omega^{-5}\tilde{\nabla}_1\Omega + \Omega^{-4}\tilde{\nabla}_2\Delta_{12} + 2\Delta_{12}\Omega^{-5}\tilde{\nabla}_2\Omega + \Omega^{-4}\tilde{\nabla}_3\Delta_{13} + 2\Delta_{13}\Omega^{-5}\tilde{\nabla}_3\Omega. \end{aligned} \tag{4.10}$$

$$= \tag{4.11}$$

$$\nu = 2 \tag{4.12}$$

$$\begin{aligned} 0 \stackrel{!}{=} & -2\Delta_{02}\dot{\Omega}\Omega^{-5} - \dot{\Delta}_{02}\Omega^{-4} + \Omega^{-4}\tilde{\nabla}_1\Delta_{12} + 2\Delta_{12}\Omega^{-5}\tilde{\nabla}_1\Omega + \Omega^{-4}\tilde{\nabla}_2\Delta_{22} + \Delta_{00}\Omega^{-5}\tilde{\nabla}_2\Omega \\ & -\Delta_{11}\Omega^{-5}\tilde{\nabla}_2\Omega + \Delta_{22}\Omega^{-5}\tilde{\nabla}_2\Omega - \Delta_{33}\Omega^{-5}\tilde{\nabla}_2\Omega + \Omega^{-4}\tilde{\nabla}_3\Delta_{23} + 2\Delta_{23}\Omega^{-5}\tilde{\nabla}_3\Omega \end{aligned} \tag{4.13}$$

$$= \tag{4.14}$$

$$\nu = 3 \tag{4.15}$$

$$\begin{aligned} 0 \stackrel{!}{=} & -2\Delta_{03}\dot{\Omega}\Omega^{-5} - \dot{\Delta}_{03}\Omega^{-4} + \Omega^{-4}\tilde{\nabla}_1\Delta_{13} + 2\Delta_{13}\Omega^{-5}\tilde{\nabla}_1\Omega + \Omega^{-4}\tilde{\nabla}_2\Delta_{23} + 2\Delta_{23}\Omega^{-5}\tilde{\nabla}_2\Omega \\ & +\Omega^{-4}\tilde{\nabla}_3\Delta_{33} + \Delta_{00}\Omega^{-5}\tilde{\nabla}_3\Omega - \Delta_{11}\Omega^{-5}\tilde{\nabla}_3\Omega - \Delta_{22}\Omega^{-5}\tilde{\nabla}_3\Omega + \Delta_{33}\Omega^{-5}\tilde{\nabla}_3\Omega \end{aligned} \tag{4.16}$$

$$= \tag{4.17}$$

Appendix A Field Equations

Substituting in background quantities p and U_μ, we have the following field equations

$$
\begin{aligned}
\Delta_{00} \;=\;& -\tfrac{1}{4}d^2X^{-6}(1+X^2)^2(-4T^2+X^2)\delta p - 12T(X^2+X^4)^{-1}\dot{\psi} + 24T^2X^{-4}(1+X^2)^{-2}\phi \\
&+24X^{-4}(1+X^2)^{-2}(T^2+X^4)\psi - 24TX^{-4}(1+X^2)^{-2}x\tilde{\nabla}_1 B - 64d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_1 V \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\psi - 4T(X^2+X^4)^{-1}\tilde{\nabla}_1\tilde{\nabla}_1 B + 4T(X^2+X^4)^{-1}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E} \\
&-8X^{-4}(1+X^2)^{-2}(X^2+X^4+3x^2)\tilde{\nabla}_1\tilde{\nabla}_1 E - 2\tilde{\nabla}_1\tilde{\nabla}_1\psi - 4(X^2+X^4)^{-1}x\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_1 E \\
&-24TX^{-4}(1+X^2)^{-2}y\tilde{\nabla}_2 B - 64d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_2 V - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\psi \\
&-48X^{-4}(1+X^2)^{-2}xy\tilde{\nabla}_2\tilde{\nabla}_1 E - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_1\tilde{\nabla}_1 E - 4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_2 B \\
&+4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E} - 8X^{-4}(1+X^2)^{-2}(X^2+X^4+3y^2)\tilde{\nabla}_2\tilde{\nabla}_2 E - 2\tilde{\nabla}_2\tilde{\nabla}_2\psi \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_1 E - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_2 E - 24TX^{-4}(1+X^2)^{-2}z\tilde{\nabla}_3 B \\
&-64d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_3 V - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\psi - 48X^{-4}(1+X^2)^{-2}xz\tilde{\nabla}_3\tilde{\nabla}_1 E \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_1\tilde{\nabla}_1 E - 48X^{-4}(1+X^2)^{-2}yz\tilde{\nabla}_3\tilde{\nabla}_2 E - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_2 E \\
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_3 B + 4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E} \\
&-8X^{-4}(1+X^2)^{-2}(X^2+X^4+3z^2)\tilde{\nabla}_3\tilde{\nabla}_3 E - 2\tilde{\nabla}_3\tilde{\nabla}_3\psi - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_1 E \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_2 E - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3 E - 24TX^{-4}(1+X^2)^{-2}xB_1 \\
&-24TX^{-4}(1+X^2)^{-2}yB_2 - 24TX^{-4}(1+X^2)^{-2}zB_3 - 64d^{-1}X(1+X^2)^{-3}xV_1 \\
&-64d^{-1}X(1+X^2)^{-3}yV_2 - 64d^{-1}X(1+X^2)^{-3}zV_3 \\
&-8X^{-4}(1+X^2)^{-2}(X^2+X^4+3x^2)\tilde{\nabla}_1 E_1 - 24X^{-4}(1+X^2)^{-2}xy\tilde{\nabla}_1 E_2 \\
&-24X^{-4}(1+X^2)^{-2}xz\tilde{\nabla}_1 E_3 - 4(X^2+X^4)^{-1}x\tilde{\nabla}_1\tilde{\nabla}_1 E_1 - 4(X^2+X^4)^{-1}y\tilde{\nabla}_1\tilde{\nabla}_1 E_2 \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_1\tilde{\nabla}_1 E_3 - 24X^{-4}(1+X^2)^{-2}xy\tilde{\nabla}_2 E_1 \\
&-8X^{-4}(1+X^2)^{-2}(X^2+X^4+3y^2)\tilde{\nabla}_2 E_2 - 24X^{-4}(1+X^2)^{-2}yz\tilde{\nabla}_2 E_3 \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_2 E_1 - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2 E_2 - 4(X^2+X^4)^{-1}z\tilde{\nabla}_2\tilde{\nabla}_2 E_3 \\
&-24X^{-4}(1+X^2)^{-2}xz\tilde{\nabla}_3 E_1 - 24X^{-4}(1+X^2)^{-2}yz\tilde{\nabla}_3 E_2 \\
&-8X^{-4}(1+X^2)^{-2}(X^2+X^4+3z^2)\tilde{\nabla}_3 E_3 - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_3 E_1 \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_3 E_2 \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3 E_3 - 8X^{-4}(1+X^2)^{-2}(X^2+X^4+3x^2)E_{11} \\
&-48X^{-4}(1+X^2)^{-2}xyE_{12} - 48X^{-4}(1+X^2)^{-2}xzE_{13} \\
&-8X^{-4}(1+X^2)^{-2}(X^2+X^4+3y^2)E_{22} - 48X^{-4}(1+X^2)^{-2}yzE_{23} \\
&-8X^{-4}(1+X^2)^{-2}(X^2+X^4+3z^2)E_{33}
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
\Delta_{11} \;=\;& -2\ddot{\psi} + \tfrac{1}{4}d^2X^{-6}(1+X^2)^2(X^2+4x^2)\delta p + 4T(X^2+X^4)^{-1}\dot{\phi} + 8T(X^2+X^4)^{-1}\dot{\psi} \\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4-T^2(3+4X^2)\big)\phi \\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+2X^4-T^2(3+4X^2)\big)\psi + 8TX^{-4}(1+X^2)^{-2}(3+4X^2)x\tilde{\nabla}_1 B \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\dot{B} - 64d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_1 V - 4(X^2+X^4)^{-1}x\tilde{\nabla}_1\phi \\
&+8(X^2+X^4)^{-1}x\tilde{\nabla}_1\psi + 8X^{-4}(1+X^2)^{-2}(3+4X^2)x^2\tilde{\nabla}_1\tilde{\nabla}_1 E \\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)y\tilde{\nabla}_2 B - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\dot{B} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\phi \\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_2\tilde{\nabla}_1 E + 4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_2 B - \tilde{\nabla}_2\tilde{\nabla}_2\dot{B} + \tilde{\nabla}_2\tilde{\nabla}_2\ddot{E} \\
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E} + 8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)\tilde{\nabla}_2\tilde{\nabla}_2 E - \tilde{\nabla}_2\tilde{\nabla}_2\phi \\
&+\tilde{\nabla}_2\tilde{\nabla}_2\psi + 4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_1 E + 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_2 E \\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)z\tilde{\nabla}_3 B - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\dot{B} - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\phi \\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_3\tilde{\nabla}_1 E + 16X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_3\tilde{\nabla}_2 E \\
&+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_2 E + 4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_3 B - \tilde{\nabla}_3\tilde{\nabla}_3\dot{B} + \tilde{\nabla}_3\tilde{\nabla}_3\ddot{E}
\end{aligned}
$$

$$
\begin{aligned}
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}+8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)\tilde{\nabla}_3\tilde{\nabla}_3E-\tilde{\nabla}_3\tilde{\nabla}_3\phi\\
&+\tilde{\nabla}_3\tilde{\nabla}_3\psi+4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_1E+4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_2E\\
&+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3E+8TX^{-4}(1+X^2)^{-2}(3+4X^2)xB_1\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)yB_2+8TX^{-4}(1+X^2)^{-2}(3+4X^2)zB_3\\
&-4(X^2+X^4)^{-1}x\dot{B}_1-4(X^2+X^4)^{-1}y\dot{B}_2-4(X^2+X^4)^{-1}z\dot{B}_3-64d^{-1}X(1+X^2)^{-3}xV_1\\
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_1B_1+\tilde{\nabla}_1\dot{B}_1-\tilde{\nabla}_1\ddot{E}_1+4T(X^2+X^4)^{-1}\tilde{\nabla}_1\dot{E}_1\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)x^2\tilde{\nabla}_1E_1+8X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_1E_2\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_1E_3+8X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_2E_1\\
&+8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)\tilde{\nabla}_2E_2+8X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_2E_3\\
&+4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_2E_1+4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2E_2+4(X^2+X^4)^{-1}z\tilde{\nabla}_2\tilde{\nabla}_2E_3\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_3E_1+8X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_3E_2\\
&+8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)\tilde{\nabla}_3E_3+4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_3E_1\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_3E_2+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3E_3-\ddot{E}_{11}+4T(X^2+X^4)^{-1}\dot{E}_{11}\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)x^2E_{11}+16X^{-4}(1+X^2)^{-2}(3+4X^2)xyE_{12}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xzE_{13}+8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)E_{22}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)yzE_{23}+8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)E_{33}\\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_1E_{11}-8(X^2+X^4)^{-1}y\tilde{\nabla}_1E_{12}-8(X^2+X^4)^{-1}z\tilde{\nabla}_1E_{13}+\tilde{\nabla}_1\tilde{\nabla}_1E_{11}\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_2E_{11}+\tilde{\nabla}_2\tilde{\nabla}_2E_{11}+4(X^2+X^4)^{-1}z\tilde{\nabla}_3E_{11}+\tilde{\nabla}_3\tilde{\nabla}_3E_{11}
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
\Delta_{22} \;=\;& -2\ddot{\psi}+\tfrac{1}{4}d^2X^{-6}(1+X^2)^2(X^2+4y^2)\delta p+4T(X^2+X^4)^{-1}\dot{\phi}+8T(X^2+X^4)^{-1}\dot{\psi}\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4-T^2(3+4X^2)\big)\phi\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+2X^4-T^2(3+4X^2)\big)\psi+8TX^{-4}(1+X^2)^{-2}(3+4X^2)x\tilde{\nabla}_1B\\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\dot{B}-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\phi+4T(X^2+X^4)^{-1}\tilde{\nabla}_1\tilde{\nabla}_1B-\tilde{\nabla}_1\tilde{\nabla}_1\dot{B}\\
&+\tilde{\nabla}_1\tilde{\nabla}_1\ddot{E}-4T(X^2+X^4)^{-1}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)\tilde{\nabla}_1\tilde{\nabla}_1E\\
&-\tilde{\nabla}_1\tilde{\nabla}_1\phi+\tilde{\nabla}_1\tilde{\nabla}_1\psi+4(X^2+X^4)^{-1}x\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_1E+8TX^{-4}(1+X^2)^{-2}(3+4X^2)y\tilde{\nabla}_2B\\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\dot{B}-64d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_2V-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\phi\\
&+8(X^2+X^4)^{-1}y\tilde{\nabla}_2\psi+16X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_2\tilde{\nabla}_1E\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_1\tilde{\nabla}_1E+8X^{-4}(1+X^2)^{-2}(3+4X^2)y^2\tilde{\nabla}_2\tilde{\nabla}_2E\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)z\tilde{\nabla}_3B-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\dot{B}-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\phi\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_3\tilde{\nabla}_1E+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_1\tilde{\nabla}_1E\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_3\tilde{\nabla}_2E+4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_3B-\tilde{\nabla}_3\tilde{\nabla}_3\dot{B}+\tilde{\nabla}_3\tilde{\nabla}_3\ddot{E}\\
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}+8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)\tilde{\nabla}_3\tilde{\nabla}_3E-\tilde{\nabla}_3\tilde{\nabla}_3\phi\\
&+\tilde{\nabla}_3\tilde{\nabla}_3\psi+4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_1E+4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_2E\\
&+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3E+8TX^{-4}(1+X^2)^{-2}(3+4X^2)xB_1\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)yB_2+8TX^{-4}(1+X^2)^{-2}(3+4X^2)zB_3\\
&-4(X^2+X^4)^{-1}x\dot{B}_1-4(X^2+X^4)^{-1}y\dot{B}_2-4(X^2+X^4)^{-1}z\dot{B}_3-64d^{-1}X(1+X^2)^{-3}yV_2\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)\tilde{\nabla}_1E_1+8X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_1E_2\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_1E_3+4(X^2+X^4)^{-1}x\tilde{\nabla}_1\tilde{\nabla}_1E_1\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_1\tilde{\nabla}_1E_2+4(X^2+X^4)^{-1}z\tilde{\nabla}_1\tilde{\nabla}_1E_3-4T(X^2+X^4)^{-1}\tilde{\nabla}_2B_2+\tilde{\nabla}_2\dot{B}_2\\
&-\tilde{\nabla}_2\ddot{E}_2+4T(X^2+X^4)^{-1}\tilde{\nabla}_2\dot{E}_2+8X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_2E_1\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)y^2\tilde{\nabla}_2E_2+8X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_2E_3
\end{aligned}
$$

$$
\begin{aligned}
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_3E_1+8X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_3E_2\\
&+8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)\tilde{\nabla}_3E_3+4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_3E_1\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_3E_2+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3E_3-\ddot{E}_{22}+4T(X^2+X^4)^{-1}\dot{E}_{22}\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)E_{11}+16X^{-4}(1+X^2)^{-2}(3+4X^2)xyE_{12}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xzE_{13}+8X^{-4}(1+X^2)^{-2}(3+4X^2)y^2E_{22}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)yzE_{23}+8X^{-4}(1+X^2)^{-2}\big(X^4+3z^2+X^2(1+4z^2)\big)E_{33}\\
&+4(X^2+X^4)^{-1}x\tilde{\nabla}_1E_{22}+\tilde{\nabla}_1\tilde{\nabla}_1E_{22}-8(X^2+X^4)^{-1}x\tilde{\nabla}_2E_{12}-4(X^2+X^4)^{-1}y\tilde{\nabla}_2E_{22}\\
&-8(X^2+X^4)^{-1}z\tilde{\nabla}_2E_{23}+\tilde{\nabla}_2\tilde{\nabla}_2E_{22}+4(X^2+X^4)^{-1}z\tilde{\nabla}_3E_{22}+\tilde{\nabla}_3\tilde{\nabla}_3E_{22} \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{33} \;=\;& -2\ddot{\psi}+\tfrac{1}{4}d^2X^{-6}(1+X^2)^2(X^2+4z^2)\delta p+4T(X^2+X^4)^{-1}\dot{\phi}+8T(X^2+X^4)^{-1}\dot{\psi}\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4-T^2(3+4X^2)\big)\phi\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+2X^4-T^2(3+4X^2)\big)\psi+8TX^{-4}(1+X^2)^{-2}(3+4X^2)x\tilde{\nabla}_1B\\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\dot{B}-4(X^2+X^4)^{-1}x\tilde{\nabla}_1\phi+4T(X^2+X^4)^{-1}\tilde{\nabla}_1\tilde{\nabla}_1B-\tilde{\nabla}_1\tilde{\nabla}_1\dot{B}\\
&+\tilde{\nabla}_1\tilde{\nabla}_1\ddot{E}-4T(X^2+X^4)^{-1}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)\tilde{\nabla}_1\tilde{\nabla}_1E\\
&-\tilde{\nabla}_1\tilde{\nabla}_1\phi+\tilde{\nabla}_1\tilde{\nabla}_1\psi+4(X^2+X^4)^{-1}x\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_1E+8TX^{-4}(1+X^2)^{-2}(3+4X^2)y\tilde{\nabla}_2B\\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\dot{B}-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\phi+16X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_2\tilde{\nabla}_1E\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_1\tilde{\nabla}_1E+4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_2B-\tilde{\nabla}_2\tilde{\nabla}_2\dot{B}+\tilde{\nabla}_2\tilde{\nabla}_2\ddot{E}\\
&-4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E}+8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)\tilde{\nabla}_2\tilde{\nabla}_2E-\tilde{\nabla}_2\tilde{\nabla}_2\phi\\
&+\tilde{\nabla}_2\tilde{\nabla}_2\psi+4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_1E+4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_2E\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)z\tilde{\nabla}_3B-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\dot{B}-64d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_3V\\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\phi+8(X^2+X^4)^{-1}z\tilde{\nabla}_3\psi+16X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_3\tilde{\nabla}_1E\\
&+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_1\tilde{\nabla}_1E+16X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_3\tilde{\nabla}_2E\\
&+4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_2E\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)z^2\tilde{\nabla}_3\tilde{\nabla}_3E+8TX^{-4}(1+X^2)^{-2}(3+4X^2)xB_1\\
&+8TX^{-4}(1+X^2)^{-2}(3+4X^2)yB_2+8TX^{-4}(1+X^2)^{-2}(3+4X^2)zB_3\\
&-4(X^2+X^4)^{-1}x\dot{B}_1-4(X^2+X^4)^{-1}y\dot{B}_2-4(X^2+X^4)^{-1}z\dot{B}_3-64d^{-1}X(1+X^2)^{-3}zV_3\\
&+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)\tilde{\nabla}_1E_1+8X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_1E_2\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_1E_3+4(X^2+X^4)^{-1}x\tilde{\nabla}_1\tilde{\nabla}_1E_1\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_1\tilde{\nabla}_1E_2+4(X^2+X^4)^{-1}z\tilde{\nabla}_1\tilde{\nabla}_1E_3\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)xy\tilde{\nabla}_2E_1+8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)\tilde{\nabla}_2E_2\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_2E_3+4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_2E_1\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2E_2+4(X^2+X^4)^{-1}z\tilde{\nabla}_2\tilde{\nabla}_2E_3-4T(X^2+X^4)^{-1}\tilde{\nabla}_3B_3+\tilde{\nabla}_3\dot{B}_3\\
&-\tilde{\nabla}_3\ddot{E}_3+4T(X^2+X^4)^{-1}\tilde{\nabla}_3\dot{E}_3+8X^{-4}(1+X^2)^{-2}(3+4X^2)xz\tilde{\nabla}_3E_1\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)yz\tilde{\nabla}_3E_2+8X^{-4}(1+X^2)^{-2}(3+4X^2)z^2\tilde{\nabla}_3E_3-\ddot{E}_{33}\\
&+4T(X^2+X^4)^{-1}\dot{E}_{33}+8X^{-4}(1+X^2)^{-2}\big(X^2+X^4+(3+4X^2)x^2\big)E_{11}\\
&+16X^{-4}(1+X^2)^{-2}(3+4X^2)xyE_{12}+16X^{-4}(1+X^2)^{-2}(3+4X^2)xzE_{13}\\
&+8X^{-4}(1+X^2)^{-2}\big(X^4+3y^2+X^2(1+4y^2)\big)E_{22}+16X^{-4}(1+X^2)^{-2}(3+4X^2)yzE_{23}\\
&+8X^{-4}(1+X^2)^{-2}(3+4X^2)z^2E_{33}+4(X^2+X^4)^{-1}x\tilde{\nabla}_1E_{33}+\tilde{\nabla}_1\tilde{\nabla}_1E_{33}\\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_2E_{33}+\tilde{\nabla}_2\tilde{\nabla}_2E_{33}-8(X^2+X^4)^{-1}x\tilde{\nabla}_3E_{13}-8(X^2+X^4)^{-1}y\tilde{\nabla}_3E_{23}\\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3E_{33}+\tilde{\nabla}_3\tilde{\nabla}_3E_{33} \qquad \text{(A.4)}
\end{aligned}
$$

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\begin{aligned}
\Delta_{01} \;=\;& -d^2TX^{-6}(1+X^2)^2x\delta p + 4(X^2+X^4)^{-1}x\dot{\psi} + 16T(X+X^3)^{-2}x\phi \\
&+16T^{-1}(T-X)(T+X)(X+X^3)^{-2}x\psi - 4(X+X^3)^{-2}(1+X^2+4x^2)\tilde{\nabla}_1 B - 2\tilde{\nabla}_1\dot{\psi} \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}(T^2+x^2)\tilde{\nabla}_1 V + 4T(X^2+X^4)^{-1}\tilde{\nabla}_1\phi - 4(X^2+X^4)^{-1}x\tilde{\nabla}_1\tilde{\nabla}_1\dot{E} \\
&-16T^{-1}(X+X^3)^{-2}x^3\tilde{\nabla}_1\tilde{\nabla}_1 E - 16(X+X^3)^{-2}xy\tilde{\nabla}_2 B + 32d^{-1}T^{-1}X(1+X^2)^{-3}xy\tilde{\nabla}_2 V \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_1\dot{E} - 32T^{-1}(X+X^3)^{-2}x^2y\tilde{\nabla}_2\tilde{\nabla}_1 E - 16T^{-1}(X+X^3)^{-2}xy^2\tilde{\nabla}_2\tilde{\nabla}_2 E \\
&-16(X+X^3)^{-2}xz\tilde{\nabla}_3 B + 32d^{-1}T^{-1}X(1+X^2)^{-3}xz\tilde{\nabla}_3 V - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_1\dot{E} \\
&-32T^{-1}(X+X^3)^{-2}x^2z\tilde{\nabla}_3\tilde{\nabla}_1 E - 32T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_3\tilde{\nabla}_2 E \\
&-16T^{-1}(X+X^3)^{-2}xz^2\tilde{\nabla}_3\tilde{\nabla}_3 E - 4(X+X^3)^{-2}(1+X^2+4x^2)B_1 - 16(X+X^3)^{-2}xyB_2 \\
&-16(X+X^3)^{-2}xzB_3 + 32d^{-1}T^{-1}X(1+X^2)^{-3}(T^2+x^2)V_1 \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}xyV_2 + 32d^{-1}T^{-1}X(1+X^2)^{-3}xzV_3 - 2(X^2+X^4)^{-1}y\tilde{\nabla}_1 B_2 \\
&-2(X^2+X^4)^{-1}z\tilde{\nabla}_1 B_3 - 4(X^2+X^4)^{-1}x\tilde{\nabla}_1\dot{E}_1 - 2(X^2+X^4)^{-1}y\tilde{\nabla}_1\dot{E}_2 \\
&-2(X^2+X^4)^{-1}z\tilde{\nabla}_1\dot{E}_3 - 16T^{-1}(X+X^3)^{-2}x^3\tilde{\nabla}_1 E_1 - 16T^{-1}(X+X^3)^{-2}x^2y\tilde{\nabla}_1 E_2 \\
&-16T^{-1}(X+X^3)^{-2}x^2z\tilde{\nabla}_1 E_3 + \tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1 B_1 - \tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}_1 + 2(X^2+X^4)^{-1}y\tilde{\nabla}_2 B_1 \\
&-2(X^2+X^4)^{-1}y\tilde{\nabla}_2\dot{E}_1 - 16T^{-1}(X+X^3)^{-2}x^2y\tilde{\nabla}_2 E_1 - 16T^{-1}(X+X^3)^{-2}xy^2\tilde{\nabla}_2 E_2 \\
&-16T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_2 E_3 + \tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2 B_1 - \tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E}_1 + 2(X^2+X^4)^{-1}z\tilde{\nabla}_3 B_1 \\
&-2(X^2+X^4)^{-1}z\tilde{\nabla}_3\dot{E}_1 - 16T^{-1}(X+X^3)^{-2}x^2z\tilde{\nabla}_3 E_1 - 16T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_3 E_2 \\
&-16T^{-1}(X+X^3)^{-2}xz^2\tilde{\nabla}_3 E_3 + \tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3 B_1 - \tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}_1 - 4(X^2+X^4)^{-1}x\dot{E}_{11} \\
&-4(X^2+X^4)^{-1}y\dot{E}_{12} - 4(X^2+X^4)^{-1}z\dot{E}_{13} - 16T^{-1}(X+X^3)^{-2}x^3E_{11} \\
&-32T^{-1}(X+X^3)^{-2}x^2yE_{12} - 32T^{-1}(X+X^3)^{-2}x^2zE_{13} - 16T^{-1}(X+X^3)^{-2}xy^2E_{22} \\
&-32T^{-1}(X+X^3)^{-2}xyzE_{23} - 16T^{-1}(X+X^3)^{-2}xz^2E_{33}
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
\Delta_{02} \;=\;& -d^2TX^{-6}(1+X^2)^2y\delta p + 4(X^2+X^4)^{-1}y\dot{\psi} + 16T(X+X^3)^{-2}y\phi \\
&+16T^{-1}(T-X)(T+X)(X+X^3)^{-2}y\psi - 16(X+X^3)^{-2}xy\tilde{\nabla}_1 B \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}xy\tilde{\nabla}_1 V - 16T^{-1}(X+X^3)^{-2}x^2y\tilde{\nabla}_1\tilde{\nabla}_1 E \\
&-4(X+X^3)^{-2}(1+X^2+4y^2)\tilde{\nabla}_2 B - 2\tilde{\nabla}_2\dot{\psi} + 32d^{-1}T^{-1}X(1+X^2)^{-3}(T^2+y^2)\tilde{\nabla}_2 V \\
&+4T(X^2+X^4)^{-1}\tilde{\nabla}_2\phi - 4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_1\dot{E} - 32T^{-1}(X+X^3)^{-2}xy^2\tilde{\nabla}_2\tilde{\nabla}_1 E \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2\dot{E} - 16T^{-1}(X+X^3)^{-2}y^3\tilde{\nabla}_2\tilde{\nabla}_2 E - 16(X+X^3)^{-2}yz\tilde{\nabla}_3 B \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}yz\tilde{\nabla}_3 V - 32T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_3\tilde{\nabla}_1 E \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_2\dot{E} - 32T^{-1}(X+X^3)^{-2}y^2z\tilde{\nabla}_3\tilde{\nabla}_2 E \\
&-16T^{-1}(X+X^3)^{-2}yz^2\tilde{\nabla}_3\tilde{\nabla}_3 E - 16(X+X^3)^{-2}xyB_1 - 4(X+X^3)^{-2}(1+X^2+4y^2)B_2 \\
&-16(X+X^3)^{-2}yzB_3 + 32d^{-1}T^{-1}X(1+X^2)^{-3}xyV_1 \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}(T^2+y^2)V_2 + 32d^{-1}T^{-1}X(1+X^2)^{-3}yzV_3 \\
&+2(X^2+X^4)^{-1}x\tilde{\nabla}_1 B_2 - 2(X^2+X^4)^{-1}x\tilde{\nabla}_1\dot{E}_2 - 16T^{-1}(X+X^3)^{-2}x^2y\tilde{\nabla}_1 E_1 \\
&-16T^{-1}(X+X^3)^{-2}xy^2\tilde{\nabla}_1 E_2 - 16T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_1 E_3 + \tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1 B_2 - \tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}_2 \\
&-2(X^2+X^4)^{-1}x\tilde{\nabla}_2 B_1 - 2(X^2+X^4)^{-1}z\tilde{\nabla}_2 B_3 - 2(X^2+X^4)^{-1}x\tilde{\nabla}_2\dot{E}_1 \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\dot{E}_2 - 2(X^2+X^4)^{-1}z\tilde{\nabla}_2\dot{E}_3 - 16T^{-1}(X+X^3)^{-2}xy^2\tilde{\nabla}_2 E_1 \\
&-16T^{-1}(X+X^3)^{-2}y^3\tilde{\nabla}_2 E_2 - 16T^{-1}(X+X^3)^{-2}y^2z\tilde{\nabla}_2 E_3 + \tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2 B_2 - \tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E}_2 \\
&+2(X^2+X^4)^{-1}z\tilde{\nabla}_3 B_2 - 2(X^2+X^4)^{-1}z\tilde{\nabla}_3\dot{E}_2 - 16T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_3 E_1 \\
&-16T^{-1}(X+X^3)^{-2}y^2z\tilde{\nabla}_3 E_2 - 16T^{-1}(X+X^3)^{-2}yz^2\tilde{\nabla}_3 E_3 + \tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3 B_2 \\
&-\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}_2 - 4(X^2+X^4)^{-1}x\dot{E}_{12} - 4(X^2+X^4)^{-1}y\dot{E}_{22} - 4(X^2+X^4)^{-1}z\dot{E}_{23} \\
&-16T^{-1}(X+X^3)^{-2}x^2yE_{11} - 32T^{-1}(X+X^3)^{-2}xy^2E_{12} - 32T^{-1}(X+X^3)^{-2}xyzE_{13} \\
&-16T^{-1}(X+X^3)^{-2}y^3E_{22} - 32T^{-1}(X+X^3)^{-2}y^2zE_{23} - 16T^{-1}(X+X^3)^{-2}yz^2E_{33}
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
\Delta_{03} \;=\;& -d^2TX^{-6}(1+X^2)^2z\delta p + 4(X^2+X^4)^{-1}z\dot{\psi} + 16T(X+X^3)^{-2}z\phi \\
&+16T^{-1}(T-X)(T+X)(X+X^3)^{-2}z\psi - 16(X+X^3)^{-2}xz\tilde{\nabla}_1 B \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}xz\tilde{\nabla}_1 V - 16T^{-1}(X+X^3)^{-2}x^2z\tilde{\nabla}_1\tilde{\nabla}_1 E - 16(X+X^3)^{-2}yz\tilde{\nabla}_2 B \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}yz\tilde{\nabla}_2 V - 32T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_2\tilde{\nabla}_1 E \\
&-16T^{-1}(X+X^3)^{-2}y^2z\tilde{\nabla}_2\tilde{\nabla}_2 E - 4(X+X^3)^{-2}(1+X^2+4z^2)\tilde{\nabla}_3 B - 2\tilde{\nabla}_3\dot{\psi} \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}(T^2+z^2)\tilde{\nabla}_3 V + 4T(X^2+X^4)^{-1}\tilde{\nabla}_3\phi - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_1\dot{E} \\
&-32T^{-1}(X+X^3)^{-2}xz^2\tilde{\nabla}_3\tilde{\nabla}_1 E - 4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_2\dot{E} - 32T^{-1}(X+X^3)^{-2}yz^2\tilde{\nabla}_3\tilde{\nabla}_2 E \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3\dot{E} - 16T^{-1}(X+X^3)^{-2}z^3\tilde{\nabla}_3\tilde{\nabla}_3 E - 16(X+X^3)^{-2}xzB_1 \\
&-16(X+X^3)^{-2}yzB_2 - 4(X+X^3)^{-2}(1+X^2+4z^2)B_3 + 32d^{-1}T^{-1}X(1+X^2)^{-3}xzV_1 \\
&+32d^{-1}T^{-1}X(1+X^2)^{-3}yzV_2 + 32d^{-1}T^{-1}X(1+X^2)^{-3}(T^2+z^2)V_3 \\
&+2(X^2+X^4)^{-1}x\tilde{\nabla}_1 B_3 - 2(X^2+X^4)^{-1}x\tilde{\nabla}_1\dot{E}_3 - 16T^{-1}(X+X^3)^{-2}x^2z\tilde{\nabla}_1 E_1 \\
&-16T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_1 E_2 - 16T^{-1}(X+X^3)^{-2}xz^2\tilde{\nabla}_1 E_3 + \tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1 B_3 - \tfrac{1}{2}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E}_3 \\
&+2(X^2+X^4)^{-1}y\tilde{\nabla}_2 B_3 - 2(X^2+X^4)^{-1}y\tilde{\nabla}_2\dot{E}_3 - 16T^{-1}(X+X^3)^{-2}xyz\tilde{\nabla}_2 E_1 \\
&-16T^{-1}(X+X^3)^{-2}y^2z\tilde{\nabla}_2 E_2 - 16T^{-1}(X+X^3)^{-2}yz^2\tilde{\nabla}_2 E_3 + \tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2 B_3 - \tfrac{1}{2}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E}_3 \\
&-2(X^2+X^4)^{-1}x\tilde{\nabla}_3 B_1 - 2(X^2+X^4)^{-1}y\tilde{\nabla}_3 B_2 - 2(X^2+X^4)^{-1}x\tilde{\nabla}_3\dot{E}_1 \\
&-2(X^2+X^4)^{-1}y\tilde{\nabla}_3\dot{E}_2 - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\dot{E}_3 - 16T^{-1}(X+X^3)^{-2}xz^2\tilde{\nabla}_3 E_1 \\
&-16T^{-1}(X+X^3)^{-2}yz^2\tilde{\nabla}_3 E_2 - 16T^{-1}(X+X^3)^{-2}z^3\tilde{\nabla}_3 E_3 + \tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3 B_3 \\
&-\tfrac{1}{2}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E}_3 - 4(X^2+X^4)^{-1}x\dot{E}_{13} - 4(X^2+X^4)^{-1}y\dot{E}_{23} - 4(X^2+X^4)^{-1}z\dot{E}_{33} \\
&-16T^{-1}(X+X^3)^{-2}x^2zE_{11} - 32T^{-1}(X+X^3)^{-2}xyzE_{12} - 32T^{-1}(X+X^3)^{-2}xz^2E_{13} \\
&-16T^{-1}(X+X^3)^{-2}y^2zE_{22} - 32T^{-1}(X+X^3)^{-2}yz^2E_{23} - 16T^{-1}(X+X^3)^{-2}z^3E_{33}
\end{aligned} \tag{A.7}
$$

$$
\begin{aligned}
\Delta_{12} \;=\;& d^2X^{-6}(1+X^2)^2xy\delta p - 32d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_1 V + 4(X^2+X^4)^{-1}y\tilde{\nabla}_1\psi \\
&-32d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_2 V + 4(X^2+X^4)^{-1}x\tilde{\nabla}_2\psi - 4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_1 B + \tilde{\nabla}_2\tilde{\nabla}_1\dot{B} \\
&-\tilde{\nabla}_2\tilde{\nabla}_1\ddot{E} + 4T(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_1\dot{E} - 8(X^2+X^4)^{-1}\tilde{\nabla}_2\tilde{\nabla}_1 E + \tilde{\nabla}_2\tilde{\nabla}_1\phi - \tilde{\nabla}_2\tilde{\nabla}_1\psi \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_1\tilde{\nabla}_1 E - 4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_1 E \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_1 E - 32d^{-1}X(1+X^2)^{-3}yV_1 - 32d^{-1}X(1+X^2)^{-3}xV_2 \\
&-2T(X^2+X^4)^{-1}\tilde{\nabla}_1 B_2 + \tfrac{1}{2}\tilde{\nabla}_1\dot{B}_2 - \tfrac{1}{2}\tilde{\nabla}_1\ddot{E}_2 + 2T(X^2+X^4)^{-1}\tilde{\nabla}_1\dot{E}_2 \\
&-4(X^2+X^4)^{-1}\tilde{\nabla}_1 E_2 - 2T(X^2+X^4)^{-1}\tilde{\nabla}_2 B_1 + \tfrac{1}{2}\tilde{\nabla}_2\dot{B}_1 - \tfrac{1}{2}\tilde{\nabla}_2\ddot{E}_1 \\
&+2T(X^2+X^4)^{-1}\tilde{\nabla}_2\dot{E}_1 - 4(X^2+X^4)^{-1}\tilde{\nabla}_2 E_1 - 4(X^2+X^4)^{-1}x\tilde{\nabla}_2\tilde{\nabla}_1 E_1 \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_2\tilde{\nabla}_1 E_2 - 4(X^2+X^4)^{-1}z\tilde{\nabla}_2\tilde{\nabla}_1 E_3 - \ddot{E}_{12} + 4T(X^2+X^4)^{-1}\dot{E}_{12} \\
&-8(X^2+X^4)^{-1}E_{12} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_1 E_{22} - 4(X^2+X^4)^{-1}z\tilde{\nabla}_1 E_{23} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{12} \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_2 E_{11} - 4(X^2+X^4)^{-1}z\tilde{\nabla}_2 E_{13} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{12} + 4(X^2+X^4)^{-1}z\tilde{\nabla}_3 E_{12} \\
&+\tilde{\nabla}_3\tilde{\nabla}_3 E_{12}
\end{aligned} \tag{A.8}
$$

$$
\begin{aligned}
\Delta_{13} \;=\;& d^2X^{-6}(1+X^2)^2xz\delta p - 32d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_1 V + 4(X^2+X^4)^{-1}z\tilde{\nabla}_1\psi \\
&-32d^{-1}X(1+X^2)^{-3}x\tilde{\nabla}_3 V + 4(X^2+X^4)^{-1}x\tilde{\nabla}_3\psi - 4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_1 B + \tilde{\nabla}_3\tilde{\nabla}_1\dot{B} \\
&-\tilde{\nabla}_3\tilde{\nabla}_1\ddot{E} + 4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_1\dot{E} - 8(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_1 E + \tilde{\nabla}_3\tilde{\nabla}_1\phi - \tilde{\nabla}_3\tilde{\nabla}_1\psi \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_1\tilde{\nabla}_1 E - 4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_1 E \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_1 E - 32d^{-1}X(1+X^2)^{-3}zV_1 - 32d^{-1}X(1+X^2)^{-3}xV_3 \\
&-2T(X^2+X^4)^{-1}\tilde{\nabla}_1 B_3 + \tfrac{1}{2}\tilde{\nabla}_1\dot{B}_3 - \tfrac{1}{2}\tilde{\nabla}_1\ddot{E}_3 + 2T(X^2+X^4)^{-1}\tilde{\nabla}_1\dot{E}_3 \\
&-4(X^2+X^4)^{-1}\tilde{\nabla}_1 E_3 - 2T(X^2+X^4)^{-1}\tilde{\nabla}_3 B_1 + \tfrac{1}{2}\tilde{\nabla}_3\dot{B}_1 - \tfrac{1}{2}\tilde{\nabla}_3\ddot{E}_1
\end{aligned}
$$

$$
\begin{aligned}
&+2T(X^2+X^4)^{-1}\tilde{\nabla}_3\dot{E}_1 - 4(X^2+X^4)^{-1}\tilde{\nabla}_3 E_1 - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_1 E_1 \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_1 E_2 - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_1 E_3 - \ddot{E}_{13} + 4T(X^2+X^4)^{-1}\dot{E}_{13} \\
&-8(X^2+X^4)^{-1}E_{13} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_1 E_{23} - 4(X^2+X^4)^{-1}z\tilde{\nabla}_1 E_{33} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{13} \\
&+4(X^2+X^4)^{-1}y\tilde{\nabla}_2 E_{13} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{13} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3 E_{11} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_3 E_{12} \\
&+\tilde{\nabla}_3\tilde{\nabla}_3 E_{13}
\end{aligned} \tag{A.9}
$$

$$
\begin{aligned}
\Delta_{23} \;=\;& d^2X^{-6}(1+X^2)^2 yz\delta p - 32d^{-1}X(1+X^2)^{-3}z\tilde{\nabla}_2 V + 4(X^2+X^4)^{-1}z\tilde{\nabla}_2\psi \\
&-32d^{-1}X(1+X^2)^{-3}y\tilde{\nabla}_3 V + 4(X^2+X^4)^{-1}y\tilde{\nabla}_3\psi - 4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_2 B + \tilde{\nabla}_3\tilde{\nabla}_2\dot{B} \\
&-\tilde{\nabla}_3\tilde{\nabla}_2\ddot{E} + 4T(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_2\dot{E} - 8(X^2+X^4)^{-1}\tilde{\nabla}_3\tilde{\nabla}_2 E + \tilde{\nabla}_3\tilde{\nabla}_2\phi - \tilde{\nabla}_3\tilde{\nabla}_2\psi \\
&-4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_1 E - 4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_2 E \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_2 E - 32d^{-1}X(1+X^2)^{-3}zV_2 - 32d^{-1}X(1+X^2)^{-3}yV_3 \\
&-2T(X^2+X^4)^{-1}\tilde{\nabla}_2 B_3 + \tfrac{1}{2}\tilde{\nabla}_2\dot{B}_3 - \tfrac{1}{2}\tilde{\nabla}_2\ddot{E}_3 + 2T(X^2+X^4)^{-1}\tilde{\nabla}_2\dot{E}_3 \\
&-4(X^2+X^4)^{-1}\tilde{\nabla}_2 E_3 - 2T(X^2+X^4)^{-1}\tilde{\nabla}_3 B_2 + \tfrac{1}{2}\tilde{\nabla}_3\dot{B}_2 - \tfrac{1}{2}\tilde{\nabla}_3\ddot{E}_2 \\
&+2T(X^2+X^4)^{-1}\tilde{\nabla}_3\dot{E}_2 - 4(X^2+X^4)^{-1}\tilde{\nabla}_3 E_2 - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3\tilde{\nabla}_2 E_1 \\
&-4(X^2+X^4)^{-1}y\tilde{\nabla}_3\tilde{\nabla}_2 E_2 - 4(X^2+X^4)^{-1}z\tilde{\nabla}_3\tilde{\nabla}_2 E_3 - \ddot{E}_{23} + 4T(X^2+X^4)^{-1}\dot{E}_{23} \\
&-8(X^2+X^4)^{-1}E_{23} + 4(X^2+X^4)^{-1}x\tilde{\nabla}_1 E_{23} + \tilde{\nabla}_1\tilde{\nabla}_1 E_{23} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_2 E_{13} \\
&-4(X^2+X^4)^{-1}z\tilde{\nabla}_2 E_{33} + \tilde{\nabla}_2\tilde{\nabla}_2 E_{23} - 4(X^2+X^4)^{-1}x\tilde{\nabla}_3 E_{12} - 4(X^2+X^4)^{-1}y\tilde{\nabla}_3 E_{22} \\
&+\tilde{\nabla}_3\tilde{\nabla}_3 E_{23}
\end{aligned} \tag{A.10}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} \;=\;& -24d^{-2}X^4(1+X^2)^{-2}\ddot{\psi} + 48d^{-2}TX^2(1+X^2)^{-3}\dot{\phi} + 144d^{-2}TX^2(1+X^2)^{-3}\dot{\psi} \\
&+96d^{-2}(1+X^2)^{-3}(-4T^2+X^2)\phi + 96d^{-2}(1+X^2)^{-3}(-4T^2+X^2)\psi \\
&+384d^{-2}T(1+X^2)^{-3}x\tilde{\nabla}_1 B - 48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_1\dot{B} - 48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_1\phi \\
&+48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_1\psi + 48d^{-2}TX^2(1+X^2)^{-3}\tilde{\nabla}_1\tilde{\nabla}_1 B - 8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_1\tilde{\nabla}_1\dot{B} \\
&+8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_1\tilde{\nabla}_1\ddot{E} - 48d^{-2}TX^2(1+X^2)^{-3}\tilde{\nabla}_1\tilde{\nabla}_1\dot{E} \\
&+96d^{-2}(1+X^2)^{-3}(X^2+4x^2)\tilde{\nabla}_1\tilde{\nabla}_1 E - 8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_1\tilde{\nabla}_1\phi \\
&+16d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_1\tilde{\nabla}_1\psi + 48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_1\tilde{\nabla}_1\tilde{\nabla}_1 E \\
&+384d^{-2}T(1+X^2)^{-3}y\tilde{\nabla}_2 B - 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\dot{B} - 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\phi \\
&+48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\psi + 768d^{-2}(1+X^2)^{-3}xy\tilde{\nabla}_2\tilde{\nabla}_1 E \\
&+48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\tilde{\nabla}_1\tilde{\nabla}_1 E + 48d^{-2}TX^2(1+X^2)^{-3}\tilde{\nabla}_2\tilde{\nabla}_2 B \\
&-8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_2\tilde{\nabla}_2\dot{B} + 8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_2\tilde{\nabla}_2\ddot{E} \\
&-48d^{-2}TX^2(1+X^2)^{-3}\tilde{\nabla}_2\tilde{\nabla}_2\dot{E} + 96d^{-2}(1+X^2)^{-3}(X^2+4y^2)\tilde{\nabla}_2\tilde{\nabla}_2 E \\
&-8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_2\tilde{\nabla}_2\phi + 16d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_2\tilde{\nabla}_2\psi \\
&+48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_1 E + 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\tilde{\nabla}_2\tilde{\nabla}_2 E \\
&+384d^{-2}T(1+X^2)^{-3}z\tilde{\nabla}_3 B - 48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\dot{B} - 48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\phi \\
&+48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\psi + 768d^{-2}(1+X^2)^{-3}xz\tilde{\nabla}_3\tilde{\nabla}_1 E \\
&+48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\tilde{\nabla}_1\tilde{\nabla}_1 E + 768d^{-2}(1+X^2)^{-3}yz\tilde{\nabla}_3\tilde{\nabla}_2 E \\
&+48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\tilde{\nabla}_2\tilde{\nabla}_2 E + 48d^{-2}TX^2(1+X^2)^{-3}\tilde{\nabla}_3\tilde{\nabla}_3 B \\
&-8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_3\tilde{\nabla}_3\dot{B} + 8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_3\tilde{\nabla}_3\ddot{E} \\
&-48d^{-2}TX^2(1+X^2)^{-3}\tilde{\nabla}_3\tilde{\nabla}_3\dot{E} + 96d^{-2}(1+X^2)^{-3}(X^2+4z^2)\tilde{\nabla}_3\tilde{\nabla}_3 E \\
&-8d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_3\tilde{\nabla}_3\phi + 16d^{-2}X^4(1+X^2)^{-2}\tilde{\nabla}_3\tilde{\nabla}_3\psi \\
&+48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_1 E + 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_2 E \\
&+48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\tilde{\nabla}_3\tilde{\nabla}_3 E + 384d^{-2}T(1+X^2)^{-3}xB_1 + 384d^{-2}T(1+X^2)^{-3}yB_2
\end{aligned}
$$

$$
\begin{aligned}
&+384d^{-2}T(1+X^2)^{-3}zB_3 - 48d^{-2}X^2(1+X^2)^{-3}x\dot{B}_1 - 48d^{-2}X^2(1+X^2)^{-3}y\dot{B}_2 \\
&-48d^{-2}X^2(1+X^2)^{-3}z\dot{B}_3 + 128d^{-2}(1+X^2)^{-3}(X^2+3x^2)\tilde{\nabla}_1 E_1 \\
&+384d^{-2}(1+X^2)^{-3}xy\tilde{\nabla}_1 E_2 + 384d^{-2}(1+X^2)^{-3}xz\tilde{\nabla}_1 E_3 \\
&+48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_1\tilde{\nabla}_1 E_1 + 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_1\tilde{\nabla}_1 E_2 \\
&+48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_1\tilde{\nabla}_1 E_3 + 384d^{-2}(1+X^2)^{-3}xy\tilde{\nabla}_2 E_1 \\
&+128d^{-2}(1+X^2)^{-3}(X^2+3y^2)\tilde{\nabla}_2 E_2 + 384d^{-2}(1+X^2)^{-3}yz\tilde{\nabla}_2 E_3 \\
&+48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_2\tilde{\nabla}_2 E_1 + 48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_2\tilde{\nabla}_2 E_2 \\
&+48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_2\tilde{\nabla}_2 E_3 + 384d^{-2}(1+X^2)^{-3}xz\tilde{\nabla}_3 E_1 + 384d^{-2}(1+X^2)^{-3}yz\tilde{\nabla}_3 E_2 \\
&+128d^{-2}(1+X^2)^{-3}(X^2+3z^2)\tilde{\nabla}_3 E_3 + 48d^{-2}X^2(1+X^2)^{-3}x\tilde{\nabla}_3\tilde{\nabla}_3 E_1 \\
&+48d^{-2}X^2(1+X^2)^{-3}y\tilde{\nabla}_3\tilde{\nabla}_3 E_2 \\
&+48d^{-2}X^2(1+X^2)^{-3}z\tilde{\nabla}_3\tilde{\nabla}_3 E_3 + 128d^{-2}(1+X^2)^{-3}(X^2+3x^2)E_{11} \\
&+768d^{-2}(1+X^2)^{-3}xyE_{12} + 768d^{-2}(1+X^2)^{-3}xzE_{13} + 128d^{-2}(1+X^2)^{-3}(X^2+3y^2)E_{22} \\
&+768d^{-2}(1+X^2)^{-3}yzE_{23} + 128d^{-2}(1+X^2)^{-3}(X^2+3z^2)E_{33}
\end{aligned}
\tag{A.11}
$$