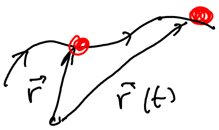
Lecture 12   03/02/2016

recap: emission of accelerated charge particles (Gauss' Units)

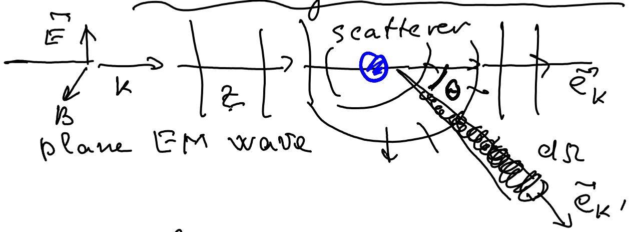


$$\boxed{P = \frac{2}{3}\frac{q^2|\ddot{\vec r}|^2}{c^3}} \leftarrow \text{for a single charge } q$$

$\ddot{\vec r} = \frac{d^2\vec r}{dt^2} \leftarrow$ particle acceleration

For system of point charges: $P = \frac{2}{3c^3}\left(\sum_i q_i \ddot{\vec r}_i\right)^2$

Scattering cross section:



$$\boxed{\frac{d\sigma}{d\Omega} = \frac{dP}{S_0} = \frac{I(\theta)\,d\Omega\cdot r^2}{S_0}}$$

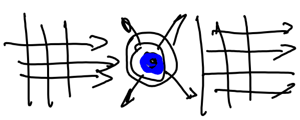
$S_0 = \frac{c}{4\pi}E^2 \leftarrow$ Poynting vector

Differential cross section

Total cross section

$$\boxed{\sigma_{tot} = \int \frac{d\sigma}{d\Omega}\cdot d\Omega = \frac{P}{S_0}}$$

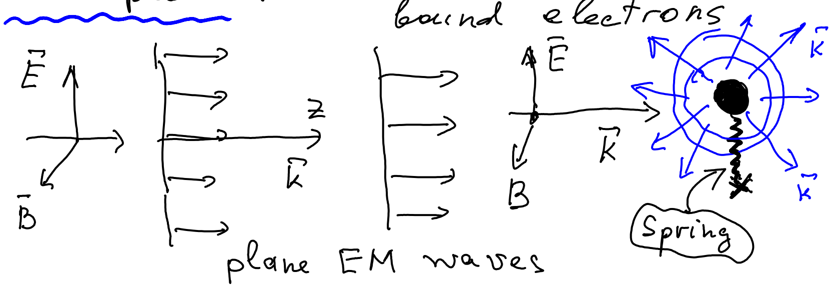
Example:
Scattering by free electrons:



Cross section of the EM-scattering by free electrons

$$\boxed{\sigma_0 = \frac{8\pi}{3}r_0^2} \qquad r_0 = \frac{e^2}{m_e c^2} \leftarrow \text{classical radius of electron}$$

Example 2: Scattering of EM waves by bound electrons



Harmonic force:

$\vec F_h = \vec e_x F_x = -\vec e_x m_e \omega_0^2 x$

Electric force

$\vec F_E = e\vec E = \hat e_x \cdot e E_0 e^{ikz - i\omega t}$

$\vec k = k\cdot \hat e_z$

Equation of the electron motion:

$$m_e \ddot x + m_e \omega_0^2 x = eE_0 e^{-i\omega t} \qquad (z=0)$$

Solution of the motion equation:

$$x = x(t) = x_h(t) + x_{non}(t)$$

solution of homogeneous equation

solution of non-homogeneous equation

$$x_{non}(t) = A \cdot e^{-i\omega t}; \quad m_e(-i\omega)^2 A \cancel{e^{-i\omega t}} + m_e \omega_0^2 A \cancel{e^{-i\omega t}} = eE_0 \cancel{e^{-i\omega t}}$$

$$A = \frac{eE_0}{m_e(\omega_0^2 - \omega^2)} \longleftarrow \boxed{A(\omega_0^2 - \omega^2) = \frac{eE_0}{m}}$$

$$\boxed{x_{non} = \frac{eE_0}{m_e(\omega_0^2 - \omega^2)} e^{-i\omega t}}$$

Electron acceleration: $\ddot{x} = -\frac{\omega^2}{\omega_0^2 - \omega^2} \frac{eE_0}{m_e} e^{-i\omega t}$

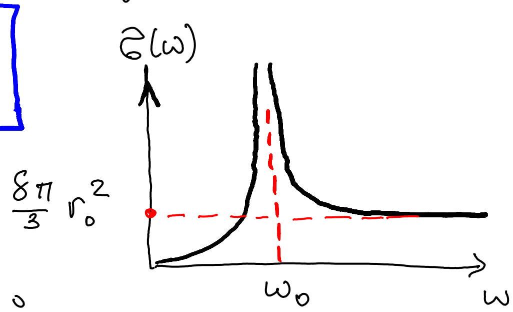
Power of the EM emisson:

$$\boxed{P = \frac{2e^2 |\ddot{x}|^2}{3c^3} = \frac{2}{3} \frac{e^4}{m_e^2 c^3} \left(\frac{\omega^2}{\omega^2 - \omega_0^2}\right)^2 \cdot E_0^2}$$

$$\sigma = \frac{P}{S} = \frac{2}{3} \frac{\left(\frac{e^2}{m_e c^2}\right)^2 \cdot c \cdot E_0^2}{\frac{c}{4\pi} E_0^2} \left(\frac{\omega^2}{\omega^2 - \omega_0^2}\right)^2 = \frac{8\pi}{3} r_0^2 \left(\frac{\omega^2}{\omega^2 - \omega_0^2}\right)^2$$

Poynting vector

Total scattering cross section $\longrightarrow$ $\boxed{\sigma = \frac{8\pi}{3} r_0^2 \left(\frac{\omega^2}{\omega^2 - \omega_0^2}\right)^2}$



Resonance cross section $\longrightarrow$ $\omega \to \omega_0$

$\sigma \to \infty$ if $\omega \to \omega_0$