Dissertation Talking Points

1 Title Page

- Thank you everyone for coming. Welcome to my dissertation defense on the topic Cosmological Perturbations as applied to both standard Einstein Gravity and Conformal gravity

2 Overview

- Alright so, to give an overview of what will be covered,
- we will first form the necessary background of cosmological perturbation theory. Talking about the geometry of the universe, introduce einstein gravity, perturbations, and coordinate invariance
- As an approach to simplify and solve the equations arising in cosmology, we're going to analyze something called the SVT decomp. in 3 dimensions and then perform some generalizations to four dimensions, demonstrating specific applications of both within a de Sitter background geometry
- Then we'll cover a discussion of cosmological perturbations in conformal gravity, and demonstrate a calculation of the fluctuations in the early universe radiation era
- Finally we'll wrap it up with overall conclusions and I'll demonstrate some of the computations involved in order to do the calculations that we'll see through this presentation

3 Cosmological Geometries

- Let's first discuss the geometry that is relevant to cosmology. If we take a look at the distribution of matter in the universe, we will see something like this image taken from the hubble telescope (which I'll add in served as my desktop background for quite a number of years).
- What we observe is that no matter where you are in the universe and no matter what direction you look, the universe is the same on large scales. More specifically, while the structure of universe may be in-homogeneous on small scales, on a large scale the universe is statistically homogeneous and isotropic. These two features are embodied in what is called the cosmological principle.
- Based solely on arguments of homogeneity and isotropy, the large scale geometry of such a universe is described through the RW metric and de Sitter metric (of which one can show that de Sitter space is actually a subset of the RW geometry).
- It of interest to note that with a proper choice of coordinates, the roberston walker geometries can all be expressed in a conformal to flat form
- with this equation here showing the spacetime line element being expressed as a minkowski spacetime multiplied by an overall conformal factor

4 Cosmological Geometries - Robertson Walker

- To see what the RW geometry entails, we have an expression for the spacetime line element in eq. (1), here in comoving coordinates. The geometry describes the expansion of space over time as characterized by the functional form of the scale factor $a(t)$
- The comoving coordinates are at rest with respect to the hubble flow, and if we look at the figure here we can see that while the comoving distance between these two galaxies remains constant, the proper distance increases as space expands according to $a(t)$
- The space itself that is expanding, referred to as the 3-space, is a space of uniform curvature, which can be represented by the curvature constant k, taking the values of $-1, 0, 1$. These values correspond to hyperbolic, flat, or spherical space respectively.
- Spaces of constant curvature are called maximally symmetric, where the curvature tensors take the specific form in eq(2). So here we have the Riemann tensor, which we might recall is the unique tensor composed of second order derivatives of the metric which measures the local curvature. We also have its contractions the ricci tensor, and the ricci scalar and we see that the ricci scalar is a constant in a max. symm. 3-space.
- As mentioned before, with a proper choice of coordinates, the RW metric can be cast into a conformal to flat form
- The simplest case is for $k = 0$, in which if we define the conformal time τ and set $k = 0$, then the line element take the form of eq (4), which we can recognize as conformal to a spherical polar flat metric
- For $k = \pm 1$, we have to perform additional coord. transformations, and we just note that for these the conformal factor is a function of both space and time.
- So this describes the geometry of the large scale universe, but in order to discuss the interaction of gravity and matter, one needs to introduce Einstein field equations, which we do now

5 Einstein Gravity

- One starts with the Einstein Hilbert action defined as the coordinate invariant integral over the Ricci scalar
- Functional variation w.r.t. the metric yields the Einstein tensor $G_{\mu\nu}$, and likewise upon specification of a matter action, one obtains the energy momentum tensor
- In requiring the sum of both the E.H. action and the matter action to be stationary with respect to arbitrary variations in the metric, we obtain the EFE's.
- With (10) showing an identity that relates the derivative of the ricci tensor to its contraction, we can see that the Einstein tensor is conserved
- In the EFE's the interaction of gravity and matter can be seen via the LHS being a pure function of the metric representing the curvature of space while the RHS defines the source of matter and energy
- We'll now look at the linearization of the Einstein field equations according to cosmological perturbation theory

6 Cosmological Perturbation Theory

- So as discussed prior, on a large scale the universe is homogeneous and isotropic which we can think of as the smooth surface of this sphere.
- Now in order to capture the departures from homogeneity and isotropy, things that are necessary in order to form localized structures in spacetime, we introduce small fluctuations on top of the otherwise smooth background. Thus we define the metric according to a background contribution and first order perturbation $h_{\mu\nu}$

- If we then substitute the metric into $G_{\mu\nu}$, it then can be split into a background piece and fluctuation tensor
- Upon similarly perturbing $T_{\mu\nu}$, we can then form the entire background field equations and first order fluctuation equations, where here we've combined them into the tensor $\Delta_{\mu\nu}$
- The background equations serve to define the rate of expansion of space given the source, whereas the fluctuation equations describe the evolution of metric perturbations due to things like over densities arising from source

7 Coordinate Transformations

- So upon perturbing the EFE's, we will need to consider the effect of coordinate transformations
- The field equations are covariant w.r.t. general coordinate transformations, with the metric transforming as in (18)
- If we now consider an infinitesimal coordinate transformation with the vector field ϵ small in the same sense that h is small, then it is conveinent to attribute the whole change in $g_{\mu\nu}$ to a change in the perturbation $h_{\mu\nu}$
- The fluctuation eqns are then to be invariant under the so called gauge transformation of eq (20), where $\Delta h_{\mu\nu}$ is given by this symmetric sum of derivatives onto epsilon
- Thus if $h_{\mu\nu}$ serves as a solution to the EFE's, then and $h'_{\mu\nu}$ defined by (20) will also serve as a solution
- Now since $h_{\mu\nu}$ is a 4x4 symmetric rank 2 tensor it has 10 components, and with the four coordinate functions that define the vector field ϵ, one can then use the coordinate freedom to reduce $h_{\mu\nu}$ to six independent components
- Its also quite instructive to look at the transformation of the fluctuation tensors themselves
- Here we note that if the background tensor vanishes, then the fluctuation tensors themselves are separately gauge invariant
- However, if the background does not vanish, then it only the entire sum of $\delta G_{\mu\nu} + \delta T_{\mu\nu}$ that is gauge invariant. In Einstein gravity the background only vanishes in spaces where the Ricci tensor itself vanishes, and thus for non-flat cosmological geometries, its only $\Delta_{\mu\nu}$ that is gauge invariant

8 Solution Methods

- Now lets take a look at what the perturbed Einstein tensor looks like in a non-flat geometry
- Here we have an expression for the spatial components of the Einstein tensor, and we can get a sense for 1) how many terms there are and 2) how tightly these terms are coupled together
- In fact, if one were to expand out the contractions over dummy indices, and try to look at a single spatial component, like the radia component δG_{rr}, one would get an expression with about 3-5 times as many terms
- So even after linearizing we require additional methods to simplify and decouple the fluctuation equations

9 SVT3 Decomposition

- One such method, called the SVT3 decomposition, is to take $h_{\mu\nu}$ and decompose it into a basis of scalars, vectors, and tensors defined according to their transformation behavior under 3D rotations
- To show this we are going to keep things generic and first factor out a conformal factor from $h_{\mu\nu}$ and express it in terms of the perturbation $f_{\mu\nu}$
- Then we form the line element, here's our background and perturbation, and then here we do a 3+1 splitting to separate the time and spatial components

- So now we want to define the time and space components of $f_{\mu\nu}$ in terms of scalars, vectors, and tensors
- Hence we view f_{00} as a 3-scalar, f_{0i} as a 3-vector and f_{ij} and a 3-tensor. f_{00} just being a scalar, we redefine it in terms of a ϕ, here we break up the 3 vector in to a transverse vector B_i and the derivatives of a scalar B, and here we have two scalars ψ and E, a transverse vector E_i and a TT tensor E_{ij}.
- If we count up the components, we have 4 scalars, two two component transverse vectors E_i and B_i, and one 2 component TT tensor E_{ij}, adding up to 10 in total
- finally here the total line element in the SVT3 basis

10 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 1/4

- To see how the SVT3 decomposition may be helpful in solving the perturbation equations, we are going to evaluate the EFE's in the de Sitter background
- As mentioned earlier, the de Sitter background can be expressed as a special case of the RW metric which here corresponds to choosing the scalar factor to have the form of $1/H\tau$
- While the RW metric consists of a 3-space that is max. symm., the de Sitter space is actually maximally symmetric w.r.t. the full 4D spacetime and so its curvature tensors take this form
- The E.M. tensor that gives rise to the desitter space is that which consists of just a cosmological constant - a simple constant background energy that drives the expansion of space
- de Sitter is chosen here just to keep things relatively simple, but the same SVT3 decomposition can be carried out in a more generic RW space where one considers an energy momentum tensor consisting of a perfect fluid
- So in perturbing the energy momentum tensor, we simply get a fluctuation proportional to $f_{\mu\nu}$

11 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 2/4

- Now we are going to insert the SVT3 decomposed $f_{\mu\nu}$ into $\delta G_{\mu\nu}$ to obtain eq 30
- We see that in the SVT3 basis the number of terms we have to deal with has been reduced a little bit, but still quite a few remain

12 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 3/4

- To form the fluctuation equations, we need to add in $\delta T_{\mu\nu}$ to form the full $\Delta_{\mu\nu}$ here
- A couple things to note: 1) if we look at the various components, we can see for example that scalars are coupled to vectors here (in Δ_{00}) and scalars, vectors, and tensors are still coupled together here (Δ_{ij}) 2) We recall that $\Delta_{\mu\nu}$ must be entirely gauge invariant, and thus the specific combinations of the SVT3 quantities that appear in each component of $\Delta_{\mu\nu}$ must themselves be gauge invariant
- Here we've identified the appropriate gauge invariant combinations, and if one counts the DOF, we see that there are two scalars, one 2-component transverse vector, and one 2 component TT tensor, totaling a set of 6 gauge invariants as expected from the gauge freedom from ϵ that we mentioned earlier
- Now in order to actually solve these, we will need to decouple them, and we find that by applying appropriate higher derivatives, we can do so

13 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 4/4

- Here we have a set of 6 decoupled equations in the 6 gauge invariants, in which we can solve
- So to give quick recap here, we first perturbed the Einstein and E.M. around a de Sitter background
- Then we decomposed $h_{\mu\nu}$ into a basis of 3-scalars, 3-vectors, and 3-tensors
- Inserted that $h_{\mu\nu}$ in the flucuation tensors
- We formed the perturbed field equations, noting that they are entirely gauge invariant
- We express the FE's in terms of the gauge invariant SVT3 combinations
- Finally applied derivatives to decouple the SVT3 modes

14 SVT3 Integral Formulation 1/4

- Prior, I simply stated that this is the decomposition of $h_{\mu\nu}$ in the SVT3 basis. However, one might ask, is such a decomposition always possible? Or even further one might ask how does a quantity like ϕ transform under gauge transformations?
- To properly address the SVT3 decomposition, we need to effectively invert eq 34 so as to express each SVT3 quantity directly in terms of $h_{\mu\nu}$ itself.
- To keep things clear and simple, we are going to analyze the following SVT3 decomposition around a flat minkowski background and we are going to start with the decomposition of the 3-vector h_{0i} into the transverse B_i and the scalar B

15 SVT3 Integral Formulation 2/4

- So we seek to decompose a general vector into two parts, a transverse part and a longitudinal part, written as the derivative of a scalar
- However, by itself the derivative of a scalar is not necessarily longitudinal; if we take the divergence of a general vector, we see that any scalar that vanishes under the laplacian will be transverse
- So to form the longitudinal part, we have to find the derivative of a scalar which could never be transverse
- To accomplish, we first introduce a flat spatial greens function and an identity, which here is just a particular ordering of derivatives and the product rule
- Upon integrating (36), we arrive at the decomposition of a generic scalar into its non-harmonic and harmonic components; here when I say harmonic, it is to mean any function that vanishes under the generalized laplacian
- Upon applying the laplacian to (37), one can show that the harmonic surface integral vanishes identically and so we are left only with the non-harmonic contribution
- So in order form the derivative of a scalar which could never be transverse, we require the scalar to be non-harmonic, and thus we require the surface contribution to vanish;
- This means that the scalar V must itself vanish asymptotically or decay sufficiently fast
- So with this being the definition of V, we can now form the longitudinal component and finally arrive at this definition for the entire decompisition
- Here we see we have defined the transverse and longitudinal components in terms of the vector field V_i itself

16 SVT3 Integral Formulation 3/4

- We can also take the decomposition we just did and cast it into the form a transverse vector project, here given by this quantity Π_{ij}. Applying Π_{ij} to a generic vector projects out its transverse component, and it obeys the expected projected algebra given in (42)
- So getting back to the decomposition of h_{0i}, we can now express B_i and B in terms of h_{0i}, and we note that the these SVT3 quantities are defined as non-local integrals over the metric perturbation, and we additionally require that the scalar B vanish asymptotically or decay sufficiently fast

17 SVT3 Integral Formulation 4/4

- So that covers the vector decomposition, but what about the tensor decomposition of h_{ij}?
- Using a similar procedure now with tensors, we introduce a vector W and present eq (45) as the transverse traceless component of h_{ij}
- Eq. (45) is automatically traceless, and to be transverse we require the vector W to obey eq. (46) here, which thus serves to define W
- Eq (45) thus represents the representation of the TT tensor E_{ij}, however we still need to obtain representations for the vectors E_i and scalars E and ψ
- To form a transverse vector, we note that we can further decompose W intro its transverse and longitudinal parts
- So finally putting everything together, we arrive at the following decomposition of h_{ij} in terms of all the SVT3 quantities
- Again one can readily check that this quantity is TT, this is transverse, and the remaining scalars are defined by the metric or derivative prefactors to match the form in (44)
- So this completes the inversion of the SVT3 basis entirely in terms $h_{\mu\nu}$

18 SVT4 Setup

- There are some shortcomings to this basis though. Recall that in the SVT3 basis, we had to do a 3+1 decomposition on both $h_{\mu\nu}$ and the fluctuation tensors themselves to give us a larger set of equations to solve. And since the full transformations of GR consist of general 4D coordinate transformations, something like this quantity ϕ which is proportional to h_{00} will transform into linear combinations of any other SVT3 quantities if we do coordinate transformations in both time and space
- It would be more natural to use a basis that is covariant with respect to the underlying transformation group of GR and it may lead to a simpler set of equations to solve. To construct such a basis, we seek to generalize the existing SVT3 basis in two ways
- A) Generalize to $D = 4$ to match GR transformations, in fact there is no additional overhead to generalize this to arbitrary dimension D, so we'll do that
- And B) Generalize to curved space backgrounds beyond the Minkowski treatement that we showed for SVT3
- When going to curved space, matters are a little more complicated as we cannot simply take the partial derivatives and carry them into covariant derivatives since curved space covariant derivatives no longer commute
- From (50) we see that the commutatation is sensitive to the Riemann curvature tensor which is non-zero in the cosmological backgrounds, so the overall decomposition is going to have to take this into account
- In four dimensions here is what the covariant decomposition would look like

- We have two scalars, one 3 component transverse vector, and one 5 component TT tensor, adding up to 10 components in total
- To address A) and B) here, we are going to show the covariant decomposition around a maximally symmetric de Sitter background. I'll add in that in our paper we've been able to formulate the decomposition around arbitrary D dimensional curved spaces as well

19 SVTD Integral Formulation - Maximally Symmetric Space 1/2

- Ok so again when we say a maximally symmetric space, we mean a space of constant curvature, where the curvature tensors take the form of eq. (53), and we see that the Ricci scalar is a constant
- To form the TT projection, we introduce the curved space Green's function of (54), and with it one can then check that (55) is traceless and tranverse where the vector W is defined in terms of h within (55)
- To show transversness, one has to make use of all these commutation relations, which are proportional to the Ricci scalar

20 SVTD Integral Formulation - Maximally Symmetric Space 2/2

- Now in order to define the transverse vectors F_μ and the scalars F and χ, similar to before we can break up the vector W into its transverse and longitudinal components, where introduce a different green's function here
- Finally, we make the appropriate definition, and complete the SVTD representation to be able to form (61)
- If we take $D = 3$ and take the background to the flat krocker delta, the two green's function become equivalent and we recover the SVT3 decomposition

21 SVT4 $\delta G_{\mu\nu}$ in a de Sitter Background 1/2

- Ok, now that we've formed a new covariant decomposition, let's go ahead an apply it to the same deSitter background case that we saw earlier in the SVT3 basis
- Here we note that the covariant derivatives are with respect to the full background, and in forming the full fluctuation equations, we get a nice compact form for $\Delta_{\mu\nu}$ given here
- If we recall that $\Delta_{\mu\nu}$ is entirely gauge invariant, we see that is it is $F_{\mu\nu}$ and χ that are the SVT4 gauge invariant quantities, and if we count the components, we see we have one scalar, one five component TT tensor, summing to 6 independent components in total
- To separate the SVT modes we can apply the trace and use this scalar commutation relation and then applying higher derivatives we get the decoupled scalar and tensor modes

22 SVT4 $\delta G_{\mu\nu}$ in a de Sitter Background 2/2

- So here we can compare the SVT4 basis to the SVT3 basis and find that not only is the SVT4 basis covariant w.r.t. the transformations of GR, it also provides a much more convenient and simpler formalism

23 Conformal Gravity Introduction

- So all the field equations presented so far have been in the context of standard Einstein gravity and we are now going to discuss the evolution of cosmological perturbations in an alternative theory of gravitation called conformal gravity

- The basis of conformal gravity is that we require the gravitational action to be locally conformal invariant, meaning that it is to be invariant under rescalings of the metric with arbitrary Ω here.
- The Weyl action here is the single unique action composed purely of the metric that is locally conformally invariant, and unlike the EH action where we had a coordinate invariant integral over the contraction of the Ricci tensor, here we have a contraction over the square of the Weyl tensor C and α_g is a dimensionless coupling constant
- So $C_{\lambda\mu\nu\kappa}$ here is in fact the traceless component of the Riemann tensor and has the unique property that it is invariant under conformal transformations
- With this theory having been advanced by my advisor Philip Mannheim, astrophysical and cosmological support for conformal gravity is motivated by the excellent agreement between fits of the accelerating universe Hubble plot data as well as galactic rotation curves of over 100 spiral galaxies, work done with Mannheim and Obrien, all without the imposition of dark energy or dark matter
- Alright so to obtain the FE's, as before, we vary the action w.r.t. the metric tensor, to obtain what is called the Bach tensor $W_{\mu\nu}$, which serves as the analog to the Einstein tensor G. $W_{\mu\nu}$ can be expressed in two ways here, firstly as derivatives onto the Weyl tensor or secondly as this entire combination $W_2 - W_1$ where these are expressed in terms of the Ricci tensor.
- We can see that vacuum solutions to the conformal gravity correspond to either a) a vanishing ricci tensor, or b), a vanishing Weyl tensor
- With the ricci tensor vanishing, this means that all vacuum solutions to Einstein gravity and also vacuum solutions to conformal gravity
- With the weyl tensor being conformally invariant, it therefore vanishes in any geometry that is conformal to flat
- and as we have seen all the RW geometries can be cast into the conformal to flat form, so therefore in cosmology, the background Bach tensor vanishes identically
- Now the field equations here are fourth order and include quite a large number of terms esepcially in comparison to the Einstein field equations here
- However, we will see that the conformal properties inherited in conformal gravity provide some very simplifications in cosmology that are not shared with einstein gravity

24 Conformal Invariance in Conformal Gravity

- So to discuss these conformal properties, we first note that the Bach tensor is traceless and conserved and that under conformal transformation, its covariant form transforms as Ω^{-2}
- As with before, we can decompose this into a background contribution and a first order fluctuation, to obtain the background and perturbative field equations

25 Trace Properties in Conformal Gravity

- In conformal gravity, its convenient to cast things in terms of $K_{\mu\nu}$, which is simply the traceless component of $h_{\mu\nu}$
- Now if we generically express the Bach fluctuation in terms of $h_{\mu\nu}$, we get one contribution from $K_{\mu\nu}$ and one contribution from the trace of $h_{\mu\nu}$
- Using the properties of the Bach tensor under conformal transformations, one can show that the perturbation as a function of the trace and the trace of the perturbation itself are both proportional to the background Bach tensor

- What this means is that in geometries where the background $W_{\mu\nu}$ vanishes, the Bach fluctuation is both traceless and can be written entirely in terms of the 9 component $K_{\mu\nu}$
- And with our freedom of four coordinate transformations, we see that in conformal cosmology $h_{\mu\nu}$ consists of 5 independent physical components

26 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 1/3

- So we are now going to construct the cosmological field equations in conformal gravity
- We start off completely generically, not having yet specified a background, and we find a pretty extensive expression here for $\delta W_{\mu\nu}$, with 52 terms in the $K_{\mu\nu}$ sector and 19 in the trace sector

27 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 2/3

- Now we evaluate these expression in a conformal to Minkowski background, with an arbitrary conformal factor
- and we get a large number of terms here, 151 in total

28 SVT4 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 3/3

- Despite its formidable form, we can express this set of terms in a rather compact form as a sequence of derivatives onto the conformal factor
- However, even with this immense simplification, we see that the various components of $K_{\mu\nu}$ are still tightly coupled to each other and the solution is not immediately obvious
- However, if we use what we have learned about the transverse traceless tensor projection, and form the integral expression in terms of $K_{\mu\nu}$, we see that these sequences of derivatives precisely gets rid of all the non-local integrals here and allows us to express the fluctuations as just one single term here
- So this equation here represents the fluctuation FE of conformal gravity and we that not only is it expressed a function of a 5 component TT tensor, but that each component of the TT tensor is completely decoupled from one another, and so the solution is readily obtainable

29 SVT4 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 3/3

- To touch basis on our earlier SVT4 decomposition, we can evaluate the conformal fluctuations in a conformal to flat background, where we then obtain the fluctuation equations here
- With $F_{\mu\nu}$ being the TT component of $h_{\mu\nu}$, we see that the SVT4 basis nicely reconciles our previous result

30 Conformal Gravity Robertson-Walker Radiation Era

- To illustrate a solution of these equations in conformal gravity, we are going look at fluctuations in the early universe described by a radiation dominated perfect fluid. In the observational fits to the spiral galaxies and hubble data plots I mentioned earlier, one finds that phenomenologically, the $k = -1$ hyperbolic RW geometry is preferred, and here is its expression in comoving coordinates
- and here is the same geometry as expressed in conformal to flat coordinates
- The plane wave solutions to this fourth order equation are given by a sum of polarization tensors $A_{\mu\nu}$ and $B_{\mu\nu}$, where here the $B_{\mu\nu}$ term has a prefactor proportional to the p' time coordinate in the conformal to flat coordinate basis

- By doing appropriate coordinate transformations, one can show that the leading order time behavior of $h^{TT}_{\mu\nu}$ resides in the product of Ω^2 time p', and upon doing the necessary coordinate transformations to convert this to the comoving time t, we find that fluctuations grow as t^4. This can be contrasted with the einstein radiation era perfect fluid solutions where fluctuations grow as $t^{1/2}$
- more comments on size of growth?

31 Conclusions

- Alright so that completes the discussion of cosmological fluctuations
- To summarize what we've covered here, uh
- We presented an integral formalism to represent the SVT3 basis components in terms of $h_{\mu\nu}$, find that the SVT components are composed of nonlocal integrals and automatically incoprporate asymptotic boundary conditions
- After forming the fluctuation equations, we could effectively decouple the SVT modes by applying higher derivatives
- We then generalized the SVT decompoisition to arbitrary doimension D and in a maximally symmetric curved space, finding that in $D = 4$, the fluctuations are both simpler and covariant
- And finally we formulated the cosmological fluctuations as applied to conformal gravity
- Here we saw that despite the rather formiddible fourth order nature of the theory, in conformal to flat backgrounds the equations take a remarkably simple form, whereby we demonstrated the growth of fluctuations going as t^4 in a radiation era early universe RW geometry

32 Computational Methods

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33 References

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35 The End