# AdS SVT3 $\Omega(z)$ v3

## 1 Background

$$ds^2 = \Omega^2(z)\left[-dt^2+dx^2+dy^2+dz^2\right] = \Omega^2(z)\tilde{g}_{\mu\nu}dx^\mu dx^\nu \tag{1.1}$$

$$\Omega(z) = \frac{1}{Hz} \tag{1.2}$$

$$R_{\lambda\mu\nu\kappa} = -H^2(g_{\mu\nu}g_{\lambda\kappa}-g_{\lambda\nu}g_{\mu\kappa}), \qquad R_{\mu\nu}=3H^2g_{\mu\nu}, \qquad R=12H^2 \tag{1.3}$$

$$G_{\mu\nu} = -3H^2g_{\mu\nu}, \qquad T_{\mu\nu}=3H^2g_{\mu\nu} \tag{1.4}$$

## 2 Perturbations

$$ds^2 = \Omega^2(z)\left(\tilde{g}_{\mu\nu}+f_{\mu\nu}\right)dx^\mu dx^\nu \tag{2.1}$$

$$f_{00} = -2\phi, \qquad f_{0i}=\tilde{\nabla}_iB+B_i \tag{2.2}$$

$$f_{ij} = -2\psi\tilde{g}_{ij}+2\tilde{\nabla}_i\tilde{\nabla}_jE+\tilde{\nabla}_iE_j+\tilde{\nabla}_iE_j+2E_{ij} \tag{2.3}$$

$$\delta T_{\mu\nu} = 3\Omega^2H^2f_{\mu\nu} \tag{2.4}$$

$$\begin{aligned}\delta G_{00} = {} & -2\tilde{\nabla}_a\tilde{\nabla}^a\psi-2\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi+\phi\Omega^{-2}(4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega-2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\\ & +\psi\Omega^{-2}(4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega-2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE+2\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega\\ & -4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a\\ & +\Omega^{-2}(2\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega-4\Omega\tilde{\nabla}_b\tilde{\nabla}_a\Omega)\tilde{\nabla}^bE^a+E_{ab}\Omega^{-2}(2\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega-4\Omega\tilde{\nabla}^b\tilde{\nabla}^a\Omega)\end{aligned} \tag{2.5}$$

$$\begin{aligned}\delta G_{0i} = {} & \Omega^{-2}(-2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega+\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_iB-2\tilde{\nabla}_i\dot{\psi}+2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i\Omega-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i\tilde{\nabla}_a\dot{E}+\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^aB_i\\ & -\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i+\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB_i-\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{E}_i+B_i\Omega^{-2}(-2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega+\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\\ & -\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_iB^a-\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i\dot{E}^a-2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a\Omega\end{aligned} \tag{2.6}$$

$$\begin{aligned}\delta G_{ij} = {} & -2\ddot{\psi}\tilde{g}_{ij}-\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}-\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi+\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi-2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B}\\ & -2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi+2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE-2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega\\ & +4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE+2\Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\psi+2\Omega^{-1}\tilde{\nabla}_i\psi\tilde{\nabla}_j\Omega+\tilde{\nabla}_j\tilde{\nabla}_i\dot{B}-\tilde{\nabla}_j\tilde{\nabla}_i\ddot{E}\\ & +\Omega^{-2}(-4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega+2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_j\tilde{\nabla}_iE+\tilde{\nabla}_j\tilde{\nabla}_i\phi-\tilde{\nabla}_j\tilde{\nabla}_i\psi\end{aligned}$$

$$
\begin{aligned}
&-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a E - 2\dot{B}^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a \\
&+2\tilde{g}_{ij}\Omega^{-2}(-\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega + 2\Omega\tilde{\nabla}_b\tilde{\nabla}_a\Omega)\tilde{\nabla}^b E^a + \tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j \\
&+\Omega^{-2}(-2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_i E_j + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i \\
&+\Omega^{-2}(-2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E_a - \ddot{E}_{ij} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij} \\
&+2\Omega^{-1}\tilde{\nabla}_a E_{ij}\tilde{\nabla}^a\Omega + E_{ij}\Omega^{-2}(-4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega) \\
&+2\tilde{g}_{ij}E_{ab}\Omega^{-2}(-\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega + 2\Omega\tilde{\nabla}^b\tilde{\nabla}^a\Omega) - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_{ja} - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_{ia}
\end{aligned} \tag{2.7}
$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} \;=\;& -6\ddot{\psi}\Omega^{-2} - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} \\
&-6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi + \phi\Omega^{-4}(-4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega) \\
&+\psi\Omega^{-4}(-4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega) + 2\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E - 4\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a E\tilde{\nabla}_b\tilde{\nabla}^b\Omega \\
&+6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E - 8\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega + 16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E - 6\dot{B}^a\Omega^{-3}\tilde{\nabla}_a\Omega \\
&+6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a \\
&+8\Omega^{-4}(-\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega + 2\Omega\tilde{\nabla}_b\tilde{\nabla}_a\Omega)\tilde{\nabla}^b E^a + 8E_{ab}\Omega^{-4}(-\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega + 2\Omega\tilde{\nabla}^b\tilde{\nabla}^a\Omega)
\end{aligned} \tag{2.8}
$$

$$
\delta T_{00} \;=\; -6H^2\phi\Omega^2 \tag{2.9}
$$

$$
\delta T_{0i} \;=\; 3H^2\Omega^2\tilde{\nabla}_i B + 3H^2 B_i\Omega^2 \tag{2.10}
$$

$$
\delta T_{ij} \;=\; -6H^2\tilde{g}_{ij}\psi\Omega^2 + 6H^2\Omega^2\tilde{\nabla}_i\tilde{\nabla}_j E + 3H^2\Omega^2\tilde{\nabla}_i E_j + 3H^2\Omega^2\tilde{\nabla}_j E_i + 6H^2 E_{ij}\Omega^2 \tag{2.11}
$$

$$
g^{\mu\nu}\delta T_{\mu\nu} \;=\; 6H^2\phi - 18H^2\psi + 6H^2\tilde{\nabla}_a\tilde{\nabla}^a E \tag{2.12}
$$

# 3 Field Equations

$$
\begin{aligned}
\Delta_{00} \;=\;& -2\tilde{\nabla}_a\tilde{\nabla}^a\psi - 2\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi + \psi\Omega^{-2}(4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega - 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega) \\
&-2\phi\Omega^{-2}(3H^2\Omega^4 - 2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega) - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\
&+2\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega - 4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a \\
&+\Omega^{-2}(2\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega - 4\Omega\tilde{\nabla}_b\tilde{\nabla}_a\Omega)\tilde{\nabla}^b E^a + E_{ab}\Omega^{-2}(2\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega - 4\Omega\tilde{\nabla}^b\tilde{\nabla}^a\Omega)
\end{aligned} \tag{3.1}
$$

$$
\begin{aligned}
\Delta_{0i} \;=\;& \Omega^{-2}(3H^2\Omega^4 - 2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_i B - 2\tilde{\nabla}_i\dot{\psi} + 2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i\Omega \\
&-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i\tilde{\nabla}_a\dot{E} + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i + \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B_i - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{E}_i \\
&+B_i\Omega^{-2}(3H^2\Omega^4 - 2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega) - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i B^a \\
&-\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i\dot{E}^a - 2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a\Omega
\end{aligned} \tag{3.2}
$$

$$
\begin{aligned}
\Delta_{ij} \;=\;& -2\ddot{\psi}\tilde{g}_{ij} - 6H^2\tilde{g}_{ij}\psi\Omega^2 - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi \\
&-2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\
&-2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E + 6H^2\Omega^2\tilde{\nabla}_i\tilde{\nabla}_j E + 2\Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\psi \\
&+2\Omega^{-1}\tilde{\nabla}_i\psi\tilde{\nabla}_j\Omega + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} + \Omega^{-2}(-4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_j\tilde{\nabla}_i E + \tilde{\nabla}_j\tilde{\nabla}_i\phi \\
&-\tilde{\nabla}_j\tilde{\nabla}_i\psi - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a E - 2\dot{B}^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a \\
&+2\tilde{g}_{ij}\Omega^{-2}(-\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega + 2\Omega\tilde{\nabla}_b\tilde{\nabla}_a\Omega)\tilde{\nabla}^b E^a + \tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j \\
&+\Omega^{-2}(3H^2\Omega^4 - 2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_i E_j + \tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i
\end{aligned}
$$

$$
\begin{aligned}
&+\Omega^{-2}(3H^2\Omega^4 - 2\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\tilde{\nabla}_j E_i - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E_a - \ddot{E}_{ij} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij}\\
&+2\Omega^{-1}\tilde{\nabla}_a E_{ij}\tilde{\nabla}^a\Omega + E_{ij}\Omega^{-2}(6H^2\Omega^4 - 4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\\
&+2\tilde{g}_{ij}E_{ab}\Omega^{-2}(-\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega + 2\Omega\tilde{\nabla}^b\tilde{\nabla}^a\Omega) - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_{ja} - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_{ia}
\end{aligned} \tag{3.3}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} &= -6\ddot{\psi}\Omega^{-2} - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi + 4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B}\\
&-6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi + \psi\big(-18H^2 + \Omega^{-4}(-4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\big)\\
&+\phi\big(6H^2 + \Omega^{-4}(-4\Omega\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 2\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega)\big) + 2\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E\\
&+\tilde{\nabla}_a\tilde{\nabla}^a E(6H^2 - 4\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}^b\Omega) + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E - 8\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega\\
&+16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E - 6\dot{B}^a\Omega^{-3}\tilde{\nabla}_a\Omega + 6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a\\
&+8\Omega^{-4}(-\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega + 2\Omega\tilde{\nabla}_b\tilde{\nabla}_a\Omega)\tilde{\nabla}^b E^a + 8E_{ab}\Omega^{-4}(-\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega + 2\Omega\tilde{\nabla}^b\tilde{\nabla}^a\Omega)
\end{aligned} \tag{3.4}
$$

# 4 Field Equations (G.I. Form)

$$
\alpha = \phi + \psi + \dot{B} - \ddot{E} \tag{4.1}
$$

$$
\begin{aligned}
\gamma &= \phi - \psi + \dot{B} - \ddot{E} + 2\Omega^{-1}\left[(B - \dot{E})\dot{\Omega} - (E_i + \tilde{\nabla}_i E)\tilde{\nabla}^i\Omega\right]\\
&= \phi - \psi + \dot{B} - \ddot{E} - 2\Omega^{-1}(E_i + \tilde{\nabla}_i E)\tilde{\nabla}^i\Omega\\
&= \phi - \psi + \dot{B} - \ddot{E} + 2z^{-1}(\tilde{\nabla}_3 E + E_3)
\end{aligned} \tag{4.2}
$$

$$
Q_i = B_i - \dot{E}_i \tag{4.3}
$$

$$
\begin{aligned}
\Delta_{00} &= 3H^2\alpha\Omega^2 - 3H^2\gamma\Omega^2 - \tilde{\nabla}_a\tilde{\nabla}^a\alpha + \tilde{\nabla}_a\tilde{\nabla}^a\gamma - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\alpha\\
&+\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\gamma - 6E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{4.4}
$$

$$
\begin{aligned}
\Delta_{0i} &= -\tilde{\nabla}_i\dot{\alpha} + \tilde{\nabla}_i\dot{\gamma} + \dot{\alpha}\Omega^{-1}\tilde{\nabla}_i\Omega - \dot{\gamma}\Omega^{-1}\tilde{\nabla}_i\Omega + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a Q_i + \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a Q_i\\
&-\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i Q^a - 2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a\Omega
\end{aligned} \tag{4.5}
$$

$$
\begin{aligned}
\Delta_{ij} &= -\tilde{g}_{ij}\ddot{\alpha} + \tilde{g}_{ij}\ddot{\gamma} - 3H^2\tilde{g}_{ij}\alpha\Omega^2 + 3H^2\tilde{g}_{ij}\gamma\Omega^2 - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\gamma - \tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\alpha\\
&-\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\gamma + \Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\alpha - \Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\gamma + \Omega^{-1}\tilde{\nabla}_i\alpha\tilde{\nabla}_j\Omega - \Omega^{-1}\tilde{\nabla}_i\gamma\tilde{\nabla}_j\Omega\\
&+\tilde{\nabla}_j\tilde{\nabla}_i\gamma - 2\tilde{g}_{ij}\dot{Q}^a\Omega^{-1}\tilde{\nabla}_a\Omega + \tfrac{1}{2}\tilde{\nabla}_i\dot{Q}_j + \tfrac{1}{2}\tilde{\nabla}_j\dot{Q}_i - \ddot{E}_{ij} + \tilde{\nabla}_a\tilde{\nabla}^a E_{ij} + 2\Omega^{-1}\tilde{\nabla}_a E_{ij}\tilde{\nabla}^a\Omega\\
&+6\tilde{g}_{ij}E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i E_{ja} - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j E_{ia}
\end{aligned} \tag{4.6}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} &= -12H^2\alpha + 12H^2\gamma - 3\ddot{\alpha}\Omega^{-2} + 3\ddot{\gamma}\Omega^{-2} + \Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\alpha - 3\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\gamma\\
&-6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\gamma - 6\dot{Q}^a\Omega^{-3}\tilde{\nabla}_a\Omega + 24E_{ab}\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{4.7}
$$

$$
\begin{aligned}
\tilde{g}^{ij}\Delta_{ij} &= -3\ddot{\alpha} + 3\ddot{\gamma} - 9H^2\alpha\Omega^2 + 9H^2\gamma\Omega^2 - 2\tilde{\nabla}_a\tilde{\nabla}^a\gamma - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\alpha\\
&-5\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\gamma - 6\dot{Q}^a\Omega^{-1}\tilde{\nabla}_a\Omega + 18E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{4.8}
$$

### Component Form

In the following, $\tilde{\nabla}^2 = \tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b$.

$$
g^{\mu\nu}\Delta_{\mu\nu} = -12H^2\alpha - 3H^2z^2\ddot{\alpha} + 3H^2z^2\ddot{\gamma} + 12H^2\gamma + H^2z^2\nabla^2\alpha - 3H^2z^2\nabla^2\gamma
$$

$$+6H^2z\tilde{\nabla}_3\gamma + 6H^2z\dot{Q}_3 + 24H^2E_{33} \tag{4.9}$$

$$\tilde{g}^{ij}\Delta_{ij} = -9z^{-2}\alpha - 3\ddot{\alpha} + 3\ddot{\gamma} + 9z^{-2}\gamma - 2\nabla^2\gamma + z^{-1}\tilde{\nabla}_3\alpha + 5z^{-1}\tilde{\nabla}_3\gamma + 6z^{-1}\dot{Q}_3 + 18z^{-2}E_{33} \tag{4.10}$$

$$\Delta_{00} = 3z^{-2}\alpha - 3z^{-2}\gamma - \nabla^2\alpha + \nabla^2\gamma + z^{-1}\tilde{\nabla}_3\alpha - z^{-1}\tilde{\nabla}_3\gamma - 6z^{-2}E_{33} \tag{4.11}$$

$$\begin{aligned}\Delta_{11} = {} & -3z^{-2}\alpha - \ddot{\alpha} + \ddot{\gamma} + 3z^{-2}\gamma - \nabla^2\gamma + \tilde{\nabla}_1\tilde{\nabla}_1\gamma + z^{-1}\tilde{\nabla}_3\alpha + z^{-1}\tilde{\nabla}_3\gamma + 2z^{-1}\dot{Q}_3 + \tilde{\nabla}_1\dot{Q}_1 - \ddot{E}_{11} \\ & +6z^{-2}E_{33} + \nabla^2E_{11} + 4z^{-1}\tilde{\nabla}_1E_{13} - 2z^{-1}\tilde{\nabla}_3E_{11}\end{aligned} \tag{4.12}$$

$$\begin{aligned}\Delta_{22} = {} & -3z^{-2}\alpha - \ddot{\alpha} + \ddot{\gamma} + 3z^{-2}\gamma - \nabla^2\gamma + \tilde{\nabla}_2\tilde{\nabla}_2\gamma + z^{-1}\tilde{\nabla}_3\alpha + z^{-1}\tilde{\nabla}_3\gamma + 2z^{-1}\dot{Q}_3 + \tilde{\nabla}_2\dot{Q}_2 - \ddot{E}_{22} \\ & +6z^{-2}E_{33} + \nabla^2E_{22} + 4z^{-1}\tilde{\nabla}_2E_{23} - 2z^{-1}\tilde{\nabla}_3E_{22}\end{aligned} \tag{4.13}$$

$$\begin{aligned}\Delta_{33} = {} & -3z^{-2}\alpha - \ddot{\alpha} + \ddot{\gamma} + 3z^{-2}\gamma - \nabla^2\gamma - z^{-1}\tilde{\nabla}_3\alpha + 3z^{-1}\tilde{\nabla}_3\gamma + \tilde{\nabla}_3\tilde{\nabla}_3\gamma + 2z^{-1}\dot{Q}_3 + \tilde{\nabla}_3\dot{Q}_3 - \ddot{E}_{33} \\ & +6z^{-2}E_{33} + \nabla^2E_{33} + 2z^{-1}\tilde{\nabla}_3E_{33}\end{aligned} \tag{4.14}$$

$$\Delta_{01} = -\tilde{\nabla}_1\dot{\alpha} + \tilde{\nabla}_1\dot{\gamma} + \tfrac{1}{2}\nabla^2Q_1 + z^{-1}\tilde{\nabla}_1Q_3 - z^{-1}\tilde{\nabla}_3Q_1 + 2z^{-1}\dot{E}_{13} \tag{4.15}$$

$$\Delta_{02} = -\tilde{\nabla}_2\dot{\alpha} + \tilde{\nabla}_2\dot{\gamma} + \tfrac{1}{2}\nabla^2Q_2 + z^{-1}\tilde{\nabla}_2Q_3 - z^{-1}\tilde{\nabla}_3Q_2 + 2z^{-1}\dot{E}_{23} \tag{4.16}$$

$$\Delta_{03} = -z^{-1}\dot{\alpha} + z^{-1}\dot{\gamma} - \tilde{\nabla}_3\dot{\alpha} + \tilde{\nabla}_3\dot{\gamma} + \tfrac{1}{2}\nabla^2Q_3 + 2z^{-1}\dot{E}_{33} \tag{4.17}$$

$$\Delta_{12} = \tilde{\nabla}_2\tilde{\nabla}_1\gamma + \tfrac{1}{2}\tilde{\nabla}_1\dot{Q}_2 + \tfrac{1}{2}\tilde{\nabla}_2\dot{Q}_1 - \ddot{E}_{12} + \nabla^2E_{12} + 2z^{-1}\tilde{\nabla}_1E_{23} + 2z^{-1}\tilde{\nabla}_2E_{13} - 2z^{-1}\tilde{\nabla}_3E_{12} \tag{4.18}$$

$$\Delta_{13} = -z^{-1}\tilde{\nabla}_1\alpha + z^{-1}\tilde{\nabla}_1\gamma + \tilde{\nabla}_3\tilde{\nabla}_1\gamma + \tfrac{1}{2}\tilde{\nabla}_1\dot{Q}_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{Q}_1 - \ddot{E}_{13} + \nabla^2E_{13} + 2z^{-1}\tilde{\nabla}_1E_{33} \tag{4.19}$$

$$\Delta_{23} = -z^{-1}\tilde{\nabla}_2\alpha + z^{-1}\tilde{\nabla}_2\gamma + \tilde{\nabla}_3\tilde{\nabla}_2\gamma + \tfrac{1}{2}\tilde{\nabla}_2\dot{Q}_3 + \tfrac{1}{2}\tilde{\nabla}_3\dot{Q}_2 - \ddot{E}_{23} + \nabla^2E_{23} + 2z^{-1}\tilde{\nabla}_2E_{33} \tag{4.20}$$

# 5 Separation

$$\begin{aligned}A &= z^{-1}\tilde{\nabla}^2(z\tilde{g}^{ab}\Delta_{ab}) - 12z^{-1}\dot{\Delta}_{03} - 4\ddot{\Delta}_{00} + 3z^{-1}\tilde{\nabla}^2(z\Delta_{00}) \\ &= \nabla^2\ddot{\alpha} - \nabla^2\ddot{\gamma} - 3\nabla^4\alpha + \nabla^4\gamma + 2z^{-1}\tilde{\nabla}_3\ddot{\alpha} - 2z^{-1}\tilde{\nabla}_3\ddot{\gamma} - 2z^{-1}\tilde{\nabla}_3\nabla^2\alpha + 4z^{-1}\tilde{\nabla}_3\nabla^2\gamma\end{aligned} \tag{5.1}$$