RW SVT4 $k \neq 0$ v3

1 Background

$$ds^2 = \Omega^2 \tilde{g}_{\mu\nu} dx^\mu dx^\nu \tag{1.1}$$

$$P_{\mu\nu} = \tilde{g}_{\mu\nu} + U_\mu U_\nu \tag{1.2}$$

$$\tilde{R}_{\lambda\mu\nu\kappa} = k(P_{\lambda\nu}P_{\mu\kappa} - P_{\mu\nu}P_{\lambda\kappa}) \qquad \tilde{R}_{\mu\nu} = -2kP_{\mu\nu}, \qquad \tilde{R} = -6k \tag{1.3}$$

$$\begin{aligned} G_{\mu\nu} &= -\tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} + \tilde{R}_{\mu\nu} - 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega \\ &= k\tilde{g}_{\mu\nu} - 2kU_\mu U_\nu - 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega - 4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega \end{aligned} \tag{1.4}$$

$$g^{\mu\nu}G_{\mu\nu} = 6k\Omega^{-2} - 6\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \tag{1.5}$$

$$T_{\mu\nu} = \Omega^2(\rho + p)U_\mu U_\nu + \Omega^2 p\tilde{g}_{\mu\nu}, \qquad U_\mu = -\delta^0_\mu, \qquad U^\mu = \delta^\mu_0 \tag{1.6}$$

$$g^{\mu\nu}T_{\mu\nu} = 3p - \rho \tag{1.7}$$

$$\begin{aligned} \Delta^{(0)}_{\mu\nu} &= k\tilde{g}_{\mu\nu} - 2kU_\mu U_\nu + \tilde{g}_{\mu\nu}p\Omega^2 + pU_\mu U_\nu\Omega^2 + U_\mu U_\nu\rho\Omega^2 - 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega \\ &\quad -4\Omega^{-2}\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega + 2\Omega^{-1}\tilde{\nabla}_\nu\tilde{\nabla}_\mu\Omega \end{aligned} \tag{1.8}$$

$$g^{\mu\nu}\Delta^{(0)}_{\mu\nu} = 3p - \rho + 6k\Omega^{-2} - 6\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \tag{1.9}$$

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= U^\alpha U^\nu\Omega^{-2}\tilde{\nabla}_\alpha p + U^\alpha U^\nu\Omega^{-2}\tilde{\nabla}_\alpha\rho + 4pU^\alpha U^\nu\Omega^{-3}\tilde{\nabla}_\alpha\Omega + 4U^\alpha U^\nu\rho\Omega^{-3}\tilde{\nabla}_\alpha\Omega + \Omega^{-2}\tilde{\nabla}^\nu p \\ &\quad +p\Omega^{-3}\tilde{\nabla}^\nu\Omega + \rho\Omega^{-3}\tilde{\nabla}^\nu\Omega \end{aligned} \tag{1.10}$$

Solving for ρ within (1.9), substituting into (1.8), and projecting $U^\mu U^\nu\Delta_{\mu\nu}$ we can solve for p as

$$\begin{aligned} p &= -k\Omega^{-2} + \tfrac{4}{3}\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \tfrac{1}{3}\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + \tfrac{4}{3}U^\alpha U^\beta\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega - \tfrac{2}{3}U^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega \\ &= -k\Omega^{-2} - \Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + 2\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega. \end{aligned} \tag{1.11}$$

From (1.9) we can then determine ρ as

$$\begin{aligned} \rho &= 3k\Omega^{-2} - 2\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega + \Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + 4U^\alpha U^\beta\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}_\beta\Omega - 2U^\alpha U^\beta\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega \\ &= 3k\Omega^{-2} - 3\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega. \end{aligned} \tag{1.12}$$

1.1 $\Omega(\tau)$ Identities

$$U^\alpha U^\beta\tilde{\nabla}_\alpha\tilde{\nabla}_\beta\Omega = -\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega \tag{1.13}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega = -\tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha \Omega \tag{1.14}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha F \tilde{\nabla}_\beta \Omega = -\tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha F \tag{1.15}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\beta F(\tilde{\nabla}^\gamma \tilde{\nabla}_\gamma \Omega) = -\tilde{\nabla}_\alpha \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\alpha \tilde{\nabla}^\beta F \tag{1.16}$$

$$U^\alpha U^\beta \tilde{\nabla}_\alpha \tilde{\nabla}_\beta F(\tilde{\nabla}^\gamma \Omega \tilde{\nabla}_\gamma \Omega) = -\tilde{\nabla}^\beta \Omega \tilde{\nabla}^\alpha \Omega \tilde{\nabla}_\alpha \tilde{\nabla}_\beta F \tag{1.17}$$

$$\tilde{\nabla}^\alpha F \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\beta \Omega = \tilde{\nabla}^\alpha F \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega \tag{1.18}$$

$$F^\alpha U_\alpha U^\beta \tilde{\nabla}_\beta \Omega = -F^\alpha \tilde{\nabla}_\alpha \Omega \tag{1.19}$$

$$U^\alpha U^\beta \tilde{\nabla}_\beta F_\alpha (\tilde{\nabla}_\gamma \Omega \tilde{\nabla}^\gamma \Omega) = -\tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta F^\alpha \tag{1.20}$$

$$U^\alpha U^\beta \tilde{\nabla}_\beta F_\alpha (\tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \Omega) = -\tilde{\nabla}_\alpha \tilde{\nabla}_\beta \Omega \tilde{\nabla}^\beta F^\alpha \tag{1.21}$$

$$F^\alpha \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\beta \Omega = F^\alpha \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}^\beta \Omega \tag{1.22}$$

$$U^\alpha U^\beta F_{\alpha\beta} \tilde{\nabla}_\gamma \Omega \tilde{\nabla}^\gamma \Omega = -F_{\alpha\beta} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega \tag{1.23}$$

$$U^\alpha U^\beta F_{\alpha\beta} \tilde{\nabla}_\gamma \tilde{\nabla}^\gamma \Omega = -F_{\alpha\beta} \tilde{\nabla}^\alpha \tilde{\nabla}^\beta \Omega \tag{1.24}$$

2 Fluctuations

$$ds^2 = \Omega^2(\tau)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu, \quad \text{with } \tilde{g}_{\mu\nu} \text{ obeying (1.3)} \tag{2.1}$$

$$f_{\mu\nu} = -2\tilde{g}_{\mu\nu}\chi + 2\tilde{\nabla}_\mu \tilde{\nabla}_\nu F + \tilde{\nabla}_\mu F_\nu + \tilde{\nabla}_\nu F_\mu + 2F_{\mu\nu} \tag{2.2}$$

$$\tilde{g}^{\mu\nu} F_{\mu\nu} = 0, \qquad \tilde{\nabla}^\mu F_{\mu\nu} = 0, \qquad \tilde{\nabla}^\mu F_\mu = 0 \tag{2.3}$$

$$U^\mu \delta U_\mu = \tfrac{1}{2} U^\mu U^\nu f_{\mu\nu} \tag{2.4}$$

$$\begin{aligned} \delta T_{\mu\nu} = {} & \delta p \tilde{g}_{\mu\nu}\Omega^2 + \delta p U_\mu U_\nu \Omega^2 + \delta\rho U_\mu U_\nu \Omega^2 - 2\tilde{g}_{\mu\nu} p\chi\Omega^2 + 2p\Omega^2 \tilde{\nabla}_\mu \tilde{\nabla}_\nu F + \delta U_\nu p U_\mu \Omega^2 + \delta U_\mu p U_\nu \Omega^2 \\ & + \delta U_\nu U_\mu \rho\Omega^2 + \delta U_\mu U_\nu \rho\Omega^2 + p\Omega^2 \tilde{\nabla}_\mu F_\nu + p\Omega^2 \tilde{\nabla}_\nu F_\mu + 2F_{\mu\nu} p\Omega^2 \end{aligned} \tag{2.5}$$

$$\begin{aligned} g^{\mu\nu}\delta T_{\mu\nu} = {} & 3\delta p - \delta\rho - 6p\chi + 2\rho\chi + 2p\tilde{\nabla}_\alpha \tilde{\nabla}^\alpha F + 2pU^\alpha U^\beta \tilde{\nabla}_\beta \tilde{\nabla}_\alpha F + 2U^\alpha U^\beta \rho \tilde{\nabla}_\beta \tilde{\nabla}_\alpha F + 2pU^\alpha U^\beta \tilde{\nabla}_\beta F_\alpha \\ & + 2U^\alpha U^\beta \rho \tilde{\nabla}_\beta F_\alpha + 2F_{\alpha\beta} p U^\alpha U^\beta + 2F_{\alpha\beta} U^\alpha U^\beta \rho \end{aligned} \tag{2.6}$$

$$\begin{aligned} \delta G_{\mu\nu} = {} & 2\tilde{g}_{\mu\nu} \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha \chi + 2k\tilde{g}_{\mu\nu}\Omega^{-1} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha F - 2kU_\mu U_\nu \Omega^{-1} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha F + 2\tilde{g}_{\mu\nu}\Omega^{-1} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\alpha \chi \\ & + 2k\tilde{g}_{\mu\nu} U^\alpha U^\beta \Omega^{-1} \tilde{\nabla}_\alpha F \tilde{\nabla}_\beta \Omega + 2\tilde{g}_{\mu\nu}\Omega^{-1} \tilde{\nabla}^\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}^\beta \tilde{\nabla}_\alpha F - 2\tilde{g}_{\mu\nu}\Omega^{-2} \tilde{\nabla}^\alpha \Omega \tilde{\nabla}_\beta \tilde{\nabla}_\alpha F \tilde{\nabla}^\beta \Omega \\ & + 4\tilde{g}_{\mu\nu}\Omega^{-1} \tilde{\nabla}_\beta \tilde{\nabla}_\alpha \Omega \tilde{\nabla}^\beta \tilde{\nabla}^\alpha F + 2kU^\alpha U_\nu \Omega^{-1} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\mu F - 2kU^\alpha U_\nu \tilde{\nabla}_\mu \tilde{\nabla}_\alpha F + 2\Omega^{-1} \tilde{\nabla}_\mu \Omega \tilde{\nabla}_\nu \chi \end{aligned}$$

$$
\begin{aligned}
&+2kU^\alpha U_\mu\Omega^{-1}\tilde\nabla_\alpha F\tilde\nabla_\nu\Omega+2k\Omega^{-1}\tilde\nabla_\mu F\tilde\nabla_\nu\Omega+2\Omega^{-1}\tilde\nabla_\mu\chi\tilde\nabla_\nu\Omega-2kU^\alpha U_\mu\tilde\nabla_\nu\tilde\nabla_\alpha F\\
&+2k\tilde\nabla_\nu\tilde\nabla_\mu F-4\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu F+2\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu F-2\tilde\nabla_\nu\tilde\nabla_\mu\chi\\
&-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu\tilde\nabla_\alpha F+2kF^\alpha\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla_\alpha\Omega+2kF_\mu U^\alpha U_\nu\Omega^{-1}\tilde\nabla_\alpha\Omega-2kF^\alpha U_\mu U_\nu\Omega^{-1}\tilde\nabla_\alpha\Omega\\
&+2kF^\alpha\tilde g_{\mu\nu}U_\alpha U^\beta\Omega^{-1}\tilde\nabla_\beta\Omega+2\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla^\beta F_\alpha-2\tilde g_{\mu\nu}\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla_\beta\Omega\tilde\nabla^\beta F^\alpha\\
&+4\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla_\beta\tilde\nabla_\alpha\Omega\tilde\nabla^\beta F^\alpha-2kU^\alpha U_\nu\tilde\nabla_\mu F_\alpha+k\tilde\nabla_\mu F_\nu-2\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\tilde\nabla_\mu F_\nu\\
&+\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\mu F_\nu-2kU^\alpha U_\mu\tilde\nabla_\nu F_\alpha+k\tilde\nabla_\nu F_\mu-2\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\tilde\nabla_\nu F_\mu\\
&+\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\nu F_\mu+2kF_\mu\Omega^{-1}\tilde\nabla_\nu\Omega+2kF^\alpha U_\alpha U_\mu\Omega^{-1}\tilde\nabla_\nu\Omega\\
&-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu F_\alpha-4kF_{\nu\alpha}U^\alpha U_\mu-4kF_{\mu\alpha}U^\alpha U_\nu+\tilde\nabla_\alpha\tilde\nabla^\alpha F_{\mu\nu}-4F_{\mu\nu}\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\\
&+2\Omega^{-1}\tilde\nabla_\alpha F_{\mu\nu}\tilde\nabla^\alpha\Omega+2F_{\mu\nu}\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega-2F_{\alpha\beta}\tilde g_{\mu\nu}\Omega^{-2}\tilde\nabla^\alpha\Omega\tilde\nabla^\beta\Omega+4F_{\alpha\beta}\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla^\beta\tilde\nabla^\alpha\Omega\\
&-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\mu F_{\nu\alpha}-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\nu F_{\mu\alpha}
\end{aligned}
\tag{2.7}
$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} &= 2k\Omega^{-2}\tilde\nabla_\alpha\tilde\nabla^\alpha F+6\Omega^{-2}\tilde\nabla_\alpha\tilde\nabla^\alpha\chi+12k\Omega^{-3}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha F+12\Omega^{-3}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\chi\\
&+12kU^\alpha U^\beta\Omega^{-3}\tilde\nabla_\alpha F\tilde\nabla_\beta\Omega-4kU^\alpha U^\beta\Omega^{-2}\tilde\nabla_\beta\tilde\nabla_\alpha F+2\Omega^{-4}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla^\beta F\\
&-4\Omega^{-3}\tilde\nabla_\alpha\tilde\nabla^\alpha F\tilde\nabla_\beta\tilde\nabla^\beta\Omega+6\Omega^{-3}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla^\beta\tilde\nabla_\alpha F-8\Omega^{-4}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla_\alpha F\tilde\nabla^\beta\Omega\\
&+16\Omega^{-3}\tilde\nabla_\beta\tilde\nabla_\alpha\Omega\tilde\nabla^\beta\tilde\nabla^\alpha F+12kF^\alpha\Omega^{-3}\tilde\nabla_\alpha\Omega-4kU^\alpha U^\beta\Omega^{-2}\tilde\nabla_\beta F_\alpha+12kF^\alpha U_\alpha U^\beta\Omega^{-3}\tilde\nabla_\beta\Omega\\
&+6\Omega^{-3}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla^\beta F_\alpha-8\Omega^{-4}\tilde\nabla_\alpha\Omega\tilde\nabla_\beta\Omega\tilde\nabla^\beta F^\alpha+16\Omega^{-3}\tilde\nabla_\beta\tilde\nabla_\alpha\Omega\tilde\nabla^\beta F^\alpha-8kF_{\alpha\beta}U^\alpha U^\beta\Omega^{-2}\\
&-8F_{\alpha\beta}\Omega^{-4}\tilde\nabla^\alpha\Omega\tilde\nabla^\beta\Omega+16F_{\alpha\beta}\Omega^{-3}\tilde\nabla^\beta\tilde\nabla^\alpha\Omega
\end{aligned}
\tag{2.8}
$$

3 Field Equations

$$
\Delta_{\mu\nu} = \delta G_{\mu\nu}+\delta T_{\mu\nu} \tag{3.1}
$$

$$
\begin{aligned}
\Delta_{\mu\nu} &= 2k\tilde g_{\mu\nu}\chi+\delta p\tilde g_{\mu\nu}\Omega^2+\delta pU_\mu U_\nu\Omega^2+\delta\rho U_\mu U_\nu\Omega^2+2\tilde g_{\mu\nu}\tilde\nabla_\alpha\tilde\nabla^\alpha\chi-4\tilde g_{\mu\nu}\chi\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\\
&+2k\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha F-2kU_\mu U_\nu\Omega^{-1}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha F+2\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\chi+2\tilde g_{\mu\nu}\chi\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\\
&+2k\tilde g_{\mu\nu}U^\alpha U^\beta\Omega^{-1}\tilde\nabla_\alpha F\tilde\nabla_\beta\Omega+2\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla^\beta\tilde\nabla_\alpha F-2\tilde g_{\mu\nu}\Omega^{-2}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla_\alpha F\tilde\nabla^\beta\Omega\\
&+4\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla_\beta\tilde\nabla_\alpha\Omega\tilde\nabla^\beta\tilde\nabla^\alpha F+2kU^\alpha U_\nu\Omega^{-1}\tilde\nabla_\alpha\Omega\tilde\nabla_\mu F+2kU_\nu\tilde\nabla_\mu V+2U_\nu\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\tilde\nabla_\mu V\\
&-4U_\nu\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\mu V-2kU^\alpha U_\nu\tilde\nabla_\mu\tilde\nabla_\alpha F-2k\tilde\nabla_\mu\tilde\nabla_\nu F+4\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\tilde\nabla_\mu\tilde\nabla_\nu F\\
&-2\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\mu\tilde\nabla_\nu F+2kU_\mu\tilde\nabla_\nu V+2U_\mu\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\tilde\nabla_\nu V-4U_\mu\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\nu V\\
&+2\Omega^{-1}\tilde\nabla_\mu\Omega\tilde\nabla_\nu\chi+2kU^\alpha U_\mu\Omega^{-1}\tilde\nabla_\alpha F\tilde\nabla_\nu\Omega+2k\Omega^{-1}\tilde\nabla_\mu F\tilde\nabla_\nu\Omega+2\Omega^{-1}\tilde\nabla_\mu\chi\tilde\nabla_\nu\Omega\\
&-2kU^\alpha U_\mu\tilde\nabla_\nu\tilde\nabla_\alpha F+2k\tilde\nabla_\nu\tilde\nabla_\mu F-4\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu F+2\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu F\\
&-2\tilde\nabla_\nu\tilde\nabla_\mu\chi-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu\tilde\nabla_\alpha F+2kU_\nu V_\mu+2kU_\mu V_\nu+2kF^\alpha\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla_\alpha\Omega\\
&+2kF_\mu U^\alpha U_\nu\Omega^{-1}\tilde\nabla_\alpha\Omega-2kF^\alpha U_\mu U_\nu\Omega^{-1}\tilde\nabla_\alpha\Omega+2U_\nu V_\mu\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega+2U_\mu V_\nu\Omega^{-1}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega\\
&-4U_\nu V_\mu\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega-4U_\mu V_\nu\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\Omega+2kF^\alpha\tilde g_{\mu\nu}U_\alpha U^\beta\Omega^{-1}\tilde\nabla_\beta\Omega\\
&+2\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla^\beta F_\alpha-2\tilde g_{\mu\nu}\Omega^{-2}\tilde\nabla_\alpha\Omega\tilde\nabla_\beta\Omega\tilde\nabla^\beta F^\alpha+4\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla_\beta\tilde\nabla_\alpha\Omega\tilde\nabla^\beta F^\alpha\\
&-2kU^\alpha U_\nu\tilde\nabla_\mu F_\alpha-2kU^\alpha U_\mu\tilde\nabla_\nu F_\alpha+2kF_\mu\Omega^{-1}\tilde\nabla_\nu\Omega+2kF^\alpha U_\alpha U_\mu\Omega^{-1}\tilde\nabla_\nu\Omega\\
&-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\nu\tilde\nabla_\mu F_\alpha-2kF_{\mu\nu}-4kF_{\nu\alpha}U^\alpha U_\mu-4kF_{\mu\alpha}U^\alpha U_\nu+\tilde\nabla_\alpha\tilde\nabla^\alpha F_{\mu\nu}\\
&+2\Omega^{-1}\tilde\nabla_\alpha F_{\mu\nu}\tilde\nabla^\alpha\Omega-2F_{\alpha\beta}\tilde g_{\mu\nu}\Omega^{-2}\tilde\nabla^\alpha\Omega\tilde\nabla^\beta\Omega+4F_{\alpha\beta}\tilde g_{\mu\nu}\Omega^{-1}\tilde\nabla^\beta\tilde\nabla^\alpha\Omega-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\mu F_{\nu\alpha}\\
&-2\Omega^{-1}\tilde\nabla^\alpha\Omega\tilde\nabla_\nu F_{\mu\alpha}
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} &= 3\delta p-\delta\rho+12k\chi\Omega^{-2}+6\Omega^{-2}\tilde\nabla_\alpha\tilde\nabla^\alpha\chi-12\chi\Omega^{-3}\tilde\nabla_\alpha\tilde\nabla^\alpha\Omega+12k\Omega^{-3}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha F\\
&+12\Omega^{-3}\tilde\nabla_\alpha\Omega\tilde\nabla^\alpha\chi+12kU^\alpha U^\beta\Omega^{-3}\tilde\nabla_\alpha F\tilde\nabla_\beta\Omega+6\Omega^{-3}\tilde\nabla^\alpha\Omega\tilde\nabla_\beta\tilde\nabla^\beta\tilde\nabla_\alpha F
\end{aligned}
$$

$$
\begin{aligned}
&-8\Omega^{-4}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}F\tilde{\nabla}^{\beta}\Omega+16\Omega^{-3}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\beta}\tilde{\nabla}^{\alpha}F+4U^{\alpha}U^{\beta}\Omega^{-3}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}F\tilde{\nabla}_{\gamma}\tilde{\nabla}^{\gamma}\Omega\\
&-8U^{\alpha}U^{\beta}\Omega^{-4}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}F\tilde{\nabla}_{\gamma}\Omega\tilde{\nabla}^{\gamma}\Omega+12kF^{\alpha}\Omega^{-3}\tilde{\nabla}_{\alpha}\Omega+12kF^{\alpha}U_{\alpha}U^{\beta}\Omega^{-3}\tilde{\nabla}_{\beta}\Omega\\
&+6\Omega^{-3}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}F_{\alpha}-8\Omega^{-4}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}_{\beta}\Omega\tilde{\nabla}^{\beta}F^{\alpha}+16\Omega^{-3}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\beta}F^{\alpha}\\
&+4U^{\alpha}U^{\beta}\Omega^{-3}\tilde{\nabla}_{\beta}F_{\alpha}\tilde{\nabla}_{\gamma}\tilde{\nabla}^{\gamma}\Omega-8U^{\alpha}U^{\beta}\Omega^{-4}\tilde{\nabla}_{\beta}F_{\alpha}\tilde{\nabla}_{\gamma}\Omega\tilde{\nabla}^{\gamma}\Omega-4kF_{\alpha\beta}U^{\alpha}U^{\beta}\Omega^{-2}\\
&-8F_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}^{\beta}\Omega+16F_{\alpha\beta}\Omega^{-3}\tilde{\nabla}^{\beta}\tilde{\nabla}^{\alpha}\Omega+4F_{\alpha\beta}U^{\alpha}U^{\beta}\Omega^{-3}\tilde{\nabla}_{\gamma}\tilde{\nabla}^{\gamma}\Omega\\
&-8F_{\alpha\beta}U^{\alpha}U^{\beta}\Omega^{-4}\tilde{\nabla}_{\gamma}\Omega\tilde{\nabla}^{\gamma}\Omega
\end{aligned}
\tag{3.3}
$$

4 Field Equations (G.I. Form)

$$
\alpha = \chi+\Omega^{-1}\tilde{\nabla}_{\alpha}\Omega(F^{\alpha}+\tilde{\nabla}^{\alpha}F)=\chi-\Omega^{-1}U^{\alpha}U^{\beta}\tilde{\nabla}_{\alpha}\Omega(F_{\beta}+\tilde{\nabla}_{\beta}F) \tag{4.1}
$$

$$
V^{GI} = V-U^{\alpha}\tilde{\nabla}_{\alpha}F-U^{\alpha}F_{\alpha} \tag{4.2}
$$

$$
\delta\rho^{GI} = \delta\rho+12\Omega^{-4}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega\chi-6k\Omega^{-2}\chi-6\Omega^{-3}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega\chi \tag{4.3}
$$

$$
\delta p^{GI} = \delta p+4\Omega^{-4}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega\chi-2\Omega^{-3}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}\Omega(\tilde{\nabla}^{\alpha}F+F^{\alpha})+2\Omega^{-2}k\chi-8\Omega^{-3}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega\chi \tag{4.4}
$$

$$
\begin{aligned}
\Delta_{\mu\nu} = {}& 4kU_{\mu}U_{\nu}\chi+\delta p^{GI}\tilde{g}_{\mu\nu}\Omega^{2}+\delta p^{GI}U_{\mu}U_{\nu}\Omega^{2}+\delta\rho^{GI}U_{\mu}U_{\nu}\Omega^{2}+2\tilde{g}_{\mu\nu}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\alpha+4\tilde{g}_{\mu\nu}\chi\Omega^{-1}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega\\
&+14U_{\mu}U_{\nu}\chi\Omega^{-1}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega-2kU_{\mu}U_{\nu}\Omega^{-1}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}F+2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\alpha\\
&-2\tilde{g}_{\mu\nu}\chi\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega-16U_{\mu}U_{\nu}\chi\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega+2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}F\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\Omega\\
&+2U_{\mu}U_{\nu}\Omega^{-1}\tilde{\nabla}^{\alpha}F\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}\Omega-2\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}F\tilde{\nabla}_{\beta}\Omega\tilde{\nabla}^{\beta}\Omega+2\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}^{\alpha}F\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\beta}\Omega\\
&+2kU^{\alpha}U_{\nu}\Omega^{-1}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}_{\mu}F+2kU_{\nu}\tilde{\nabla}_{\mu}V^{GI}+2U_{\nu}\Omega^{-1}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\mu}V^{GI}\\
&-4U_{\nu}\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\mu}V^{GI}+2U^{\alpha}U_{\nu}\Omega^{-1}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\mu}\tilde{\nabla}_{\alpha}F-4U^{\alpha}U_{\nu}\Omega^{-2}\tilde{\nabla}_{\beta}\Omega\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\mu}\tilde{\nabla}_{\alpha}F\\
&+2kU_{\mu}\tilde{\nabla}_{\nu}V^{GI}+2U_{\mu}\Omega^{-1}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\nu}V^{GI}-4U_{\mu}\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\nu}V^{GI}+2\Omega^{-1}\tilde{\nabla}_{\mu}\Omega\tilde{\nabla}_{\nu}\chi\\
&+2kU^{\alpha}U_{\mu}\Omega^{-1}\tilde{\nabla}_{\alpha}F\tilde{\nabla}_{\nu}\Omega+2k\Omega^{-1}\tilde{\nabla}_{\mu}F\tilde{\nabla}_{\nu}\Omega+2\Omega^{-1}\tilde{\nabla}_{\mu}\chi\tilde{\nabla}_{\nu}\Omega+2U^{\alpha}U_{\mu}\Omega^{-1}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\nu}\tilde{\nabla}_{\alpha}F\\
&-4U^{\alpha}U_{\mu}\Omega^{-2}\tilde{\nabla}_{\beta}\Omega\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\nu}\tilde{\nabla}_{\alpha}F-2\tilde{\nabla}_{\nu}\tilde{\nabla}_{\mu}\chi-2\Omega^{-1}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\nu}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\alpha}F+2kU_{\nu}V_{\mu}+2kU_{\mu}V_{\nu}\\
&+2kF_{\mu}U^{\alpha}U_{\nu}\Omega^{-1}\tilde{\nabla}_{\alpha}\Omega-2kF^{\alpha}U_{\mu}U_{\nu}\Omega^{-1}\tilde{\nabla}_{\alpha}\Omega+2U_{\nu}V_{\mu}\Omega^{-1}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega+2U_{\mu}V_{\nu}\Omega^{-1}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega\\
&-4U_{\nu}V_{\mu}\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega-4U_{\mu}V_{\nu}\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\Omega+2F^{\alpha}\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\Omega\\
&+2F^{\alpha}U_{\mu}U_{\nu}\Omega^{-1}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\tilde{\nabla}_{\alpha}\Omega-2F^{\alpha}\tilde{g}_{\mu\nu}\Omega^{-3}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}_{\beta}\Omega\tilde{\nabla}^{\beta}\Omega+2F^{\alpha}\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\beta}\Omega\\
&+2U^{\alpha}U_{\nu}\Omega^{-1}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\mu}F_{\alpha}-4U^{\alpha}U_{\nu}\Omega^{-2}\tilde{\nabla}_{\beta}\Omega\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\mu}F_{\alpha}+2U^{\alpha}U_{\mu}\Omega^{-1}\tilde{\nabla}_{\beta}\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\nu}F_{\alpha}\\
&-4U^{\alpha}U_{\mu}\Omega^{-2}\tilde{\nabla}_{\beta}\Omega\tilde{\nabla}^{\beta}\Omega\tilde{\nabla}_{\nu}F_{\alpha}+2kF_{\mu}\Omega^{-1}\tilde{\nabla}_{\nu}\Omega+2kF^{\alpha}U_{\alpha}U_{\mu}\Omega^{-1}\tilde{\nabla}_{\nu}\Omega\\
&-2\Omega^{-1}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\nu}\tilde{\nabla}_{\mu}F_{\alpha}-2kF_{\mu\nu}-4kF_{\nu\alpha}U^{\alpha}U_{\mu}-4kF_{\mu\alpha}U^{\alpha}U_{\nu}+\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}F_{\mu\nu}\\
&+2\Omega^{-1}\tilde{\nabla}_{\alpha}F_{\mu\nu}\tilde{\nabla}^{\alpha}\Omega+4F^{\alpha\beta}\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}\Omega-2F_{\alpha\beta}\tilde{g}_{\mu\nu}\Omega^{-2}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}^{\beta}\Omega-2\Omega^{-1}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\mu}F_{\nu\alpha}\\
&-2\Omega^{-1}\tilde{\nabla}^{\alpha}\Omega\tilde{\nabla}_{\nu}F_{\mu\alpha}
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
g^{\mu\nu}\Delta_{\mu\nu} = {}& 3\delta p^{GI}-\delta\rho^{GI}+6\Omega^{-2}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\alpha+6\alpha\Omega^{-3}\tilde{\nabla}_{\alpha}\tilde{\nabla}^{\alpha}\Omega+12\Omega^{-3}\tilde{\nabla}_{\alpha}\Omega\tilde{\nabla}^{\alpha}\alpha-4kF_{\alpha\beta}U^{\alpha}U^{\beta}\Omega^{-2}\\
&+12F^{\alpha\beta}\Omega^{-3}\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}\Omega
\end{aligned}
\tag{4.6}
$$