

dS SVT4 Conformal Trace

In reference to *dS4_Ein_SVT4_Conformal*, computationally I calculated $g^{\mu\nu}\Delta_{\mu\nu}$ not by $-\Delta_{00} + \delta^{ij}\Delta_{ij}$ but rather by $g^{\mu\nu}\delta G_{\mu\nu} + g^{\mu\nu}\delta T_{\mu\nu}$. With this in mind, the lack of \dot{F}_{00} term originates in eq. (1.10). To check if this is in fact the correct trace of $\delta G_{\mu\nu}$, I recalculated it directly and looked at its trace in terms of $h_{\mu\nu}$:

$$\begin{aligned} \delta G_{\mu\nu} = & \frac{1}{2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha h_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha h - 2h_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - \frac{1}{2}\tilde{\nabla}_\alpha\tilde{\nabla}_\mu h_\nu{}^\alpha - \frac{1}{2}\tilde{\nabla}_\alpha\tilde{\nabla}_\nu h_\mu{}^\alpha \\ & - \tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha h + \Omega^{-1}\tilde{\nabla}_\alpha h_{\mu\nu}\tilde{\nabla}^\alpha\Omega + h_{\mu\nu}\Omega^{-2}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega + 2\tilde{g}_{\mu\nu}\Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta h_\alpha{}^\beta \\ & + \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha h^{\alpha\beta} + 2\tilde{g}_{\mu\nu}h^{\alpha\beta}\Omega^{-1}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega - \tilde{g}_{\mu\nu}h_{\alpha\beta}\Omega^{-2}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega - \Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\mu h_{\nu\alpha} \\ & - \Omega^{-1}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\nu h_{\mu\alpha} + \frac{1}{2}\tilde{\nabla}_\nu\tilde{\nabla}_\mu h. \end{aligned} \quad (0.1)$$

$$\begin{aligned} \Omega^{-2}g^{\mu\nu}\delta G_{\mu\nu} = & -\Omega^{-2}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha h - 2h\Omega^{-3}\tilde{\nabla}_\alpha\tilde{\nabla}^\alpha\Omega - 4\Omega^{-3}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha h + \Omega^{-3}\tilde{\nabla}_\alpha h\tilde{\nabla}^\alpha\Omega + h\Omega^{-4}\tilde{\nabla}_\alpha\Omega\tilde{\nabla}^\alpha\Omega \\ & + 6\Omega^{-3}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}_\beta h_\alpha{}^\beta + \Omega^{-2}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha h^{\alpha\beta} + 8h^{\alpha\beta}\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega - 4h_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega. \end{aligned} \quad (0.2)$$

Since $F_{\mu\nu}$ cannot appear in h nor $\tilde{\nabla}^\alpha h_{\alpha\beta}$, this leaves only

$$8h^{\alpha\beta}\Omega^{-3}\tilde{\nabla}_\beta\tilde{\nabla}_\alpha\Omega - 4h_{\alpha\beta}\Omega^{-4}\tilde{\nabla}^\alpha\Omega\tilde{\nabla}^\beta\Omega = -8\dot{\Omega}^2 F_{00}\Omega^{-4} + 16\ddot{\Omega} F_{00}\Omega^{-3} = 24H^2 F_{00} \quad (0.3)$$

This doesn't yet resolve the apparent discrepancy with $-\Delta_{00} + \delta^{ij}\Delta_{ij}$, but it does tell us that if there is an error it would lie within the calculation of δG_{ij} and/or δG_{00} .