Web address: http://www.phys.uconn.edu/~kharchenko/Courses/P5302_16S/

Electrodynamics II

Monday and Wednesday
12pm – 1:15pm
Room: P121

01/20/16

Lecture 1

Home work – 30%
Midterm – 30%
Final – 40%

(1) Introduction: Maxwell Equations and Electromagnetic Waves
Wave equations for $\vec{A}$ and φ potentials

(2) Radiating system (retarded potentials, multipole radiation)

(3) Special Relativity and EM theory

(4) Relativistic Mechanics for Particles and Waves

(5) Wave scattering and Diffraction

(6) EM wave propagation: Wavelets, Eikonal and Geometrical Optics

(7) Radiation by Moving Charges, relativistic effects

(8) Cherenkov's and Transition Radiations

(9) Bremsstrahlung and Cyclotron/synchrotron Radiation

(10) Radiation Damping (classical and relativistic)

(11) EM waves in plasmas, non-linear effects.

Maxwell Equations (recap)

$$\boxed{\begin{array}{ll}(1) & \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \\ (2) & \vec{\nabla}\cdot\vec{B} = 0\end{array}} \qquad \boxed{\begin{array}{ll}(3) & \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ (4) & \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{j} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\end{array}}$$

\+ continuity equation: $\boxed{\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0}$ $\quad \vec{j} = \rho\vec{v}$

Lorentz force: $\vec{F}_L = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$ ← Gaussian system

Scalar and vector potentials: $\{\varphi, \vec{A}\}$

$$\begin{cases}\vec{E} = -\vec{\nabla}\varphi \\ \vec{B} = -\vec{\nabla}\times\vec{A}\end{cases} \qquad \boxed{\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\varphi}$$

$\frac{v}{c} \ll 1$

EM Waves: Simple example of the EM waves in vacuum $\boxed{\rho = 0;\ \vec{j} = 0}$

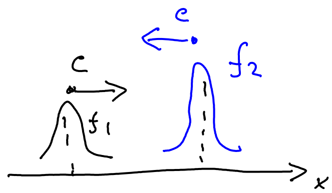
$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\vec{\nabla}(\cancel{\vec{\nabla}\vec{E}}) - \vec{\nabla}^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) = -\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

0 $(\rho = 0)$

Wave equation:

$$\begin{cases}\vec{\nabla}^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \\ \vec{\nabla}^2\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0\end{cases}$$



Simplest case: 1D propagation → $\boxed{\vec{E}(\vec{r},t) = \vec{E}(x,t)}$ $\quad \vec{E} = (E_x, E_y, E_z)$ $\vec{E}$ - vector

Math. Appendix

General solution of the wave equation:

$$\boxed{\frac{\partial^2 f(x,t)}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2} = 0} \quad \text{Eq. } (*)$$

We can introduce new variables:

$$\begin{cases}\xi = x - vt \\ \eta = x + vt\end{cases}$$

Operator "$\frac{\partial}{\partial x}$": $\frac{\partial}{\partial x} = \underbrace{\frac{\partial\xi}{\partial x}}_{1}\frac{\partial}{\partial\xi} + \underbrace{\frac{\partial\eta}{\partial x}}_{1}\frac{\partial}{\partial\eta} = \left(\frac{\partial}{\partial\eta} + \frac{\partial}{\partial\xi}\right)$

Operator "$\frac{\partial}{\partial t}$": $\frac{\partial}{\partial t} = \underbrace{\frac{\partial\xi}{\partial t}}_{-v}\frac{\partial}{\partial\xi} + \underbrace{\frac{\partial\eta}{\partial t}}_{v}\frac{\partial}{\partial\eta} = \left(\frac{\partial}{\partial\eta} - \frac{\partial}{\partial\xi}\right)v$

Wave equation operators:

$$\left.\begin{cases}\frac{\partial^2}{\partial x^2} = \left(\frac{\partial}{\partial\eta} + \frac{\partial}{\partial\xi}\right)^2 = \frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\xi^2} + 2\frac{\partial^2}{\partial\eta\partial\xi} \\ \frac{\partial^2}{\partial t^2} = v^2\left(\frac{\partial}{\partial\eta} - \frac{\partial}{\partial\xi}\right)^2 = v^2\left(\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\xi^2} - 2\frac{\partial^2}{\partial\eta\partial\xi}\right)\end{cases}\right\} \Rightarrow \frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} \equiv 4\frac{\partial^2}{\partial\eta\partial\xi}$$

Solution of the wave equation (*):

$$4\frac{\partial^2 f}{\partial\xi\partial\eta} = 0 \Rightarrow f(x,t) = f(\eta,\xi) = u(\eta) + g(\xi) \Rightarrow \boxed{f(x,t) = f_+(x - vt) + f_-(x + vt)}$$

for any functions $u(\eta)$ and $g(\xi)$