RW SVT4 Decomposition v1

1 Background

$$ds^2 = \Omega^2(\tau)\left(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j\right), \qquad R_{ij} = -2k\tilde{g}_{ij} \tag{1.1}$$

$$R_{\lambda\mu\nu\kappa} = -\tfrac{1}{6}g_{\lambda\nu}g_{\mu\kappa}R + \tfrac{1}{6}g_{\lambda\kappa}g_{\mu\nu}R - \tfrac{1}{2}g_{\mu\nu}R_{\lambda\kappa} + \tfrac{1}{2}g_{\mu\kappa}R_{\lambda\nu} + \tfrac{1}{2}g_{\lambda\nu}R_{\mu\kappa} - \tfrac{1}{2}g_{\lambda\kappa}R_{\mu\nu} \tag{1.2}$$

$$R_{\mu\nu} = (A+B)U_\mu U_\nu + g_{\mu\nu}B, \qquad R = 3B - A \tag{1.3}$$

$$\begin{aligned} A &= -\tfrac{1}{2}(3p+\rho) \\ &= -3\dot{\Omega}^2\Omega^{-4} + 3\ddot{\Omega}\Omega^{-3} \end{aligned} \tag{1.4}$$

$$\begin{aligned} B &= \tfrac{1}{2}(p-\rho) \\ &= -\dot{\Omega}^2\Omega^{-4} - \ddot{\Omega}\Omega^{-3} - 2k\Omega^{-2} \end{aligned} \tag{1.5}$$

$$\begin{aligned} \rho &= \tfrac{1}{2}(-A-3B) \\ &= 3\dot{\Omega}^2\Omega^{-4} + 3k\Omega^{-2} \end{aligned} \tag{1.6}$$

$$\begin{aligned} p &= \tfrac{1}{2}(-A+B) \\ &= \dot{\Omega}^2\Omega^{-4} - 2\ddot{\Omega}\Omega^{-3} - k\Omega^{-2} \end{aligned} \tag{1.7}$$

1.1 Identities

A, B, ρ, p are functions only of coordinate x^0.

$$U^\alpha U^\beta \nabla_\alpha F \nabla_\beta A = -\nabla^\alpha F \nabla_\alpha A \tag{1.8}$$

$$F^\alpha U_\alpha U^\beta \nabla_\beta A = -F^\alpha \nabla_\alpha A \tag{1.9}$$

$$U^\alpha \nabla_\alpha U^\mu = 0 \tag{1.10}$$

$$\nabla_\mu U_\nu = \dot{\Omega}\Omega^{-2}(g_{\mu\nu} + U_\mu U_\nu) \tag{1.11}$$

2 Fluctuations

$$ds^2 = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \tag{2.1}$$

$$g^{\mu\nu}F_{\mu\nu} = 0, \qquad \nabla^\mu F_{\mu\nu} = 0, \qquad \nabla^\mu F_\mu = 0 \tag{2.2}$$

$$h_{\mu\nu} = -2g_{\mu\nu}\chi + 2\nabla_\mu\nabla_\nu F + \nabla_\mu F_\nu + \nabla_\nu F_\mu + 2F_{\mu\nu} \tag{2.3}$$

$$g^{\alpha\beta}h_{\alpha\beta} = -8\chi + 2\nabla_\alpha\nabla^\alpha F \tag{2.4}$$

$$\begin{aligned}\nabla^\mu h_{\mu\nu} = {} & 2U^\alpha U_\nu(p+\rho)\nabla_\alpha F + (-p+\rho)\nabla_\nu F - 2\nabla_\nu\chi + 2\nabla_\nu\nabla_\alpha\nabla^\alpha F + \tfrac{1}{2}F_\nu(-p+\rho) \\ & + F^\alpha U_\alpha U_\nu(p+\rho) + \nabla_\alpha\nabla^\alpha F_\nu\end{aligned} \tag{2.5}$$

$$\begin{aligned}\nabla^\mu\nabla^\nu h_{\mu\nu} = {} & (-p+\rho)\nabla_\alpha\nabla^\alpha F - 2\nabla_\alpha\nabla^\alpha\chi + (-3\nabla_\alpha p - \nabla_\alpha\rho)\nabla^\alpha F + 2U^\alpha(p+\rho)\nabla_\alpha F\nabla_\beta U^\beta \\ & + 2U^\alpha U^\beta(p+\rho)\nabla_\beta\nabla_\alpha F + 2\nabla_\beta\nabla^\beta\nabla_\alpha\nabla^\alpha F + 2U^\alpha U^\beta(p+\rho)\nabla_\beta F_\alpha \\ & + F^\alpha\big(-3\nabla_\alpha p - \nabla_\alpha\rho + 2U_\alpha(p+\rho)\nabla_\beta U^\beta\big)\end{aligned} \tag{2.6}$$

$$U^\mu U^\nu h_{\mu\nu} = 2\chi + 2U^\alpha U^\beta\nabla_\beta\nabla_\alpha F + 2U^\alpha U^\beta\nabla_\beta F_\alpha + 2F_{\alpha\beta}U^\alpha U^\beta \tag{2.7}$$

$$(U^\mu U^\nu + g^{\mu\nu})h_{\mu\nu} = -6\chi + 2\nabla_\alpha\nabla^\alpha F + 2U^\alpha U^\beta\nabla_\beta\nabla_\alpha F + 2U^\alpha U^\beta\nabla_\beta F_\alpha + 2F_{\alpha\beta}U^\alpha U^\beta \tag{2.8}$$

$$(U^\mu\nabla^\nu + U^\nu\nabla^\mu)h_{\mu\nu} = -4U^\alpha\nabla_\alpha\chi + 4U^\alpha\nabla_\beta\nabla^\beta\nabla_\alpha F - F^\alpha U_\alpha(3p+\rho) + 2U^\alpha\nabla_\beta\nabla^\beta F_\alpha \tag{2.9}$$

$$\begin{aligned}\Delta'_{\mu\nu} = {} & 2(p+\rho)U_\mu U_\nu U^\alpha\nabla_\alpha V^{GI} + (\tfrac{2}{3}g_{\mu\nu} + \tfrac{2}{3}U_\mu U_\nu)\nabla_\alpha\nabla^\alpha\chi + (\tfrac{2}{3}g_{\mu\nu}U^\alpha U^\beta + \tfrac{8}{3}U_\mu U_\nu U^\alpha U^\beta)\nabla_\beta\nabla_\alpha\chi \\ & + (p+\rho)U_\nu\nabla_\mu V^{GI} + (p+\rho)U_\mu\nabla_\nu V^{GI} - 2\nabla_\nu\nabla_\mu\chi + 2(p+\rho)U_\mu U_\nu U^\alpha V_\alpha + (p+\rho)U_\nu V_\mu \\ & + (p+\rho)U_\mu V_\nu + \big(\tfrac{2}{9}\rho g_{\mu\nu}U^\alpha U^\beta - \tfrac{4}{9}(9p+7\rho)U_\mu U_\nu U^\alpha U^\beta\big)F_{\alpha\beta} - 2(p+\rho)U_\nu U^\alpha F_{\mu\alpha} - \tfrac{2}{3}\rho F_{\mu\nu} \\ & - 2(p+\rho)U_\mu U^\alpha F_{\nu\alpha} + \nabla_\alpha\nabla^\alpha F_{\mu\nu} + (-\tfrac{1}{3}g_{\mu\nu}U^\alpha U^\beta - \tfrac{4}{3}U_\mu U_\nu U^\alpha U^\beta)\nabla_\gamma\nabla^\gamma F_{\alpha\beta}\end{aligned} \tag{2.10}$$