Bach SVT Matthew v1

The Bach tensor is perturbed according to

$$ds^2 = \Omega^2(x)\big\{-(1+2\phi)d\tau^2 + 2(\tilde{\nabla}_i B + B_i)d\tau dx^i + [(1-2\psi)\gamma_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij}]dx^i dx^j\big\} \tag{1}$$

where

$$\gamma^{ij}\tilde{\nabla}_i B_j = 0, \gamma^{ij}\tilde{\nabla}_i E_j = 0,\ \gamma^{ij}\tilde{\nabla}_i E_{kj} = 0,\ \gamma^{ij} E_{ij} = 0. \tag{2}$$

Covariant derivatives are defined with respect to the flat 3-space background γ_{ij} and are indicated as $\tilde{\nabla}_i$.

$\Omega(x)$

$$\delta W^{(S)}_{00} = \Omega^{-2}\big[-\tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - \tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi - \tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi\big]. \tag{3}$$

$$\delta W^{(S)}_{0i} = \Omega^{-2}\big[-\tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{B} + \tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dddot{E} - \tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{\phi} - \tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{\psi}\big]. \tag{4}$$

$$\delta W^{(V)}_{0i} = \Omega^{-2}\big[-\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{B}_i + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dddot{E}_i + \tfrac{1}{2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i\big]. \tag{5}$$

$$\begin{aligned}\delta W^{(S)}_{ij} = \Omega^{-2}\big[&\tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dddot{B} - \tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddddot{E} + \tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{\phi} + \tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{\psi} - \tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}\\ &+ \tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - \tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi - \tfrac{1}{3}\gamma_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi - \tilde{\nabla}_j\tilde{\nabla}_i\dddot{B} + \tilde{\nabla}_j\tilde{\nabla}_i\ddddot{E}\\ &- \tilde{\nabla}_j\tilde{\nabla}_i\ddot{\phi} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{\psi} + \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} - \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\phi\\ &+ \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\psi\big].\end{aligned} \tag{6}$$

$$\begin{aligned}\delta W^{(V)}_{ij} = \Omega^{-2}\big[&-\tfrac{1}{2}\tilde{\nabla}_i\dddot{B}_j + \tfrac{1}{2}\tilde{\nabla}_i\ddddot{E}_j + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_j - \tfrac{1}{2}\tilde{\nabla}_j\dddot{B}_i + \tfrac{1}{2}\tilde{\nabla}_j\ddddot{E}_i\\ &+ \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}_i - \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_i\big].\end{aligned} \tag{7}$$

$$\delta W^{(T)}_{ij} = \Omega^{-2}\big[\ddddot{E}_{ij} - 2\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_{ij} + \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a E_{ij}\big]. \tag{8}$$