Gauge Invariant $\delta G_{\mu\nu} = \delta T_{\mu\nu}$

Perturbed metric:

$$ds^2 = \Omega^2 \left\{-(1-2\phi)d\tau^2 + 2(\nabla_i B - B_i)d\tau dx^i + \left[(1-2\psi)\delta_{ij} + 2\nabla_i\nabla_j E + \nabla_i E_j + \nabla_j E_i + 2E_{ij}\right] dx^i dx^j\right\} \tag{1}$$

where

$$\nabla^i B_i = 0,\ \nabla^i E_i = 0,\ \nabla^i E_{ij} = 0,\ \delta^{ij} E_{ij} = 0.$$

Under coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \epsilon^\mu \tag{2}$$

where

$$\epsilon^\mu = (\epsilon^0, \nabla^i \epsilon + \epsilon^i), \qquad \nabla^i \epsilon_i = 0$$

the components of the metric transform as

$$\tilde{\phi} = \phi - H\epsilon^0 - \dot{\epsilon}^0 \tag{3}$$

$$\tilde{\psi} = \psi + H\epsilon^0 \tag{4}$$

$$\tilde{B} = B + \epsilon^0 - \dot{\epsilon} \tag{5}$$

$$\tilde{E} = E - \epsilon \tag{6}$$

$$\tilde{E}_i = E_i - \epsilon_i \tag{7}$$

$$\tilde{B}_i = B_i + \dot{\epsilon}_i \tag{8}$$

$$\tilde{E}_{ij} = E_{ij} \tag{9}$$

From the above, we may form gauge invariant combinations (adding to 6 DOF):

$$\Phi = \phi - H(\dot{E} - B) - (\ddot{E} - \dot{B}) \tag{10}$$

$$\Psi = \psi + H(\dot{E} - B) \tag{11}$$

$$Q_i = B_i + \dot{E}_i \tag{12}$$

$$E_{ij} = E_{ij} \tag{13}$$

By orthogonal and parallel projections to the four velocity u^μ, a generic symmetric $T_{\mu\nu}$ may be decomposed as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + u_\nu q_\mu + u_\mu q_\nu + \pi_{\mu\nu} \tag{14}$$

where

$$u^\mu q_\mu = 0,\ g^{\mu\nu}\pi_{\mu\nu} = 0,\ u^\mu u_\nu u^\rho u_\sigma \pi_{\nu\sigma} = 0.$$

The conditions on $\pi_{\mu\nu}$ specify that it is traceless and orthogonal to the four velocity u^μ, i.e. $\pi_{\mu\nu} = \pi_{ij}$. We may further decompose π_{ij} as

$$\pi_{ij} = \nabla_i\nabla_j\Pi - \frac{1}{3}\nabla^2\Pi\delta_{ij} + \frac{1}{2}\nabla_i\Pi_j + \frac{1}{2}\nabla_j\Pi_i + \Pi_{ij} \tag{15}$$

where as expected,

$$\nabla^i\Pi_i = 0,\ \nabla^i\Pi_{ij} = 0,\ \delta^{ij}\Pi_{ij} = 0.$$

We have 2 degrees of freedom from ρ and p, 3 from q_μ, and 5 from $\pi_{\mu\nu}$ adding to 10 in total. We decompose $T_{\mu\nu}$ into a background piece and first order fluctuations (mixed tensor to match Ellis):

$$T^\mu{}_\nu = {}^{(0)}T^\mu{}_\nu + \delta T^\mu{}_\nu.$$

We start by separating scalars, where according to homogeneity and isotropy, the background may only depend on τ,

$$\rho(x^\mu) = \bar{\rho}(\tau) + \delta\rho(x^\mu) \tag{16}$$
$$p(x^\mu) = \bar{p}(\tau) + \delta p(x^\mu). \tag{17}$$

The four velocity is also perturbed

$$u^\mu = \frac{1}{a}\frac{dx^\mu}{d\tau} = \bar{u}^\mu + \delta u^\mu \tag{18}$$

where $\bar{u}^\mu = a^{-1}\delta^\mu{}_0$ and $\delta u^i = \nabla^i v + v^i$ with $\nabla_i v^i = 0$. By normalization of the four velocity $-1 = g_{\mu\nu}u^\mu u^\nu$, we may derive the background and perturbed components of u^μ:

$$u^\mu = \frac{1}{a}\left(1-\phi,\ \nabla^i v + v^i\right), \qquad u_\mu = a\left(-1-\phi,\ \nabla_i v + v_i + \nabla_i B - B_i\right). \tag{19}$$

Since the background of interest (FLRW) is homogeneous and isotropic, there is no anisotropic stress $\pi_{\mu\nu}$ at zeroth order and so π_{ij} itself is first order. We may now form the perturbed E-M tensor:

$$\delta T^0{}_0 = -\delta\rho \tag{20}$$
$$\delta T^0{}_i = (\rho+p)(\nabla_i v + v_i + \nabla_i B - B_i) \tag{21}$$
$$\delta T^i{}_j = \delta p \delta^i{}_j + \pi^i{}_j. \tag{22}$$

Under gauge transformation (2), scalars transform as (see A.1)

$$\delta\tilde{\rho} = \delta\rho - \epsilon^0\dot{\bar{\rho}} \tag{23}$$
$$\delta\tilde{p} = \delta p - \epsilon^0\dot{\bar{p}} \tag{24}$$

and the velocity transforms as (see A.2)

$$\tilde{v} = v + \dot{\epsilon}, \quad \tilde{v}^i = v^i + \dot{\epsilon}^i. \tag{25}$$

The components of π_{ij}, that is Π, Π_i and Π_{ij} are all gauge invariant since they vanish in the background (A.3). From these transformation laws, we may form many gauge invariant quantities (omitting the bars on all background quantities now):

$$\Delta = \frac{\delta\rho}{\rho} + \frac{\dot{\rho}}{\rho}(v+B) \tag{26}$$
$$\delta\rho_\sigma = \delta\rho + \dot{\rho}(B-\dot{E}) \tag{27}$$
$$\delta p_\psi = \delta\rho + \frac{\dot{\rho}}{H}\psi \tag{28}$$
$$\mathcal{V} = v + \dot{E} \tag{29}$$
$$\delta p_{nad} = \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho \tag{30}$$
$$\zeta = -\psi - H\frac{\rho}{\dot{\rho}}\Delta \tag{31}$$
$$q_i = (\rho+p)(v_i - B_i) \tag{32}$$
$$Q_i = B_i + \dot{E}_i \tag{33}$$

and Π, Π_i, Π_{ij}.

From the Mathematica program, we explicity calculated $\delta(G^\mu{}_\nu)$ in the metric of (1), the result is:

$\delta(G^{\mu}{}_{\nu})$

00	$\Omega^{-2}[-6\frac{\Omega'}{\Omega}\partial_0\psi + 2\nabla^2\psi - 6(\frac{\Omega'}{\Omega})^2\phi - 2\frac{\Omega'}{\Omega}\nabla^2(B-\partial_0 E)]$
11	$\Omega^{-2}[-2\partial_0\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_0(\phi+2\psi+E_{11}) + 2[(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}]\phi - (\nabla^2-\partial_1\partial_1)(\phi-\psi+\partial_0 B-\partial_0\partial_0 E)$ $- 2\frac{\Omega'}{\Omega}(\nabla^2-\partial_1\partial_1)(B-\partial_0 E) - (\partial_1\partial_0+2\frac{\Omega'}{\Omega}\partial_1)(B_1+\partial_0 E_1) + \Box E_{11}]$
22	$\Omega^{-2}[-2\partial_0\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_0(\phi+2\psi+E_{22}) + 2[(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}]\phi - (\nabla^2-\partial_2\partial_2)(\phi-\psi+\partial_0 B-\partial_0\partial_0 E)$ $- 2\frac{\Omega'}{\Omega}(\nabla^2-\partial_2\partial_2)(B-\partial_0 E) - (\partial_2\partial_0+2\frac{\Omega'}{\Omega}\partial_2)(B_2+\partial_0 E_2) + \Box E_{22}]$
33	$\Omega^{-2}[-2\partial_0\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_0(\phi+2\psi+E_{33}) + 2[(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}]\phi - (\nabla^2-\partial_3\partial_3)(\phi-\psi+\partial_0 B-\partial_0\partial_0 E)$ $- 2\frac{\Omega'}{\Omega}(\nabla^2-\partial_3\partial_3)(B-\partial_0 E) - (\partial_3\partial_0+2\frac{\Omega'}{\Omega}\partial_3)(B_3+\partial_0 E_3) + \Box E_{33}]$
01	$\Omega^{-2}[2\partial_1\partial_0\psi + 2\frac{\Omega'}{\Omega}\partial_1\phi + \frac{1}{2}\nabla^2(B_1+\partial_0 E_1)]$
02	$\Omega^{-2}[2\partial_2\partial_0\psi + 2\frac{\Omega'}{\Omega}\partial_2\phi + \frac{1}{2}\nabla^2(B_2+\partial_0 E_2)]$
03	$\Omega^{-2}[2\partial_3\partial_0\psi + 2\frac{\Omega'}{\Omega}\partial_3\phi + \frac{1}{2}\nabla^2(B_3+\partial_0 E_3)]$
12	$\Omega^{-2}[\partial_1\partial_2(\phi-\psi+\partial_0 B-\partial_0\partial_0 E) -$ $(\frac{1}{2}\partial_0+\frac{\Omega'}{\Omega})(\partial_1 B_2+\partial_1\partial_0 E_2+\partial_2 B_1-\partial_2\partial_0 E_1) + 2\frac{\Omega'}{\Omega}(\partial_1\partial_2 B-\partial_1\partial_2\partial_0 E-\partial_0 E_{12}) + \Box E_{12}]$
13	$\Omega^{-2}[\partial_1\partial_3(\phi-\psi+\partial_0 B-\partial_0\partial_0 E) -$ $(\frac{1}{2}\partial_0+\frac{\Omega'}{\Omega})(\partial_1 B_3+\partial_1\partial_0 E_3+\partial_3 B_1-\partial_3\partial_0 E_1) + 2\frac{\Omega'}{\Omega}(\partial_1\partial_3 B-\partial_1\partial_3\partial_0 E-\partial_0 E_{13}) + \Box E_{13}]$
23	$\Omega^{-2}[\partial_2\partial_3(\phi-\psi+\partial_0 B-\partial_0\partial_0 E) -$ $(\frac{1}{2}\partial_0+\frac{\Omega'}{\Omega})(\partial_2 B_3+\partial_2\partial_0 E_3+\partial_3 B_2-\partial_3\partial_0 E_2) + 2\frac{\Omega'}{\Omega}(\partial_2\partial_3 B-\partial_2\partial_3\partial_0 E-\partial_0 E_{23}) + \Box E_{23}]$

Now we will equate $\delta G^{\mu}{}_{\nu} = -8\pi G \delta T^{\mu}{}_{\nu}$.

Scalars:

$\underline{\delta G^0{}_0 = \delta T^0{}_0}$:

$$\nabla^2\psi - 3H(\dot{\psi} + H\phi) + H\nabla^2(\dot{E} - B) = 4\pi G\Omega^2\delta\rho \tag{34}$$

If we use the Freidman (background) equation $G_{00} = -8\pi G T_{00}$, which implies

$$3H^2 = 8\pi G\rho\Omega^2$$

then we may express (34) in terms of gauge invariant variables

$$\boxed{-\nabla^2\Psi + 3H(\dot{\Psi} + H\Phi) = -4\pi G\Omega^2\delta\rho_\sigma} \tag{35}$$

$\underline{\delta G^0{}_i = \delta T^0{}_i}$:

From the Mathematica result, we get

$$\nabla_i(\dot{\psi} + H\phi) = -4\pi G(\rho + p)\nabla_i(v + B). \tag{36}$$

Ellis drops the ∇_i common to both sides, though it seems we may add an arbitrary function of time. Ellis's result is then

$$\dot{\psi} + H\phi = -4\pi G\Omega^2(\rho + p)(v + B). \tag{37}$$

If we use the Freidman trace equation for the background, which implies

$$3\frac{\ddot{\Omega}}{\Omega} = 4\pi G(\rho + 3p),$$

then we can express (37) as the gauge invariant equation:

$$\boxed{\dot{\Psi} + H\Phi = -4\pi G\Omega^2(\rho + p)\mathcal{V}} \tag{38}$$

<u>$\delta G^i{}_j = \delta T^i{}_j \quad i \neq j$:</u>

From the Mathematica result, we get

$$\nabla_i\nabla_j\left[(\ddot{E} - \dot{B}) + 2H(\dot{E} - B) - \phi + \psi\right] = 8\pi G\Omega^2\nabla_i\nabla_j\Pi. \tag{39}$$

Ellis again drops the $\nabla_i\nabla_j$ to obtain

$$(\ddot{E} - \dot{B}) + 2H(\dot{E} - B) - \phi + \psi = 8\pi G\Omega^2\Pi. \tag{40}$$

This one may be expressed easily in gauge invariant form

$$\boxed{\Psi - \Phi = 8\pi G\Omega^2\Pi} \tag{41}$$

<u>$\delta^j{}_i\delta G^i{}_j = \delta^j{}_i\delta T^i{}_j$:</u>

From the Mathematica result of the spatial trace, we get

$$\ddot{\psi} + H(\dot{\phi} + 2\dot{\psi}) + (2\dot{H} + H^2)\phi + \frac{1}{3}\nabla^2[\phi - \psi + \dot{B} - \ddot{E} + 2H(B - \dot{E})] = \frac{4}{3}\pi G\Omega^2\delta p. \tag{42}$$

Substituting the Laplacian of (40) into the above, we get

$$\ddot{\psi} + H(\dot{\phi} + 2\dot{\psi}) + (2\dot{H} + H^2)\phi = 4\pi G\Omega^2\left(\delta p + \frac{2}{3}\nabla^2\Pi\right). \tag{43}$$

The gauge invariant form given in Ellis for the spatial trace needs further inspection.

Vectors:

<u>$\delta G^0{}_i = \delta T^0{}_i$:</u>
From the Mathematica result, we get

$$\nabla^2(B_i + \dot{E}_i) = -16\pi G\Omega^2(\rho + p)(v_i - B_i) \tag{44}$$

which is easily expressed in gauge invariant form

$$\boxed{\nabla^2 Q_i = -16\pi G\Omega^2 q_i} \tag{45}$$

<u>$\delta G^i{}_j = \delta T^i{}_j \quad i \neq j$:</u>
From the Mathematica result, we get

$$\nabla_i(\dot{B}_j + \ddot{E}_j) + \nabla_j(\dot{B}_i + \ddot{E}_i) + 2H\nabla_i(B_j + \dot{E}_j) + 2H\nabla_j(B_i + \dot{E}_i) = 8\pi G\Omega^2(\nabla_i\Pi_j + \nabla_j\Pi_i). \tag{46}$$

Ellis equates the ∇_i and ∇_j quantities with each other as in

$$\dot{B}_i + \ddot{E}_i + 2HB_i + \dot{E}_i = 8\pi G\Omega^2\Pi_i$$

in which the gauge invariance manifests as

$$\boxed{\dot{Q}_i + 2HQ_i = 8\pi G\Omega^2\Pi_i}. \tag{47}$$

Tensors:

<u>$\delta G^i{}_j = \delta T^i{}_j$:</u>
From the Mathematica result, we get a result that is already gauge invariant

$$\boxed{2H\dot{E}_{ij} - \Box E_{ij} = 8\pi G\Omega^2\Pi_{ij}} \tag{48}$$

Appendix

* Apparent sign error in vector quantity E_i in $\delta G^\mu{}_\nu$.

In RW K=0 space, $\Omega(t)$, we have the covariant Einstein tensor and Weyl tensor:

$\delta G_{\mu\nu}$

00	$6\frac{\Omega'}{\Omega}\partial_0\psi - 2\nabla^2\psi + 2\frac{\Omega'}{\Omega}\nabla^2(B-\partial_0 E)$
11	$-2\partial_0\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_0(\phi+2\psi+E_{11})$ $+ 2[(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\phi+\psi-\partial_1\partial_1E-\partial_1E_1-E_{11}) -$ $(\nabla^2-\partial_1\partial_1)(\phi-\psi+\partial_0B-\partial_0\partial_0E) - 2\frac{\Omega'}{\Omega}(\nabla^2-\partial_1\partial_1)(B-\partial_0E)$ $- (\partial_1\partial_0+2\frac{\Omega'}{\Omega}\partial_1)(B_1+\partial_0E_1) + \Box E_{11}$
22	$-2\partial_0\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_0(\phi+2\psi+E_{22})$ $+ 2[(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\phi+\psi-\partial_2\partial_2E-\partial_2E_2-E_{22}) -$ $(\nabla^2-\partial_2\partial_2)(\phi-\psi+\partial_0B-\partial_0\partial_0E) - 2\frac{\Omega'}{\Omega}(\nabla^2-\partial_2\partial_2)(B-\partial_0E)$ $- (\partial_2\partial_0+2\frac{\Omega'}{\Omega}\partial_2)(B_2+\partial_0E_2) + \Box E_{22}$
33	$-2\partial_0\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_0(\phi+2\psi+E_{33})$ $+ 2[(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\phi+\psi-\partial_3\partial_3E-\partial_3E_3-E_{33}) -$ $(\nabla^2-\partial_3\partial_3)(\phi-\psi+\partial_0B-\partial_0\partial_0E) - 2\frac{\Omega'}{\Omega}(\nabla^2-\partial_3\partial_3)(B-\partial_0E)$ $- (\partial_3\partial_0+2\frac{\Omega'}{\Omega}\partial_3)(B_3+\partial_0E_3) + \Box E_{33}$
01	$-2\partial_1\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_1\phi - [(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\partial_1B-B_1) - \frac{1}{2}\nabla^2(B_1+\partial_0E_1)$
02	$-2\partial_2\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_2\phi - [(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\partial_2B-B_2) - \frac{1}{2}\nabla^2(B_2+\partial_0E_2)$
03	$-2\partial_3\partial_0\psi - 2\frac{\Omega'}{\Omega}\partial_3\phi - [(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\partial_3B-B_3) - \frac{1}{2}\nabla^2(B_3+\partial_0E_3)$
12	$\partial_1\partial_2(\phi-\psi+\partial_0B-\partial_0\partial_0E) + 2\frac{\Omega'}{\Omega}\partial_1\partial_2(B-\partial_0E)$ $- (\frac{1}{2}\partial_0+\frac{\Omega'}{\Omega})(\partial_1B_2+\partial_1\partial_0E_2+\partial_2B_1+\partial_2\partial_0E_1)$ $- [(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\partial_1E_2+\partial_2E_1-2\partial_1\partial_2E) + \Box E_{12}$
13	$\partial_1\partial_3(\phi-\psi+\partial_0B-\partial_0\partial_0E) + 2\frac{\Omega'}{\Omega}\partial_1\partial_3(B-\partial_0E)$ $- (\frac{1}{2}\partial_0+\frac{\Omega'}{\Omega})(\partial_1B_3+\partial_1\partial_0E_3+\partial_3B_1+\partial_3\partial_0E_1)$ $- [(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\partial_1E_3+\partial_3E_1-2\partial_1\partial_3E) + \Box E_{13}$
23	$\partial_2\partial_3(\phi-\psi+\partial_0B-\partial_0\partial_0E) + 2\frac{\Omega'}{\Omega}\partial_2\partial_3(B-\partial_0E)$ $- (\frac{1}{2}\partial_0+\frac{\Omega'}{\Omega})(\partial_2B_3+\partial_2\partial_0E_3+\partial_3B_2+\partial_3\partial_0E_2)$ $- [(\frac{\Omega'}{\Omega})^2-2\frac{\Omega''}{\Omega}](\partial_2E_3+\partial_3E_2-2\partial_2\partial_3E) + \Box E_{23}$

$\delta W_{\mu\nu}$

00	$\Omega^{-2}[-\frac{2}{3}\nabla^4(\psi+\phi+\partial_0 B-\partial_0\partial_0 E)]$
11	$\Omega^{-2}[-\frac{1}{3}[\Box^2+\Box(\partial_0\partial_0-\partial_1\partial_1)+2\partial_1\partial_1\partial_0\partial_0](\psi+\phi+\partial_0 B-\partial_0\partial_0 E)$ $+\ \Box\partial_1(\partial_0 B_1-\partial_0\partial_0 E_1)\ +\ \Box^2 E_{11}]$
22	$\Omega^{-2}[-\frac{1}{3}[\Box^2+\Box(\partial_0\partial_0-\partial_2\partial_2)+2\partial_2\partial_2\partial_0\partial_0](\psi+\phi+\partial_0 B-\partial_0\partial_0 E)$ $+\ \Box\partial_2(\partial_0 B_2-\partial_0\partial_0 E_2)\ +\ \Box^2 E_{22}]$
33	$\Omega^{-2}[-\frac{1}{3}[\Box^2+\Box(\partial_0\partial_0-\partial_3\partial_3)+2\partial_3\partial_3\partial_0\partial_0](\psi+\phi+\partial_0 B-\partial_0\partial_0 E)$ $+\ \Box\partial_3(\partial_0 B_3-\partial_0\partial_0 E_3)\ +\ \Box^2 E_{33}]$
01	$\Omega^{-2}[-\frac{2}{3}\nabla^2\partial_1(\partial_0\psi+\partial_0\phi+\partial_0\partial_0 B-\partial_0\partial_0\partial_0 E)+\frac{1}{2}(\nabla^4-\nabla^2\partial_0\partial_0)(B_1-\partial_0 E_1)]$
02	$\Omega^{-2}[-\frac{2}{3}\nabla^2\partial_2(\partial_0\psi+\partial_0\phi+\partial_0\partial_0 B-\partial_0\partial_0\partial_0 E)+\frac{1}{2}(\nabla^4-\nabla^2\partial_0\partial_0)(B_2-\partial_0 E_2)]$
03	$\Omega^{-2}[-\frac{2}{3}\nabla^2\partial_3(\partial_0\psi+\partial_0\phi+\partial_0\partial_0 B-\partial_0\partial_0\partial_0 E)+\frac{1}{2}(\nabla^4-\nabla^2\partial_0\partial_0)(B_3-\partial_0 E_3)]$
12	$\Omega^{-2}[\frac{1}{3}(\Box-2\partial_0\partial_0)\partial_1\partial_2(\psi+\phi+\partial_0 B-\partial_0\partial_0 E)\ +$ $\frac{1}{2}\Box\partial_1\partial_0(B_2-\partial_0 E_2)\ +\ \frac{1}{2}\Box\partial_2\partial_0(B_1-\partial_0 E_1)\ +\ \Box^2 E_{12}]$
13	$\Omega^{-2}[\frac{1}{3}(\Box-2\partial_0\partial_0)\partial_1\partial_3(\psi+\phi+\partial_0 B-\partial_0\partial_0 E)\ +$ $\frac{1}{2}\Box\partial_1\partial_0(B_3-\partial_0 E_3)\ +\ \frac{1}{2}\Box\partial_3\partial_0(B_1-\partial_0 E_1)\ +\ \Box^2 E_{13}]$
23	$\Omega^{-2}[\frac{1}{3}(\Box-2\partial_0\partial_0)\partial_2\partial_3(\psi+\phi+\partial_0 B-\partial_0\partial_0 E)\ +$ $\frac{1}{2}\Box\partial_2\partial_0(B_3-\partial_0 E_3)\ +\ \frac{1}{2}\Box\partial_3\partial_0(B_2-\partial_0 E_2)\ +\ \Box^2 E_{23}]$