Applications of Standard Gravity and Conformal Gravity

James G. O'Brien, Ph.D.

University of Connecticut,

In this Thesis, we explore a number of interesting problems in the study of gravitation. First, we explore the open question of the validity of alternative theories of gravitation through applying the Conformal Gravity theory to galactic rotation curve data. The Curves generated are fits that require no Dark Matter of external fitting parameters, and are of a universal nature with striking results. Next, we apply the standard linearized Einstein Gravity to the Ring Laser of Mallett to calculate the gravitational Faraday effect produced by the setup. Third, we apply the mathematics of general relativity to the field of transformation optics to yield a straight forward way to compute the geodesics for light rays traveling through transformative media. Lastly, we explore the standard gravitational formulation of covarientizing a space-time perfect fluid. We show that the calculation when performed in a curved space time using an incoherent averaging procedure does not return the standard perfect fluid form of the energy momentum tensor.

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James G. O'Brien

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Applications of Standard Gravity and Conformal Gravity

Presented by

James G. O'Brien,

Major Advisor Philip Mannheim

Associate Advisor Ronald Mallett

Associate Advisor Alexander Kovner

University of Connecticut

This Thesis is dedicated to my family. Without your loving support, none of the work enclosed would be possible. To my Mother Martha, who has taught me to have endless patience for whatever obstacles the world may hurl at me. To my Father Jim, who has been a role model of service, and has proven to me time and time again the endless rewards of helping others. To my sister Kathleen, who has taught me to have the strength to be yourself, no matter the circumstances, and showed me how to sail through all rough waters. To my best friend and brother Scott, who has taught me to be a leader and showed how to inspire others by leading through example. To my godmother Dorathy, who has showed endless generosity to all around her. To my Grandpa Vincent and my Grammies Dorathy, who showed the support of how strong a family bond can be, and that through love, one can accomplish anything. To my Grandma Elenor, and my Grandfather "'Pop"' William, who is the person I desire to be, a person with the purist of souls, with the spirit of an angel, the smile of a king, and the heart of a saint. And my loving, beautiful and angelic wife Natalie, who has been my harbor, my inspiration, and helped me to grow into the person I am today. Without all of you, I have no well defined existence.

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and everyone else whom I have had the pleasure to encounter, thank you.

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Chapter 1

General Introduction

The Thesis presented here is the culmination of the various works in which I had the pleasure to be a part of during my graduate career at the University of Connecticut. Upon arrival, I had the priveledge of working with two highly creative and influential physicists in the world of Cosmology and Gravitation. The work I have done under the guidance of Dr. Philip Mannheim, presented in chapters two, three and four, are works that challenge the standard theory of Gravitation and Cosmology. It is a truly exiting path to walk, when you are challenging the physics community at large on such a topic as to the validity of dark matter. The topics covered in chapters two and three challenge the dark matter theory by fitting rotation curves to a wide range of galaxies, varying in their respective morphologies with striking accuracy. It is the hope that this work will further the conformal theory as a candidate for an alternative theory of gravitation, and hopefully attract more people in the community to consider taking a step back from the standard theory, and entertain not only the conformal theory, but others as well. In Chapter four, we address the validity of the current covarientization methods of standard gravitation. This topic, is one in which we usually take for granted in standard workings, and thus, some of the implications are usually overlooked. We show that when performing this calculation in a curved space, some information is lost, and thus for strongly interacting gravitational systems. With current work being done with precise study of neutron stars and even theoretical models of quark stars, much work for this topic is left for the future. In chapter five, we explore work done with Dr. Ronald Mallett. At an early stage of my graduate career, Mallett had suggested the calculation of the gravitational effect on the plane of polarization of an incident laser through the Mallett ring laser setup. We show that the plane of polarization is shifted due to the interaction with the ring laser, and thus we could in the future provide the grounds for a terrestrial experiment for testing for gravitational frame dragging. Currently, this work is being carried out by Dr. Ronald Mallett through a joint collaboration with Penn. State, known as the STL project (Space Twisting by Light). Lastly, we explore the application of General Relativity to the field of Transformation optics. This project was initially begun by another graduate student, Richard Crudo, who got me involved due to the highly interesting aspects of linking these two seemingly unrelated fields. The results that followed were not only instructive to the field of transformation optics, but also allowed for us to explore the possibility of a new two dimensional cloaking device which could have implications in astrophysics. All of the above have been a pleasure to be a part of, and as a physicist moving to the next stage of my career, I am forever grateful to have worked on such exciting and new topics as a thesis.

Chapter 2

Impact of a global quadratic potential on galactic rotation curves

We have made a conformal gravity fit to an available sample of 110 spiral galaxies, and report here on the 20 of those galaxies whose rotation curve data points extend the furthest from galactic centers. We identify the impact on the 20 galaxy data set of a universal de Sitter-like potential term $V(r) = -\kappa c^2 r^2/2$ that is induced by inhomogeneities in the cosmic background. This quadratic term accompanies a universal linear potential term $V(r) = \gamma_0 c^2 r/2$ that is associated with the cosmic background itself. We find that when these two potential terms are taken in conjunction with the contribution generated by the local luminous matter within the galaxies, the conformal theory is able to account for the rotation curve systematics that is observed in the entire 110 galaxy sample, without the need for any dark matter whatsoever. With the two universal coefficients being found to be of global magnitude, viz. $\kappa = 9.54 \times 10^{-54}$ cm^{-2} and $\gamma_0 = 3.06 \times 10^{-30}$cm^{-1}, our study suggests that invoking the presence of dark matter may be nothing more than an attempt to describe global effects in purely local galactic terms. With the quadratic potential term having negative sign, galaxies are only able to support bound orbits up to distances of order $\gamma_0/\kappa = 3.21 \times 10^{23}$ cm, with global physics thus imposing a natural limit on the size of galaxies.

I Introduction

At the present time it is widely believed that on scales much larger than solar-system-sized ones astrophysical and cosmological phenomena are controlled by dark matter and dark energy, with luminous matter being only a minor contributor. However, given the lack to date of either direct detection of dark matter particles or of a solution to the cosmological constant problem, a few authors (see e.g. [1] for a recent review) have ventured to suggest that the standard dark matter/dark energy picture may be incorrect, and that one instead needs to modify the standard Newton-Einstein gravitational theory that leads to that picture in the first place. In this paper we study one specific alternative to Einstein gravity that has been advanced, namely conformal gravity. We report here on the results of a conformal gravity study of the instructive 17 largest of a full sample of 110 galaxies, all of whose rotation curves we have readily been able to fit without the need for any dark matter at all.

In seeking an alternative to Einstein gravity that is to address both the dark matter and dark energy problems, our strategy is to seek some alternate, equally metric-based theory of gravity that possesses all of the general coordinate invariance and equivalence principle structure of Einstein gravity, that yields a geometry that is described by the Ricci-flat Schwarzschild metric on solar-system-sized distance scales while departing from it on larger ones, and that has a symmetry that can control the cosmological constant Λ. All of these criteria are met in the conformal gravity theory (see e.g. [1]) that was first developed by Weyl. Specifically, as well as coordinate invariance, in addition one requires that the action be left invariant under local conformal transformations of the form $g_{\mu\nu}(x) \to e^{2\alpha(x)}g_{\mu\nu}(x)$ with arbitrary local phase $\alpha(x)$. Given this requirement, the gravitational action is then uniquely prescribed to be of the form $I_{\rm W} = -\alpha_g \int d^4x(-g)^{1/2}C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa} = -2\alpha_g \int d^4x(-g)^{1/2}\left[R_{\mu\kappa}R^{\mu\kappa} - (1/3)(R^{\alpha}{}_{\alpha})^2\right]$ where $C^{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor and α_g is a dimensionless gravitational coupling constant.

With the conformal symmetry forbidding the presence of any fundamental Λ term in $I_{\rm W}$, conformal gravity has a control on Λ that is not possessed by Einstein gravity; and through this control conformal gravity is then able to solve the cosmological constant problem [2,3]. In addition, the conformal gravity equations of motion are given by [1]

$$
\begin{aligned}
4\alpha_g W^{\mu\nu} &= 4\alpha_g\left[2C^{\mu\lambda\nu\kappa}{}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = 4\alpha_g\left[W^{\mu\nu}_{(2)} - \frac{1}{3}W^{\mu\nu}_{(1)}\right] = T^{\mu\nu}, \\
W^{\mu\nu}_{(1)} &= 2g^{\mu\nu}(R^{\alpha}{}_{\alpha})^{;\beta}{}_{;\beta} - 2(R^{\alpha}{}_{\alpha})^{;\mu;\nu} - 2R^{\alpha}{}_{\alpha}R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}(R^{\alpha}{}_{\alpha})^2, \\
W^{\mu\nu}_{(2)} &= \frac{1}{2}g^{\mu\nu}(R^{\alpha}{}_{\alpha})^{;\beta}{}_{;\beta} + R^{\mu\nu;\beta}{}_{;\beta} - R^{\mu\beta;\nu}{}_{;\beta} - R^{\nu\beta;\mu}{}_{;\beta} - 2R^{\mu\beta}R^{\nu}{}_{\beta} + \frac{1}{2}g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}
\end{aligned}
\tag{2.1}
$$

with Schwarzschild thus being a vacuum solution to conformal gravity, just as required [4].

II Universal Potentials from the Rest of the Universe

Given its structure, $W^{\mu\nu}$ could potentially vanish even if the geometry is not Ricci flat, and the conformal theory could thus have non-Schwarzschild solutions as well. To identify any such solutions, Mannheim and Kazanas solved for the metric in a vacuum region exterior to a static, spherically symmetric source of radius r_0, to find [5] that the exact, all-order line element is given by $ds^2 = -B(r)dt^2 + dr^2/B(r) + r^2 d\Omega_2$, with

$$B(r > r_0) = 1 - \frac{2\beta}{r} + \gamma r - kr^2. \tag{2.2}$$

In the γr and $-kr^2$ terms we see that the conformal gravity metric departs from the $B(r > r_0) = 1 - 2\beta/r$ Schwarzschild metric only at large r, just as we want.

In seeking to relate the various constants in (6.2) to properties of the energy-momentum tensor $T_{\mu\nu}$ of the source, Mannheim and Kazanas found [6] that in the static, spherically symmetric case the quantity $(3/B(r))\,(W^0_{\ 0} - W^r_{\ r})$ evaluates exactly to $\nabla^4 B(r)$; and that, in terms of the general source function $f(r) = (3/4\alpha_g B(r))(T^0_{\ 0} - T^r_{\ r})$, the exact fourth-order equation of motion of the conformal theory reduced to the remarkably simple

$$\nabla^4 B = \left[\frac{d^4}{dr^4} + \frac{4}{r}\frac{d^3}{dr^3}\right] B(r) = f(r), \tag{2.3}$$

without any approximation whatsoever. Since $\nabla^4(r^2)$ vanishes identically everywhere while $\nabla^4(1/r)$ and $\nabla^4(r)$ evaluate to delta functions and their derivatives, we see that of the constants given in (2.2), only β and γ, but not k, can be associated with properties of a local source of radius r_0; with the matching of the interior and exterior metrics then yielding [6]

$$\gamma = -\frac{1}{2}\int_0^{r_0} dr' r'^2 f(r'), \qquad 2\beta = \frac{1}{6}\int_0^{r_0} dr' r'^4 f(r'). \tag{2.4}$$

Thus despite the presence of the $-kr^2$ term in the exterior vacuum solution in (2.2), the above analysis provides no specific basis for considering it further, as it is associated with the trivial solution to $\nabla^4 B = 0$, to thereby be devoid of dynamical content.

In conformal gravity a local gravitational source generates a gravitational potential

$$V^*(r) = -\frac{\beta^* c^2}{r} + \frac{\gamma^* c^2 r}{2} \tag{2.5}$$

per unit solar mass, with β^* being given by the familiar $M_\odot G/c^2 = 1.48 \times 10^5$ cm, and with the numerical value of the solar γ^* needing to be determined by data fitting. In the theory the visible local material in a given galaxy would generate a net local gravitational potential $V_{\rm LOC}(r)$ given by integrating $V^*(r)$ over the visible galactic mass distribution. In disk galaxies luminous matter is typically distributed with a surface brightness $\Sigma(R) = \Sigma_0 e^{-R/R_0}$ with scale length R_0 and total luminosity $L = 2\pi\Sigma_0 R_0^2$, with most of the surface brightness being contained in the $R \leq 4R_0$ or so optical disk region. For a galactic mass to light ratio M/L

one can define the number of solar mass units N^* in the galaxy to be $M = N^* M_\odot$. Then, on integrating $V^*(r)$ over this visible matter distribution, one obtains [1]

$$\begin{aligned} \frac{v_{\rm LOC}^2}{R} &= \frac{N^*\beta^* c^2 R}{2R_0^3}\left[I_0\left(\frac{R}{2R_0}\right)K_0\left(\frac{R}{2R_0}\right) - I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right)\right] \\ &\quad + \frac{N^*\gamma^* c^2 R}{2R_0} I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right) \end{aligned} \tag{2.6}$$

for the net local contribution to the centripetal accelerations of particles orbiting in the plane of the galactic disk.

However, unlike the situation that obtains in standard second-order gravity, one cannot simply use (2.6) as is to fit galactic rotation curve data, as there are two additional global effects coming from the rest of the material in the universe that need to be taken into consideration as well, one associated with the homogeneous cosmological background and the other with the inhomogeneities in it. As regards first the effect of inhomogeneities, we recall for the standard second order Poisson equation $\nabla^2\phi(r) = g(r)$, the force associated with a general static, spherically symmetric source $g(r)$ is given by

$$\frac{d\phi(r)}{dr} = \frac{1}{r^2}\int_0^r dr' r'^2 g(r'). \tag{2.7}$$

As such, the import of (2.7) is that even though $g(r)$ could continue globally all the way to infinity, the force at any radial point r is determined only by the material in the local $0 < r' < r$ region. In this sense Newtonian gravity is local, since to explain a gravitational effect in some local region one only needs to consider the material in that region. Thus in Newtonian gravity, if one wishes to explain the behavior of galactic rotation curves through the use of dark matter, one must locate the dark matter where the problem is and not elsewhere. Since the discrepancy problem in galaxies occurs primarily in the region beyond the optical disk, one must thus locate galactic dark matter in precisely the region in galaxies where there is little or no visible matter.

However, this local character to Newtonian gravity is not a generic property of any gravitational potential. In particular for the fourth-order Poisson equation $\nabla^4\phi(r) = h(r) = f(r)c^2/2$ of interest to conformal gravity, the potential and the force evaluate to

$$\begin{aligned}
\phi(r) &= -\frac{r}{2}\int_0^r dr' r'^2 h(r') - \frac{1}{6r}\int_0^r dr' r'^4 h(r') - \frac{1}{2}\int_r^\infty dr' r'^3 h(r') - \frac{r^2}{6}\int_r^\infty dr' r' h(r'), \\
\frac{d\phi(r)}{dr} &= -\frac{1}{2}\int_0^r dr' r'^2 h(r') + \frac{1}{6r^2}\int_0^r dr' r'^4 h(r') - \frac{r}{3}\int_r^\infty dr' r' h(r'),
\end{aligned} \qquad (2.8)$$

so that this time we do find a global contribution to the force coming from material that is beyond the radial point of interest. Hence in conformal gravity one cannot ignore the rest of the universe, with a test particle in orbit in a galaxy being able to sample both the local field due to the matter in the galaxy and the global field due to the rest of the universe.

In the presence of inhomogeneities $W^{\mu\nu}$ does not vanish, as the very presence of a localized source prevents a geometry from being conformal to flat, with inhomogeneities in the universe thus leading to integrals in (2.8) that can extend to very large distances. However, this is not the only global effect that we need to take into consideration, as one can also add on to (2.8) any terms that would cause

$W^{\mu\nu}$ to vanish, provided they make it do so non-trivially. Since the cosmological Robertson-Walker (RW) metric is homogeneous and isotropic, it is conformal to flat, and thus its geometry obeys $W^{\mu\nu} = 0$. For the cosmological background the vanishing of $W^{\mu\nu}$ entails that conformal cosmology be described by $T^{\mu\nu} = 0$. As discussed in [7,1] the equation $T^{\mu\nu} = 0$ can be satisfied non-trivially, and leads to a non-trivial RW cosmology, with its contribution to $W^{\mu\nu}$ then indeed vanishing non-trivially, just as desired.

Since cosmology is written in comoving Hubble flow coordinates while rotation curves are measured in galactic rest frames, to ascertain the impact of cosmology on rotation curves one needs to transform the RW metric to static coordinates. As noted in [5], the transformation

$$\rho = \frac{4r}{2(1+\gamma_0 r - kr^2)^{1/2} + 2 + \gamma_0 r}, \qquad \tau = \int dt R(t) \tag{2.9}$$

effects the metric transformation

$$\begin{aligned} &-(1+\gamma_0 r - kr^2)c^2 dt^2 + \frac{dr^2}{(1+\gamma_0 r - kr^2)} + r^2 d\Omega_2 = \\ &\frac{1}{R^2(\tau)} \frac{[1-\rho^2(\gamma_0^2/16 + k/4)]^2}{[(1-\gamma_0\rho/4)^2 + k\rho^2/4]^2} \left[-c^2 d\tau^2 + \frac{R^2(\tau)}{[1-\rho^2(\gamma_0^2/16+k/4)]^2}(d\rho^2 + \rho^2 d\Omega_2)\right] \end{aligned} \tag{2.10}$$

Since an RW geometry is conformally flat and since it remains so under a conformal transformation, we see that when written in a static coordinate system a comoving conformal cosmology with 3-space spatial curvature K looks just like a static metric with universal linear and quadratic terms with coefficients that obey $K = -\gamma_0^2/4 - k$. However, since there was only one spatial scale in the RW metric (viz. K), its decomposition into two static coordinate system scales (γ_0 and k) was artificial, and so in [8] the k term was dropped. Then, without the k term we see that in a static coordinate system a topologically open RW cosmology looks like a universal linear potential with cosmological strength $\gamma_0/2 = (-K)^{1/2}$.

In the conformal theory then we recognize not one but two linear potential terms, a local $N^*\gamma^*$ dependent one associated with the matter within a galaxy and a global cosmological one $\gamma_0 c^2 r/2$ associated with cosmological background. Thus in the weak gravity limit one can add the two potentials and replace (2.6) by [8]

$$\frac{v_{\mathrm{TOT}}^2}{R} = \frac{v_{\mathrm{LOC}}^2}{R} + \frac{\gamma_0 c^2}{2}. \tag{2.11}$$

In [8] (2.11) was used to fit the galactic rotation curve data of a sample of 11 galaxies (of which only NGC 2841 and NGC 3198 are in the sample considered here), and very good fits were found, with the two universal linear potential parameters being fixed to the values

$$\gamma^* = 5.42 \times 10^{-41}\mathrm{cm}^{-1}, \qquad \gamma_0 = 3.06 \times 10^{-30}\mathrm{cm}^{-1}. \tag{2.12}$$

The value obtained for γ^* entails that the linear potential of the Sun is so small that there are no modifications to standard solar system phenomenology, with the values obtained for $N^*\gamma^*$ and γ_0 being such that one has to go to galactic scales before their effects can become as big as the Newtonian contribution. The value obtained for γ_0 shows that it is indeed of cosmological magnitude. In the fitting to the 110 galaxy sample (2.12) does not change.