Perturbations in conformal to flat geometry

In[102]:=
$$R_{\mu\nu} = -\frac{\eta^{\alpha\beta}\ \eta_{\mu\nu}\ \partial_\alpha\Omega\,\partial_\beta\Omega}{\Omega^2} - \frac{\eta^{\alpha\beta}\ \eta_{\mu\nu}\ \partial_\beta\partial_\alpha\Omega}{\Omega} + \frac{4\,\partial_\mu\Omega\,\partial_\nu\Omega}{\Omega^2} - \frac{2\,\partial_\nu\partial_\mu\Omega}{\Omega}$$

$$R = -\frac{6\ \eta^{\alpha\beta}\ \partial_\beta\partial_\alpha\Omega}{\Omega^3}$$

In[107]:=
$$\delta W_1 = 2\ K^{\alpha}{}_{\mu}\ R_{\alpha\beta}\ R_{\nu}{}^{\beta} - g_{\mu\nu}\ K^{\alpha}{}_{\alpha}\ R^2 + \frac{1}{2}\ K_{\mu\nu}\ R^2 + R\ K_{\nu\mu}{}^{;\alpha}{}_{;\alpha} + 2\ K_{\mu\nu}\ \nabla_\alpha\nabla^\alpha R -$$
$$2\ R_{\mu\nu}\ K^{\alpha\beta}{}_{;\beta;\alpha} + g_{\mu\nu}\ R\ K^{\alpha\beta}{}_{;\beta;\alpha} - R\ K^{\alpha}{}_{\mu;\nu;\alpha} - \nabla_\alpha R\ K_{\nu\mu}{}^{;\alpha} - 2\ g_{\mu\nu}\ \nabla_\alpha R\ K^{\alpha\beta}{}_{;\beta} +$$
$$g_{\mu\nu}\ \nabla_\alpha R\ K^{\beta\alpha}{}_{;\beta} + 2\ g^{\alpha\beta}\ g_{\mu\nu}\ K^{\gamma\delta}{}_{;\delta;\gamma;\alpha;\beta} - g^{\alpha\beta}\ g_{\mu\nu}\ K_{\gamma}{}^{\gamma;\delta}{}_{;\delta;\alpha;\beta} -$$
$$g^{\alpha\beta}\ g_{\mu\nu}\ K^{\gamma}{}_{\gamma}{}^{;\delta}{}_{;\delta;\alpha;\beta} + R_{\mu\nu}\ K_{\alpha}{}^{\alpha;\beta}{}_{;\beta} - \frac{1}{2}\ g_{\mu\nu}\ R\ K_{\alpha}{}^{\alpha;\beta}{}_{;\beta} + R_{\mu\nu}\ K^{\alpha}{}_{\alpha}{}^{;\beta}{}_{;\beta} -$$
$$\frac{1}{2}\ g_{\mu\nu}\ R\ K^{\alpha}{}_{\alpha}{}^{;\beta}{}_{;\beta} - 2\ g_{\mu\nu}\ K^{\alpha}{}_{\alpha}\ \nabla_\beta\nabla^\beta R - 2\ g^{\alpha\beta}\ g_{\mu\nu}\ K^{\gamma}{}_{\alpha}\ \nabla_\gamma\nabla_\beta R - 2\ g_{\mu\nu}\ R_{\alpha\beta}\ K^{\alpha\beta;\gamma}{}_{;\gamma} -$$
$$4\ g_{\mu\nu}\ R_{\alpha\beta;\gamma}\ K^{\alpha\beta;\gamma} + 2\ K^{\alpha}{}_{\nu}{}^{;\beta}\ R_{\alpha\beta;\mu} + R\ K^{\alpha}{}_{\nu;\alpha;\mu} - R\ K_{\nu}{}^{\alpha}{}_{;\alpha;\mu} + 2\,\nabla_\alpha R\ K^{\alpha}{}_{\mu;\nu} +$$
$$2\ K^{\alpha}{}_{\mu}{}^{;\beta}\ R_{\alpha\beta;\nu} + 2\ K^{\alpha}{}_{\mu}\ \nabla_\nu\nabla_\alpha R + 2\ R_{\alpha\beta}\ K^{\alpha}{}_{\mu}{}^{;\beta}{}_{;\nu} - 2\ K^{\alpha\beta}{}_{;\beta;\alpha;\mu;\nu} + K_{\alpha}{}^{\alpha;\beta}{}_{;\beta;\mu;\nu} + K^{\alpha}{}_{\alpha}{}^{;\beta}{}_{;\beta;\mu;\nu}$$

$$\delta W_2 = \frac{1}{2} K_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} + 2 K^{\alpha}{}_{\mu} R_{\nu\alpha} R - g_{\mu\nu} K^{\alpha}{}_{\alpha} R^2 + \frac{1}{2} K_{\mu\nu} \nabla_\alpha \nabla^\alpha R - K^{\alpha}{}_{\mu} R_{\nu}{}^{\beta}{}_{;\beta;\alpha} -$$
$$\frac{1}{2} R^{\alpha}{}_{\beta} K_{\mu\nu}{}^{;\beta}{}_{;\alpha} - \frac{1}{2} R^{\alpha}{}_{\beta} K_{\nu\mu}{}^{;\beta}{}_{;\alpha} + \frac{1}{4} g_{\mu\nu} R^{\alpha}{}_{\beta} K^{\beta\gamma}{}_{;\gamma;\alpha} - \frac{1}{4} g_{\mu\nu} R^{\alpha}{}_{\beta} K^{\gamma\beta}{}_{;\gamma;\alpha} +$$
$$\frac{1}{2} K^{\alpha}{}_{\nu}{}^{;\beta}{}_{;\beta;\mu;\alpha} - \frac{1}{2} K^{\alpha\beta}{}_{;\beta;\nu;\mu;\alpha} + 2 R^{\alpha}{}_{\beta} K^{\beta}{}_{\mu;\nu;\alpha} + \frac{1}{2} K^{\alpha}{}_{\mu}{}^{;\beta}{}_{;\beta;\nu;\alpha} - \frac{1}{2} K^{\alpha\beta}{}_{;\beta;\mu;\nu;\alpha} -$$
$$R_{\mu\nu;\alpha} K^{\alpha\beta}{}_{;\beta} - \frac{1}{2} g_{\mu\nu} \nabla_\alpha R K^{\alpha\beta}{}_{;\beta} + R_{\nu}{}^{\beta}{}_{;\alpha} K^{\alpha}{}_{\mu;\beta} + R_{\mu}{}^{\beta}{}_{;\alpha} K^{\alpha}{}_{\nu;\beta} + \frac{1}{4} g_{\mu\nu} \nabla_\alpha R K^{\beta\alpha}{}_{;\beta} -$$
$$\frac{1}{2} K_{\mu\nu}{}^{;\alpha} R_{\alpha}{}^{\beta}{}_{;\beta} - \frac{1}{2} K_{\nu\mu}{}^{;\alpha} R_{\alpha}{}^{\beta}{}_{;\beta} - g^{\alpha\beta} K^{\gamma}{}_{\nu;\alpha} R_{\mu\gamma;\beta} - g^{\alpha\beta} K^{\gamma}{}_{\mu;\alpha} R_{\nu\gamma;\beta} -$$
$$R_{\nu}{}^{\alpha} K^{\beta}{}_{\mu;\alpha;\beta} - R_{\mu}{}^{\alpha} K^{\beta}{}_{\nu;\alpha;\beta} - \frac{1}{2} g^{\alpha\beta} K_{\nu\mu}{}^{;\gamma}{}_{;\gamma;\alpha;\beta} + \frac{1}{2} g^{\alpha\beta} g_{\mu\nu} K^{\gamma\delta}{}_{;\delta;\gamma;\alpha;\beta} +$$
$$\frac{1}{2} g^{\alpha\beta} K^{\gamma}{}_{\mu;\nu;\gamma;\alpha;\beta} - \frac{1}{4} g^{\alpha\beta} g_{\mu\nu} K_{\gamma}{}^{\gamma;\delta}{}_{;\delta;\alpha;\beta} - \frac{1}{4} g^{\alpha\beta} g_{\mu\nu} K^{\gamma}{}_{\gamma}{}^{;\delta}{}_{;\delta;\alpha;\beta} -$$
$$\frac{1}{2} g^{\alpha\beta} K^{\gamma}{}_{\nu;\gamma;\mu;\alpha;\beta} + \frac{1}{2} g^{\alpha\beta} K_{\nu}{}^{\gamma}{}_{;\gamma;\mu;\alpha;\beta} + \frac{1}{2} R_{\nu\alpha} K^{\beta}{}_{\mu}{}^{;\alpha}{}_{;\beta} + \frac{1}{2} R_{\mu\alpha} K^{\beta}{}_{\nu}{}^{;\alpha}{}_{;\beta} +$$
$$R_{\mu}{}^{\alpha} K_{\alpha\nu}{}^{;\beta}{}_{;\beta} + R_{\nu\alpha} K^{\alpha}{}_{\mu}{}^{;\beta}{}_{;\beta} - \frac{1}{2} R_{\nu\alpha} K_{\mu}{}^{\alpha;\beta}{}_{;\beta} - \frac{1}{2} R_{\mu\alpha} K_{\nu}{}^{\alpha;\beta}{}_{;\beta} - \frac{1}{2} g_{\mu\nu} K^{\alpha}{}_{\alpha} \nabla_\beta \nabla^\beta R +$$
$$\frac{1}{4} g_{\mu\nu} R^{\alpha\beta} K_{\alpha}{}^{\gamma}{}_{;\gamma;\beta} - \frac{1}{4} g_{\mu\nu} R^{\alpha\beta} K^{\gamma}{}_{\alpha;\gamma;\beta} - \frac{1}{2} g^{\alpha\beta} K^{\gamma}{}_{\nu;\alpha;\gamma;\mu;\beta} + \frac{1}{2} K^{\alpha}{}_{\alpha}{}^{;\beta}{}_{;\nu;\mu;\beta} -$$
$$\frac{1}{2} g^{\alpha\beta} K^{\gamma}{}_{\mu;\alpha;\gamma;\nu;\beta} + \frac{1}{2} K^{\alpha}{}_{\alpha}{}^{;\beta}{}_{;\mu;\nu;\beta} + \frac{1}{2} R_{\mu\nu;\beta} K_{\alpha}{}^{\alpha;\beta} - \frac{1}{2} R_{\nu}{}^{\alpha}{}_{;\beta} K_{\alpha\mu}{}^{;\beta} - \frac{1}{2} R_{\mu}{}^{\alpha}{}_{;\beta} K_{\alpha\nu}{}^{;\beta} +$$
$$R_{\nu\beta;\alpha} K^{\alpha}{}_{\mu}{}^{;\beta} + R_{\mu\beta;\alpha} K^{\alpha}{}_{\nu}{}^{;\beta} - \frac{1}{2} R_{\nu\alpha;\beta} K_{\mu}{}^{\alpha;\beta} - \frac{1}{2} R_{\mu\alpha;\beta} K_{\nu}{}^{\alpha;\beta} + \frac{1}{4} g_{\mu\nu} R^{\alpha\beta} K^{\gamma}{}_{\alpha;\beta;\gamma} -$$
$$\frac{1}{2} g^{\alpha\beta} g_{\mu\nu} K^{\gamma}{}_{\alpha} \nabla_\gamma \nabla_\beta R + \frac{1}{4} g_{\mu\nu} R^{\alpha}{}_{\beta} K^{\gamma}{}_{\alpha}{}^{;\beta}{}_{;\gamma} - \frac{1}{4} g_{\mu\nu} R^{\alpha\beta} K_{\alpha\beta}{}^{;\gamma}{}_{;\gamma} - \frac{3}{4} g_{\mu\nu} R_{\alpha\beta} K^{\alpha\beta;\gamma}{}_{;\gamma} -$$
$$g_{\mu\nu} R_{\alpha\beta;\gamma} K^{\alpha\beta;\gamma} - \frac{1}{2} K^{\alpha}{}_{\nu}{}^{;\beta} R_{\alpha\beta;\mu} + K^{\alpha}{}_{\nu;\beta} R_{\alpha}{}^{\beta}{}_{;\mu} + K^{\alpha\beta}{}_{;\beta} R_{\nu\alpha;\mu} - \frac{1}{2} K_{\alpha}{}^{\alpha;\beta} R_{\nu\beta;\mu} -$$
$$R_{\nu\alpha} K^{\alpha\beta}{}_{;\beta;\mu} + R_{\nu\alpha} K^{\beta\alpha}{}_{;\beta;\mu} + 2 R_{\alpha}{}^{\beta}{}_{;\beta} K^{\alpha}{}_{\mu;\nu} - \frac{1}{2} K^{\alpha}{}_{\mu}{}^{;\beta} R_{\alpha\beta;\nu} + K^{\alpha}{}_{\mu;\beta} R_{\alpha}{}^{\beta}{}_{;\nu} +$$
$$K^{\alpha\beta}{}_{;\beta} R_{\mu\alpha;\nu} - \frac{1}{2} K_{\alpha}{}^{\alpha;\beta} R_{\mu\beta;\nu} + 2 K^{\alpha}{}_{\mu} \nabla_\nu \nabla_\alpha R - R_{\mu}{}^{\alpha} K_{\alpha}{}^{\beta}{}_{;\beta;\nu} + R_{\mu}{}^{\alpha} K^{\beta}{}_{\alpha;\beta;\nu}$$