

Phys. 6430 - THEORY OF RELATIVITY - HOMEWORK 1

Due in class on Thursday, September 15.

1. The Barn and Pole paradox.

A lightweight pole 20m long lies on the ground next to a barn 15m long. An athlete picks up the pole, carries it far away, and runs with it towards the end of the barn at a speed $0.8c$ (some athlete!). His friend remains at rest, standing by the door of the barn.

a). How long does the friend measure the pole to be as it approaches the barn?

b). The barn door is initially open and, immediately after the runner and the pole are entirely inside the barn, the friend closes the door. How long after the door is shut does the front of the pole hit the other end of the barn, as measured by the friend? Compute the interval between the events of shutting the door and hitting the wall. Is it spacelike, timelike or lightlike?

c). In the reference frame of the runner, what is the length of the barn and the pole?

d). Does the runner believe that the pole is entirely inside the barn when its front hits the front of the barn? Can you explain why?

e). After the collision, the pole and runner come to rest relative to the barn. From the friend's point of view, the 20m pole is now inside the 15m barn, since the barn door was shut before the pole stopped. How is this possible? Alternatively from the runner's point of view, the collision should have stopped the pole *before* the door closed, so the door could not have closed at all. Was or was not the door closed with the pole inside?

2. **Spacewalk.** A rocket is travelling at a constant velocity v_0 relative to an inertial frame S . The engines are off and an astronaut must do a space walk to repair them. After leaving the cockpit, she must travel from the tip of the rocket to the tail, which is of proper length L . If her speed relative to the rocket is v , how much time elapses in S during the walk?

3. **Addition of velocities.** Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity of $v = \frac{v_1+v_2}{1+v_1v_2}$, which is the parallel-velocity addition law.
4. **Who won the race?** Two athletes run on a track in the z direction. The lanes they run in are separated by a distance Δx in the x direction. The first runner crosses the finish line by time ΔT earlier than the second one.
 - a). Is there an inertial system K' in which the two runners cross the finish line simultaneously? Does your answer depend on the values of Δx and ΔT ?
 - b). If such a system exists, find explicitly the Lorentz transformation between the original system K and K' . If in the system K the two athletes started running simultaneously, who started off earlier and by how much from the point of view of an observer moving with K' ?