Coordinate Transformation RW SVT3 v1

1 RW $\Omega(\tau)$

$$ds^2 = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = \Omega^2(\tau)(\tilde{g}_{\mu\nu} + f_{\mu\nu})dx^\mu dx^\nu \quad (1.1)$$

$$\tilde{g}_{\mu\nu} = \mathrm{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right) \qquad \tilde{\Gamma}^\lambda_{\alpha\beta} = \delta^\lambda_i \delta^j_\alpha \delta^k_\beta \tilde{\Gamma}^i_{jk} \quad (1.2)$$

1.1 $f_{\mu\nu}(SVT3)$

$$\begin{aligned}
f_{00} &= -2\phi \\
f_{0i} &= B_i + \tilde{\nabla}_i B \\
f_{ij} &= -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \\
\tilde{g}^{ij} f_{ij} &= -6\psi + 2\tilde{\nabla}^k\tilde{\nabla}_k E \\
\tilde{g}^{\mu\nu} f_{\mu\nu} &= 2\phi - 6\psi + 2\tilde{\nabla}^k\tilde{\nabla}_k E
\end{aligned} \quad (1.3)$$

$$-\tilde{\nabla}^a\tilde{\nabla}^\alpha \Omega f_{a\alpha} = \dot{\Omega}\tilde{\nabla}'_a\tilde{\nabla}'^a B \quad (1.4)$$

1.2 $SVT3(f_{\mu\nu})$

$$\phi = -\tfrac{1}{2} f_{00} \quad (1.5)$$

$$\tilde{\nabla}_a\tilde{\nabla}^a B = \tilde{\nabla}^a f_{0a} \quad (1.6)$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)B_i = (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)f_{0i} - \tilde{\nabla}_i\tilde{\nabla}^a f_{0a} \quad (1.7)$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\psi = \frac{1}{4}\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} - (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{g}^{bc} f_{bc})\right] \quad (1.8)$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b E = \frac{3}{4}\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} - \frac{1}{3}\tilde{\nabla}_a\tilde{\nabla}^a(\tilde{g}^{bc} f_{bc})\right] \quad (1.9)$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)E_i = (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\tilde{\nabla}^b f_{ib} - \tilde{\nabla}_i\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} \quad (1.10)$$

$$\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)(2E_{ij}) &= (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)f_{ij} + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} + (\tilde{\nabla}_a\tilde{\nabla}^a + 4k)(\tilde{g}^{bc} f_{bc})\right] \\
&+ \tfrac{1}{2}\tilde{g}_{ij}\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 4k)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - (\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b - 2k\tilde{\nabla}_a\tilde{\nabla}^a + 4k^2)(\tilde{g}^{bc} f_{bc})\right] \\
&- (\tilde{\nabla}_a\tilde{\nabla}^a - 3k)(\tilde{\nabla}_i\tilde{\nabla}^b f_{jb} + \tilde{\nabla}_j\tilde{\nabla}^b f_{ib})
\end{aligned} \quad (1.11)$$

1.3 $\Delta_\epsilon[SVT3]$

$$\bar{x}^\mu = x^\mu - \epsilon^\mu(x) \quad \Longrightarrow \quad \bar{h}_{\mu\nu} = h_{\mu\nu} + \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu \tag{1.12}$$

$$\Delta_\epsilon\left[\phi\right] = \dot{\Omega}\Omega^{-1}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)f_0 \tag{1.13}$$

$$\Delta_\epsilon\left[\tilde{\nabla}_a\tilde{\nabla}^a B\right] = \tilde{\nabla}_a\dot{f}^a + \tilde{\nabla}_a\tilde{\nabla}^a f_0 \tag{1.14}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)B_i\right] = (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\dot{f}_i - \tilde{\nabla}_i\tilde{\nabla}_a\dot{f}^a \tag{1.15}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\psi\right] = -\dot{f}_0 - \dot{\Omega}f_0\Omega^{-1} \tag{1.16}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b E\right] = (\tilde{\nabla}_b\tilde{\nabla}^b + 3k)\tilde{\nabla}_a f^a \tag{1.17}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)E_i\right] = (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)f_i - \tilde{\nabla}_i(\tilde{\nabla}_b\tilde{\nabla}^b + 4k)\tilde{\nabla}_a f^a \tag{1.18}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)(2E_{ij})\right] = 0 \tag{1.19}$$

1.4 Gauge Invariants

We mix time derivative notation a bit, using ∂_0 upon $f_{\mu\nu}$ and dot upon Ω and SVT3 quantities.

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\phi + \psi + \dot{B} - \ddot{E}] &= (\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}^b(\partial_0 f_{0b}) - \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k - \partial_0^2)\tilde{\nabla}_b\tilde{\nabla}^b(\tilde{g}^{cd}f_{cd}) \\ &\quad + \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a - 3\partial_0^2)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - \tfrac{1}{2}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b f_{00}\end{aligned} \tag{1.20}$$

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\psi - \dot{\Omega}\Omega^{-1}(B - \dot{E})] &= -\dot{\Omega}\Omega^{-1}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}^b f_{0b} + \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 3\dot{\Omega}\Omega^{-1}\partial_0)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} \\ &\quad - \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k + \dot{\Omega}\Omega^{-1}\partial_0)\tilde{\nabla}_b\tilde{\nabla}^b(\tilde{g}^{cd}f_{cd})\end{aligned} \tag{1.21}$$

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)[B_i - \dot{E}_i] &= (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)f_{0i} - (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\tilde{\nabla}^b(\partial_0 f_{ib}) \\ &\quad - \tilde{\nabla}_i(\tilde{\nabla}_a\tilde{\nabla}^a + 4k)\tilde{\nabla}^b f_{0b} + \tilde{\nabla}_i\tilde{\nabla}^a\tilde{\nabla}^b(\partial_0 f_{ab})\end{aligned} \tag{1.22}$$

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)[2E_{ij}] &= (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)f_{ij} + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} + (\tilde{\nabla}_a\tilde{\nabla}^a + 4k)(\tilde{g}^{bc}f_{bc})\right] \\ &\quad + \tfrac{1}{2}\tilde{g}_{ij}\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 4k)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - (\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b - 2k\tilde{\nabla}_a\tilde{\nabla}^a + 4k^2)(\tilde{g}^{bc}f_{bc})\right] \\ &\quad - (\tilde{\nabla}_a\tilde{\nabla}^a - 3k)(\tilde{\nabla}_i\tilde{\nabla}^b f_{jb} + \tilde{\nabla}_j\tilde{\nabla}^b f_{ib})\end{aligned} \tag{1.23}$$

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2 RW $\Omega(T, R)$

$$ds^2 = (g'_{\mu\nu} + h'_{\mu\nu})dx'^\mu dx'^\nu = \Omega^2(T,R)(\tilde{g}'_{\mu\nu} + f'_{\mu\nu})dx'^\mu dx'^\nu \tag{2.1}$$

$$\tilde{g}'_{\mu\nu} = \text{diag}\left(-1, 1, R^2, R^2\sin^2\theta\right) \tag{2.2}$$

2.1 $f'_{\mu\nu}(SVT3)$

$$
\begin{aligned}
f'_{00} &= -2\phi \\
f'_{0i} &= B_i + \tilde{\nabla}'_i B \\
f'_{ij} &= -2\tilde{g}'_{ij}\psi + 2\tilde{\nabla}'_i\tilde{\nabla}'_j E + \tilde{\nabla}'_i E_j + \tilde{\nabla}'_j E_i + 2E_{ij} \\
\tilde{g}'^{ij} f'_{ij} &= -6\psi + 2\tilde{\nabla}'^k\tilde{\nabla}'_k E \\
\tilde{g}'^{\mu\nu} f'_{\mu\nu} &= 2\phi - 6\psi + 2\tilde{\nabla}'^k\tilde{\nabla}'_k E
\end{aligned} \tag{2.3}
$$

2.2 $SVT3(f'_{\mu\nu})$

These quantities mimic (1.5)-(1.11) with $k=0$.

$$\phi = -\tfrac{1}{2} f'_{00} \tag{2.4}$$

$$\tilde{\nabla}'_a\tilde{\nabla}'^a B = \tilde{\nabla}'^a f'_{0a} \tag{2.5}$$

$$\tilde{\nabla}'_a\tilde{\nabla}'^a B_i = \tilde{\nabla}'_a\tilde{\nabla}'^a f'_{0i} - \tilde{\nabla}'_i\tilde{\nabla}'^a f'_{0a} \tag{2.6}$$

$$\tilde{\nabla}'_a\tilde{\nabla}'^a \psi = \frac{1}{4}\left[\tilde{\nabla}'^a\tilde{\nabla}'^b f'_{ab} - \tilde{\nabla}'_a\tilde{\nabla}'^a(\tilde{g}'^{bc} f'_{bc})\right] \tag{2.7}$$

$$\tilde{\nabla}'_a\tilde{\nabla}'^a\tilde{\nabla}'_b\tilde{\nabla}'^b E = \frac{3}{4}\left[\tilde{\nabla}'^a\tilde{\nabla}'^b f'_{ab} - \frac{1}{3}\tilde{\nabla}'_a\tilde{\nabla}'^a(\tilde{g}'^{bc} f'_{bc})\right] \tag{2.8}$$

$$\tilde{\nabla}'_a\tilde{\nabla}'^a\tilde{\nabla}'_b\tilde{\nabla}'^b E_i = \tilde{\nabla}'_a\tilde{\nabla}'^a\tilde{\nabla}'^b f'_{ib} - \tilde{\nabla}'_i\tilde{\nabla}'^a\tilde{\nabla}'^b f'_{ab} \tag{2.9}$$

$$
\begin{aligned}
\tilde{\nabla}'_a\tilde{\nabla}'^a\tilde{\nabla}'_b\tilde{\nabla}'^b(2E_{ij}) &= \tilde{\nabla}'_a\tilde{\nabla}'^a\tilde{\nabla}'_b\tilde{\nabla}'^b f'_{ij} + \tfrac{1}{2}\tilde{\nabla}'_i\tilde{\nabla}'_j\left[\tilde{\nabla}'^a\tilde{\nabla}'^b f'_{ab} + \tilde{\nabla}'_a\tilde{\nabla}'^a(\tilde{g}'^{bc} f'_{bc})\right] \\
&\quad + \tfrac{1}{2}\tilde{g}'_{ij}\left[\tilde{\nabla}'_a\tilde{\nabla}'^a\tilde{\nabla}'^b\tilde{\nabla}'^c f'_{bc} - \tilde{\nabla}'_a\tilde{\nabla}'^a\tilde{\nabla}'_b\tilde{\nabla}'^b(\tilde{g}'^{bc} f'_{bc})\right] \\
&\quad - \tilde{\nabla}'_a\tilde{\nabla}'^a(\tilde{\nabla}'_i\tilde{\nabla}'^b f'_{jb} + \tilde{\nabla}'_j\tilde{\nabla}'^b f'_{ib})
\end{aligned} \tag{2.10}
$$

2.3 $\Delta_\epsilon[f'_{\mu\nu}]$

$$\bar{x}^\mu = x'^\mu - \epsilon^\mu(x') \implies \Delta_\epsilon\left[h'_{\mu\nu}\right] = \nabla'_\mu\epsilon_\nu + \nabla'_\nu\epsilon_\mu \tag{2.11}$$

$$f'_\mu = \Omega^2\epsilon_\mu, \qquad f'^\mu = \epsilon^\mu \tag{2.12}$$

$$\Delta_\epsilon\left[f'_{\mu\nu}\right] = \tilde{\nabla}'_\mu f'_\nu + \tilde{\nabla}'_\nu f'_\mu + 2f'^\gamma\tilde{g}'_{\mu\nu}\Omega^{-1}\tilde{\nabla}'_\gamma\Omega \tag{2.13}$$

$$\Delta_\epsilon\left[\tilde{f}'_{00}\right] = 2\dot{f}'_0 - 2\Omega^{-1}\left(-f'_0\dot{\Omega} + f^a\tilde{\nabla}'_a\Omega\right) \tag{2.14}$$

$$\Delta_\epsilon\left[\tilde{f}'_{0i}\right] = \dot{f}'_i + \tilde{\nabla}'_i f'_0 \tag{2.15}$$

$$\Delta_\epsilon\left[\tilde{f}'_{ij}\right] = \tilde{\nabla}'_i f'_j + \tilde{\nabla}'_j f'_i + 2\Omega^{-1}\tilde{g}_{ij}\left(-f'_0\dot{\Omega} + f^a\tilde{\nabla}'_a\Omega\right) \tag{2.16}$$

$$\Delta_\epsilon\left[\tilde{g}'^{ab} f'_{ab}\right] = 2\tilde{\nabla}'^a f'_a + 6\Omega^{-1}\left(-f'_0\dot{\Omega} + f^a\tilde{\nabla}'_a\Omega\right) \tag{2.17}$$

$$\Delta_\epsilon\left[\tilde{g}'^{\alpha\beta} f'_{\alpha\beta}\right] = -2\dot{f}'_0 + 2\tilde{\nabla}'^a f'_a + 8\Omega^{-1}\left(-f'_0\dot{\Omega} + f^a\tilde{\nabla}'_a\Omega\right) \tag{2.18}$$

2.4 $\Delta_\epsilon[SVT3]$

$$\Delta_\epsilon\left[\phi\right] = -\dot{f'}_0-\dot\Omega f'_0\Omega^{-1}+f'^a\Omega^{-1}\tilde\nabla_a\Omega \tag{2.19}$$

$$\Delta_\epsilon\left[\tilde\nabla'_a\tilde\nabla'^a B\right] = \tilde\nabla'_a\dot{f'}^a+\tilde\nabla'_a\tilde\nabla'^a f'_0 \tag{2.20}$$

$$\Delta_\epsilon\left[\tilde\nabla'_a\tilde\nabla'^a B_i\right] = \tilde\nabla'_a\tilde\nabla'^a\dot{f'}_i-\tilde\nabla'_a\tilde\nabla'_i\dot{f'}^a \tag{2.21}$$

$$\begin{aligned}\Delta_\epsilon\left[\tilde\nabla'_a\tilde\nabla'^a\psi\right] = {} & f'_0\Omega^{-1}\tilde\nabla'_a\tilde\nabla'^a\dot\Omega+\dot\Omega\Omega^{-1}\tilde\nabla'_a\tilde\nabla'^a f'_0-\dot\Omega f'_0\Omega^{-2}\tilde\nabla'_a\tilde\nabla'^a\Omega+2\Omega^{-1}\tilde\nabla'_a f'_0\tilde\nabla'^a\dot\Omega-2f'_0\Omega^{-2}\tilde\nabla'_a\Omega\tilde\nabla'^a\dot\Omega\\ & -2\dot\Omega\Omega^{-2}\tilde\nabla'_a\Omega\tilde\nabla'^a f'_0+2\dot\Omega f'_0\Omega^{-3}\tilde\nabla'_a\Omega\tilde\nabla'^a\Omega-\Omega^{-1}\tilde\nabla'^a\Omega\tilde\nabla'_b\tilde\nabla'^b f'_a+f'^a\Omega^{-2}\tilde\nabla'_a\Omega\tilde\nabla'_b\tilde\nabla'^b\Omega\\ & -f'^a\Omega^{-1}\tilde\nabla'_b\tilde\nabla'^b\tilde\nabla'_a\Omega+2\Omega^{-2}\tilde\nabla'_a\Omega\tilde\nabla'_b\Omega\tilde\nabla'^b f'^a-2\Omega^{-1}\tilde\nabla'_b\tilde\nabla'_a\Omega\tilde\nabla'^b f'^a-2f'^a\Omega^{-3}\tilde\nabla'_a\Omega\tilde\nabla'_b\Omega\tilde\nabla'^b\Omega\\ & +2f'^a\Omega^{-2}\tilde\nabla'_b\tilde\nabla'_a\Omega\tilde\nabla'^b\Omega\end{aligned} \tag{2.22}$$

$$\Delta_\epsilon\left[\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b E\right] = \tilde\nabla'_b\tilde\nabla'^b\tilde\nabla'_a f'^a \tag{2.23}$$

$$\Delta_\epsilon\left[\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b E_i\right] = \tilde\nabla'_b\tilde\nabla'^b\tilde\nabla'_a\tilde\nabla'^a f'_i-\tilde\nabla'_b\tilde\nabla'^b\tilde\nabla'_i\tilde\nabla'_a f'^a \tag{2.24}$$

$$\Delta_\epsilon\left[\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b(2E_{ij})\right] = 0 \tag{2.25}$$

2.5 Gauge Invariants

We mix time derivative notation a bit, using ∂_0 upon $f'_{\mu\nu}$ and dot upon Ω and SVT3 quantities.

$$\begin{aligned}\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b[\phi+\psi+\dot B-\ddot E] = {} & \tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'^b(\partial_0 f'_{0b})-\tfrac{1}{4}(\tilde\nabla'_a\tilde\nabla'^a-\partial_0^2)\tilde\nabla'_b\tilde\nabla'^b(\tilde g'^{cd}f'_{cd})\\ & +\tfrac{1}{4}(\tilde\nabla'_a\tilde\nabla'^a-3\partial_0^2)\tilde\nabla'^b\tilde\nabla'^c f'_{bc}-\tfrac{1}{2}\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b f'_{00}\end{aligned} \tag{2.26}$$

$$\begin{aligned}&\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b\times\\ &\left[\psi-\Omega^{-1}[(B-\dot E)\dot\Omega-(\tilde\nabla'_a E+E_a)\tilde\nabla'^a\Omega]\right] = ?\end{aligned} \tag{2.27}$$

$$\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b[B'_i-\dot E'_i] = \tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b f'_{0i}-\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'^b(\partial_0 f'_{ib})-\tilde\nabla'_i\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'^b f'_{0b}+\tilde\nabla'_i\tilde\nabla'^a\tilde\nabla'^b(\partial_0 f'_{ab}) \tag{2.28}$$

$$\begin{aligned}\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b[2E_{ij}] = {} & \tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b f'_{ij}+\tfrac{1}{2}\tilde\nabla'_i\tilde\nabla'_j\left[\tilde\nabla'^a\tilde\nabla'^b f'_{ab}+\tilde\nabla'_a\tilde\nabla'^a(\tilde g'^{bc}f'_{bc})\right]\\ & +\tfrac{1}{2}\tilde g'_{ij}\left[\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'^b\tilde\nabla'^c f'_{bc}-\tilde\nabla'_a\tilde\nabla'^a\tilde\nabla'_b\tilde\nabla'^b(\tilde g'^{bc}f'_{bc})\right]\\ & -\tilde\nabla'_a\tilde\nabla'^a(\tilde\nabla'_i\tilde\nabla'^b f'_{jb}+\tilde\nabla'_j\tilde\nabla'^b f'_{ib})\end{aligned} \tag{2.29}$$

2.6 On the G.I. of $\psi-\Omega^{-1}[(B-\dot E)\dot\Omega-(\tilde\nabla'_a E+E_a)\tilde\nabla'^a\Omega]$

In the conformal to flat decomposition, E_i is given by the integral

$$E_i = \int D\tilde\nabla^k f_{ik}-\tilde\nabla_i\int D\tilde\nabla^k\tilde\nabla^l f_{kl},\qquad \tilde\nabla_a\tilde\nabla^a D(x,x')=\delta(x-x'). \tag{2.30}$$

As given in (2.9), the lowest derivative relation in terms of $f_{\mu\nu}$ for E_i is

$$\tilde\nabla_a\tilde\nabla^a\tilde\nabla_b\tilde\nabla^b E_i = \tilde\nabla_a\tilde\nabla^a\tilde\nabla^b f_{ib}-\tilde\nabla_i\tilde\nabla^a\tilde\nabla^b f_{ab}. \tag{2.31}$$

E_i can also be found as a single derivative within f_{ij}

$$f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{2.32}$$

When we take any derivative upon the gauge invariant

$$\psi - \Omega^{-1}[(B - \dot{E})\dot{\Omega} - (\tilde{\nabla}'_a E + E_a)\tilde{\nabla}'^a\Omega], \tag{2.33}$$

from the product rule we will necessarily generate terms that depend on E_a alone; i.e. terms that could only be expressed as integrals over f_{ij} and not derivatives of f_{ij}. Consequently, it would not seem possible to construct this gauge invariant based on any combination of $f_{\mu\nu}$ or derivatives thereof.

It would then seem puzzling how we were able to express $\Delta_{\mu\nu}$ in terms of the gauge invariant $\gamma = \psi - \Omega^{-1}[(B - \dot{E})\dot{\Omega} - (\tilde{\nabla}'_a E + E_a)\tilde{\nabla}'^a\Omega]$. Looking at *RW_Radiation_SVT3_Conformal_Flat_-k_Cartesian_v2.pdf*, it turns out that neither $\delta G_{\mu\nu}$ nor $\delta T_{\mu\nu}$ have any terms that depend on E_i without derivatives. When forming the gauge invariant combinations, we made substitutions like

$$\psi = \gamma + \Omega^{-1}[(B - \dot{E})\dot{\Omega} - (\tilde{\nabla}'_a E + E_a)\tilde{\nabla}'^a\Omega]. \tag{2.34}$$

All contributions of E_a that we originally introduce end up canceling after simplifying all relevant terms.