

# SVT

$$h_{00} = -2\phi$$

$$h_{0i} = w_i = \partial_i B + B_i$$

$$h_{ij} = -2\psi + S_{ij} = -2\psi + 2\partial_i \partial_j E + \partial_i E_j + \partial_j E_i + 2E_{ij}$$

where

$$\partial_i B^i = \partial_i E^i = 0$$

$$\partial_i E^{ij} = 0$$

$$\delta_{ij} E^{ij} = 0$$

No gauge,  $\delta G_{\mu\nu}$  flat:

00	$\frac{\partial_1 \partial_1 h}{2} + \frac{\partial_2 \partial_2 h}{2} + \frac{\partial_3 \partial_3 h}{2} + \frac{1}{2} \partial_1 \partial_1 h_{00} - \frac{1}{2} \partial_1 \partial_1 h_{11} - \partial_2 \partial_1 h_{12} +$ $\frac{1}{2} \partial_2 \partial_2 h_{00} - \frac{1}{2} \partial_2 \partial_2 h_{22} - \partial_3 \partial_1 h_{13} - \partial_3 \partial_2 h_{23} + \frac{1}{2} \partial_3 \partial_3 h_{00} - \frac{1}{2} \partial_3 \partial_3 h_{33}$
11	$\frac{1}{2} \partial_0 \partial_0 h_{00} - \frac{1}{2} \partial_0 \partial_0 h_{11} + \frac{\partial_0 \partial_0 h}{2} - \partial_2 \partial_0 h_{02} + \frac{1}{2} \partial_2 \partial_2 h_{11} +$ $\frac{1}{2} \partial_2 \partial_2 h_{22} - \frac{\partial_2 \partial_2 h}{2} - \partial_3 \partial_0 h_{03} + \partial_3 \partial_2 h_{23} + \frac{1}{2} \partial_3 \partial_3 h_{11} + \frac{1}{2} \partial_3 \partial_3 h_{33} - \frac{\partial_3 \partial_3 h}{2}$
22	$\frac{1}{2} \partial_0 \partial_0 h_{00} - \frac{1}{2} \partial_0 \partial_0 h_{22} + \frac{\partial_0 \partial_0 h}{2} - \partial_1 \partial_0 h_{01} + \frac{1}{2} \partial_1 \partial_1 h_{11} +$ $\frac{1}{2} \partial_1 \partial_1 h_{22} - \frac{\partial_1 \partial_1 h}{2} - \partial_3 \partial_0 h_{03} + \partial_3 \partial_1 h_{13} + \frac{1}{2} \partial_3 \partial_3 h_{22} + \frac{1}{2} \partial_3 \partial_3 h_{33} - \frac{\partial_3 \partial_3 h}{2}$
33	$\frac{1}{2} \partial_0 \partial_0 h_{00} - \frac{1}{2} \partial_0 \partial_0 h_{33} + \frac{\partial_0 \partial_0 h}{2} - \partial_1 \partial_0 h_{01} + \frac{1}{2} \partial_1 \partial_1 h_{11} +$ $\frac{1}{2} \partial_1 \partial_1 h_{33} - \frac{\partial_1 \partial_1 h}{2} - \partial_2 \partial_0 h_{02} + \partial_2 \partial_1 h_{12} + \frac{1}{2} \partial_2 \partial_2 h_{22} + \frac{1}{2} \partial_2 \partial_2 h_{33} - \frac{\partial_2 \partial_2 h}{2}$
01	$\frac{1}{2} \partial_1 \partial_0 h_{00} - \frac{1}{2} \partial_1 \partial_0 h_{11} + \frac{\partial_1 \partial_0 h}{2} - \frac{1}{2} \partial_2 \partial_0 h_{12} -$ $\frac{1}{2} \partial_2 \partial_1 h_{02} + \frac{1}{2} \partial_2 \partial_2 h_{01} - \frac{1}{2} \partial_3 \partial_0 h_{13} - \frac{1}{2} \partial_3 \partial_1 h_{03} + \frac{1}{2} \partial_3 \partial_3 h_{01}$
02	$-\frac{1}{2} \partial_1 \partial_0 h_{12} + \frac{1}{2} \partial_1 \partial_1 h_{02} + \frac{1}{2} \partial_2 \partial_0 h_{00} - \frac{1}{2} \partial_2 \partial_0 h_{22} +$ $\frac{\partial_2 \partial_0 h}{2} - \frac{1}{2} \partial_2 \partial_1 h_{01} - \frac{1}{2} \partial_3 \partial_0 h_{23} - \frac{1}{2} \partial_3 \partial_2 h_{03} + \frac{1}{2} \partial_3 \partial_3 h_{02}$
03	$-\frac{1}{2} \partial_1 \partial_0 h_{13} + \frac{1}{2} \partial_1 \partial_1 h_{03} - \frac{1}{2} \partial_2 \partial_0 h_{23} + \frac{1}{2} \partial_2 \partial_2 h_{03} +$ $\frac{1}{2} \partial_3 \partial_0 h_{00} - \frac{1}{2} \partial_3 \partial_0 h_{33} + \frac{\partial_3 \partial_0 h}{2} - \frac{1}{2} \partial_3 \partial_1 h_{01} - \frac{1}{2} \partial_3 \partial_2 h_{02}$
12	$-\frac{1}{2} \partial_0 \partial_0 h_{12} + \frac{1}{2} \partial_1 \partial_0 h_{02} + \frac{1}{2} \partial_2 \partial_0 h_{01} - \frac{1}{2} \partial_2 \partial_1 h_{11} -$ $\frac{1}{2} \partial_2 \partial_1 h_{22} + \frac{\partial_2 \partial_1 h}{2} - \frac{1}{2} \partial_3 \partial_1 h_{23} - \frac{1}{2} \partial_3 \partial_2 h_{13} + \frac{1}{2} \partial_3 \partial_3 h_{12}$
13	$-\frac{1}{2} \partial_0 \partial_0 h_{13} + \frac{1}{2} \partial_1 \partial_0 h_{03} - \frac{1}{2} \partial_2 \partial_1 h_{23} + \frac{1}{2} \partial_2 \partial_2 h_{13} +$ $\frac{1}{2} \partial_3 \partial_0 h_{01} - \frac{1}{2} \partial_3 \partial_1 h_{11} - \frac{1}{2} \partial_3 \partial_1 h_{33} + \frac{\partial_3 \partial_1 h}{2} - \frac{1}{2} \partial_3 \partial_2 h_{12}$
23	$-\frac{1}{2} \partial_0 \partial_0 h_{23} + \frac{1}{2} \partial_1 \partial_1 h_{23} + \frac{1}{2} \partial_2 \partial_0 h_{03} - \frac{1}{2} \partial_2 \partial_1 h_{13} +$ $\frac{1}{2} \partial_3 \partial_0 h_{02} - \frac{1}{2} \partial_3 \partial_1 h_{12} - \frac{1}{2} \partial_3 \partial_2 h_{22} - \frac{1}{2} \partial_3 \partial_2 h_{33} + \frac{\partial_3 \partial_2 h}{2}$

Now decompose into the form with  $\phi$ ,  $\psi$ ,  $w_i$ , and  $S_{ij}$ .

00	$\partial_1 \partial_1 \nabla^2 E - \frac{1}{2} \partial_1 \partial_1 S_{11} - 2 \partial_1 \partial_1 \psi - \partial_2 \partial_1 S_{12} + \partial_2 \partial_2 \nabla^2 E -$ $\frac{1}{2} \partial_2 \partial_2 S_{22} - 2 \partial_2 \partial_2 \psi - \partial_3 \partial_1 S_{13} - \partial_3 \partial_2 S_{23} + \partial_3 \partial_3 \nabla^2 E - \frac{1}{2} \partial_3 \partial_3 S_{33} - 2 \partial_3 \partial_3 \psi$
11	$\partial_0 \partial_0 \nabla^2 E - \frac{1}{2} \partial_0 \partial_0 S_{11} - 2 \partial_0 \partial_0 \psi - \partial_2 \partial_0 w_2 - \partial_2 \partial_2 \nabla^2 E + \frac{1}{2} \partial_2 \partial_2 S_{11} + \frac{1}{2} \partial_2 \partial_2 S_{22} -$ $\partial_2 \partial_2 \phi + \partial_2 \partial_2 \psi - \partial_3 \partial_0 w_3 + \partial_3 \partial_2 S_{23} - \partial_3 \partial_3 \nabla^2 E + \frac{1}{2} \partial_3 \partial_3 S_{11} + \frac{1}{2} \partial_3 \partial_3 S_{33} - \partial_3 \partial_3 \phi + \partial_3 \partial_3 \psi$
22	$\partial_0 \partial_0 \nabla^2 E - \frac{1}{2} \partial_0 \partial_0 S_{22} - 2 \partial_0 \partial_0 \psi - \partial_1 \partial_0 w_1 - \partial_1 \partial_1 \nabla^2 E + \frac{1}{2} \partial_1 \partial_1 S_{11} + \frac{1}{2} \partial_1 \partial_1 S_{22} -$ $\partial_1 \partial_1 \phi + \partial_1 \partial_1 \psi - \partial_3 \partial_0 w_3 + \partial_3 \partial_1 S_{13} - \partial_3 \partial_3 \nabla^2 E + \frac{1}{2} \partial_3 \partial_3 S_{22} + \frac{1}{2} \partial_3 \partial_3 S_{33} - \partial_3 \partial_3 \phi + \partial_3 \partial_3 \psi$
33	$\partial_0 \partial_0 \nabla^2 E - \frac{1}{2} \partial_0 \partial_0 S_{33} - 2 \partial_0 \partial_0 \psi - \partial_1 \partial_0 w_1 - \partial_1 \partial_1 \nabla^2 E + \frac{1}{2} \partial_1 \partial_1 S_{11} + \frac{1}{2} \partial_1 \partial_1 S_{33} -$ $\partial_1 \partial_1 \phi + \partial_1 \partial_1 \psi - \partial_2 \partial_0 w_2 + \partial_2 \partial_1 S_{12} - \partial_2 \partial_2 \nabla^2 E + \frac{1}{2} \partial_2 \partial_2 S_{22} + \frac{1}{2} \partial_2 \partial_2 S_{33} - \partial_2 \partial_2 \phi + \partial_2 \partial_2 \psi$
01	$\partial_1 \partial_0 \nabla^2 E - \frac{1}{2} \partial_1 \partial_0 S_{11} - 2 \partial_1 \partial_0 \psi - \frac{1}{2} \partial_2 \partial_0 S_{12} -$ $\frac{1}{2} \partial_2 \partial_1 w_2 + \frac{1}{2} \partial_2 \partial_2 w_1 - \frac{1}{2} \partial_3 \partial_0 S_{13} - \frac{1}{2} \partial_3 \partial_1 w_3 + \frac{1}{2} \partial_3 \partial_3 w_1$
02	$- \frac{1}{2} \partial_1 \partial_0 S_{12} + \frac{1}{2} \partial_1 \partial_1 w_2 + \partial_2 \partial_0 \nabla^2 E - \frac{1}{2} \partial_2 \partial_0 S_{22} -$ $2 \partial_2 \partial_0 \psi - \frac{1}{2} \partial_2 \partial_1 w_1 - \frac{1}{2} \partial_3 \partial_0 S_{23} - \frac{1}{2} \partial_3 \partial_2 w_3 + \frac{1}{2} \partial_3 \partial_3 w_2$
03	$- \frac{1}{2} \partial_1 \partial_0 S_{13} + \frac{1}{2} \partial_1 \partial_1 w_3 - \frac{1}{2} \partial_2 \partial_0 S_{23} + \frac{1}{2} \partial_2 \partial_2 w_3 +$ $\partial_3 \partial_0 \nabla^2 E - \frac{1}{2} \partial_3 \partial_0 S_{33} - 2 \partial_3 \partial_0 \psi - \frac{1}{2} \partial_3 \partial_1 w_1 - \frac{1}{2} \partial_3 \partial_2 w_2$
12	$- \frac{1}{2} \partial_0 \partial_0 S_{12} + \frac{1}{2} \partial_1 \partial_0 w_2 + \frac{1}{2} \partial_2 \partial_0 w_1 + \partial_2 \partial_1 \nabla^2 E - \frac{1}{2} \partial_2 \partial_1 S_{11} -$ $\frac{1}{2} \partial_2 \partial_1 S_{22} + \partial_2 \partial_1 \phi - \partial_2 \partial_1 \psi - \frac{1}{2} \partial_3 \partial_1 S_{23} - \frac{1}{2} \partial_3 \partial_2 S_{13} + \frac{1}{2} \partial_3 \partial_3 S_{12}$
13	$- \frac{1}{2} \partial_0 \partial_0 S_{13} + \frac{1}{2} \partial_1 \partial_0 w_3 - \frac{1}{2} \partial_2 \partial_1 S_{23} + \frac{1}{2} \partial_2 \partial_2 S_{13} +$ $\frac{1}{2} \partial_3 \partial_0 w_1 + \partial_3 \partial_1 \nabla^2 E - \frac{1}{2} \partial_3 \partial_1 S_{11} - \frac{1}{2} \partial_3 \partial_1 S_{33} + \partial_3 \partial_1 \phi - \partial_3 \partial_1 \psi - \frac{1}{2} \partial_3 \partial_2 S_{12}$
23	$- \frac{1}{2} \partial_0 \partial_0 S_{23} + \frac{1}{2} \partial_1 \partial_1 S_{23} + \frac{1}{2} \partial_2 \partial_0 w_3 - \frac{1}{2} \partial_2 \partial_1 S_{13} +$ $\frac{1}{2} \partial_3 \partial_0 w_2 - \frac{1}{2} \partial_3 \partial_1 S_{12} + \partial_3 \partial_2 \nabla^2 E - \frac{1}{2} \partial_3 \partial_2 S_{22} - \frac{1}{2} \partial_3 \partial_2 S_{33} + \partial_3 \partial_2 \phi - \partial_3 \partial_2 \psi$

Further decompose  $S_{ij}$  and  $w_i$  as given in the first equations.

00	$-2 \nabla^2 \psi$
11	$(-2 \partial_0 \partial_0 - \partial_1 \partial_1 + \nabla^2) \psi - (\nabla^2 - \partial_1 \partial_1) (\phi + \partial_0 B - \partial_0 \partial_0 E) + -\partial_1 \partial_0 (-B_1 + \partial_0 E_1) + \square E_{11}$
22	$(-2 \partial_0 \partial_0 - \partial_2 \partial_2 + \nabla^2) \psi - (\nabla^2 - \partial_2 \partial_2) (\phi + \partial_0 B - \partial_0 \partial_0 E) + -\partial_2 \partial_0 (-B_2 + \partial_0 E_2) + \square E_{22}$
33	$(-2 \partial_0 \partial_0 - \partial_3 \partial_3 + \nabla^2) \psi - (\nabla^2 - \partial_3 \partial_3) (\phi + \partial_0 B - \partial_0 \partial_0 E) + -\partial_3 \partial_0 (-B_3 + \partial_0 E_3) + \square E_{33}$
01	$-2 \partial_1 \partial_0 \psi - \frac{1}{2} \nabla^2 (-B_1 + \partial_0 E_1)$
02	$-2 \partial_2 \partial_0 \psi - \frac{1}{2} \nabla^2 (-B_2 + \partial_0 E_2)$
03	$-2 \partial_3 \partial_0 \psi - \frac{1}{2} \nabla^2 (-B_3 + \partial_0 E_3)$
12	$-\partial_2 \partial_1 \psi + \partial_2 \partial_1 (\phi + \partial_0 B - \partial_0 \partial_0 E) - \frac{1}{2} \partial_1 \partial_0 (\partial_0 E_2 - B_2) - \frac{1}{2} \partial_2 \partial_0 (\partial_0 E_1 - B_1) + \square E_{12}$
13	$-\partial_3 \partial_1 \psi + \partial_3 \partial_1 (\phi + \partial_0 B - \partial_0 \partial_0 E) - \frac{1}{2} \partial_1 \partial_0 (\partial_0 E_3 - B_3) - \frac{1}{2} \partial_3 \partial_0 (\partial_0 E_1 - B_1) + \square E_{13}$
23	$-\partial_3 \partial_2 \psi + \partial_3 \partial_2 (\phi + \partial_0 B - \partial_0 \partial_0 E) - \frac{1}{2} \partial_2 \partial_0 (\partial_0 E_3 - B_3) - \frac{1}{2} \partial_3 \partial_0 (\partial_0 E_2 - B_2) + \square E_{23}$

Now expressed in terms of the gauge invariant quantities:

$$\psi = \psi$$

$$\mathcal{P} = \phi + \partial_0 \mathbf{B} - \partial_0 \partial_0 \mathbf{E}$$

$$\mathcal{F}_i = \partial_0 \mathbf{E}_i - \mathbf{B}_i$$

00	$-2\nabla^2\psi$
11	$(-2\partial_0\partial_0-\partial_1\partial_1+\nabla^2)\psi - (\nabla^2-\partial_1\partial_1)\mathcal{P} + -\partial_1\partial_0\mathcal{F}_1 + \square\mathbf{E}_{11}$
22	$(-2\partial_0\partial_0-\partial_2\partial_2+\nabla^2)\psi - (\nabla^2-\partial_2\partial_2)\mathcal{P} + -\partial_2\partial_0\mathcal{F}_2 + \square\mathbf{E}_{22}$
33	$(-2\partial_0\partial_0-\partial_3\partial_3+\nabla^2)\psi - (\nabla^2-\partial_3\partial_3)\mathcal{P} + -\partial_3\partial_0\mathcal{F}_3 + \square\mathbf{E}_{33}$
01	$-2\partial_1\partial_0\psi - \frac{1}{2}\nabla^2\mathcal{F}_1$
02	$-2\partial_2\partial_0\psi - \frac{1}{2}\nabla^2\mathcal{F}_2$
03	$-2\partial_3\partial_0\psi - \frac{1}{2}\nabla^2\mathcal{F}_3$
12	$-\partial_2\partial_1\psi + \partial_2\partial_1\mathcal{P} - \frac{1}{2}\partial_1\partial_0\mathcal{F}_2 - \frac{1}{2}\partial_2\partial_0\mathcal{F}_1 + \square\mathbf{E}_{12}$
13	$-\partial_3\partial_1\psi + \partial_3\partial_1\mathcal{P} - \frac{1}{2}\partial_1\partial_0\mathcal{F}_3 - \frac{1}{2}\partial_3\partial_0\mathcal{F}_1 + \square\mathbf{E}_{13}$
23	$-\partial_3\partial_2\psi + \partial_3\partial_2\mathcal{P} - \frac{1}{2}\partial_2\partial_0\mathcal{F}_3 - \frac{1}{2}\partial_3\partial_0\mathcal{F}_2 + \square\mathbf{E}_{23}$