Special Gauge Matthew v10

Setup

Metric decomposed to first order:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} = \Omega^2(\tau)(\eta_{\mu\nu} + f_{\mu\nu}). \tag{1}$$

We then split $f_{\mu\nu}$ into its traceless and trace components, i.e.

$$f_{\mu\nu} = k_{\mu\nu} + \frac{1}{4}\eta_{\mu\nu} f \tag{2}$$

where $f = \eta^{\mu\nu} f_{\mu\nu}$. We impose a generalized gauge of the form

$$\eta^{\alpha\beta}\partial_\alpha k_{\beta\nu} = \Omega^{-1} J \eta^{\alpha\beta} k_{\nu\alpha}\partial_\beta\Omega + P\partial_\nu f + R\Omega^{-1} f\partial_\nu\Omega. \tag{3}$$

and take

$$J = -4, \qquad R = 2P - \frac{3}{2}. \tag{4}$$

Now we will further express our results in terms of

$$K_{\mu\nu} = \Omega^2 k_{\mu\nu} \qquad \text{and} \qquad d = \Omega^2 f \tag{5}$$

$\Omega(\tau)$

Working with a time dependent conformal factor, $\Omega(\tau)$, the fluctuations are evaluated as

$$\begin{aligned} \eta^{\mu\nu}\delta G_{\mu\nu} &= (-8\Omega^{-4}\dot{\Omega}^2 + 4\Omega^{-3}\ddot{\Omega})K_{00} + (-\tfrac{3}{4}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + P\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu)d \\ &= \Omega^{-2}(-8\Omega^{-2}\dot{\Omega}^2 + 4\Omega^{-1}\ddot{\Omega})K_{00} + (P - \tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta d \end{aligned} \tag{6}$$

$$\begin{aligned} \delta G_{00} = {}& (5\Omega^{-4}\dot{\Omega}^2 - \Omega^{-3}\ddot{\Omega} + \tfrac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-3}\dot{\Omega}\partial_0)K_{00} + (-\tfrac{3}{4}\Omega^{-4}\dot{\Omega}^2 + \tfrac{3}{4}\Omega^{-3}\ddot{\Omega} \\ & + \tfrac{1}{4}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}P\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + P\Omega^{-3}\dot{\Omega}\partial_0 + \tfrac{1}{4}\Omega^{-2}\partial_0\partial_0 - P\Omega^{-2}\partial_0\partial_0)d. \end{aligned} \tag{7}$$

$$\begin{aligned} \delta G_{01} = {}& -\Omega^{-3}\dot{\Omega}\partial_1 K_{00} + (2\Omega^{-4}\dot{\Omega}^2 + \Omega^{-3}\ddot{\Omega} + \tfrac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu)K_{01} + (P\Omega^{-3}\dot{\Omega}\partial_1 + \tfrac{1}{4}\Omega^{-2}\partial_1\partial_0 \\ & - P\Omega^{-2}\partial_1\partial_0)d. \end{aligned} \tag{8}$$

$$\begin{aligned} \delta G_{11} = {}& \Omega^{-4}\dot{\Omega}^2 K_{00} - 2\Omega^{-3}\dot{\Omega}\partial_1 K_{01} + (-2\Omega^{-4}\dot{\Omega}^2 + 3\Omega^{-3}\ddot{\Omega} + \tfrac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \Omega^{-3}\dot{\Omega}\partial_0)K_{11} \\ & + (-\tfrac{5}{4}\Omega^{-4}\dot{\Omega}^2 + \tfrac{1}{4}\Omega^{-3}\ddot{\Omega} - \tfrac{1}{4}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}P\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + P\Omega^{-3}\dot{\Omega}\partial_0 + \tfrac{1}{4}\Omega^{-2}\partial_1\partial_1 \\ & - P\Omega^{-2}\partial_1\partial_1)d. \end{aligned} \tag{9}$$

$$\begin{aligned} \delta G_{12} = {}& -\Omega^{-3}\dot{\Omega}\partial_2 K_{01} - \Omega^{-3}\dot{\Omega}\partial_1 K_{02} + (-2\Omega^{-4}\dot{\Omega}^2 + 3\Omega^{-3}\ddot{\Omega} + \tfrac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \Omega^{-3}\dot{\Omega}\partial_0)K_{12} \\ & + (\tfrac{1}{4}\Omega^{-2}\partial_2\partial_1 - P\Omega^{-2}\partial_2\partial_1)d. \end{aligned} \tag{10}$$

$\Omega(\tau) = \frac{1}{H\tau}$

Now set $\Omega(\tau) = \frac{1}{H\tau}$, with the fluctuations being evaluated as

$$\begin{aligned} \eta^{\mu\nu}\delta G_{\mu\nu} =& (-\tfrac{3}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu)d \\ =& (P-\tfrac{3}{4})\Omega^{-2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta d \end{aligned} \tag{11}$$

$$\begin{aligned} \delta G_{00} = {}& (3H^2 + \tfrac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + H^2\tau\partial_0)K_{00} + (\tfrac{3}{4}H^2 + \tfrac{1}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - \tfrac{1}{2}H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & - H^2P\tau\partial_0 + \tfrac{1}{4}H^2\tau^2\partial_0\partial_0 - H^2P\tau^2\partial_0\partial_0)d. \end{aligned} \tag{12}$$

$$\begin{aligned} \delta G_{01} = {}& H^2\tau\partial_1 K_{00} + (4H^2 + \tfrac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu)K_{01} + (-H^2P\tau\partial_1 + \tfrac{1}{4}H^2\tau^2\partial_1\partial_0 \\ & - H^2P\tau^2\partial_1\partial_0)d. \end{aligned} \tag{13}$$

$$\begin{aligned} \delta G_{11} = {}& H^2K_{00} + 2H^2\tau\partial_1K_{01} + (4H^2 + \tfrac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - H^2\tau\partial_0)K_{11} + (-\tfrac{3}{4}H^2 \\ & - \tfrac{1}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + \tfrac{1}{2}H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - H^2P\tau\partial_0 + \tfrac{1}{4}H^2\tau^2\partial_1\partial_1 - H^2P\tau^2\partial_1\partial_1)d. \end{aligned} \tag{14}$$

$$\begin{aligned} \delta G_{12} = {}& H^2\tau\partial_2K_{01} + H^2\tau\partial_1K_{02} + (4H^2 + \tfrac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - H^2\tau\partial_0)K_{12} + (\tfrac{1}{4}H^2\tau^2\partial_2\partial_1 \\ & - H^2P\tau^2\partial_2\partial_1)d. \end{aligned} \tag{15}$$