Polar $\delta G_{\mu\nu}$ SVT

Background Metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{r^2}{1-p^2} & 0 \\ 0 & 0 & 0 & p^2 r^2 \end{pmatrix}$$

Metric

$$h_{00} = -2\phi$$

$$h_{0i} = \nabla_i B + B_i$$

$$h_{ij} = -2\psi + 2\nabla_i\nabla_j E + \nabla_i E_j + \nabla_j E_i + 2E_{ij}$$

Conditions

$$\nabla^i E_{i1} = \frac{2\,E_{11}}{r} + \frac{E_{12}}{p\,r^2} - \frac{2\,p\,E_{12}}{r^2} - \frac{E_{22}}{r^3} + \frac{p^2\,E_{22}}{r^3} - \frac{E_{33}}{p^2\,r^3} + \partial_1 E_{11} + \frac{\partial_2 E_{12}}{r^2} - \frac{p^2\,\partial_2 E_{12}}{r^2} + \frac{\partial_3 E_{13}}{p^2\,r^2}$$

$$\nabla^i E_{i2} = \frac{2\,E_{12}}{r} + \frac{E_{22}}{p\,r^2} - \frac{3\,p\,E_{22}}{r^2} - \frac{E_{33}}{p^3\,r^2} + \partial_1 E_{21} + \frac{\partial_2 E_{22}}{r^2} - \frac{p^2\,\partial_2 E_{22}}{r^2} + \frac{\partial_3 E_{23}}{p^2\,r^2}$$

$$\nabla^i E_{i3} = \frac{2\,E_{13}}{r} + \frac{E_{23}}{p\,r^2} - \frac{2\,p\,E_{23}}{r^2} + \partial_1 E_{31} + \frac{\partial_2 E_{23}}{r^2} - \frac{p^2\,\partial_2 E_{23}}{r^2} + \frac{\partial_3 E_{33}}{p^2\,r^2}$$

$$\nabla^i E_i = \frac{2\,E_1}{r} + \frac{E_2}{p\,r^2} - \frac{2\,p\,E_2}{r^2} + \partial_1 E_1 + \frac{\partial_2 E_2}{r^2} - \frac{p^2\,\partial_2 E_2}{r^2} + \frac{\partial_3 E_3}{p^2\,r^2} = 0$$

$$g_{polar}{}^{\mu\nu} E_{\mu\nu} = E_{11} + \frac{E_{22}}{r^2} - \frac{p^2\,E_{22}}{r^2} + \frac{E_{33}}{p^2\,r^2} = 0$$

Scalar Laplacian

$$\nabla^2 = \frac{2\,\partial_1}{r} + \partial_1\partial_1 + \frac{\partial_2}{p\,r^2} - \frac{2\,p\,\partial_2}{r^2} + \frac{\partial_2\partial_2}{r^2} - \frac{p^2\,\partial_2\partial_2}{r^2} + \frac{\partial_3\partial_3}{p^2\,r^2}$$

$\delta G_{\mu\nu}$ Ω=1

00	$\left(-\frac{4\,\partial_1\psi}{r} - 2\,\partial_1\partial_1\psi - \frac{2\,\partial_2\psi}{p\,r^2} + \frac{4\,p\,\partial_2\psi}{r^2} - \frac{2\,\partial_2\partial_2\psi}{r^2} + \frac{2\,p^2\,\partial_2\partial_2\psi}{r^2} - \frac{2\,\partial_3\partial_3\psi}{p^2\,r^2}\right) + (0) + (0)$
11	$\left(-2\,\partial_0\partial_0\psi - \frac{2\,\partial_1\phi}{r} + \frac{2\,\partial_1\psi}{r} - \frac{2\,\partial_1\partial_0 B}{r} + \frac{2\,\partial_1\partial_0\partial_0 E}{r} - \frac{\partial_2\phi}{p\,r^2} + \frac{2\,p\,\partial_2\phi}{r^2} + \frac{\partial_2\psi}{p\,r^2} - \frac{2\,p\,\partial_2\psi}{r^2} - \frac{\partial_2\partial_0 B}{p\,r^2} + \frac{2\,p\,\partial_2\partial_0 B}{r^2} + \frac{\partial_2\partial_0\partial_0 E}{p\,r^2} - \frac{2\,p\,\partial_2\partial_0\partial_0 E}{r^2} - \frac{\partial_2\partial_2\phi}{r^2} + \frac{p^2\,\partial_2\partial_2\phi}{r^2} + \frac{\partial_2\partial_2\psi}{r^2} - \frac{p^2\,\partial_2\partial_2\psi}{r^2} - \frac{\partial_2\partial_2\partial_0 B}{r^2} + \frac{p^2\,\partial_2\partial_2\partial_0 B}{r^2} + \frac{\partial_2\partial_2\partial_0\partial_0 E}{r^2} - \frac{p^2\,\partial_2\partial_2\partial_0\partial_0 E}{r^2} - \frac{\partial_3\partial_3\phi}{p^2\,r^2} + \frac{\partial_3\partial_3\psi}{p^2\,r^2} - \frac{\partial_3\partial_3\partial_0 B}{p^2\,r^2} + \frac{\partial_3\partial_3\partial_0\partial_0 E}{p^2\,r^2}\right) + (\partial_1\partial_0 B_1 - \partial_1\partial_0\partial_0 E_1) + \left(-\frac{4\,E_{11}}{r^2} - \frac{4\,E_{12}}{p\,r^3} + \frac{8\,p\,E_{12}}{r^3} + \frac{2\,E_{22}}{r^4} - \frac{2\,p^2\,E_{22}}{r^4} + \frac{2\,E_{33}}{p^2\,r^4} - \partial_0\partial_0 E_{11} + \frac{2\,\partial_1 E_{11}}{r} + \partial_1\partial_1 E_{11} + \frac{\partial_2 E_{11}}{p\,r^2} - \frac{2\,p\,\partial_2 E_{11}}{r^2} - \frac{4\,\partial_2 E_{12}}{r^3} + \frac{4\,p^2\,\partial_2 E_{12}}{r^3} + \frac{\partial_2\partial_2 E_{11}}{r^2} - \frac{p^2\,\partial_2\partial_2 E_{11}}{r^2} - \frac{4\,\partial_3 E_{13}}{p^2\,r^3} + \frac{\partial_3\partial_3 E_{11}}{p^2\,r^2}\right)$

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$$\left(\frac{2 r^2 \partial_0\partial_0\psi}{-1+p^2}+\frac{r \partial_1\phi}{-1+p^2}-\frac{r \partial_1\psi}{-1+p^2}+\frac{r \partial_1\partial_0 B}{-1+p^2}-\frac{r \partial_1\partial_0\partial_0 E}{-1+p^2}+\frac{r^2 \partial_1\partial_1\phi}{-1+p^2}-\frac{r^2 \partial_1\partial_1\psi}{-1+p^2}+\frac{r^2 \partial_1\partial_1\partial_0 B}{-1+p^2}-\frac{r^2 \partial_1\partial_1\partial_0\partial_0 E}{-1+p^2}+\frac{\partial_2\phi}{p\left(-1+p^2\right)}-\frac{p \partial_2\phi}{-1+p^2}-\frac{\partial_2\psi}{p\left(-1+p^2\right)}+\frac{p \partial_2\psi}{-1+p^2}+\frac{\partial_2\partial_0 B}{p\left(-1+p^2\right)}-\frac{p \partial_2\partial_0 B}{-1+p^2}-\frac{\partial_2\partial_0\partial_0 E}{p\left(-1+p^2\right)}+\frac{p \partial_2\partial_0\partial_0 E}{-1+p^2}+\frac{\partial_3\partial_3\phi}{p^2\left(-1+p^2\right)}-\frac{\partial_3\partial_3\psi}{p^2\left(-1+p^2\right)}+\frac{\partial_3\partial_3\partial_0 B}{p^2\left(-1+p^2\right)}-\frac{\partial_3\partial_3\partial_0\partial_0 E}{p^2\left(-1+p^2\right)}\right)+\left(-\frac{r \partial_0 B_1}{-1+p^2}+\frac{p \partial_0 B_2}{-1+p^2}+\frac{r \partial_0\partial_0 E_1}{-1+p^2}-\frac{p \partial_0\partial_0 E_2}{-1+p^2}-\frac{\partial_2\partial_0 B_2}{-1+p^2}+\frac{p^2 \partial_2\partial_0 B_2}{-1+p^2}+\frac{\partial_2\partial_0\partial_0 E_2}{-1+p^2}-\frac{p^2 \partial_2\partial_0\partial_0 E_2}{-1+p^2}\right)+\left(-\frac{2 E_{11}}{-1+p^2}+\frac{4 p E_{12}}{\left(-1+p^2\right) r}+\frac{2 E_{22}}{p^2\left(-1+p^2\right) r^2}-\frac{4 p^2 E_{22}}{\left(-1+p^2\right) r^2}-\frac{2 E_{33}}{p^4\left(-1+p^2\right) r^2}+\frac{2 E_{33}}{p^2\left(-1+p^2\right) r^2}+\frac{\partial_0\partial_0 E_{22}}{-1+p^2}-\frac{p^2 \partial_0\partial_0 E_{22}}{-1+p^2}+\frac{2 \partial_1 E_{22}}{\left(-1+p^2\right) r}-\frac{2 p^2 \partial_1 E_{22}}{\left(-1+p^2\right) r}-\frac{\partial_1\partial_1 E_{22}}{-1+p^2}+\frac{p^2 \partial_1\partial_1 E_{22}}{-1+p^2}-\frac{4 \partial_2 E_{12}}{\left(-1+p^2\right) r}+\frac{4 p^2 \partial_2 E_{12}}{\left(-1+p^2\right) r}-\frac{\partial_2 E_{22}}{p\left(-1+p^2\right) r^2}+\frac{7 p \partial_2 E_{22}}{\left(-1+p^2\right) r^2}-\frac{6 p^3 \partial_2 E_{22}}{\left(-1+p^2\right) r^2}-\frac{\partial_2\partial_2 E_{22}}{\left(-1+p^2\right) r^2}+\frac{2 p^2 \partial_2\partial_2 E_{22}}{\left(-1+p^2\right) r^2}-\frac{p^4 \partial_2\partial_2 E_{22}}{\left(-1+p^2\right) r^2}+\frac{4 \partial_3 E_{23}}{p^3\left(-1+p^2\right) r^2}-\frac{4 \partial_3 E_{23}}{p\left(-1+p^2\right) r^2}+\frac{\partial_3\partial_3 E_{22}}{\left(-1+p^2\right) r^2}-\frac{\partial_3\partial_3 E_{22}}{p^2\left(-1+p^2\right) r^2}\right)$$

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$$\left(-2 p^2 r^2 \partial_0\partial_0\psi-p^2 r \partial_1\phi+p^2 r \partial_1\psi-p^2 r \partial_1\partial_0 B+p^2 r \partial_1\partial_0\partial_0 E-p^2 r^2 \partial_1\partial_1\phi+p^2 r^2 \partial_1\partial_1\psi-p^2 r^2 \partial_1\partial_1\partial_0 B+p^2 r^2 \partial_1\partial_1\partial_0\partial_0 E+p^3 \partial_2\phi-p^3 \partial_2\psi+p^3 \partial_2\partial_0 B-p^3 \partial_2\partial_0\partial_0 E-p^2 \partial_2\partial_2\phi+p^4 \partial_2\partial_2\phi+p^2 \partial_2\partial_2\psi-p^4 \partial_2\partial_2\psi-p^2 \partial_2\partial_2\partial_0 B+p^4 \partial_2\partial_2\partial_0 B+p^2 \partial_2\partial_2\partial_0\partial_0 E-p^4 \partial_2\partial_2\partial_0\partial_0 E\right)+\left(p^2 r \partial_0 B_1+p \partial_0 B_2-p^3 \partial_0 B_2-p^2 r \partial_0\partial_0 E_1-p \partial_0\partial_0 E_2+p^3 \partial_0\partial_0 E_2+\partial_3\partial_0 B_3-\partial_3\partial_0\partial_0 E_3\right)+\left(2 p^2 E_{11}+\frac{4 p E_{12}}{r}-\frac{4 p^3 E_{12}}{r}+\frac{2 E_{22}}{r^2}-\frac{4 p^2 E_{22}}{r^2}+\frac{2 p^4 E_{22}}{r^2}+\frac{2 E_{33}}{p^2 r^2}-\partial_0\partial_0 E_{33}-\frac{2 \partial_1 E_{33}}{r}+\partial_1\partial_1 E_{33}-\frac{3 \partial_2 E_{33}}{p r^2}+\frac{2 p \partial_2 E_{33}}{r^2}+\frac{\partial_2\partial_2 E_{33}}{r^2}-\frac{p^2 \partial_2\partial_2 E_{33}}{r^2}+\frac{4 \partial_3 E_{13}}{r}+\frac{4 \partial_3 E_{23}}{p r^2}-\frac{4 p \partial_3 E_{23}}{r^2}+\frac{\partial_3\partial_3 E_{33}}{p^2 r^2}\right)$$

01

$$\left(-2 \partial_1\partial_0\psi\right)+\left(-\frac{B_1}{r^2}-\frac{B_2}{p r^3}+\frac{2 p B_2}{r^3}+\frac{\partial_0 E_1}{r^2}+\frac{\partial_0 E_2}{p r^3}-\frac{2 p \partial_0 E_2}{r^3}+\frac{\partial_1 B_1}{r}-\frac{\partial_1\partial_0 E_1}{r}+\frac{1}{2}\partial_1\partial_1 B_1-\frac{1}{2}\partial_1\partial_1\partial_0 E_1+\frac{\partial_2 B_1}{2 p r^2}-\frac{p \partial_2 B_1}{r^2}-\frac{\partial_2 B_2}{r^3}+\frac{p^2 \partial_2 B_2}{r^3}-\frac{\partial_2\partial_0 E_1}{2 p r^2}+\frac{p \partial_2\partial_0 E_1}{r^2}+\frac{\partial_2\partial_0 E_2}{r^3}-\frac{p^2 \partial_2\partial_0 E_2}{r^3}+\frac{\partial_2\partial_2 B_1}{2 r^2}-\frac{p^2 \partial_2\partial_2 B_1}{2 r^2}-\frac{\partial_2\partial_2\partial_0 E_1}{2 r^2}+\frac{p^2 \partial_2\partial_2\partial_0 E_1}{2 r^2}-\frac{\partial_3 B_3}{p^2 r^3}+\frac{\partial_3\partial_0 E_3}{p^2 r^3}+\frac{\partial_3\partial_3 B_1}{2 p^2 r^2}-\frac{\partial_3\partial_3\partial_0 E_1}{2 p^2 r^2}\right)+(0)$$

02

$$\left(-2 \partial_2\partial_0\psi\right)+\left(-\frac{B_2}{r^2}-\frac{B_2}{2 p^2 r^2}+\frac{\partial_0 E_2}{r^2}+\frac{\partial_0 E_2}{2 p^2 r^2}+\frac{1}{2}\partial_1\partial_1 B_2-\frac{1}{2}\partial_1\partial_1\partial_0 E_2+\frac{\partial_2 B_1}{r}+\frac{\partial_2 B_2}{2 p r^2}-\frac{2 p \partial_2 B_2}{r^2}-\frac{\partial_2\partial_0 E_1}{r}-\frac{\partial_2\partial_0 E_2}{2 p r^2}+\frac{2 p \partial_2\partial_0 E_2}{r^2}+\frac{\partial_2\partial_2 B_2}{2 r^2}-\frac{p^2 \partial_2\partial_2 B_2}{2 r^2}-\frac{\partial_2\partial_2\partial_0 E_2}{2 r^2}+\frac{p^2 \partial_2\partial_2\partial_0 E_2}{2 r^2}-\frac{\partial_3 B_3}{p^3 r^2}+\frac{\partial_3\partial_0 E_3}{p^3 r^2}+\frac{\partial_3\partial_3 B_2}{2 p^2 r^2}-\frac{\partial_3\partial_3\partial_0 E_2}{2 p^2 r^2}\right)+(0)$$

03

$$\left(-2 \partial_3\partial_0\psi\right)+\left(\frac{1}{2}\partial_1\partial_1 B_3-\frac{1}{2}\partial_1\partial_1\partial_0 E_3-\frac{\partial_2 B_3}{2 p r^2}+\frac{\partial_2\partial_0 E_3}{2 p r^2}+\frac{\partial_2\partial_2 B_3}{2 r^2}-\frac{p^2 \partial_2\partial_2 B_3}{2 r^2}-\frac{\partial_2\partial_2\partial_0 E_3}{2 r^2}+\frac{p^2 \partial_2\partial_2\partial_0 E_3}{2 r^2}+\frac{\partial_3 B_1}{r}+\frac{\partial_3 B_2}{p r^2}-\frac{p \partial_3 B_2}{r^2}-\frac{\partial_3\partial_0 E_1}{r}-\frac{\partial_3\partial_0 E_2}{p r^2}+\frac{p \partial_3\partial_0 E_2}{r^2}+\frac{\partial_3\partial_3 B_3}{2 p^2 r^2}-\frac{\partial_3\partial_3\partial_0 E_3}{2 p^2 r^2}\right)+(0)$$

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$$\left(-\frac{\partial_2\phi}{r}+\frac{\partial_2\psi}{r}-\frac{\partial_2\partial_0 B}{r}+\frac{\partial_2\partial_0\partial_0 E}{r}+\partial_2\partial_1\phi-\partial_2\partial_1\psi+\partial_2\partial_1\partial_0 B-\partial_2\partial_1\partial_0\partial_0 E\right)+\left(-\frac{\partial_0 B_2}{r}+\frac{\partial_0\partial_0 E_2}{r}+\frac{1}{2}\partial_1\partial_0 B_2-\frac{1}{2}\partial_1\partial_0\partial_0 E_2+\frac{1}{2}\partial_2\partial_0 B_1-\frac{1}{2}\partial_2\partial_0\partial_0 E_1\right)+\left(-\frac{6 E_{12}}{r^2}-\frac{E_{12}}{p^2 r^2}-\frac{2 E_{22}}{p r^3}+\frac{6 p E_{22}}{r^3}+\frac{2 E_{33}}{p^3 r^3}-\partial_0\partial_0 E_{12}+\partial_1\partial_1 E_{12}+\frac{2 \partial_2 E_{11}}{r}+\frac{\partial_2 E_{12}}{p r^2}-\frac{4 p \partial_2 E_{12}}{r^2}-\frac{2 \partial_2 E_{22}}{r^3}+\frac{2 p^2 \partial_2 E_{22}}{r^3}+\frac{\partial_2\partial_2 E_{12}}{r^2}-\frac{p^2 \partial_2\partial_2 E_{12}}{r^2}-\frac{2 \partial_3 E_{13}}{p^3 r^2}-\frac{2 \partial_3 E_{23}}{p^2 r^3}+\frac{\partial_3\partial_3 E_{12}}{p^2 r^2}\right)$$

13

$$\left(-\frac{\partial_3\phi}{r}+\frac{\partial_3\psi}{r}-\frac{\partial_3\partial_0 B}{r}+\frac{\partial_3\partial_0\partial_0 E}{r}+\partial_3\partial_1\phi-\partial_3\partial_1\psi+\partial_3\partial_1\partial_0 B-\partial_3\partial_1\partial_0\partial_0 E\right)+\left(-\frac{\partial_0 B_3}{r}+\frac{\partial_0\partial_0 E_3}{r}+\frac{1}{2}\partial_1\partial_0 B_3-\frac{1}{2}\partial_1\partial_0\partial_0 E_3+\frac{1}{2}\partial_3\partial_0 B_1-\frac{1}{2}\partial_3\partial_0\partial_0 E_1\right)+\left(-\frac{4 E_{13}}{r^2}-\frac{2 E_{23}}{p r^3}+\frac{4 p E_{23}}{r^3}-\partial_0\partial_0 E_{13}+\partial_1\partial_1 E_{13}-\frac{\partial_2 E_{13}}{p r^2}-\frac{2 \partial_2 E_{23}}{r^3}+\frac{2 p^2 \partial_2 E_{23}}{r^3}+\frac{\partial_2\partial_2 E_{13}}{r^2}-\frac{p^2 \partial_2\partial_2 E_{13}}{r^2}+\frac{2 \partial_3 E_{11}}{r}+\frac{2 \partial_3 E_{12}}{p r^2}-\frac{2 p \partial_3 E_{12}}{r^2}-\frac{2 \partial_3 E_{33}}{p^2 r^3}+\frac{\partial_3\partial_3 E_{13}}{p^2 r^2}\right)$$

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$$\left(-\frac{\partial_3\phi}{p}+\frac{\partial_3\psi}{p}-\frac{\partial_3\partial_0 B}{p}+\frac{\partial_3\partial_0\partial_0 E}{p}+\partial_3\partial_2\phi-\partial_3\partial_2\psi+\partial_3\partial_2\partial_0 B-\partial_3\partial_2\partial_0\partial_0 E\right)+\left(-\frac{\partial_0 B_3}{p}+\frac{\partial_0\partial_0 E_3}{p}+\frac{1}{2}\partial_2\partial_0 B_3-\frac{1}{2}\partial_2\partial_0\partial_0 E_3+\frac{1}{2}\partial_3\partial_0 B_2-\frac{1}{2}\partial_3\partial_0\partial_0 E_2\right)+\left(-\frac{4E_{13}}{p\,r}+\frac{4E_{23}}{r^2}-\frac{3E_{23}}{p^2\,r^2}-\partial_0\partial_0 E_{23}-\frac{2\partial_1 E_{23}}{r}+\partial_1\partial_1 E_{23}+\frac{2\partial_2 E_{13}}{r}-\frac{\partial_2 E_{23}}{p\,r^2}-\frac{2p\,\partial_2 E_{23}}{r^2}+\frac{\partial_2\partial_2 E_{23}}{r^2}-\frac{p^2\,\partial_2\partial_2 E_{23}}{r^2}+\frac{2\partial_3 E_{12}}{r}+\frac{2\partial_3 E_{22}}{p\,r^2}-\frac{2p\,\partial_3 E_{22}}{r^2}-\frac{2\partial_3 E_{33}}{p^3\,r^2}+\frac{\partial_3\partial_3 E_{23}}{p^2\,r^2}\right)$$