

First order C^2 Fluctuations

$W_{\mu\nu}$

$$\begin{aligned} W_{\mu\nu}^{(1)} &= \frac{1}{2} g_{\mu\nu} R^2 - 2 R R_{\mu\nu} + 2 g_{\mu\nu} \nabla_\alpha \nabla^\alpha R - 2 \nabla_\nu \nabla_\mu R \\ W_{\mu\nu}^{(2)} &= \frac{1}{2} g_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} - 2 R^{\alpha\beta} R_{\alpha\mu\beta\nu} + \frac{1}{2} g_{\mu\nu} \nabla_\alpha \nabla^\alpha R + \nabla_\alpha \nabla^\alpha R_{\mu\nu} - \nabla_\mu \nabla^\alpha R_{\nu\alpha} - \nabla_\nu \nabla^\alpha R_{\mu\alpha} \\ W_{\mu\nu} &= -\frac{1}{6} g_{\mu\nu} R^2 + \frac{1}{2} g_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} + \frac{2}{3} R R_{\mu\nu} - 2 R^{\alpha\beta} R_{\mu\alpha\nu\beta} - \frac{1}{6} g_{\mu\nu} \nabla_\alpha \nabla^\alpha R + \nabla_\alpha \nabla^\alpha R_{\mu\nu} - \frac{\nabla_\nu \nabla_\mu R}{3} \end{aligned}$$

$\delta W_{\mu\nu}$

$$\begin{aligned} \delta W_{\mu\nu}^{(1)} &= -\frac{1}{8} g_{\mu\nu} h^\alpha{}_\alpha R^2 + \frac{1}{2} K_{\mu\nu} R^2 - g_{\mu\nu} K^{\alpha\beta} R R_{\alpha\beta} - K_{\nu}{}^\alpha R R_{\mu\alpha} + \frac{1}{2} h^\alpha{}_\alpha R R_{\mu\nu} + 2 \\ &\quad K^{\alpha\beta} R_{\alpha\beta} R_{\mu\nu} - K_{\mu}{}^\alpha R R_{\nu\alpha} + 2 K^{\alpha\beta} R R_{\mu\alpha\nu\beta} - R \nabla_\alpha \nabla^\alpha K_{\mu\nu} + 2 K_{\mu\nu} \nabla_\alpha \nabla^\alpha R - \frac{3}{4} g_{\mu\nu} \\ &\quad \nabla_\alpha h^\beta{}_\beta \nabla^\alpha R - \nabla_\alpha K_{\mu\nu} \nabla^\alpha R - 2 g_{\mu\nu} \nabla^\alpha R \nabla_\beta K_{\alpha}{}^\beta - g_{\mu\nu} R \nabla_\beta \nabla_\alpha K^{\alpha\beta} + 2 R_{\mu\nu} \nabla_\beta \nabla_\alpha K^{\alpha\beta} - 2 g_{\mu\nu} \\ &\quad K^{\alpha\beta} \nabla_\beta \nabla_\alpha R - \frac{3}{2} R_{\mu\nu} \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - \frac{1}{2} g_{\mu\nu} h^\alpha{}_\alpha \nabla_\beta \nabla^\beta R - 2 g_{\mu\nu} R^{\alpha\beta} \nabla_\gamma \nabla^\gamma K_{\alpha\beta} - 2 g_{\mu\nu} K^{\alpha\beta} \\ &\quad \nabla_\gamma \nabla^\gamma R_{\alpha\beta} - 2 g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla_\alpha K^{\alpha\beta} + \frac{3}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - 4 g_{\mu\nu} \nabla_\gamma R_{\alpha\beta} \nabla^\gamma K^{\alpha\beta} + \nabla^\alpha R \nabla_\mu K_{\nu\alpha} + R \\ &\quad \nabla_\mu \nabla_\alpha K_{\nu}{}^\alpha + \frac{3}{4} \nabla_\mu R \nabla_\nu h^\alpha{}_\alpha + 2 \nabla_\mu R_{\alpha\beta} \nabla_\nu K^{\alpha\beta} + \nabla^\alpha R \nabla_\nu K_{\mu\alpha} + \frac{3}{4} \nabla_\mu h^\alpha{}_\alpha \nabla_\nu R + 2 \nabla_\mu K^{\alpha\beta} \nabla_\nu R_{\alpha\beta} + R \\ &\quad \nabla_\nu \nabla_\alpha K_{\mu}{}^\alpha + 2 R^{\alpha\beta} \nabla_\nu \nabla_\mu K_{\alpha\beta} + \frac{1}{2} h^\alpha{}_\alpha \nabla_\nu \nabla_\mu R + 2 K^{\alpha\beta} \nabla_\nu \nabla_\mu R_{\alpha\beta} + 2 \nabla_\nu \nabla_\mu \nabla_\beta \nabla_\alpha K^{\alpha\beta} - \frac{3}{2} \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha \\ \delta W_{\mu\nu}^{(2)} &= \frac{1}{2} K_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} - \frac{1}{8} g_{\mu\nu} h^\alpha{}_\alpha R_{\beta\gamma} R^{\beta\gamma} - \frac{1}{2} K_{\nu}{}^\alpha R_{\alpha\beta} R_{\mu}{}^\beta + K^{\alpha\beta} R_{\mu\alpha} R_{\nu\beta} - \frac{1}{2} K_{\mu}{}^\alpha R_{\alpha\beta} \\ &\quad R_{\nu}{}^\beta - g_{\mu\nu} K^{\alpha\beta} R^{\gamma\eta} R_{\alpha\gamma\beta\eta} - K_{\nu}{}^\alpha R^{\beta\gamma} R_{\mu\beta\alpha\gamma} + \frac{1}{2} h^\alpha{}_\alpha R^{\beta\gamma} R_{\mu\beta\gamma\alpha} + 2 K^{\alpha\beta} R_{\alpha}{}^\gamma R_{\mu\gamma\nu\beta} + 2 \\ &\quad K^{\alpha\beta} R_{\alpha\gamma\beta\eta} R_{\mu}{}^\gamma{}_\nu{}^\eta - K_{\mu}{}^\alpha R^{\beta\gamma} R_{\nu\beta\alpha\gamma} + \frac{1}{2} K_{\mu\nu} \nabla_\alpha \nabla^\alpha R + \frac{1}{2} R_{\nu}{}^\alpha \nabla_\alpha \nabla_\beta K_{\mu}{}^\beta + \frac{1}{2} R_{\mu}{}^\alpha \nabla_\alpha \nabla_\beta K_{\nu}{}^\beta - \frac{1}{4} \\ &\quad g_{\mu\nu} \nabla_\alpha h^\beta{}_\beta \nabla^\alpha R - \frac{1}{2} \nabla_\alpha K_{\mu\nu} \nabla^\alpha R - \frac{1}{2} g_{\mu\nu} \nabla^\alpha R \nabla_\beta K_{\alpha}{}^\beta - \nabla_\alpha K^{\alpha\beta} \nabla_\beta R_{\mu\nu} - R^{\alpha\beta} \nabla_\beta \nabla_\alpha K_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \\ &\quad K^{\alpha\beta} \nabla_\beta \nabla_\alpha R - K^{\alpha\beta} \nabla_\beta \nabla_\alpha R_{\mu\nu} - \frac{1}{2} R_{\mu\nu} \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - \frac{1}{8} g_{\mu\nu} h^\alpha{}_\alpha \nabla_\beta \nabla^\beta R + \frac{1}{2} K_{\nu}{}^\alpha \nabla_\beta \nabla^\beta R_{\mu\alpha} - \frac{1}{4} h^\alpha{}_\alpha \\ &\quad \nabla_\beta \nabla^\beta R_{\mu\nu} + \frac{1}{2} K_{\mu}{}^\alpha \nabla_\beta \nabla^\beta R_{\nu\alpha} + \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\alpha \nabla^\alpha K_{\mu\nu} - \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\mu \nabla_\alpha K_{\nu}{}^\alpha - \frac{1}{2} \nabla_\beta \nabla^\beta \nabla_\nu \nabla_\alpha K_{\mu}{}^\alpha - g_{\mu\nu} R^{\alpha\beta} \\ &\quad \nabla_\beta \nabla_\gamma K_{\alpha}{}^\gamma - \frac{1}{4} \nabla_\beta R_{\mu\nu} \nabla^\beta h^\alpha{}_\alpha + \nabla_\alpha R_{\nu\beta} \nabla^\beta K_{\mu}{}^\alpha + \nabla_\alpha R_{\mu\beta} \nabla^\beta K_{\nu}{}^\alpha + \frac{1}{4} \nabla^\beta h^\alpha{}_\alpha \nabla_\gamma R_{\mu}{}^\gamma{}_\nu{}^\beta - 2 R_{\mu\alpha\nu\beta} \\ &\quad \nabla_\gamma \nabla^\gamma K^{\alpha\beta} - \frac{1}{2} g_{\mu\nu} K^{\alpha\beta} \nabla_\gamma \nabla^\gamma R_{\alpha\beta} - K^{\alpha\beta} \nabla_\gamma \nabla^\gamma R_{\mu\alpha\nu\beta} - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla_\alpha K^{\alpha\beta} + \frac{1}{2} g_{\mu\nu} \nabla_\gamma \nabla^\gamma \nabla_\beta \nabla^\beta h^\alpha{}_\alpha - g_{\mu\nu} \\ &\quad \nabla_\gamma R_{\alpha\beta} \nabla^\gamma K^{\alpha\beta} - 2 \nabla_\gamma R_{\mu\alpha\nu\beta} \nabla^\gamma K^{\alpha\beta} + R_{\mu\beta\gamma\alpha} \nabla^\gamma \nabla_\alpha K^{\alpha\beta} + R_{\mu\gamma\nu\beta} \nabla^\gamma \nabla_\alpha K^{\alpha\beta} - \frac{1}{4} \nabla_\beta R_{\nu}{}^\beta \nabla_\mu h^\alpha{}_\alpha - \nabla_\beta R_{\nu\alpha} \\ &\quad \nabla_\mu K^{\alpha\beta} + \frac{1}{2} \nabla^\alpha R \nabla_\mu K_{\nu\alpha} - \frac{1}{2} R_{\nu}{}^\alpha \nabla_\mu \nabla_\beta K_{\alpha}{}^\beta + R^{\alpha\beta} \nabla_\mu \nabla_\beta K_{\nu\alpha} - \frac{1}{4} \nabla_\beta R_{\mu}{}^\beta \nabla_\nu h^\alpha{}_\alpha + \frac{3}{8} \nabla_\mu R \nabla_\nu h^\alpha{}_\alpha - \nabla_\beta R_{\mu\alpha} \\ &\quad \nabla_\nu K^{\alpha\beta} + \nabla_\mu R_{\alpha\beta} \nabla_\nu K^{\alpha\beta} + \frac{1}{2} \nabla^\alpha R \nabla_\nu K_{\mu\alpha} + \frac{3}{8} \nabla_\mu h^\alpha{}_\alpha \nabla_\nu R + \nabla_\mu K^{\alpha\beta} \nabla_\nu R_{\alpha\beta} + \frac{1}{4} \nabla^\beta h^\alpha{}_\alpha \nabla_\nu R_{\mu\beta} - \frac{1}{2} \\ &\quad R_{\mu}{}^\alpha \nabla_\nu \nabla_\beta K_{\alpha}{}^\beta + R^{\alpha\beta} \nabla_\nu \nabla_\beta K_{\mu\alpha} + \frac{1}{4} h^\alpha{}_\alpha \nabla_\nu \nabla_\mu R + K^{\alpha\beta} \nabla_\nu \nabla_\mu R_{\alpha\beta} + \nabla_\nu \nabla_\mu \nabla_\beta \nabla_\alpha K^{\alpha\beta} - \frac{1}{2} \nabla_\nu \nabla_\mu \nabla_\beta \nabla^\beta h^\alpha{}_\alpha \end{aligned}$$

$$g^{(0)\mu\nu} \delta W_{\mu\nu}$$

$$\begin{aligned} g^{(0)\mu\nu} \delta W_{\mu\nu} &= -\frac{1}{6} g^{\alpha\beta} h_{\alpha\beta} R^2 + \frac{2}{3} h^{\alpha\beta} R R_{\alpha\beta} + \frac{1}{2} g^{\alpha\beta} h_{\alpha\beta} R_{\mu\nu} \\ &\quad R^{\mu\nu} - 2 h^{\alpha\beta} R^{\mu\nu} R_{\alpha\mu\beta\nu} - \frac{1}{3} h^{\alpha\beta} \nabla_\beta \nabla_\alpha R - \frac{1}{6} g^{\alpha\beta} h_{\alpha\beta} \nabla_\mu \nabla^\mu R + h^{\alpha\beta} \nabla_\mu \nabla^\mu R_{\alpha\beta} \\ &= h^{\mu\nu} \left(-\frac{1}{6} g_{\mu\nu} R^2 + \frac{1}{2} g_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} + \frac{2}{3} R \right. \\ &\quad \left. R_{\mu\nu} - 2 R^{\alpha\beta} R_{\mu\alpha\nu\beta} - \frac{1}{6} g_{\mu\nu} \nabla_\alpha \nabla^\alpha R + \nabla_\alpha \nabla^\alpha R_{\mu\nu} - \frac{\nabla_\nu \nabla_\mu R}{3} \right) \\ &= h^{\mu\nu} W_{\mu\nu} (g^{(0)})_{\mu\nu} \end{aligned}$$

$$\delta W_{\mu\nu} \left(\frac{h}{4} g^{(0)}_{\mu\nu} \right)$$

$$\text{Setting } K_{\mu\nu} = 0 \text{ in } \delta W_{\mu\nu}(h_{\mu\nu}) = \delta W_{\mu\nu} \left(K_{\mu\nu} + \frac{h}{4} g_{\mu\nu} \right),$$

$$\begin{aligned} \overline{\delta W}_{\mu\nu} \left(\frac{h}{4} g^{(0)}_{\mu\nu} \right) &= \frac{1}{24} g_{\mu\nu} h^\alpha{}_\alpha R^2 - \frac{1}{8} g_{\mu\nu} h^\alpha{}_\alpha R^{\beta\gamma} R_{\beta\gamma} - \frac{1}{6} h^\alpha{}_\alpha R \\ &\quad R_{\mu\nu} + \frac{1}{2} h^\alpha{}_\alpha R^{\beta\gamma} R_{\mu\beta\nu\gamma} + \frac{1}{24} g_{\mu\nu} h^\alpha{}_\alpha \nabla_\beta \nabla^\beta R - \frac{1}{4} h^\alpha{}_\alpha \nabla_\beta \nabla^\beta R_{\mu\nu} + \frac{1}{12} h^\alpha{}_\alpha \nabla_\nu \nabla_\mu R \\ &= -\frac{h}{4} W_{\mu\nu} (g^{(0)})_{\mu\nu} \end{aligned}$$

$$\overline{\delta W}_{\mu\nu}(\overline{K}_{\mu\nu})$$

No gauge (151 terms):

$$\begin{aligned} \overline{\delta W}_{\mu\nu}(\overline{K}_{\mu\nu}) &= \\ &- \frac{48 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\beta \Omega \partial_\gamma \Omega \partial_\eta \overline{K}_{\mu\nu}}{\Omega^7} + \frac{24 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\gamma \partial_\beta \Omega \partial_\eta \overline{K}_{\mu\nu}}{\Omega^6} + \frac{1}{\Omega^8} 20 \eta^{\alpha\beta} \eta^{\gamma\kappa} \eta^{\eta\lambda} \eta_{\mu\nu} \overline{K}_{\kappa\lambda} \partial_\alpha \\ &\quad \Omega \partial_\beta \Omega \partial_\gamma \Omega \partial_\eta \Omega + \frac{60 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\mu\gamma} \partial_\alpha \Omega \partial_\beta \Omega \partial_\gamma \Omega \partial_\eta \Omega}{\Omega^8} - \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\gamma \partial_\alpha \Omega \partial_\eta \partial_\beta \overline{K}_{\mu\nu}}{\Omega^5} + \\ &\frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\mu\gamma} \partial_\gamma \partial_\alpha \Omega \partial_\eta \partial_\beta \Omega}{\Omega^6} + \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\eta \partial_\beta \partial_\mu \overline{K}_{\gamma\gamma}}{\Omega^5} + \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\eta \partial_\beta \partial_\nu \overline{K}_{\mu\gamma}}{\Omega^5} + \\ &\frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\eta \partial_\beta \partial_\nu \partial_\mu \overline{K}_{\alpha\gamma}}{3 \Omega^4} - \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\alpha\gamma} \partial_\eta \partial_\beta \partial_\nu \partial_\mu \Omega}{3 \Omega^5} + \frac{12 \eta^{\alpha\gamma} \eta^{\beta\eta} \partial_\alpha \Omega \partial_\beta \Omega \partial_\eta \partial_\gamma \overline{K}_{\mu\nu}}{\Omega^6} + \\ &\frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\beta \Omega \partial_\eta \partial_\gamma \overline{K}_{\mu\nu}}{\Omega^6} - \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\beta \partial_\alpha \Omega \partial_\eta \partial_\gamma \overline{K}_{\mu\nu}}{\Omega^5} + \frac{12 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\beta \overline{K}_{\mu\nu} \partial_\eta \partial_\gamma \Omega}{\Omega^6} - \\ &\frac{48 \eta^{\alpha\gamma} \eta^{\beta\eta} \overline{K}_{\mu\gamma} \partial_\alpha \Omega \partial_\beta \Omega \partial_\eta \partial_\gamma \Omega}{\Omega^7} - \frac{24 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\mu\gamma} \partial_\alpha \Omega \partial_\beta \Omega \partial_\eta \partial_\gamma \Omega}{\Omega^7} + \frac{3 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\mu\gamma} \partial_\beta \partial_\alpha \Omega \partial_\eta \partial_\gamma \Omega}{\Omega^6} \\ &- \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\eta \partial_\gamma \partial_\beta \overline{K}_{\mu\nu}}{\Omega^5} - \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \overline{K}_{\mu\gamma} \partial_\eta \partial_\gamma \partial_\beta \Omega}{\Omega^5} + \frac{12 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\mu\gamma} \partial_\alpha \Omega \partial_\eta \partial_\gamma \partial_\beta \Omega}{\Omega^6} + \\ &\frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\eta \partial_\gamma \partial_\beta \partial_\alpha \overline{K}_{\mu\nu}}{2 \Omega^4} - \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\mu\gamma} \partial_\eta \partial_\gamma \partial_\beta \partial_\alpha \Omega}{\Omega^5} - \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\eta \partial_\gamma \partial_\beta \partial_\mu \overline{K}_{\gamma\alpha}}{2 \Omega^4} - \\ &\frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\eta \partial_\gamma \partial_\beta \partial_\nu \overline{K}_{\mu\alpha}}{2 \Omega^4} + \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\eta \partial_\gamma \partial_\mu \overline{K}_{\gamma\beta}}{\Omega^5} + \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\gamma\alpha} \partial_\eta \partial_\gamma \partial_\mu \partial_\beta \Omega}{\Omega^5} + \\ &\frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\eta \partial_\gamma \partial_\nu \overline{K}_{\mu\beta}}{\Omega^5} + \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\mu\alpha} \partial_\eta \partial_\gamma \partial_\nu \partial_\beta \Omega}{\Omega^5} + \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\gamma \partial_\alpha \Omega \partial_\eta \partial_\mu \overline{K}_{\gamma\beta}}{\Omega^5} - \\ &\frac{6 \eta^{\alpha\gamma} \eta^{\beta\eta} \partial_\alpha \Omega \partial_\beta \Omega \partial_\eta \partial_\mu \overline{K}_{\gamma\gamma}}{\Omega^6} - \frac{3 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\alpha \Omega \partial_\beta \Omega \partial_\eta \partial_\mu \overline{K}_{\gamma\gamma}}{\Omega^6} + \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\beta \partial_\alpha \Omega \partial_\eta \partial_\mu \overline{K}_{\gamma\gamma}}{\Omega^5} - \\ &\frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\gamma\gamma} \partial_\alpha \Omega \partial_\eta \partial_\mu \partial_\beta \Omega}{\Omega^6} + \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_\gamma \overline{K}_{\gamma\alpha} \partial_\eta \partial_\mu \partial_\beta \Omega}{\Omega^5} - \frac{3 \eta^{\alpha\beta} \eta^{\gamma\eta} \overline{K}_{\gamma\beta} \partial_\alpha \Omega \partial_\eta \partial_\mu \partial_\gamma \Omega}{\Omega^6} + \end{aligned}$$

$$\begin{aligned}
& \frac{40 \eta^{\alpha\gamma} \eta^{\beta\eta} \bar{K}_{\gamma\eta} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\mu\Omega} \partial_{\nu\Omega}}{\Omega^8} - \frac{16 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\eta} \bar{K}_{\beta\gamma} \partial_{\mu\Omega} \partial_{\nu\Omega}}{\Omega^7} + \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \partial_{\beta} \bar{K}_{\alpha\gamma} \partial_{\mu\Omega} \partial_{\nu\Omega}}{\Omega^6} - \\
& \frac{8 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\alpha\gamma} \partial_{\eta} \partial_{\beta\Omega} \partial_{\mu\Omega} \partial_{\nu\Omega}}{\Omega^7} + \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \bar{K}_{\beta\gamma} \partial_{\mu} \partial_{\alpha\Omega} \partial_{\nu\Omega}}{\Omega^6} - \frac{16 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\beta\gamma} \partial_{\alpha\Omega} \partial_{\mu} \partial_{\eta\Omega} \partial_{\nu\Omega}}{\Omega^7} + \\
& \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \partial_{\beta} \bar{K}_{\mu\gamma} \partial_{\nu} \partial_{\alpha\Omega}}{\Omega^5} + \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \partial_{\gamma} \bar{K}_{\mu\beta} \partial_{\nu} \partial_{\alpha\Omega}}{\Omega^5} - \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \partial_{\mu} \bar{K}_{\beta\gamma} \partial_{\nu} \partial_{\alpha\Omega}}{3 \Omega^5} + \\
& \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \bar{K}_{\beta\gamma} \partial_{\mu\Omega} \partial_{\nu} \partial_{\alpha\Omega}}{\Omega^6} - \frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\eta} \bar{K}_{\mu\gamma} \partial_{\nu} \partial_{\beta\Omega}}{\Omega^6} - \frac{3 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\alpha} \partial_{\eta} \partial_{\gamma\Omega} \partial_{\nu} \partial_{\beta\Omega}}{\Omega^6} - \\
& \frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\beta} \bar{K}_{\mu\eta} \partial_{\nu} \partial_{\gamma\Omega}}{\Omega^6} - \frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\eta} \bar{K}_{\mu\beta} \partial_{\nu} \partial_{\gamma\Omega}}{\Omega^6} - \frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\alpha} \partial_{\eta} \partial_{\beta\Omega} \partial_{\nu} \partial_{\gamma\Omega}}{\Omega^6} + \\
& \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\mu} \bar{K}_{\beta\eta} \partial_{\nu} \partial_{\gamma\Omega}}{\Omega^6} + \frac{24 \eta^{\alpha\gamma} \eta^{\beta\eta} \bar{K}_{\mu\gamma} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\nu} \partial_{\eta\Omega}}{\Omega^7} + \frac{12 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\gamma} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\nu} \partial_{\eta\Omega}}{\Omega^7} - \\
& \frac{16 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\beta\gamma} \partial_{\alpha\Omega} \partial_{\mu\Omega} \partial_{\nu} \partial_{\eta\Omega}}{\Omega^7} + \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\alpha\gamma} \partial_{\mu} \partial_{\beta\Omega} \partial_{\nu} \partial_{\eta\Omega}}{\Omega^6} - \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\gamma} \partial_{\alpha\Omega} \partial_{\nu} \partial_{\mu} \bar{K}_{\beta\eta}}{3 \Omega^5} + \\
& \frac{2 \eta^{\alpha\gamma} \eta^{\beta\eta} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\nu} \partial_{\mu} \bar{K}_{\gamma\eta}}{\Omega^6} - \frac{8 \eta^{\alpha\gamma} \eta^{\beta\eta} \bar{K}_{\gamma\eta} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\nu} \partial_{\mu\Omega}}{\Omega^7} + \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\eta} \bar{K}_{\beta\gamma} \partial_{\nu} \partial_{\mu\Omega}}{\Omega^6} - \\
& \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \partial_{\beta} \bar{K}_{\alpha\gamma} \partial_{\nu} \partial_{\mu\Omega}}{3 \Omega^5} + \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\alpha\gamma} \partial_{\eta} \partial_{\beta\Omega} \partial_{\nu} \partial_{\mu\Omega}}{\Omega^6}
\end{aligned}$$

Now working within the gauge

$$\bar{\nabla}_{\nu} \bar{K}^{\mu\nu} = 4 \Omega^{-1} \bar{K}^{\mu\nu} \partial_{\nu} \Omega,$$

or equivalently

$$\eta^{\alpha\beta} \partial_{\alpha} \bar{K}_{\mu\beta} = 2 \Omega^{-1} \eta^{\alpha\beta} \bar{K}_{\mu\beta} \partial_{\alpha} \Omega,$$

we have

$$\begin{aligned}
\delta \bar{W}_{\mu\nu}(\bar{K}_{\mu\nu}) &= - \frac{48 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\gamma\Omega} \partial_{\eta} \bar{K}_{\mu\nu}}{\Omega^7} + \frac{24 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\gamma} \partial_{\beta\Omega} \partial_{\eta} \bar{K}_{\mu\nu}}{\Omega^6} + \\
& \frac{60 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\gamma} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\gamma\Omega} \partial_{\eta\Omega}}{\Omega^8} - \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\gamma} \partial_{\alpha\Omega} \partial_{\eta} \partial_{\beta} \bar{K}_{\mu\gamma}}{\Omega^5} + \frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\gamma} \partial_{\gamma} \partial_{\alpha\Omega} \partial_{\eta} \partial_{\beta\Omega}}{\Omega^6} + \\
& \frac{12 \eta^{\alpha\gamma} \eta^{\beta\eta} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\eta} \partial_{\gamma} \bar{K}_{\mu\nu}}{\Omega^6} + \frac{6 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\eta} \partial_{\gamma} \bar{K}_{\mu\nu}}{\Omega^6} - \frac{2 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\beta} \partial_{\alpha\Omega} \partial_{\eta} \partial_{\gamma} \bar{K}_{\mu\nu}}{\Omega^5} + \\
& \frac{12 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\beta} \bar{K}_{\mu\gamma} \partial_{\eta} \partial_{\gamma\Omega}}{\Omega^6} - \frac{48 \eta^{\alpha\gamma} \eta^{\beta\eta} \bar{K}_{\mu\gamma} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\eta} \partial_{\gamma\Omega}}{\Omega^7} - \frac{24 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\gamma} \partial_{\alpha\Omega} \partial_{\beta\Omega} \partial_{\eta} \partial_{\gamma\Omega}}{\Omega^7} + \\
& + \frac{3 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\gamma} \partial_{\beta} \partial_{\alpha\Omega} \partial_{\eta} \partial_{\gamma\Omega}}{\Omega^6} - \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha\Omega} \partial_{\eta} \partial_{\gamma} \partial_{\beta} \bar{K}_{\mu\gamma}}{\Omega^5} - \frac{4 \eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\alpha} \bar{K}_{\mu\gamma} \partial_{\eta} \partial_{\gamma} \partial_{\beta\Omega}}{\Omega^5} + \\
& \frac{12 \eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\gamma} \partial_{\alpha\Omega} \partial_{\eta} \partial_{\gamma} \partial_{\beta\Omega}}{\Omega^6} + \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \partial_{\eta} \partial_{\gamma} \partial_{\beta} \partial_{\alpha} \bar{K}_{\mu\gamma}}{2 \Omega^4} - \frac{\eta^{\alpha\beta} \eta^{\gamma\eta} \bar{K}_{\mu\gamma} \partial_{\eta} \partial_{\gamma} \partial_{\beta} \partial_{\alpha\Omega}}{\Omega^5} \\
&= \frac{1}{2} \Omega^{-2} \eta^{\sigma\rho} \eta^{\alpha\beta} \partial_{\sigma} \partial_{\rho} \partial_{\alpha} \partial_{\beta} (\Omega^{-2} \bar{K}_{\mu\nu}) \\
&= \frac{1}{2} \Omega^{-2} \square^2 (\Omega^{-2} \bar{K}_{\mu\nu})
\end{aligned}$$