SVT3 RW Radiation $\Omega(x)$ $k<0$ Cartesian

1 Background

1.1 Comoving $a(t)$

First, determine the form of $a(t)$ for $\rho = 3p$ radiation in comoving coordinates

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2) = -dt^2 + a(t)^2\tilde{g}_{ij}dx^i dx^j \tag{1.1}$$

$$T_{\mu\nu} = p(4U_\mu U_\nu + g_{\mu\nu}), \qquad U_\mu = -\delta^0_\mu \tag{1.2}$$

$$T_{00} = 3p, \qquad T_{ij} = a^2(t)p\tilde{g}_{ij} \tag{1.3}$$

$$G_{00} = -3ka^{-2} - 3\dot{a}^2 a^{-2}, \qquad G_{ij} = \tilde{g}_{ij}(k + \dot{a}^2 + 2a\ddot{a}) \tag{1.4}$$

$$\Delta_{\mu\nu} = G_{\mu\nu} + T_{\mu\nu} = 0 \tag{1.5}$$

$$\Delta_{00} = 3(p - ka^{-2} - \dot{a}^2 a^{-2}), \qquad \Delta_{ij} = \tilde{g}_{ij}(a^2 p + k + \dot{a}^2 + 2a\ddot{a}) \tag{1.6}$$

$$\rightarrow \qquad \boxed{p = ka^{-2} + \dot{a}^2 a^{-2}} \qquad \boxed{0 = k + \dot{a}^2 + a\ddot{a}} \tag{1.7}$$

With $k = -1/L^2$, we will follow APM (B1) and take

$$a^2(t) = \frac{d^2}{L^2}\left(1 + \frac{t^2}{d^2}\right) \tag{1.8}$$

$$p = -\frac{1}{d^2(1+t^2/d^2)^2} = -\frac{d^2}{L^4 a^4} \tag{1.9}$$

1.2 Conformal T, R Coordinates

Given $a(t)$ in the form (1.8), we will transform the metric from

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2), \tag{1.10}$$

to the conformal flat form

$$ds^2 = \Omega^2(T,R)(-dT^2 + dR^2 + R^2 d\theta^2 + R^2\sin^2\theta d\phi^2). \tag{1.11}$$

First define some auxiliary variables for reference with APM (in this context p is a time coordinate, not pressure),

$$p = \frac{\tau}{L} = \int_0^t \frac{dt}{a(t)} = \sinh^{-1}\frac{t}{d}, \qquad \sin\chi = \frac{r}{L} \tag{1.12}$$

$$u \equiv \frac{t}{d}, \qquad v \equiv \frac{r}{L} \tag{1.13}$$

The transformation equations are then given as

$$T = \frac{\sinh p}{\cosh p + \cosh \chi} = \frac{u}{(1+u^2)^{1/2} + (1+v^2)^{1/2}} \tag{1.14}$$

$$R = \frac{\sinh \chi}{\cosh p + \cosh \chi} = \frac{v}{(1+u^2)^{1/2} + (1+v^2)^{1/2}} \tag{1.15}$$

$$\begin{aligned}
u &= \left(\frac{4T^2}{[1-(T+R)^2][1-(T-R)^2]}\right)^{1/2} \\
v &= \left(\frac{4R^2}{[1-(T+R)^2][1-(T-R)^2]}\right)^{1/2} \\
L^2 a^2 &= d^2(1+u^2) = \frac{d^2(1+T^2-R^2)^2}{[1-(T+R)^2][1-(T-R)^2]} \qquad (1.16)
\end{aligned}$$

$$\begin{aligned}
\Omega^2 &= \frac{4L^2a^2}{[1-(T+R)^2][1-(T-R)^2]} \\
&= \frac{4d^2(1+T^2-R^2)^2}{[1-(T+R)^2]^2[1-(T-R)^2]^2} \qquad (1.17)
\end{aligned}$$

1.3 $T'_{\mu\nu}(T, R, \theta, \phi)$

$$T'_{\mu\nu} = p(4U'_\mu U'_\nu + g'_{\mu\nu}) \tag{1.18}$$

$$p = -\frac{1}{d^2}\left(\frac{[1-(T+R)^2][1-(T-R)^2]}{(1+T^2-R^2)^2}\right)^2 = -\frac{4}{\Omega^2(1+T^2-R^2)^2} \tag{1.19}$$

$$U'_\mu = \frac{\partial x^\alpha}{\partial x'^\mu}U_\alpha = -\frac{\partial t}{\partial x'^\mu} = -d\left(\frac{\partial u}{\partial T}, \frac{\partial u}{\partial R}, 0, 0\right) \tag{1.20}$$

$$U_T = \Omega\left(\frac{T^2+R^2-1}{[1-(T+R)^2]^{1/2}[1-(T-R)^2]^{1/2}}\right) \tag{1.21}$$

$$U_R = -\Omega\left(\frac{2TR}{[1-(T+R)^2]^{1/2}[1-(T-R)^2]^{1/2}}\right) \tag{1.22}$$

$$g'_{\mu\nu} = \Omega^2 \mathrm{diag}\left(-1, 1, R^2, R^2\sin^2\theta\right) \tag{1.23}$$

1.4 $T'_{\mu\nu}(T, x, y, z)$

$$T'_{\mu\nu} = p(4U'_\mu U'_\nu + g'_{\mu\nu}) \tag{1.24}$$

$$p = -\frac{4}{\Omega^2(1+T^2-R^2)^2} \tag{1.25}$$

$$U'_\mu = \frac{\partial x^\alpha}{\partial x'^\mu}U_\alpha = \left(U_T, \frac{\partial R}{\partial x}U_R, \frac{\partial R}{\partial y}U_R, \frac{\partial R}{\partial z}U_R\right) \tag{1.26}$$

$$U_0 = \Omega\left(\frac{T^2+R^2-1}{[1-(T+R)^2]^{1/2}[1-(T-R)^2]^{1/2}}\right) \tag{1.27}$$

$$U_1 = -\Omega\left(\frac{2Tx}{[1-(T+R)^2]^{1/2}[1-(T-R)^2]^{1/2}}\right) \tag{1.28}$$

$$U_2 = -\Omega\left(\frac{2Ty}{[1-(T+R)^2]^{1/2}[1-(T-R)^2]^{1/2}}\right) \tag{1.29}$$

$$U_3 = -\Omega\left(\frac{2Tz}{[1-(T+R)^2]^{1/2}[1-(T-R)^2]^{1/2}}\right) \tag{1.30}$$

$$g'_{\mu\nu} = \Omega^2 \mathrm{diag}\,(-1,1,1,1) \tag{1.31}$$

2 Fluctuations

$$ds^2 = \Omega^2(x)(-d\tau^2 + \tilde{g}_{ij}dx^i dx^j + f_{\mu\nu}dx^\mu dx^\nu) \tag{2.1}$$

$$\tilde{g}_{ij} = \mathrm{diag}(-1, R^2, R^2\sin^2\theta) \tag{2.2}$$

$$f_{00} = -2\phi, \qquad f_{0i} = \tilde{\nabla}_i B + B_i, \qquad f_{ij} = -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{2.3}$$

2.1 $\delta G_{\mu\nu}$

$$\begin{aligned}
\delta G_{00} = {} & 6\dot{\psi}\dot{\Omega}\Omega^{-1} + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} - 2\tilde{\nabla}_a\tilde{\nabla}^a\psi + 4\phi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega \\
& +4\psi\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega + 4\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^a B - 2\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B - 2\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi \\
& -2\phi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - 2\psi\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\
& +2\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega - 4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E \\
& +4B^a\Omega^{-1}\tilde{\nabla}_a\dot{\Omega} - 2B^a\dot{\Omega}\Omega^{-2}\tilde{\nabla}_a\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b E_a + 2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^b E^a \\
& -4\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b E^a - 4E^{ab}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega + 2E_{ab}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
\delta G_{0i} = {} & -\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_i B + 2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_i B - 2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_i B + \Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_i B - 2\tilde{\nabla}_i\dot{\psi} \\
& -2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\phi + 2\dot{\psi}\Omega^{-1}\tilde{\nabla}_i\Omega - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_i\tilde{\nabla}_a\dot{E} - B_i\dot{\Omega}^2\Omega^{-2} + 2B_i\ddot{\Omega}\Omega^{-1} \\
& +\tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i - 2B_i\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega + \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B_i - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{E}_i \\
& +B_i\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i B^a - \Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}_i\dot{E}^a - 2\dot{E}_{ia}\Omega^{-1}\tilde{\nabla}^a\Omega
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
\delta G_{ij} = {} & -2\ddot{\psi}\tilde{g}_{ij} + 2\dot{\Omega}^2\tilde{g}_{ij}\phi\Omega^{-2} + 2\dot{\Omega}^2\tilde{g}_{ij}\psi\Omega^{-2} - 2\dot{\phi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\dot{\psi}\dot{\Omega}\tilde{g}_{ij}\Omega^{-1} - 4\ddot{\Omega}\tilde{g}_{ij}\phi\Omega^{-1} \\
& -4\ddot{\Omega}\tilde{g}_{ij}\psi\Omega^{-1} - 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a B - \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E} \\
& -\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi - 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^a B + 2\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a B \\
& -2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B} - 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi + 2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a E \\
& -2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_a E\tilde{\nabla}^b\Omega + 4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^a E + 2\Omega^{-1}\tilde{\nabla}_i\Omega\tilde{\nabla}_j\psi \\
& +2\Omega^{-1}\tilde{\nabla}_i\psi\tilde{\nabla}_j\Omega + 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i B + \tilde{\nabla}_j\tilde{\nabla}_i\dot{B} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{E} - 2\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i\dot{E} \\
& -2\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i E + 4\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_j\tilde{\nabla}_i E - 4\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E + 2\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i E \\
& +\tilde{\nabla}_j\tilde{\nabla}_i\phi - \tilde{\nabla}_j\tilde{\nabla}_i\psi - 2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a E
\end{aligned}$$

$$
\begin{aligned}
&-4B^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\dot{\Omega}+2B^a\dot{\Omega}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega-2\dot{B}^a\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_a\Omega+2\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a\\
&-2\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a+4\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_iB_j+\tfrac{1}{2}\tilde{\nabla}_i\dot{B}_j-\tfrac{1}{2}\tilde{\nabla}_i\ddot{E}_j\\
&-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_i\dot{E}_j-\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_iE_j+2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_iE_j-2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_iE_j\\
&+\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_iE_j+\dot{\Omega}\Omega^{-1}\tilde{\nabla}_jB_i+\tfrac{1}{2}\tilde{\nabla}_j\dot{B}_i-\tfrac{1}{2}\tilde{\nabla}_j\ddot{E}_i-\dot{\Omega}\Omega^{-1}\tilde{\nabla}_j\dot{E}_i-\dot{\Omega}^2\Omega^{-2}\tilde{\nabla}_jE_i\\
&+2\ddot{\Omega}\Omega^{-1}\tilde{\nabla}_jE_i-2\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\tilde{\nabla}_jE_i+\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_jE_i-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_j\tilde{\nabla}_iE_a\\
&-\ddot{E}_{ij}-2\dot{\Omega}^2E_{ij}\Omega^{-2}-2\dot{E}_{ij}\dot{\Omega}\Omega^{-1}+4\ddot{\Omega}E_{ij}\Omega^{-1}+\tilde{\nabla}_a\tilde{\nabla}^aE_{ij}-4E_{ij}\Omega^{-1}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\\
&+2\Omega^{-1}\tilde{\nabla}_aE_{ij}\tilde{\nabla}^a\Omega+2E_{ij}\Omega^{-2}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega+4E^{ab}\tilde{g}_{ij}\Omega^{-1}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\\
&-2E_{ab}\tilde{g}_{ij}\Omega^{-2}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_iE_{ja}-2\Omega^{-1}\tilde{\nabla}^a\Omega\tilde{\nabla}_jE_{ia}
\end{aligned}
\tag{2.6}
$$

$$
\begin{aligned}
g^{\mu\nu}\delta G_{\mu\nu} \;&=\; \Omega^{-2}(-\delta G_{00}+\tilde{g}^{ab}\delta G_{ab})\\
&=\; 6\dot{\Omega}^2\phi\Omega^{-4}+6\dot{\Omega}^2\psi\Omega^{-4}-6\dot{\phi}\dot{\Omega}\Omega^{-3}-18\dot{\psi}\dot{\Omega}\Omega^{-3}-12\ddot{\Omega}\phi\Omega^{-3}-12\ddot{\Omega}\psi\Omega^{-3}-6\ddot{\psi}\Omega^{-2}\\
&\quad -6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aB-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}+2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}+6\dot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}\\
&\quad -2\dot{\Omega}^2\Omega^{-4}\tilde{\nabla}_a\tilde{\nabla}^aE+4\ddot{\Omega}\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE-2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi+4\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi-4\phi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega\\
&\quad -4\psi\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^a\Omega-16\Omega^{-3}\tilde{\nabla}_a\dot{\Omega}\tilde{\nabla}^aB+8\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^aB-6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\dot{B}\\
&\quad -6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\phi+6\Omega^{-3}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\psi+2\phi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega+2\psi\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\\
&\quad +2\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE-4\Omega^{-3}\tilde{\nabla}_a\tilde{\nabla}^aE\tilde{\nabla}_b\tilde{\nabla}^b\Omega+6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_aE\\
&\quad -8\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}_aE\tilde{\nabla}^b\Omega+16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^b\tilde{\nabla}^aE.-16B^a\Omega^{-3}\tilde{\nabla}_a\dot{\Omega}\\
&\quad +8B^a\dot{\Omega}\Omega^{-4}\tilde{\nabla}_a\Omega-6\dot{B}^a\Omega^{-3}\tilde{\nabla}_a\Omega+6\Omega^{-3}\tilde{\nabla}^a\Omega\tilde{\nabla}_b\tilde{\nabla}^bE_a-8\Omega^{-4}\tilde{\nabla}_a\Omega\tilde{\nabla}_b\Omega\tilde{\nabla}^bE^a\\
&\quad +16\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega\tilde{\nabla}^bE^a+16E^{ab}\Omega^{-3}\tilde{\nabla}_b\tilde{\nabla}_a\Omega-8E_{ab}\Omega^{-4}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega
\end{aligned}
\tag{2.7}
$$

2.2 $\delta T_{\mu\nu}$

From $\delta(g^{\mu\nu}U_\mu U_\nu)=0$ we find

$$
U^\mu\delta U_\mu \;=\; \frac{1}{2}\left(h_{\mu\nu}U^\mu U^\nu\right)
\tag{2.8}
$$

Within U_μ let us define the denominator

$$
W\equiv[1-(T+R)^2]^{1/2}[1-(T-R)^2]^{1/2}.
\tag{2.9}
$$

Then from (2.8) we have (summation implied)

$$
-U_0\delta U_0+U_i\delta U_i \;=\; \frac{1}{2}\left(f_{00}U_0U_0-2f_{0i}U_0U_i+f_{ij}U_iU_j\right).
\tag{2.10}
$$

$$
\begin{aligned}
\frac{1}{2}\left(f_{00}U_0U_0-2f_{0i}U_0U_i+f_{ij}U_iU_j\right) \;=\; \frac{\Omega^2}{W^2}\big[&-2\phi(T^2+R^2-1)^2+4(\tilde{\nabla}_iB+B_i)(T^2+R^2-1)(Tx^i)\\
&+(-2\psi g_{ij}+2\tilde{\nabla}_i\tilde{\nabla}_jE+\tilde{\nabla}_iE_j+\tilde{\nabla}_jE_i+2E_{ij})4T^2x^ix^j\big]
\end{aligned}
\tag{2.11}
$$