Dissertation Talking Points

1 Title Page

- Thank you everyone for coming. Welcome to my dissertation defense on the topic Cosmological Perturbations as applied to both standard Einstein Gravity and Conformal gravity

2 Overview

- Alright so, to give an overview of what will be covered,
- we will first form the necessary background of cosmological perturbation theory. Talking about the geometry of the universe, introduce einstein gravity, perturbations, and coordinate invariance
- As an approach to simplify and solve the equations arising in cosmology, we're going to analyze something called the SVT decomp. in 3 dimensions and then perform some generalizations to four dimensions, demonstrating specific applications of both within a de Sitter background geometry
- Then we'll cover a discussion of cosmological perturbations in conformal gravity, and demonstrate a calculation of the fluctuations in the early universe radiation era
- Finally we'll wrap it up with overall conclusions and I'll demonstrate some of the computations involved in order to do the calculations that we'll see through this presentation

3 Cosmological Geometries

- Let's first discuss the geometry that is relevant to cosmology. If we take a look at the distribution of matter in the universe, we will see something like this image taken from the hubble telescope (which I'll add in served as my desktop background for quite a number of years).
- What we observe is that no matter where you are in the universe and no matter what direction you look, the universe is the same on large scales. More specifically, while the structure of universe may be in-homogeneous on small scales, on a large scale the universe is statistically homogeneous and isotropic. These two features are embodied in what is called the cosmological principle.
- Based solely on arguments of homogeneity and isotropy, the large scale geometry of such a universe is described through the RW metric and de Sitter metric (of which one can show that de Sitter space is actually a subset of the RW geometry).
- It of interest to note that with a proper choice of coordinates, the roberston walker geometries can all be expressed in a conformal to flat form
- with this equation here showing the spacetime line element being expressed as a minkowski spacetime multiplied by an overall conformal factor

4 Cosmological Geometries - Robertson Walker

- To see what the RW geometry entails, we have an expression for the spacetime line element in eq. (1), here in comoving coordinates. The geometry describes the expansion of space over time as characterized by the functional form of the scale factor $a(t)$
- The comoving coordinates are at rest with respect to the hubble flow, and if we look at the figure here we can see that while the comoving distance between these two galaxies remains constant, the proper distance increases as space expands according to $a(t)$
- The space itself that is expanding, referred to as the 3-space, is a space of uniform curvature, which can be represented by the curvature constant k, taking the values of $-1, 0, 1$. These values correspond to hyperbolic, flat, or spherical space respectively.
- Spaces of constant curvature are called maximally symmetric, where the curvature tensors take the specific form in eq(2). So here we have the Riemann tensor, which we might recall is the unique tensor composed of second order derivatives of the metric which measures the local curvature. We also have its contractions the ricci tensor, and the ricci scalar and we see that the ricci scalar is a constant in a max. symm. 3-space.
- As mentioned before, with a proper choice of coordinates, the RW metric can be cast into a conformal to flat form
- The simplest case is for $k = 0$, in which if we define the conformal time τ and set $k = 0$, then the line element take the form of eq (4), which we can recognize as conformal to a spherical polar flat metric
- For $k = \pm 1$, we have to perform additional coord. transformations, and we just note that for these the conformal factor is a function of both space and time.
- So this describes the geometry of the large scale universe, but in order to discuss the interaction of gravity and matter, one needs to introduce Einstein field equations, which we do now

5 Einstein Gravity

- One starts with the Einstein Hilbert action defined as the coordinate invariant integral over the Ricci scalar
- Functional variation w.r.t. the metric yields the Einstein tensor $G_{\mu\nu}$, and likewise upon specification of a matter action, one obtains the energy momentum tensor
- In requiring the sum of both the E.H. action and the matter action to be stationary with respect to arbitrary variations in the metric, we obtain the EFE's.
- With (10) showing an identity that relates the derivative of the ricci tensor to its contraction, we can see that the Einstein tensor is conserved
- In the EFE's the interaction of gravity and matter can be seen via the LHS being a pure function of the metric representing the curvature of space while the RHS defines the source of matter and energy
- We'll now look at the linearization of the Einstein field equations according to cosmological perturbation theory

6 Cosmological Perturbation Theory

- So as discussed prior, on a large scale the universe is homogeneous and isotropic which we can think of as the smooth surface of this sphere.
- Now in order to capture the departures from homogeneity and isotropy, things that are necessary in order to form localized structures in spacetime, we introduce small fluctuations on top of the otherwise smooth background. Thus we define the metric according to a background contribution and first order perturbation $h_{\mu\nu}$

- If we then substitute the metric into $G_{\mu\nu}$, it then can be split into a background piece and fluctuation tensor
- Upon similarly perturbing $T_{\mu\nu}$, we can then form the entire background field equations and first order fluctuation equations, where here we've combined them into the tensor $\Delta_{\mu\nu}$
- The background equations serve to define the rate of expansion of space given the source, whereas the fluctuation equations describe the evolution of metric perturbations due to things like over densities arising from source

7 Coordinate Transformations

- So upon perturbing the EFE's, we will need to consider the effect of coordinate transformations
- The field equations are covariant w.r.t. general coordinate transformations, with the metric transforming as in (18)
- If we now consider an infinitesimal coordinate transformation with the vector field ϵ small in the same sense that h is small, then it is conveinent to attribute the whole change in $g_{\mu\nu}$ to a change in the perturbation $h_{\mu\nu}$
- The fluctuation eqns are then to be invariant under the so called gauge transformation of eq (20), where $\Delta h_{\mu\nu}$ is given by this symmetric sum of derivatives onto epsilon
- Thus if $h_{\mu\nu}$ serves as a solution to the EFE's, then and $h'_{\mu\nu}$ defined by (20) will also serve as a solution
- Now since $h_{\mu\nu}$ is a 4x4 symmetric rank 2 tensor it has 10 components, and with the four coordinate functions that define the vector field ϵ, one can then use the coordinate freedom to reduce $h_{\mu\nu}$ to six independent components
- Its also quite instructive to look at the transformation of the fluctuation tensors themselves
- Here we note that if the background tensor vanishes, then the fluctuation tensors themselves are separately gauge invariant
- However, if the background does not vanish, then it only the entire sum of $\delta G_{\mu\nu} + \delta T_{\mu\nu}$ that is gauge invariant. In Einstein gravity the background only vanishes in spaces where the Ricci tensor itself vanishes, and thus for non-flat cosmological geometries, its only $\Delta_{\mu\nu}$ that is gauge invariant

8 Solution Methods

- Now lets take a look at what the perturbed Einstein tensor looks like in a non-flat geometry
- Here we have an expression for the spatial components of the Einstein tensor, and we can get a sense for 1) how many terms there are and 2) how tightly these terms are coupled together
- In fact, if one were to expand out the contractions over dummy indices, and try to look at a single spatial component, like the radia component δG_{rr}, one would get an expression with about 3-5 times as many terms
- So even after linearizing we require additional methods to simplify and decouple the fluctuation equations

9 SVT3 Decomposition

- One such method, called the SVT3 decomposition, is to take $h_{\mu\nu}$ and decompose it into a basis of scalars, vectors, and tensors defined according to their transformation behavior under 3D rotations
- To show this we are going to keep things generic and first factor out a conformal factor from $h_{\mu\nu}$ and express it in terms of the perturbation $f_{\mu\nu}$
- Then we form the line element, here's our background and perturbation, and then here we do a 3+1 splitting to separate the time and spatial components

- So now we want to define the time and space components of $f_{\mu\nu}$ in terms of scalars, vectors, and tensors
- Hence we view f_{00} as a 3-scalar, f_{0i} as a 3-vector and f_{ij} and a 3-tensor. f_{00} just being a scalar, we redefine it in terms of a ϕ, here we break up the 3 vector in to a transverse vector B_i and the derivatives of a scalar B, and here we have two scalars ψ and E, a transverse vector E_i and a TT tensor E_{ij}.
- If we count up the components, we have 4 scalars, two two component transverse vectors E_i and B_i, and one 2 component TT tensor E_{ij}, adding up to 10 in total
- finally here the total line element in the SVT3 basis

10 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 1/4

- To see how the SVT3 decomposition may be helpful in solving the perturbation equations, we are going to evaluate the EFE's in the de Sitter background
- As mentioned earlier, the de Sitter background can be expressed as a special case of the RW metric which here corresponds to choosing the scalar factor to have the form of $1/H\tau$
- While the RW metric consists of a 3-space that is max. symm., the de Sitter space is actually maximally symmetric w.r.t. the full 4D spacetime and so its curvature tensors take this form
- The E.M. tensor that gives rise to the desitter space is that which consists of just a cosmological constant - a simple constant background energy that drives the expansion of space
- de Sitter is chosen here just to keep things relatively simple, but the same SVT3 decomposition can be carried out in a more generic RW space where one considers an energy momentum tensor consisting of a perfect fluid
- So in perturbing the energy momentum tensor, we simply get a fluctuation proportional to $f_{\mu\nu}$

11 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 2/4

- Now we are going to insert the SVT3 decomposed $f_{\mu\nu}$ into $\delta G_{\mu\nu}$ to obtain eq 30
- We see that in the SVT3 basis the number of terms we have to deal with has been reduced a little bit, but still quite a few remain

12 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 3/4

- To form the fluctuation equations, we need to add in $\delta T_{\mu\nu}$ to form the full $\Delta_{\mu\nu}$ here
- A couple things to note: 1) if we look at the various components, we can see for example that scalars are coupled to vectors here (in Δ_{00}) and scalars, vectors, and tensors are still coupled together here (Δ_{ij}) 2) We recall that $\Delta_{\mu\nu}$ must be entirely gauge invariant, and thus the specific combinations of the SVT3 quantities that appear in each component of $\Delta_{\mu\nu}$ must themselves be gauge invariant
- Here we've identified the appropriate gauge invariant combinations, and if one counts the DOF, we see that there are two scalars, one 2-component transverse vector, and one 2 component TT tensor, totaling a set of 6 gauge invariants as expected from the gauge freedom from ϵ that we mentioned earlier
- Now in order to actually solve these, we will need to decouple them, and we find that by applying appropriate higher derivatives, we can do so

13 SVT3 $\delta G_{\mu\nu}$ in a de Sitter Background 4/4

- Here we have a set of 6 decoupled equations in the 6 gauge invariants, in which we can solve
- So to give quick recap here, we first perturbed the Einstein and E.M. around a de Sitter background
- Then we decomposed $h_{\mu\nu}$ into a basis of 3-scalars, 3-vectors, and 3-tensors
- Inserted that $h_{\mu\nu}$ in the flucuation tensors
- We formed the perturbed field equations, noting that they are entirely gauge invariant
- We express the FE's in terms of the gauge invariant SVT3 combinations
- Finally applied derivatives to decouple the SVT3 modes

14 SVT3 Integral Formulation 1/4

- Prior, I simply stated that this is the decomposition of $h_{\mu\nu}$ in the SVT3 basis. However, one might ask, is such a decomposition always possible? Or even further one might ask how does a quantity like ϕ transform under gauge transformations?
- To properly address the SVT3 decomposition, we need to effectively invert eq 34 so as to express each SVT3 quantity directly in terms of $h_{\mu\nu}$ itself.
- To keep things clear and simple, we are going to analyze the following SVT3 decomposition around a flat minkowski background and we are going to start with the decomposition of the 3-vector h_{0i} into the transverse B_i and the scalar B

15 SVT3 Integral Formulation 2/4

- Wording: Harmonic function, generalized Laplacian. divergence of gradient.

16 SVT3 Integral Formulation 3/4

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17 SVT3 Integral Formulation 4/4

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18 SVT4 Setup

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19 SVTD Integral Formulation - Maximally Symmetric Space 1/2

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20 SVTD Integral Formulation - Maximally Symmetric Space 2/2

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21 SVT4 $\delta G_{\mu\nu}$ in a de Sitter Background 1/2

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22 SVT4 $\delta G_{\mu\nu}$ in a de Sitter Background 2/2

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23 Conformal Gravity Introduction

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24 Conformal Invariance in Conformal Gravity

- a

25 Trace Properties in Conformal Gravity

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26 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 1/3

- a

27 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 2/3

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28 SVT4 $\delta W_{\mu\nu}$ in Conformal to Flat Backgrounds 3/3

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29 Conformal Gravity Robertson-Walker Radiation Era

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30 Conclusions

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31 Computational Methods

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32 References

- a

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34 The End