Bach SVT3 Separation

1 Fluctuations

$$ds^2 = \Omega^2(x)[\tilde{g}_{\mu\nu} + f_{\mu\nu}]dx^\mu dx^\nu \tag{1.1}$$

$$\tilde{g}_{\mu\nu} = \mathrm{diag}\,(-1,1,1,1) \qquad \text{(or some coordinate transformation of this)} \tag{1.2}$$

$$f_{00} = -2\phi, \qquad f_{0i} = \tilde{\nabla}_i B + B_i, \qquad f_{ij} = -2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{1.3}$$

$$\delta W_{\mu\nu} = 0 \tag{1.4}$$

$$\Omega^2\delta W_{00} = -\tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - \tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi - \tfrac{2}{3}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi \tag{1.5}$$

$$\begin{aligned}\Omega^2\delta W_{0i} = {} & -\tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{B} + \tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dddot{E} - \tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{\phi} - \tfrac{2}{3}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{\psi} - \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{B}_i + \tfrac{1}{2}\tilde{\nabla}_a\tilde{\nabla}^a\dddot{E}_i \\ & + \tfrac{1}{2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a B_i - \tfrac{1}{2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i\end{aligned} \tag{1.6}$$

$$\begin{aligned}\Omega^2\delta W_{ij} = {} & \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dddot{B} - \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddddot{E} + \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{\phi} + \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{\psi} - \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} \\ & + \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi - \tfrac{1}{3}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{B} + \tilde{\nabla}_j\tilde{\nabla}_i\dddot{E} - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{\phi} \\ & - \tilde{\nabla}_j\tilde{\nabla}_i\ddot{\psi} + \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} - \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tfrac{1}{3}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\psi - \tfrac{1}{2}\tilde{\nabla}_i\dddot{B}_j \\ & + \tfrac{1}{2}\tilde{\nabla}_i\dddot{E}_j + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}_j - \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_j - \tfrac{1}{2}\tilde{\nabla}_j\ddot{B}_i + \tfrac{1}{2}\tilde{\nabla}_j\dddot{E}_i + \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}_i \\ & - \tfrac{1}{2}\tilde{\nabla}_j\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_i + \ddddot{E}_{ij} - 2\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_{ij} + \tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a E_{ij}\end{aligned} \tag{1.7}$$

$$g^{\mu\nu}\delta W_{\mu\nu} = 0 \tag{1.8}$$

1.1 Separation

Scalar equation:

$$\delta W_{00} = -\tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a(\phi + \psi + \dot{B} - \ddot{E}) \tag{1.9}$$

Vector Equation:

$$\tilde{\nabla}_a\tilde{\nabla}^a(\Omega^2\delta W_{0i}) - \tilde{\nabla}_i\tilde{\nabla}^a(\Omega^2\delta W_{0a}) = -\tfrac{1}{2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a(\ddot{B}_i - \dddot{E}_i) + \tfrac{1}{2}\tilde{\nabla}_c\tilde{\nabla}^c\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a(B_i - \dot{E}_i) \tag{1.10}$$

Tensor Equation:

$$\begin{aligned}& \tilde{\nabla}_a\tilde{\nabla}^a\left[\tilde{\nabla}_b\tilde{\nabla}^b\delta(\Omega^2 W_{ij}) - \tilde{\nabla}_i\tilde{\nabla}^l(\Omega^2\delta W_{jl}) - \tilde{\nabla}_j\tilde{\nabla}^l(\Omega^2\delta W_{il})\right] + \tfrac{1}{2}g_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\left[\tilde{\nabla}^k\tilde{\nabla}^l(\Omega^2\delta W_{kl}) - \tilde{\nabla}_b\tilde{\nabla}^b(\Omega^2\tilde{g}^{ab}\delta W_{ab})\right] \\ & + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\left[\tilde{\nabla}^k\tilde{\nabla}^l(\Omega^2\delta W_{kl}) + \tilde{\nabla}_a\tilde{\nabla}^a(\Omega^2\tilde{g}^{ab}\delta W_{ab})\right] \\ & = \tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b\left[\tilde{\nabla}_c\tilde{\nabla}^c - \partial_t^2\right]^2 E_{ij}\end{aligned} \tag{1.11}$$