Bach SVT3 RW $k \neq 0$

1 Fluctuations

$$ds^2 = \Omega^2(\tau)\,[\tilde{g}_{\mu\nu} + f_{\mu\nu}]\,dx^\mu dx^\nu, \tag{1.1}$$

$$\tilde{g}_{\mu\nu} = \mathrm{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right) \tag{1.2}$$

$$f_{00} = -2\phi, \qquad f_{0i} = \tilde{\nabla}_i B + B_i, \qquad f_{ij} = -2\psi\tilde{g}_{ij} + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \tag{1.3}$$

$$\alpha = \phi + \psi + \dot{B} - \ddot{E}, \qquad Q_i = B_i - \dot{E}_i \tag{1.4}$$

$$\delta W_{00} = -\tfrac{2}{3}\Omega^{-2}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b\alpha \tag{1.5}$$

$$\delta W_{0i} = -\tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_i(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\dot{\alpha} + \tfrac{1}{2}\Omega^{-2}\left[\tilde{\nabla}_a\tilde{\nabla}^a(\tilde{\nabla}_b\tilde{\nabla}^b - \partial_0^2)Q_i - 2k(2k + \partial_0^2)Q_i\right] \tag{1.6}$$

$$\begin{aligned}\delta W_{ij} = &-\tfrac{1}{3}\Omega^{-2}\left[\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a(\tilde{\nabla}_b\tilde{\nabla}^b + 2k - \partial_0^2)\alpha - \tilde{\nabla}_i\tilde{\nabla}_j(\tilde{\nabla}_a\tilde{\nabla}^a - 3\partial_0^2)\alpha\right]\\ &+\tfrac{1}{2}\Omega^{-2}\left[\tilde{\nabla}_i(\tilde{\nabla}_a\tilde{\nabla}^a - 2k - \partial_0^2)\partial_0 Q_j + \tilde{\nabla}_j(\tilde{\nabla}_a\tilde{\nabla}^a - 2k - \partial_0^2)\partial_0 Q_i\right]\\ &+\Omega^{-2}\left[(\tilde{\nabla}_a\tilde{\nabla}^a - \partial_0)^2 E_{ij} - 4k(\tilde{\nabla}_a\tilde{\nabla}^a - k - 2\partial_0^2)E_{ij}\right]\end{aligned} \tag{1.7}$$

$$g^{\mu\nu}\delta W_{\mu\nu} = 0 \tag{1.8}$$

2 Separation

Scalar:

$$\delta W_{00} = -\tfrac{2}{3}\Omega^{-2}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b\alpha \tag{2.1}$$

Vector:

$$(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\delta W_{0i} - \tilde{\nabla}_i\tilde{\nabla}^a\delta W_{0a} = \tfrac{1}{2}\Omega^{-2}(\tilde{\nabla}_a\tilde{\nabla}^a - 2k - \partial_0^2)(\tilde{\nabla}_b\tilde{\nabla}^b + 2k)(\tilde{\nabla}_c\tilde{\nabla}^c - 2k)Q_i \tag{2.2}$$

Tensor:

$$\begin{aligned}&(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)\delta W_{ij} + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\left[\tilde{\nabla}^a\tilde{\nabla}^b\delta W_{ab} + (\tilde{\nabla}_a\tilde{\nabla}^a + 4k)(\tilde{g}^{bc}\delta W_{bc})\right]\\ &+\tfrac{1}{2}\tilde{g}_{ij}\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 4k)\tilde{\nabla}^b\tilde{\nabla}^c\delta W_{bc} - (\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b - 2k\tilde{\nabla}_a\tilde{\nabla}^a + 4k^2)(\tilde{g}^{bc}\delta W_{bc})\right]\\ &-(\tilde{\nabla}_a\tilde{\nabla}^a - 3k)(\tilde{\nabla}_i\tilde{\nabla}^b\delta W_{jb} + \tilde{\nabla}_j\tilde{\nabla}^b\delta W_{ib})\\ &=\\ &\Omega^{-2}(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)\left[(\tilde{\nabla}_a\tilde{\nabla}^a - \partial_0)^2 E_{ij} - 4k(\tilde{\nabla}_a\tilde{\nabla}^a - k - 2\partial_0^2)E_{ij}\right]\end{aligned} \tag{2.3}$$

Appendix A Mathematica Output

Confirmed transverse.

$$\begin{aligned}\delta W_{00} &= -2k\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + 2k\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - 2k\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\phi - 2k\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\psi - \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} \\ &\quad + \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi - \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi \qquad \text{(A.1)}\end{aligned}$$

$$\begin{aligned}\delta W_{0i} &= -2k\Omega^{-2}\tilde{\nabla}_i\ddot{B} + 2k\Omega^{-2}\tilde{\nabla}_i\dddot{E} - 2k\Omega^{-2}\tilde{\nabla}_i\dot{\phi} - 2k\Omega^{-2}\tilde{\nabla}_i\dot{\psi} - \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{B} \\ &\quad + \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dddot{E} - \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{\phi} - \tfrac{2}{3}\Omega^{-2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{\psi} - 2k^2\Omega^{-2}B_i - k\Omega^{-2}\ddot{B}_i + k\Omega^{-2}\dddot{E}_i \\ &\quad + 2k^2\Omega^{-2}\dot{E}_i - \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{B}_i + \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\dddot{E}_i + \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a B_i \\ &\quad - \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{E}_i \qquad \text{(A.2)}\end{aligned}$$

$$\begin{aligned}\delta W_{ij} &= -\tfrac{2}{3}k\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{B} - \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\dddot{E} + \tfrac{2}{3}k\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} \\ &\quad + \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{\phi} + \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{\psi} - \tfrac{2}{3}k\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\phi - \tfrac{2}{3}k\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_a\tilde{\nabla}^a\psi \\ &\quad - \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} + \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} - \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\phi \\ &\quad - \tfrac{1}{3}\Omega^{-2}\tilde{g}_{ij}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a\psi - \Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\ddot{B} + \Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\dddot{E} - \Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\ddot{\phi} - \Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\ddot{\psi} \\ &\quad + \tfrac{1}{3}\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{B} - \tfrac{1}{3}\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E} + \tfrac{1}{3}\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\phi + \tfrac{1}{3}\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\psi \\ &\quad - k\Omega^{-2}\tilde{\nabla}_i\dot{B}_j - \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_i\ddot{B}_j + \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_i\dddot{E}_j + k\Omega^{-2}\tilde{\nabla}_i\ddot{E}_j + \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}_j \\ &\quad - \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_i\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_j - k\Omega^{-2}\tilde{\nabla}_j\dot{B}_i - \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_j\ddot{B}_i + \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_j\dddot{E}_i + k\Omega^{-2}\tilde{\nabla}_j\ddot{E}_i \\ &\quad + \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_a\tilde{\nabla}^a\dot{B}_i - \tfrac{1}{2}\Omega^{-2}\tilde{\nabla}_j\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_i + \Omega^{-2}\dddot{E}_{ij} + 8k\Omega^{-2}\ddot{E}_{ij} + 4k^2\Omega^{-2}E_{ij} \\ &\quad - 2\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a\ddot{E}_{ij} - 4k\Omega^{-2}\tilde{\nabla}_a\tilde{\nabla}^a E_{ij} + \Omega^{-2}\tilde{\nabla}_b\tilde{\nabla}^b\tilde{\nabla}_a\tilde{\nabla}^a E_{ij} \qquad \text{(A.3)}\end{aligned}$$

$$g^{\mu\nu}\delta W_{\mu\nu} = 0 \qquad \text{(A.4)}$$