$h_{\mu\nu}$ SVT3 Decomposition v3

1 Background and Fluctuations

$$ds^2 = \Omega^2(\tau)(\tilde{g}_{\mu\nu} + f_{\mu\nu})dx^\mu dx^\nu = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \tag{1.1}$$

$$\tilde{g}_{\mu\nu} = \mathrm{diag}\left(-1, \frac{1}{1-kr^2}, r^2, r^2\sin^2\theta\right), \qquad \tilde{\Gamma}^\lambda_{\alpha\beta} = \delta^\lambda_i \delta^j_\alpha \delta^k_\beta \tilde{\Gamma}^i_{jk} \tag{1.2}$$

$$x'^\mu = x^\mu - \epsilon^\mu(x) \implies h'_{\mu\nu} = h_{\mu\nu} + \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu \tag{1.3}$$

$$f_\mu = \Omega^2 \epsilon_\mu, \qquad f^\mu = \epsilon^\mu \tag{1.4}$$

$$\Delta_\epsilon\left[f_{\mu\nu}\right] = \tilde{\nabla}_\alpha f_\beta + \tilde{\nabla}_\beta f_\alpha + 2f^\gamma \tilde{g}_{\alpha\beta}\Omega^{-1}\tilde{\nabla}_\gamma \Omega \tag{1.5}$$

$$\Delta_\epsilon\left[\tilde{g}^{\mu\nu} f_{\mu\nu}\right] = 2\tilde{\nabla}_\alpha f^\alpha + 8 f^\alpha \Omega^{-1}\tilde{\nabla}_\alpha \Omega \tag{1.6}$$

$$\Delta_\epsilon\left[\tilde{f}_{00}\right] = 2\dot{f}_0 + 2f_0\Omega^{-1}\dot{\Omega} \tag{1.7}$$

$$\Delta_\epsilon\left[\tilde{f}_{0i}\right] = \dot{f}_i + \tilde{\nabla}_i f_0 \tag{1.8}$$

$$\Delta_\epsilon\left[\tilde{f}_{ij}\right] = \tilde{\nabla}_i f_j + \tilde{\nabla}_j f_i - 2\tilde{g}_{ij} f_0 \Omega^{-1}\dot{\Omega} \tag{1.9}$$

$$\Delta_\epsilon\left[\tilde{f}\right] = -2\dot{f}_0 + 2\tilde{\nabla}^k f_k - 8 f_0 \Omega^{-1}\dot{\Omega} \tag{1.10}$$

2 SVT3

2.1 $f_{\mu\nu}(SVT3)$

$$\begin{aligned}
f_{00} &= -2\phi \\
f_{0i} &= B_i + \tilde{\nabla}_i B \\
f_{ij} &= -2\tilde{g}_{ij}\psi + 2\tilde{\nabla}_i\tilde{\nabla}_j E + \tilde{\nabla}_i E_j + \tilde{\nabla}_j E_i + 2E_{ij} \\
\tilde{g}^{ij} f_{ij} &= -6\psi + 2\tilde{\nabla}^k\tilde{\nabla}_k E \\
\tilde{g}^{\mu\nu} f_{\mu\nu} &= 2\phi - 6\psi + 2\tilde{\nabla}^k\tilde{\nabla}_k E
\end{aligned} \tag{2.1}$$

2.2 $SVT3(f_{\mu\nu})$

$$\phi = -\tfrac{1}{2} f_{00} \tag{2.2}$$

$$\tilde{\nabla}_a\tilde{\nabla}^a B = \tilde{\nabla}^a f_{0a} \tag{2.3}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)B_i = (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)f_{0i} - \tilde{\nabla}_i\tilde{\nabla}^a f_{0a} \tag{2.4}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\psi = \frac{1}{4}\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} - (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{g}^{bc}f_{bc})\right] \tag{2.5}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b E = \frac{3}{4}\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} - \frac{1}{3}\tilde{\nabla}_a\tilde{\nabla}^a(\tilde{g}^{bc}f_{bc})\right] \tag{2.6}$$

$$(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)E_i = (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\tilde{\nabla}^b f_{ib} - \tilde{\nabla}_i\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} \tag{2.7}$$

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)(2E_{ij}) = {}& (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)f_{ij} + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\left[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} + (\tilde{\nabla}_a\tilde{\nabla}^a + 4k)(\tilde{g}^{bc}f_{bc})\right] \\ &+\tfrac{1}{2}\tilde{g}_{ij}\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 4k)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - (\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b - 2k\tilde{\nabla}_a\tilde{\nabla}^a + 4k^2)(\tilde{g}^{bc}f_{bc})\right] \\ &-(\tilde{\nabla}_a\tilde{\nabla}^a - 3k)(\tilde{\nabla}_i\tilde{\nabla}^b f_{jb} + \tilde{\nabla}_j\tilde{\nabla}^b f_{ib})\end{aligned} \tag{2.8}$$

2.3 Gauge Transformation

Under gauge transformation (1.3), the SVT3 quantities (2.2)-(2.8) transform as

$$\Delta_\epsilon[\phi] = \dot{\Omega}\Omega^{-1}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)f_0 \tag{2.9}$$

$$\Delta_\epsilon\left[\tilde{\nabla}_a\tilde{\nabla}^a B\right] = \tilde{\nabla}_a\dot{f}^a + \tilde{\nabla}_a\tilde{\nabla}^a f_0 \tag{2.10}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)B_i\right] = (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\dot{f}_i - \tilde{\nabla}_i\tilde{\nabla}_a\dot{f}^a \tag{2.11}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\psi\right] = -\dot{f}_0 - \dot{\Omega}f_0\Omega^{-1} \tag{2.12}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b E\right] = (\tilde{\nabla}_b\tilde{\nabla}^b + 3k)\tilde{\nabla}_a f^a \tag{2.13}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)E_i\right] = (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)f_i - \tilde{\nabla}_i(\tilde{\nabla}_b\tilde{\nabla}^b + 4k)\tilde{\nabla}_a f^a \tag{2.14}$$

$$\Delta_\epsilon\left[(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)(2E_{ij})\right] = 0 \tag{2.15}$$

2.4 Gauge Invariants

We mix time derivative notation a bit, using ∂_0 upon $f_{\mu\nu}$ and dot upon Ω and SVT3 quantities.

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\phi + \psi + \dot{B} - \ddot{E}] = {}& (\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}^b(\partial_0 f_{0b}) - \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k - \partial_0^2)\tilde{\nabla}_b\tilde{\nabla}^b(\tilde{g}^{cd}f_{cd}) \\ &+\tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a - 3\partial_0^2)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - \tfrac{1}{2}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b f_{00}\end{aligned} \tag{2.16}$$

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[\psi - \dot{\Omega}\Omega^{-1}(B - \dot{E})] = {}& -\dot{\Omega}\Omega^{-1}(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}^b f_{0b} + \tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 3\dot{\Omega}\Omega^{-1}\partial_0)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} \\ &-\tfrac{1}{4}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k + \dot{\Omega}\Omega^{-1}\partial_0)\tilde{\nabla}_b\tilde{\nabla}^b(\tilde{g}^{cd}f_{cd})\end{aligned} \tag{2.17}$$

$$\begin{aligned}(\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)[B_i - \dot{E}_i] = {}& (\tilde{\nabla}_a\tilde{\nabla}^a + 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 2k)f_{0i} - (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)\tilde{\nabla}^b(\partial_0 f_{ib}) \\ &-\tilde{\nabla}_i(\tilde{\nabla}_a\tilde{\nabla}^a + 4k)\tilde{\nabla}^b f_{0b} + \tilde{\nabla}_i\tilde{\nabla}^a\tilde{\nabla}^b(\partial_0 f_{ab})\end{aligned} \tag{2.18}$$

$$
\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)[2E_{ij}] \quad = \quad & (\tilde{\nabla}_a\tilde{\nabla}^a - 2k)(\tilde{\nabla}_b\tilde{\nabla}^b - 3k)f_{ij} + \tfrac{1}{2}\tilde{\nabla}_i\tilde{\nabla}_j\big[\tilde{\nabla}^a\tilde{\nabla}^b f_{ab} + (\tilde{\nabla}_a\tilde{\nabla}^a + 4k)(\tilde{g}^{bc}f_{bc})\big] \\
& + \tfrac{1}{2}\tilde{g}_{ij}\big[(\tilde{\nabla}_a\tilde{\nabla}^a - 4k)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} - (\tilde{\nabla}_a\tilde{\nabla}^a\tilde{\nabla}_b\tilde{\nabla}^b - 2k\tilde{\nabla}_a\tilde{\nabla}^a + 4k^2)(\tilde{g}^{bc}f_{bc})\big] \\
& - (\tilde{\nabla}_a\tilde{\nabla}^a - 3k)(\tilde{\nabla}_i\tilde{\nabla}^b f_{jb} + \tilde{\nabla}_j\tilde{\nabla}^b f_{ib})
\end{aligned} \qquad (2.19)
$$

Appendix A γ Alternative

$$
\begin{aligned}
(\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}_b\tilde{\nabla}^b[-\dot{\Omega}^{-1}\Omega\psi + B - \dot{E}] \quad &= \quad (\tilde{\nabla}_a\tilde{\nabla}^a + 3k)\tilde{\nabla}^b f_{0b} - \tfrac{1}{4}(\dot{\Omega}^{-1}\Omega\tilde{\nabla}_a\tilde{\nabla}^a + 3\partial_0)\tilde{\nabla}^b\tilde{\nabla}^c f_{bc} \\
&\quad + \tfrac{1}{4}\left[\dot{\Omega}^{-1}\Omega(\tilde{\nabla}_a\tilde{\nabla}^a + 2k) + \partial_0\right]\tilde{\nabla}_b\tilde{\nabla}^b(\tilde{g}^{cd} f_{cd}) \qquad \text{(A.1)}
\end{aligned}
$$

Appendix B SVTD in Max. Sym. Space

$$
\left(\nabla_\alpha\nabla^\alpha - \frac{R}{D-1}\right)\chi \quad = \quad \frac{1}{2(D-1)}\left[\nabla^\alpha\nabla^\beta h_{\alpha\beta} - \left(\nabla_\alpha\nabla^\alpha - \frac{R}{D}\right)h\right] \qquad \text{(B.1)}
$$

$$
\left(\nabla_\alpha\nabla^\alpha - \frac{R}{D-1}\right)\nabla_\beta\nabla^\beta F \quad = \quad \frac{D}{2(D-1)}\left(\nabla^\alpha\nabla^\beta h_{\alpha\beta} - \frac{1}{D}\nabla_\alpha\nabla^\alpha h\right) \qquad \text{(B.2)}
$$

$$
\left(\nabla_\alpha\nabla^\alpha - \frac{R}{D}\right)\left(\nabla_\beta\nabla^\beta + \frac{R}{D}\right)F_\mu \quad = \quad \left(\nabla_\alpha\nabla^\alpha + \frac{R}{D}\right)\nabla^\sigma h_{\sigma\mu} - \nabla_\mu\nabla^\alpha\nabla^\beta h_{\alpha\beta}, \qquad \text{(B.3)}
$$