

Special Gauge Trace Matthew v6

Special $K_{\mu\nu}$ Gauge for Trace

The perturbed Einstein tensor $\delta G_{\mu\nu}(h_{\mu\nu})$ evaluated in the metric decomposed to first order

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + g_{\mu\nu}^{(1)} = \Omega^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}) \quad (1)$$

is calculated as

$$\begin{aligned} \delta G_{\mu\nu} = & \eta^{\alpha\beta}\Omega^{-1}\partial_\alpha\Omega\partial_\beta h_{\mu\nu} - \eta^{\alpha\gamma}\eta^{\beta\zeta}\eta_{\mu\nu}h_{\gamma\zeta}\Omega^{-2}\partial_\alpha\Omega\partial_\beta\Omega + \eta^{\alpha\beta}h_{\mu\nu}\Omega^{-2}\partial_\alpha\Omega\partial_\beta\Omega \\ & + \frac{1}{2}\eta^{\alpha\beta}\partial_\beta\partial_\alpha h_{\mu\nu} - 2\eta^{\alpha\beta}h_{\mu\nu}\Omega^{-1}\partial_\beta\partial_\alpha\Omega - \frac{1}{2}\eta^{\alpha\beta}\partial_\beta\partial_\mu h_{\nu\alpha} - \frac{1}{2}\eta^{\alpha\beta}\partial_\beta\partial_\nu h_{\mu\alpha} \\ & + 2\eta^{\alpha\beta}\eta^{\gamma\zeta}\eta_{\mu\nu}\Omega^{-1}\partial_\alpha\Omega\partial_\zeta h_{\beta\gamma} + \frac{1}{2}\eta^{\alpha\beta}\eta^{\gamma\zeta}\eta_{\mu\nu}\partial_\zeta\partial_\beta h_{\alpha\gamma} + 2\eta^{\alpha\beta}\eta^{\gamma\zeta}\eta_{\mu\nu}h_{\alpha\gamma}\Omega^{-1}\partial_\zeta\partial_\beta\Omega \\ & - \eta^{\alpha\beta}\Omega^{-1}\partial_\alpha\Omega\partial_\mu h_{\nu\beta} - \eta^{\alpha\beta}\Omega^{-1}\partial_\alpha\Omega\partial_\nu h_{\mu\beta} - \eta^{\alpha\beta}\eta_{\mu\nu}\Omega^{-1}\partial_\alpha h\partial_\beta\Omega - \frac{1}{2}\eta^{\alpha\beta}\eta_{\mu\nu}\partial_\beta\partial_\alpha h \\ & + \frac{1}{2}\partial_\nu\partial_\mu h. \end{aligned} \quad (2)$$

Now we split $h_{\mu\nu}$ into its traceless and trace components, i.e.

$$h_{\mu\nu} = K_{\mu\nu} + \frac{1}{4}\eta_{\mu\nu}h \quad (3)$$

where $h = \eta^{\mu\nu}h_{\mu\nu}$. With this substitution, (2) takes the form

$$\begin{aligned} \delta G_{\mu\nu} = & -2\eta^{\alpha\beta}K_{\mu\beta}\Omega^{-1}\partial_\alpha\partial_\nu\Omega + \eta^{\alpha\beta}\Omega^{-1}\partial_\alpha\Omega\partial_\beta K_{\mu\nu} - \eta^{\alpha\gamma}\eta^{\beta\zeta}\eta_{\mu\nu}K_{\gamma\zeta}\Omega^{-2}\partial_\alpha\Omega\partial_\beta\Omega \\ & + \eta^{\alpha\beta}K_{\mu\nu}\Omega^{-2}\partial_\alpha\Omega\partial_\beta\Omega + \frac{1}{2}\eta^{\alpha\beta}\partial_\beta\partial_\alpha K_{\mu\nu} + 2\eta^{\alpha\gamma}\eta^{\beta\zeta}\eta_{\mu\nu}K_{\gamma\zeta}\Omega^{-1}\partial_\beta\partial_\alpha\Omega \\ & - 2\eta^{\alpha\beta}K_{\mu\nu}\Omega^{-1}\partial_\beta\partial_\alpha\Omega + 2\eta^{\alpha\beta}\eta^{\gamma\zeta}\eta_{\mu\nu}\Omega^{-1}\partial_\alpha\Omega\partial_\zeta K_{\beta\gamma} + \frac{1}{2}\eta^{\alpha\beta}\eta^{\gamma\zeta}\eta_{\mu\nu}\partial_\zeta\partial_\beta K_{\alpha\gamma} \\ & - \eta^{\alpha\beta}\Omega^{-1}\partial_\alpha\Omega\partial_\mu K_{\nu\beta} - \frac{1}{2}\eta^{\alpha\beta}\partial_\mu\partial_\beta K_{\nu\alpha} - \eta^{\alpha\beta}\Omega^{-1}\partial_\alpha\Omega\partial_\nu K_{\mu\beta} + 2\eta^{\alpha\beta}K_{\mu\beta}\Omega^{-1}\partial_\nu\partial_\alpha\Omega \\ & - \frac{1}{2}\eta^{\alpha\beta}\partial_\nu\partial_\beta K_{\mu\alpha} + \frac{3}{4}\eta^{\alpha\beta}\eta_{\mu\nu}\Omega^{-1}\partial_\alpha\Omega\partial_\beta h - \eta^{\alpha\beta}\eta_{\mu\nu}\Omega^{-1}\partial_\alpha h\partial_\beta\Omega - \frac{1}{4}\eta^{\alpha\beta}\eta_{\mu\nu}\partial_\beta\partial_\alpha h \\ & - \frac{1}{8}\partial_\mu\partial_\nu h - \frac{1}{4}\Omega^{-1}\partial_\mu\Omega\partial_\nu h - \frac{1}{4}\Omega^{-1}\partial_\mu h\partial_\nu\Omega + \frac{3}{8}\partial_\nu\partial_\mu h. \end{aligned} \quad (4)$$

Now we impose a generalized gauge of the form

$$\eta^{\alpha\beta}\partial_\alpha K_{\beta\nu} = \Omega^{-1}J\eta^{\alpha\beta}K_{\nu\alpha}\partial_\beta\Omega + P\partial_\nu h + R\Omega^{-1}h\partial_\nu\Omega. \quad (5)$$

For a strictly time dependent conformal factor $\Omega(\tau)$, we find the fluctuation trace takes the form

$$\begin{aligned} \Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = & (-4\Omega^{-4}\dot{\Omega}^2 + 5J\Omega^{-4}\dot{\Omega}^2 + J^2\Omega^{-4}\dot{\Omega}^2 + 8\Omega^{-3}\ddot{\Omega} + J\Omega^{-3}\ddot{\Omega})K_{00} + (-5R\Omega^{-4}\dot{\Omega}^2 \\ & - JR\Omega^{-4}\dot{\Omega}^2 - R\Omega^{-3}\ddot{\Omega} - \frac{3}{4}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + P\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{2}\Omega^{-3}\dot{\Omega}\partial_0 - 6P\Omega^{-3}\dot{\Omega}\partial_0 \\ & - JP\Omega^{-3}\dot{\Omega}\partial_0 - R\Omega^{-3}\dot{\Omega}\partial_0)h. \end{aligned} \quad (6)$$

In the deSitter background, we take $\Omega(\tau) = \frac{1}{H\tau}$, in which the trace reduces to

$$\begin{aligned} \Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = & (12H^2 + 7H^2J + H^2J^2)K_{00} + (-7H^2R - H^2JR - \frac{3}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & - \frac{3}{2}H^2\tau\partial_0 + 6H^2P\tau\partial_0 + H^2JP\tau\partial_0 + H^2R\tau\partial_0)h. \end{aligned} \quad (7)$$

Here in deSitter we may take $J = -3$ or $J = -4$ to allow the trace of the perturbation to be proportional to h . The rest of $\delta G_{\mu\nu}$ in deSitter is:

$$\begin{aligned}\delta G_{00} = & (-\frac{3}{2}J\tau^{-2} - \frac{1}{2}J^2\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \tau^{-1}\partial_0 - J\tau^{-1}\partial_0)K_{00} + (\frac{3}{2}R\tau^{-2} + \frac{1}{2}JR\tau^{-2} \\ & + \frac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{4}\tau^{-1}\partial_0 - 2P\tau^{-1}\partial_0 - \frac{1}{2}JP\tau^{-1}\partial_0 + \frac{1}{2}R\tau^{-1}\partial_0 + \frac{1}{4}\partial_0\partial_0 \\ & - P\partial_0\partial_0)h.\end{aligned}\quad (8)$$

$$\begin{aligned}\delta G_{01} = & (-\tau^{-1}\partial_1 - \frac{1}{2}J\tau^{-1}\partial_1)K_{00} + (3\tau^{-2} + \frac{1}{2}J\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{2}J\tau^{-1}\partial_0)K_{01} + (\frac{1}{4}\tau^{-1}\partial_1 \\ & + \frac{1}{2}R\tau^{-1}\partial_1 + \frac{1}{4}\partial_1\partial_0 - P\partial_1\partial_0)h.\end{aligned}\quad (9)$$

$$\begin{aligned}\delta G_{11} = & (3\tau^{-2} + \frac{5}{2}J\tau^{-2} + \frac{1}{2}J^2\tau^{-2})K_{00} + (-2\tau^{-1}\partial_1 - J\tau^{-1}\partial_1)K_{01} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & + \tau^{-1}\partial_0)K_{11} + (-\frac{5}{2}R\tau^{-2} - \frac{1}{2}JR\tau^{-2} - \frac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{4}\tau^{-1}\partial_0 \\ & + 2P\tau^{-1}\partial_0 + \frac{1}{2}JP\tau^{-1}\partial_0 + \frac{1}{2}R\tau^{-1}\partial_0 + \frac{1}{4}\partial_1\partial_1 - P\partial_1\partial_1)h.\end{aligned}\quad (10)$$

$$\begin{aligned}\delta G_{12} = & (-\tau^{-1}\partial_2 - \frac{1}{2}J\tau^{-1}\partial_2)K_{01} + (-\tau^{-1}\partial_1 - \frac{1}{2}J\tau^{-1}\partial_1)K_{02} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & + \tau^{-1}\partial_0)K_{12} + (\frac{1}{4}\partial_2\partial_1 - P\partial_2\partial_1)h.\end{aligned}\quad (11)$$

Since the trace simplifies completely particularly in deSitter, we will try to simplify the equations most in deSitter, and then find their form in the general $\Omega(\tau)$. Within deSitter, for $J = -3$ and arbitrary P, R , we have

$$\Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = (-4H^2R - \frac{3}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{3}{2}H^2\tau\partial_0 + 3H^2P\tau\partial_0 + H^2R\tau\partial_0)h. \quad (12)$$

$$\begin{aligned}\delta G_{00} = & (\frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 2\tau^{-1}\partial_0)K_{00} + (\frac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{4}\tau^{-1}\partial_0 - \frac{1}{2}P\tau^{-1}\partial_0 \\ & + \frac{1}{2}R\tau^{-1}\partial_0 + \frac{1}{4}\partial_0\partial_0 - P\partial_0\partial_0)h.\end{aligned}\quad (13)$$

$$\begin{aligned}\delta G_{01} = & \frac{1}{2}\tau^{-1}\partial_1K_{00} + (\frac{3}{2}\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{2}\tau^{-1}\partial_0)K_{01} + (\frac{1}{4}\tau^{-1}\partial_1 + \frac{1}{2}R\tau^{-1}\partial_1 + \frac{1}{4}\partial_1\partial_0 \\ & - P\partial_1\partial_0)h.\end{aligned}\quad (14)$$

$$\begin{aligned}\delta G_{11} = & \tau^{-1}\partial_1K_{01} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{11} + (-R\tau^{-2} - \frac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & - \frac{1}{4}\tau^{-1}\partial_0 + \frac{1}{2}P\tau^{-1}\partial_0 + \frac{1}{2}R\tau^{-1}\partial_0 + \frac{1}{4}\partial_1\partial_1 - P\partial_1\partial_1)h.\end{aligned}\quad (15)$$

$$\delta G_{12} = \frac{1}{2}\tau^{-1}\partial_2K_{01} + \frac{1}{2}\tau^{-1}\partial_1K_{02} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{12} + (\frac{1}{4}\partial_2\partial_1 - P\partial_2\partial_1)h. \quad (16)$$

Within deSitter, for $J = -4$ and arbitrary P, R , we have

$$\Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = (-3H^2R - \frac{3}{4}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu + H^2P\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{3}{2}H^2\tau\partial_0 + 2H^2P\tau\partial_0 + H^2R\tau\partial_0)h. \quad (17)$$

$$\begin{aligned}\delta G_{00} = & (-2\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 3\tau^{-1}\partial_0)K_{00} + (-\frac{1}{2}R\tau^{-2} + \frac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{4}\tau^{-1}\partial_0 \\ & + \frac{1}{2}R\tau^{-1}\partial_0 + \frac{1}{4}\partial_0\partial_0 - P\partial_0\partial_0)h.\end{aligned}\quad (18)$$

$$\begin{aligned}\delta G_{01} = & \tau^{-1}\partial_1K_{00} + (\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 2\tau^{-1}\partial_0)K_{01} + (\frac{1}{4}\tau^{-1}\partial_1 + \frac{1}{2}R\tau^{-1}\partial_1 + \frac{1}{4}\partial_1\partial_0 \\ & - P\partial_1\partial_0)h.\end{aligned}\quad (19)$$

$$\delta G_{11} = \tau^{-2}K_{00} + 2\tau^{-1}\partial_1 K_{01} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{11} + (-\frac{1}{2}R\tau^{-2} - \frac{1}{4}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{2}P\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{4}\tau^{-1}\partial_0 + \frac{1}{2}R\tau^{-1}\partial_0 + \frac{1}{4}\partial_1\partial_1 - P\partial_1\partial_1)h. \quad (20)$$

$$\delta G_{12} = \tau^{-1}\partial_2 K_{01} + \tau^{-1}\partial_1 K_{02} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{12} + (\frac{1}{4}\partial_2\partial_1 - P\partial_2\partial_1)h. \quad (21)$$

It appears that a simplifying choice for $J = -3$ would be $P = \frac{1}{4}$ and $R = -\frac{1}{2}$. With these coefficients $\delta G_{\mu\nu}$ becomes

$$\Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = (2H^2 - \frac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{5}{4}H^2\tau\partial_0)h. \quad (22)$$

$$\delta G_{00} = (\frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 2\tau^{-1}\partial_0)K_{00} + (\frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{8}\tau^{-1}\partial_0)h. \quad (23)$$

$$\delta G_{01} = \frac{1}{2}\tau^{-1}\partial_1 K_{00} + (\frac{3}{2}\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{2}\tau^{-1}\partial_0)K_{01}. \quad (24)$$

$$\delta G_{11} = \tau^{-1}\partial_1 K_{01} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{11} + (\frac{1}{2}\tau^{-2} - \frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{3}{8}\tau^{-1}\partial_0)h. \quad (25)$$

$$\delta G_{12} = \frac{1}{2}\tau^{-1}\partial_2 K_{01} + \frac{1}{2}\tau^{-1}\partial_1 K_{02} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{12}. \quad (26)$$

It appears that a simplifying choice for $J = -4$ would be again $P = \frac{1}{4}$ and $R = -\frac{1}{2}$. With these coefficients $\delta G_{\mu\nu}$ becomes

$$\Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = (\frac{3}{2}H^2 - \frac{1}{2}H^2\tau^2\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{3}{2}H^2\tau\partial_0)h. \quad (27)$$

$$\delta G_{00} = (-2\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 3\tau^{-1}\partial_0)K_{00} + (\frac{1}{4}\tau^{-2} + \frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{2}\tau^{-1}\partial_0)h. \quad (28)$$

$$\delta G_{01} = \tau^{-1}\partial_1 K_{00} + (\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + 2\tau^{-1}\partial_0)K_{01}. \quad (29)$$

$$\delta G_{11} = \tau^{-2}K_{00} + 2\tau^{-1}\partial_1 K_{01} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{11} + (\frac{1}{4}\tau^{-2} - \frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{2}\tau^{-1}\partial_0)h. \quad (30)$$

$$\delta G_{12} = \tau^{-1}\partial_2 K_{01} + \tau^{-1}\partial_1 K_{02} + (3\tau^{-2} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \tau^{-1}\partial_0)K_{12}. \quad (31)$$

Going back to arbitrary $\Omega(\tau)$, we calculate for $J = -3$, $P = \frac{1}{4}$, and $R = -\frac{1}{2}$:

$$\Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = (-10\Omega^{-4}\dot{\Omega}^2 + 5\Omega^{-3}\ddot{\Omega})K_{00} + (\Omega^{-4}\dot{\Omega}^2 + \frac{1}{2}\Omega^{-3}\ddot{\Omega} - \frac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{5}{4}\Omega^{-3}\dot{\Omega}\partial_0)h. \quad (32)$$

$$\delta G_{00} = (3\Omega^{-2}\dot{\Omega}^2 - \frac{3}{2}\Omega^{-1}\ddot{\Omega} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - 2\Omega^{-1}\dot{\Omega}\partial_0)K_{00} + (-\frac{1}{2}\Omega^{-2}\dot{\Omega}^2 + \frac{1}{4}\Omega^{-1}\ddot{\Omega} + \frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{3}{8}\Omega^{-1}\dot{\Omega}\partial_0)h. \quad (33)$$

$$\delta G_{01} = -\frac{1}{2}\Omega^{-1}\dot{\Omega}\partial_1 K_{00} + (\frac{1}{2}\Omega^{-2}\dot{\Omega}^2 + \frac{1}{2}\Omega^{-1}\ddot{\Omega} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \frac{3}{2}\Omega^{-1}\dot{\Omega}\partial_0)K_{01}. \quad (34)$$

$$\delta G_{11} = (-\Omega^{-2}\dot{\Omega}^2 + \frac{1}{2}\Omega^{-1}\ddot{\Omega})K_{00} - \Omega^{-1}\dot{\Omega}\partial_1 K_{01} + (-\Omega^{-2}\dot{\Omega}^2 + 2\Omega^{-1}\ddot{\Omega} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-1}\dot{\Omega}\partial_0)K_{11} + (\frac{1}{4}\Omega^{-1}\ddot{\Omega} - \frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{8}\Omega^{-1}\dot{\Omega}\partial_0)h. \quad (35)$$

$$\delta G_{12} = -\frac{1}{2}\Omega^{-1}\dot{\Omega}\partial_2 K_{01} - \frac{1}{2}\Omega^{-1}\dot{\Omega}\partial_1 K_{02} + (-\Omega^{-2}\dot{\Omega}^2 + 2\Omega^{-1}\ddot{\Omega} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-1}\dot{\Omega}\partial_0)K_{12}. \quad (36)$$

For arbitrary $\Omega(\tau)$, we calculate for $J = -4$, $P = \frac{1}{4}$, and $R = -\frac{1}{2}$:

$$\Omega^{-2}\eta^{\mu\nu}\delta G_{\mu\nu} = (-8\Omega^{-4}\dot{\Omega}^2 + 4\Omega^{-3}\ddot{\Omega})K_{00} + (\frac{1}{2}\Omega^{-4}\dot{\Omega}^2 + \frac{1}{2}\Omega^{-3}\ddot{\Omega} - \frac{1}{2}\Omega^{-2}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{3}{2}\Omega^{-3}\dot{\Omega}\partial_0)h. \quad (37)$$

$$\begin{aligned} \delta G_{00} = & (2\Omega^{-2}\dot{\Omega}^2 - 2\Omega^{-1}\ddot{\Omega} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - 3\Omega^{-1}\dot{\Omega}\partial_0)K_{00} + (-\frac{1}{4}\Omega^{-2}\dot{\Omega}^2 + \frac{1}{4}\Omega^{-1}\ddot{\Omega} + \frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu \\ & - \frac{1}{2}\Omega^{-1}\dot{\Omega}\partial_0)h. \end{aligned} \quad (38)$$

$$\delta G_{01} = -\Omega^{-1}\dot{\Omega}\partial_1K_{00} + (\Omega^{-2}\dot{\Omega}^2 + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - 2\Omega^{-1}\dot{\Omega}\partial_0)K_{01}. \quad (39)$$

$$\begin{aligned} \delta G_{11} = & \Omega^{-2}\dot{\Omega}^2K_{00} - 2\Omega^{-1}\dot{\Omega}\partial_1K_{01} + (-\Omega^{-2}\dot{\Omega}^2 + 2\Omega^{-1}\ddot{\Omega} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-1}\dot{\Omega}\partial_0)K_{11} \\ & + (-\frac{1}{4}\Omega^{-2}\dot{\Omega}^2 + \frac{1}{4}\Omega^{-1}\ddot{\Omega} - \frac{1}{8}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{2}\Omega^{-1}\dot{\Omega}\partial_0)h. \end{aligned} \quad (40)$$

$$\delta G_{12} = -\Omega^{-1}\dot{\Omega}\partial_2K_{01} - \Omega^{-1}\dot{\Omega}\partial_1K_{02} + (-\Omega^{-2}\dot{\Omega}^2 + 2\Omega^{-1}\ddot{\Omega} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - \Omega^{-1}\dot{\Omega}\partial_0)K_{12}. \quad (41)$$