Polar $\delta G_{\mu\nu}$ SVT

Metric

$h_{00} = -2\phi$

$h_{0i} = \nabla_i B + B_i$

$h_{ij} = -2\psi + 2\nabla_i\nabla_j E + \nabla_i E_j + \nabla_j E_i + 2E_{ij}$

Conditions

$$\nabla^i E_{i1} = \frac{2\,E_{11}}{r} + \frac{Cot[\theta]\,E_{12}}{r^2} - \frac{E_{22}}{r^3} - \frac{Csc[\theta]^2\,E_{33}}{r^3} + \partial_1 E_{11} + \frac{\partial_2 E_{12}}{r^2} + \frac{Csc[\theta]^2\,\partial_3 E_{13}}{r^2} = 0$$

$$\nabla^i E_{i2} = \frac{2\,E_{12}}{r} + \frac{Cot[\theta]\,E_{22}}{r^2} - \frac{Cot[\theta]\,Csc[\theta]^2\,E_{33}}{r^2} + \partial_1 E_{21} + \frac{\partial_2 E_{22}}{r^2} + \frac{Csc[\theta]^2\,\partial_3 E_{23}}{r^2} = 0$$

$$\nabla^i E_{i3} = \frac{2\,E_{13}}{r} + \frac{Cot[\theta]\,E_{23}}{r^2} + \partial_1 E_{31} + \frac{\partial_2 E_{23}}{r^2} + \frac{Csc[\theta]^2\,\partial_3 E_{33}}{r^2} = 0$$

$$\nabla^i E_i = \frac{2\,E_1}{r} + \frac{Cot[\theta]\,E_2}{r^2} + \partial_1 E_1 + \frac{\partial_2 E_2}{r^2} + \frac{Csc[\theta]^2\,\partial_3 E_3}{r^2} = 0$$

$$g_{polar}{}^{\mu\nu} E_{\mu\nu} = E_{11} + \frac{E_{22}}{r^2} + \frac{Csc[\theta]^2\,E_{33}}{r^2} = 0$$

Scalar Laplacian

$$\nabla^2 = \frac{2\,\partial_1}{r} + \partial_1\partial_1 + \frac{Cot[\theta]\,\partial_2}{r^2} + \frac{\partial_2\partial_2}{r^2} + \frac{Csc[\theta]^2\,\partial_3\partial_3}{r^2}$$

$\delta G_{\mu\nu}$ Ω=1

Computer output

00	$(-2\,\nabla^2\,\psi) + (0) + (0)$
11	$(-\nabla^2\,\phi + \nabla^2\,\psi - \nabla^2\,\partial_0 B + \nabla^2\,\partial_0\partial_0 E - 2\,\partial_0\partial_0\psi + \partial_1\partial_1\phi - \partial_1\partial_1\psi + \partial_1\partial_1\partial_0 B + \partial_1\partial_1\partial_0\partial_0 E) + (\partial_1\partial_0 B_1)$
	$+ \left(-\frac{4\,E_{11}}{r^2} + \nabla^2\,E_{11} - \frac{4\,Cot[\theta]\,E_{12}}{r^3} + \frac{2\,E_{22}}{r^4} + \frac{2\,Csc[\theta]^2\,E_{33}}{r^4} - \partial_0\partial_0 E_{11} - \frac{4\,\partial_2 E_{12}}{r^3} - \frac{4\,Csc[\theta]^2\,\partial_3 E_{13}}{r^3}\right)$
22	$(2\,\nabla^2\,E + 2\,Cot[\theta]^2\,\nabla^2\,E - 2\,Csc[\theta]^2\,\nabla^2\,E - r^2\,\nabla^2\,\phi + r^2\,\nabla^2\,\psi - r^2\,\nabla^2\,\partial_0 B +$
	$r^2\,\nabla^2\,\partial_0\partial_0 E - 2\,r^2\,\partial_0\partial_0\psi - \frac{4\,\partial_1 E}{r} - \frac{4\,Cot[\theta]^2\,\partial_1 E}{r} + \frac{4\,Csc[\theta]^2\,\partial_1 E}{r} + r\,\partial_1\phi - r\,\partial_1\psi + r\,\partial_1\partial_0 B +$
	$r\,\partial_1\partial_0\partial_0 E - 2\,\partial_1\partial_1 E - 2\,Cot[\theta]^2\,\partial_1\partial_1 E + 2\,Csc[\theta]^2\,\partial_1\partial_1 E + \frac{Cot[\theta]\,\partial_2 E}{r^2} + \frac{Cot[\theta]^3\,\partial_2 E}{r^2} -$
	$\frac{Cot[\theta]\,Csc[\theta]^2\,\partial_2 E}{r^2} - \frac{5\,\partial_2\partial_2 E}{r^2} - \frac{5\,Cot[\theta]^2\,\partial_2\partial_2 E}{r^2} + \frac{5\,Csc[\theta]^2\,\partial_2\partial_2 E}{r^2} + \partial_2\partial_2\phi - \partial_2\partial_2\psi + \partial_2\partial_2\partial_0 B + \partial_2\partial_2\partial_0\partial_0 E$
	$) + \left(\frac{2\,Cot[\theta]\,E_2}{r^2} + \frac{2\,Cot[\theta]^3\,E_2}{r^2} - \frac{2\,Cot[\theta]\,Csc[\theta]^2\,E_2}{r^2} + r\,\partial_0 B_1 - \frac{2\,\partial_2 E_2}{r^2} - \frac{2\,Cot[\theta]^2\,\partial_2 E_2}{r^2} +\right.$
	$\frac{2\,Csc[\theta]^2\,\partial_2 E_2}{r^2} + \partial_2\partial_0 B_2 + \frac{2\,Csc[\theta]^2\,\partial_3 E_3}{r^2} + \frac{2\,Cot[\theta]^2\,Csc[\theta]^2\,\partial_3 E_3}{r^2} - \frac{2\,Csc[\theta]^4\,\partial_3 E_3}{r^2}\bigg) + ($
	$2\,E_{11} - \frac{2\,Cot[\theta]^2\,E_{22}}{r^2} + \nabla^2\,E_{22} + \frac{2\,Cot[\theta]^2\,Csc[\theta]^2\,E_{33}}{r^2} - \partial_0\partial_0 E_{22} - \frac{4\,\partial_1 E_{22}}{r} + \frac{4\,\partial_2 E_{12}}{r} - \frac{4\,Cot[\theta]\,Csc[\theta]^2\,\partial_3 E_{23}}{r^2}\bigg)$

33	$(-r^2 \sin[\theta]^2 \nabla^2 \phi + r^2 \sin[\theta]^2 \nabla^2 \psi - r^2 \sin[\theta]^2 \nabla^2 \partial_0 B + r^2 \sin[\theta]^2 \nabla^2 \partial_0\partial_0 E - 2 r^2 \sin[\theta]^2 \partial_0\partial_0\psi + \frac{2\,\partial_1 E}{r} -$ $\frac{2\cos[\theta]^2\,\partial_1 E}{r} - \frac{2\sin[\theta]^2\,\partial_1 E}{r} + r \sin[\theta]^2\,\partial_1\phi - r \sin[\theta]^2\,\partial_1\psi + r \sin[\theta]^2\,\partial_1\partial_0 B + r \sin[\theta]^2\,\partial_1\partial_0\partial_0 E +$ $\frac{2\cot[\theta]\,\partial_2 E}{r^2} - \frac{2\cos[\theta]^2\cot[\theta]\,\partial_2 E}{r^2} - \frac{2\cos[\theta]\sin[\theta]\,\partial_2 E}{r^2} + \cos[\theta]\sin[\theta]\,\partial_2\phi - \cos[\theta]\sin[\theta]\,\partial_2\psi +$ $\cos[\theta]\sin[\theta]\,\partial_2\partial_0 B + \cos[\theta]\sin[\theta]\,\partial_2\partial_0\partial_0 E + \partial_3\partial_3\phi - \partial_3\partial_3\psi + \partial_3\partial_3\partial_0 B + \partial_3\partial_3\partial_0\partial_0 E$ $) \; + \; (\frac{2\,E_1}{r} - \frac{2\cos[\theta]^2\,E_1}{r} - \frac{2\sin[\theta]^2\,E_1}{r} + \frac{3\cot[\theta]\,E_2}{r^2} - \frac{3\cos[\theta]^2\cot[\theta]\,E_2}{r^2} - \frac{3\cos[\theta]\sin[\theta]\,E_2}{r^2} +$ $r \sin[\theta]^2\,\partial_0 B_1 + \cos[\theta]\sin[\theta]\,\partial_0 B_2 - \frac{2\,\partial_3 E_3}{r^2} - \frac{2\cot[\theta]^2\,\partial_3 E_3}{r^2} + \frac{2\csc[\theta]^2\,\partial_3 E_3}{r^2} + \partial_3\partial_0 B_3$ $) \; + \; (2\sin[\theta]^2\,E_{11} + \frac{4\cos[\theta]\sin[\theta]\,E_{12}}{r} + \frac{2\cos[\theta]^2\,E_{22}}{r^2} + \frac{2\csc[\theta]^2\,E_{33}}{r^2} +$ $\nabla^2\,E_{33} - \partial_0\partial_0 E_{33} - \frac{4\,\partial_1 E_{33}}{r} - \frac{4\cot[\theta]\,\partial_2 E_{33}}{r^2} + \frac{4\,\partial_3 E_{13}}{r} + \frac{4\cot[\theta]\,\partial_3 E_{23}}{r^2})$
01	$(-2\,\partial_1\partial_0\psi + \partial_1\partial_0\partial_0\partial_0 E) \; + \; (-\frac{B_1}{r^2} + \frac{\nabla^2\,B_1}{2} - \frac{\cot[\theta]\,B_2}{r^3} + \frac{\partial_0 E_1}{r^2} - \frac{1}{2}\,\nabla^2\,\partial_0 E_1 +$ $\frac{\cot[\theta]\,\partial_0 E_2}{r^3} + \frac{1}{2}\,\partial_0\partial_0\partial_0 E_1 - \frac{\partial_2 B_2}{r^3} + \frac{\partial_2\partial_0 E_2}{r^3} - \frac{\csc[\theta]^2\,\partial_3 B_3}{r^3} + \frac{\csc[\theta]^2\,\partial_3\partial_0 E_3}{r^3}) \; + \; (0)$
02	$(-2\,\partial_2\partial_0\psi + \partial_2\partial_0\partial_0\partial_0 E) \; + \; (-\frac{B_2}{2\,r^2} - \frac{\cot[\theta]^2\,B_2}{2\,r^2} + \frac{\nabla^2\,B_2}{2} + \frac{\csc[\theta]^2\,\partial_0 E_2}{2\,r^2} - \frac{1}{2}\,\nabla^2\,\partial_0 E_2 +$ $\frac{1}{2}\,\partial_0\partial_0\partial_0 E_2 - \frac{\partial_1 B_2}{r} + \frac{\partial_1\partial_0 E_2}{r} + \frac{\partial_2 B_1}{r} - \frac{\partial_2\partial_0 E_1}{r} - \frac{\cot[\theta]\csc[\theta]^2\,\partial_3 B_3}{r^2} + \frac{\cot[\theta]\csc[\theta]^2\,\partial_3\partial_0 E_3}{r^2}) \; + \; (0)$
03	$(-\frac{\partial_3 B}{2\,r^2} - \frac{\cot[\theta]^2\,\partial_3 B}{2\,r^2} + \frac{\csc[\theta]^2\,\partial_3 B}{2\,r^2} + \frac{\partial_3\partial_0 E}{r^2} + \frac{\cot[\theta]^2\,\partial_3\partial_0 E}{r^2} - \frac{\csc[\theta]^2\,\partial_3\partial_0 E}{r^2} - 2\,\partial_3\partial_0\psi + \partial_3\partial_0\partial_0\partial_0 E$ $) \; + \; (-\frac{B_3}{2\,r^2} - \frac{\cot[\theta]^2\,B_3}{2\,r^2} + \frac{\csc[\theta]^2\,B_3}{2\,r^2} + \frac{\nabla^2\,B_3}{2} - \frac{1}{2}\,\nabla^2\,\partial_0 E_3 + \frac{1}{2}\,\partial_0\partial_0\partial_0 E_3 - \frac{\partial_1 B_3}{r} +$ $\frac{\partial_1\partial_0 E_3}{r} - \frac{\cot[\theta]\,\partial_2 B_3}{r^2} + \frac{\cot[\theta]\,\partial_2\partial_0 E_3}{r^2} + \frac{\partial_3 B_1}{r} + \frac{\cot[\theta]\,\partial_3 B_2}{r^2} - \frac{\partial_3\partial_0 E_1}{r} - \frac{\cot[\theta]\,\partial_3\partial_0 E_2}{r^2}) \; + \; (0)$
12	$(\frac{2\,\partial_2 E}{r^3} + \frac{2\cot[\theta]^2\,\partial_2 E}{r^3} - \frac{2\csc[\theta]^2\,\partial_2 E}{r^3} - \frac{\partial_2\phi}{r} + \frac{\partial_2\psi}{r} - \frac{\partial_2\partial_0 B}{r} - \frac{\partial_2\partial_0\partial_0 E}{r} -$ $\frac{\partial_2\partial_1 E}{r^2} - \frac{\cot[\theta]^2\,\partial_2\partial_1 E}{r^2} + \frac{\csc[\theta]^2\,\partial_2\partial_1 E}{r^2} + \partial_2\partial_1\phi - \partial_2\partial_1\psi + \partial_2\partial_1\partial_0 B + \partial_2\partial_1\partial_0\partial_0 E) \; + \; ($ $\frac{3\,E_2}{r^3} + \frac{3\cot[\theta]^2\,E_2}{r^3} - \frac{3\csc[\theta]^2\,E_2}{r^3} - \frac{\partial_0 B_2}{r} - \frac{\partial_1 E_2}{2\,r^2} - \frac{\cot[\theta]^2\,\partial_1 E_2}{2\,r^2} + \frac{\csc[\theta]^2\,\partial_1 E_2}{2\,r^2} + \frac{1}{2}\,\partial_1\partial_0 B_2 - \frac{\partial_2 E_1}{2\,r^2} -$ $\frac{\cot[\theta]^2\,\partial_2 E_1}{2\,r^2} + \frac{\csc[\theta]^2\,\partial_2 E_1}{2\,r^2} + \frac{1}{2}\,\partial_2\partial_0 B_1) \; + \; (-\frac{5\,E_{12}}{r^2} - \frac{\cot[\theta]^2\,E_{12}}{r^2} + \nabla^2\,E_{12} - \frac{2\cot[\theta]\,E_{22}}{r^3} +$ $\frac{2\cot[\theta]\csc[\theta]^2\,E_{33}}{r^3} - \partial_0\partial_0 E_{12} - \frac{2\,\partial_1 E_{12}}{r} + \frac{2\,\partial_2 E_{11}}{r} - \frac{2\,\partial_2 E_{22}}{r^3} - \frac{2\cot[\theta]\csc[\theta]^2\,\partial_3 E_{13}}{r^2} - \frac{2\csc[\theta]^2\,\partial_3 E_{23}}{r^3})$
13	$(-\frac{\partial_3 E}{r^3} - \frac{\cot[\theta]^2\,\partial_3 E}{r^3} + \frac{\csc[\theta]^2\,\partial_3 E}{r^3} - \frac{\partial_3\phi}{r} + \frac{\partial_3\psi}{r} - \frac{\partial_3\partial_0 B}{r} - \frac{\partial_3\partial_0\partial_0 E}{r} + \partial_3\partial_1\phi - \partial_3\partial_1\psi + \partial_3\partial_1\partial_0 B + \partial_3\partial_1\partial_0\partial_0 E$ $) \; + \; (\frac{3\,E_3}{r^3} + \frac{3\cot[\theta]^2\,E_3}{r^3} - \frac{3\csc[\theta]^2\,E_3}{r^3} - \frac{\partial_0 B_3}{r} - \frac{\partial_1 E_3}{2\,r^2} - \frac{\cot[\theta]^2\,\partial_1 E_3}{2\,r^2} + \frac{\csc[\theta]^2\,\partial_1 E_3}{2\,r^2} + \frac{1}{2}\,\partial_1\partial_0 B_3 -$ $\frac{\partial_3 E_1}{2\,r^2} - \frac{\cot[\theta]^2\,\partial_3 E_1}{2\,r^2} + \frac{\csc[\theta]^2\,\partial_3 E_1}{2\,r^2} + \frac{1}{2}\,\partial_3\partial_0 B_1) \; + \; (-\frac{5\,E_{13}}{r^2} - \frac{\cot[\theta]^2\,E_{13}}{r^2} + \frac{\csc[\theta]^2\,E_{13}}{r^2} + \nabla^2\,E_{13} -$ $\frac{2\cot[\theta]\,E_{23}}{r^3} - \partial_0\partial_0 E_{13} - \frac{2\,\partial_1 E_{13}}{r} - \frac{2\cot[\theta]\,\partial_2 E_{13}}{r^2} - \frac{2\,\partial_2 E_{23}}{r^3} + \frac{2\,\partial_3 E_{11}}{r} + \frac{2\cot[\theta]\,\partial_3 E_{12}}{r^2} - \frac{2\csc[\theta]^2\,\partial_3 E_{33}}{r^3})$
23	$(\frac{2\cot[\theta]\,\partial_3 E}{r^2} + \frac{2\cot[\theta]^3\,\partial_3 E}{r^2} - \frac{2\cot[\theta]\csc[\theta]^2\,\partial_3 E}{r^2} - \cot[\theta]\,\partial_3\phi + \cot[\theta]\,\partial_3\psi - \cot[\theta]\,\partial_3\partial_0 B -$ $\cot[\theta]\,\partial_3\partial_0\partial_0 E - \frac{3\,\partial_3\partial_2 E}{r^2} - \frac{3\cot[\theta]^2\,\partial_3\partial_2 E}{r^2} + \frac{3\csc[\theta]^2\,\partial_3\partial_2 E}{r^2} + \partial_3\partial_2\phi - \partial_3\partial_2\psi + \partial_3\partial_2\partial_0 B + \partial_3\partial_2\partial_0\partial_0 E$ $) \; + \; (\frac{4\cot[\theta]\,E_3}{r^2} + \frac{4\cot[\theta]^3\,E_3}{r^2} - \frac{4\cot[\theta]\csc[\theta]^2\,E_3}{r^2} - \cot[\theta]\,\partial_0 B_3 - \frac{2\,\partial_2 E_3}{r^2} - \frac{2\cot[\theta]^2\,\partial_2 E_3}{r^2} +$ $\frac{2\csc[\theta]^2\,\partial_2 E_3}{r^2} + \frac{1}{2}\,\partial_2\partial_0 B_3 - \frac{2\,\partial_3 E_2}{r^2} - \frac{2\cot[\theta]^2\,\partial_3 E_2}{r^2} + \frac{2\csc[\theta]^2\,\partial_3 E_2}{r^2} + \frac{1}{2}\,\partial_3\partial_0 B_2$ $) \; + \; (-\frac{4\cot[\theta]\,E_{13}}{r} - \frac{4\cot[\theta]^2\,E_{23}}{r^2} + \frac{\csc[\theta]^2\,E_{23}}{r^2} + \nabla^2\,E_{23} - \partial_0\partial_0 E_{23} - \frac{4\,\partial_1 E_{23}}{r} +$ $\frac{2\,\partial_2 E_{13}}{r} - \frac{2\cot[\theta]\,\partial_2 E_{23}}{r^2} + \frac{2\,\partial_3 E_{12}}{r} + \frac{2\cot[\theta]\,\partial_3 E_{22}}{r^2} - \frac{2\cot[\theta]\csc[\theta]^2\,\partial_3 E_{33}}{r^2})$

$\delta G_{\mu\nu}\,\Omega{=}1$

After simplification

00	$(-2\,\nabla^2\,\psi)\ +\ (0)\ +\ (0)$
11	$(-\nabla^2\,\phi + \nabla^2\,\psi - \nabla^2\,\partial_0 B + \nabla^2\,\partial_0\partial_0 E - 2\,\partial_0\partial_0\psi + \partial_1\partial_1\phi - \partial_1\partial_1\psi + \partial_1\partial_1\partial_0 B + \partial_1\partial_1\partial_0\partial_0 E$ $)\ +\ (\partial_1\partial_0 B_1)\ +\ (\frac{6\,E_{11}}{r^2} + \nabla^2\,E_{11} - \partial_0\partial_0 E_{11} + \frac{4\,\partial_1 E_{11}}{r})$
22	$(2\,\nabla^2\,E + 2\,\mathrm{Cot}[\theta]^2\,\nabla^2\,E - 2\,\mathrm{Csc}[\theta]^2\,\nabla^2\,E - r^2\,\nabla^2\,\phi + r^2\,\nabla^2\,\psi - r^2\,\nabla^2\,\partial_0 B +$ $r^2\,\nabla^2\,\partial_0\partial_0 E - 2\,r^2\,\partial_0\partial_0\psi - \frac{4\,\partial_1 E}{r} - \frac{4\,\mathrm{Cot}[\theta]^2\,\partial_1 E}{r} + \frac{4\,\mathrm{Csc}[\theta]^2\,\partial_1 E}{r} + r\,\partial_1\phi - r\,\partial_1\psi + r\,\partial_1\partial_0 B +$ $r\,\partial_1\partial_0\partial_0 E - 2\,\partial_1\partial_1 E - 2\,\mathrm{Cot}[\theta]^2\,\partial_1\partial_1 E + 2\,\mathrm{Csc}[\theta]^2\,\partial_1\partial_1 E + \frac{\mathrm{Cot}[\theta]\,\partial_2 E}{r^2} + \frac{\mathrm{Cot}[\theta]^3\,\partial_2 E}{r^2} -$ $\frac{\mathrm{Cot}[\theta]\,\mathrm{Csc}[\theta]^2\,\partial_2 E}{r^2} - \frac{5\,\partial_2\partial_2 E}{r^2} - \frac{5\,\mathrm{Cot}[\theta]^2\,\partial_2\partial_2 E}{r^2} + \frac{5\,\mathrm{Csc}[\theta]^2\,\partial_2\partial_2 E}{r^2} + \partial_2\partial_2\phi - \partial_2\partial_2\psi + \partial_2\partial_2\partial_0 B + \partial_2\partial_2\partial_0\partial_0 E$ $)\ +\ (\frac{2\,\mathrm{Cot}[\theta]\,E_2}{r^2} + \frac{2\,\mathrm{Cot}[\theta]^3\,E_2}{r^2} - \frac{2\,\mathrm{Cot}[\theta]\,\mathrm{Csc}[\theta]^2\,E_2}{r^2} + r\,\partial_0 B_1 - \frac{2\,\partial_2 E_2}{r^2} - \frac{2\,\mathrm{Cot}[\theta]^2\,\partial_2 E_2}{r^2} +$ $\frac{2\,\mathrm{Csc}[\theta]^2\,\partial_2 E_2}{r^2} + \partial_2\partial_0 B_2 + \frac{2\,\mathrm{Csc}[\theta]^2\,\partial_3 E_3}{r^2} + \frac{2\,\mathrm{Cot}[\theta]^2\,\mathrm{Csc}[\theta]^2\,\partial_3 E_3}{r^2} - \frac{2\,\mathrm{Csc}[\theta]^4\,\partial_3 E_3}{r^2})\ +\ ($ $2\,E_{11} - 2\,\mathrm{Cot}[\theta]^2\,E_{11} - \frac{4\,\mathrm{Cot}[\theta]^2\,E_{22}}{r^2} + \nabla^2\,E_{22} - \partial_0\partial_0 E_{22} - \frac{4\,\partial_1 E_{22}}{r} + \frac{4\,\partial_2 E_{12}}{r} - \frac{4\,\mathrm{Cot}[\theta]\,\mathrm{Csc}[\theta]^2\,\partial_3 E_{23}}{r^2})$
33	$(-r^2\,\mathrm{Sin}[\theta]^2\,\nabla^2\,\phi + r^2\,\mathrm{Sin}[\theta]^2\,\nabla^2\,\psi - r^2\,\mathrm{Sin}[\theta]^2\,\nabla^2\,\partial_0 B + r^2\,\mathrm{Sin}[\theta]^2\,\nabla^2\,\partial_0\partial_0 E - 2\,r^2\,\mathrm{Sin}[\theta]^2\,\partial_0\partial_0\psi + \frac{2\,\partial_1 E}{r} -$ $\frac{2\,\mathrm{Cos}[\theta]^2\,\partial_1 E}{r} - \frac{2\,\mathrm{Sin}[\theta]^2\,\partial_1 E}{r} + r\,\mathrm{Sin}[\theta]^2\,\partial_1\phi - r\,\mathrm{Sin}[\theta]^2\,\partial_1\psi + r\,\mathrm{Sin}[\theta]^2\,\partial_1\partial_0 B + r\,\mathrm{Sin}[\theta]^2\,\partial_1\partial_0\partial_0 E +$ $\frac{2\,\mathrm{Cot}[\theta]\,\partial_2 E}{r^2} - \frac{2\,\mathrm{Cos}[\theta]^2\,\mathrm{Cot}[\theta]\,\partial_2 E}{r^2} - \frac{2\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,\partial_2 E}{r^2} + \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,\partial_2\phi - \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,\partial_2\psi +$ $\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,\partial_2\partial_0 B + \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,\partial_2\partial_0\partial_0 E + \partial_3\partial_3\phi - \partial_3\partial_3\psi + \partial_3\partial_3\partial_0 B + \partial_3\partial_3\partial_0\partial_0 E$ $)\ +\ (\frac{2\,E_1}{r} - \frac{2\,\mathrm{Cos}[\theta]^2\,E_1}{r} - \frac{2\,\mathrm{Sin}[\theta]^2\,E_1}{r} + \frac{3\,\mathrm{Cot}[\theta]\,E_2}{r^2} - \frac{3\,\mathrm{Cos}[\theta]^2\,\mathrm{Cot}[\theta]\,E_2}{r^2} - \frac{3\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,E_2}{r^2} +$ $r\,\mathrm{Sin}[\theta]^2\,\partial_0 B_1 + \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,\partial_0 B_2 - \frac{2\,\partial_3 E_3}{r^2} - \frac{2\,\mathrm{Cot}[\theta]^2\,\partial_3 E_3}{r^2} + \frac{2\,\mathrm{Csc}[\theta]^2\,\partial_3 E_3}{r^2} + \partial_3\partial_0 B_3$ $)\ +\ (2\,\mathrm{Sin}[\theta]^2\,E_{11} + \frac{4\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,E_{12}}{r} + \frac{2\,\mathrm{Cos}[\theta]^2\,E_{22}}{r^2} + \frac{2\,\mathrm{Csc}[\theta]^2\,E_{33}}{r^2} +$ $\nabla^2\,E_{33} - \partial_0\partial_0 E_{33} - \frac{4\,\partial_1 E_{33}}{r} - \frac{4\,\mathrm{Cot}[\theta]\,\partial_2 E_{33}}{r^2} + \frac{4\,\partial_3 E_{13}}{r} + \frac{4\,\mathrm{Cot}[\theta]\,\partial_3 E_{23}}{r^2})$
01	$(-2\,\partial_1\partial_0\psi + \partial_1\partial_0\partial_0\partial_0 E)\ +\ (\frac{B_1}{r^2} + \frac{\nabla^2\,B_1}{2} - \frac{\partial_0 E_1}{r^2} - \frac{1}{2}\,\nabla^2\,\partial_0 E_1 + \frac{1}{2}\,\partial_0\partial_0\partial_0 E_1 + \frac{\partial_1 B_1}{r} - \frac{\partial_1\partial_0 E_1}{r})\ +\ (0)$
02	$(-2\,\partial_2\partial_0\psi + \partial_2\partial_0\partial_0\partial_0 E)\ +\ ($ $\frac{2\,\mathrm{Cot}[\theta]\,B_1}{r} - \frac{B_2}{2\,r^2} + \frac{\mathrm{Cot}[\theta]^2\,B_2}{2\,r^2} + \frac{\nabla^2\,B_2}{2} - \frac{2\,\mathrm{Cot}[\theta]\,\partial_0 E_1}{r} - \frac{\mathrm{Cot}[\theta]^2\,\partial_0 E_2}{r^2} + \frac{\mathrm{Csc}[\theta]^2\,\partial_0 E_2}{2\,r^2} - \frac{1}{2}\,\nabla^2\,\partial_0 E_2 + \frac{1}{2}\,\partial_0\partial_0\partial_0 E_2 +$ $\mathrm{Cot}[\theta]\,\partial_1 B_1 - \frac{\partial_1 B_2}{r} - \mathrm{Cot}[\theta]\,\partial_1\partial_0 E_1 + \frac{\partial_1\partial_0 E_2}{r} + \frac{\partial_2 B_1}{r} + \frac{\mathrm{Cot}[\theta]\,\partial_2 B_2}{r^2} - \frac{\partial_2\partial_0 E_1}{r} - \frac{\mathrm{Cot}[\theta]\,\partial_2\partial_0 E_2}{r^2})\ +\ (0)$
03	$(-\frac{\partial_3 B}{2\,r^2} - \frac{\mathrm{Cot}[\theta]^2\,\partial_3 B}{2\,r^2} + \frac{\mathrm{Csc}[\theta]^2\,\partial_3 B}{2\,r^2} + \frac{\partial_3\partial_0 E}{r^2} + \frac{\mathrm{Cot}[\theta]^2\,\partial_3\partial_0 E}{r^2} - \frac{\mathrm{Csc}[\theta]^2\,\partial_3\partial_0 E}{r^2} - 2\,\partial_3\partial_0\psi + \partial_3\partial_0\partial_0\partial_0 E$ $)\ +\ (-\frac{B_3}{2\,r^2} - \frac{\mathrm{Cot}[\theta]^2\,B_3}{2\,r^2} + \frac{\mathrm{Csc}[\theta]^2\,B_3}{2\,r^2} + \frac{\nabla^2\,B_3}{2} - \frac{1}{2}\,\nabla^2\,\partial_0 E_3 + \frac{1}{2}\,\partial_0\partial_0\partial_0 E_3 - \frac{\partial_1 B_3}{r} +$ $\frac{\partial_1\partial_0 E_3}{r} - \frac{\mathrm{Cot}[\theta]\,\partial_2 B_3}{r^2} + \frac{\mathrm{Cot}[\theta]\,\partial_2\partial_0 E_3}{r^2} + \frac{\partial_3 B_1}{r} + \frac{\mathrm{Cot}[\theta]\,\partial_3 B_2}{r^2} - \frac{\partial_3\partial_0 E_1}{r} - \frac{\mathrm{Cot}[\theta]\,\partial_3\partial_0 E_2}{r^2})\ +\ (0)$
12	$(\frac{2\,\partial_2 E}{r^3} + \frac{2\,\mathrm{Cot}[\theta]^2\,\partial_2 E}{r^3} - \frac{2\,\mathrm{Csc}[\theta]^2\,\partial_2 E}{r^3} - \frac{\partial_2\phi}{r} + \frac{\partial_2\psi}{r} - \frac{\partial_2\partial_0 B}{r} - \frac{\partial_2\partial_0\partial_0 E}{r} - \frac{\partial_2\partial_1 E}{r^2} - \frac{\mathrm{Cot}[\theta]^2\,\partial_2\partial_1 E}{r^2} + \frac{\mathrm{Csc}[\theta]^2\,\partial_2\partial_1 E}{r^2} +$ $\partial_2\partial_1\phi - \partial_2\partial_1\psi + \partial_2\partial_1\partial_0 B + \partial_2\partial_1\partial_0\partial_0 E)\ +\ (\frac{3\,E_2}{r^3} + \frac{3\,\mathrm{Cot}[\theta]^2\,E_2}{r^3} - \frac{3\,\mathrm{Csc}[\theta]^2\,E_2}{r^3} - \frac{\partial_0 B_2}{r} - \frac{\partial_1 E_2}{2\,r^2} -$ $\frac{\mathrm{Cot}[\theta]^2\,\partial_1 E_2}{2\,r^2} + \frac{\mathrm{Csc}[\theta]^2\,\partial_1 E_2}{2\,r^2} + \frac{1}{2}\,\partial_1\partial_0 B_2 - \frac{\partial_2 E_1}{2\,r^2} - \frac{\mathrm{Cot}[\theta]^2\,\partial_2 E_1}{2\,r^2} + \frac{\mathrm{Csc}[\theta]^2\,\partial_2 E_1}{2\,r^2} + \frac{1}{2}\,\partial_2\partial_0 B_1)\ +\ ($ $\frac{6\,\mathrm{Cot}[\theta]\,E_{11}}{r} - \frac{E_{12}}{r^2} + \frac{\mathrm{Cot}[\theta]^2\,E_{12}}{r^2} + \nabla^2\,E_{12} - \partial_0\partial_0 E_{12} + 2\,\mathrm{Cot}[\theta]\,\partial_1 E_{11} + \frac{2\,\partial_2 E_{11}}{r} + \frac{2\,\mathrm{Cot}[\theta]\,\partial_2 E_{12}}{r^2})$

13	$\left(-\dfrac{\partial_3 E}{r^3} - \dfrac{\text{Cot}[\theta]^2 \partial_3 E}{r^3} + \dfrac{\text{Csc}[\theta]^2 \partial_3 E}{r^3} - \dfrac{\partial_3 \phi}{r} + \dfrac{\partial_3 \psi}{r} - \dfrac{\partial_3 \partial_0 B}{r} - \dfrac{\partial_3 \partial_0 \partial_0 E}{r} + \partial_3 \partial_1 \phi - \partial_3 \partial_1 \psi + \partial_3 \partial_1 \partial_0 B + \partial_3 \partial_1 \partial_0 \partial_0 E\right)$ $+ \left(\dfrac{3 E_3}{r^3} + \dfrac{3 \text{Cot}[\theta]^2 E_3}{r^3} - \dfrac{3 \text{Csc}[\theta]^2 E_3}{r^3} - \dfrac{\partial_0 B_3}{r} - \dfrac{\partial_1 E_3}{2 r^2} - \dfrac{\text{Cot}[\theta]^2 \partial_1 E_3}{2 r^2} + \dfrac{\text{Csc}[\theta]^2 \partial_1 E_3}{2 r^2} + \dfrac{1}{2} \partial_1 \partial_0 B_3 - \dfrac{\partial_3 E_1}{2 r^2} - \dfrac{\text{Cot}[\theta]^2 \partial_3 E_1}{2 r^2} + \dfrac{\text{Csc}[\theta]^2 \partial_3 E_1}{2 r^2} + \dfrac{1}{2} \partial_3 \partial_0 B_1\right)$ $+ \left(-\dfrac{E_{13}}{r^2} - \dfrac{\text{Cot}[\theta]^2 E_{13}}{r^2} + \dfrac{\text{Csc}[\theta]^2 E_{13}}{r^2} + \nabla^2 E_{13} - \partial_0 \partial_0 E_{13} - \dfrac{2 \text{Cot}[\theta] \partial_2 E_{13}}{r^2} + \dfrac{2 \partial_3 E_{11}}{r} + \dfrac{2 \text{Cot}[\theta] \partial_3 E_{12}}{r^2}\right)$
23	$\left(\dfrac{2 \text{Cot}[\theta] \partial_3 E}{r^2} + \dfrac{2 \text{Cot}[\theta]^3 \partial_3 E}{r^2} - \dfrac{2 \text{Cot}[\theta] \text{Csc}[\theta]^2 \partial_3 E}{r^2} - \text{Cot}[\theta] \partial_3 \phi + \text{Cot}[\theta] \partial_3 \psi - \text{Cot}[\theta] \partial_3 \partial_0 B - \text{Cot}[\theta] \partial_3 \partial_0 \partial_0 E - \dfrac{3 \partial_3 \partial_2 E}{r^2} - \dfrac{3 \text{Cot}[\theta]^2 \partial_3 \partial_2 E}{r^2} + \dfrac{3 \text{Csc}[\theta]^2 \partial_3 \partial_2 E}{r^2} + \partial_3 \partial_2 \phi - \partial_3 \partial_2 \psi + \partial_3 \partial_2 \partial_0 B + \partial_3 \partial_2 \partial_0 \partial_0 E\right)$ $+ \left(\dfrac{4 \text{Cot}[\theta] E_3}{r^2} + \dfrac{4 \text{Cot}[\theta]^3 E_3}{r^2} - \dfrac{4 \text{Cot}[\theta] \text{Csc}[\theta]^2 E_3}{r^2} - \text{Cot}[\theta] \partial_0 B_3 - \dfrac{2 \partial_2 E_3}{r^2} - \dfrac{2 \text{Cot}[\theta]^2 \partial_2 E_3}{r^2} + \dfrac{2 \text{Csc}[\theta]^2 \partial_2 E_3}{r^2} + \dfrac{1}{2} \partial_2 \partial_0 B_3 - \dfrac{2 \partial_3 E_2}{r^2} - \dfrac{2 \text{Cot}[\theta]^2 \partial_3 E_2}{r^2} + \dfrac{2 \text{Csc}[\theta]^2 \partial_3 E_2}{r^2} + \dfrac{1}{2} \partial_3 \partial_0 B_2\right)$ $+ \left(-\dfrac{2 \text{Cot}[\theta]^2 E_{23}}{r^2} + \dfrac{\text{Csc}[\theta]^2 E_{23}}{r^2} + \nabla^2 E_{23} - \partial_0 \partial_0 E_{23} + 2 \text{Cot}[\theta] \partial_1 E_{13} - \dfrac{4 \partial_1 E_{23}}{r} + \dfrac{2 \partial_2 E_{13}}{r} + \dfrac{2 \partial_3 E_{12}}{r} + \dfrac{2 \text{Cot}[\theta] \partial_3 E_{22}}{r^2}\right)$

Substitutions:

$$\delta G_{11} + \frac{4}{r} \nabla^i E_{i1} + \frac{2}{r^2} E^i{}_i$$

$$\delta G_{22} - 2 \text{Cot}^2[\theta] E^i{}_i$$

$$\delta G_{01} - \frac{1}{r} \partial_0 \nabla^i E_i + \frac{1}{r} \nabla^i B_i$$

$$\delta G_{02} - \text{Cot}[\theta] \partial_0 \nabla^i E_i + \text{Cot}[\theta] \nabla^i B_i$$

$$\delta G_{12} + \frac{2}{r} \nabla^i E_{i2} + 2 \text{Cot}[\theta] \nabla^i E_{i1} + \frac{2 \text{Cot}[\theta]}{r} E^i{}_i$$

$$\delta G_{13} + \frac{2}{r} \nabla^i E_{i3}$$

$$\delta G_{23} + 2 \text{Cot}[\theta] \nabla^i E_{i3}$$