# How To Find the Time of Impact (TOI) of a Ball colliding with the border:

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Background: Similar to ball-ball collisions, the physics engine of the Collision Lab simulation uses a time-discretization approach to detecting and processing ball-to-border collisions. It checks and processes collisions *after* they've happened. So in one frame, a Ball might not be colliding with the border, but in the next frame, the Ball is overlapping with the border.

For instance, the position of a ball at a given frame might look like:
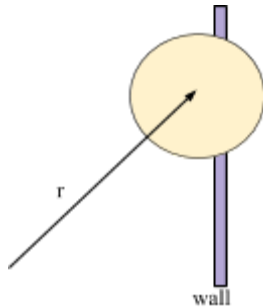
To provide a more realistic collision experience, on the exact frame where a Ball is overlapping (colliding) with the border, the Physics engine must reconstruct the collision and find the exact position of where the Ball collided, which was likely in between the current frame and the last frame.
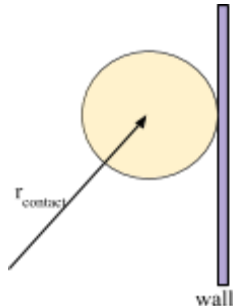
The question, answered in this document, is how the physics engine calculates the time from when the collision first happened (which occurred in between a frame) to the current overlapping position of the ball? This method is called "getBallToBorderCollisionOverlapTime" in the model.

Note that the pictures above show the right wall, when in the actual model, the ball can collide with any of the walls. The challenge of this derivation is to generalize a formula to make the calculation the same for all walls.

Known Quantities:

$\vec{r}$ - the position of the Ball involved in the collision.

$\vec{v}$ - the velocity of the Ball involved in the collision.

radius - the radius of the Ball involved in the collision.

wall
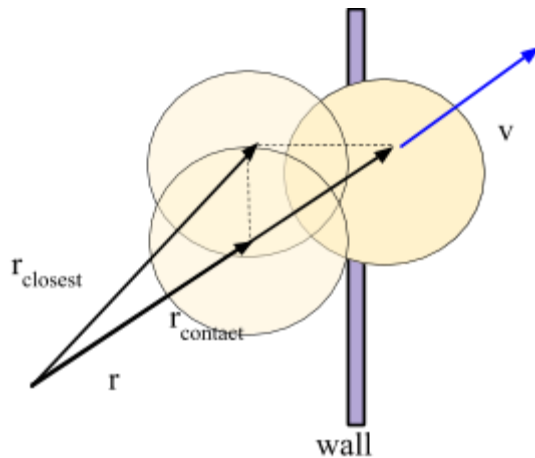
$\vec{r}_{closest}$ - the position of a ball with the same radius as the Ball involved in the collision that is closest to

the current position of the ball and *fully* inside of the border. This is computed with
Bounds2.prototype.closestPointTo()

Unknown Quantities:

$\vec{r}_{contact}$ - the position of the Ball when it first collided with the wall (in between frames).
time - The elapsed time from when the Ball first collided to its current colliding position. This is the quantity we are solving for.
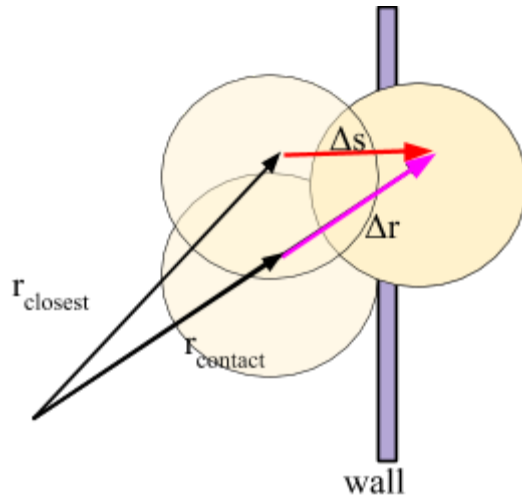
Derivation:
Based on a uniform motion model, the ball is not accelerating. C is a time quantity.
Thus, we know:

$\vec{r}_{contact} = \vec{r} - time \cdot \vec{v}$

$\Rightarrow \vec{r} - \vec{r}_{contact} = time \cdot \vec{v}$

We define $\Delta \vec{s}$ as the red vector in the picture, which would be $\vec{r} - \vec{r}_{closest}$.

We also define $\Delta \vec{r}$ as the pink vector, which would be $\vec{r} - \vec{r}_{contact} = time \cdot \vec{v}$

Using a dot product projection,
$$|\Delta \vec{s}| = \text{projection}(\Delta \vec{r}, \Delta \vec{s})$$

The idea behind this is that the x coordinates of $\vec{r}_{closest}$ and $\vec{r}_{contact}$ are the same.

$$\Rightarrow |\Delta \vec{s}| = \frac{(\Delta \vec{r} \cdot \Delta \vec{s})}{|\Delta \vec{s}|}$$

$$\Rightarrow |\Delta \vec{s}| = \frac{(time\ \vec{v} \cdot \Delta \vec{s})}{|\Delta \vec{s}|} \quad \text{(by substitution)}$$

Finally time $= \dfrac{|\Delta \vec{s}|^2}{\vec{v} \cdot \Delta \vec{s}}$ which are all known quantities.

This formula yields a positive result for forward collisions and a negative result for reverse collisions, which matches the behavior of ball-to-ball impact times.