How To Find the Time of Impact (TOI) of two Balls:

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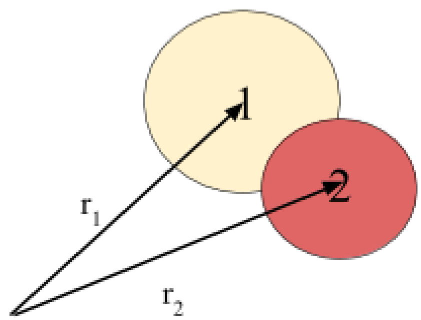
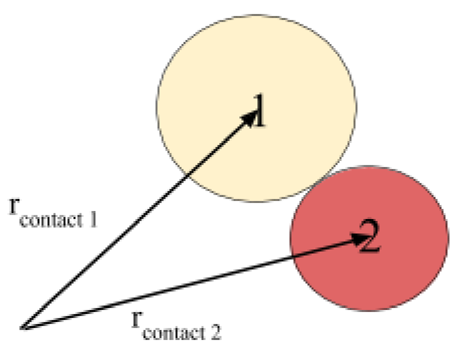
Background: The physics engine of the Collision Lab simulation uses a time-discretization approach to detecting and processing ball collisions. It checks and processes collisions *after* they've happened. So in one frame, the Balls might not be colliding (but are about to), and in the next frame, the Balls are overlapping each other.

For instance, the position of two Balls at a given frame might look like:



To provide a more realistic collision experience, on the exact frame where Balls are overlapping (colliding), the Physics engine must reconstruct the collision and find the positions where the Balls exactly collided, which was likely in between the current frame and the last frame



The question, answered in this document, is how would the physics engine calculate the time from when the collision first happened (which occurred in between a frame) to the current overlapping position of two Balls? This method is called "getBallToBallCollisionOverlapTime" in the model.

Known Quantities:

$\vec{r}_1$ - the position of the first Ball involved in the collision.

$\vec{r}_2$ - the position of the second Ball involved in the collision.

$\vec{v}_1$ - the velocity of the first Ball involved in the collision.

$\vec{v}_2$ - the velocity of the second Ball involved in the collision.

$radius_1$ - the radius of the first Ball involved in the collision.

$radius_2$ - the radius of the first Ball involved in the collision.

Unknown Quantities:

$\vec{r}_{\text{contact 1}}$ - the position of the first Ball when the Balls first collided (in between frames)

$\vec{r}_{\text{contact 2}}$ - the position of the second Ball when the Balls first collided (in between frames)

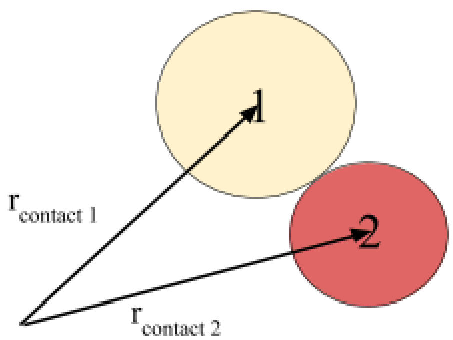
C - The elapsed time from when the Balls first collided to their current colliding positions. This is the quantity we are solving for.

Derivation:

Based on a ballistic motion model, the balls are not accelerating. C is a time quantity.
Thus, we know:

$\vec{r}_{\text{contact 1}} = \vec{r}_1 - C \cdot \vec{v}_1$

$\vec{r}_{\text{contact 2}} = \vec{r}_2 - C \cdot \vec{v}_2$



Additionally, based on this picture, we know that when the Balls are exactly colliding, the distance between the centers of the ball are equal to the sum of the radii of the ball:

$|\vec{r}_{\text{contact 2}} - \vec{r}_{\text{contact 1}}| = radius_1 + radius_2$

Substituting:

$|(\vec{r}_2 - C \cdot \vec{v}_1) - (\vec{r}_1 - C \cdot \vec{v}_1)| = radius_1 + radius_2$

=> $|(\vec{r}_2 - \vec{r}_1) - C \cdot (\vec{v}_2 - \vec{v}_1)| = radius_1 + radius_2$

=> $|\Delta\vec{r} - C \cdot \Delta\vec{v}| = radius_1 + radius_2$

Use the Vector Property: $\vec{u} \cdot \vec{u} = |\vec{u}|^2$

Thus,

$|\Delta\vec{r} - C \cdot \Delta\vec{v}|^2 = (radius_1 + radius_2)^2 = (\Delta\vec{r} - C \cdot \Delta\vec{v}) \cdot (\Delta\vec{r} - C \cdot \Delta\vec{v})$

Finally:

$|\Delta\vec{r}|^2 - 2C(\Delta\vec{v} \cdot \Delta\vec{r}) + C^2|\Delta\vec{v}|^2 = (radius_1 + radius_2)^2$, which is quadratic in-form for C.

We pick the positive value of the two roots for forward collisions and the negative value for negative collisions.