How To Find the Time of Impact (TOI) of a Ball colliding with a border:

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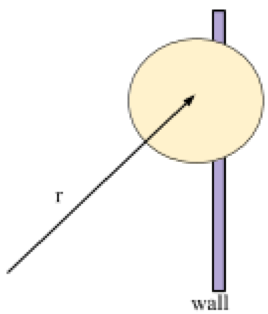
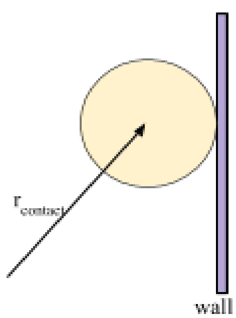
Background: Similar to ball-ball collisions, the physics engine of the Collision Lab simulation uses a time-discretization approach to detecting and processing ball-to-border collisions. It checks and processes collisions *after* they've happened. So in one frame, a Ball might not be colliding with the border, but in the next frame, the Ball is overlapping with the border.

For instance, the position of a ball at a given frame might look like:



To provide a more realistic collision experience, on the exact frame where a Ball is overlapping (colliding) with the border, the Physics engine must reconstruct the collision and find the positions where the Ball exactly collided, which was likely in between the current frame and the last frame.



The question, answered in this document, is how would the physics engine calculate the time from when the collision first happened (which occurred in between a frame) to the current overlapping position of the ball? This method is called "getBallToBorderCollisionOverlapTime" in the model.

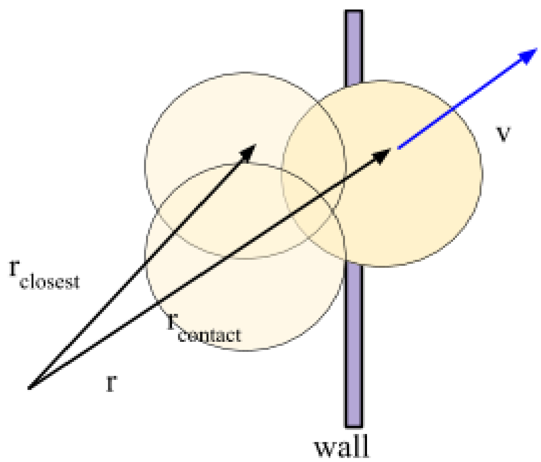
Note that the pictures above show the right wall, when in the actual model, the ball can collide with any of the walls. The challenge of this derivation is to make the calculation the same for all walls.

Known Quantities:

$\vec{r}$ - the position of the Ball involved in the collision.

$\vec{v}$ - the velocity of the Ball involved in the collision.

radius - the radius of the Ball involved in the collision.



$\vec{r}_{\text{closest}}$ - the position of a ball with the same radius as the Ball involved in the collision that is closest to the current position of the ball and *fully* inside of the border. This is computed with Bounds2.prototype.closestPointTo()

Unknown Quantities:

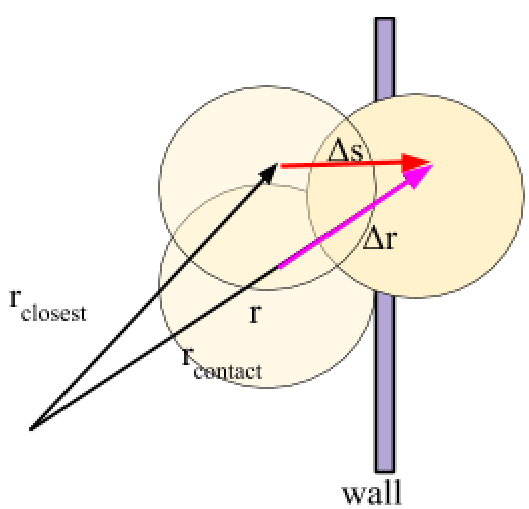
$\vec{r}_{\text{contact}}$ - the position of the Ball when the Ball first collided with the wall (in between frames).
C - The elapsed time from when the Ball first collided to its current colliding position. This is the quantity we are solving for.

Derivation:

Based on a ballistic motion model, the ball is not accelerating. C is a time quantity.
Thus, we know:

$$\vec{r}_{\text{contact}} = \vec{r} - C \cdot \vec{v}$$

$$\Rightarrow \vec{r} - \vec{r}_{\text{contact}} = C \cdot \vec{v}$$



We define $\Delta\vec{s}$ as the red vector in the picture, which would be $\vec{r}$ - $\vec{r}_{\text{closest}}$.
We also define $\Delta\vec{r}$ as the pink vector, which would be $\vec{r}$ - $\vec{r}_{\text{contact}} = \mathrm{C}\cdot\vec{v}$

Using a dot product projection,
$|\Delta\vec{s}| =$ projection$(\Delta\vec{r}, \Delta\vec{s})$

The idea behind this is that the x coordinates of $\vec{r}_{\text{closest}}$ and $\vec{r}_{\text{contact}}$ are the same.

=> $|\Delta\vec{s}| = \dfrac{(\Delta\vec{r}\cdot\Delta\vec{s})}{|\Delta\vec{s}|}$

=> $|\Delta\vec{s}| = \dfrac{(C\,\vec{v}\cdot\Delta\vec{s})}{|\Delta\vec{s}|}$ (by substitution)

Finally C = $\dfrac{|\Delta\vec{s}|^2}{\vec{v}\cdot\Delta\vec{s}}$ which are all known quantities.