# Collaborative Insurance Sustainability & Network Structure

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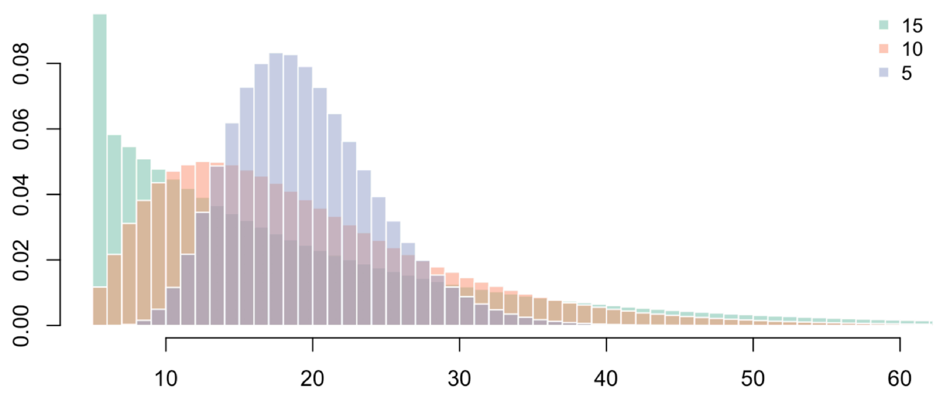
Online Appendix
Contains mathematical derivations and details on simulations

## Introduction

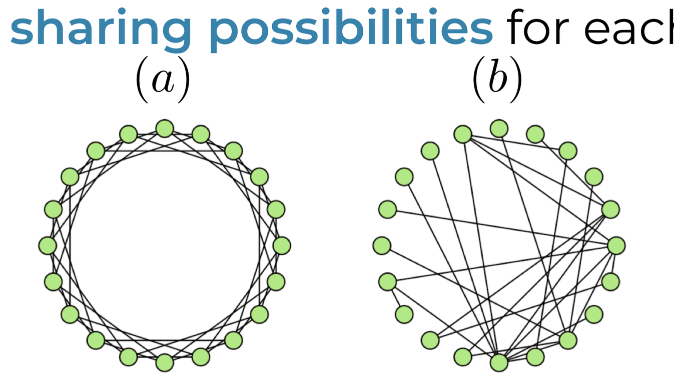
The **peer-to-peer** (P2P) economy is characterized by its decentralization - individuals interact directly with each other rather than via a third party. Instead, a **third party** might be **used to expedite the connection** between individuals, where *Uber* or *Airbnb* are prominent examples of such facilitators. Recent advances in the **actuarial literature** demonstrated how such a **P2P approach based on networks** might be incorporated into the insurance industry. Whereas previous research such as Feng et al. (2021) or Denuit and Robert (2020) studied risk sharing on **homogenous networks** with **heterogeneous risks**, in this article (Charpentier et al., 2021) we instead consider the sustainability of such a system by considering **different structures of networks** themselves and assume **homogenous risks** based on the concept of **homophily**. We then develop a mechanism that supports a range of network structures.

## Graph Representation

Our goal is to derive a risk sharing mechanism based on variety of different networks of agents. To cover the general case, we express a network as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of **vertices** or **nodes** that will represent agents that would like insure themselves against a risk and $\mathcal{E}$ is the set of **edges** or **links** that represent ties between the agents. We will assume undirected edges, $i \leftrightarrow j$ meaning that if $i$ has ties to $j$, $j$ has ties to $i$. An important measure to quantify how regular a graph is its **degree vector** $d$ which contains the number of adjacent vertices for every node in the graph $\mathcal{G}$ (that is $d_i = |\{j : (i,j) \in \mathcal{E}\}|$). Simply changing the **dispersion of the degree vector** results in a array of different networks . On one extreme we have low-variance regular graphs such as encountered in Feng et al. (2021) and in the **high-variance case** almost star shaped networks such as they are often encountered in **social media**. Ensuring the **sustainability of an insurance mechanism** is highly dependent on the shape of a network. Even in the case of homogenous risk *within* a network, when risk is to be shared via edges, only **regular networks guarantee a stable number of sharing possibilities** for each agent. In the **high-variance case**, as depicted right in subfigure (b), some **agents only have a single connection** and efficient risk sharing via edges becomes more difficult. In what follows we simulate different networks with the degree distribution simulated as:




*Degree distribution for networks same mean but different dispersion*




*Networks with low (a) and high (b) degree variance*

$$D = \min\{5 + \lfloor \Delta \rfloor, n\}, \quad \Delta \sim \Gamma(\bar{d} - 5, \sigma^2)$$

## Risk Sharing with Friends

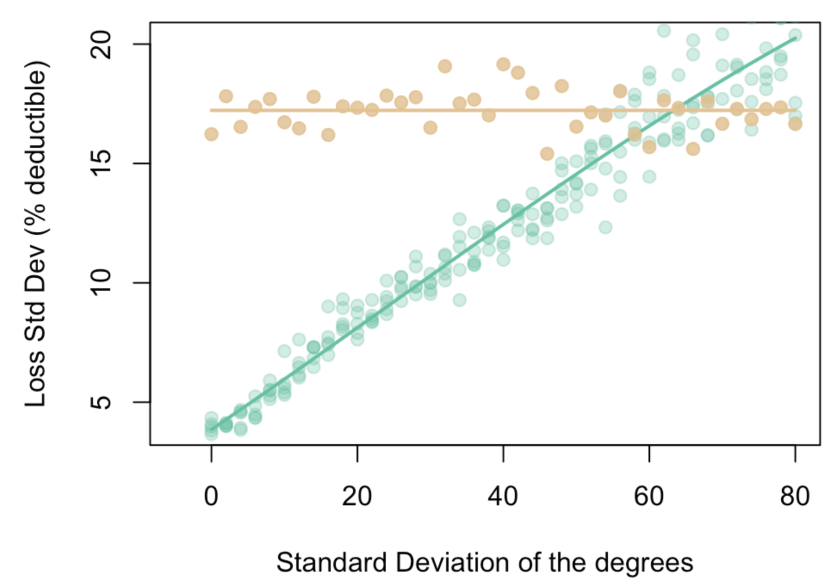
Here we consider the simple case where risk is exchanged on a network via the nodes in form of **reciprocal contracts between agents**. For the example presented here, consider $n = 5000$ agents, with average degree $\bar{d} = 20$, who are exposed to the same **risk that either materializes into a claim or not** within a given period, according to an i.i.d. Bernoulli variable $Z_i$ with $p = 0.1$. If a claim is recorded, a cost $Y_i \sim \Gamma(900, 2000^2)$ is associated to it. Further, we consider the case where the **total cost is capped above** at $s = 1000$ (any amount above might be covered by a traditional insurer). This setup will result in an expected loss of around $4.5\%$ of the deductible and a loss standard deviation of around $17\%$ of the deductible. A desirable risk sharing mechanism should result in a **lower standard deviation of losses**.

### Identical contracts across the network

The simplest case is to issue identical contracts for every edge in the network. A **natural bound on the amount** would be $\gamma = \frac{s}{\bar{d}} = 50$, such that on average, every agent could cover their entire deductible in a regular network. This **proper risk-sharing mechanism** will then assign the following cost to every agent $i$ with connections $V_i$ at the end of each period:

$$\xi_i = Z_i \min\{s, Y_i\} + \sum_{j \in V_i} Z_j \min\left\{\gamma, \frac{\min\{s, Y_j\}}{d_j}\right\} - Z_i \min\{d_i\gamma, \min\{s, Y_i\}\}$$

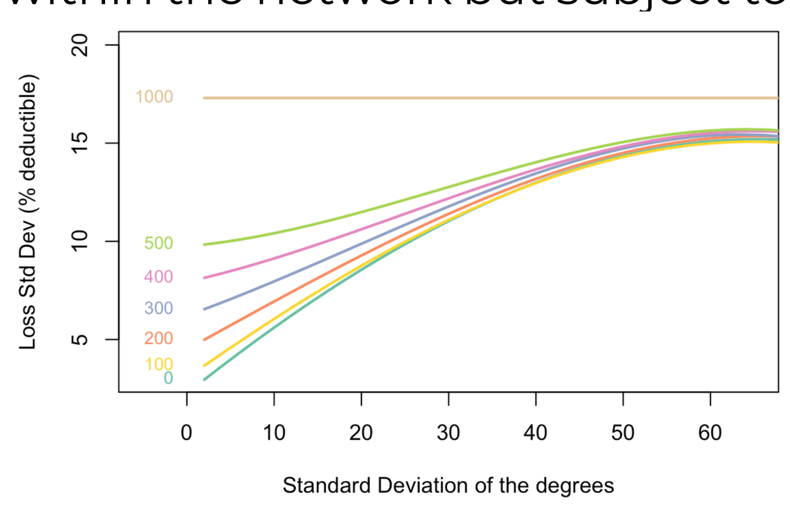
Where the gray part corresponds to the non-insurance case and the two terms on the right correspond to the debits and credits to $i$ due to the transfers. Results from simulations are depicted to the right. Clearly the simple mechanism **works well for rather regular networks**, but it results in outcomes even worse than the no insurance case for high-variance networks. This is due to the **concentration of some nodes** that will assume more than their optimal share of contracts.




*Results of simulation of no-insurance case vs simple case*

### Formulation as LP

Instead of having fixed amount contracts, we can introduce the **amounts into an optimization problem**. We formulate a **linear program** that maximizes total coverage within the network but subject to the constraint that every agent only wants to cover at most $s$. In the high variance case this will naturally lead to **some contracts (edges) having** $\gamma_k = 0$. We further introduce some self-contribution to alleviate the issues from above. Results of the **simulations** are depicted to the left with indicators for the different levels of **self-contribution**.



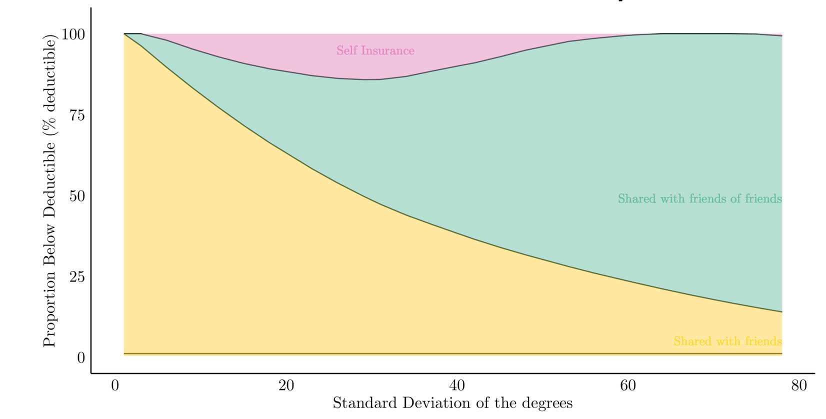

*Results of personalized amounts*

## Extensions to the simple Model

The mechanism, even with optimized reciprocal contracts, still **struggles to perform well in the high-variance cases**. Further, the assumption of **i.i.d. risks** across the whole network might seem overly **simplistic**. Here we propose two extensions to mitigate these issues.

### Higher order connections

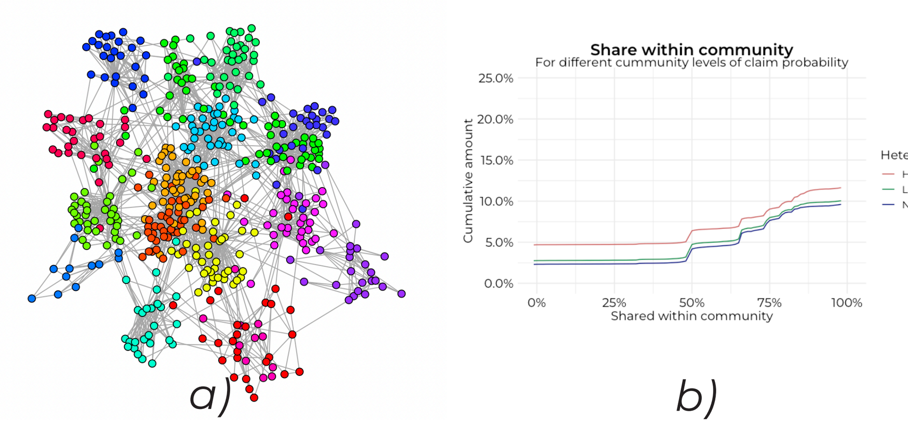
Instead of only considering sharing risk with friends, the framework can be extended to include friends-of-friends. This just corresponds to the **connections of the second degree**, which can easily be calculated using matrix algebra. We assume **trust between parties is smaller in the second degree** and hence the agents will want to **share less of their risk** with the second degree connections. In the example to the left we assumed that $\gamma_1$, the amount shared with first order connections is equal to 25 and $\gamma_2 = 5$. The **amount sharable** by first degree connections **decreases with higher degree variance** but the amount sharable by **second degree** connections actually **increases** (and enables more agents to fully insure their entire risk). This arises because high variance networks generate **hubs** that act as pass-through fares similar to the central nodes in star shaped networks.




*Relative amount of total insured*

### Risk sharing on subnetworks (heterogeneous risks)

Instead of assuming **homophily** on the whole network, we can also pose the **assumption on a smaller level**. An example for such a case would be a P2P household insurance within a city. Consider for example theft insurance, usually theft occurences will be **homogenous within a community** but less so across. Such connections can again be modelled by a network. Instead of directly optimizing the LP as before, we can employ an algorithm to **find more densely connected subgroups** and impose additional constraints on the LP. This in turn should alleviate issues concerning the **sustainability in heterogeneous risk cases**. One measure of success could be the total amount shared within each true subgroup as depicted above in b), or in **future work, dynamics** of such solutions could be analysed.




*A network with correlated subgroups and intragroup share*

## References

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