Collaborative Insurance Sustainability & Network Structure

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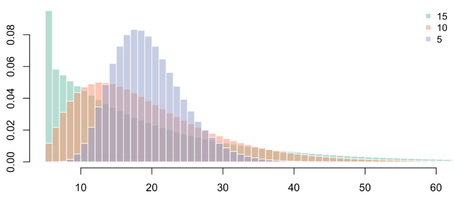
Online Appendix
Contains mathematical derivations and details on simulations

Introduction

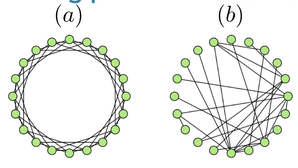
The **peer-to-peer** (P2P) economy is characterized by its decentralization - individuals interact directly with each other rather than via a third party. Instead, a **third party** might be **used to expedite the connection** between individuals, where *Uber* or *Airbnb* are prominent examples of such facilitators. Recent advances in the **actuarial literature** demonstrated how such a **P2P approach based on networks** might be incorporated into the insurance industry. Whereas previous research such as Feng et al. (2021) or Denuit and Robert (2020) studied risk sharing on **homogenous networks** with **heterogeneous risks**, in this article (Charpentier et al., 2021) we instead consider the sustainability of such a system by considering **different structures of networks** themselves and assume **homogenous risks** based on the concept of **homophily**. We then develop a mechanism that supports a range of network structures.

Graph Representation

Our goal is to derive a risk sharing mechanism based on variety of different networks of agents. To cover the general case, we express a network as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of **vertices** or **nodes** that will represent agents that would like insure themselves against a risk and $\mathcal{E}$ is the set of **edges** or **links** that represent ties between the agents. We will assume undirected edges, $i \leftrightarrow j$ meaning that if i has ties to j, j has ties to i. An important measure to quantify how regular a graph is its **degree vector** d which contains the number of adjacent vertices for every node in the graph $\mathcal{G}$ (that is $d_i = |\{j : (i,j) \in \mathcal{E}\}|$). Simply changing the **dispersion of the degree vector** results in a array of different networks . On one extreme we have low-variance regular graphs such as encountered in Feng et al. (2021) and in the **high-variance case** almost star shaped networks such as they are often encountered in **social media**. Ensuring the **sustainability of an insurance mechanism** is highly dependent on the shape of a network. Even in the case of homogenous risk *within* a network, when risk is to be shared via edges, only **regular networks guarantee a stable number of sharing possibilities** for each agent. In the **high-variance case**, as depicted right in subfigure (b), some **agents only have a single connection** and efficient risk sharing via edges becomes more difficult. In what follows we simulate different networks with the degree distribution simulated as:



Degree distribution for networks same mean but different dispersion



Networks with low (a) and high (b) degree variance

$$D = \min\{5 + \lfloor \Delta \rfloor, n\}, \quad \Delta \sim \Gamma(\overline{d} - 5, \sigma^2)$$

Risk Sharing with Friends

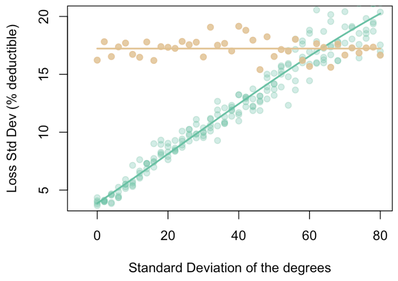
Here we consider the simple case where risk is exchanged on a network via the nodes in form of **reciprocal contracts between agents**. For the example presented here, consider $n = 5000$ agents, with average degree $\overline{d} = 20$, who are exposed to the same **risk that either materializes into a claim or not** within a given period, according to an i.i.d. Bernoulli variable Z_i with $p = 0.1$. If a claim is recorded, a cost $Y_i \sim \Gamma(900, 2000^2)$ is associated to it. Further, we consider the case where the **total cost is capped above** at $s = 1000$ (any amount above might be covered by a traditional insurer). This setup will result in an expected loss of around 4.5% of the deductible and a loss standard deviation of around 17% of the deductible. A desirable risk sharing mechanism should result in a **lower standard deviation of losses**.

Identical contracts across the network

The simplest case is to issue identical contracts for every edge in the network. A **natural bound on the amount** would be $\gamma = \frac{s}{\overline{d}} = 50$, such that on average, every agent could cover their entire deductible in a regular network. This **proper risk-sharing mechanism** will then assign the following cost to every agent i with connections V_i at the end of each period:

$$\xi_i = Z_i \min\{s, Y_i\} + \sum_{j \in V_i} Z_j \min\left\{\gamma, \frac{\min\{s, Y_j\}}{d_j}\right\} - Z_i \min\{d_i \gamma, \min\{s, Y_i\}\}$$

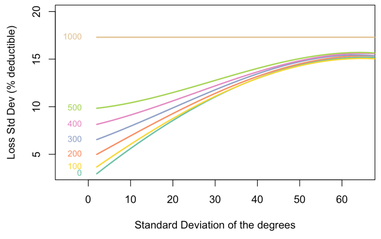
Where the gray part corresponds to the non-insurance case and the two terms on the right correspond to the debits and credits to i due to the transfers. Results from simulations are depicted to the right. Clearly the simple mechanism **works well for rather regular networks**, but it results in outcomes even worse than the no insurance case for high-variance networks. This is due to the **concentration of some nodes** that will assume more than their optimal share of contracts.



Results of simulation of no-insurance case vs simple case

Formulation as LP

Instead of having fixed amount contracts, we can introduce the **amounts into an optimization problem**. We formulate a **linear program** that maximizes total coverage within the network but subject to the constraint that every agent only wants to cover at most s. In the high variance case this will naturally lead to **some contracts (edges) having** $\gamma_k = 0$. We further introduce some self-contribution to alleviate the issues from above. Results of the **simulations** are depicted to the left with indicators for the different levels of **self-contribution**.



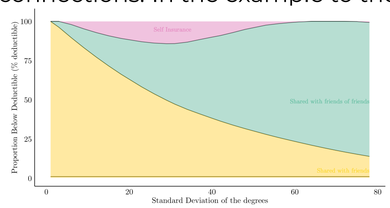
Results of personalized amounts

Extensions to the simple Model

The mechanism, even with optimized reciprocal contracts, still **struggles to perform well in the high-variance cases**. Further, the assumption of **i.i.d. risks** across the whole network might seem overly **simplistic**. Here we propose two extensions to mitigate these issues.

Higher order connections

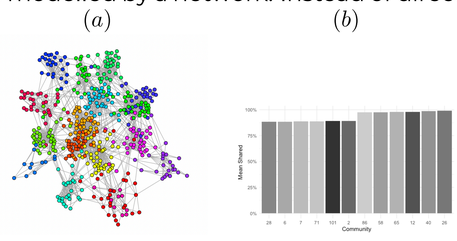
Instead of only considering sharing risk with friends, the framework can be extended to include friends-of-friends. This just corresponds to the **connections of the second degree**, which can easily be calculated using matrix algebra. We assume **trust between parties is smaller in the second degree** and hence the agents will want to **share less of their risk** with the second degree connections. In the example to the left we assumed that γ_1, the amount shared with first order connections is equal to 25 and $\gamma_2 = 5$. The **amount sharable** by first degree connections **decreases with higher degree variance** but the amount sharable by **second degree** connections actually **increases** (and enables more agents to fully insure their entire risk). This arises because high variance networks generate **hubs** that act as pass-through fares similar to the central nodes in star shaped networks.



Relative amount of total insured

Risk sharing on subnetworks (heterogeneous risks)

Instead of assuming **homophily** on the whole network, we can also pose the **assumption on a smaller level**. An example for such a case would be a P2P household insurance within a city. Consider for example theft insurance, usually theft occurences will be **homogenous within a community** but less so across. Such connections can again be modelled by a network. Instead of directly optimizing the LP as before, we can employ an algorithm to **find more densely connected subgroups** and impose additional constraints on the LP. This in turn should alleviate issues concerning the **sustainability in heterogeneous risk cases**. One measure of success could be the total amount shared within each true subgroup as depicted above in (b), or in **future work, dynamics** of such solutions could be analysed.



A network with correlated subgroups and intragroup share

References

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