Collaborative Insurance Sustainability

Today's Goal

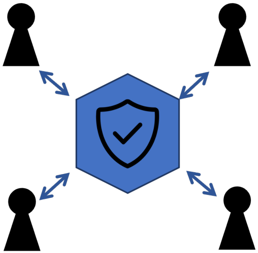
- Present a new way to consider P2P insurance

  ...and show how network structure influences its sustainability

Based on Collaborative Insurance sustainability and network structure

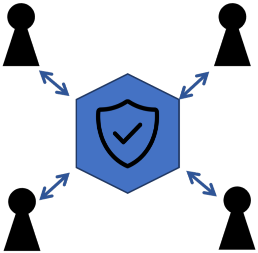
Joint work with Arthur Charpentier (UQAM), Lariosse Kouakou (EURIA), Matthias Löwe (Universität Münster), Franck Vermet (EURIA)

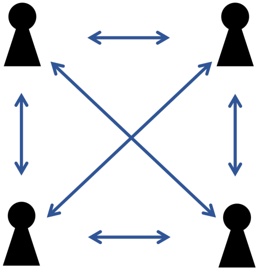
Different Insurance Models



Traditional Model

Different Insurance Models





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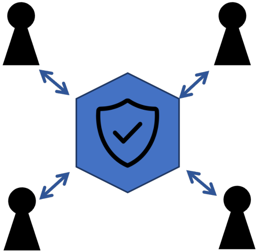

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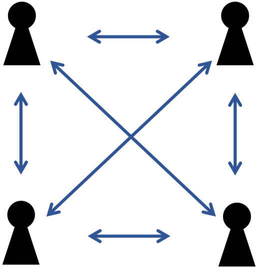
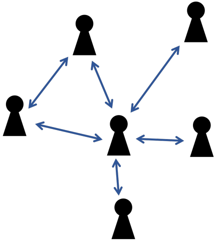


P2P ”Classic”



P2P ”Network”

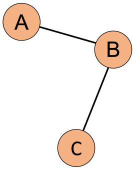
Outline - A new angle

1. Consider *arbitrary* networks
   i. Common language for networks
   ii. Simulation approaches
2. Set up basic risk-sharing scheme
   i. Evaluate using simulations
3. Improve mechanism

Graph Terminology I

Graph

A graph $\mathcal{G}$ is an ordered pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ represents a set of vertices (or nodes) and $\mathcal{E}$ a set of edges (or links) such that $\mathcal{E} \subseteq \{\{x, y\} | x, y, \in \mathcal{V}, x \neq y\}$



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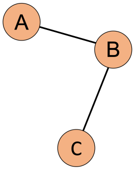
Adjacency Matrix

The adjacency matrix A for a set of vertices $\mathcal{V} = \{v_1, \ldots, v_n\}$ is a matrix of size $n \times n$, such that $A_{i,j} = 1$ if there is an edge from vertex v_i to v_j.



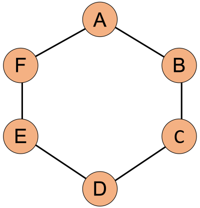
$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

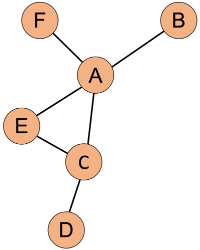
Graph Terminology II

Degree distribution

The degree of a vertex is the number of edges that are incident to it. We can construct the degree vector from elements d_i as $d = A\mathbb{1}$. The average degree of a network can be calculated as:

$$\bar{d} = \mathbb{E}[d] = \frac{1}{|\mathcal{V}|}\sum_{v\in\mathcal{V}} d(v) = \frac{2|\mathcal{E}|}{|\mathcal{V}|} = \frac{1}{|\mathcal{V}|}\,\|d\|_1$$



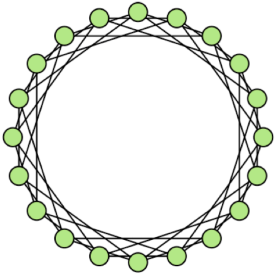


Graph Simulation - I

Regular Network

Every node in $\mathcal{V}$ has degree equal to some constant $k < n$. As a technical condition least one of k and the number of vertices must be even.

$$\mathbb{E}[d] = k$$
$$\mathbb{V}[d] = 0$$

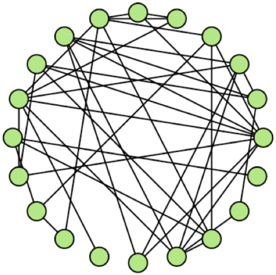


Graph Simulation - II

Erdős–Rényi Network

Each possible edge $i, j \in \mathcal{V} \times \mathcal{V}$ is included in the network with probability p independent from other edges. The Network then has a degree distribution that follows a binomial distribution $\mathcal{B}(n-1, p)$. Also, if p not too large but n_V: $p \sim \lambda / n_V, d(v) \sim \mathcal{P}(\lambda)$

$$\mathbb{E}[d] \approx \mathbb{V}[d]$$



Graph Simulation - III

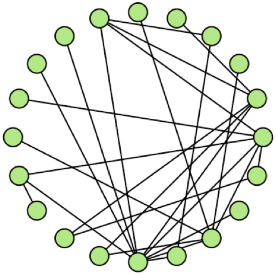
Preferential attachment Network

To start with, consider a small network, and create a new node at every time-step. During every time-step, the new node is connected to the existing nodes with a probability p that is proportional to the degree of an existing node d_i. This way, every new node is more likely to connect to existing "popular" nodes. Here the degree distribution follows a power law.

$$\mathbb{E}[d] = ?$$

$$\mathbb{V}[d] = ?$$

usually fat tailed

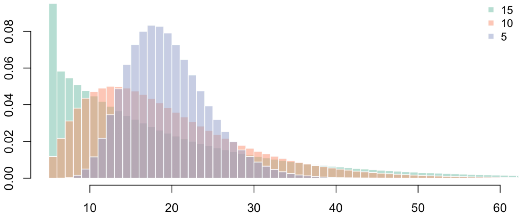


Graph Simulation - IV

Instead of the mentioned algorithms, we use a flexible method:

$$D \stackrel{\mathcal{L}}{=} \min\{5 + [\Delta], n - 1\}, \quad \Delta \sim \mathcal{G}(\alpha, \beta)$$

Where we vary $\frac{\alpha}{\beta^2}$ but not $\frac{\alpha}{\beta}$ (fix the mean $\overline{d}$ but change variance $\mathbb{V}[d]$)



The Core: Risk Sharing with Friends

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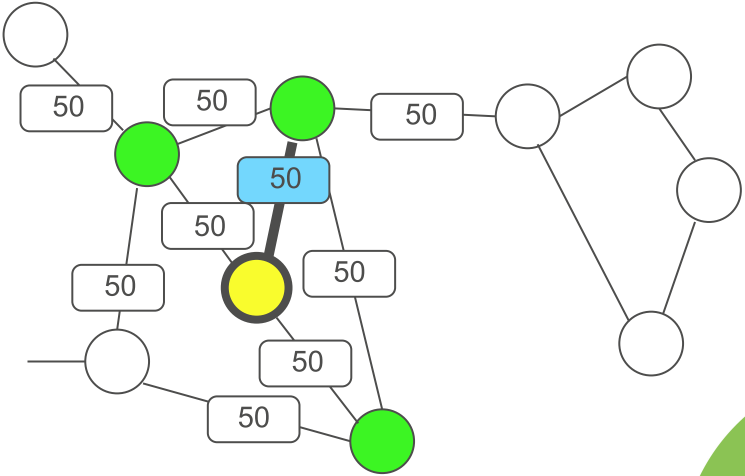
- We consider a homogenous population where a risk materialises at most once per period
- With homogenous risks usually implies the $\frac{1}{n}$-rule (and everybody gets the same contract). We refer to the contract amount γ

Risk Sharing on a network



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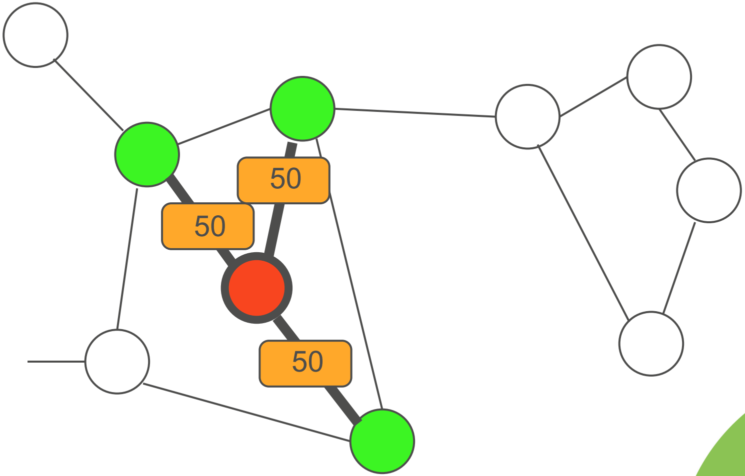

The Core: Risk Sharing with Friends

- Here we consider the case where the homogenous population only wants to insure an amount below a deductible

Deductible:	$s_i = s \quad \forall i$
Risk (i.i.d.):	$Y_i \sim 100 + \mathcal{G}(a, b)$
Helper:	$Z_i \sim \mathcal{B}(p)$
Loss:	$L_i = Z_i \min\{s, Y_i\}$
Engagement:	$\gamma_{(i,j)} = \gamma_{(j,i)} = \frac{s}{d}$

Single period wealth

Let $\mathcal{V}_i$ denote the set of "friends" of a node i. We denote the wealth after one period:

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1. **Simulation:** Draw a graph, fix s, draw from $\mathcal{B}$, $\mathcal{G}$
2. **Optimization:** $\xi_i = f(s, Y_i, Z_i, d_i, \gamma)$
3. **Compare:** ξ vs. $\min\{s, Y\}$

The simple framework

Consider a network with $\overline{d} = 20$, $s = 1000$, $\mathbb{E}[L] = 4.5\% \times s$, $\sqrt{\mathbb{V}[L]} = 17\% \times s$

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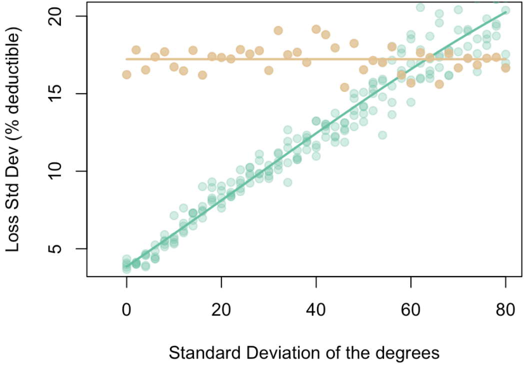

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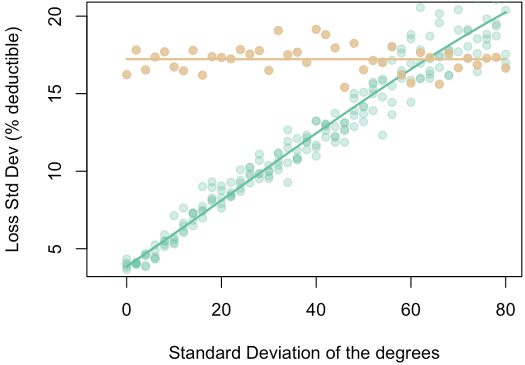

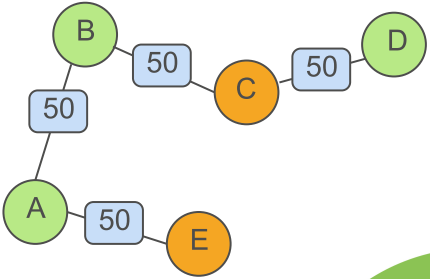

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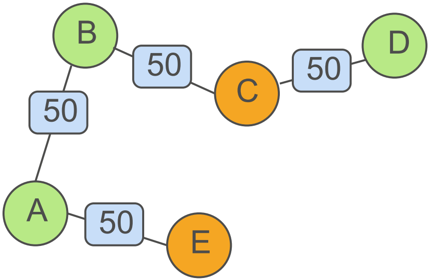

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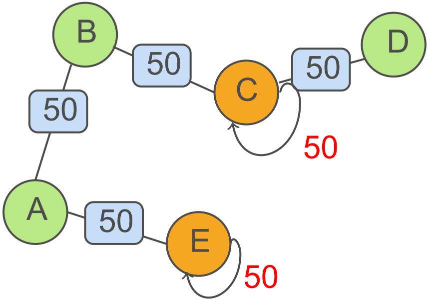

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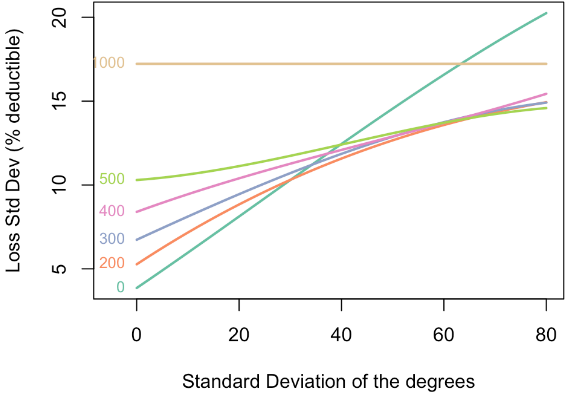

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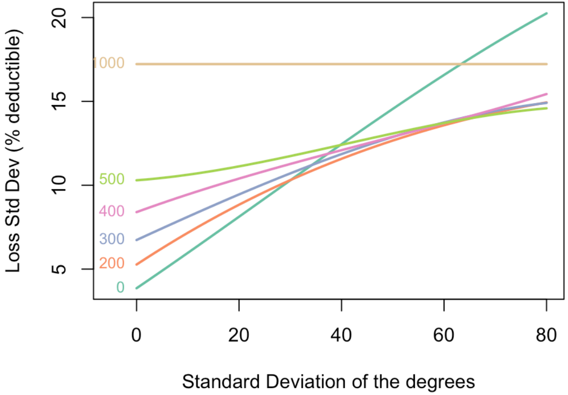

Optimal reciprocal contributions

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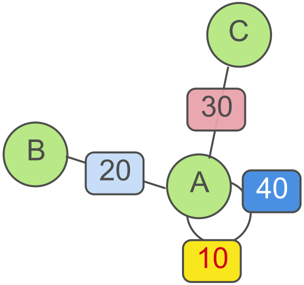
- Can γ be optimized for any $\boldsymbol{d}$?
- Here: consider a "global" coverage

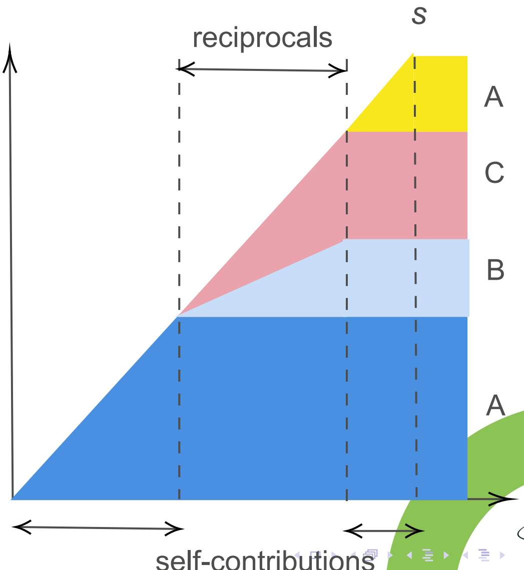
$$\gamma_1^* = \operatorname{argmax} \left\{ \sum_{(i,j)\in\mathcal{E}} \gamma_{(i,j)} \right\}$$

$$\text{s.t. } \gamma_{(i,j)} \in [0, \gamma],\ \forall (i,j) \in \mathcal{E}$$

$$\sum_{j\in\mathcal{V}_i} \gamma_{(i,j)} \leq s,\ \forall i \in \mathcal{V}$$

A P2P-Network mechanism





Results

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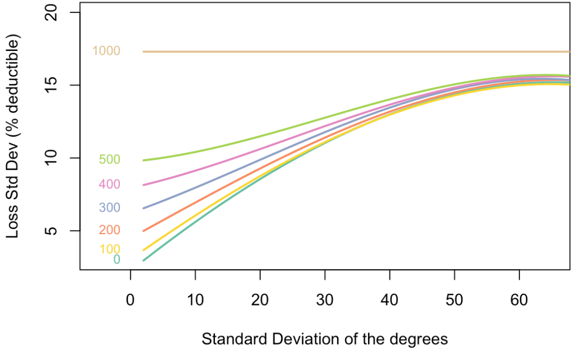

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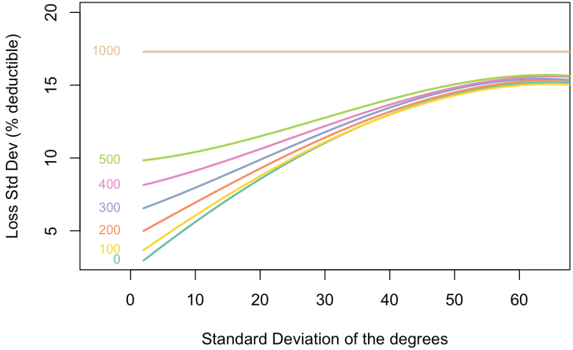
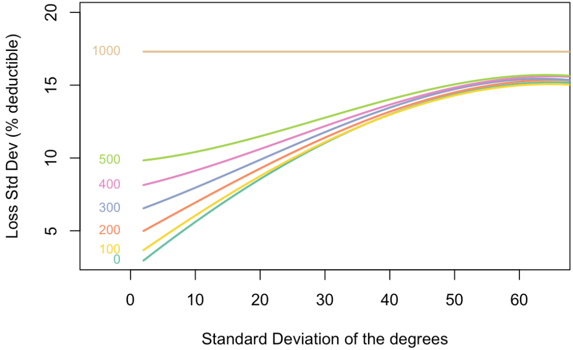


Results

How did we do?

- ☑ Risk sharing works
- ☑ Ex-Post fairness
- ☑ Works on arbitrary networks



Extensions

Optimal reciprocal contributions - II

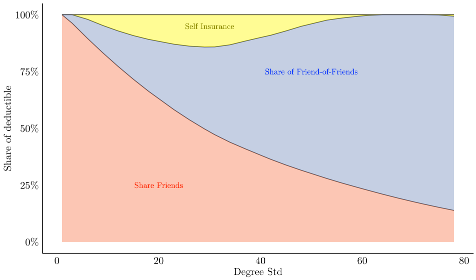
Why stop at friends?
We can also consider friends-of-friends!

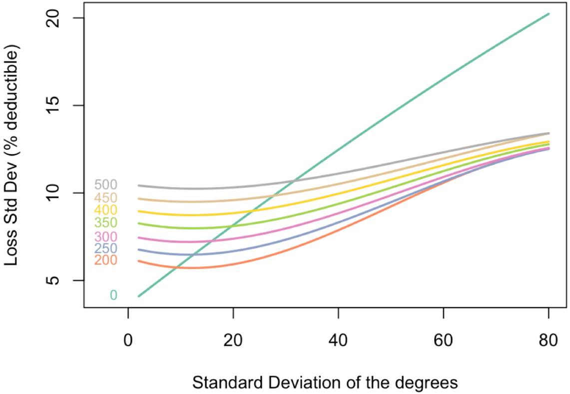
$$\gamma_2^\star = \text{argmax} \left\{ \sum_{(i,j)\in\mathcal{E}^{(2)}} \gamma_{(i,j)} \right\}$$

$$\text{s.t. } \gamma_{(i,j)} \in [0, \gamma_2],\ \forall (i,j) \in \mathcal{E}^{(2)},\ \mathcal{E}^{(2)} \text{ from } A^2$$

$$\sum_{j\in\mathcal{V}_i^{(1)}} \gamma^\star_{1:(i,j)} + \sum_{j\in\mathcal{V}_i^{(2)}} \gamma_{(i,j)} \le s,\ \forall i \in \mathcal{V}$$

Optimal reciprocal contributions - II





If you want to learn more..

- https://phi-ra.github.io/
- arxiv.org/abs/2107.02764
- github.com/phi-ra/collaborative_insurance