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NO.1 state space:

Handin

No.1

$$\begin{pmatrix} r_t^p \\ \theta_t^p \\ V_{\theta t}^p \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{t-1}^p \\ \theta_{t-1}^p \\ V_{\theta t-1}^p \end{pmatrix} + \begin{pmatrix} \varepsilon_{rt}^p \\ \varepsilon_{\theta t}^p \\ \varepsilon_{v\theta t}^p \end{pmatrix} \qquad \Sigma_1 = \begin{pmatrix} 500^2 & 0 & 0 \\ 0 & 0.005^2 & 0 \\ 0 & 0 & 0.005^2 \end{pmatrix}$$

$$\begin{pmatrix} Y_{rt} \\ Y_{\theta t} \end{pmatrix} = \begin{pmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \end{pmatrix} \begin{pmatrix} r_t^p \\ \theta_t^p \\ V_{\theta t}^p \end{pmatrix} + \begin{pmatrix} \varepsilon_{rt}^p \\ \varepsilon_{\theta t}^p \end{pmatrix} \qquad \Sigma_2 = \begin{pmatrix} 2000^2 & 0 \\ 0 & 0.03^2 \end{pmatrix}$$

No.2

Implementation of Kalman filter (see appendix)

No.3

Reconstruction of the trajectory:

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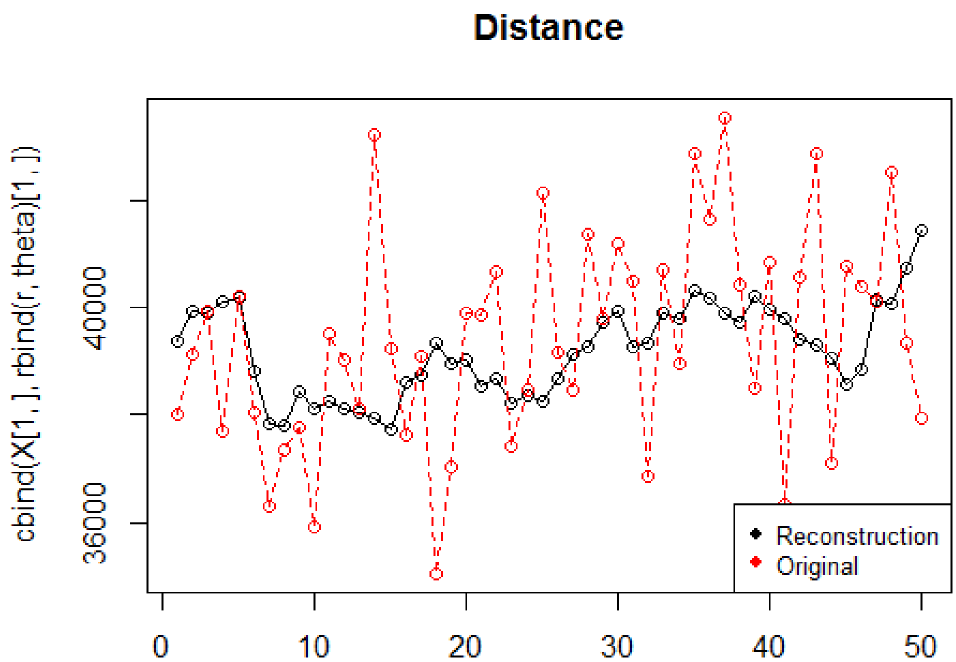

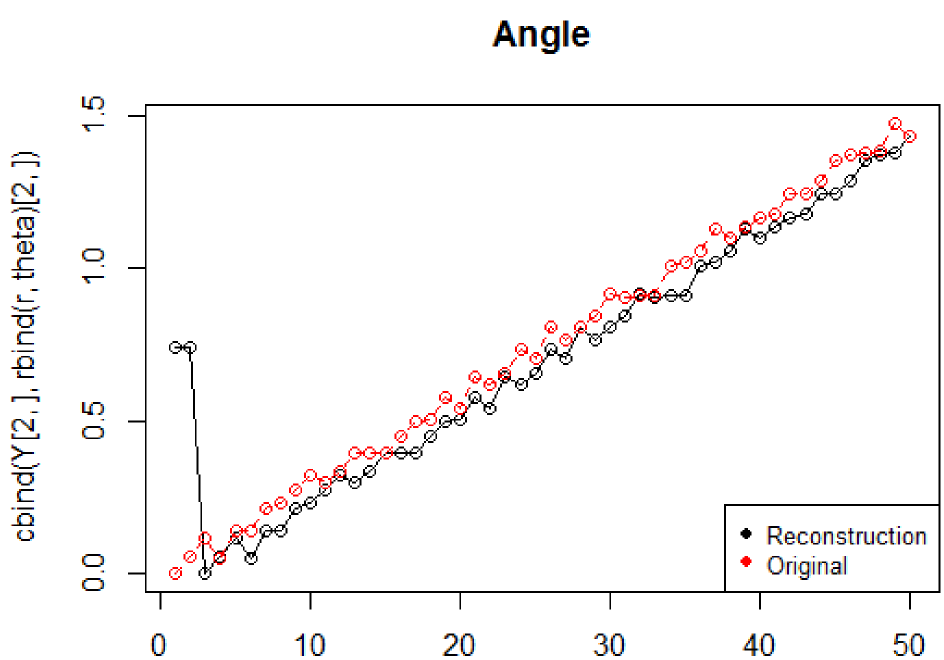

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Appendix

```
r <- read.csv("Satelliteorbit.csv")[[1]]

theta<- read.csv("Satelliteorbit.csv")[[2]]

n=50

A<-matrix(c(1, 0, 0, 0, 1, 1, 0, 0, 1), nrow=3,ncol=3,byrow=TRUE)

C<-matrix(c(1, 0, 0,0, 1, 0), nrow=2,ncol=3,byrow=TRUE)


sigma1<-matrix(c(500^2, 0, 0, 0, 0.005^2, 0, 0, 0, 0.005^2), nrow=3,ncol=3,byrow=TRUE)

sigma2<-matrix(c(2000^2, 0, 0,0, 0, 0.03^2), nrow=2,ncol=2,byrow=TRUE)


#initial condition

X <- matrix(nrow=3,ncol=n)

X[,1] = rbind(mean(r),mean(theta),.03)

Y <- matrix(nrow=2,ncol=n)

Y <- rbind(r,theta)

sigmaxx<-diag(c(1,.01,.01))


library(MASS)

set.seed(286)

for (I in 2:n){

  #pre

  Xpre <- matrix(nrow=3,ncol=n)

  Xpre[,I] <- A %*% X[,I-1]
```

Ran Wang
S111503

```
  sigmaxx_pre <-A %*% sigmaxx %*% t(A) + sigma1

  sigmayy=C %*% sigmaxx_pre %*% t(C)+sigma2



  #updating

  k = sigmaxx_pre %*% t(C) %*% solve(sigmayy);

  X[,I]<-Xpre[,I] + k %*% (Y[,I-1] - C%*%Xpre[,I])

  sigmaxx <-sigmaxx_pre - k %*%C %*% sigmaxx_pre

  Y[,I-1] <- C %*% X[,I-1] + mvrnorm(mu=cbind(0,0),Sigma=sigma2)

}



matplot(cbind(X[1,],rbind(r,theta)[1,]),type="o",pch=c(1,1),col=1:2, main="Distance")

legend("bottomright", c("Reconstruction", "Original"), cex=0.8, col=c("black","red"), pch=c(16,16),
lty=c(0,0))

matplot(cbind(Y[2,],rbind(r,theta)[2,]),type="o",pch=c(1,1),col=1:2,main="Angle")
legend("bottomright", c("Reconstruction", "Original"), cex=0.8, col=c("black","red"), pch=c(16,16),
lty=c(0,0))
```