Technical University of Denmark

42401: Introduction to Planning

PROJECT 1

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Introduction to planning

Question One

Part A

1. **Formulate a mathematical model that can be used to optimize GroundCo's staff scheduling and shift selection problem.**

The following mathematical model aims to find the optimal solution for the staff scheduling and shift selection problem. Therefore, the objective is to minimize the total cost of their ground crew schedules for any 24-hour period.

The objective function, which must be minimized, represents the hourly payment for 7 hours of work (shifts have 8 hours but an hour of meal-break must be taken and it is not payed) which has different costs according to the time of the day and a fixed administrative cost for every shift which is opened. The condition for t just wants to represent the cycle of t. Since we are considering data from one typical day we would like to consider that when t is greater than 23 it must be followed by 0, as the late hours of a day are followed by the early hours of the following day.

Decision variables:

y_{ts} ={1,0}

$X_{ts} =$ amount of staffs assigned to a shift, start at t, break at s

t = {0,1,...23} starting time

s ={3,4} type of shift, 3 refers to taking break after 3 hours and 4 refers to taking the break after 4 hours of work.

Objective function:

$$Min \sum_{t=23}^{3} \sum_{s=3}^{4} 1.25 \cdot C \cdot X_{ts} + \sum_{t=4}^{22} \sum_{s=3}^{4} C \cdot X_{ts} + \sum_{t=0}^{23} \sum_{s=3}^{4} S \cdot y_{ts}$$

$$\textbf{for } t = \begin{cases} t & \leq 23 \\ t-24 > 23 \end{cases}$$

S=1000 shift cost

C=7 * 240 hourly wage

s.t.

$$\sum_{t=0}^{23} \sum_{s=3}^{4} X_{ts} \leq \quad 155 \qquad (1)$$

$$\sum_{t=i-7}^{i} \sum_{s=3}^{4} X_{ts} \geq \quad d_i \qquad \text{for } \{t=i-4 | s\neq4\} \text{ and } \{t=i-3 | s\neq3\} \qquad (2)$$

i = {0,1,...23} hour

$$X_{ts} \leq M \cdot y_{ts} \qquad (3)$$

Given,

d_i to be a vector of a {24x1} dimension where each of the cells is the hourly required staff according to the following data table.

00:00 - 01:00	4	06:00 - 07:00	58	12:00 - 13:00	64	18:00 - 19:00	56
01:00 - 02:00	4	07:00 - 08:00	64	13:00 - 14:00	61	19:00 - 20:00	38
02:00 - 03:00	4	08:00 - 09:00	64	14:00 - 15:00	60	20:00 - 21:00	19
03:00 - 04:00	4	09:00 - 10:00	54	15:00 - 16:00	58	21:00 - 22:00	11
04:00 - 05:00	18	10:00 - 11:00	54	16:00 - 17:00	53	22:00 - 23:00	4
05:00 - 06:00	52	11:00 - 12:00	62	17:00 - 18:00	55	23:00 - 24:00	4

M to be a large number that restricts the maximum amount of employees per shift. Then, the most conservative value for M would be 155 because it is the maximum amount of employees that can be allocated in any 24-hour period. However, as the maximum hourly required staff is 64, then we can use M=70 under the hypothesis that no shift will need to allocate more than this amount of employees regarding the hourly required staff shown in the previous table. Once we have solved the problem we can verify that the largest shift allocates 22 employees. Knowing this, we can use whichever M greater than 22 (Part A) and the obtained results will be the same while the algorithm is optimized while we reduce the number for M.

2. How many decision variables and constraints are there?

In the mathematical model previously formulated, there are 96 decision variables.

- 48 decision variables (X_{ts}) are integer and correspond to the amount of staff assigned for a shift that starts at time t and has the meal-break after the s hours of work.
- 48 decision variables (Y_{ts}) are binary and represent if a shift with start time t and meal-break after the s hours of work is opened or not. When Y_{ts} = 1, the shift is opened and X_{ts} employees are assigned for it; when Y_{ts} = 0, the shift is closed because no employee has been scheduled for it.

In the mathematical model previously formulated, there are 73 constraints.

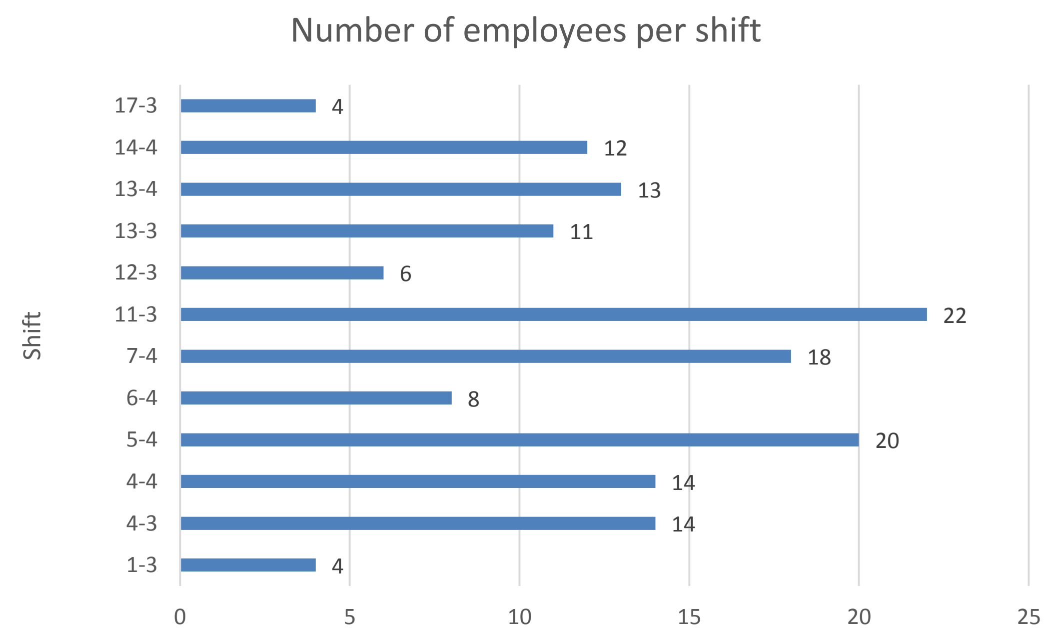
- The first constraint represents that no more than 155 employees can be allocated in any 24-hour period.
- The second type of constraint represents that the total amount of employees that are working at hour i must be equal or larger to the required staff for hour i. The total amount of employees working at hour i are those who started to work at that same hour plus those who started within the previous 7 hours disregarding the employees who are at the meal-break. Then, 24 constraints have been included and each one represents one hour.
- The third type of constraint represents that the total amount of employees assigned to a shift with start time t and meal-break after the s hours of work is smaller or equal to the maximum number of employees per shift if the shift is opened. When the shift is closed the total amount of employees assigned to it will be 0. Then, 48 constraints have been

included and each one represents a shift with start time t and meal-break after the s hours of work.

3. Using OpenSolver, find the optimal solution to GroundCo's problem. You should report the number of shifts used, the number of employees assigned to each shift and the total minimum cost.

The mathematical model previously formulated was solved using OpenSolver, the calculations are included in Question1.xls. The optimal solution is:

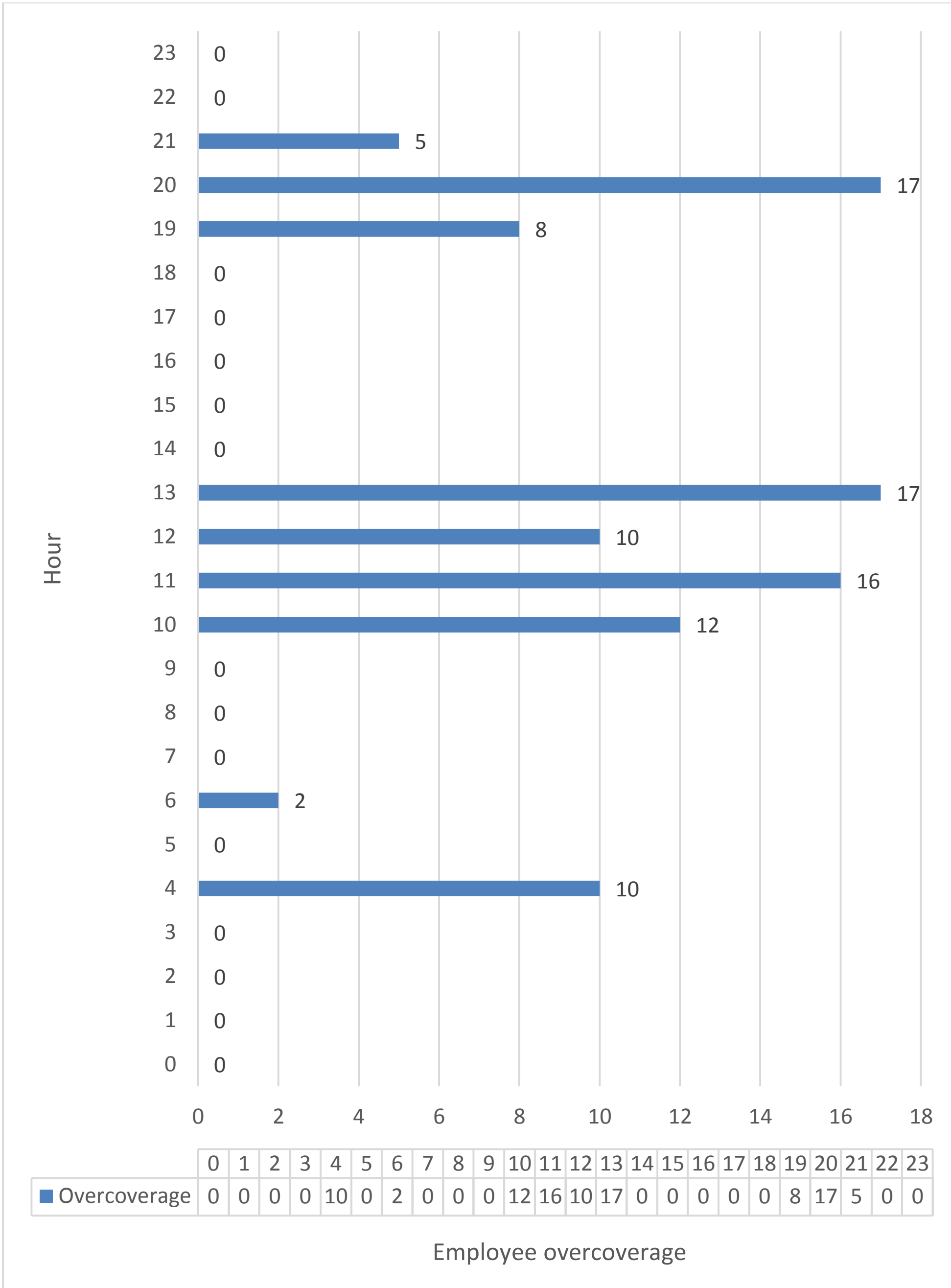
- A total number of 12 shifts. It is considered a shift a group of employees who start to work in a determined hour with the same time for the meal-break.
- The total minimum cost is: 258.960 DKK
- The total number of employees is 146 and they are assigned to the shifts according to the following graph:



	1-3	4-3	4-4	5-4	6-4	7-4	11-3	12-3	13-3	13-4	14-4	17-3
N° Employees	4	14	14	20	8	18	22	6	11	13	12	4

4. How much employee overcoverage is incurred in your solution? Overcoverage is incurred whenever the number of staff in a given period exceeds the level required.

The total amount of employee overcoverage is 97. The following graph and table represent the overcoverage incurred in each hour.



Part B

Recent negotiations with the employee's union have given GroundCo permission to also run four hour shifts. A four hour shift must adhere to the same requirements as an eight hour shift, with the exception that no meal break is required.

6. Extend your formulation from Part A2 to account for this possibility.

The formulation of the mathematical problem is similar to the one presented in Part A. In this case shifts of 4 hours were taken into account in our model by adding a new type of shift s=5.
The objective function was also modified adding two new terms that consider the hourly costs of shifts of 4 hours, while the term which refers to the fixed administrative cost for every opened shift was extended to the new type of shift s=5.

Decision variables:
y_{ts} ={1,0}
X_{ts} = amount of staffs assigned to a shift, start at t, shift type s

t = {0,1,...23} starting time
s ={3,4,5} type of shift, 3 refers to taking break after 3 hour of work, 4 refers to taking the break after 4 hours of work and 5 refers to 4-hours shift.

Objective function:

$$Min \sum_{t=23}^{3} \sum_{s=3}^{4} 1{,}25 * C * X_{ts} + \sum_{t=4}^{22} \sum_{s=3}^{4} C * X_{ts} + \sum_{t=23}^{3} \sum_{s=5}^{5} 1.25 * C4 * X_{ts} + \sum_{t=4}^{22} \sum_{s=5}^{5} C4 * X_{ts} + \sum_{t=0}^{23} \sum_{s=3}^{5} S * y_{ts}$$

$$\textbf{for } t = \begin{cases} t & \leq 23 \\ t - 24 > 23 \end{cases}$$

S=1000 shift cost
C=7 * 240 hourly wage for 8-hour shift
C4=4*240 hourly wage for 4-hour shift

s.t.

$$\sum_{t=0}^{23} \sum_{s=3}^{5} X_{ts} \leq 155 \qquad (1)$$

$$\sum_{t=i-7}^{i} \sum_{s=3}^{5} X_{ts} \geq d_i$$ for {t=i-7|s≠5}, {t=i-6|s≠5}, {t=i-5|s≠5}, {t=i-4|s≠5,s≠4} and {t=i-3|s≠3} (2)

i = {0,1,...23} hour

$$X_{ts} \leq M \cdot y_{ts} \qquad (3)$$

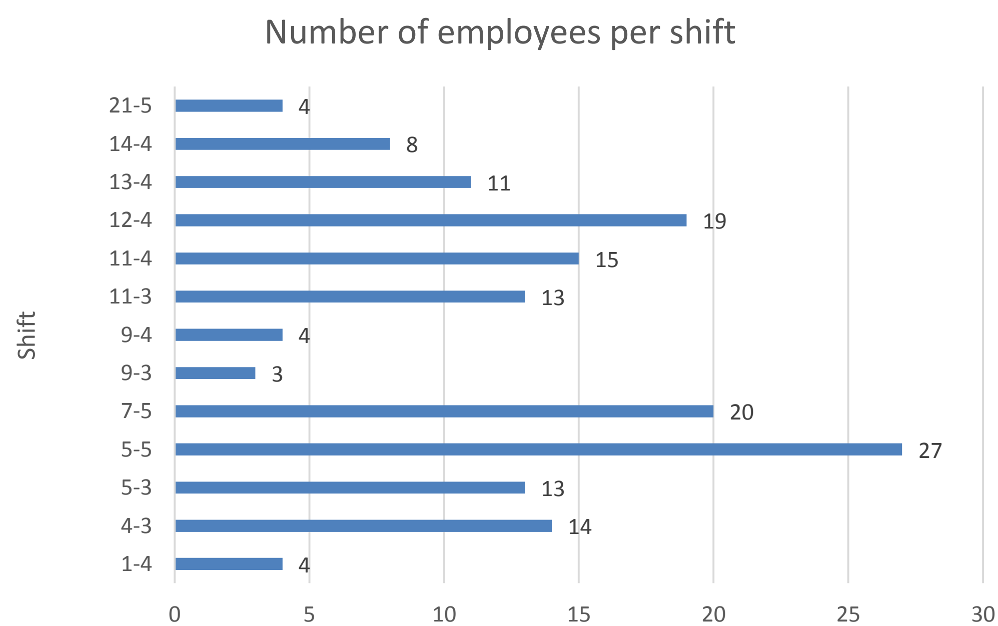
7. What is the optimal mix of 8-hour and 4-hour shifts.

The mathematical model extended for Part B was solved using OpenSolver, the calculations are included in Question1.xls.
A total number of 13 shifts resulted to be optimal for this case: 3 4-hours shift and 10 8-hours shift. It is considered a shift a group of employees who start to work in a determined hour with the same time for the meal-break (only considered for the 8 hours shifts) and who finish their working time simultaneously.
According to the results, the shifts that should be opened are:

Type Shift	Shift	N° Employees
8-hour shift	1-4	4
8-hour shift	4-3	14
8-hour shift	5-3	13
4-hour shift	5-5	27
4-hour shift	7-5	20
8-hour shift	9-3	3
8-hour shift	9-4	4
8-hour shift	11-3	13
8-hour shift	11-4	15
8-hour shift	12-4	19
8-hour shift	13-4	11
8-hour shift	14-4	8
4-hour shift	21-5	4

8. How many staff are required in this case, and what level of overcoverage is incurred in your solution?

The total amount of required staff is 155 employees, they are assigned to 13 different shifts according to the following graph. The total overcoverage incurred is 7 and the overcoverage incurred in each hour is pictured in the graph of question 9.



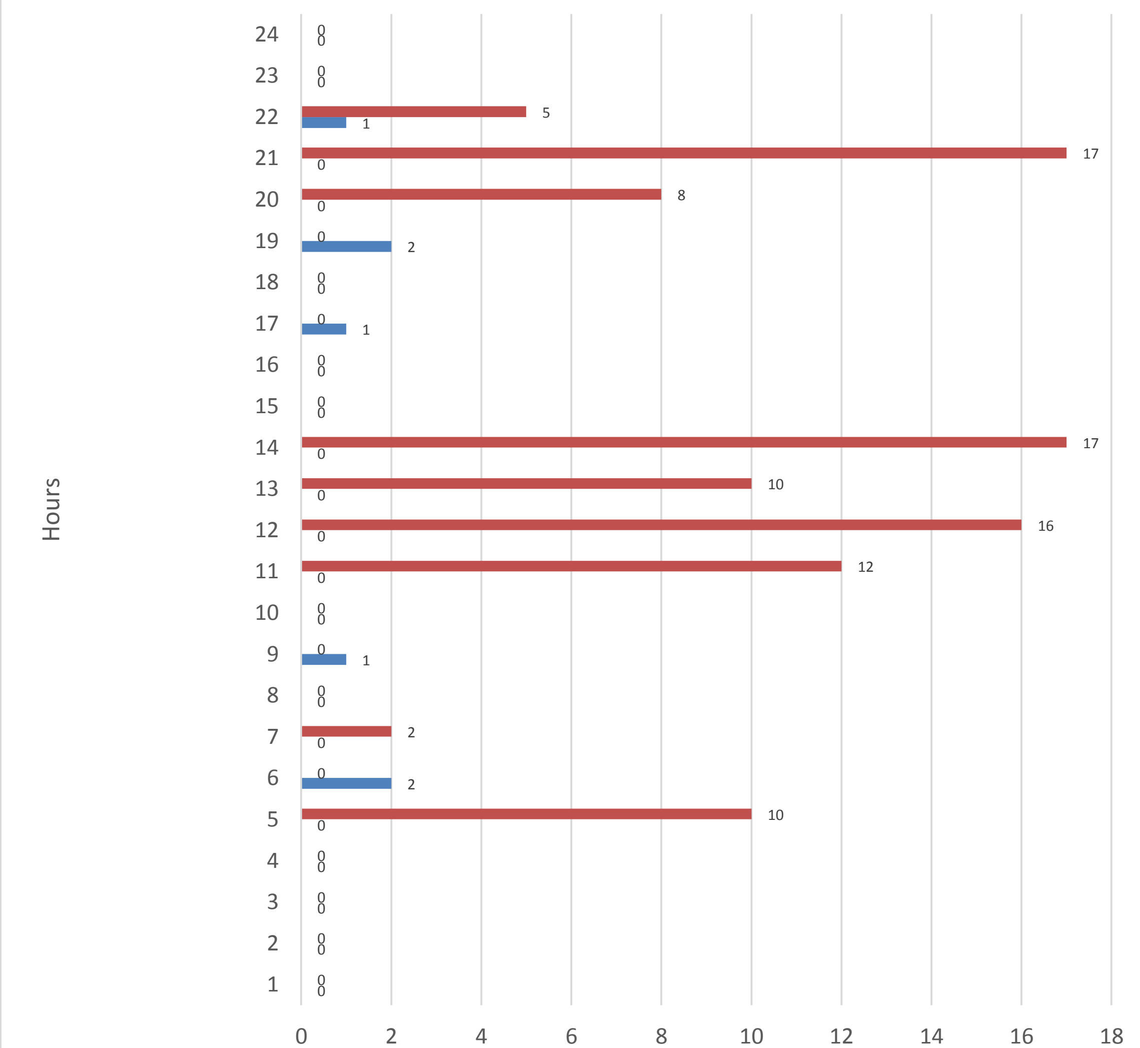
	1-4	4-3	5-3	5-5	7-5	9-3	9-4	11-3	11-4	12-4	13-4	14-4	21-5
N° Employees	4	14	13	27	20	3	4	13	15	19	11	8	4

Number of employees

9. Explain your observations to Questions A4 and B8.

The optimal solution for Part A which only considers 8-hours shift incurs in a 97 employee*hour overcoverage while in Part B the overcoverage is reduced to 7 employee*hour. We can see that 4-hours shift adapt better to the demand requirements and therefore the employee overcoverage is reduced. The correct mix of 8-hours shift and 4-hours shifts allows us to get a better solution, reducing the total cost from 258.960DKK to 238.360 DKK.

The following graphs shows the comparison of overcoverage per hour for both analyzed situations.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
8 hour shift	0	0	0	0	10	0	2	0	0	0	12	16	10	17	0	0	0	0	0	8	17	5	0	0
4 and 8 hour mixed shift	0	0	0	0	0	2	0	0	1	0	0	0	0	0	0	0	1	0	2	0	0	1	0	0

It is also interesting to consider when comparing the results of Part A and B that although Part B arrives to a lower total cost, it also requires a higher number of employees and shifts and also a new type of shift. This can be seen as a complication from an administrative point of view.

	Total Cost	N° of Shifts	N° of Employees
A	258960	12	146
B	238360	13	155

Part C

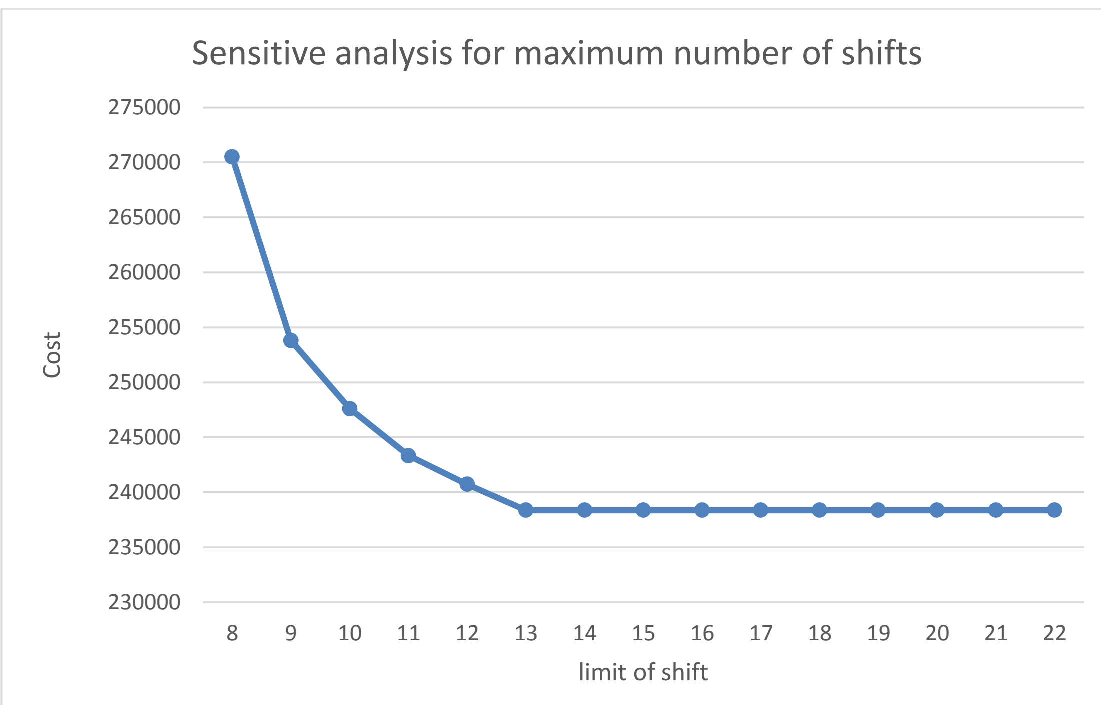
GroundCo is of the opinion that too many different shifts is, from an administration perspective, difficult to oversee. They would like to limit the number of shifts to at most 10.

10. What do you recommend?

To evaluate the administration opinion we entered a new constraint to our previous formulated model, which limits the total amount of shifts allowed in any 24-hour period. The constraint is stated:

$$\sum_{t=0}^{23}\sum_{s=3}^{5} y_{ts} \leq \textit{limit of shifts}$$

We carried out a sensitive analysis for different limits of shifts and calculated the total cost for each of the situations.



From the graph we can clearly see that 13 shifts is the optimal solution and therefore the cost remains constant for shift limits of 13 or more. We can also observe that the cost increases exponentially as the shift limit decreases.
The cost when the shift limit is reduced to 10 is 1.04 times higher than the optimal. Although it is not the best solution from a financial point of view, this solution can be considered suitable because it makes the administrative process easier. However, it is not recommended to reduce the shift limit beneath 10 as the cost will increase considerably.

11. Discuss briefly how you could extend your model to ensure that certain time intervals are more preferable to overcover than others.

In order to extend the model to ensure that certain time intervals are more preferable to overcover than others, we insert an adjusting variable α_t as a weight to the employee overcoverage at each hour. The vector α_t can have different weights for each hour or time interval.

Then the objective function that will be minimized will be the following:

$$Min \sum_{t=23}^{3} \sum_{s=3}^{4} 1{,}25 * C * X_{ts} + \sum_{t=4}^{22} \sum_{s=3}^{4} C * X_{ts} + \sum_{t=23}^{3} \sum_{s=5}^{5} 1.25 * C4 * X_{ts} + \sum_{t=4}^{22} \sum_{s=5}^{5} C4 * X_{ts} + \sum_{t=0}^{23} \sum_{s=3}^{5} S * y_{ts}$$
$$+ \sum_{t=0}^{23} \sum_{s=3}^{5} \alpha_t * (X_{ts} - d_t)$$

Question two

Table 1: *Job information*

Job i	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Release Date	2	5	4	0	0	8	9
Processing Time	5	6	8	4	2	4	2
Due Date	10	21	15	10	6	26	22

1. How many unique sequences are there for the seven jobs?

We are searching for the unique sequences of the jobs without putting into consideration due dates. The different jobs are 7 in number and since each job can be fixed only once in the sequence, so the number of the unique different sequences will be given of the factorial of the number of the jobs. So:

Possible sequences = 7! = 5040

2. Formulate a MIP model which minimizes the makespan of the seven jobs

In this problem we have:

- R_i release date
- P_i Processing time
- D_i Due date

We have to minimize makespan (which is the total length of the schedule, which will be symbolized as C_{max}), using one machine for all 7 jobs. In order to do that, we formulate the following linear problem:

Decision Variables:

$Y_{ij} \in \{0,1\}$	$\forall i,j \in \{1...7\}$	Y_{ij} binary variable
S_j	$\forall j \in \{1...7\}$	S_j integer variable

Formulation:

Objective function:

$$Min \sum_{i=1}^{7} C_{max} = S_{j=7} + \sum_{i=1}^{7} Y_{ij=7} * P_i$$

s.t.

$\Sigma_i Y_{ij} = 1$	$\forall i \in \{1...7\}$	
$\Sigma_j Y_{ij} = 1$	$\forall j \in \{1...7\}$	
$S_j \geq Y_{ij} * R_i$	$\forall i,j \in \{1...7\}$	(1)
$S_{j=1} \geq 0$		
$S_{j+1} \geq S_j + Y_{ij} * P_i$	$\forall i,j \in \{1...7\}$	(2)

We indicate whether or not a job i is set on a position j by using the binary variable Y_{ij}. Analyzing the constrains listed, by using the constraint (1), we assure that the start time of the position j is greater or equal the release time of the job i completed in position j. Using constraint (2) we define that the job i at position j will start only after the previous job at position j-1 has been completed.

3. Solve your model using OpenSolver. Report the optimal solution. Is this unique?

The previous model was solved using Open Solver which is attach in Question2.xls. We got the optimal solution equal to 31 for this specific sequence of jobs below:

5 – 4 – 1 – 6 – 7 – 3 – 2

This is NOT a unique sequence because we may get into the same optimal solution equal 31 changing jobs order. For example:

5 – 4 – 1 – 7 – 6 – 2 – 3

4. For each of the following objective functions, find a job sequence that minimizes it. For each objective function, you should indicate any changes that are required to your initial model.

(a) The minimum sum of the completion times for each of the seven jobs

We need to minimize the sum of the completion times for each of the 7 jobs. If C_{ij} is the completion time of each of jobs at the position j, then the constraints for the previous model will remain the same and the objective function will change into:

$$Min \sum_{j=1}^{7} \sum_{i=1}^{7} C_{ij} = \sum_{j=1}^{7} S_j + \sum_{j=1}^{7} \sum_{i=1}^{7} Y_{ij} * P_i$$

In this case, the minimum of the sum of the completion times is equal to 103 and the completion time is 31.

The job sequence results to be: 5 – 4 – 1 – 7 – 6 – 2 – 3

(b) The minimum number of tardy Jobs

We need to minimize the sum of the number of tardy jobs. Each job i corresponds to a specific position j in the sequence. Then, the number of tardy jobs will relate to the number of tardy sequences. If U_j is the number of tardy sequences, then the problem will change into:

Decision Variables:

$U_j \in \{0,1\}$	$\forall j \in \{1...7\}$	U_j binary variable
$Y_{ij} \in \{0,1\}$	$\forall i,j \in \{1...7\}$	Y_{ij} binary variable
S_j	$\forall j \in \{1...7\}$	S_j integer variable

Formulation:

Objective function:

$$Min \sum_{j=1}^{7} U_j$$

s.t.

$\Sigma_i Y_{ij} = 1$ $\quad \forall i \in \{1...7\}$

$\Sigma_j Y_{ij} = 1$ $\quad \forall j \in \{1...7\}$

$S_j \geq Y_{ij} * R_i$ $\quad \forall i,j \in \{1...7\}$

$S_{j=1} \geq 0$

$S_{j+1} \geq S_j + Y_{ij} * P_i$ $\quad \forall i,j \in \{1...7\}$

$M * U_j \geq S_j + Y_{ij} * P_i - Y_{ij} * D_i$ $\quad \forall i,j \in \{1...7\}$

$U_j \in \{0,1\}$ $\quad \forall j \in \{1...7\}$

$Y_{ij} \in \{0,1\}$ $\quad \forall i,j \in \{1...7\}$

Minimum of tardy jobs for our problem is 2 (job 1 and job 6).

The job sequence results to be: 5 – 4 – 3 – 2 – 7 – 1 – 6

(c) The total minimum tardiness

We need to minimize the total tardiness. If T_i is the tardiness, then the problem will change into:

Decision Variables:

T_j	$\forall j \in \{1...7\}$	T_j integer variable
$Y_{ij} \in \{0,1\}$	$\forall i,j \in \{1...7\}$	Y_{ij} binary variable
S_j	$\forall j \in \{1...7\}$	S_j integer variable

Formulation:

Objective function:

$$Min\ \sum_{j=1}^{7} T_j$$

s.t.

$\sum_i Y_{ij} = 1 \quad \forall i \in \{1...7\}$

$\sum_j Y_{ij} = 1 \quad \forall j \in \{1...7\}$

$S_j \geq Y_{ij}*R_i \quad \forall i,j \in \{1...7\}$

$S_{j=1} \geq 0$

$S_{j+1} \geq S_j + Y_{ij}*P_i \quad \forall i,j \in \{1...7\}$

$T_j \geq S_j + Y_{ij}*P_i - Y_{ij}*D_i \quad \forall i,j \in \{1...7\}$

$T_j \geq 0 \quad \forall i \in \{1...7\}$

Minimum total tardiness is equal to 18 with 3 tardy jobs (job 1, 7 and 3).

The job sequence results to be: 5 – 4 – 1 – 6 – 2 – 7 – 3

Question three

1. Calculate

(a) rb of old line = 160 (parts/hr), rb of new line = 420 (parts/hr)

For old line:

$$\text{Rate in} = \frac{1260}{8} = 157.5$$

$$\text{Utilization of cutting} = \frac{rate\ in}{capacity} = \frac{157.5}{30\times 8} = 0.65$$

$$\text{Utilization of bending} = \frac{rate\ in}{capacity} = \frac{157.5}{24\times 8} = 0.82$$

$$\text{Utilization of assembly} = \frac{rate\ in}{capacity} = \frac{157.5}{40\times 4} = 0.98$$

(the highest utilization defines the bottleneck)

$$\text{Utilization of finishing} = \frac{rate\ in}{capacity} = \frac{157.5}{100\times 2} = 0.78$$

$\rightarrow \text{Rb of old line} = 40 \times 4 = 160$

For new line:

$\text{Rate in} = \frac{2720}{8} = 340$

$\text{Utilization of cutting} = \frac{rate\ in}{capacity} = \frac{340}{240\times2} = 0.71$

$\text{Utilization of bending} = \frac{rate\ in}{capacity} = \frac{340}{210\times2} = 0.81$

(the highest utilization defines the bottleneck)

$\text{Utilization of assembly} = \frac{rate\ in}{capacity} = \frac{340}{250\times2} = 0.68$

$\text{Utilization of assembly} = \frac{rate\ in}{capacity} = \frac{340}{250\times2} = 0.68$

→$\text{Rb of old line} = 210 \times 2 = 420$

(b) To of old line = 0.44 (hrs) , To of new line = 0.07 (hrs)

For old line:

$\text{To} = \frac{(8.0+10.0+6.0+2.4)}{60} = 0.44$

For new line:

$\text{To} = \frac{(1.00+1.00+0.96+0.96)}{60} = 0.07$

(c) Wo of old line = 70.40 , Wo of new line = 29.40

For old line:

$\text{Wo} = \text{rb} \times \text{To} = 160 \times 0.44 = 70.40$

For new line:

$\text{Wo} = \text{rb} \times \text{To} = 420 \times 0.07 = 29.40$

(d) Old line has the larger critical WIP.

It is the result of relatively lower throughput and higher cycle time.

2. Compare the performance of the two production lines to the practical worst case. What can you conclude about the relative performance of the two production lines compared to their underlying capabilities? Is management correct in criticizing the old line for inefficiency?

For old line:

Actual throughput = $\frac{1260}{8} = 157.5$

THpwc = $\frac{W}{(Wo+W-1)} \times rb = \frac{200}{(70.40+200-1)} \times 160 = 118.78$

Capacity (rb) $= 160$

- ➔ The fact that actual throughput of old line is higher than this level indicates that the performance of old line is better than that in the practical worst case.
- ➔ Actual throughput old line is 98% of capacity. Compared to the new line, the old line's actual throughput has higher achievement to its capacity.

For new line:

Actual throughput = $\frac{2720}{8} = 340$

THpwc = $\frac{W}{(Wo+W-1)} \times rb = \frac{175}{(29.40+175-1)} \times 420 = 361.36$

Capacity (rb) $= 420$

- ➔ The fact that actual throughput of new line is lower than this level indicates that the performance of new line is more worse than that in the practical worst case.
- ➔ Actual throughput of new line is 80% of capacity

Conclusion : Management is not correct in criticizing the old line for inefficiency because of the fact we expressed above.

3. If you were the manager in charge of these lines, what option would you consider first to improve throughput:

a) of the old line?

If we have not constraints with working time and extra payment after 8 hours, we would increase the working shifts by 2 hours. If we do that, we improve throughput by 25% which will be 200 parts/hour. If we cannot, we will buy a new machine in the process with the bottleneck, so the throughput will be improved to *192 parts/hour.

b) of the new line?

Compared to the old line, the new line has bigger potential to be improved. If we increased the shifts by 2 hours the throughput will be improved to 525 parts per hour. If we buy a new machine, the throughput will improve to 480 parts/hour.

Concluding, the best solution is to add 2 hours in the working shift for both lines, otherwise we will buy new machines and will put them at the processes with the bottlenecks.

*If we buy a new machine for the **old** line, the new bottleneck will be in Assembly
Capacity of the Assembly process=24*8=**192 parts/hour**
If we buy a new machine for the **new** line, the new bottleneck will be in Cutting
Capacity of the Cutting process=240*2=**480 parts/hour**