Technical University of Denmark

42401: Introduction to Planning

PROJECT 2

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Introduction to planning

1. **Develop a Linear Programming (LP) model to determine the aggregate plan for *MacPherson*. Describe the variables you use, your objective function and the constraints.**

The purpose of the following LP model is to find the minimum total cost of the aggregate plan regarding to the human capital cost (which includes salaries, hiring cost and layoff cost) and inventory holding cost.

Objective function:

$\text{Min. } 2400 \cdot W_i + 1800 \cdot H_i + 1200 \cdot L_i + 3300 \cdot O_i + 8 \cdot I_i$

Decision variables:

$W_i = Number\ of\ workers\ in\ month\ i$

$H_i = Number\ of\ worker\ hired\ in\ month\ i$

$L_i = Number\ of\ worker\ fired\ in\ month\ i$

$O_i = Number\ of\ overtime\ worker\ in\ month\ i$

$I_i = Inventory\ in\ month\ i$

$X_i = Production\ units\ in\ month\ i$

$S_i = Number\ of\ shipment\ forecast\ per\ month\ i \quad (demand\ in\ month\ i)$

$for\ i = \{1,2,3, \ldots ..12\}$

$W_i , H_i , L_i , O_i , I_i , X_i , S_i \ \geq 0$
$W_i , H_i , L_i , O_i , I_i , X_i , S_i \ \in I$

Constraints:

$X_i \leq 13000$ --Production units in month i shall not exceed the capacity of the plant in certain month

$(W_i + O_i) \cdot 40 \geq X_i$ --Production units in month i shall not exceed the maximum production units that all the workers can produce

$I_i = I_{i-1} + X_i - S_i$ –Inventory in month i is regarding to inventory level of month (i-1), production units in month i, and shipment forecast in month i

$I_1 = 240 + X_1 - S_1$ --Inventory in the 1st month is regarding to inventory level in the end of last December, production units in the 1st month , and shipment forecast in the 1st month

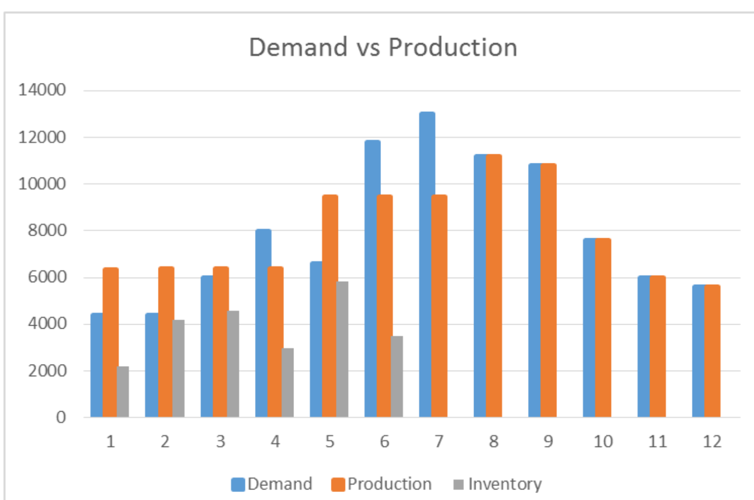
$W_i = W_{i-1} + H_i - L_i$ –Number of workers in month i is regarding to number of workers in month $(i-1)$, number of workers hired in month i, and number of workers fired in month i

$W_1 = 160 + H_1 - L_1$ –Number of workers in the 1st month is regarding to the initial human power level, number of workers hired in the 1st month , and number of workers fired in the 1st month

2. Provide the aggregate plan, similar to the tables in the case study. Evaluate the resulting plan. What are its advantages? What are its disadvantages?

With the results of our LP model solved by Opensolver, an aggregate plan is built as following,

Month		Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Totals
Production Plan															
Shipment forecast			4400	4400	6000	8000	6600	11800	13000	11200	10800	7600	6000	5600	95400
Production Plan			6347	6400	6400	6400	9453	9480	9480	11200	10800	7600	6000	5600	95160
Shipments			4400	4400	6000	8000	6600	11800	13000	11200	10800	7600	6000	5600	95400
Inventory		240	2187	4187	4587	2987	5840	3520	0	0	0	0	0	0	23308
Extraordinary Labour Costs															
Number of Workers		160	160	160	160	160	237	237	237	237	237	191	151	151	2318
Hirings			0	0	0	0	77	0	0	0	0	0	0	0	77
Layoffs			0	0	0	0	0	0	0	0	0	46	40	0	86
Worker Months Overtime			0	0	0	0	0	0	0	43	33	0	0	0	76
Cost of Alternative 4															
Hiring Costs	77X1800= 138600														
Layoff Costs	86X1200= 103200														
Inventory Holding Costs	23308X8= 186464														
Labour Costs															
Regular	2318X2400= 5563200														
Overtime	76X3300= 250800														
Total Costs	6242264														



According to the resulting plan, the advantages and disadvantages are as following,

Advantages:

1. Low risk of aging stock

During the second half of the year, units produced in each month could be sold out within the same month, which means the age of stock would keep no more than one month.

2. Employee-friendly labour policy

Low frequency of layoff and over-time (only twice each within the year) would keep the employee morale and good union relations.

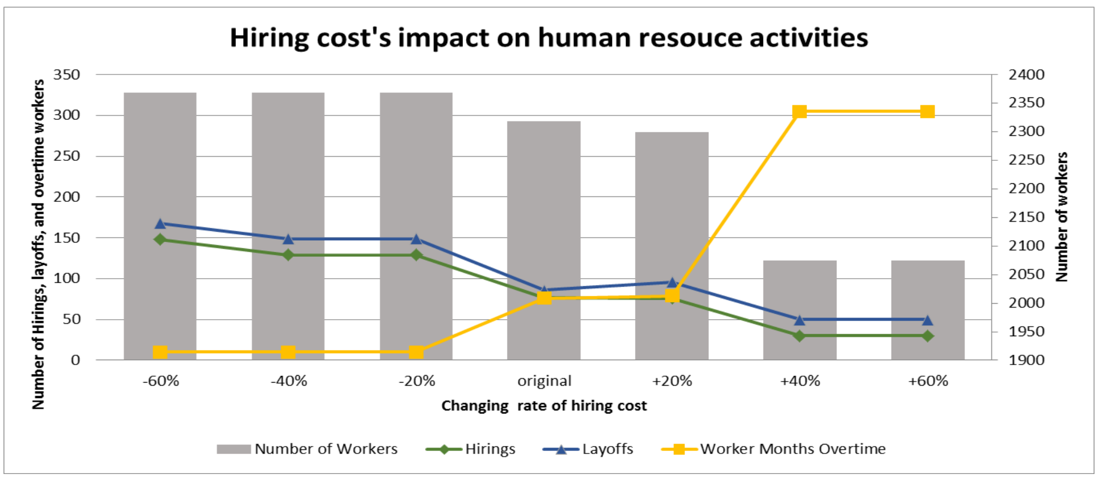
Disadvantages:

Risk of inventory shortage. The inventory level in the second half of the year might not able to react to unexpected increase of demand.

3. Analyze the impact of the flexibility of the labour market.

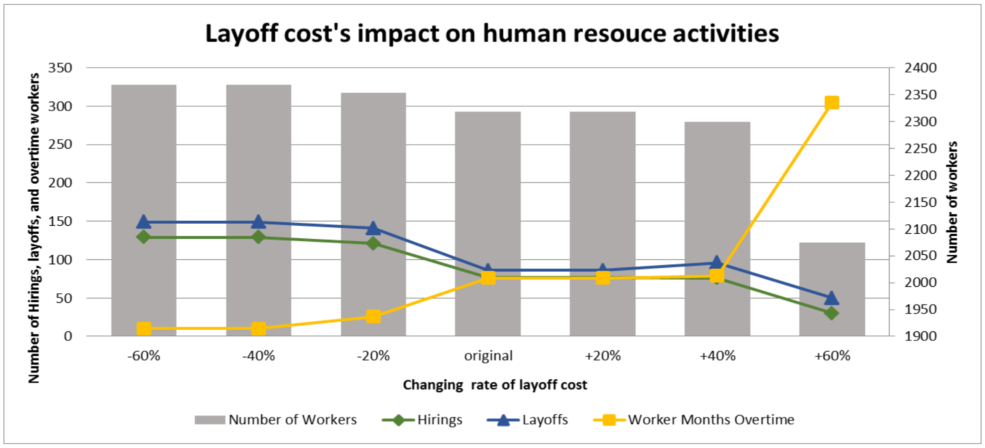
With the aggregate plan as a result of question 2, we analyze the impact of flexibility of the labour market on the company's human resource activities, which is reflected in the numbers of workers, hirings, layoffs, and worker months overtime, as following.

1. Change in hiring cost



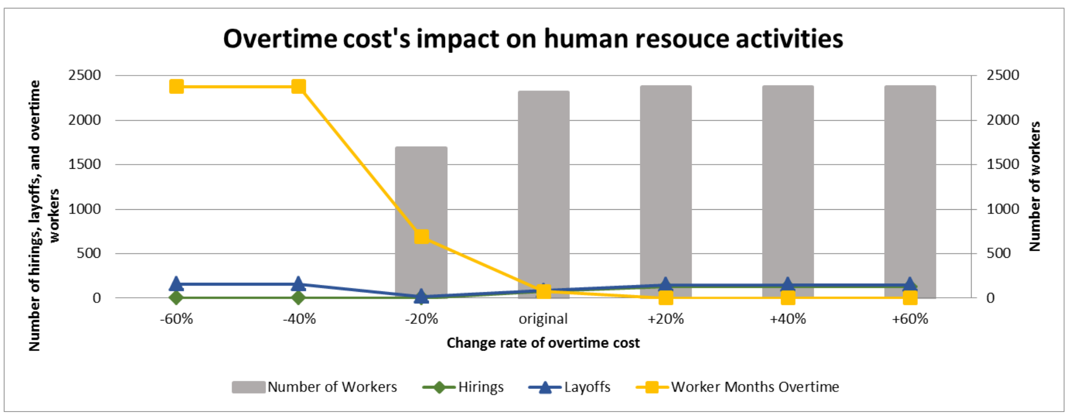
Generally speaking, the numbers of hirings and layoffs are inversely proportional to the hiring cost. Consequently, the frequency of hiring and firing brings corresponding number of workers, which also varies directly with hiring cost. Worker overtime and hiring cost are basically in direct proportion, which means the company would tend to hire less workers and increase the overtime amount of them while hiring cost increases. However, these four factors are not sensitive to the changes of hiring cost in it's changing rate intervals of -20% to -40% and +40% to +60%, which means there are ceilings existing in the impacts of hiring cost on the changes of these four factors.

2. Change in layoff cost



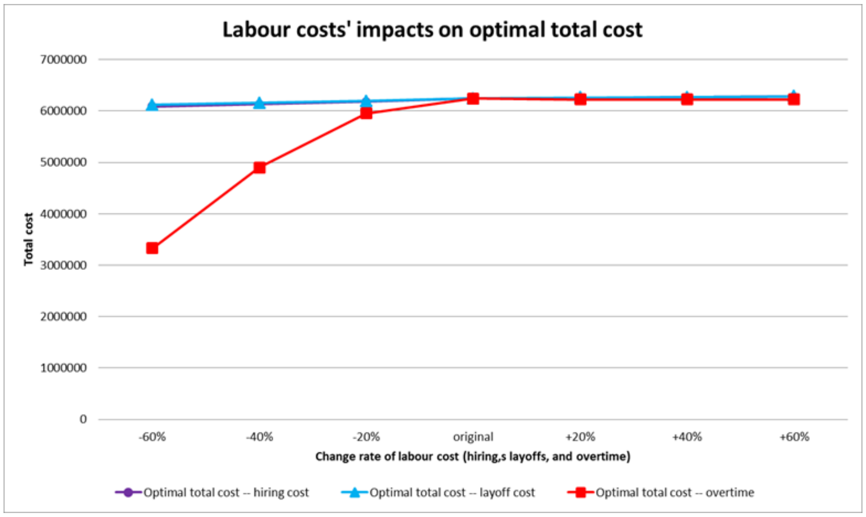
Compared with the impact of hiring cost, change in layoff cost influences the company in a similar way. The main difference between them is in the ceiling point. Whilst the change of hiring cost cause non-sensitive intervals in -20% to -40% and +40% to +60%, the change of layoff cost leads to non-sensitive intervals in -40% to -60% and 0% (original) to +40%. It means that when the hiring/layoff cost decrease, the company's reaction would be more sensitive to the change of layoff cost. However, when the hiring/layoff cost increase, it would be more sensitive to the change of hiring cost.

3. Change in overtime cost



Decrease in overtime cost would makes the company sharply reduce its number of workers and transfer its human resource budgets to pay for overtime. The company is also sensitive to the increase of overtime cost, which inflected in the plan without any overtime workers while the overtime cost increases more than 20%.

Furthermore, comparisons between these three labour costs' impacts on the optimal total cost as following charts indicate that the change of overtime cost is more beneficial to the company due to it would lead to the most significant reduction of the total cost.



However, in reality, the possible reactions of the company to labour market flexibility might not be the same as we analyzed according to the aggregate plan. Dramatic reduction in worker number, increase in overtime, and frequent layoffs will bring the company pressures from the unions and also lower productivity of workers.

4. Explain how your LP model could be adapted to consider that inventory should not be older than two weeks. What it would mean for the total cost. Would you consider Linda's idea a good solution?

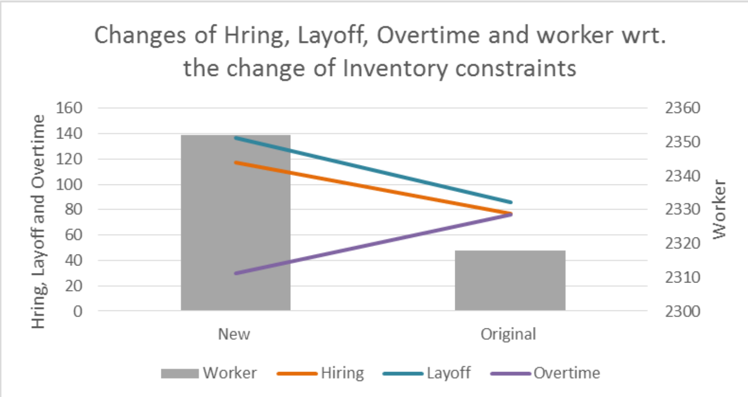
Considering Linda's idea, the LP model needs constraints such that products can be in inventory at maximum two weeks. Since we do not have weekly information about inventory, production and demand, we can only assume that production and demand within one month are equally distributed, which makes the problem easier to solve. The intuition is to consider one month has two weeks, and I_i is the amount of inventory at the end of a month. Firstly, we think forwards that the I_i should not be bigger than half of the demand of next month. Which ensures that all of the I_i will be shipped out before the middle of the next month, thus we meet our purpose that the product in inventory is not older than two weeks. This idea induces a constraint:

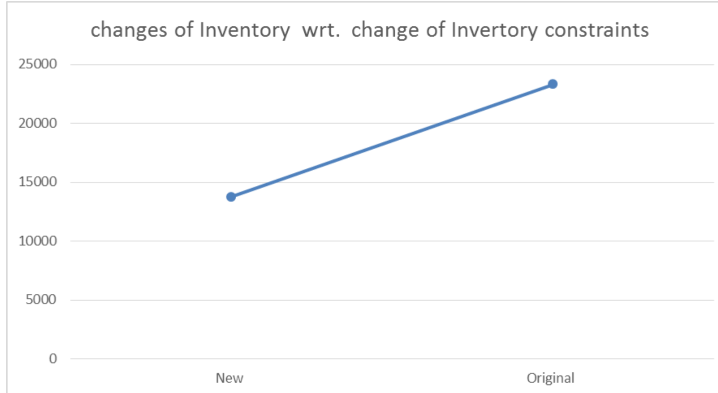
$$I_i \leq \frac{S_{i+1}}{2} \tag{4.1}$$

However, it is not enough, we have to think backwards about what happens in the earlier two weeks. Bearing in mind the recursive process that there will be no products left in inventory by the middle of a month due to the constraints 4.1, as well as some of the products might be shipped to meet the demand if there were any. So the only inventory left must be less than or equal to the amount of the products by the end of the same month. This leads to another constraint:

$$I_i \leq \frac{X_i}{2} \tag{4.2}$$

With the two new constraints added the total cost lowers to \$6228816 in comparison with the original \$6242264, which is 0.25% drop. I_i drops about \$10000 that is almost half of original inventory, whereas the H, L increases double so much, O drops a lot as well.





This makes sense, as the drastically dropping inventory cuts considerable amount of cost, which in turn results in more hiring and layoff and less overtime. The LP model is adapted in the way that the total cost is lower, but it would be difficult to execute, as hiring 117 in May and firing 70 in October would not be an easy task for the HR department, thus Linda's idea should be thought carefully.

	New	Original
Optimal	6228816	6242264
Inventory	13752	23308
Worker	2352	2318
Hiring	117	77
Layoff	137	86
Overtime	30	76

5. Design the Kanban system for the described case and answer the sub-questions below. You can also make graphical illustration of the system showing the order and number of machines as well as the component number and demand, to create a better understanding it. Use minutes as your standard time unit.

5.1. How many parallel machines are required at each workstation to meet the daily demand, considering the machine availability?

In this part we are going to design the Kanban system for our case. A one-card Kanban system of 5 units of container capacity is used. We calculate the required number of parallel machines for every workstation to meet the daily demand.

First we compute the total amount of components and total time for each workstation (Blanking, Blending and Foaming), while we also compute the total amount of products that will be assembled and the total time for the Assembly. We calculate the total available time per machine for each workstation considering the working shifts and the machine availability, for blanking 80 %, bending and foaming equal to 85 % whereas availability of machines used for assembly we assumed to be 100%. To meet the demand we must set up at the blanking station 3 parallel machines, for bending station 7 and for foaming and assembly stations 12 parallel machines have to be arranged.

WS	Number of units	Time (min)	Available Time	Required Machines	MACHINES
Blanking	1076	1836,1	720	2,55	**3,00**
Bending	1076	4900,6	765	6,41	**7,00**
Foaming	918	8984	765	11,74	**12,00**
Assembly	422	10592	900	11,77	**12,00**

5.2. How many Kanban cards are required at each workstation including the sourced components?

The number of Kanban cards was calculated for each component of each workstation using the following equation:

$$y = \frac{D*(Tw+Tp)(1+\alpha)}{a}$$

Where,

$$y = Number\ of\ Kanban$$

$$D = Demand\ per\ unit\ time$$

$$T_w = Waiting\ time\ of\ Kanban$$

$$T_p = Processing\ time$$

$$a = Container\ capacity$$

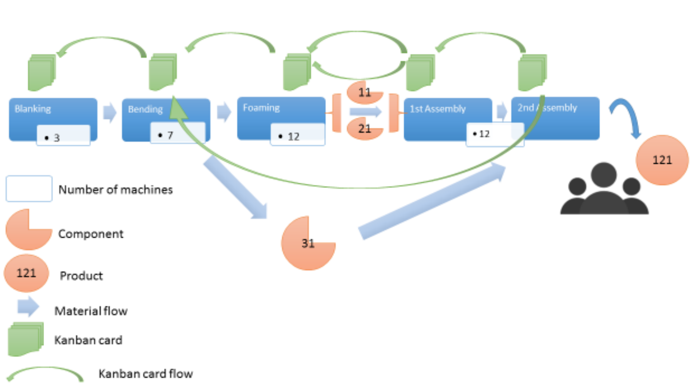
$$\alpha = Policy\ variable$$

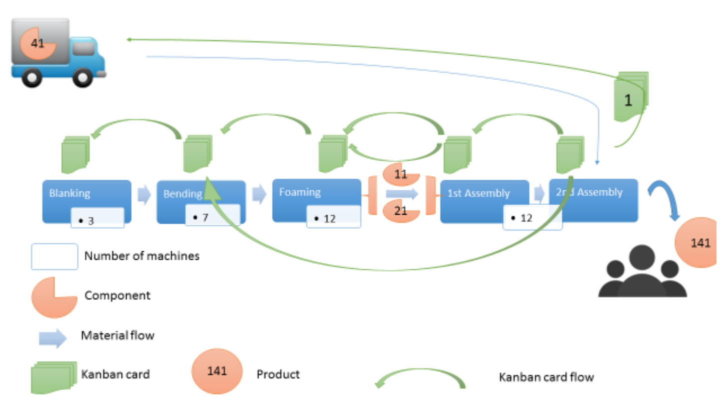
The results for number of cards were rounded up since it would make no sense to have half a card. Then, the calculated cards were added up to know the total amount of required cards for each workstation. The required number of cards are:

Blanking station = 7
Bending station = 8
Foaming station = 12
Assembly station = 15
and 2 Kanban cards used to transport outsourced components.

The total number of Kanban card for the system is 44.

The graphical illustration of the system to produce final products 121 and 141 follow:





5.3. What is the maximum WIP level of the system, considering the current number of Kanban cards and container capacity?

Considering the current number of Kanban cards equal to 44 and container capacity equal to 5 units, the maximum work in process (WIP) level of the system will be equal to their product.

WIP = 220 units

6. Linda would like to improve her current setup of the production system. For that she estimates that the current performance can be calculated based on the practical worst case (PWC) scenario, by using the average process times (processing time + setup time) for each work station in [minutes/ unit]. Help Linda to identify her current performance and to further improve it, by answering the following sub-questions.

6.1 Calculate the critical work in process W0 of the current production system:

6.1.1 Which workstation is the bottleneck of the production system and what is the amount of r_b in [units/ minute]?

The bottleneck of the production system will be the workstation with the lowest throughput and highest utilization. As it displayed at table 6.1.1.1, Assembly is the workstation with the lowest throughput and highest utilization, and thus the bottleneck of the system. Moreover, having calculated the number of machines per workstation as well as the average process time, the Throughput (units per minute) for each workstation was calculated.

Table 6.1.1.1: *Identification of the bottleneck and calculation of Throughput for each workstation*

WS	Utilization	Average Process Time per WS	TH
Blanking	85%	1,76	1,71
Bending	92%	5,26	1,32
Foaming	97,86%	11	1,09
Assembly	**98,07%**	**29,22**	**0,41**

Thus, **r_b = 0,41 units/minute**

6.1.2 What is the raw process Time T_0 [minutes/unit] of the production system?

The raw process time of the production system will be the sum of the long term average process times of each workstation (presented at the table 6.1.1.1). The sum will be:

T_0 = 47,24 min/unit

6.1.3 How much is the critical work in process W_0 ?

Knowing the maximum throughput (r_b) as well as the minimum cycle time (T_0), the Critical WIP will be equal to their product. So:

W_0 = 19,40 units

6.2 Calculate the throughout TH(PWC) of the current production system and compare it to the initially planned throughput to meet the demand:

6.2.1 What is the current throughput TH(PWC) [units/ day] of the system according to the practical worst case (PWC) scenario, when using the current number of Kanban cards and container capacity (i.e. at maximum WIP level)?

Knowing the maximum WIP level of the system (question 5.3) as well as the critical work in progress (question 6.1.3), the Throughput to the practical worst case scenario will be:

$$\mathbf{TH_{PWC}} = \frac{\mathrm{w}}{\mathrm{W_0} + \mathrm{w} - 1}\mathrm{r_b} = \mathbf{0{,}378\ comp/min}$$

6.2.2 Does the current throughput TH(PWC) meet the estimated demand?

With the current throughput, as calculated on 6.2.1, and a total of 900 minutes of production, the total number of components made will be:

TH_{PWC} * 900mins = 340comp

The total demand is known, and equal to 422 comp. Thus, the total demand is not met (81% is met).

6.2.3 Which of the three alternative changes would you suggest to Linda, assuming all of them are equally possible, in order to increase the current throughput TH(PWC)?

A: Increase the container capacity to 10 units for all component at each workstation?
B: Increase the machine availability at each work station to 100%?
C: Reduce all process times by 10%.

The three alternatives will be analyzed separately. We assume the number of the machines fixed and equal to (3,7,12,12) for (Blanking, Bending, Foaming, Assembly) respectively. Starting with ***alternative A***, after increasing the container capacity to 10 units for all components at each station, TH_{PWC} increases to 0,389 units/min, meeting an increased percentage of 83% of the demand met.

By increasing the machine availability at each work station to 100% at ***alternative B***, a TH_{PWC} value equals to 0.378 and gives a total 81% coverage of the demand (the same as the initial values before the change).

Finally, as for the ***alternative C***, two scenarios were examined. Firstly, the reduction of all process times by 10% was applied to the processing time of the components as well as at the assembly time per product, leading to a TH_{PWC} value equal to 0,419 unit/min, and a 89% demand coverage. Then, the reduction was applied to the average time for a component of a batch (reducing by a 10% processing times and also the setup times) while the assembly time per product was also reduced. In this case as well, the TH_{PWC} was equal to 0,419 unit/min, with 89% demand coverage. It is observed that both cases result the same as the bottleneck of the system corresponds to the assembly and this workstation does not have setup times.

Concluding, Alternative C should be chosen. Thus, Linda would be suggested to reduce the process times by 10% in order to increase the current throughput.

Table 6.2.3.1: *TH_{PWC} values, bottleneck workstations and demand coverage for the three alternatives*

		TH_{PWC}	Bottleneck WS	Demand Coverage
Alternative A		0,389	Assembly	83%
Alternative B		0,378	Assembly	81%
Alternative C	**Case 1**	0,419	Assembly	89%
	Case 2	0,419	Assembly	89%

7. Evaluate how well the initial production system from question 5 would meet a demand with a seasonal pattern, where the number of machines per workstation is fixed and only the number of Kanban cards can be adjusted, based on the varying demand

7.1 How well does the throughput TH(PWC) meet the demand (coverage Co), if in high seasons (hs) the demand for each variant increases with 20%? Provide the answer in percent of the actual demand, i.e. coverage

Co(hs) = TH(PWC)(hs)/Demand(hs)?

In this case, the TH_{PWC} is equal to 0.380 units/min, which for the 900 minutes of operation will lead to 342 produced components. The demand increased by 20% in the high season will be equal to 506.4 components. Thus, the coverage will be:

$$\textbf{Coverage}_{\textbf{High Season}} = 342/506.4 = \mathbf{67\%}$$

7.1 How well does the throughput TH(PWC) meet the demand (coverage Co), if in low seasons (ls) the demand for each variant decreases with 20%? Provide the answer in percent of the actual demand, i.e. coverage

Co(ls) = TH(PWC)(ls)/Demand(ls)?

In this case, the TH_{PWC} is equal to 0.374 units/min, which for the 900 minutes of operation will lead to 336 produced components. The, by 20% decreased, demand in the low season will be equal to 337.6 components. Thus, the coverage will be:

$$\textbf{Coverage}_{\textbf{Low Season}} = 336/337.6 \approx \mathbf{100\%}$$