42104 Introduction to Financial Engineering
Final Assignment

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December 8, 2015

1 Portfolio Optimization

1.1 Diversification

In search for an optimal portfolio, based on historical performance, the sector indices listed in Table 1 have been chosen as basis for further analysis. Consequently, the asset allocation for our optimal portfolios will therefore consist of a combination of these seven stock indices. Twenty years of historical data has been collected in monthly intervals. They are all listed in one currency (USD), so we do not need to worry about currency conversions.

Ticker	Name
ˆDJI	Dow Jones Industrial Average
ˆDJT	Dow Jones Transportation Average
ˆDJU	Dow Jones Utility Average
ˆNBI	NASDAQ Biotechnology
ˆXMI	NYSE ARCA MAJOR MARKET INDEX
ˆSOX	PHLX Semiconductor
ˆBKX	KBW Nasdaq Bank Index

Table 1: Set of sector indices selected for further analysis.

In hopes of benefiting from diversification, we chose a selection of indices that represent different market sectors. In hindsight, we know that sectors such as semiconductors peaked along with the dot-com boom and that biotech has had its best years so far in the recent years. As of that, it would be interesting to see how an optimal portfolio, based on historical data, performs the following year - especially protecting profits during a bear market and whether or not a significant turnover favoring a bullish sector will take place. Unfortunately, an index covering precious metals with the required time span was not found, though it could also be of interest to see is the stock market tends to move in the opposite direction compare to metals such as gold.

This section is tagged 'A)' within the source file `project.m`.

1.2 Estimate

We are interested in constructing rolling windows of ten years in annual steps in order to compare the composition of the optimal portfolios over the years. As the first step, we compute the expected yearly return and covariance matrix in each rolling window. Table 2 contains the annualized expected returns for each of the sector indices in each ten-year period. The expected returns and covariances in each rolling window are also plotted as a line graph in Figure 1 and as a heat map in Figure 2, respectively. The corresponding code can be found under section 'B)' in `project.m`.
This section is tagged 'B)' within `project.m`.

	(%)						
Period	^DJI	^DJT	^DJU	^NBI	^XMI	^SOX	^BK
2005–2014	4.925	8.770	6.180	15.35	4.821	4.810	-4.151
2004–2013	4.033	9.265	6.215	11.99	4.743	0.288	-3.882
2003–2012	5.429	9.823	8.249	11.40	5.574	4.184	-3.165
2002–2011	2.417	6.425	4.517	4.361	3.136	-3.124	-6.874
2001–2010	0.882	4.805	0.948	-0.680	2.085	-5.081	-5.921
2000–2009	0.832	4.153	1.825	-1.663	-0.067	9.008	4.971
1999–2008	-1.568	-0.768	1.996	4.133	-1.029	-7.025	-10.43
1998–2007	4.700	3.619	6.467	9.803	4.421	2.164	2.684
1997–2006	6.166	7.493	6.703	9.055	5.669	4.634	7.097
1996–2005	7.000	8.080	5.837	9.214	6.440	10.60	9.356
1995–2004	10.04	8.794	5.758	14.54	10.30	10.63	13.21

Table 2: Annualized expected returns.

1.3 Efficient Frontier

By means of an interest rate R_f that allows risk-less lending and borrowing, we can determine the optimal portfolio by solving a system of linear equations of the following form:

$$\begin{aligned}
\overline{R}_1 - R_f &= Z_1\sigma_1^2 + Z_2\sigma_{12} + Z_3\sigma_{13} + ... + Z_N\sigma_{1N} \\
\overline{R}_2 - R_f &= Z_1\sigma_{12} + Z_2\sigma_2^2 + Z_3\sigma_{23} + ... + Z_N\sigma_{2N} \\
\overline{R}_3 - R_f &= Z_1\sigma_{13} + Z_2\sigma_{23} + Z_3\sigma_3^2 + ... + Z_N\sigma_{3N} \\
&\vdots \\
\overline{R}_N - R_f &= Z_1\sigma_{1N} + Z_2\sigma_{2N} + Z_3\sigma_{3N} + ... + Z_N\sigma_N^2
\end{aligned}$$

where Z_j for $1 \leq j \leq N$ denotes the proportion that should be invested in an asset j.

In our case, $N = 7$, meaning that we have seven equations and seven unknowns for each of the 10-year periods. The expected returns and covariances for each period were found in the previous subsection, and $R_f = 0.03$ is specified as a fixed constant for the scope of this assignment although the exact value is not important in this step. The source code that covers this is `highest_slope_portfolio.m`.

The specific portfolio found represents one of all of the portfolios on the efficient frontier. Another portfolio can be found by simply changing the risk-free rate. Now, treating these two portfolios as assets allows us to find other efficient portfolios and essential trace out the total efficient frontier by

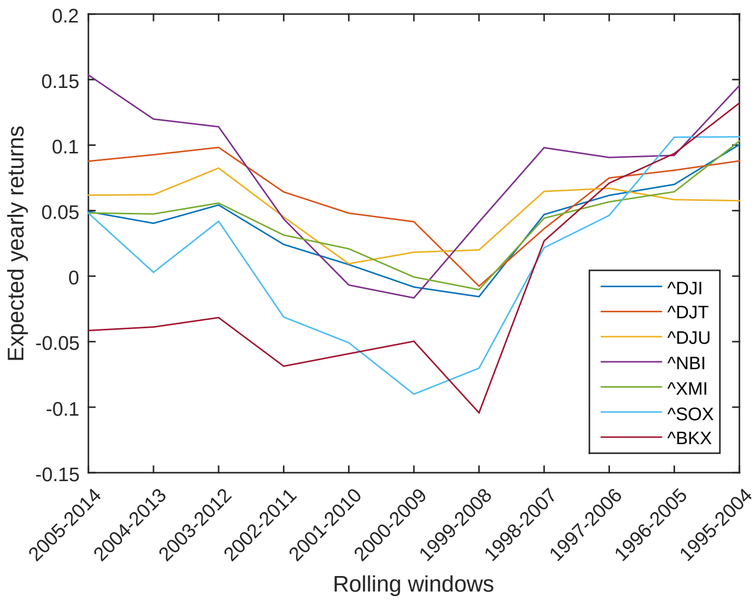

Figure 1: The yearly means of the sector indices in each rolling window.

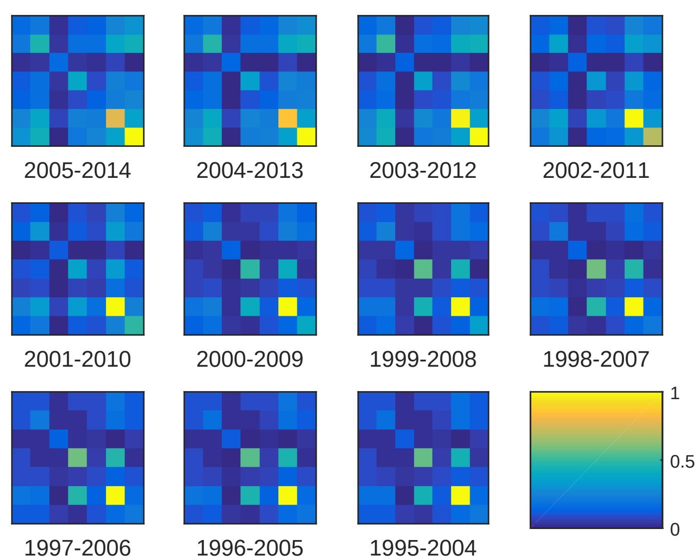

Figure 2: The yearly covariances of the sector indices in each rolling window shown as a heat map.

varying the amount invested in each of them; i.e. we calculate a large quantity of linear combinations with the below relationship between the allocation where w denotes the amount invested in w in percent:

$$opt_x = w \cdot opt_{R_f=3\%} + (1-w) \cdot opt_{R_f=y \cdot 3\%} \quad \text{, where } y \neq 1.$$

In Figure 3, the efficient frontiers for each of the 11 moving windows are shown graphically. These plots illustrate the benefit of diversification. When the securities are perfectly correlated, there is no diversification benefit, i.e. the particular graph would simply be a straight line. Otherwise, having combinations of securities always reduces risk with the magnitude of payoff inversely proportional to their correlation coefficient.

When comparing the frontiers with each other, it seems like the Sharpe ratio gradually increases as we move forward into time. To some extent, this can be explained by the early 2000's recession, the dot-com bubble, and the global crisis in 2007-2008 along with the recovery following such a crisis. Imagining that an investment had to be fixed for a duration of 10 years, the most recent periods would have been the best choice simply because entering the market after the dot-com bobble result in the best upside — again, assuming a fixed investment period of 10 years.

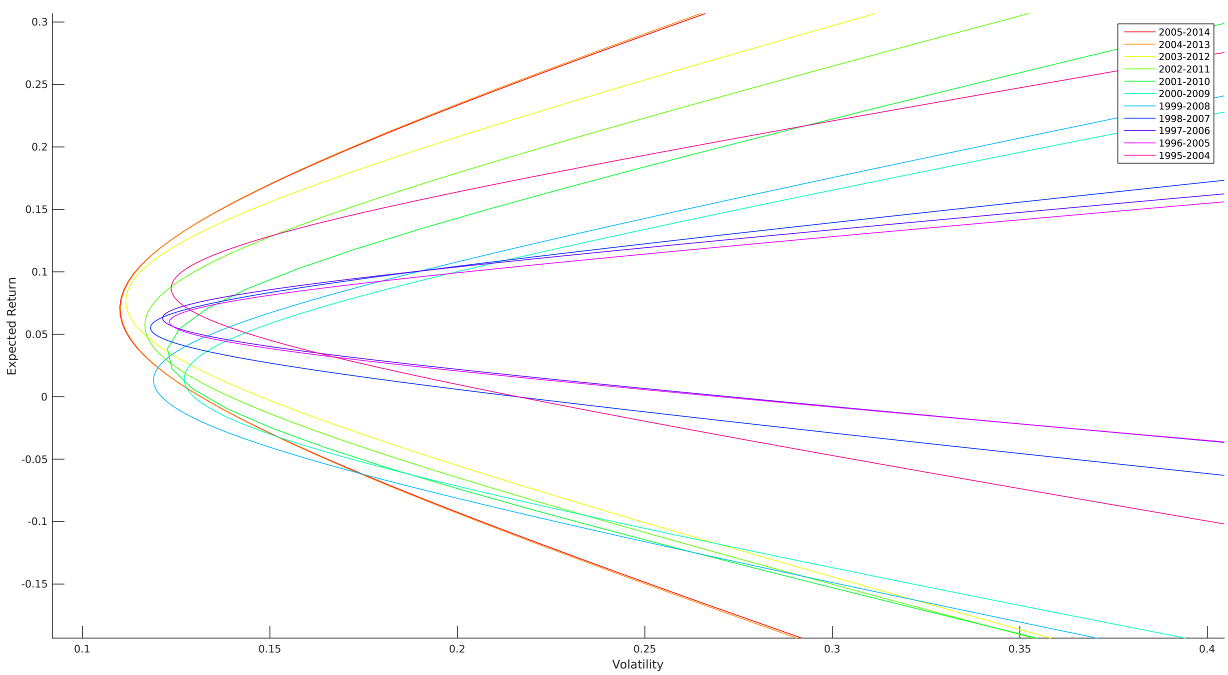

Figure 3: Efficient frontiers for each of the rolling windows

This specific section is tagged 'C)' within `project.m`.

1.4 Tobin Separation

In general, also given the opportunity to invest in a risk-free asset, the efficient frontier becomes the line intersecting the risk-free rate $R_f = 0.03$ and the optimal tangency portfolio. Figure 4 depicts this nicely as all portfolios on the old efficient frontier (with no lending or borrowing at the risk-free rate) except for the tangency one lie below the capital market line. The red crosses represent asset allocations with 100% in the risk-free asset or 100% in the optimal portfolio with $R_f = 0.03$. The green dots depicts the same, but with $R_f = 0.10$. This plot simply illustrates that these new portfolios on the line are superior to those on the curve as measured by the expected return per unit of risk. The particular asset allocation chosen among all the portfolios on the new efficient frontier reflects how risk-averse the investor is and of course also if borrowing at the risk-free rate is allowed. Graphically, the farther to the right a portfolio is positioned, the greater the expected return, but also the greater the risk.

The frontiers for years 1999-2008 and 2000-2009, respectively, does not seem to follow the above description. This is assumed to be due to a general bear market across all the sectors covered by our indices. The negative slope of the capital market line basically states that the larger the amount put in stocks the larger the expected loss will be — which the yearly returns in Table 2 and the yearly asset allocation in Table 3 also seem to suggest. In these particular cases, 100% in the risk-free asset is preferable over any other allocation.

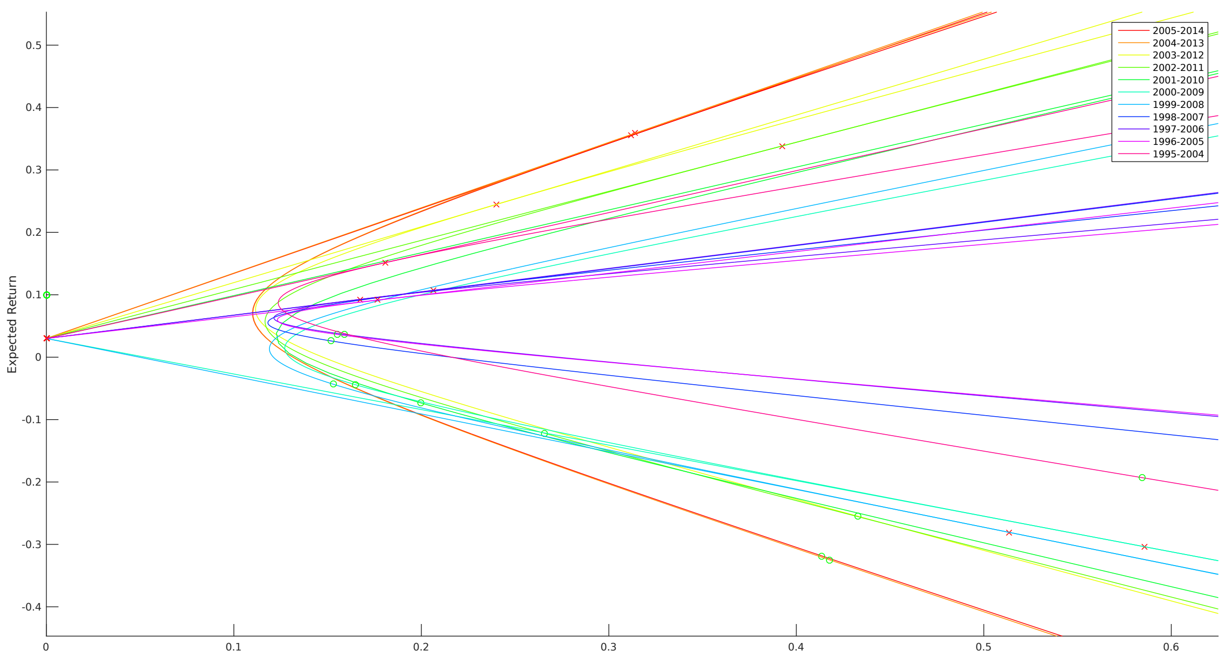

Figure 4: Efficient frontiers for each of the rolling windows when a risk-less asset is introduced.

The code associated with this section can be found under section 'D)' in `project.m`.

1.5 Asset Allocation

Given a required return of 10%, a possible allocation for each year is shown below. Here it is assumed that a risk-less asset at $R_f = 0.03$ is available.

Period	Weight in Risk-free Asset (%)	Weight in $opt_{R_f=3\%}$ (%)	Turnover rate (%)
2005-2014	78.46	21.54	0.263
2004-2013	78.73	21.27	11.33
2003-2012	67.40	32.60	9.911
2002-2011	77.31	22.69	18.01
2001-2010	95.32	4.683	25.65
2000-2009	121.0	-20.97	1.563
1999-2008	122.5	-22.53	113.6
1998-2007	8.965	91.03	20.66
1997-2006	-11.70	111.7	2.122
1996-2005	-13.82	113.8	56.18
1995-2004	42.36	57.64	–

Table 3: Turnover rates in each of the rolling windows.

The most notable entry in Table 3 is the turnover rate observed from the portfolio chosen in 1998 to one chosen in 1999. Here the asset allocation complete flips around with all funds (and more) being put in the risk-less asset by shorting the stock market. Momentarily jumping back to Figure 4, we see that the optimal portfolio for year 1998 has an expected return $\overline{R}_{1998-2007} = -28\%$ with $\sigma_{1998-2007} = 0.51$. This is rather significant as the market is relatively unstable, but the decline in the market is still expected to outweigh this. Considering that the only major difference between these two periods is the inclusion of the year 2008, we can confidently attribute this sudden change to the global financial crisis that year. Notice also that this position is roughly left unchanged the following year. Starting 2001 and 2002 the allocation is rebalanced and the new weights are kept fairly stable throughout the remaining years included in this dataset.

The code associated with this section can be found under 'E)' in `project.m`.

1.6 Backtest

Next, we study the performance of our optimized portfolios by performing a back test. For each optimal portfolio, we check the returns out of sample by holding it for a year. This allows us to compare the estimated returns based on the asset allocations of the previous year's portfolio with the expected returns of the current year. This exercise provides some insight into market performance and how well the optimal portfolios hold over time.

Taking investment year 2005 as an example, we see that the specific allocation based on historical data results in a loss in profit this particular year. The overall allocation is 42.36% in the risk-free asset and 57.64% in stocks (see Table 3). In general the state of major indices such as the S&P 500 where bullish this year, so to understand the results of the back test we need to look specifically at

Historical Data	Investment Year	$\overline{R_{BT}}$ (%)	Relative Error (%)
2004-2013	2014	11.01	100.2
2003-2012	2013	15.22	114.9
2002-2011	2012	2.097	-63.28
2001-2010	2011	13.70	368.7
2000-2009	2010	3.901	5.817
1999-2008	2009	-17.49	873.7
1998-2007	2008	4.343	-70.54
1997-2006	2007	1.861	-59.99
1996-2005	2006	10.11	41.46
1995-2004	2005	-10.14	-20.77

Table 4: Backtest performance of the optimal portfolios.

the positions taken within the portfolio. The specific breakdown is:

	ˆDJI	ˆDJT	ˆDJU	ˆNBI	ˆXMI	ˆSOX	ˆBK
Allocation (%)	-215.7	14.05	-35.11	36.66	264.5	-11.72	47.39
Market value increase (%)	0.1	11.6	22.5	4.3	-5.4	13.5	0.5

Although the exact calculation will not be performed, the above table should give the gist of the unfortunate positions taken in each of the sector indices. For instance, shorting ˆDJI proved to be a very cheap loan — much more attractive than common bank loans. However, using these funds to go long in ˆXMI was the wrong decision. Another significant factor to the total lost in profit can be attributed to shorts in ˆDJU, which had bullish characteristics throughout most of the year. Based on the results presented in Table 4, the back-test does not seem to support using this particular model as an investments strategy — at least not within the time period presented here.

The code associated with this section can be found under 'F)' in `project.m`.

1.7 Beta

Taking the S&P 500 as representative of the entire market, we can check if our portfolios follow the CAPM model.

$$\overline{R}_P = R_F + \beta_P(\overline{R}_M - R_F) \tag{1}$$

We calculate Jensen's alpha to determine the difference between our expected returns and the theoretical expected returns according to the CAPM prediction.

$$\alpha_J = \overline{R}_P - R_F - \beta_P(\overline{R}_M - R_F) \tag{2}$$

This can be accomplished by solving a simple linear regression of the expected returns of the optimal portfolios over the expected market returns. For clarity, Jensen's alpha can be rewritten as:

$$\overline{R}_P - R_F = \alpha_J + \beta_P(\overline{R}_M - R_F) \tag{3}$$

We are interested in the α_J term, which signifies how close our estimate is to the value predicted by the CAPM. Using the optimal portfolios from each rolling window and their corresponding market returns, we get:

$$\alpha_J = 0.2127$$
$$\beta_P = 0.5135$$

We see that our portfolio has created Jensen's alpha α_J, though it is not very big. However, the R^2 statistic is 0.00203 and the p value of the F statistic is 0.8954. These values indicate that the linear model we constructed does not fit the data very well. As a result, we cannot determine if the alpha is significant or not because the model is bad.

The code associated with this section can be found under 'G)' in `project.m`.

1.8 Black-Litterman (BL)

The Black-Litterman model combines our subjective views with the "neutral" market expected returns to form a better estimate of expected returns. Since we found in our set of optimal portfolios that NASDAQ Biotechnology, PHLX Semiconductor, and KBW Nasdaq Bank Index generally have higher returns than the other sectors, we want to invest more in them. We formalize these views in the Black-Litterman model by assigning weights to the different indices according to our beliefs.

For each of our views, we set each index to a value in set $\{-1, 0, 1\}$, where a 1 is assigned to sectors expected to outperform the others and a -1 is assigned to those expected to underperform the others. We form the matrix P indicating these views and the matrix V indicating the weights on each of these views.

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \text{ and } V = \begin{bmatrix} 0.02 \\ 0.02 \\ 0.03 \end{bmatrix}$$

Next, we set the values of the risk aversion coefficient (gamma) and precision coefficient (tau) based on our opinions.

$$\gamma = 1.1$$
$$\tau = 0.8$$

Then we can compute expected returns based on the BL model as:

$$E(R) = \Pi + \tau\Omega P'(P\tau\Omega P')^{-1}(V - P\Pi) \tag{4}$$

where

$$\Pi = \gamma\Omega w \tag{5}$$

Based on gamma, covariance and the return, we can get the new weights for our portfolio:

$$w = (\gamma\Omega)^{-1}E(R); \tag{6}$$

'NASDAQ Biotechnology', ' PHLX Semiconductor' and 'KBW Nasdaq Bank' are the fourth, sixth, and seventh sector indices, respectively. In Figure 5, we see that after adding our views to the model, those three columns are relatively higher than their counterparts in the original one. The opposite can be said for the indices that have been assigned negative weights.

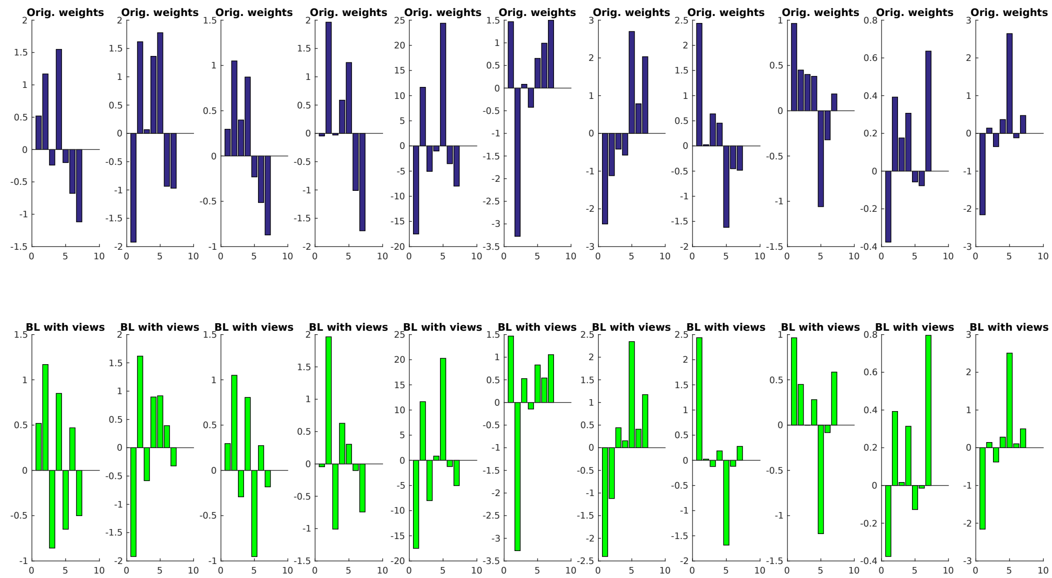

Figure 5: Sector allocation before and after Black-Litterman.

In order to see how our asset allocation changes, the returns computed by Black-Litterman model are compared to those calculated earlier. In Figure 6, the first column indicates how much we allocate to our risky portfolios and the second column indicates how much we allocate to the risk-free asset. Since we weigh the sectors using Black-Litterman model, the new asset allocation generally puts greater investments in the portfolios while decreasing the allocation to the risk-free assets, which implies that these new portfolios are better with these views than without.

We would also like to know how using the Black-Litterman approach affects the results of our back test, namely the mean relative error and sigma relative error. We do this by applying the same algorithms as before, except we replace the expected return and the asset allocation with those computed by Black-Litterman approach. The upper part of Figure 7 shows that the sigma error is generally much lower than the original. It also changes the overall profile of the mean relative error. Rather than having large peaks around the financial crisis, we have a more uniform relative error across the years. From these two measures, it seems like the model benefited from the inclusion of our three predictions.

The code associated with this section can be found under 'H)' in `project.m`.

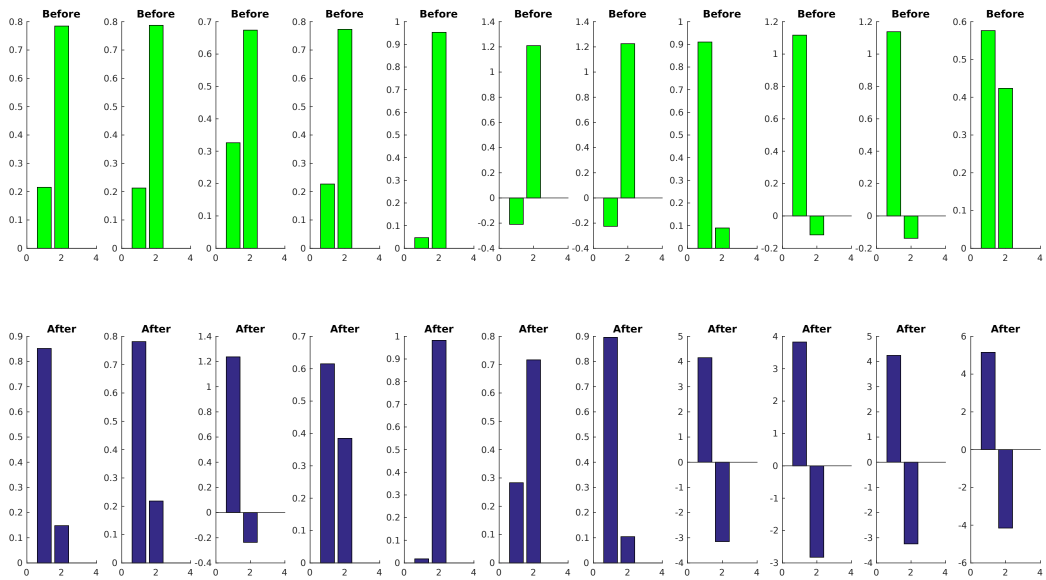

Figure 6: Positions in stocks (left) and the risk-free asset (right) before and after Black-Litterman.

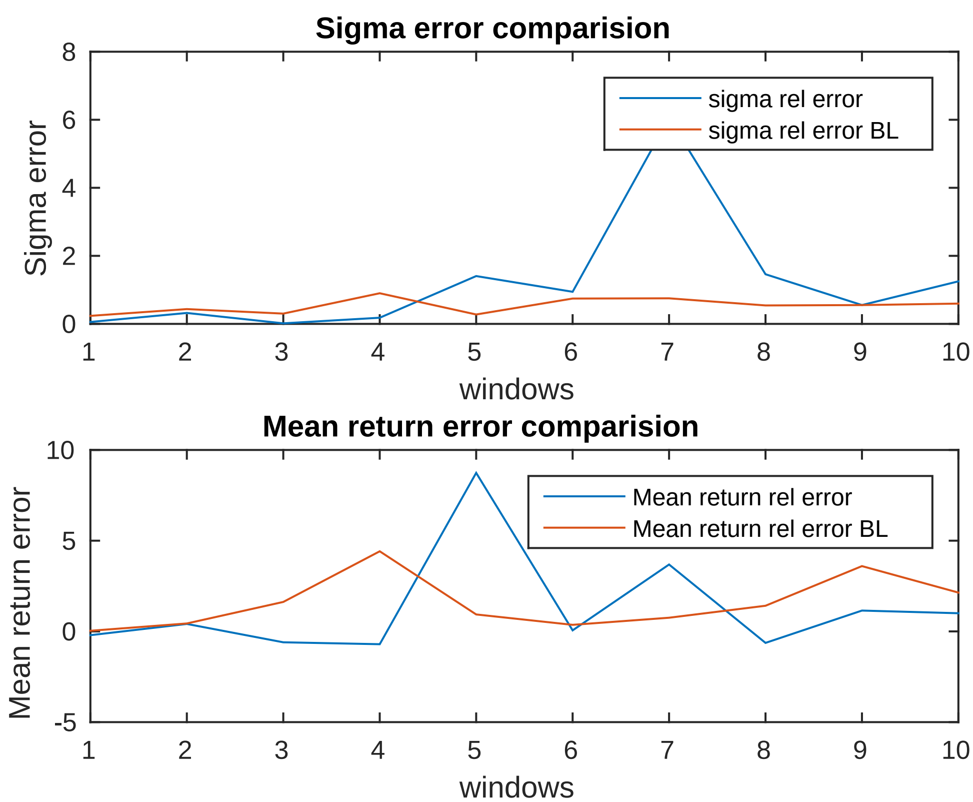

Figure 7: Error characteristics for the backtest before and after Black-Litterman.

1.9 Timing

We calculate the Treynor-Mazuy measure to determine how well the portfolio anticipates changes in the market. Investors buy in anticipation of a bull market, and they sell in preparation for a bear market. Thus, they seek a β_P that grows proportional to the market return. To consider this additional timing factor, a quadratic term is added to the Jensen's alpha equation.

As with Jensen's alpha, the Treynor-Mazuy measure can be computed by solving a simple linear regression of the expected returns of the optimal portfolios over the expected market returns.

$$\overline{R}_P - R_F = \alpha_P + \beta_P(\overline{R}_M - R_F) + \gamma_P(\overline{R}_M - R_F)^2 \tag{7}$$

We are interested in the γ_P term, which measures the convexity of the portfolio's return in terms of the market's return. As before, we use the optimal portfolios from each rolling window and their corresponding market returns to get:

$$\alpha_P = 0.4281$$
$$\beta_P = -1.639$$
$$\gamma_P = -129.8$$

We see that the convexity coefficient γ_P is quite large. However, the R^2 statistic is 0.1918 and the p value of the F statistic is 0.4266, which indicates that the model does a very poor job of fitting the data. Therefore, the results are inconclusive and we cannot say anything about the timing.

The code associated with this section can be found under 'I)' in `project.m`.

2 Bonds

The bonds in Table 5 have been selected for further analysis.

Issuer	Price	Coupon (%)	Maturity Date
US Treasury (Stripped Prin.)	78.11	—	15-Aug-2025
General Electric Corp	99.94	3.000	15-Nov-2025
Morgan Stanley	102.50	4.000	01-Nov-2024
Verizon Communications Inc	100.87	3.500	01-Nov-2024
North Las Vegas, NV	94.10	6.122	01-Jun-2025
Wendys Intl Inc	108.75	7.000	15-Dec-2025
Penney J C Inc	90.00	6.900	15-Aug-2026

Table 5: Bond portfolio

All of these fixed-income securities have the same characteristic and (lack of) embedded options. In other words, these bonds all pay semi-annual coupons (the zero-coupon bond being an exception); they are not callable, and for the scope of this assignment, they are assumed to be neither puttable, extendable or exchangeable. In short, these are "plain vanilla" bonds. It should also be noted that the price listed is in relation to 100 of the actual face value.

In terms of the source code, the top-level script covering this section is `bondPortfolio.m`. The file `bondTestRunner.m` represents an automated, test-suite runner that verifies different parts of the implementation at unit level.

2.1 Return of Investment and Risk

Yield to maturity In order to determine the annual profit, given that bonds are held to maturity, we calculate the yield to maturity for each of them. This can be done be solving for ytm in the equation

$$price = \sum_{t=1}^{n} \frac{C(t)}{(1 + \frac{ytm}{k})^t} \tag{8}$$

where $C(t)$ expresses the cash flow at time t, n the remaining number of payments, and k the number of annual coupon payments. Essentially this follow the characteristics of the time value of money.

Equation (8) calculates the clean price of a bond. I.e. it does not take accrued interest into account in the event that a bond is purchase between payment dates. For all of the bonds listed above the settlement dates are December 4^{th} 2014, with the exception of the zero-coupon Treasury bond requiring settlement a negligible two days earlier. For the sake of simplicity, day count and day count convention have not been taking into consideration; even more so, the settlement date is simply said to be equal to time 0 such that forward rates do not come into play and particularly leaving the dirty price equal to the clean price. As the chosen bonds all mature in roughly 10 years these simplification are not assumed to make any significant difference. Furthermore this also allows cross-referencing results with Yahoo! Finance.

For the selected bonds, equation (8) produces Table 6. In addition, the credit ratings by Fitch has been included to the table as well. Though the bond issued by the US Treasury has not been rated we will assume a rating of AAA as it is backed by the government and therefore considered default free [2]. Although this small sample is not sufficient to infer any relationship, it does seem that yield increases as credit ratings decreases. ”Wendys Intl Inc” does, however, seem to object to this description to some extent.

As a final note it should be stated that the yield of a zero-coupon bond is also calculated using Equation (8) with $k = 2$.

The source specifically calculating yield to maturity can be found in the file `calcYieldToMaturity.m`. It has the related test suite `testCalcYieldToMaturity.m` exercising the implementation.

Issuer	Yield To Maturity (%)	Fitch Rating
US Treasury (Stripped Prin.)	2.37	—
General Electric Corp	3.01	AA
Morgan Stanley	3.70	A
Verizon Communications Inc	3.40	BBB
North Las Vegas, NV	6.92	BB
Wendys Intl Inc	5.91	B
Penney J C Inc	8.26	CCC

Table 6: Yield to maturity

Duration For bonds the term duration has an special meaning. Here duration is a measure of sensitivity of a bond’s price in regards to changes to the yield. A perhaps less conventional definition is that duration expresses the time it takes for the price of a bond to be repaid by its payments (assuming everything else is held constant) [3]. At the very least, the latter definition gives some intuition what this value should be since duration is measured in years. E.g. the (Macaulay) duration for a zero-coupon bond must be equal to the time of maturity as this type of bond does not have payouts before maturity. Building on top of this notion, bonds that do pay coupons must have a duration that is less then its time to maturity — simply because the bondholder receive some money earlier on. The prior definition, however, is of more interest when determining the unexpected return as will we do later.

Let the Macaulay Duration[1] be

$$dur_{mac} = \frac{\sum_{t=1}^{n} \frac{t \cdot C(t)}{(1+\frac{ytm}{k})^t}}{price} \tag{9}$$

where	$C(t)$	payment at time t
	n	remaining number of payments
	k	number of annual coupon payments
	ytm	yield to maturity

[1] This differs from the course handout material which refers to the formula as the ‘Modified Duration’. The above definition, however, seems to be better in line with e.g. the textbook and Wolfram Alpha which was used to cross-check results. In any case it will only differ by a “constant” factor based on the interest rate.

In words, this formula has similarities to a weighted average with respect to the price of the bond and each payment expressed as its equivalent present value.

An adjusted version of the Macaulay duration is the modified duration. It is believed that this measure can be used in regards to the price sensitively for a unit change in the yield to maturity. Let the modified duration be defined as:

$$dur_{mod} = \frac{dur_{mac}}{(1 + \frac{ytm}{k})} \tag{10}$$

The source file calculating duration is `calcDuration.m` and the relevant unit tests can be found in `testCalcDuration.m`. Using the data from the previous tables as input on `calcDuration` results in Table 7.

	(in years)		
Issuer	Time to Maturity	Macaulay Duration	Modified Duration
US Treasury (Stripped Prin.)	10.5	10.5	10.4
General Electric Corp	11.0	9.45	9.3
Morgan Stanley	10.0	8.36	8.2
Verizon Communications Inc	10.0	8.53	8.4
North Las Vegas, NV	10.5	7.80	7.5
Wendys Intl Inc	11.0	7.99	7.8
Penney J C Inc	11.5	7.91	7.6

Table 7: Bond Duration

With a means of calculating the duration of a bond, we can now estimate the unanticipated return R_u, given a change in yield, as: $R_u = -dur_{mod} \cdot \Delta ytm$.

Convexity [2] Although the modified duration can be used to describe the change in a bond's price, given a shift in yield to maturity, it assumes a linear relationship between the two. The true bond price, however, is convex, which means that the effect of a significant change in yield, results in an overestimated change in price. As of which a convexity term will be introduced to account for this inaccuracy. The definition is as following:

$$con = \frac{1}{k^2} \cdot \frac{1}{(1 + \frac{ytm}{k})^2} \cdot \frac{\sum_{t=1}^{n} \frac{t(t+1) \cdot C(t)}{(1+\frac{ytm}{k})^t}}{price} \tag{11}$$

$C(t)$	payment at time t
n	remaining number of payments
k	number of annual coupon payments
ytm	yield to maturity

This definition differs from Equation (22.4) in the textbook [2], where the most important term is $\frac{1}{k^2}$, which annualizes the convexity for our semi-annual coupon bonds. The second term $\frac{1}{(1+\frac{ytm}{k})^2}$ is believed to make this the modified variant of convexity. The constant factor $\frac{1}{2}$ seen in Equation (22.4) has instead been "embedded" into Equation 12, which happens to be the only place where

[2] TODO: Is the unit for convexity correct?

convexity is used. Where this constant truly belongs is said to be of "immense disagreement" [1] as of which we allow ourselves to make this modification. At the very least, the above definition produces comparable result to other sources used to cross-check the results. The tests exercising the implementation can be found in the file `testConvexity.m`.

Using our selected bonds as input data for the function described in `convexity.m`, we get the results listed in Table 8.

Issuer	Convexity (years/‱)
US Treasury (Stripped Prin.)	113
General Electric Corp	100
Morgan Stanley	79
Verizon Communications Inc	82
North Las Vegas, NV	72
Wendys Intl Inc	77
Penney J C Inc	76

Table 8: Bond Convexity

With this convexity term, we can now give a better estimate for the unanticipated return, namely:

$$R_u = -dur_{mod} \cdot \Delta ytm + \frac{1}{2} \cdot con \cdot (\Delta ytm)^2 \tag{12}$$

2.2 Bond portfolio

For the sake of completeness, the characteristics found in the previous section is summarized in Table 9.

Issuer	Yield to Maturity (%)	Modified Duration (years)	Convexity (years/‱)
US Treasury (Stripped Prin.)	2.37	10.4	113
General Electric Corp	3.01	9.3	100
Morgan Stanley	3.70	8.2	79
Verizon Communications Inc	3.40	8.4	82
North Las Vegas, NV	6.92	7.5	72
Wendys Intl Inc	5.91	7.8	77
Penney J C Inc	8.26	7.6	76

Table 9: Bond characteristics

The duration and convexity of a portfolio is simply the weighted average of the individual bonds in the portfolio. Assuming an equal amount is invested in each of the bonds, we get:

$$dur_{port} = \tfrac{1}{7} \cdot 10.4 + \tfrac{1}{7} \cdot 9.3 + \tfrac{1}{7} \cdot 8.2 + \tfrac{1}{7} \cdot 8.4 + \tfrac{1}{7} \cdot 7.5 + \tfrac{1}{7} \cdot 7.8 + \tfrac{1}{7} \cdot 7.6 = 8.46 \text{ years.}$$

$$con_{port} = \tfrac{1}{7} \cdot 113 + \tfrac{1}{7} \cdot 100 + \tfrac{1}{7} \cdot 79 + \tfrac{1}{7} \cdot 82 + \tfrac{1}{7} \cdot 72 + \tfrac{1}{7} \cdot 77 + \tfrac{1}{7} \cdot 76 = 85.6 \text{ years/‱.}$$

All of the aforementioned values were taken from the script `bondPortfolio.m`.

2.3 Valuation of Portfolio

Finally, we estimate of the portfolio market value in the event that the yield increases by 150 ‱. Assume again the an equal amount is invested in each of the bonds, and the total investments amounts to 700,000 EUR. By equation (12), we calculate the unanticipated return as:

$$R_{u_{port}} = -dur_{port} \cdot \Delta ytm + \tfrac{1}{2} \cdot con_{port} \cdot (\Delta ytm)^2 = -8.46 \cdot 0.015 + \tfrac{1}{2} \cdot 85.6 \cdot 0.015^2 = -11.72\%.$$

The value of the portfolio is therefore estimated to decrease by $700000 \cdot (-0.1172) = -82055$ EUR, revealing a new market value of 617945 EUR.

These final calculations can also be found in `bondPortfolio.m`.

3 Appendix

References

[1] B. Campbell. *Financial Exam Help 123: Convexity*, 2014 (accessed December, 2015).

[2] E. Elton, M. Gruber, S. Brown, and W. Goetzmann. *Modern Portfolio Theory and Investment Analysis*. Wiley, 2011.

[3] www.investopedia.com. *Advanced Bond Concepts: Duration*, 2015 (accessed December, 2015).