Assignment 02

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Part 1: Theoretical part

Exercise1

Question1:

An AR process is stationary if the roots for the characteristic equation lie within the unit circle, which means the coefficients in the equation (Page 121 ii) satisfy the following three conditions:

1. $(\lambda_1 + \lambda_2) < 1$
2. $(\lambda_2 - \lambda_1) < 1$
3. $|\lambda_2| < 1$

When all the conditions are satisfied, the roots of an AR(2) will fall into a triangle (see figure 1)

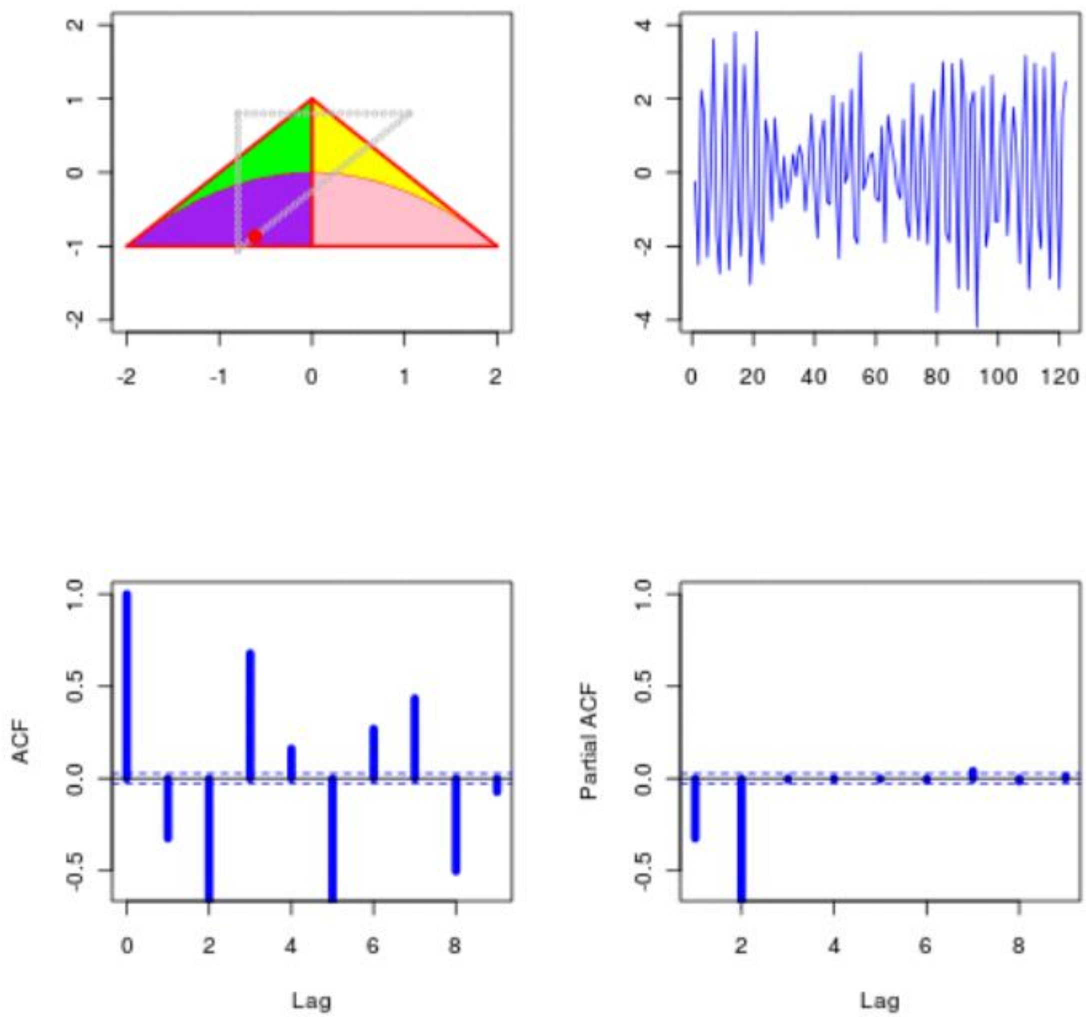

Figure 1 AR(2) allowed area [1]

[1] http://freakonometrics.hypotheses.org/12081

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Question 2:

When the three conditions are fulfilled and the roots lie into the triangle below the dash line(figure2), the ACF can give damping harmonic oscillations, actually figure 1 is one of them.

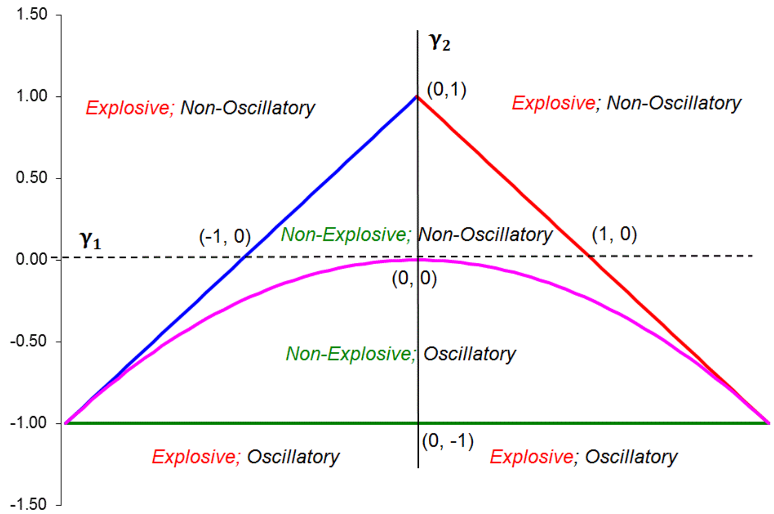

Figure 2 AR(2) roots area in detail[2]

Exercise 2

The given linear process is a multiplicative (1,1,0) x $(1,0,0)_6$ seasonal model.

R code:

```
waterLevel = arima(c(2,1,-1,-2,-3,1,4,4,0,-3),order=c(1,1,0),seasonal=list(order=c(1,0,0),kappa=0.31,
frequency=6))     ##kappa is variance

predict(waterLevel, n.ahead = 2)

plot(y,xlim=c(0,12),ylim=c(-6,6),xlab="hour", ylab="dm", main="Observed water levels and predictions")

points(p$pred, col=2, pch=19)

legend(x=0,y=6,c("prediction","observed"),cex=.6, col=c("red","black"),pch=c(16,1))
```

Output:

```
$pred
Time Series:
Start = 11
End = 12
Frequency = 1
```

[2] http://davegiles.blogspot.dk/2013/06/when-is-autoregressive-model.html

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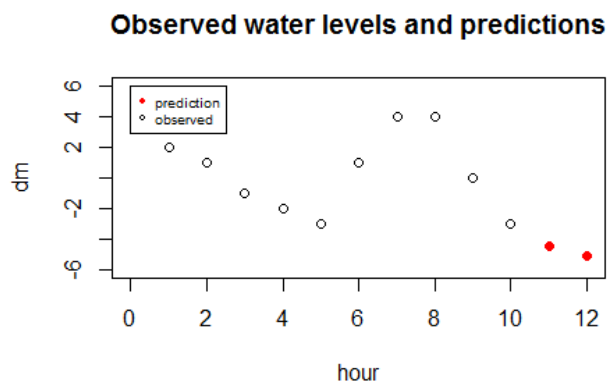
[1] -4.482220 -5.114968

$se
Time Series:
Start = 11
End = 12
Frequency = 1
[1] 2.105722 4.015600

The output shows the predictions $Y_{11}=-4.48$, $Y_{12}=-5.11$ with standard errors 2.105 and 4.015 respectivel y.



Part 2. Simulating AR process

Data simulation and ACF using “filter” for AR(1) when a1= 0.1, -0.1, 0.9, -0.9. When the coefficient is bigger than 1, the process goes infinitive, which makes the process not stationary (see figure 3).

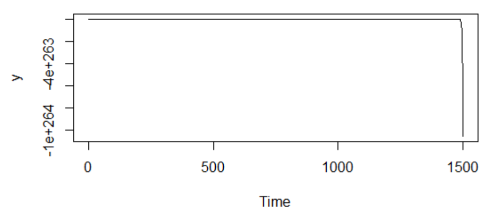


Figure 3 when a is bigger than 1

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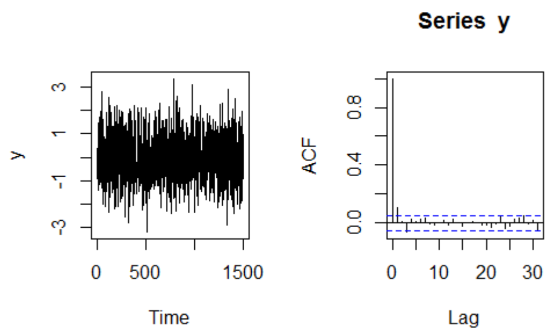

Figure 4 a1=0.1

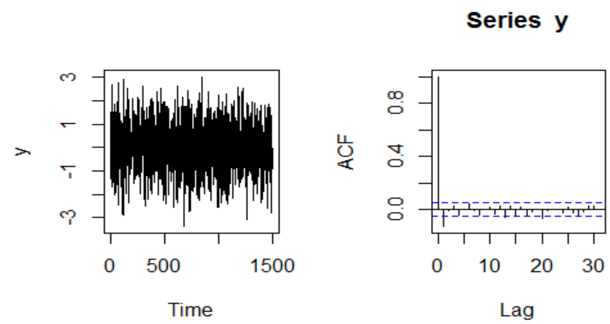

Figure 5 a1=-0.1

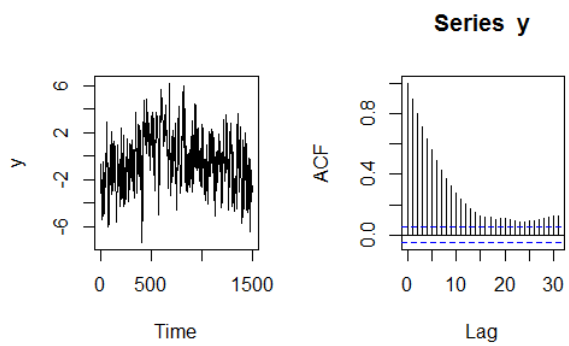

Figure 6 Figure 4 a1=0.9

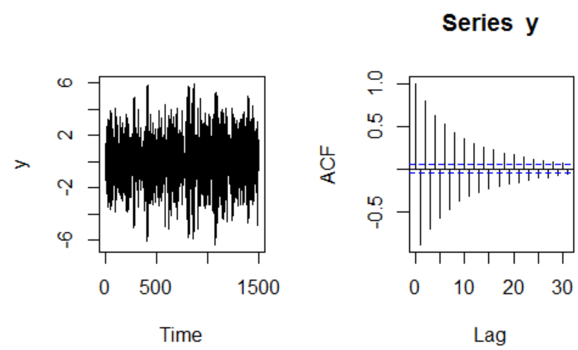

Figure 7 Figure 4 a1=-0.9

From the figures 4 to 7 we can see that when a1 is negative, the ACF is exponentially decreasing from both sides of x axis where y is 0. The bigger a1 is, the slower the ACF regress exponentially.

Data simulation and ACF using “filter” for AR(2) when a1= 0.1, a2=0.5, a1= -0.2, a2= -0.7 and a1= -0.6, a2=0.3. The ACF figures are all additively and exponentially decreasing. When both a1 and a2 are positive, the wave is primarily above 0. When one of the parameters is negative or both of them are negative, the wave is on both side of x axis along y=0.

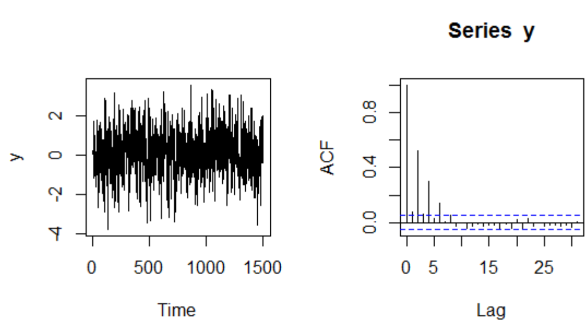

Figure 8 AR(2) a1=0.1 a2=0.5

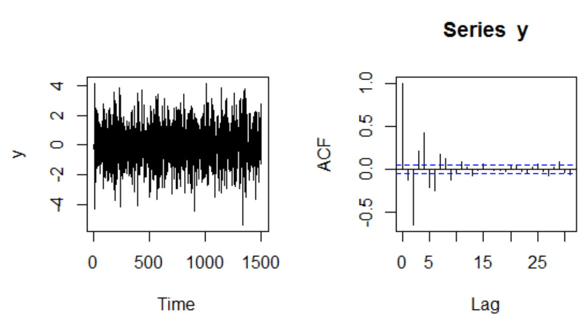

Figure 9 AR(2) a1=-0.2 a2=-0.7

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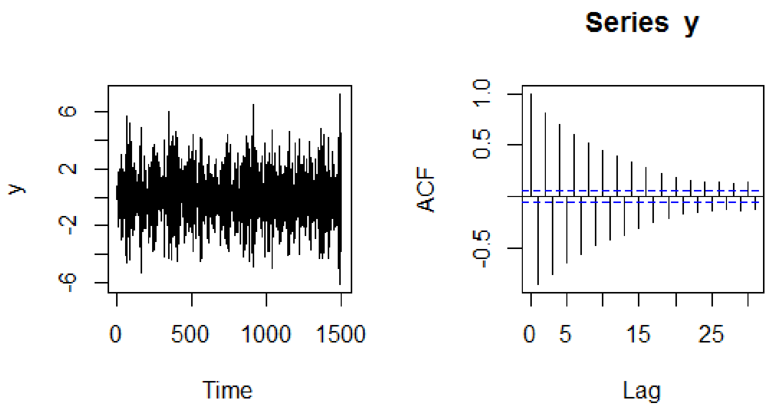


Figure 10 AR(2) a1= -0.6   a2=0.3

Using “arima.sim” simulate AR(2) when a1= 0.3, a2= -0.6, a1= 0.6, a2=0.2 and , a1= 0.1, a2=0.8. From figure you can see that when the coefficients are positive and big, the exponentially decreased wave is also big, in which when a1>a2, the exponential additive effect is slight (see figure 12), but when a1<a2 the additive exponential effect is strong(see figure13).

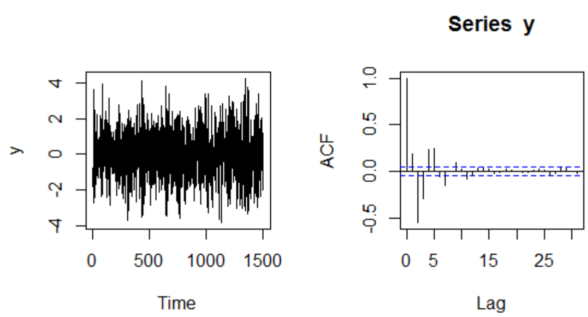


Figure 11 AR(2) a1= 0.3   a2=-0.6

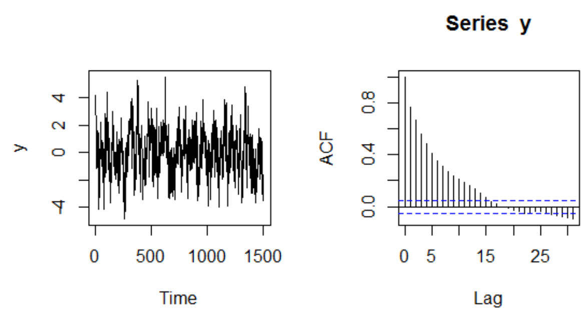


Figure 12 AR(2) a1= 0.6   a2=0.2

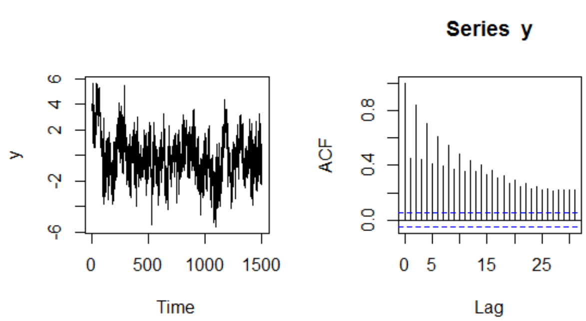
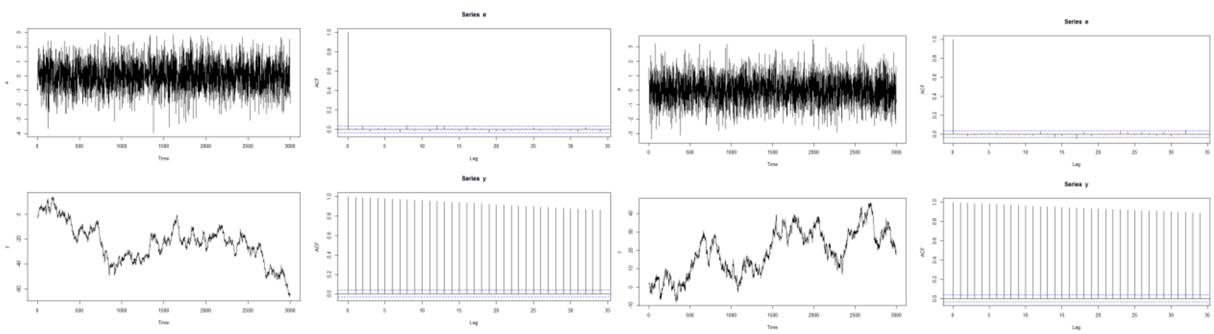
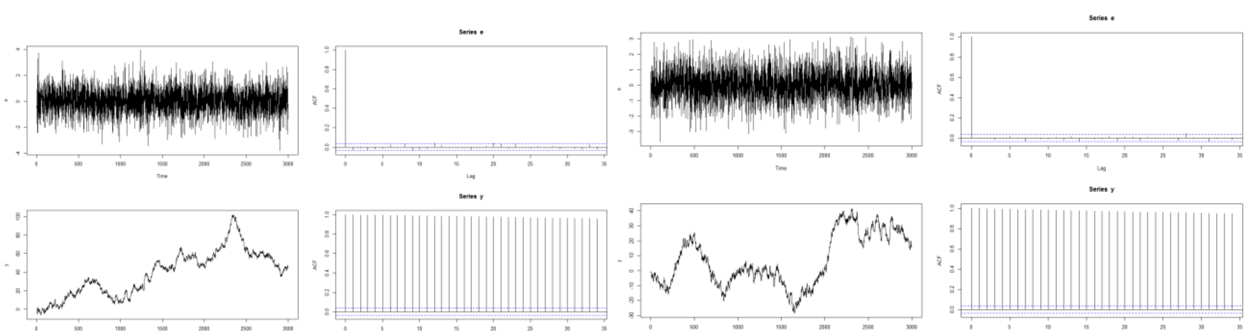


Figure 13 AR(2) a1= 0.1   a2=0.8

Part 3: Random walk

{e_s} is white noise meaning that it is a sequence of mutually uncorrelated identically distributed random variables with mean is 0 and a constant variance. This process is stationary as the covariance function is 1 only when the lag is 0, otherwise it is always 0.(Page108, Definition5.9) It is very clear from figure 14 that indicates white noise process is stationary, but the Y_t process has clear trend, which is not stationary. Because the white noise covariance does not change over time, and Y_t decline rate is extremely small, so that the correlation function is does not depend on the time difference (Page 103, 5.2.2).





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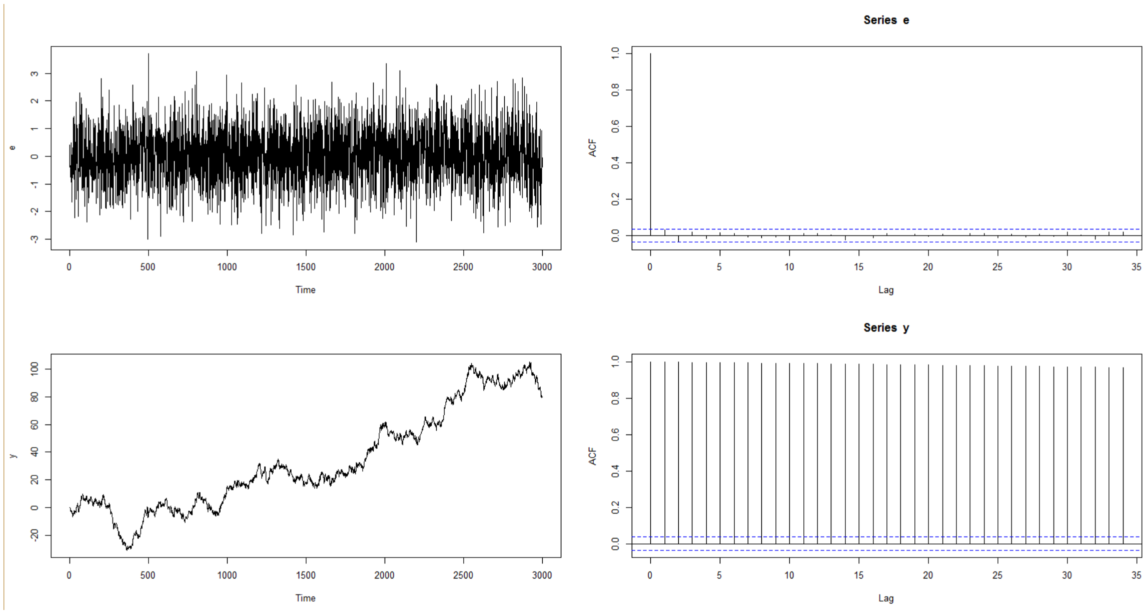

Figure 14 white noise processes compare with its cumsum processes

$X_t = Y_t - Y_{t-1}$ is a MA(1) process, according to Theorem 5.8 (Page118) MA is always stationary. Figure 15 shows that all the X_t MA(1) processes are stationary. The interesting finding is that even the Y_t process itself is not stationary, but when using it as a parameter in X_t (a first order MA process), the X_t is stationary.

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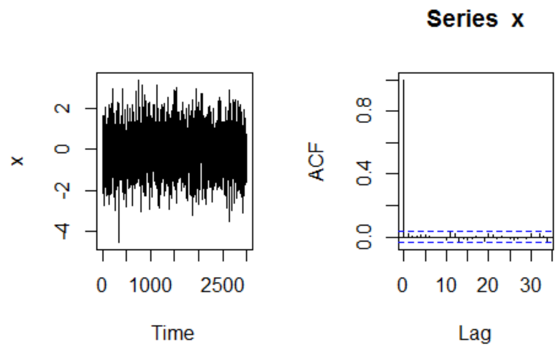

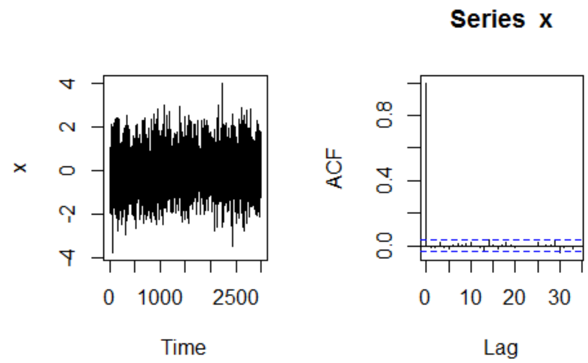

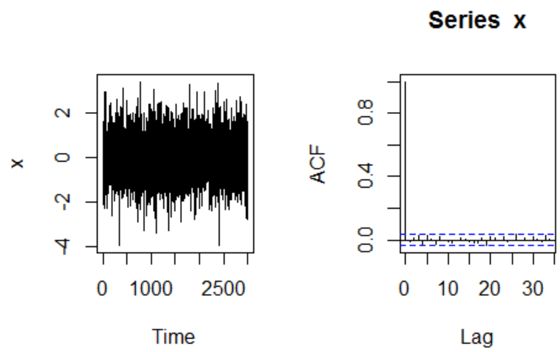

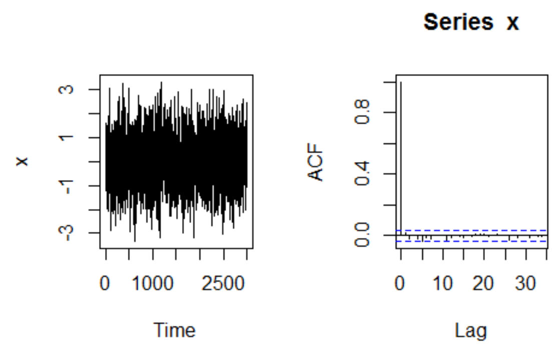

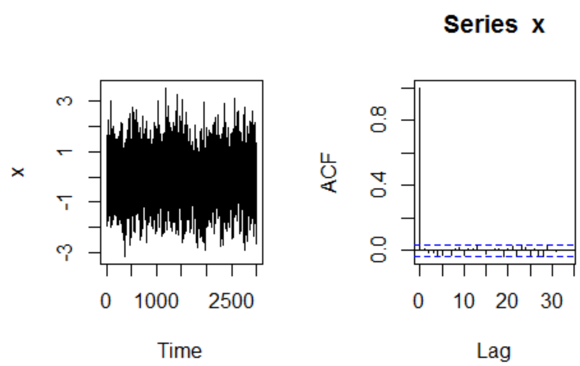

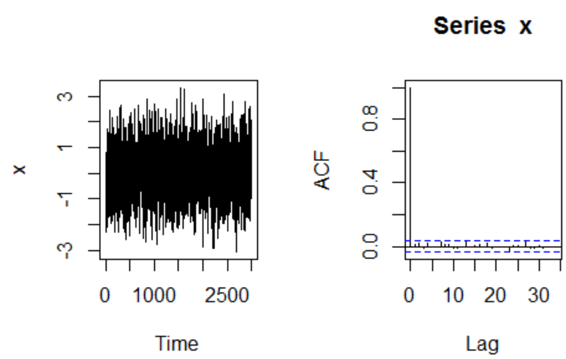

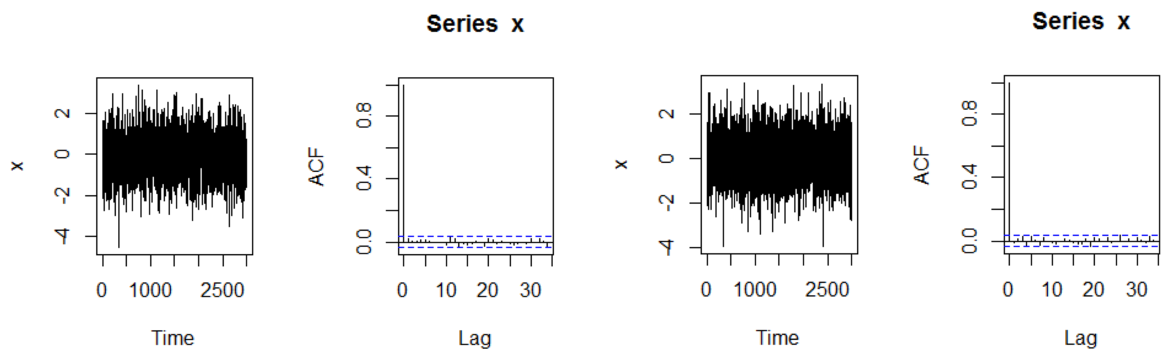

Figure 15 MA(1) process

Part 4 Seasonal processes

1. (1, 0, 0) × (0, 0, 0)12 is the autocorrelation function at lags in seasonal part are empty, meaning no order from seasonal part will multiply 12, thus the function will be determined by the ordinary ARIMA part. (p.132, section 5.6.2) Figure 16 shows that autocorrelation begins exponential decreasing from lag 1, whereas the partial autocorrelation is 0 when lag k > p.

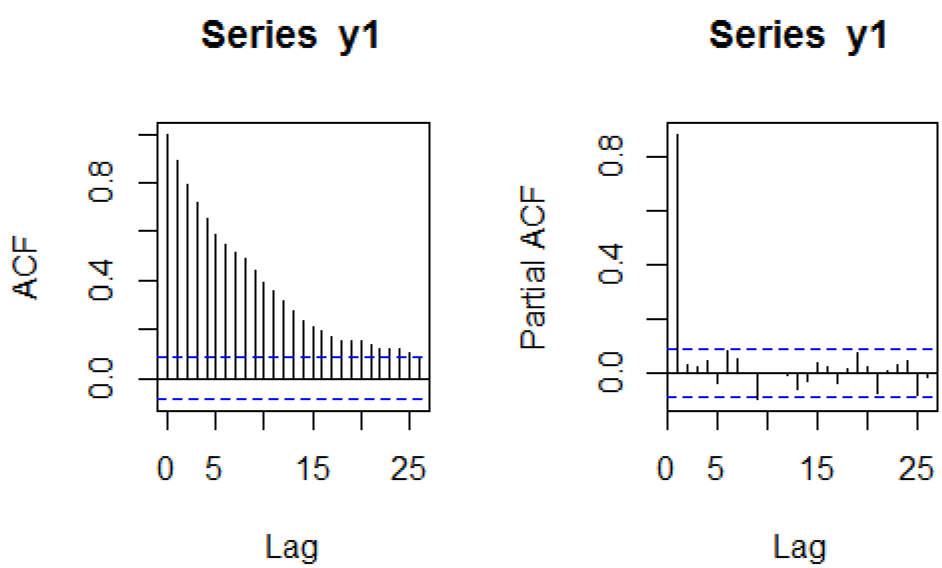

Figure 16 Autocorrelation (left) & partial autocorrelation (right) for y1

2. (0, 0, 0) × (1, 0, 0)12 is a pure seasonal model as the ordinary ARIMA part is empty. Figure 17 indicates a seasonal pattern with s =12. Inside each period the function follows ARIMA (P,0,Q). P is multiplied to 12, when k > 12, the autocorrelation function is 0 (Page 155, Table 6.1), when k=12, p(12)= Φp(12-k)= Φ=0.7 (Page133), so the PACF is 0.7.

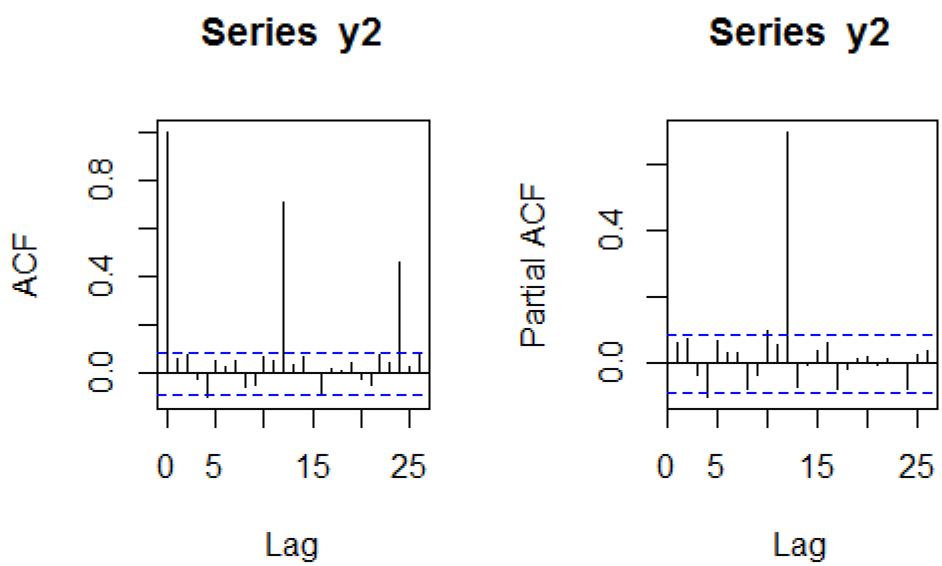

Figure 17 Autocorrelation (left) & partial autocorrelation (right) for y2

3. $(1, 0, 0) \times (0, 0, 1)_{12}$ is a typical mixed seasonal model, thus it suggests two polynomials:
   AR(1): $(1 - \phi_1 B) Y_t = e_t$
   MA(1): $Yt = (1 + \Theta_1 B^{12}) e_t$
   $Yt = (1 - \phi_1 B)^{-1} * (1 + \Theta_1 B^{12}) e_t = (1 + \phi_1 B + B^{12} + \phi_1 \Theta_1 B^{13})$
   ACF (see figure 18 left) shows that two spikes stick out when at lags 1 and 12.

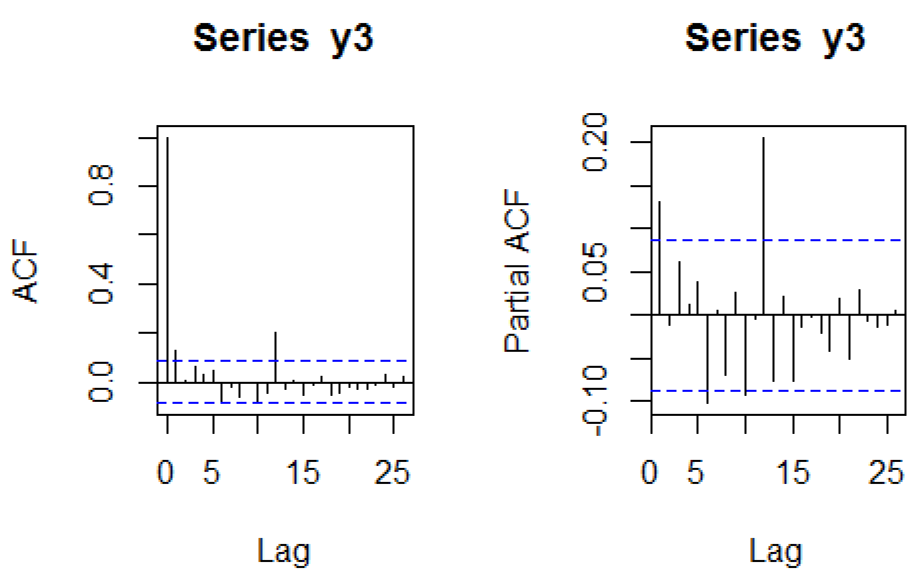

Figure 18 Autocorrelation (left) & partial autocorrelation (right) for y3

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4. (1, 0, 0) × (1, 0, 0)12 is a mixed seasonal model with ϕ_1 = 0.9 and Φ_1 = 0.7, which implies two polynomials that can be multiplied to get the final model:
   $(1 - \Phi_1 B^{12}) \times (1 - \phi_1 B)\, Y_t = e_t$ $\quad Y_t = (1 + \phi_1 B + \Phi_1 B^{12} + \phi_1 \Phi_1 B^{13}) e_t$
   Plug ϕ_1 = 0.9 , Φ_1 = 0.7 and $\phi_1\,\Phi$ =0.63 into the model, we have Figure19 (left) that indicates a nice seasonal pattern with interval of 12 lags, within each period it gives an exponential sine wave. The one on the right side indicates that PACF is 0 when k>12 (see figure 19).

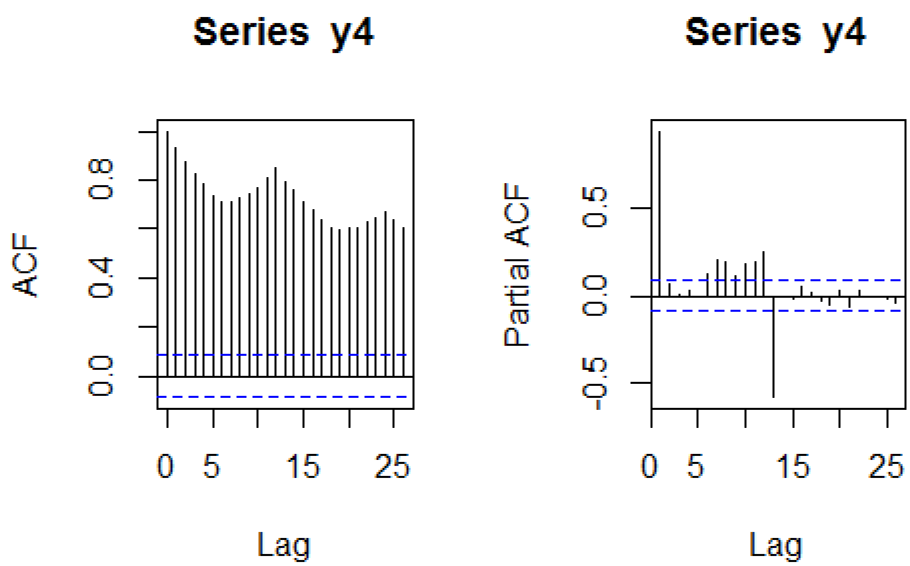

Figure 19 Autocorrelation (left) & partial autocorrelation (right) for y4

5. (0, 0, 1) × (0, 0, 1)12 is a mixed seasonal mode that has both orders on MA, which implies two polynomials that can be multiplied to get the final model:

   $Y_t = (1 + \Theta_1 B^{12})*(1 + \theta_1 B) * e_t$ $\quad Y_t = (1 + \theta_1(B) + \Theta_1(B^{12}) + \theta_1\Theta_1 B^{13}) e_t$
   Bring θ_1=0.4, Θ_1=0.3 and $\theta_1\Theta_1$=0.12 into the model, we have the ACF and PACF for the simulated seasonal model. The autocorrelation function shows a slight damped sine wave in each period. Four spikes on lag 1, 11, 12 and 13stick out in PACF (see figure 20).

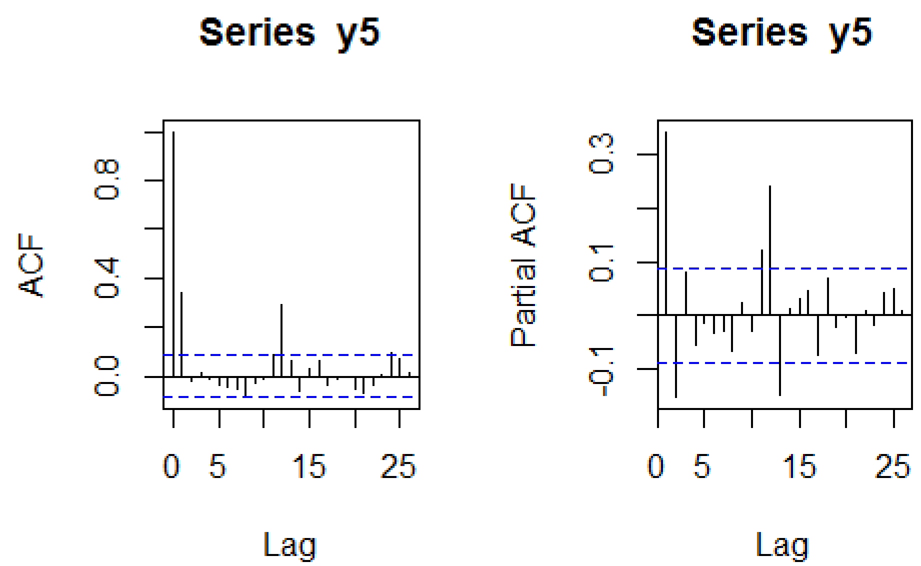

Figure 20 Autocorrelation (left) & partial autocorrelation (right) for y5

6. (0, 0, 1) × (1, 0, 0)12 is a mixed seasonal model with both AR and MA, which implies a couple of polynomials that can combind:
   $Y_t = (1 + \theta_1 B)*e_t$
   $(1 - \Phi_1 B^{12})*Y_t = e_t$
   $Y_t = (1 - \Phi_1 B^{12})^{-1} * (1 + \theta_1 B)*e_t = (1 + \theta_1 B + \Phi_1 B^{12} + \theta\ \Phi_1 B^{13})e_t$

   Bring $\theta_1=0.4$, $\Phi_1=0.7$ and $\theta\ \Phi_1=0.28$ into the model, The autocorrelation (see figure21) gives an exponential decrease among the seasonal pattern.

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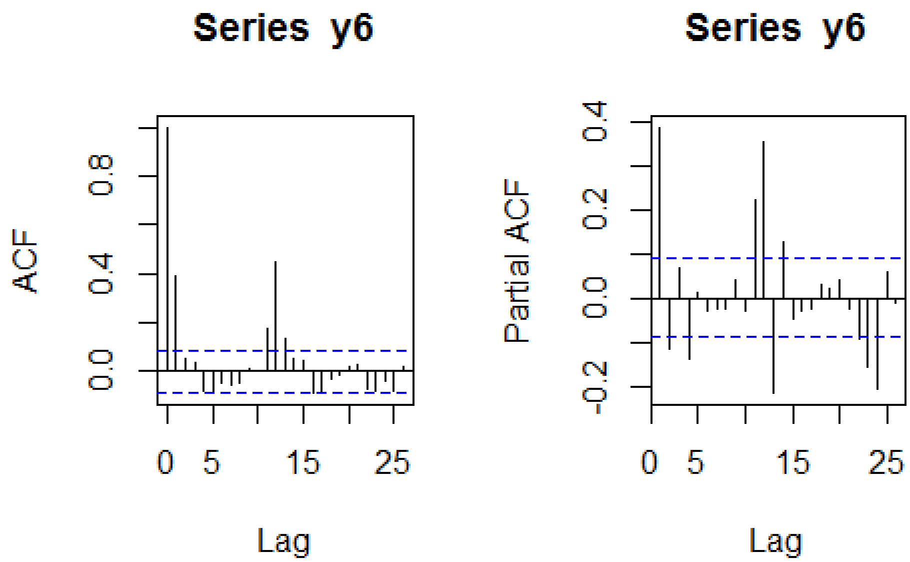

Figure 21 Autocorrelation (left) & partial autocorrelation (right) for y6

Appendix

R code for Part2, 3 and 4

##Part 2 AR(1)

```
e=rnorm(1500)

a=0.9

#for(i in 1000:1500) {y[i]=filter(e[i],filter=a,method="recursive")}

y=filter(e,filter=a,method="recursive")

par(mfrow=c(1,2))

ts.plot(y)

acf(y)


e=rnorm(1500)

a=1.5

y=filter(e,filter=a,method="recursive")

par(mfrow=c(1,2))

ts.plot(y)

acf(y)
```

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#AR(2)

```
e=rnorm(1500)

a=c(0.6,0.1)

y=filter(e,filter=a,method="recursive")

par(mfrow=c(1,2))

ts.plot(y)

acf(y)


y=arima.sim(model=list(ar=c(0.1,0.8)),n=1500)

par(mfrow=c(1,2))

ts.plot(y)

acf(y)
```

##part3 Random walk

```
e<-arima.sim(model = list(), n = 3000)

y<-cumsum(e)

par(mfrow=c(2,2))

ts.plot(e)

acf(e)

ts.plot(y)

acf(y)
```

##part3 diff

```
e<-arima.sim(model = list(), n = 3000)

y<-cumsum(e)

x<-diff(y,lag=1, differences = 1)

par(mfrow=c(1,2))

ts.plot(x)

acf(x)
```

##part 4 Seasonal processes

```
y1<-arima.sim(model = list(ar = 0.9, order = c(1,0,0)), n = 500)

ts.plot(y1)
```

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```
par(mfrow=c(1,2))

acf(y1)

pacf(y1)


y2<-arima.sim(model=list(ar=c(rep(0,11),.7)), n = 500)

ts.plot(y2)

par(mfrow=c(1,2))

acf(y2)

pacf(y2)


y3<-arima.sim(model=list(ar=c(-.9,0,0,0,0,0,0,0,0,0,0,.4,-.36)), n=500)

ts.plot(y3)

par(mfrow=c(1,2))

acf(y3)

pacf(y3)


y4<-arima.sim(model=list(ar=c(.9,0,0,0,0,0,0,0,0,0,0,.7,.63)), n=500)

ts.plot(y4)

par(mfrow=c(1,2))

acf(y4)

pacf(y4)


y5<-arima.sim(model=list(ma=c(.4,0,0,0,0,0,0,0,0,0,0,.3,.12)), n=500)

ts.plot(y5)

par(mfrow=c(1,2))

acf(y5)

pacf(y5)


y6<-arima.sim(model=list(ma=c(.4,0,0,0,0,0,0,0,0,0,0,.7,.28)), n=500)

ts.plot(y6)
```

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```
par(mfrow=c(1,2))

acf(y6)

pacf(y6)
```