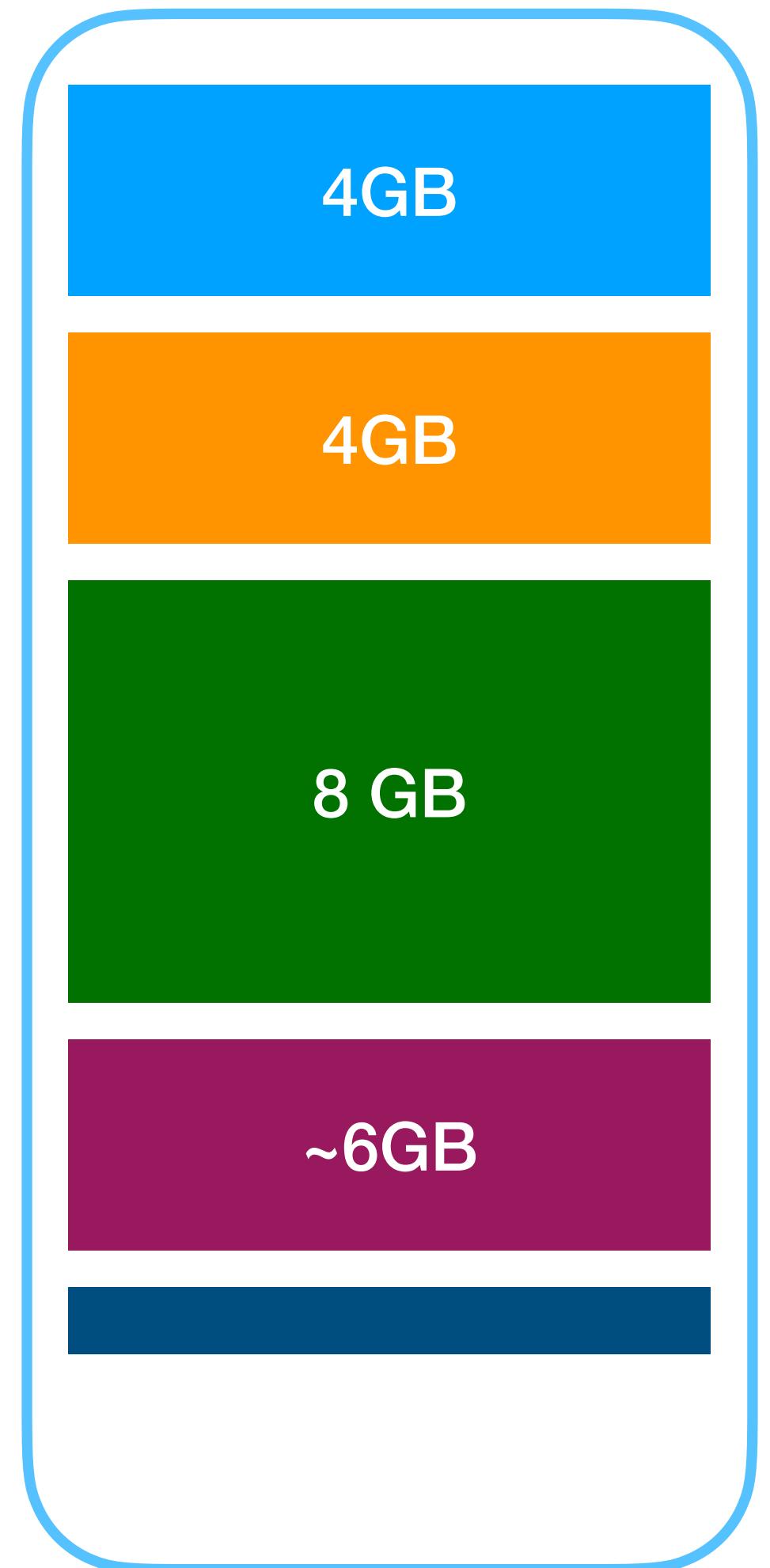


# Fine tune a LLM: how much memory do I need?

**RTX 4090**  
**24 GB**

**tuning a 2B model**



**Original model**

**2B (FP16) =  $2 \times 16 / 8 = 4\text{GB}$**

**Gradient**

**Same size with the model**

**Optimizer**

**Two tensors: mean + variance**

**Activation**

**Model architecture, batch size, sequence length etc, ~1.5x model size**

**Something else**



# Understand temperature in LLM

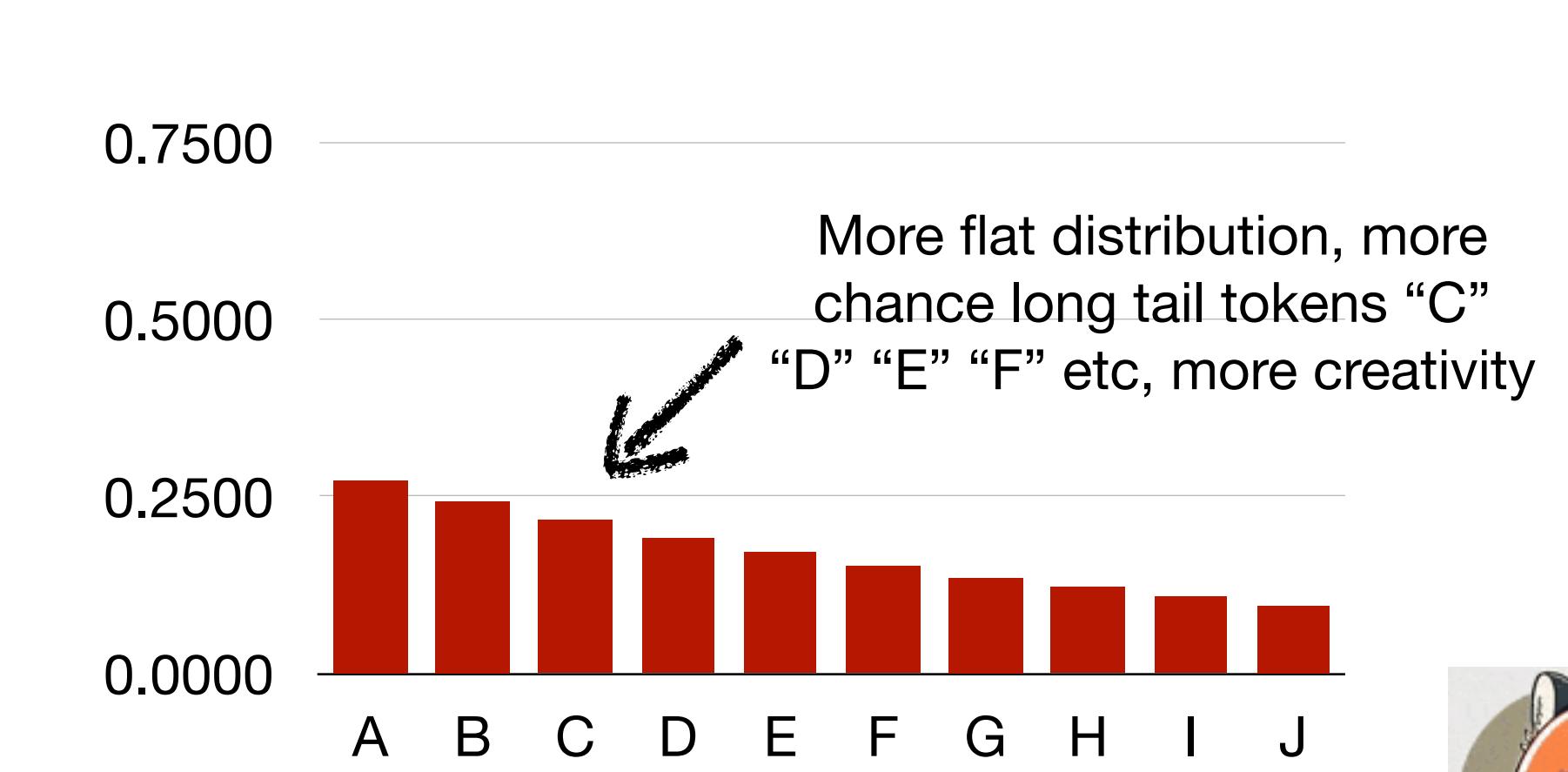
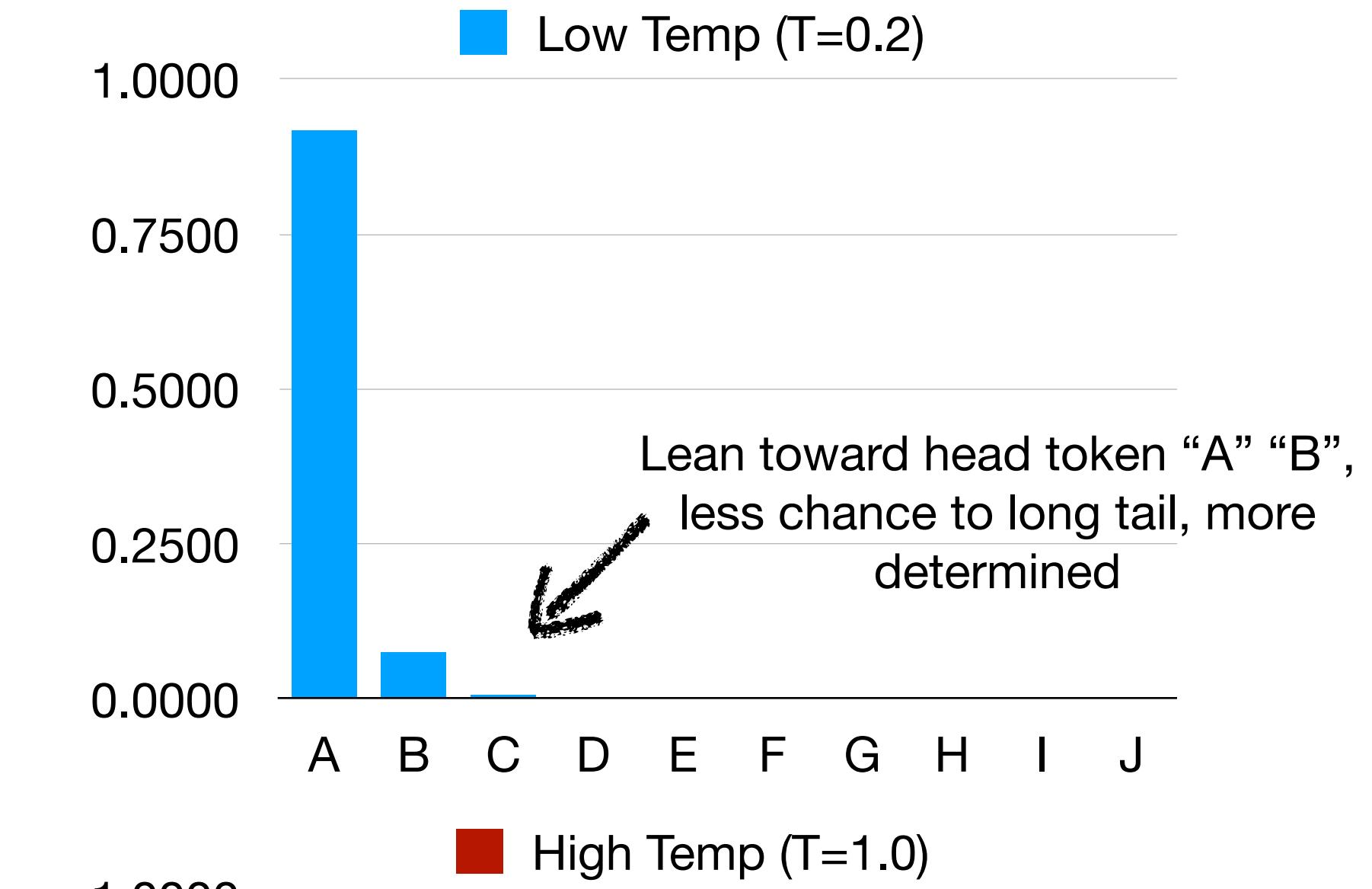
```
response = openai.ChatCompletion.create(  
    model='gpt-4o',  
    temperature=0.7,  
    max_tokens=30,  
    messages=[{  
        'role': 'user',  
        'content': question  
    }],  
)
```

$$P(x_i) = \frac{\exp(z_i/T)}{\sum_{j=1}^N \exp(z_j/T)}$$

Softmax with Temperature  
Hinton et al "Distilling the Knowledge in a Neural Network" 2015  
<https://arxiv.org/abs/1503.02531>

T = 0.2

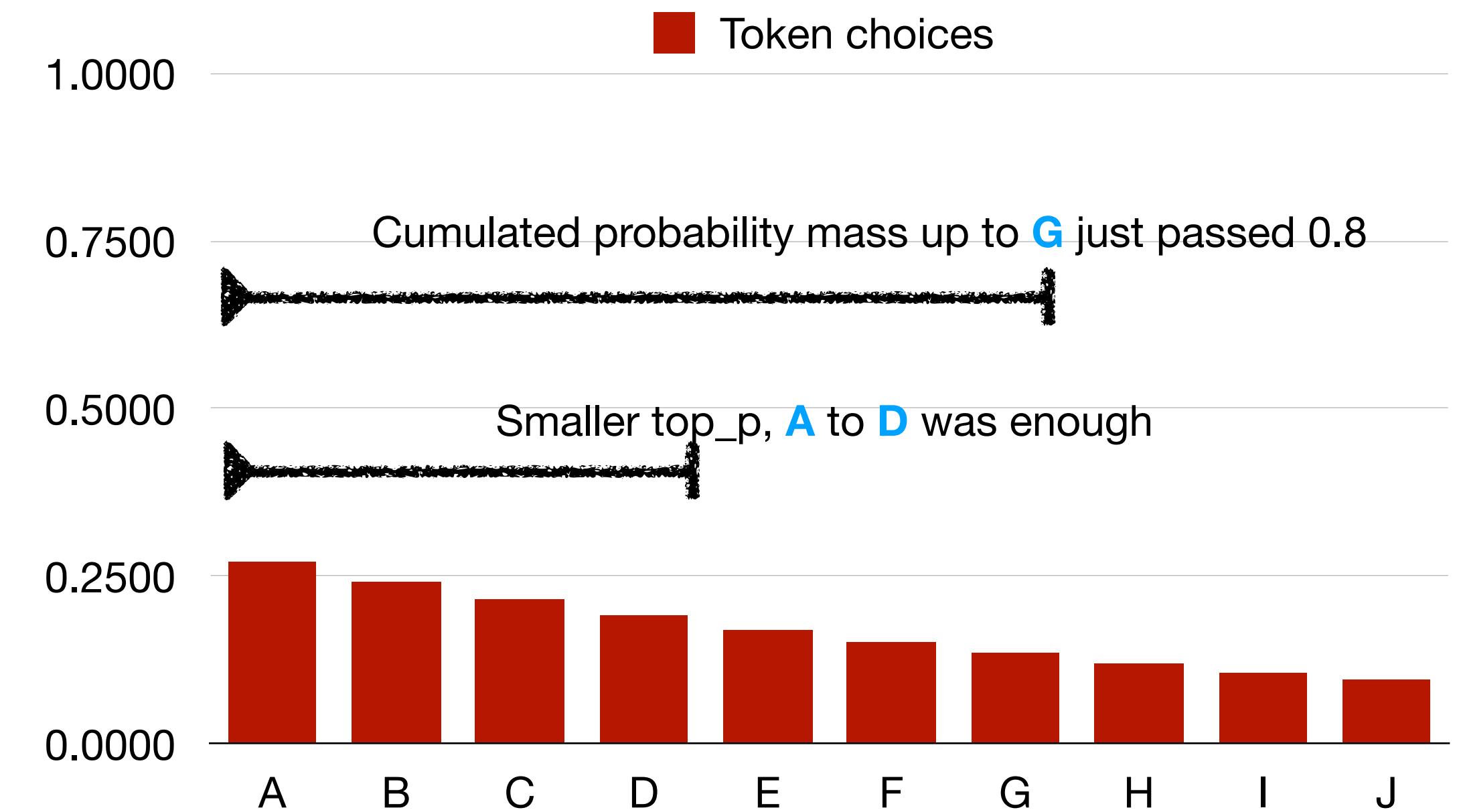
T = 1.0



# Understand top\_p in LLM

```
response = openai.ChatCompletion.create(  
    model='gpt-4o',  
    max_tokens=30,  
    top_p=0.8,  
    messages=[ {  
        'role': 'user',  
        'content': question  
    } ],  
)
```

$$\sum_{x \in V^{(p)}} P(x|x_{1:i-1}) \geq p$$



Top P or "Nucleus sampling"

Holtzman et al, "The Curious Case of Neural Text Degeneration" 2019

<https://arxiv.org/abs/1904.09751>

More flexible than top K to the shape of the probability distribution, across different contexts.



# Understand Boltzmann distribution and neural networks

## What computer scientists see

Probability of class **i** given input **x** in a neural network

$$P(\text{class } i|x) = \text{softmax}(z_i) = \frac{e^{z_i}}{\sum_j e^{z_j}}$$

logits (pre-softmax activations)

interpret neural network input as probabilities instead of numbers



## What physicists see

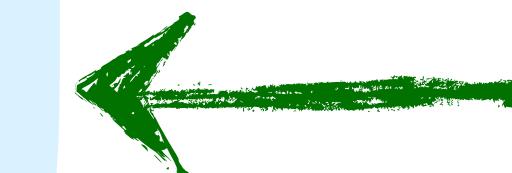
Probability of a certain state

$$P(s) = \frac{1}{Z} \exp\left(-\frac{E(s; \theta)}{T}\right)$$

Partition function (normalization)

Energy of the state

Temperature parameter



Optimization and generalization can make statistical sense.



# Understand prompting

Prompting introduces a **task-specific bias** to the model's output distribution by its **activation patterns**, effectively **aligning the target task** with the LLM's pre-trained task manifold.

$$P(y_t | \text{prompt}, x) = \text{softmax}(f(h_L + \beta \nabla_{\text{task}})/\tau)$$
$$h_L = g_L(\dots g_2(g_1([E_{\text{prompt}}; E_x])))$$

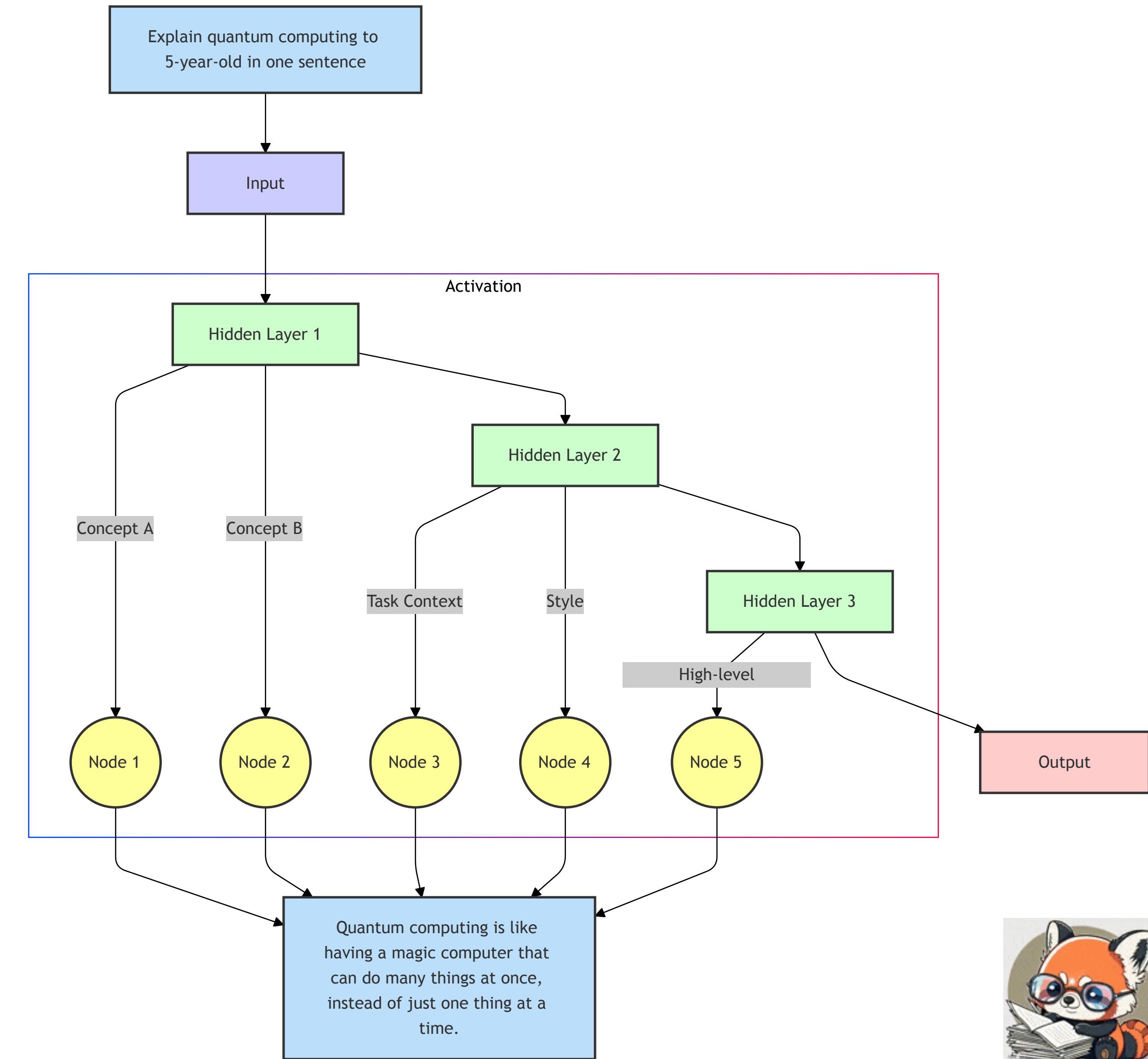
Hidden states

Task gradient

$$\nabla_{\text{task}} = \frac{\partial \mathcal{L}_{\text{task}}}{\partial h_L}$$

Alignment strength

$$\beta = \gamma \cdot \text{sim}(h_L, h_{\text{task}})$$



# Understand tool using

LLMs translate language tasks into tool actions by computing  $P(\text{tool}|\text{context})$  through **attention-based alignment** between task requirements and tool capabilities.

$$P(\text{action}|\text{state}) = \int P(\text{action}|\text{intention})P(\text{intention}|\text{state})dI$$

**Connect state observation to action selection**

$$P(\text{tool}|\text{context}) = \text{softmax}\left(\frac{h_{\text{tool}}^{\text{ctx}} W_{\text{out}}}{\tau}\right)$$

**Context-aware tool selection**

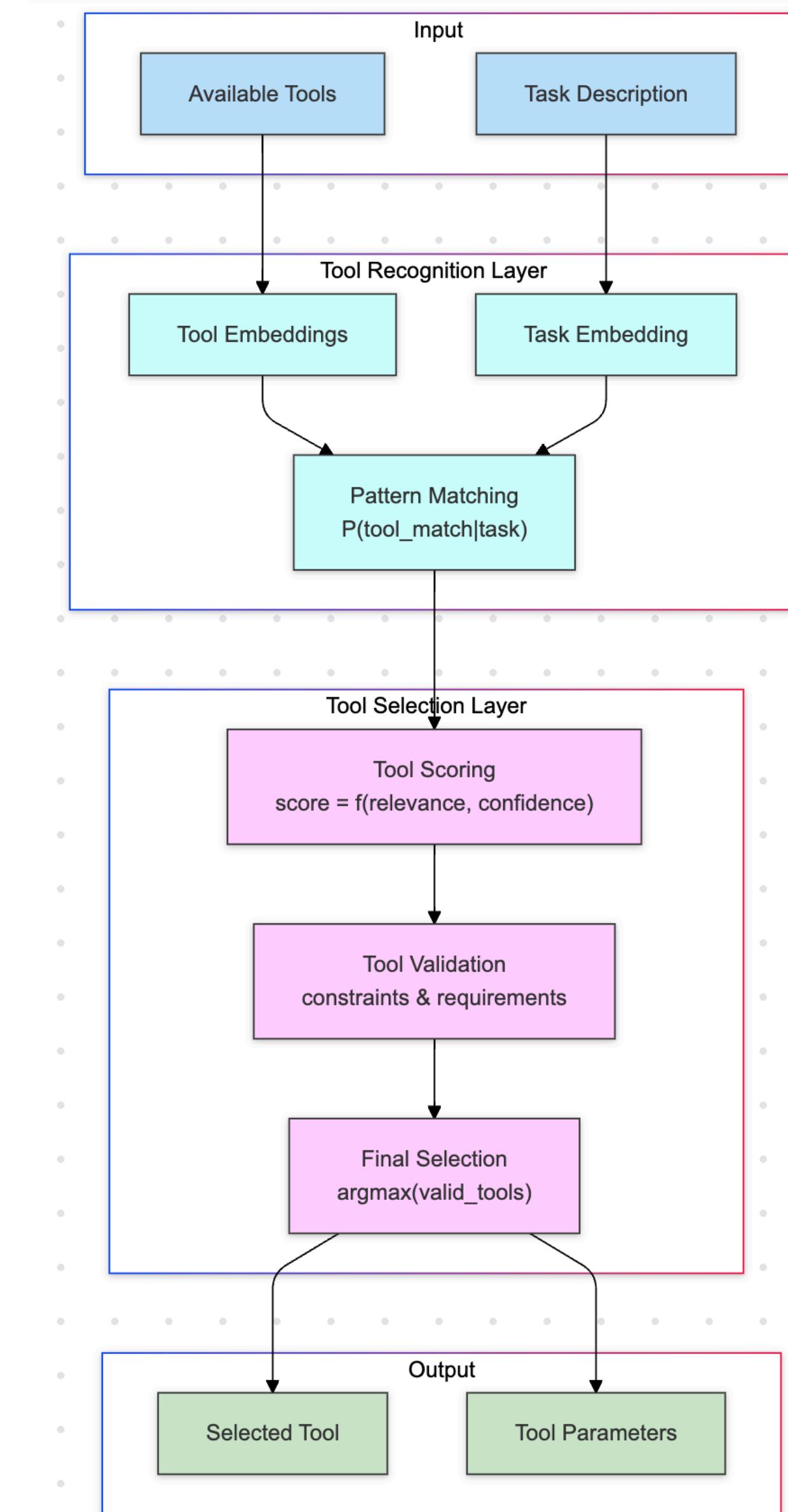
Temperature control

$$\alpha_{\text{tool}} = \text{softmax}\left(\frac{h_{\text{context}} W_Q (h_{\text{tool}} W_K)^T}{\sqrt{d_k}}\right)$$

**Attention of tools**

$$h_{\text{context}} = \text{LayerNorm}([h_{\text{task}}; h_{\text{tools}}])$$

**Hidden states of context**



# Understand chain-of-thoughts (CoT)

Chain-of-thought (CoT) emerges from attention mechanism building up **a working memory** of key-value pairs from each reasoning step, while **hidden states evolve** by attending to this memory to compute next steps, creating **a computational cycle** where each step can query and build upon previous computations.

Attention-based Memory Access

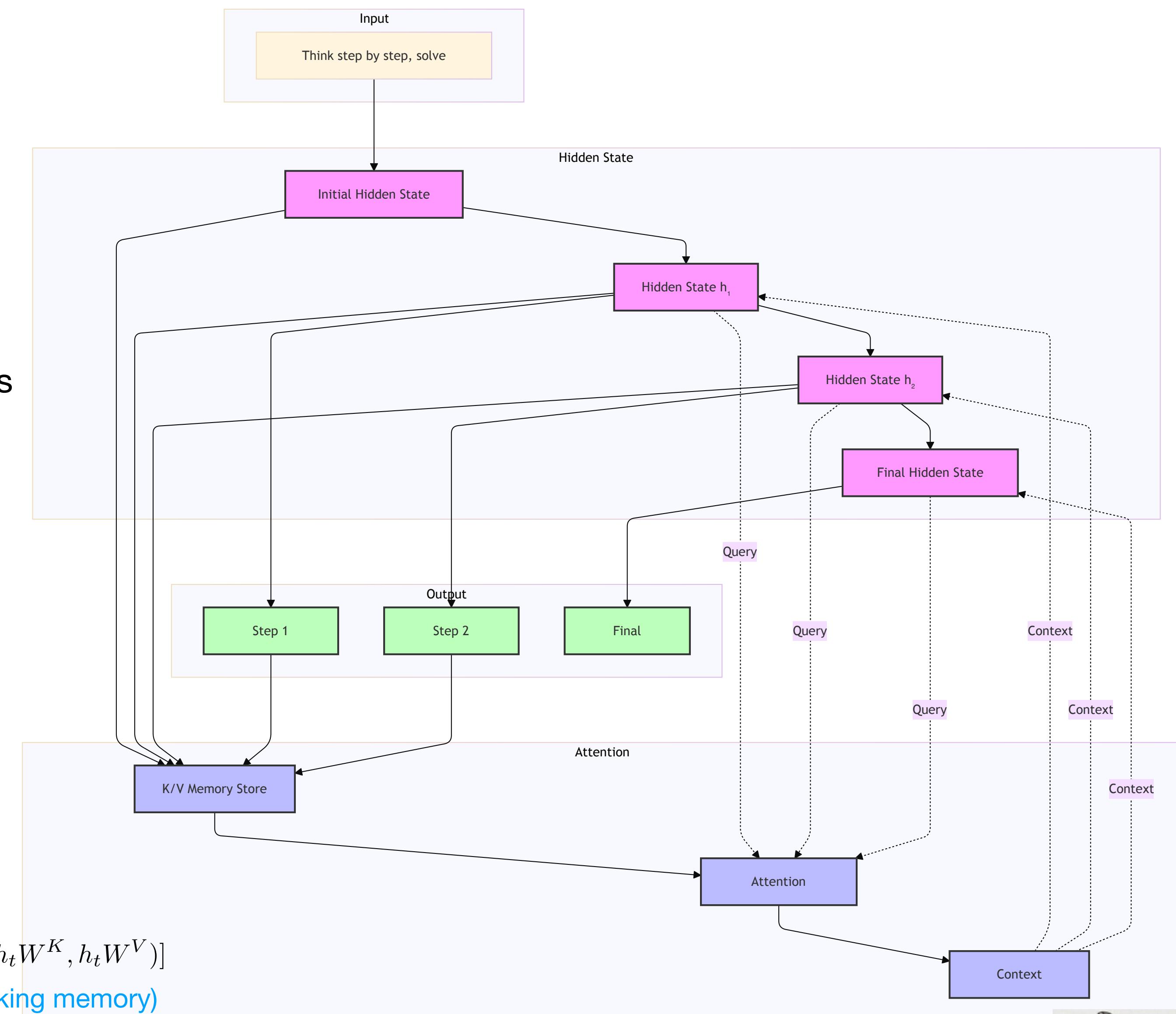
$$c_t = \text{softmax} \left( \frac{h_t W^Q (M_t^K)^T}{\sqrt{d}} \right) M_t^V$$

Hidden State Evolution

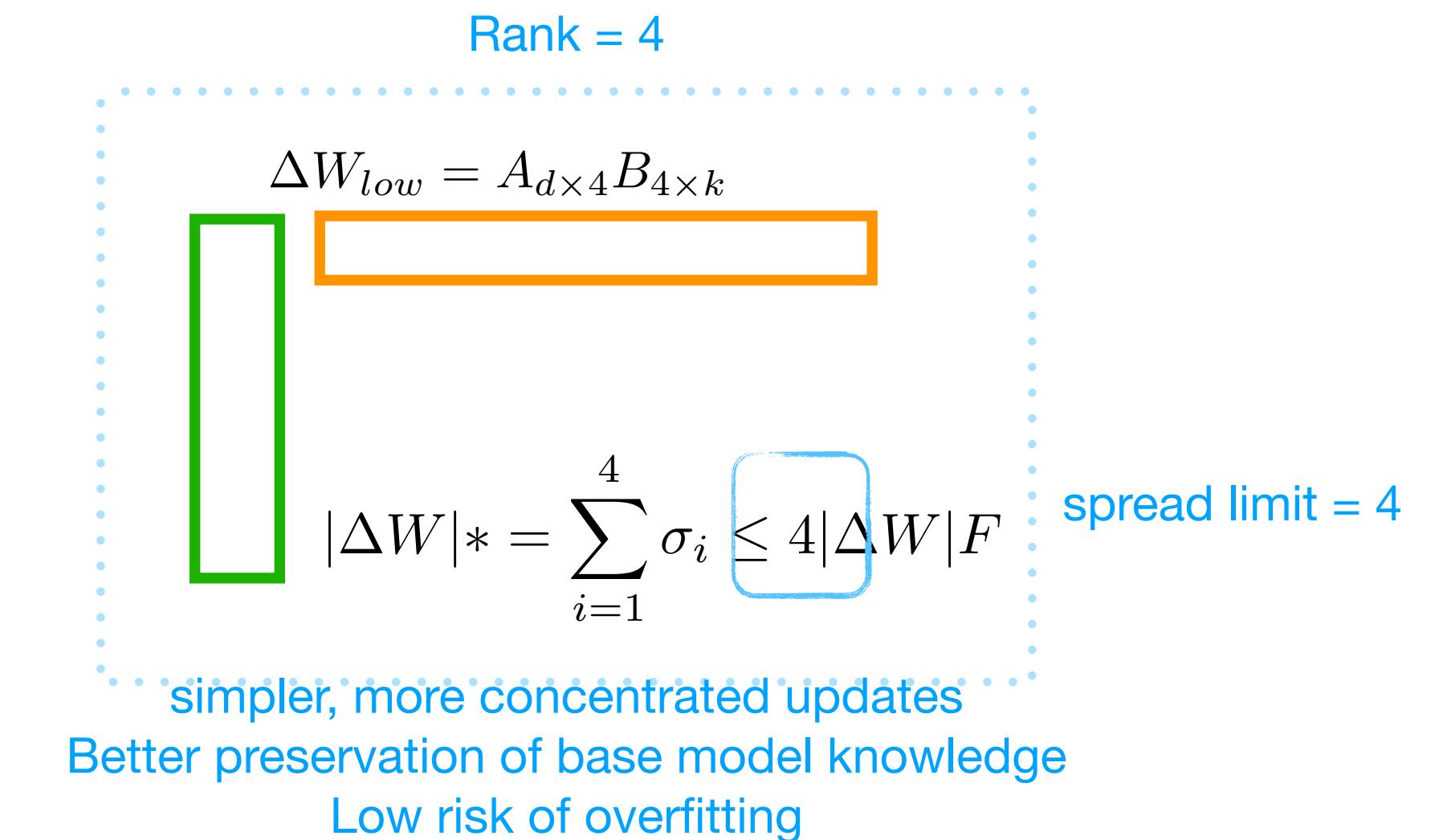
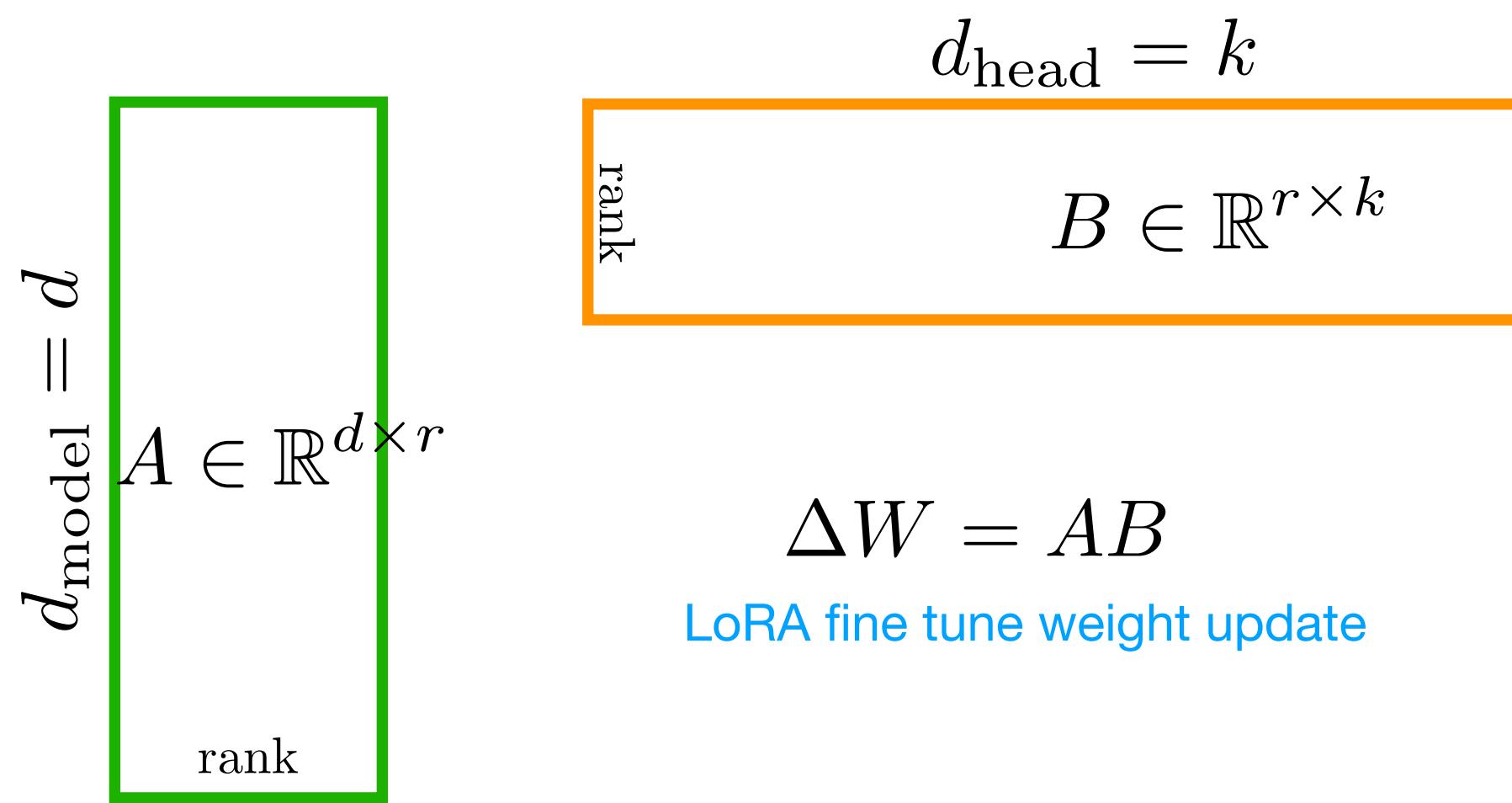
$$h_t = f(\text{attention}(h_{t-1}, [h_1, \dots, h_{t-1}]))$$

$$M_t = [K_t, V_t] = [(h_1 W^K, h_1 W^V), (h_2 W^K, h_2 W^V), \dots, (h_t W^K, h_t W^V)]$$

KV Memory Collection (working memory)



# Understand LoRA ranks



Nuclear Norm (sum all singular values)  $\min(d, k)$

$$|\Delta W|* = \sum_{i=1}^{\min(d, k)} \sigma_i$$

spread limit or “rank” (power of LoRA fine-tune)

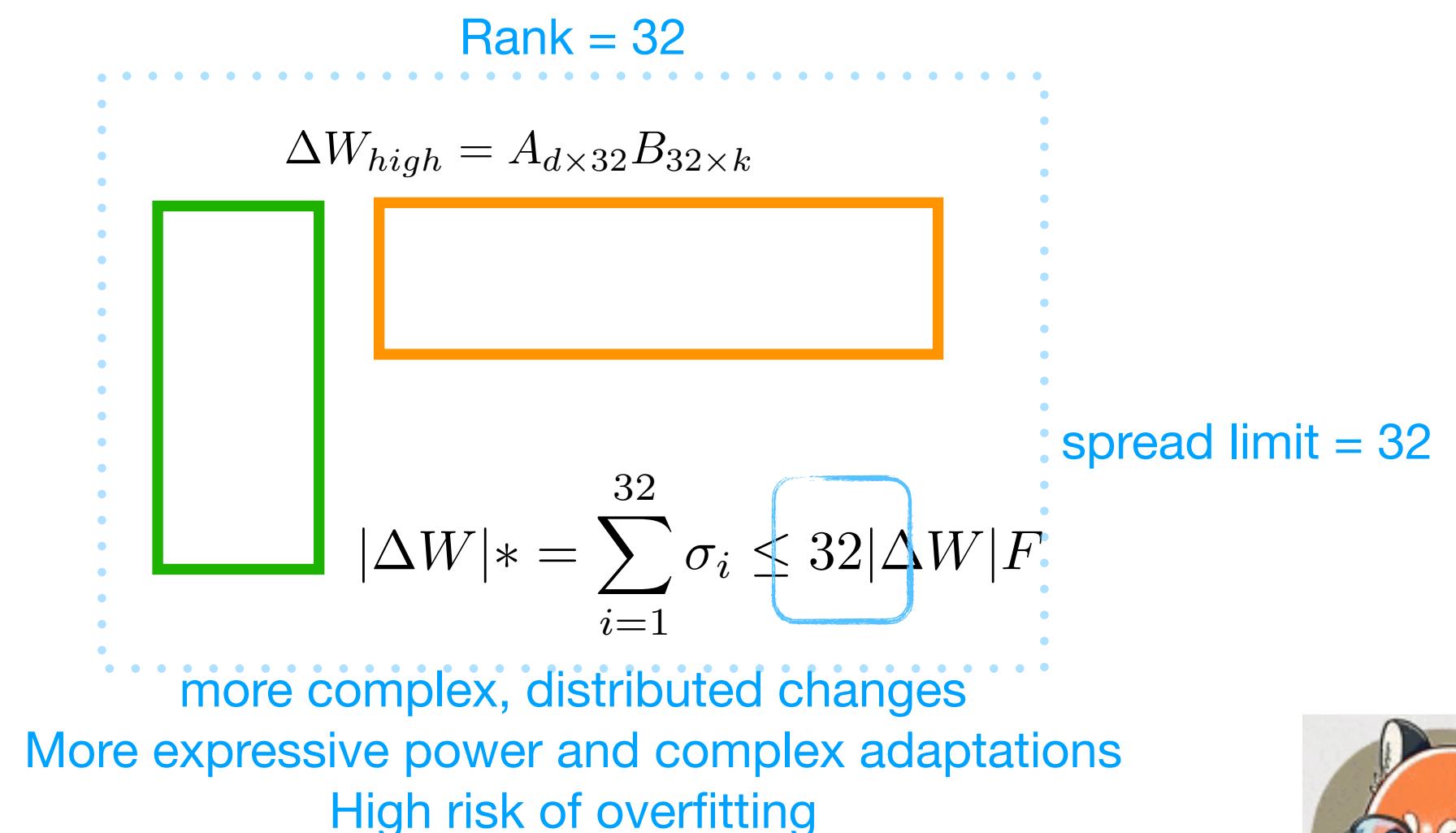
---

Frobenius Norm as sqrt(sum of squared singular values)

$$|\Delta W|F = \sqrt{\sum_{i=1}^{\min(d, k)} \sigma_i^2}$$

$\sum_{i=1}^r \sigma_i \leq \sqrt{r} \sqrt{\sum_{i=1}^r \sigma_i^2}$

Cauchy-Schwarz Interpretation



# Understand LLM inference time

Total time

Position embeddings

$$T_{pos} = \mathcal{O}(s \cdot d)$$

+

Self-attention

$$T_{attn} = \mathcal{O}(s \cdot d \cdot h)$$

Number of layers ✖

32 layers in Llama 8B

Feed-forward network

$$T_{ffn} = \mathcal{O}(s \cdot d \cdot 4d)$$

Layer norm

$$T_{ln} = \mathcal{O}(s \cdot d)$$

+

Final layer norm

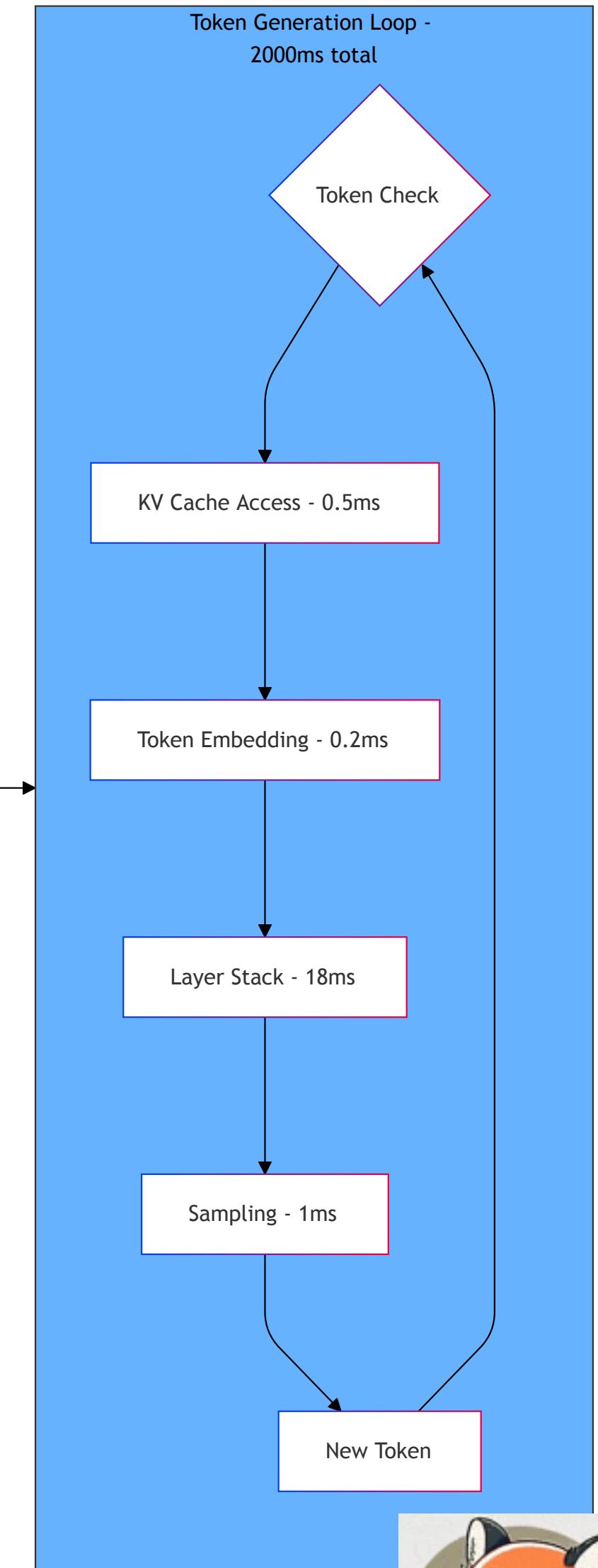
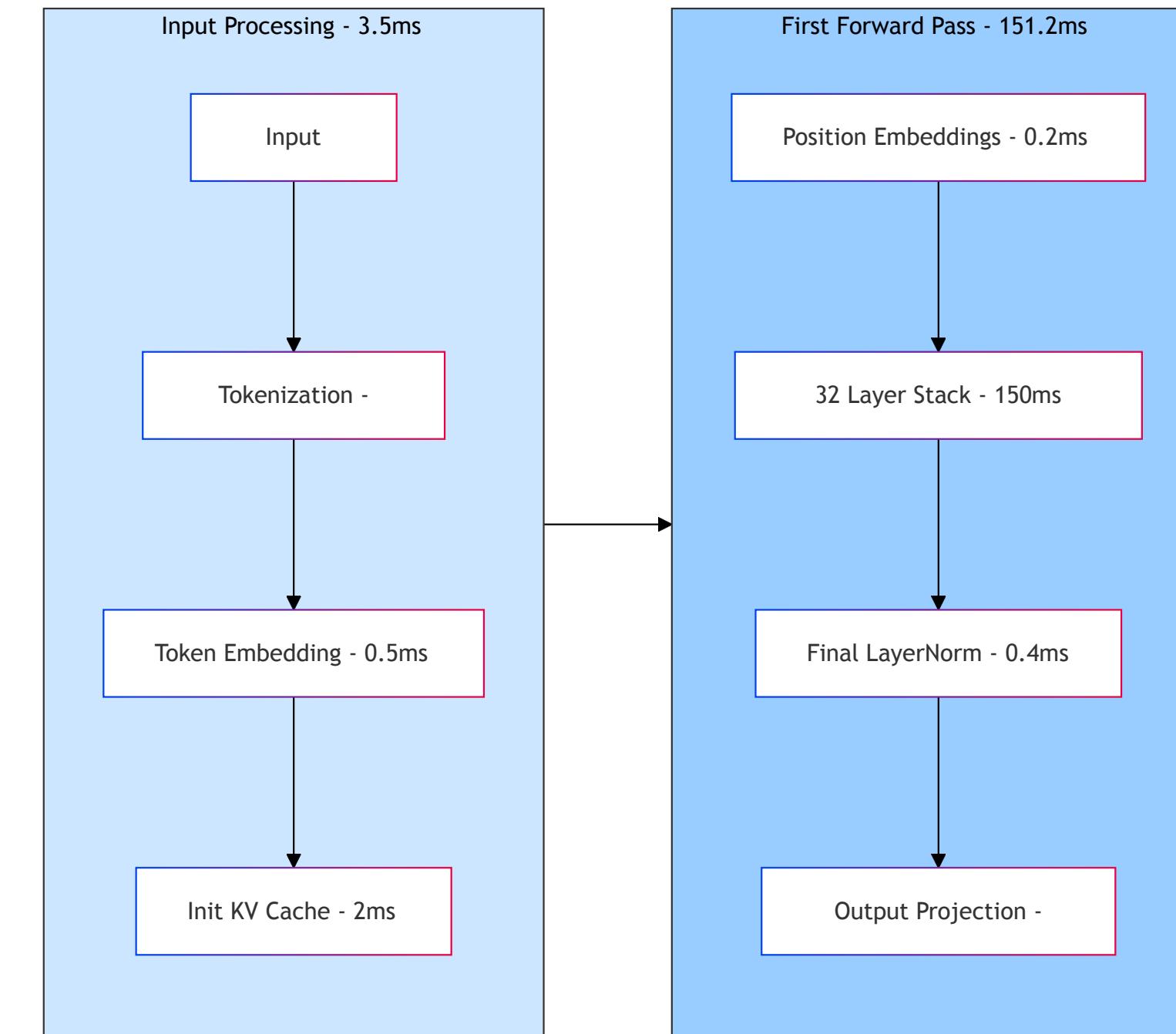
$$T_{ln} = \mathcal{O}(s \cdot d)$$

+

Output projection

$$T_{proj} = \mathcal{O}(s \cdot d \cdot v)$$

Example: Llama 8B (FP32) on T4 GPU  
512 tokens input 100 tokens output



$s$  = sequence length (e.g. 512)

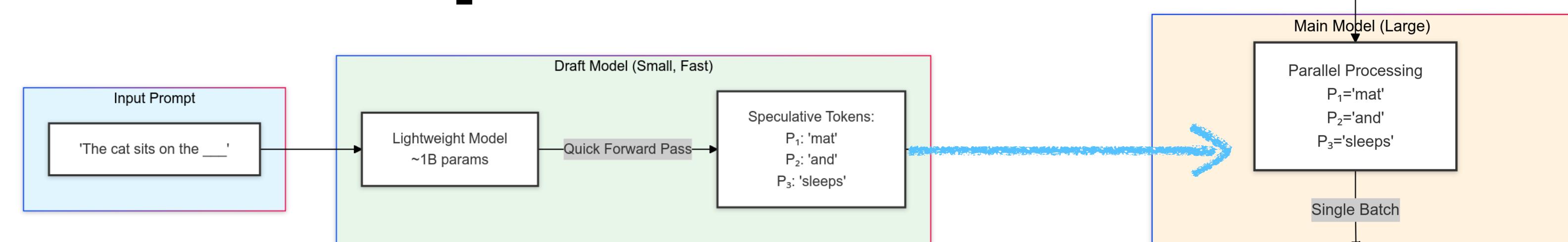
$d$  = hidden dimension (4096)

$h$  = number of attention heads (32)

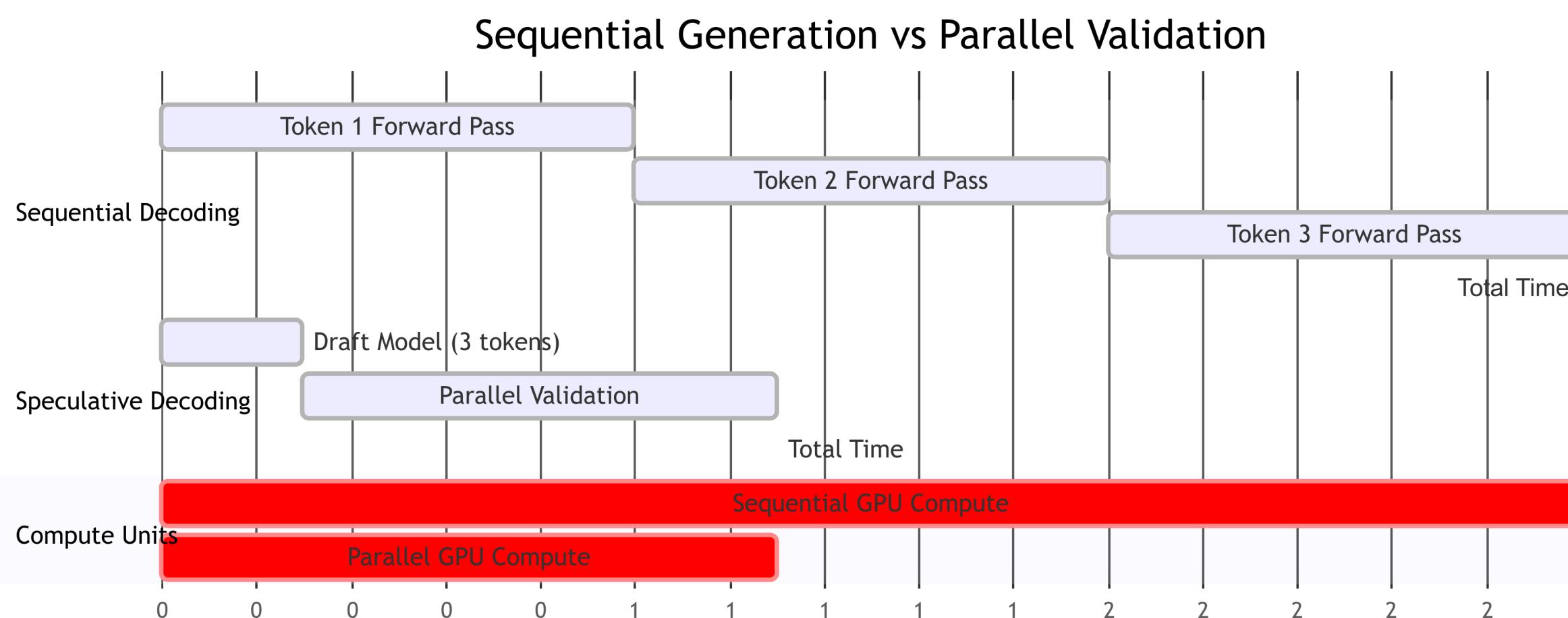
$v$  = vocabulary size (e.g. 32k)



# Understand speculative decoding



Speculative decoding uses a small model to quickly guess multiple next tokens that a large model can verify in parallel, replacing sequential token generation with batch processing when predictions are correct.



# Understand RoPE and lost-in-the-middle

RoPE enables **longer context** through its **geometric frequency progression** that preserves **relative positional** information via rotational embeddings

It suffers from **attention score decay** proportional to  $\exp(-|m-n|\lambda)$ , leading to **degraded performance for middle tokens** as they compete for attention without strong positional anchors like sequence endpoints.

attention(q,k) only needs relative positions

$$\begin{aligned} QK_{m,n}^T &= \sum_d \frac{e^{im\theta_d} x_q \cdot e^{in\theta_d} x_k}{\sqrt{d}} \\ &= \sum_d \frac{|x_q||x_k| \cos((m-n)\theta_d)}{\sqrt{d}} \end{aligned}$$

$\cos((m-n)\theta_d)$  oscillates much for large  $|m-n|$  of large open angle

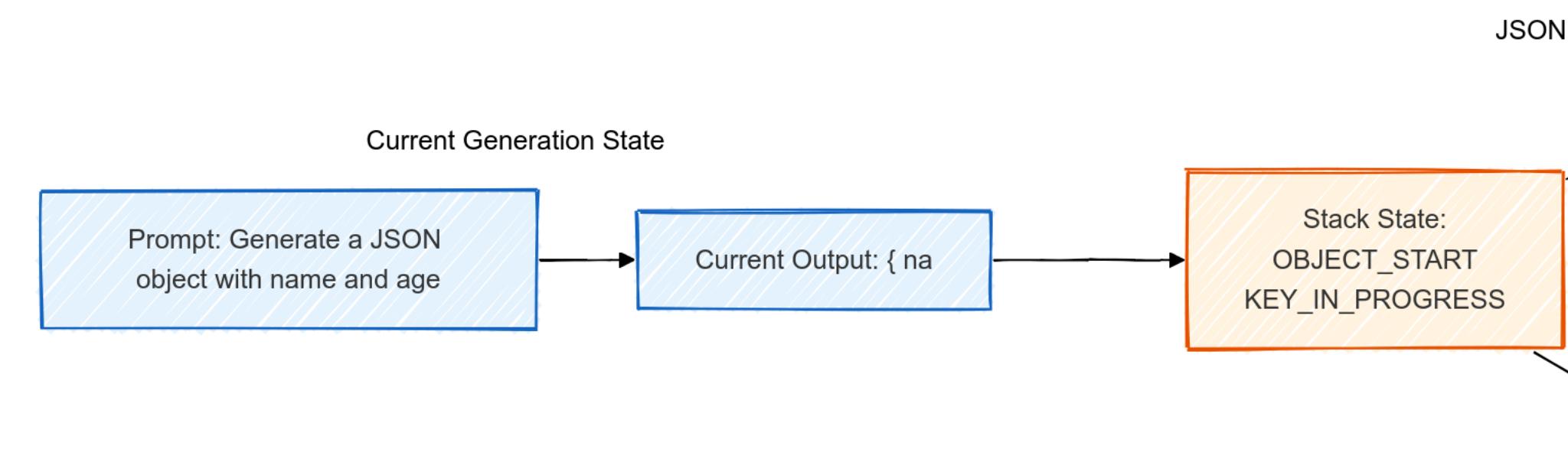
$\theta_d = \omega^{d/D} \theta_0$  high dimension decays faster, so decay score is

$$\text{score}(m, n) \propto \exp\left(-\frac{|m-n|}{\lambda}\right)$$

$$\begin{aligned} Q_{RoPE} &= e^{im\theta} x_q W_q \\ &= [\cos(m\theta)x_q - \sin(m\theta)y_q] W_q \\ K_{RoPE} &= e^{in\theta} x_k W_k \\ &= [\cos(n\theta)x_k - \sin(n\theta)y_k] W_k \end{aligned}$$

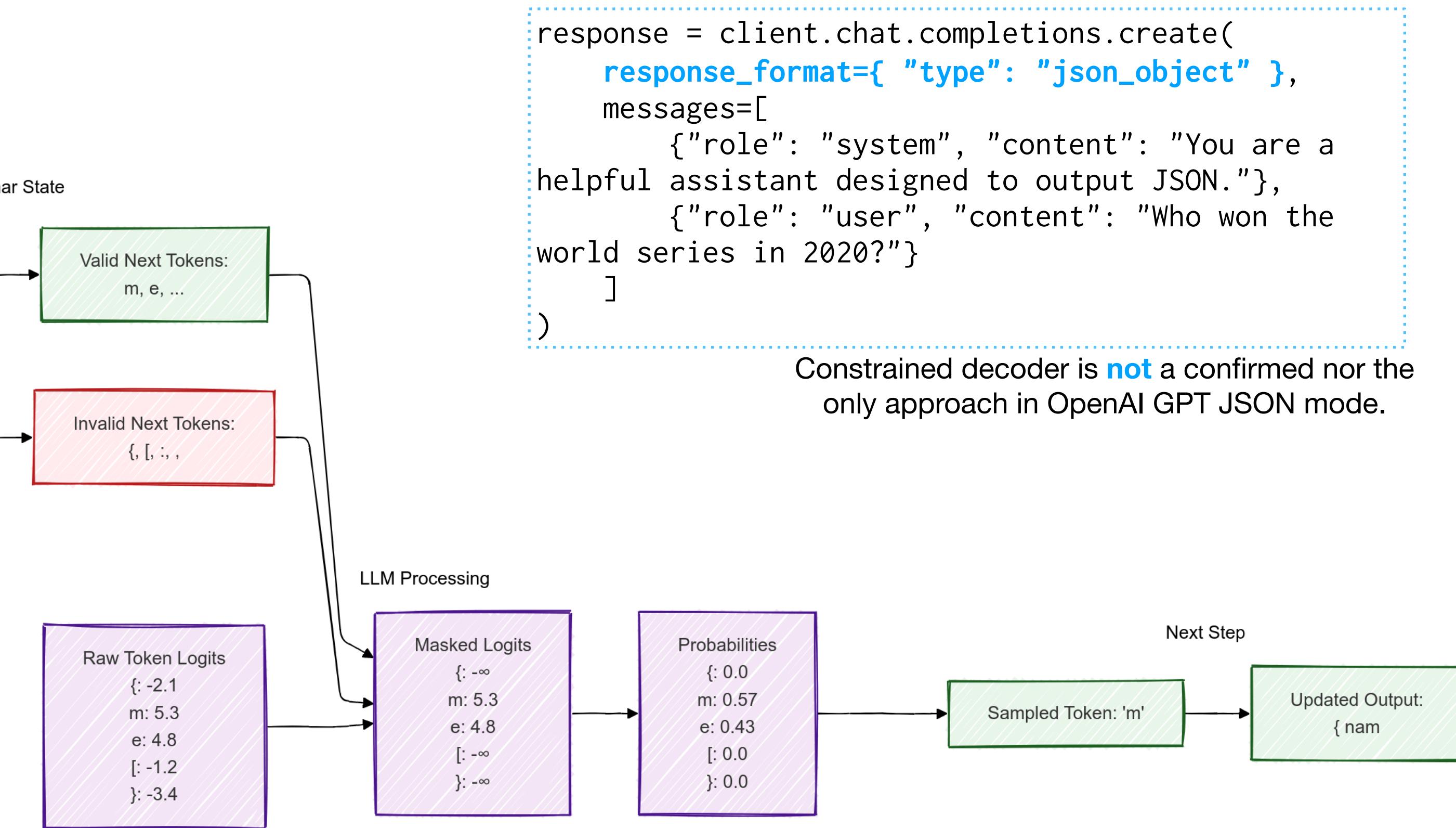


# Understand constrained decoder and JSON mode



Constrained decoding **filters LLM's token probabilities using grammar rules**, ensuring valid structure while preserving relative probabilities among valid tokens.

Prompt engineering, fine-tuning, context-free generations etc are other techniques of structured generation for JSON mode.



# Understand ReAct and cross-attention

You are a helpful math assistant that solves problems step by step. Use the following format:

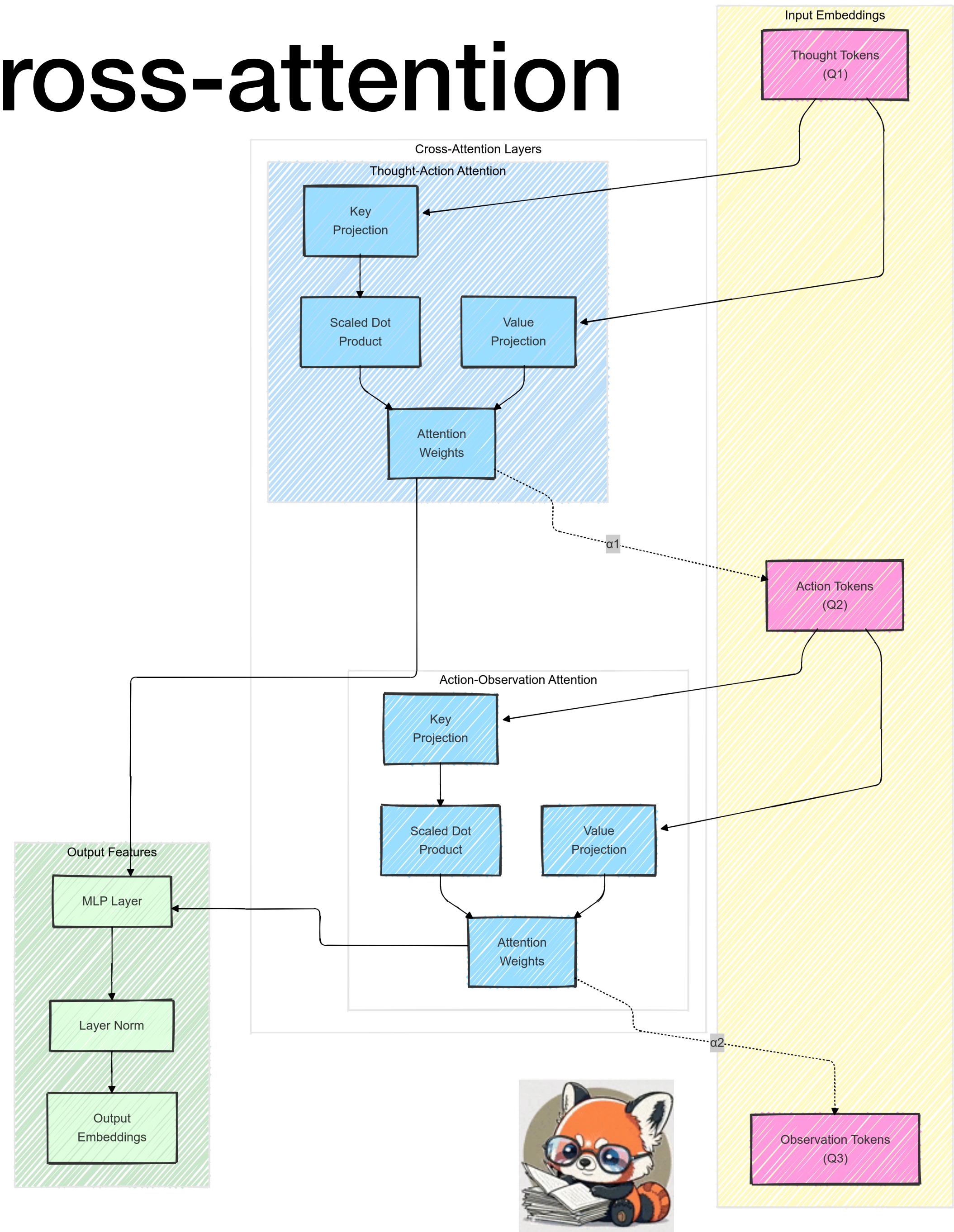
**Thought**: Analyze the problem and consider solution steps

**Action**: Choose a specific integration technique or step to take

**Observation**: Note the results of applying that step ...  
(continue this cycle until solution is reached)

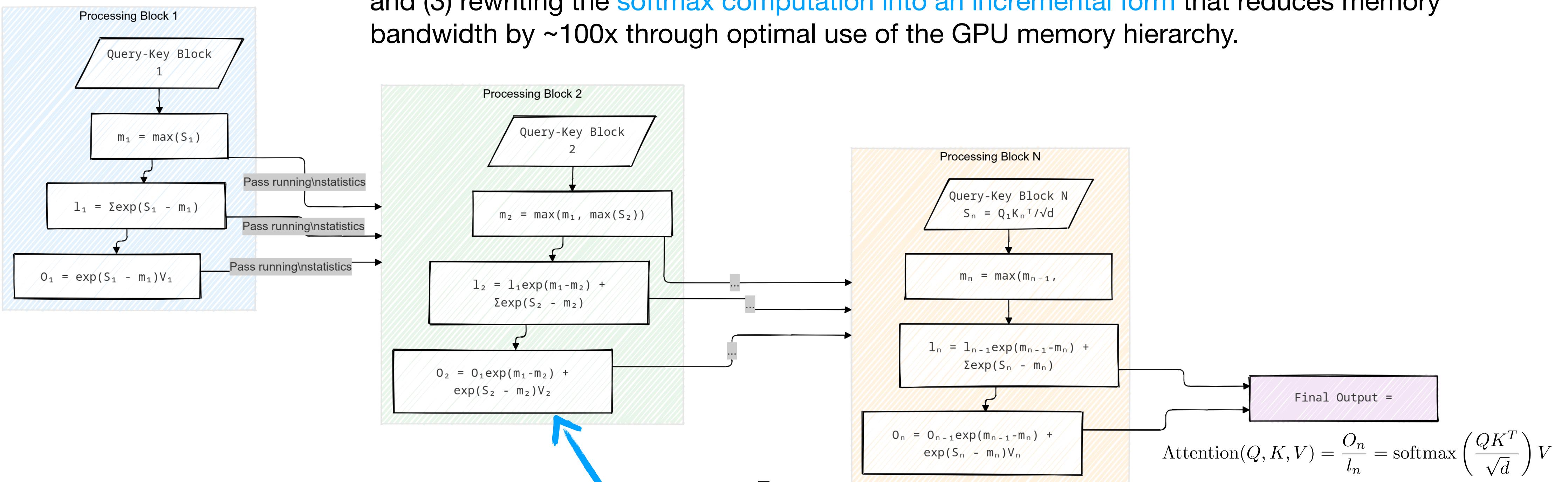
**TASK**: Solve this integral:  $\int x^2 \cos(x) dx$

ReAct uses **cross-attention** to create learnable neural bridges between its three core steps (Reasoning, Acting, and Observation), enabling the transformer to translate abstract thoughts into concrete actions, process their outcomes, and update its reasoning strategies through focused attention weights.



# Understand flash attention, part I

Flash Attention achieves standard attention's computation **with O(N)** rather than O( $N^2$ ) memory usage by leveraging block-wise computations in fast SRAM memory – specifically by (1) rescaling previous results with exponential factor to maintain stability, (2) processing **attention computations in blocks** that fit in fast GPU memory without materializing the full attention matrix, and (3) rewriting the **softmax computation into an incremental form** that reduces memory bandwidth by ~100x through optimal use of the GPU memory hierarchy.



# Understand reinforced fine tuning

ReFT enhances LLM reasoning through a two-stage process: supervised warm-up followed by reinforcement learning that explores multiple Chain-of-Thought paths while maintaining stability via KL-constrained rewards.

The KL divergence term prevents catastrophic forgetting of pre-trained knowledge while allowing controlled exploration of new reasoning paths, ensuring the model maintains useful capabilities from the warm-up phase while discovering diverse solution strategies.

$$r(s_t, a_t, s_{t+1}) = \begin{cases} 1, & \text{EXTRACT}(s_{t+1}) = y \\ 0.1, & \text{EXTRACT}(s_{t+1}) \neq \text{null}, \neq y \\ 0, & \text{EXTRACT}(s_{t+1}) = \text{null} \end{cases}$$

$$r_{total}(s_t, a_t, s_{t+1}) = r(s_t, a_t, s_{t+1}) - \beta KL(\pi_\theta(\cdot|s_t), \pi_\theta^{(0)}(\cdot|s_t))$$

Total Reward with KL Constraint

