Project GRASP – primes 1

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1 Administrative Stuff

1.1 Mailing list

Please sign up to the mailing list: https://groups.google.com/forum/#!forum/project-grasp.

1.2 British Mathematical Olympiad

2 Problems

1. Bezout's lemma. Let a and b be any natural numbers, with $\gcd(a, b) = 1$. Show that there exist integers n and k, such that $a \cdot n + b \cdot k = 1$.

2. Prove Euclid's lemma. A prime number p divides ab if and only if p divides a or p divides b.

3. State and prove Fundamental Theorem of Arithmetic. The idea behind the theorem is that most natural numbers can be decomposed into prime factors, which are in some sense unique. But it's a good exercise to try and "dress up" it into a good wording. You should read a treatment of this subject on Timothy Gower's (a prominent British Mathematician) blog: https://gowers.wordpress.com/2011/11/18/proving-the-fundamental-theorem-of-arithmetic/

4. Let a and b be any positive integers, and let p be any prime number $p > 2$, such that p divides $a+b$ and p divides a^2+b^2. Show that p^2 divides a^2+b^2.

5. A positive integer is called charming if it is equal to 2 or is of the form $3^i 5^j$, where i and j are non-negative integers. Prove that every positive integer can be written as a sum of different charming integers.

6. Let m and n be such integers, that in the set $\{1, 2, \ldots, n\}$ there are exactly m prime numbers. Show that, among any $m + 1$ numbers from this set

one can find a number which is a divisor of the product of the other m numbers.