Project GRASP – squares 1

Adam Kurkiewicz
Univeristy of Glasgow
adam@kurkiewicz.pl

September 26, 2017

1 Administrative Stuff

1.1 Mailing list

Please sign up to the mailing list: https://groups.google.com/forum/#!forum/project-grasp.

1.2 British Mathematical Olympiad

British Mathematical Olympiad (BMO) is the most challenging high school maths competition in the UK, with 6 best participants going to IMO (International Mathematical Olympiad), which is the most challenging international high school competition.

There are 2 ways to sign up for BMO:

- Qualifying through Senior Challenge. Your maths teacher has to sign you up, I can't do this. This is the form your teacher has to fill in:

  https://www.ukmt.org.uk/docs/EF%201718%20webform%20UK.pdf

  The deadline is 16th of October. You should all try to qualify in this way, as it is the cheapest way to qualify, and solving the Senior Challenge is a form of practice. Don't treat the Senior Challenge too seriously though – BMO is very different and requires different type of skills.

- If you don't qualify through Senior Challenge, your teacher can still sign you up, but we can figure that out later.

2 Problems

1. Find x.

$$x = \frac{1}{1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\dots}}}}$$

2. Determine all pairs (m, n) of positive integers which satisfy the equation $n^2 - 6n = m^2 + m - 10$.

3. Naomi and Tom play a game, with Naomi going first. They take it in turns to pick an integer from 1 to 100, each time selecting an integer which no-one has chosen before. A player loses the game if, after their turn, the sum of all the integers chosen since the start of the game (by both of them) cannot be written as the difference of two square numbers. Determine if one of the players has a winning strategy, and if so, which.

4. Three integers a, b, c are given. Show that there exists an integer n such that $n^3 + an^2 + bn + c$ is not a square of of any integer.

5. Determine whether there exist such three different non-zero real numbers a, b, c, such that among the numbers $\frac{a+b}{a^2+ab+b^2}$, $\frac{b+c}{b^2+bc+c^2}$, $\frac{c+a}{c^2+ca+a^2}$, two numbers are equal to each other and the third number is not equal to the others.

6. Show that the equation $(x^2 + 2y^2)^2 - 2(z^2 + 2t^2)^2 = 1$, where t, x, y, z are integers has infinitely many solutions.