

Project GRASP – squares 2

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1 Administrative Stuff

1.1 Mailing list

Please sign up to the mailing list: <https://groups.google.com/forum/#!forum/project-grasp>.

1.2 British Mathematical Olympiad

2 Problems

1. Prove Euclid's lemma – if a prime number p divides ab , then p divides a or p divides b .
2. State and prove Fundamental Theorem of Arithmetic. The idea behind the theorem is that most natural numbers can be decomposed into prime factors, which are in some sense unique. But it's a good exercise to try and "dress up" it into a good wording.
3. Let a and b be any positive integers, and let p be any prime number $p > 2$, such that p divides $a+b$ and p divides a^2+b^2 . Show that p^2 divides a^2+b^2 .
4. A positive integer is called charming if it is equal to 2 or is of the form $3^i 5^j$, where i and j are non-negative integers. Prove that every positive integer can be written as a sum of different charming integers.
5. Let m and n be such integers, that in the set $\{1, 2, \dots, n\}$ there are exactly m prime numbers. Show that, among any $m+1$ numbers from this set one can find a number which is a divisor of the product of the other m numbers.