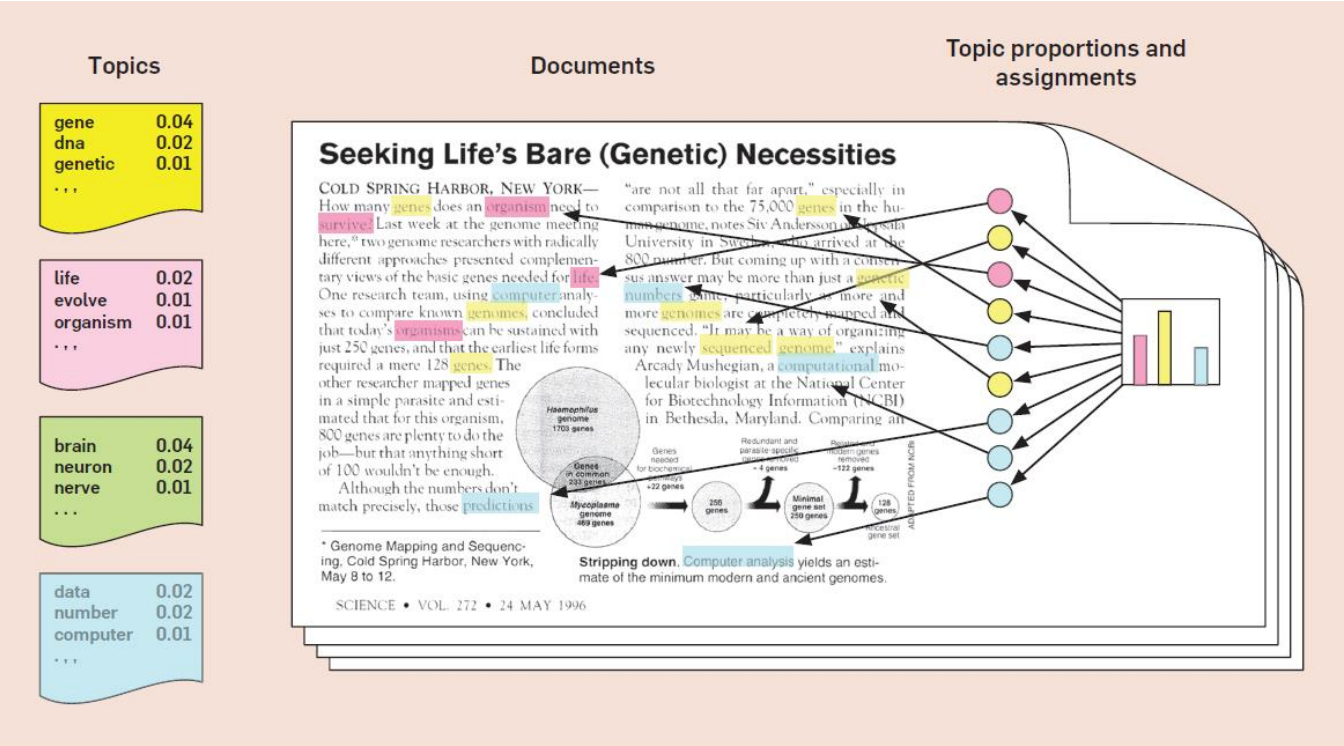

Lecture 9: Topic Modeling

Pilsung Kang

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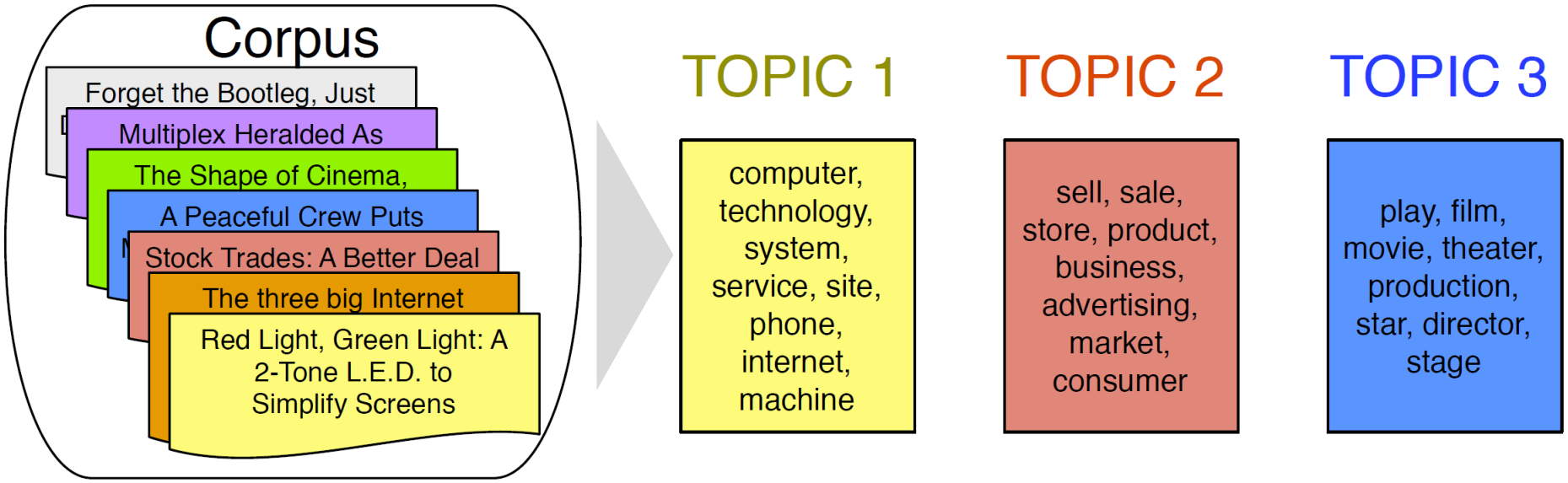
AGENDA

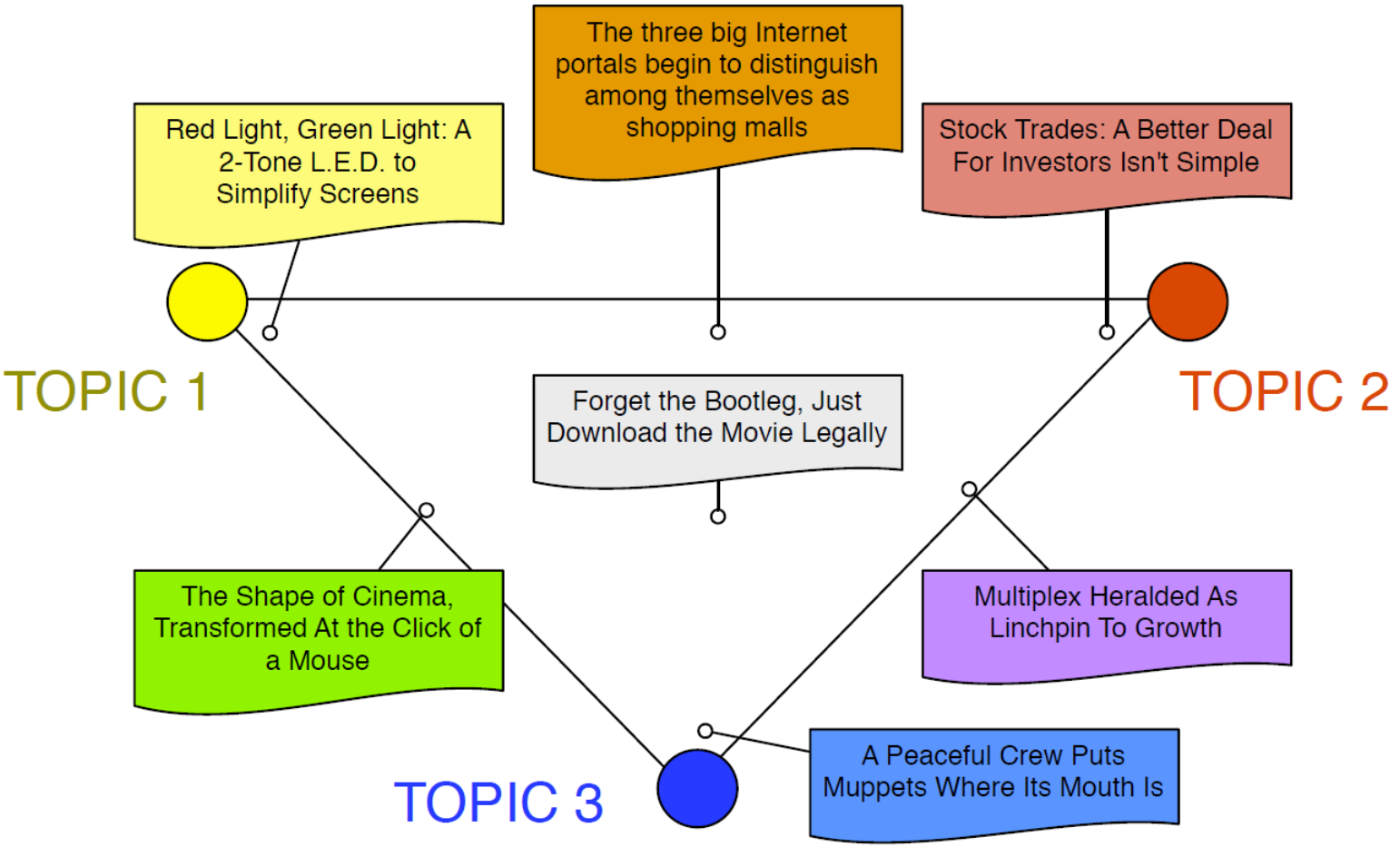
Topic Model: Conceptual Approach

- Topic Model
  - ✓ From an input corpus and the number of topics K → words to topics
  - ✓ From an input corpus and the number of topics K → words to topics



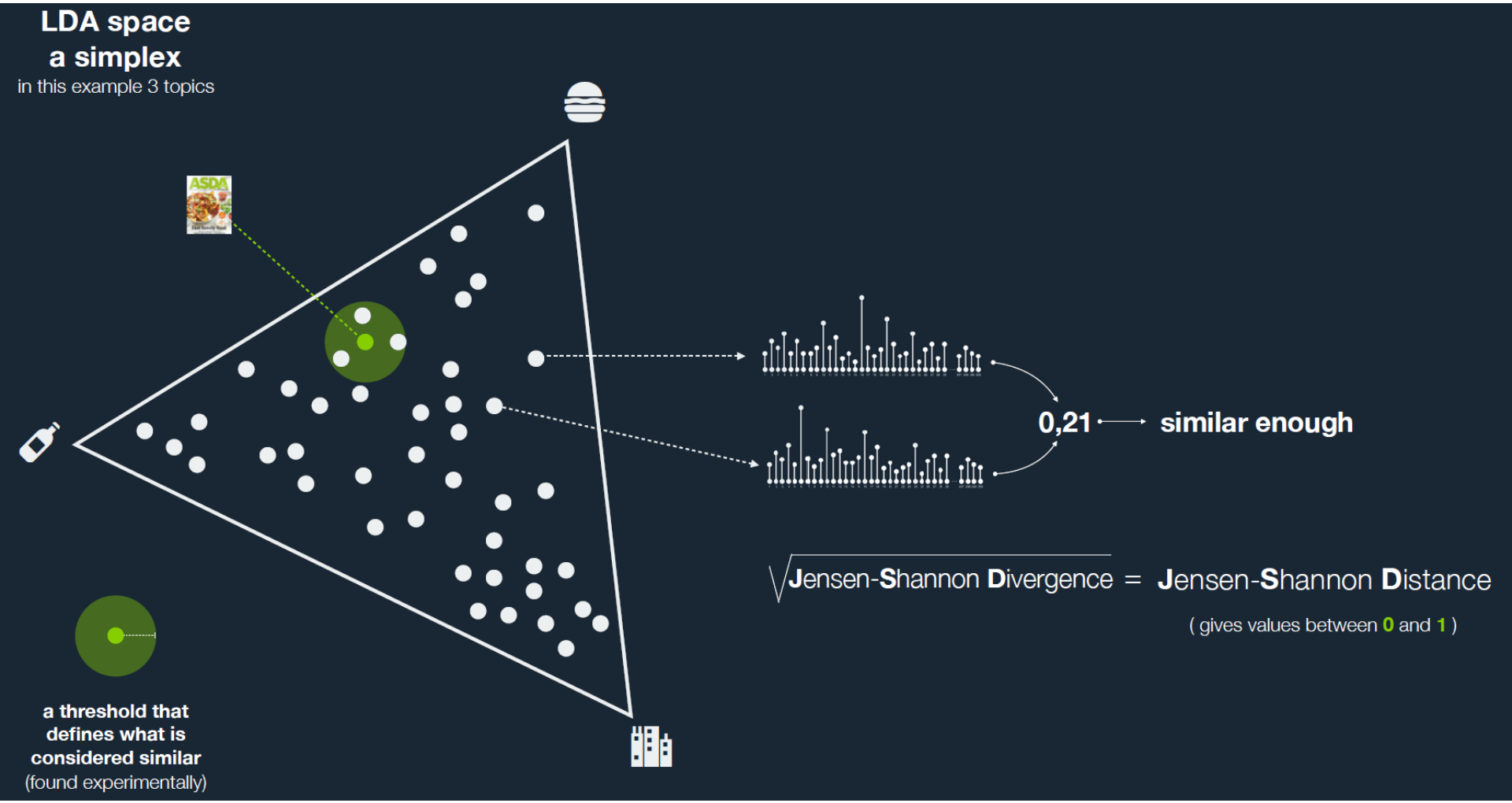
Topic Model: Conceptual Approach

- Topic Model
  - ✓ For each document, what topics are expressed by that document?



Topic Model: Conceptual Approach

Knispelis (2015)



Topic Models: Topic Extraction

Kim et al. (2016)

- Topic Extraction
  - ✓ 30 Topics discovered for "Deep Learning"

Fault detection with DBN	Convolutional neural network	Network Learning	Representation learning	Face Recognition	Speech Recognition	Acoustic Modeling	Extreme Learning	Deep learning architecture	Image Segmentation
deep	neural	layer	feature	face	speaker	speech	deep	deep	image
belief	convolutional	input	level	recognition	speech	recognition	learn	architecture	scene
network	pool	output	extract	estimation	noise	acoustic	algorithm	neural	scale
dbn	convolution	unit	learn	facial	adaptation	hmm	structure	standard	segmentation
fault	convnet	hide	extraction	shape	source	neural	extreme	explore	pixel
		function							

Long-short term memory	Predictive analytics	Signal processing	Classification models	Large-scale computing	Image quality assessment	Visual recognition	NLP	Detection using CNN	Action recognition
term	data	analysis	classification	application	domain	pattern	word	cnn	video
recurrent	prediction	filter	classifier	implementation	state	process	text	detection	human
long	technique	signal	class	efficient	quality	compute	language	convolutional	temporal
lstm	information	component	vector	process	resolution	visual	representation	neural	action
network	research	audio	support	power	relationship	field	semantic	detect	track

Image retrieval	Medical image diagnosis	Reinforcement learning	Parameter optimization	Auto encoder	RBM and variations	Learning with few labeled data	Fast learning complexity reduction	Applications for vehicles & robots	Character recognition
image	image	learn	train	representation	machine	train	fast	time	recognition
visual	segmentation	question	algorithm	learn	boltzmann	data	reduce	real	system
retrieval	disease	state	gradient	sparse	rbm	label	parameter	application	character
descriptor	cell	answer	sample	encode	restrict	few	weight	drive	network
attribute	medical	reinforcement	optimization	stack	distribution	transfer	complexity	Vehicle	neural

Topic Models: Topic Extraction

- Topic Extraction
  - ✓ 50 Topics discovered for "Ultrasound/Ultrasonography"

Vascular	Prostate	heart	CAD	MSK	nerve	tumor	OB	surgery	intervention
plaque	biopsy	artery	image	joint	block	case	ultrasound	surgery	guide
ivus	prostate	carotid	ultrasound	patient	nerve	lesion	fetal	patient	patient
coronary	cancer	patient	method	disease	ultrasound	diagnosis	infant	intraoperative	complication
intravascular	patient	stenosis	base	score	guide	ultrasound	abnormality	preoperative	treatment
stent	transrectal	plaque	propose	arthritis	patient	cyst	prenatal	surgical	percutaneous
patient	trus	ultrasound	feature	ultrasound	pain	mass	case	ultrasound	ultrasound
lesion	guide	cardiac	algorithm	clinical	anesthesia	tumor	fetus	localization	drainage
mm.	core	dus	segmentation	inflammatory	surgery	finding	anomaly	operative	month
ultrasound	ultrasound	stroke	analysis	activity	plexus	ultrasonography	diagnosis	resection	rate
area	rate	arterial	result	study	technique	present	congenital	surgeon	procedure

osteoporosis	cerebral	ER&ICU	cancer	Lab test	US general	vein	lymph node	lung	Healthcare
age	brain	patient	cancer	extraction	ultrasound	vein	node	lung	patient
ultrasound	dog	emergency	patient	assist	imaging	venous	lymph	chest	risk
child	fus	care	tumor	ultrasound	technique	patient	patient	ultrasound	ultrasound
bone	bbb	ultrasound	stage	method	clinical	internal	biopsy	patient	year
year	ultrasound	department	eus	liquid	review	ultrasound	metastasis	pulmonary	study
study	blood	bedside	gastric	sample	application	jugular	ultrasound	lus	follow
fat	study	perform	ovarian	time	diagnostic	thrombosis	cancer	pleural	clinical
qus	day	physician	endoscopic	solvent	disease	central	guide	line	factor
body	follicle	point	ultrasonography	determination	article	dvt	negative	radiography	month
measure	barrier	cardiac	invasion	extract	role	femoral	positive	diagnosis	age

Topic Models: Topic Extraction

- Topic Extraction
  - ✓ 10 Topics discovered for "Insider Threat"

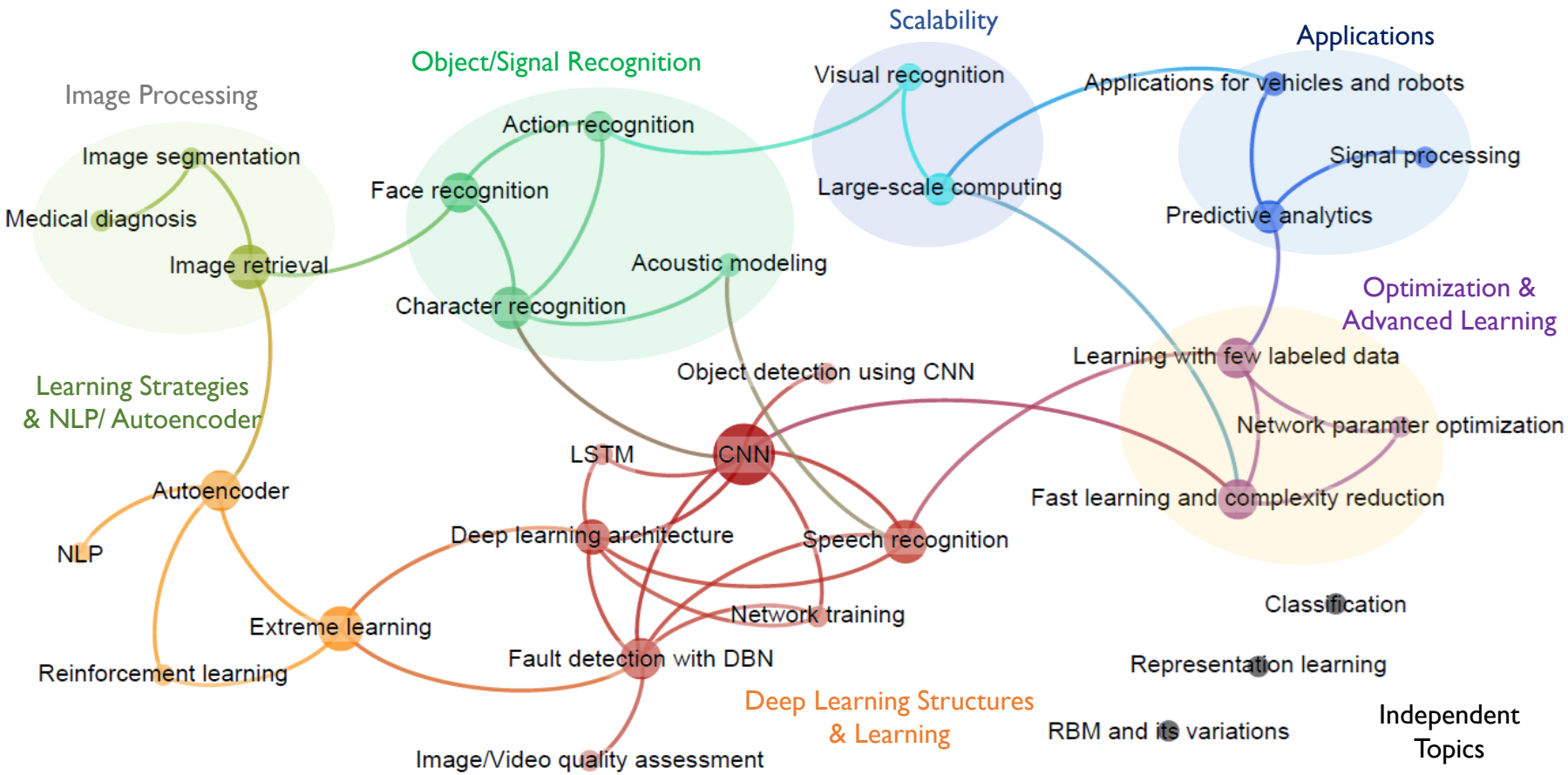
No.	Insider threat in relational database	Assessment of insider threat	Insider attacks on Communication protocol	Modeling and system framework for insider threat	Masquerade detection
1	data	measure	attack	insider	user
2	information	assess	agent	threat	behavior
3	database	security	scheme	social	detect
4	leakage	behavior	protocol	analysis	activity
5	access	analysis	monitor	framework	malicious
6	detect	management	mitigation	mitigate	masquerade
7	transaction	privacy	fraud	monitor	attack
8	confidential	policy	damage	factor	legitimate
9	document	risk	psychological	technical	abnormal
10	file	threat	financial	business	decoy

No.	Access control for insider threat mitigation	Network intrusion detection systems	Feature selection for intrusion detection	Miscellaneous	Malicious domain detection
1	insider	network	detection	software	attack
2	access	detection	algorithm	security	malicious
3	user	intrusion	feature	system	domain
4	control	malicious	classification	device	event
5	cloud	traffic	accuracy	server	scenario
6	misuse	log	dataset	malicious	human
7	trust	event	performance	protect	knowledge
8	risk	packet	pattern	web	ontology
9	abuse	internet	learning	architecture	represent
10	attacker	resource	random	electronic	generate

Topic Models: Relation between Topics

Kim et al. (2016)

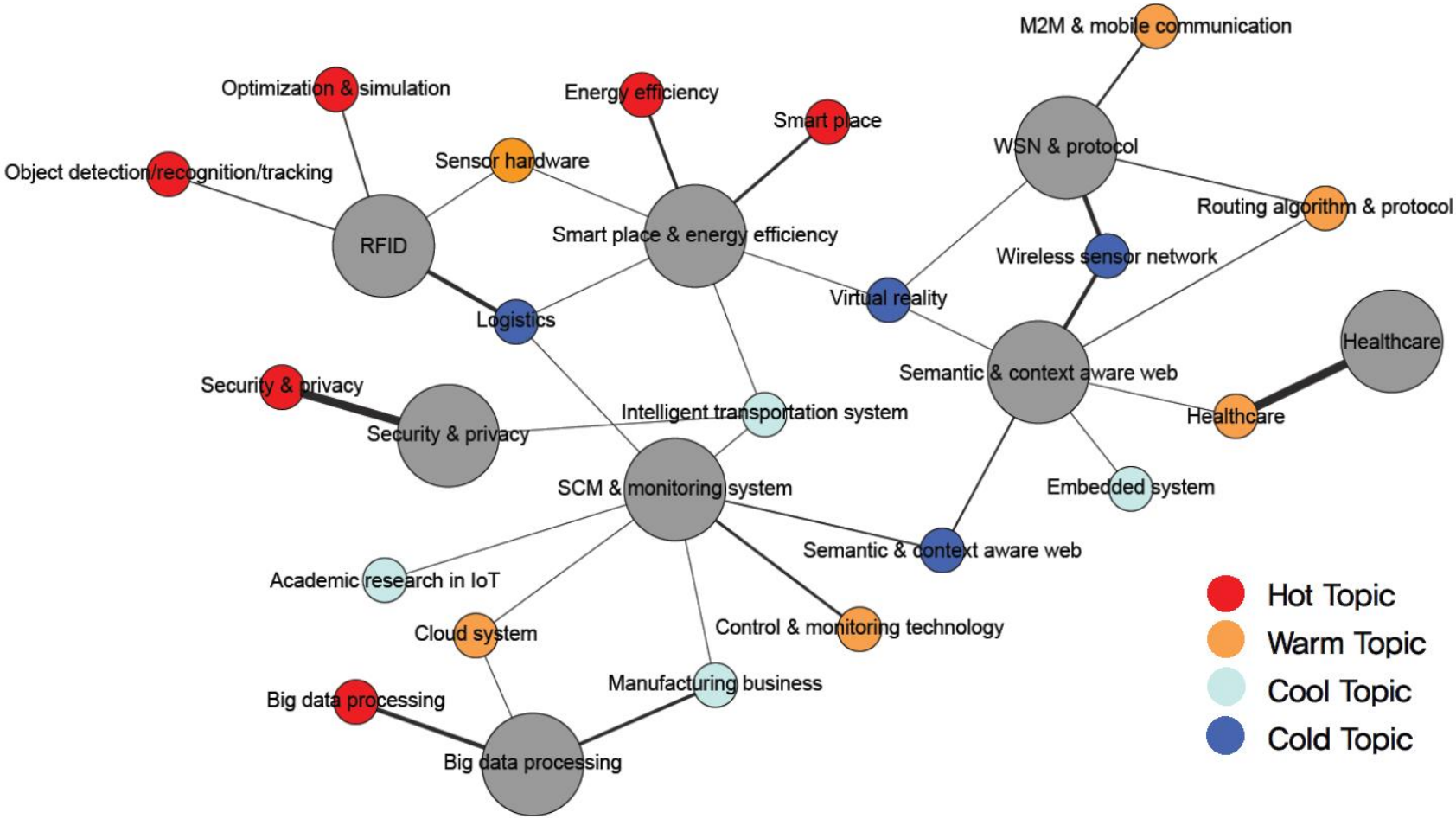
- Relation between Topics: Deep Learning



Topic Models: Relation between Topics

Kim and Kang (2018+)

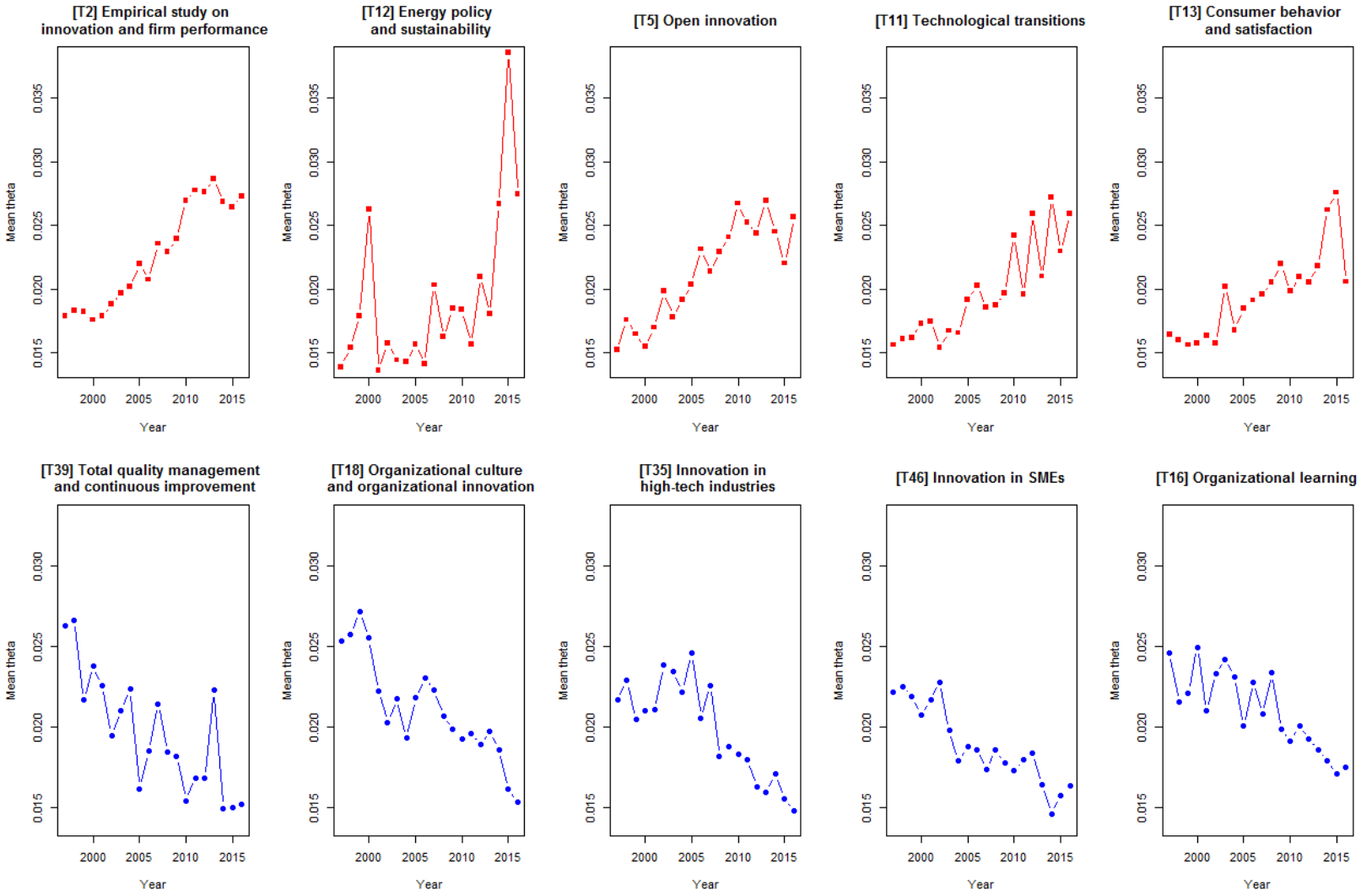
- Relation between Topics: Internet of Things



Topic Models: Trend Analysis

Lee and Kang (2017)

- Topic trends for "technology and innovation management"



Topic Model: Document Retrieval

Knispelis (2015)

Topic Model: Document Retrieval

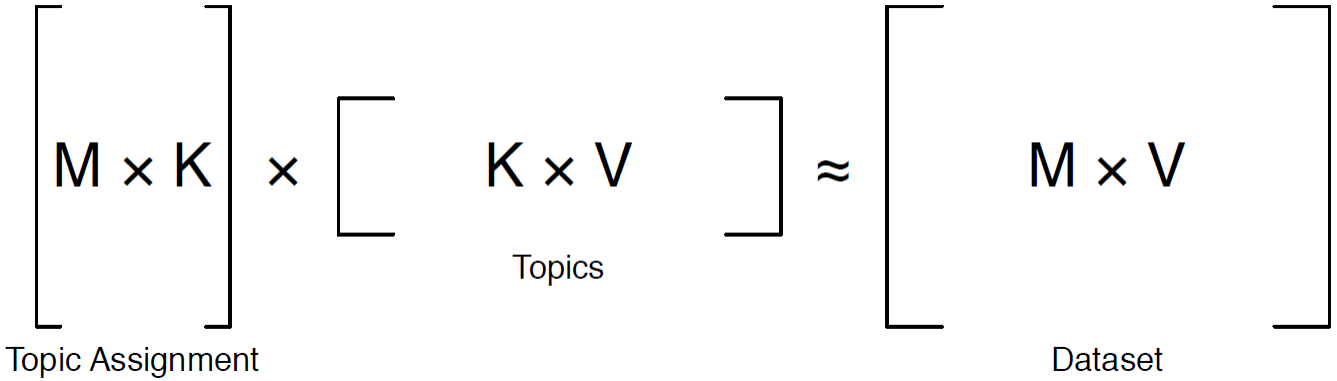
Knispelis (2015)

Topic Model: Document Retrieval

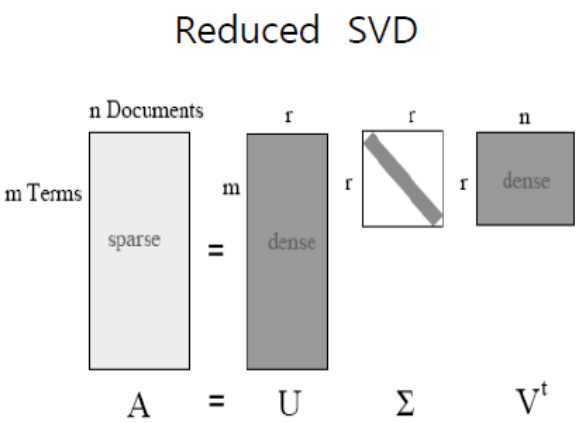
Knispelis (2015)

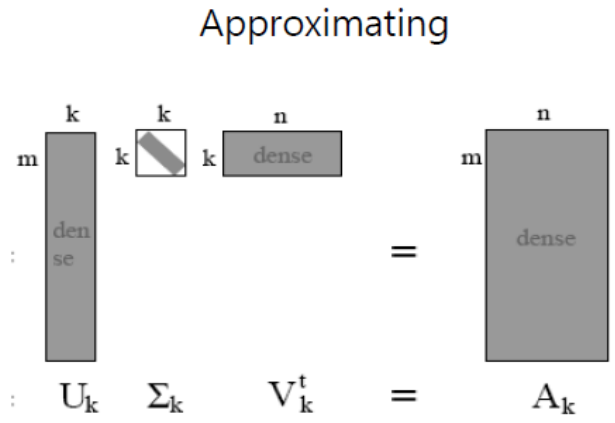
Topic Model

- Matrix Factorization Approach



✓ If we use singular value decomposition (SVD), it is called latent semantic analysis (LSA)





Topic Model

Helic (2014)

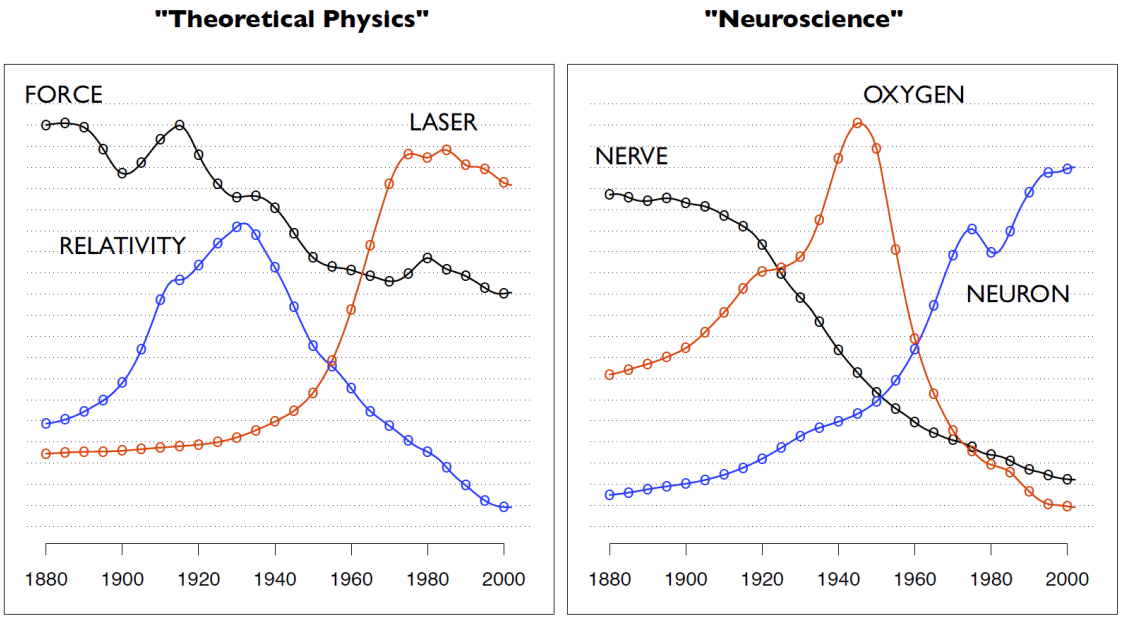
- Disadvantage of LSA
  - ✓ Statistical foundation is missing
  - ✓ SVD assumes normally distributed data
  - ✓ Term occurrence is not normally distributed
  - ✓ Still, often it works remarkably good because matrix entries are weighted (e.g. tf-idf) and those weighted entries may be normally distributed

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad U = \begin{bmatrix} 0.82 & -0.58 & 0 & 0 \\ 0.58 & 0.82 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 5.47 & 0 & 0 & 0 \\ 0 & 0.37 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} 0.40 & -0.91 \\ 0.91 & 0.40 \end{bmatrix}$$

Topic Model

Helic (2014)

- Probabilistic Topic Model: Generative Approach
  - ✓ Each document is a probability distribution over topics
  - ✓ Distribution over topics represents the essence of a given document
  - ✓ Each topic is a probability distribution over words
    - Topic "Education": school, students, education, university,…
    - Topic "Budget": million, finance, tax, program, …



Topic Model: Generative Approach

Helic (2014)

- Model-based methods
  - ✓ Statistical inference is based on fitting a probabilistic model of data
  - ✓ The idea is based on a probabilistic or generative model
  - ✓ Such models assign a probability for observing specific data examples
    - Observing words in a text document
  - ✓ Generative models are powerful method to encode specific assumptions of how unknown parameters interact to create data
- How it work?
  - ✓ It defines a conditional probability distribution over data given a hypothesis P(D|h)
  - ✓ Given h, we generate data from the conditional distribution P(D|h)
  - ✓ Has many advantages but the main disadvantage is that fitting the model can be more complicated than an algorithmic approach

Topic Model: Generative Approach

Helic (2014)

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  - ✓ It defines a conditional probability distribution over data given a hypothesis P(D|h)
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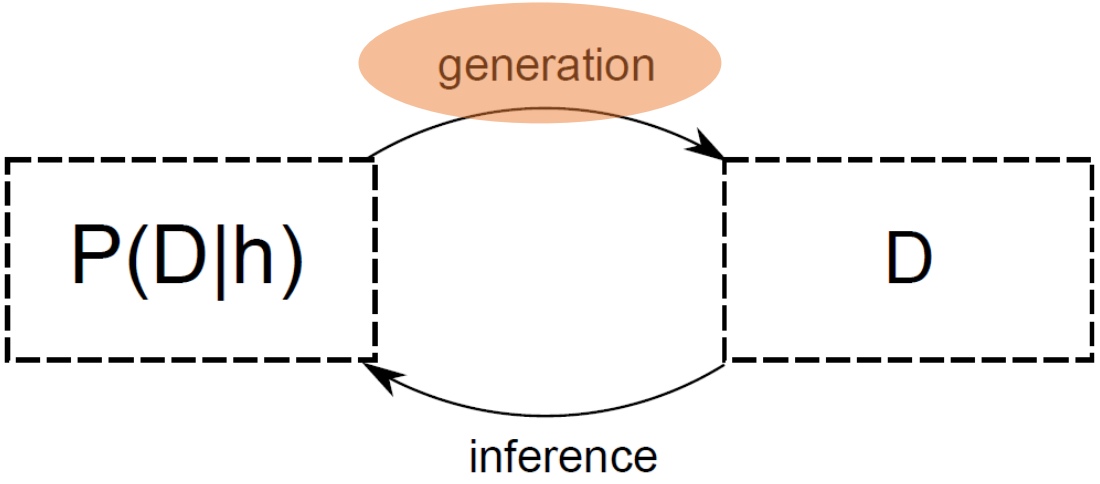
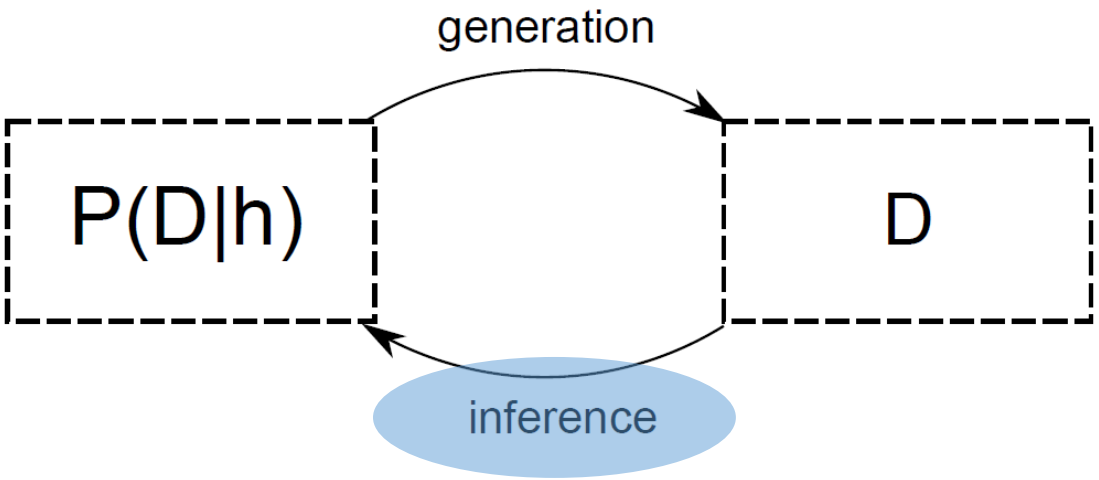


Topic Model: Generative Approach

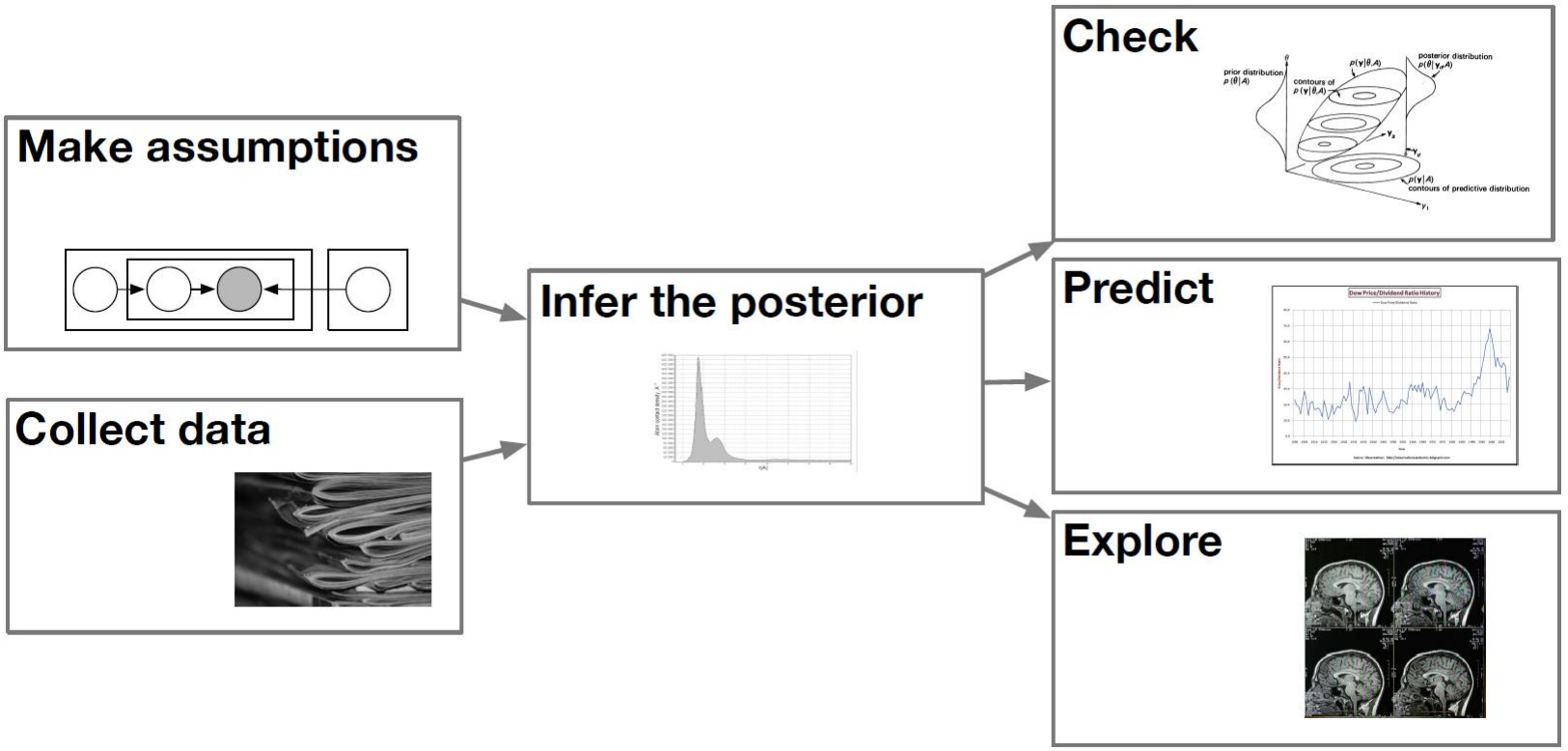
Helic (2014)

- (Statistical) inference is the reverse of the generation process
  - ✓ We are given some data D, e.g. a collection of documents
  - ✓ We want to estimate the model, or more precisely the parameters of the hypothesis h that are most likely to have generated data



Topic Model: Generative Approach

- Process of generative model



AGENDA

Latent Structure

Hofmann (2005)

- Given a matrix that "encodes" data (e.g. term-document matrix), we have following potential problems
  - ✓ Too large
  - ✓ Too complicated
  - ✓ Lack of structure
  - ✓ Missing Entries
  - ✓ Noisy Entries, …

$$\mathbf{A} = \begin{pmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1m} \\ \dots & \dots & \dots & \dots & \dots \\ a_{i1} & \dots & a_{ij} & \dots & a_{im} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{nj} & \dots & a_{nm} \end{pmatrix}$$

- Questions
  - ✓ Is there a simpler way to explain entities?
  - ✓ There might be a latent structure underlying the data
  - ✓ How can we reveal or discover this structure?

Matrix Decomposition

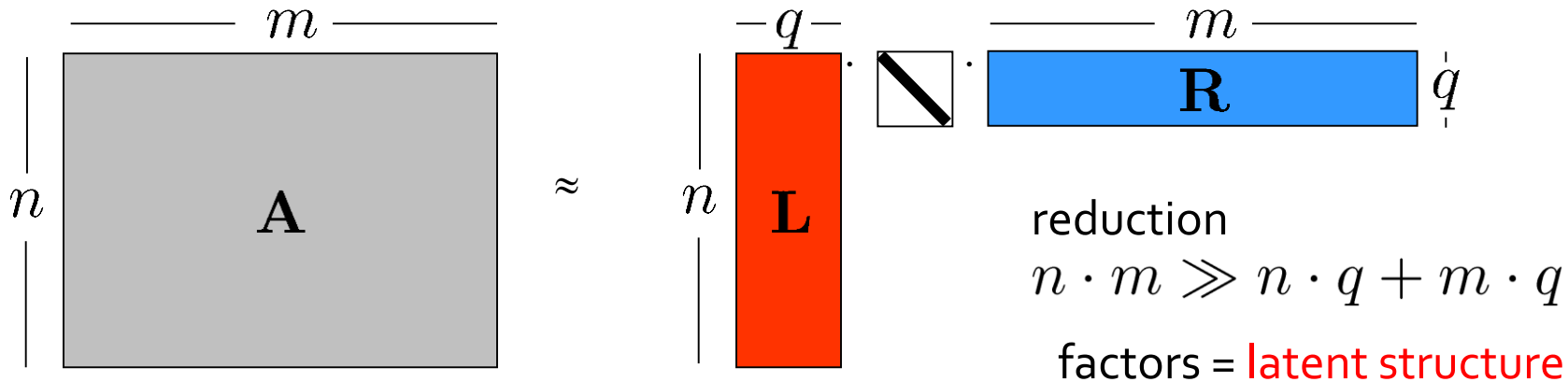
Hofmann (2005)

- Common approach: approximately factorize matrix

$$\mathbf{A} \approx \hat{\mathbf{A}} = \mathbf{L} \cdot \mathbf{R}$$

approximation left factor right factor

- Factors are typically constrained to be "thin"



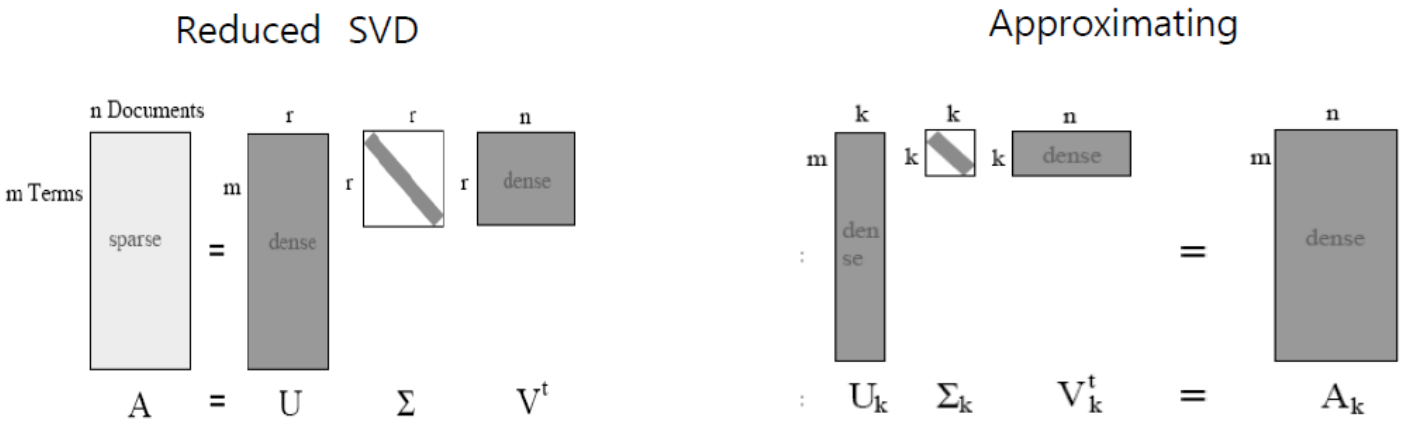
reduction

$$n \cdot m \gg n \cdot q + m \cdot q$$

factors = latent structure

LSA Decomposition (revisited)

- Reduce the dimensions using SVD



✓ Step 1) Construct the approximated matrix A_k from the original term-document matrix A using SVD

$$\mathbf{A} \approx \mathbf{A}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^T$$

✓ Step 2) Multiply the transpose of U_k to obtain k (<<m) by n term-document matrix

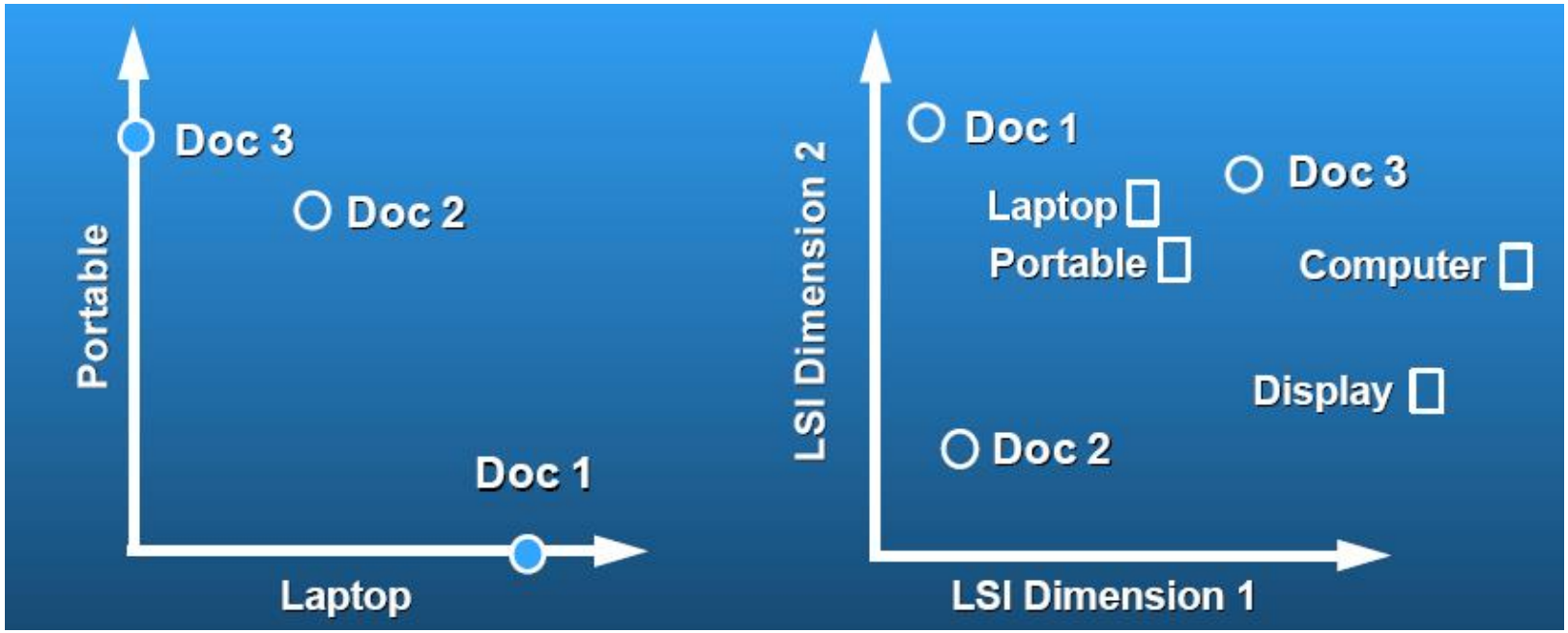
$$\mathbf{U}_k^T \mathbf{A}_k = \mathbf{U}_k^T \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^T = \mathbf{I} \mathbf{\Sigma}_k \mathbf{V}_k^T = \mathbf{\Sigma}_k \mathbf{V}_k^T$$

✓ Step 3: Apply data mining algorithms

LSA Decomposition

Hofmann (2005)

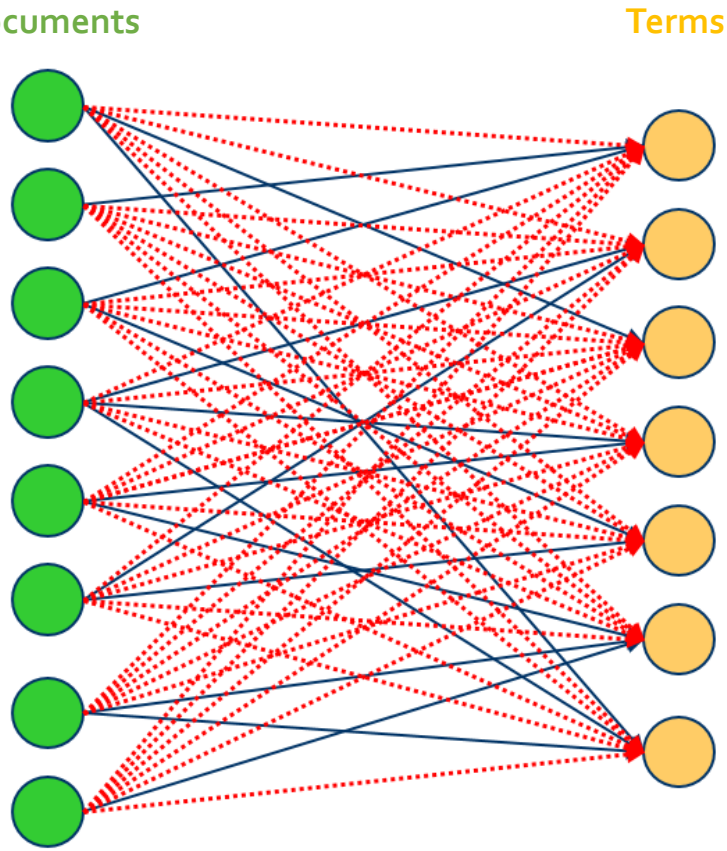
- Illustrative Example



Language Model: Naïve Approach

Hofmann (2005)

- Maximum likelihood estimation (MLE)



Number of occurrences of term w in document d

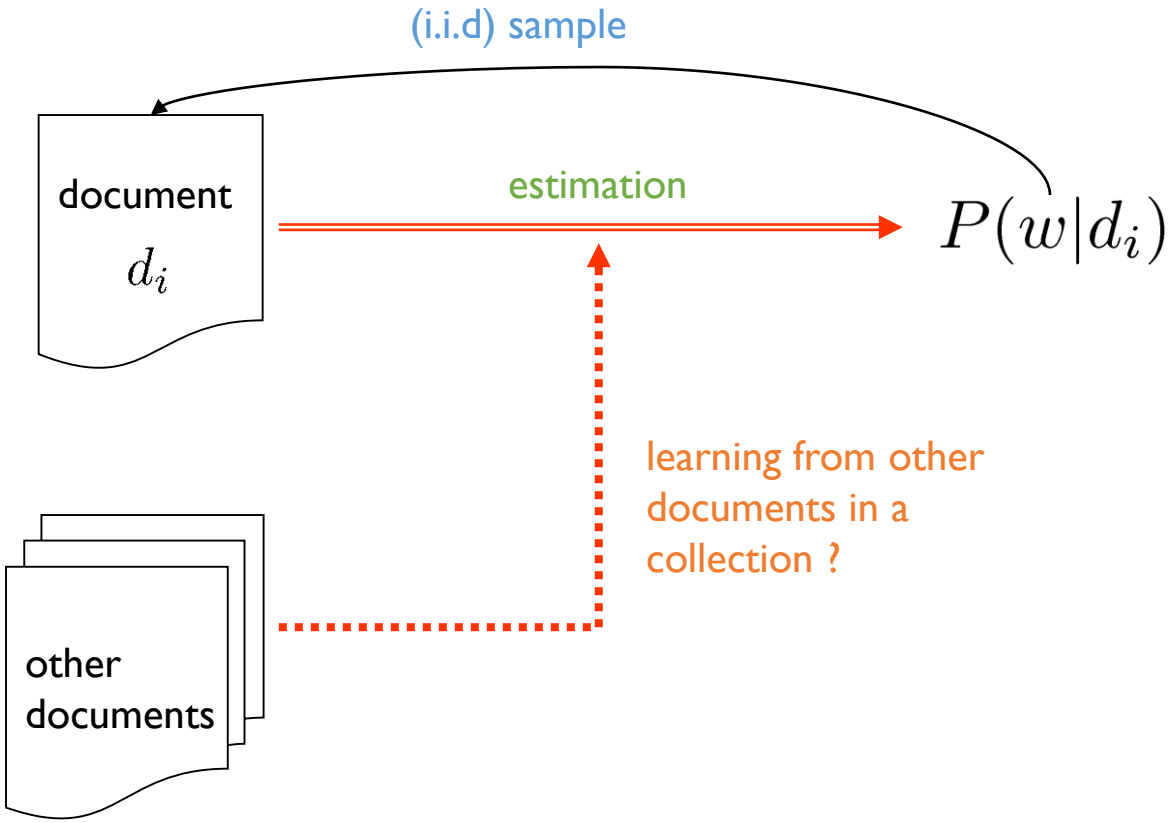
$$\hat{P}_{\mathrm{ML}}(w|d) = \frac{n(d,w)}{\sum_{w'} n(d,w')}$$

Zero frequency problem: terms not occurring in a document get zero probability

Language Model: Estimation Problem

Hofmann (2005)

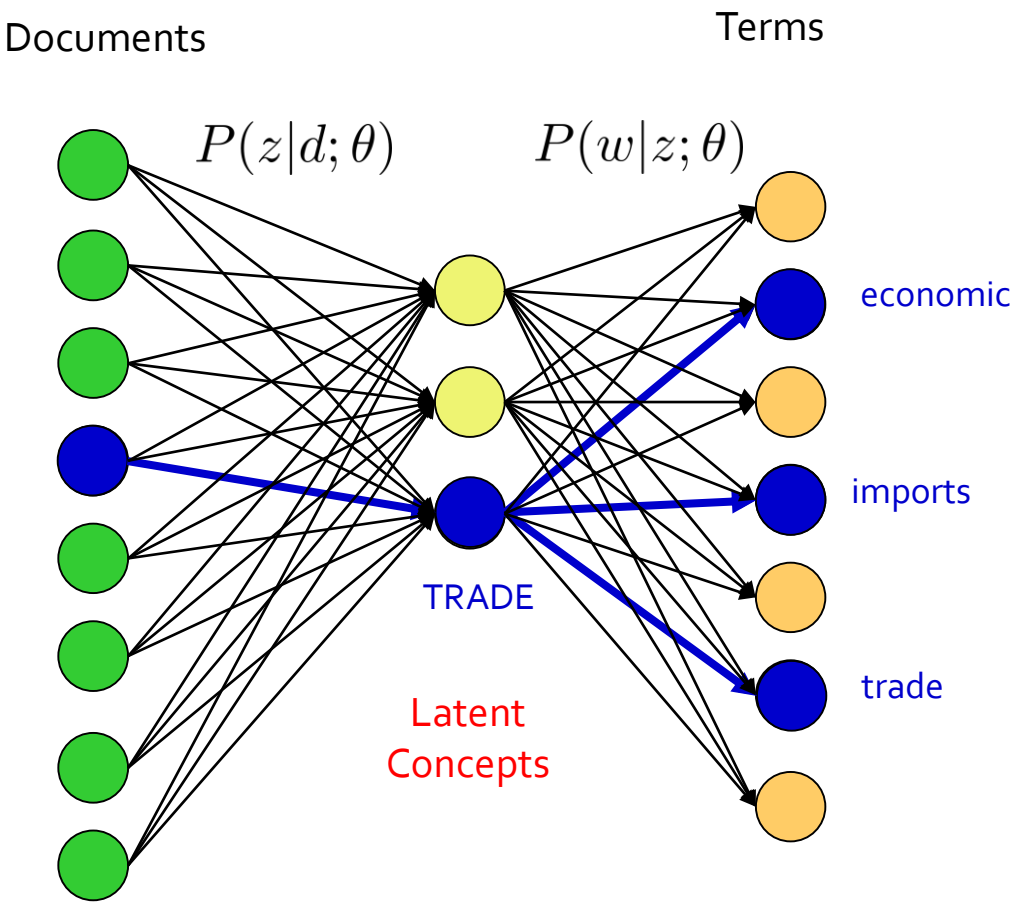
- Crucial question
  - ✓ In which way can the document collection be utilized to improve estimates?



Probabilistic Latent Semantic Analysis (pLSA)

Hofmann (2005)

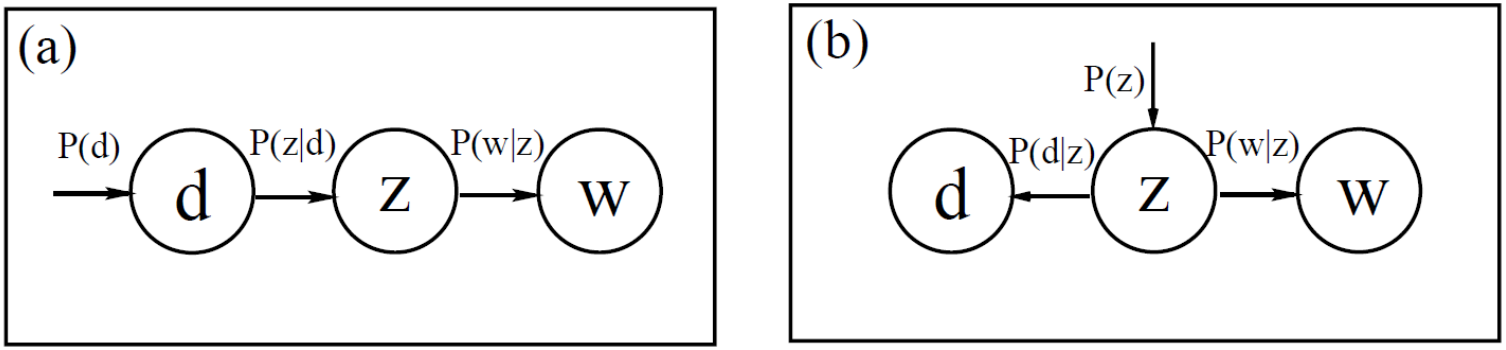
- Concept expression probability
  - ✓ Estimated based on all documents that are dealing with a concept
  - ✓ "Unmixing" of superimposed concepts is achieved by statistical learning algorithm
  - ✓ No prior knowledge about concepts required, context and term co-occurrences are exploited



pLSA: Latent Variable Model

Hofmann (2005)

- Structural modeling assumption (mixture model)



$$\hat{P}_{\mathrm{LSA}}(w|d) = \sum_z P(w|z;\theta)P(z|d;\pi)$$

Document language model

Latent concepts or topics

Concept expression probabilities

Document-specific mixture proportions

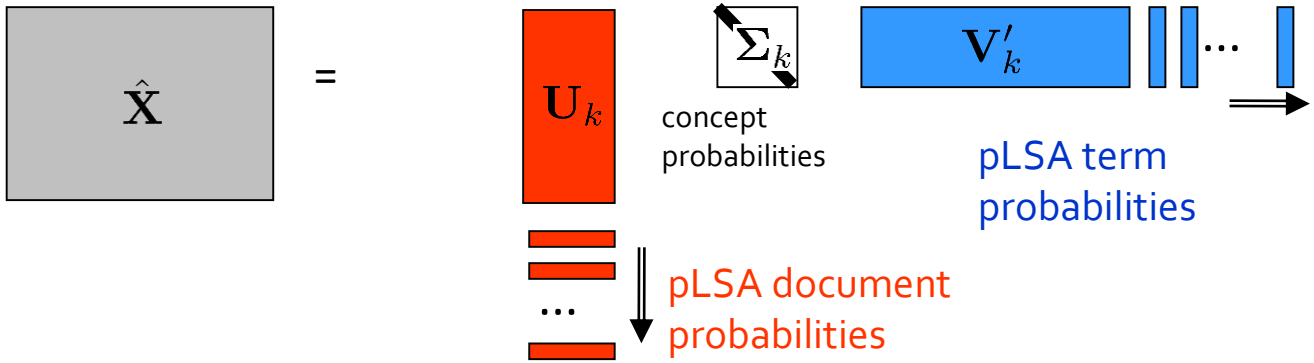
Model fitting

pLSA: Matrix Decomposition

Hofmann (2005)

- Mixture model can be written as a matrix factorization

$$\hat{P}_{\text{LSA}}(d,w) = \sum_z P(d|z)\,P(z)\,P(w|z) \qquad = P(d)\sum_z P(w|z)P(z|d)$$



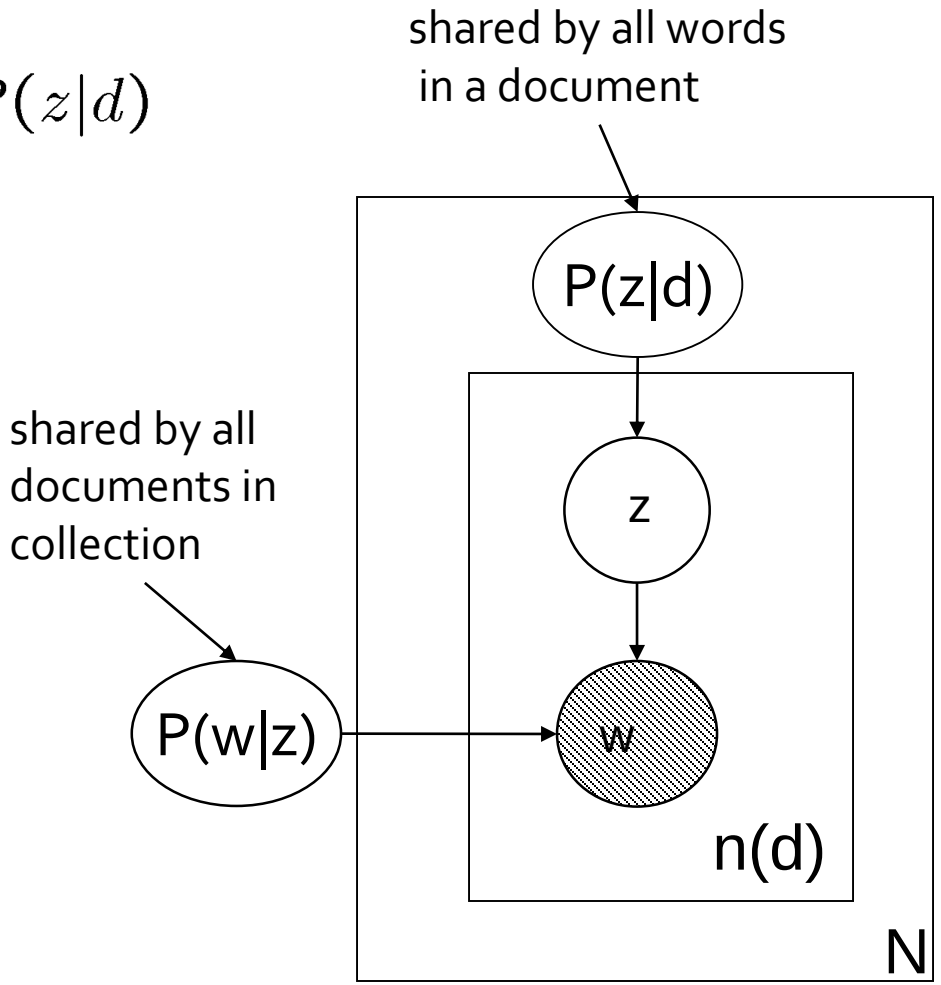
- Contrast to LSA
  - ✓ Non-negativity: every element in U & V is non-negative
  - ✓ Normalization: Each document vector in U and each term vector in V has sum 1

pLSA: Graphical Model

Hofmann (2005)

- Graphical Representation

$$P(w|d) = \sum_z P(w|z)P(z|d)$$



pLSA: Parameter Inference

Helic (2014)

- Parameter inference
  - ✓ We will infer parameters using Maximum Likelihood Estimator (MLE)
  - ✓ First, we need to write down the likelihood function
  - ✓ Let $n(w_i, d_j)$ be the number of occurrences of word w_i in document d_j
  - ✓ $p(w_i, d_j)$ is the probability of observing a single occurrence word w_i in document d_j
  - ✓ Then, the probability of observing $n(w_i, d_j)$ occurrence of word w_i in document d_j is give by:

$$p(w_i, d_j)^{n(w_i, d_j)}$$

pLSA: Parameter Inference

Helic (2014)

- Parameter Inference
  - ✓ The probability of observing the compete document collection is then given by the product of probabilities of observing every single word in every document with corresponding number of occurrences
  - ✓ Then, the likelihood function becomes

$$L = \prod_{i=1}^{m} \prod_{j=1}^{n} p(w_i, d_j)^{n(w_i, d_j)}$$

  - ✓ The log-likelihood function becomes

$$\mathcal{L} = \sum_{i=1}^{m} \sum_{j=1}^{n} n(w_i, d_j) log\big(p(w_i, d_j)\big)$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} n(w_i, d_j) log\big(\sum_{l=1}^{k} p(w_i|z_l)p(z_l)p(d_j|z_l)\big)$$

pLSA: Parameter Inference

Helic (2014)

- Parameter Inference
  - ✓ We can not maximize the likelihood analytically because of the logarithm of the sum
  - ✓ A standard procedure is to use an algorithm called Expectation-Maximization (EM)
  - ✓ This is an iterative method to estimate parameters of the models with latent variables
  - ✓ Each iteration consists of two steps: expectation step (E) and maximization step (M)

pLSA: EM Algorithm

- E-Step: Posterior probability of latent variables (concepts)

$$p(z|d,w) = \frac{P(z)P(d|z)P(w|z)}{\sum_{z' \in Z} P(z')P(d|z')P(w|z')}$$

Probability that the occurence of term w in document d can be "explained" by concept z

- M-Step: Parameter estimation based on "completed" statistics

$$P(w|z) = \frac{\sum_{d \in D} n(d,w)P(z|d,w)}{\sum_{d \in D, w' \in W} n(d,w')P(z|d,w')}$$

how often is term w associated with concept z ?

$$P(d|z) = \frac{\sum_{w \in W} n(d,w)P(z|d,w)}{\sum_{d' \in D, w \in W} n(d',w)P(z|d',w)}$$

how often is document d associated with concept z ?

$$P(z) = \frac{\sum_{d \in D, w \in W} n(d,w)P(z|d,w)}{\sum_{d \in D, w \in W} n(d,w)}$$

how prevalent is the concept z ?

pLSA: A Simple Example

- Raw Data

	Doc 1	Doc 2	Doc 3	Doc 4	Doc 5	Doc 6
Baseball	1	2	0	0	0	0
Basketball	3	1	0	0	0	0
Boxing	2	0	0	0	0	0
Money	3	3	2	3	2	4
Interest	0	0	3	2	0	0
Rate	0	0	4	1	0	0
Democrat	0	0	0	0	4	3
Republican	0	0	0	0	2	1
Cocus	0	0	0	0	3	2
President	0	0	1	0	2	3

pLSA: A Simple Example

- Parameter Initialization

$P(z)$

Topic 1	Topic 2	Topic 3
0.525	0.407	0.068

$P(d|z)$

	Topic 1	Topic 2	Topic 3
Doc 1	0.020	0.008	0.048
Doc 2	0.294	0.255	0.329
Doc 3	0.204	0.138	0.178
Doc 4	0.200	0.146	0.007
Doc 5	0.186	0.196	0.233
Doc 6	0.096	0.257	0.205

$P(w|z)$

	Topic 1	Topic 2	Topic 3
Term 1	0.022	0.016	0.010
Term 2	0.018	0.133	0.166
Term 3	0.242	0.058	0.133
Term 4	0.123	0.088	0.145
Term 5	0.016	0.030	0.044
Term 6	0.020	0.167	0.056
Term 7	0.147	0.129	0.201
Term 8	0.188	0.156	0.039
Term 9	0.146	0.114	0.008
Term 10	0.077	0.110	0.199

pLSA: A Simple Example

- After 1 EM step

Initialization

Topic 1	Topic 2	Topic 3
0.525	0.407	0.068

	Topic 1	Topic 2	Topic 3
Doc 1	0.020	0.008	0.048
Doc 2	0.294	0.255	0.329
Doc 3	0.204	0.138	0.178
Doc 4	0.200	0.146	0.007
Doc 5	0.186	0.196	0.233
Doc 6	0.096	0.257	0.205

After 1 EM step

Topic 1	Topic 2	Topic 3
0.459	0.430	0.111

	Topic 1	Topic 2	Topic 3
Doc 1	0.180	0.077	0.382
Doc 2	0.124	0.089	0.091
Doc 3	0.147	0.213	0.149
Doc 4	0.125	0.110	0.004
Doc 5	0.266	0.204	0.167
Doc 6	0.158	0.308	0.207

pLSA: A Simple Example

- After 1 EM step

Initialization

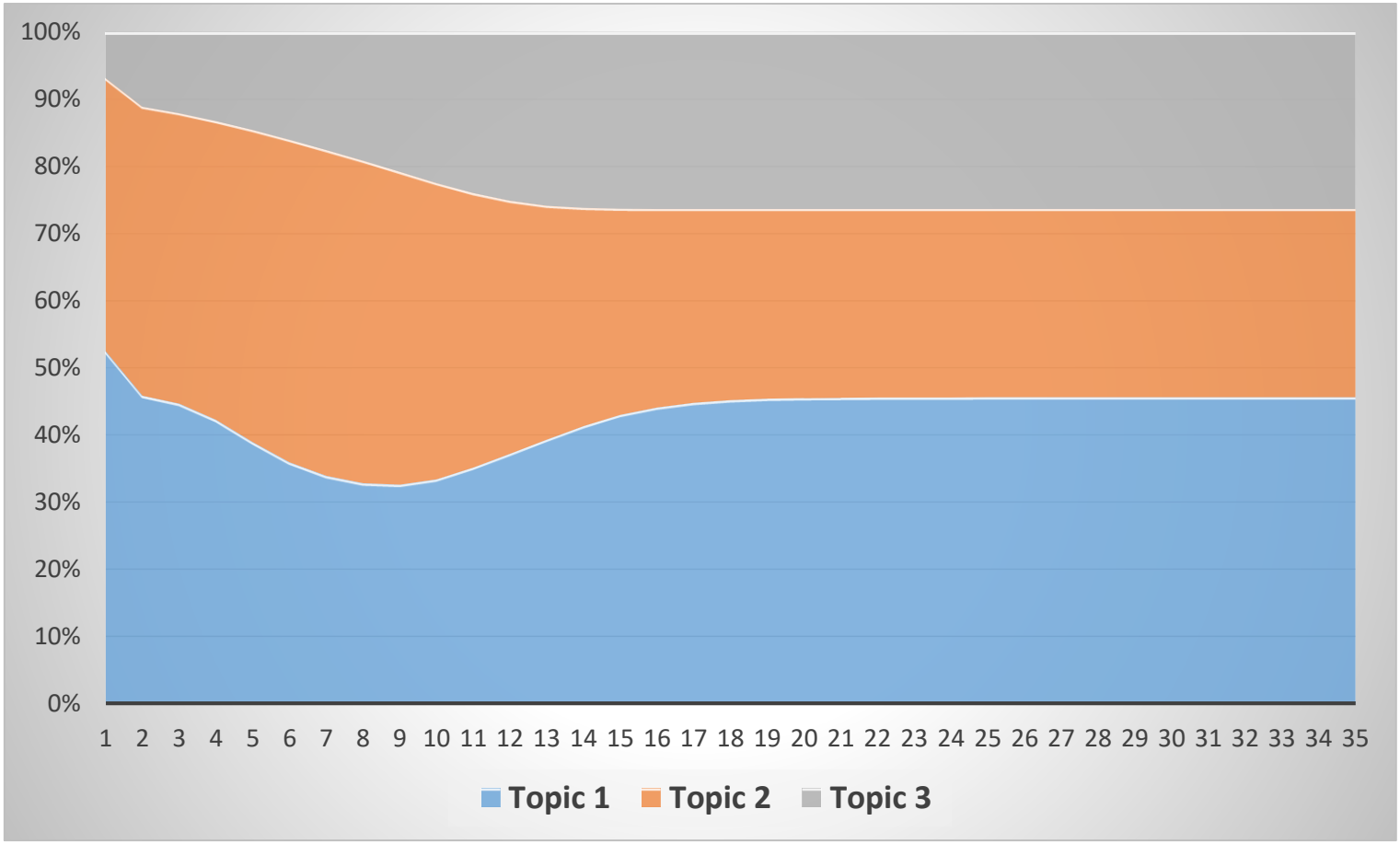
	Topic 1	Topic 2	Topic 3
Term 1	0.022	0.016	0.010
Term 2	0.018	0.133	0.166
Term 3	0.242	0.058	0.133
Term 4	0.123	0.088	0.145
Term 5	0.016	0.030	0.044
Term 6	0.020	0.167	0.056
Term 7	0.147	0.129	0.201
Term 8	0.188	0.156	0.039
Term 9	0.146	0.114	0.008
Term 10	0.077	0.110	0.199

After 1 EM step

	Topic 1	Topic 2	Topic 3
Term 1	0.077	0.033	0.028
Term 2	0.024	0.074	0.245
Term 3	0.061	0.005	0.043
Term 4	0.370	0.222	0.295
Term 5	0.088	0.093	0.065
Term 6	0.033	0.159	0.035
Term 7	0.115	0.129	0.129
Term 8	0.058	0.058	0.010
Term 9	0.099	0.098	0.004
Term 10	0.073	0.129	0.146

pLSA: A Simple Example

- Topic Distribution
  - ✓ Topic distribution changes w.r.t. the EM iterations



pLSA: A Simple Example

- Final result

	Doc 1	Doc 2	Doc 3	Doc 4	Doc 5	Doc 6
Baseball	1	2	0	0	0	0
Basketball	3	1	0	0	0	0
Boxing	2	0	0	0	0	0
Money	3	3	2	3	2	4
Interest	0	0	3	2	0	0
Rate	0	0	4	1	0	0
Democrat	0	0	0	0	4	3
Republican	0	0	0	0	2	1
Cocus	0	0	0	0	3	2
President	0	0	1	0	2	3

Topic 1	Topic 2	Topic 3
0.456	0.281	0.263

	Topic 1	Topic 2	Topic 3
Doc 1	0.000	0.000	0.600
Doc 2	0.000	0.000	0.400
Doc 3	0.000	0.625	0.000
Doc 4	0.000	0.375	0.000
Doc 5	0.500	0.000	0.000
Doc 6	0.500	0.000	0.000

	Topic 1	Topic 2	Topic 3
Baseball	0.000	0.000	0.200
Basketball	0.000	0.000	0.267
Boxing	0.000	0.000	0.133
Money	0.231	0.313	0.400
Interest	0.000	0.312	0.000
Rate	0.000	0.312	0.000
Democrat	0.269	0.000	0.000
Republican	0.115	0.000	0.000
Cocus	0.192	0.000	0.000
President	0.192	0.063	0.000

pLSA: Example

- Concepts extracted from Science Magazine articles

P(w|z)

universe	0.0439	drug	0.0672	cells	0.0675	sequence	0.0818	years	0.156
galaxies	0.0375	patients	0.0493	stem	0.0478	sequences	0.0493	million	0.0556
clusters	0.0279	drugs	0.0444	human	0.0421	genome	0.033	ago	0.045
matter	0.0233	clinical	0.0346	cell	0.0309	dna	0.0257	time	0.0317
galaxy	0.0232	treatment	0.028	gene	0.025	sequencing	0.0172	age	0.0243
cluster	0.0214	trials	0.0277	tissue	0.0185	map	0.0123	year	0.024
cosmic	0.0137	therapy	0.0213	cloning	0.0169	genes	0.0122	record	0.0238
dark	0.0131	trial	0.0164	transfer	0.0155	chromosome	0.0119	early	0.0233
light	0.0109	disease	0.0157	blood	0.0113	regions	0.0119	billion	0.0177
density	0.01	medical	0.00997	embryos	0.0111	human	0.0111	history	0.0148

P(w|z)

bacteria	0.0983	male	0.0558	theory	0.0811	immune	0.0909	stars	0.0524
bacterial	0.0561	females	0.0541	physics	0.0782	response	0.0375	star	0.0458
resistance	0.0431	female	0.0529	physicists	0.0146	system	0.0358	astrophys	0.0237
coli	0.0381	males	0.0477	einstein	0.0142	responses	0.0322	mass	0.021
strains	0.025	sex	0.0339	university	0.013	antigen	0.0263	disk	0.0173
microbiol	0.0214	reproductive	0.0172	gravity	0.013	antigens	0.0184	black	0.0161
microbial	0.0196	offspring	0.0168	black	0.0127	immunity	0.0176	gas	0.0149
strain	0.0165	sexual	0.0166	theories	0.01	immunology	0.0145	stellar	0.0127
salmonella	0.0163	reproduction	0.0143	aps	0.00987	antibody	0.014	astron	0.0125
resistant	0.0145	eggs	0.0138	matter	0.00954	autoimmune	0.0128	hole	0.00824

pLSA: Example

- Example
  - ✓ Polysemy: a word may have multiple senses and multiple types of usage in different context

"segment 1"	"segment 2"	"matrix 1"	"matrix 2"	"line 1"	"line 2"	"power 1"	power 2"
imag	speaker	robust	manufactur	constraint	alpha	POWER	load
SEGMENT	speech	MATRIX	cell	LINE	redshift	spectrum	memori
texture	recogni	eigenvalu	part	match	LINE	omega	vlsi
color	signal	uncertainti	MATRIX	locat	galaxi	mpc	POWER
tissue	train	plane	cellular	imag	quasar	hsup	systolic
brain	hmm	linear	famili	geometr	absorp	larg	input
slice	source	condition	design	impos	high	redshift	complex
cluster	speakerind.	perturb	machinepart	segment	ssup	galaxi	arrai
mri	SEGMENT	root	format	fundament	densiti	standard	present
volume	sound	suffici	group	recogn	veloc	model	implement

Document 1, $P\{z_k|d_1, w_j = \text{`}segment\text{`}\} = (0.951, 0.0001, \ldots)$
$P\{w_j = \text{`}segment\text{`}|d_1\} = 0.06$

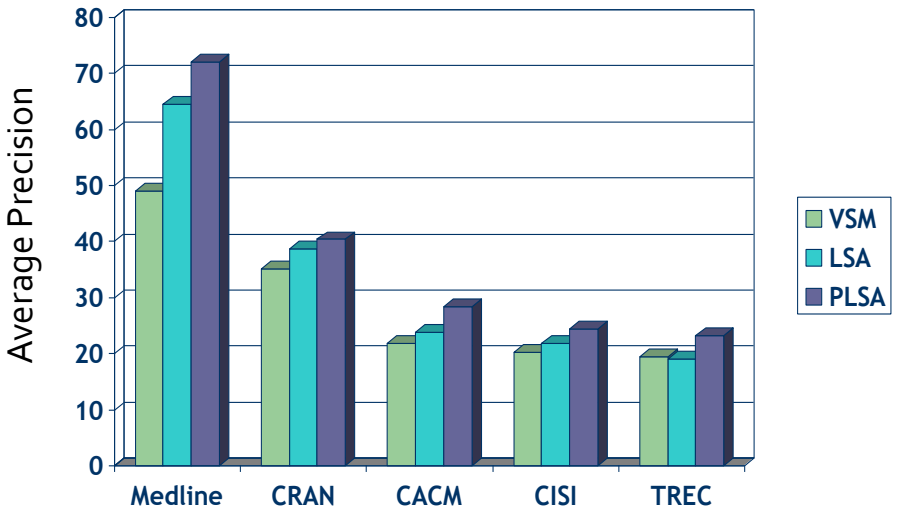
SEGMENT medic imag challeng problem field imag analysi diagnost base proper **SEGMENT** digit imag **SEGMENT** medic imag need applic involv estim boundari object classif tissu abnorm shape analysi contour detec textur **SEGMENT** despit exist techniqu **SEGMENT** specif medic imag remain crucial problem [...]

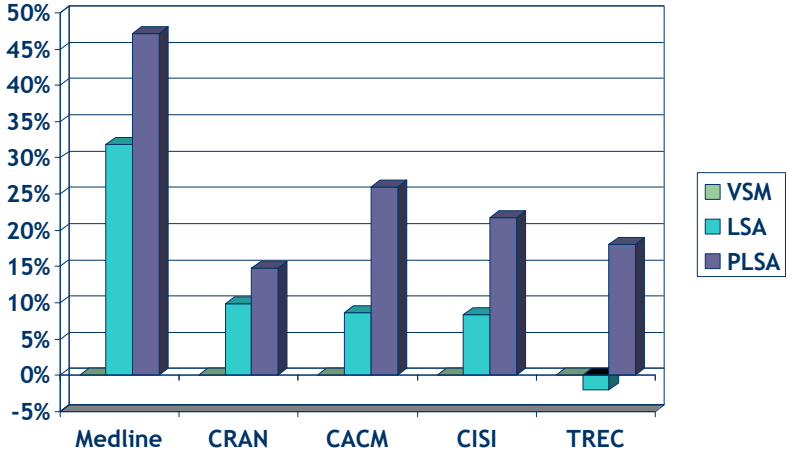
Document 2, $P\{z_k|d_2, w_j = \text{`}segment\text{`}\} = (0.025, 0.867, \ldots)$
$P\{w_j = \text{`}segment\text{`}|d_2\} = 0.010$

consid signal origin sequenc sourc specif problem **SEGMENT** signal relat **SEGMENT** sourc address issu wide applic field report describ resolu method ergod hidden markov model hmm hmm state correspond signal sourc signal sourc sequenc determin decod procedur viterbi algorithm forward algorithm observ sequenc baumwelch train estim hmm paramet train materi applic multipl signal sourc identif problem experi perform unknown speaker identif [...]

pLSA: Example

- Experimental Evaluation





✓ Consistent improvements of retrieval accuracy

✓ Relative improvement of average precision: 15-45%

AGENDA

LDA: Intuition

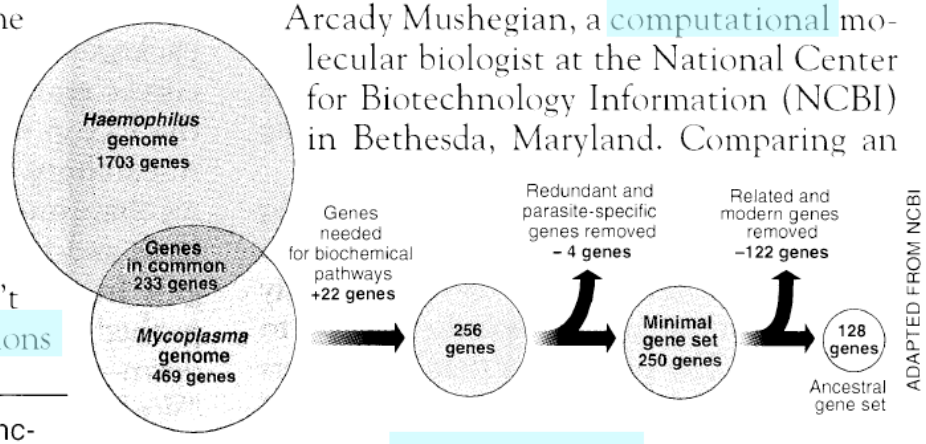
Blei (2012)

- Documents exhibit multiple topics

Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an



Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

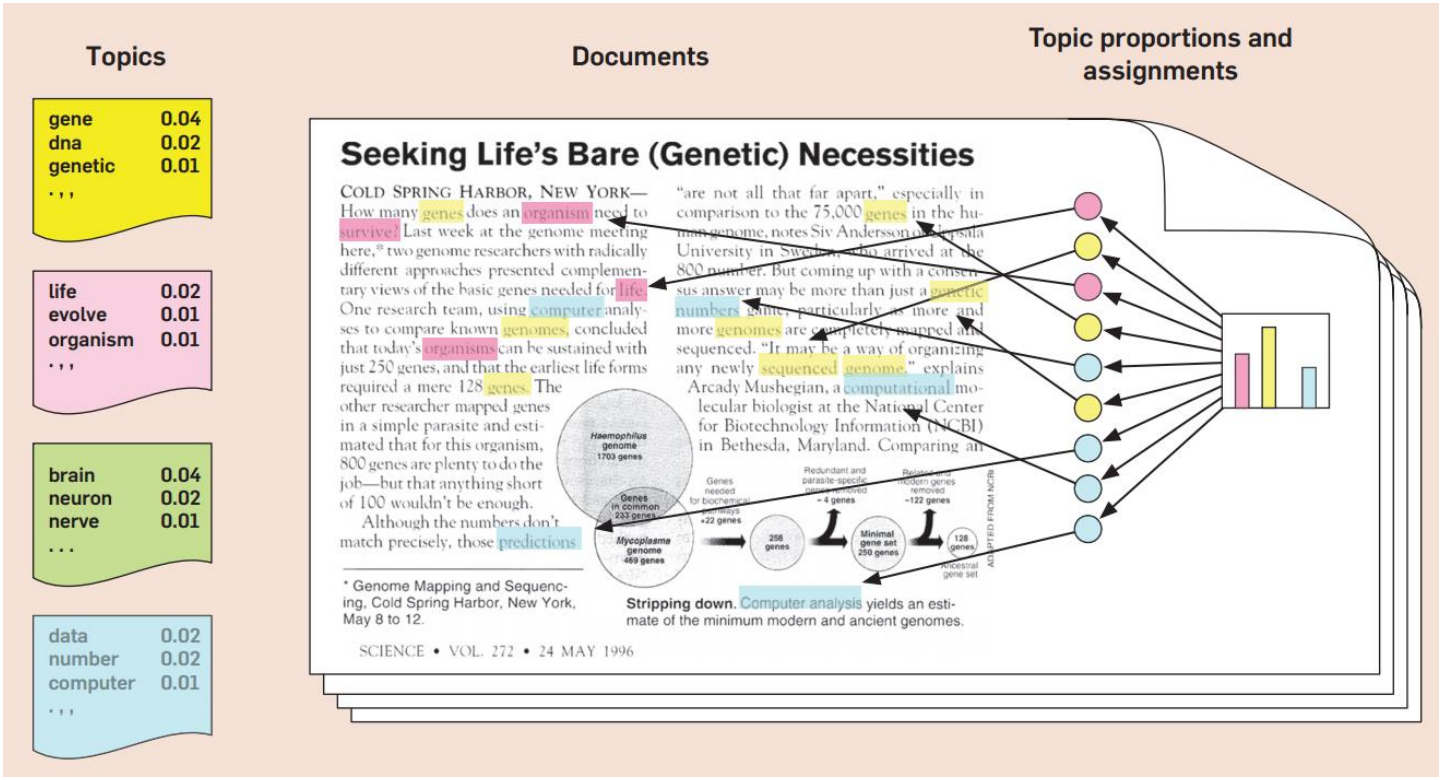
* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.

SCIENCE • VOL. 272 • 24 MAY 1996

LDA: Intuition

Blei (2012)

- Documents exhibit multiple topics



✓ Each topic is a distribution over words

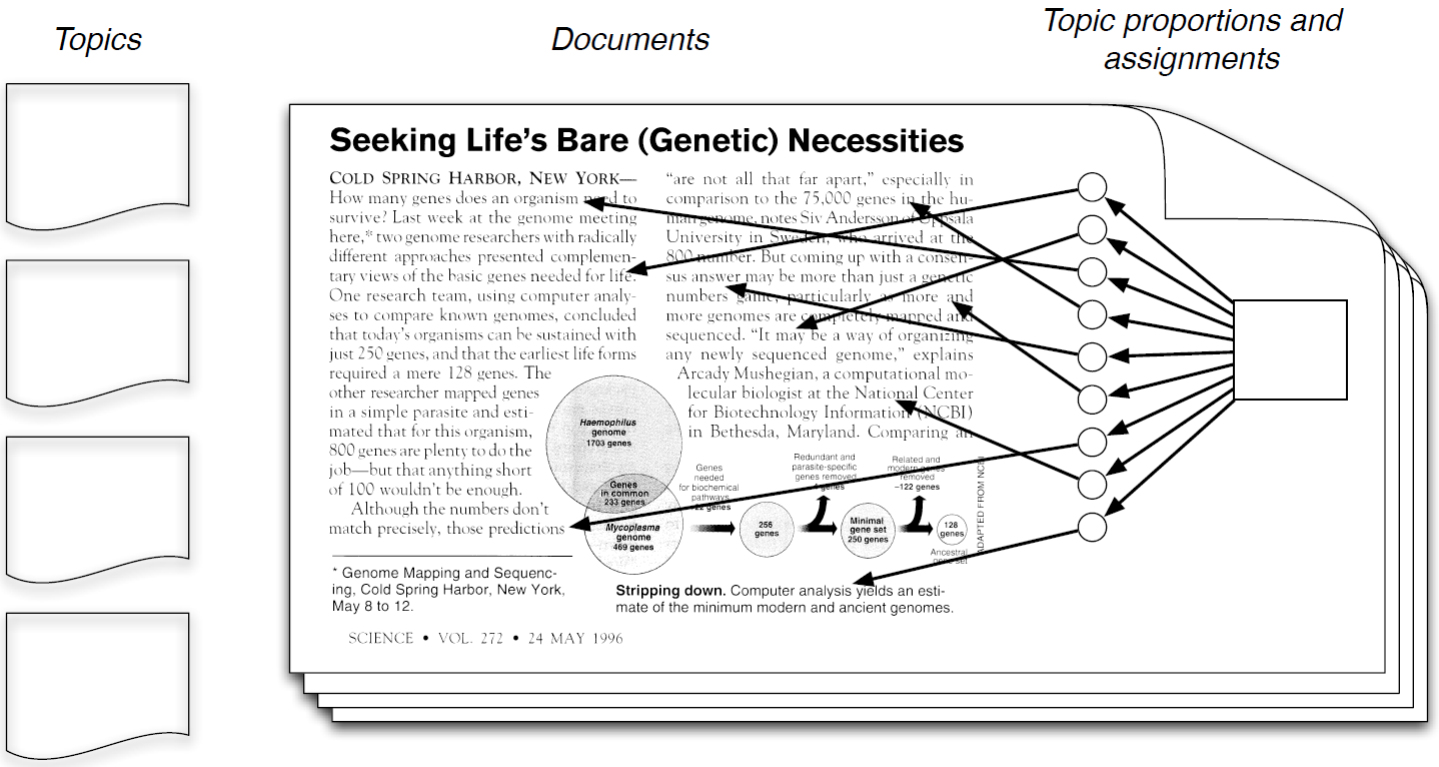
✓ Each document is a mixture of corpus-wide topics

✓ Each word is drawn from one of those topics

LDA: Intuition

Blei (2012)

- Documents exhibit multiple topics



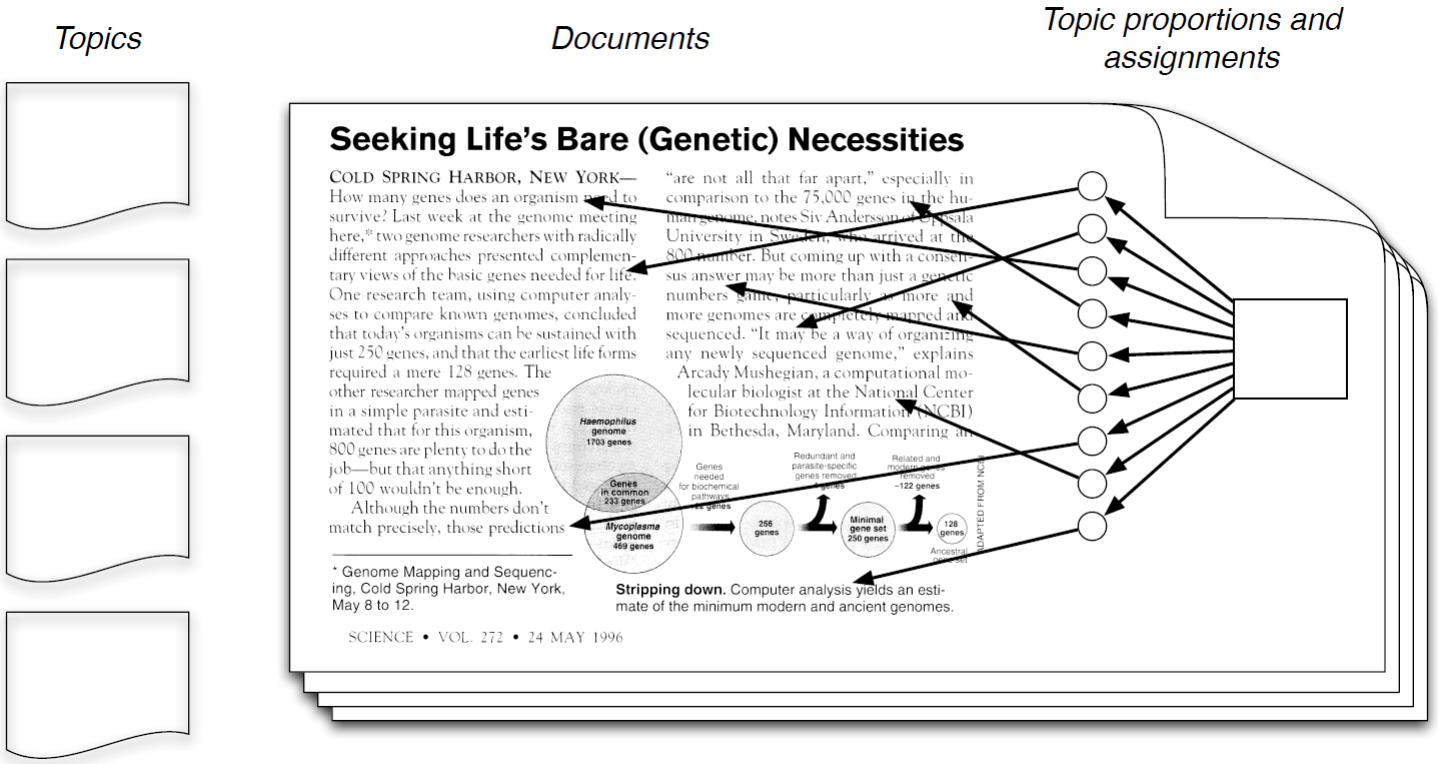
✓ In reality, we only observe the documents

✓ The other structure are hidden variables

LDA: Intuition

Blei (2012)

- Documents exhibit multiple topics



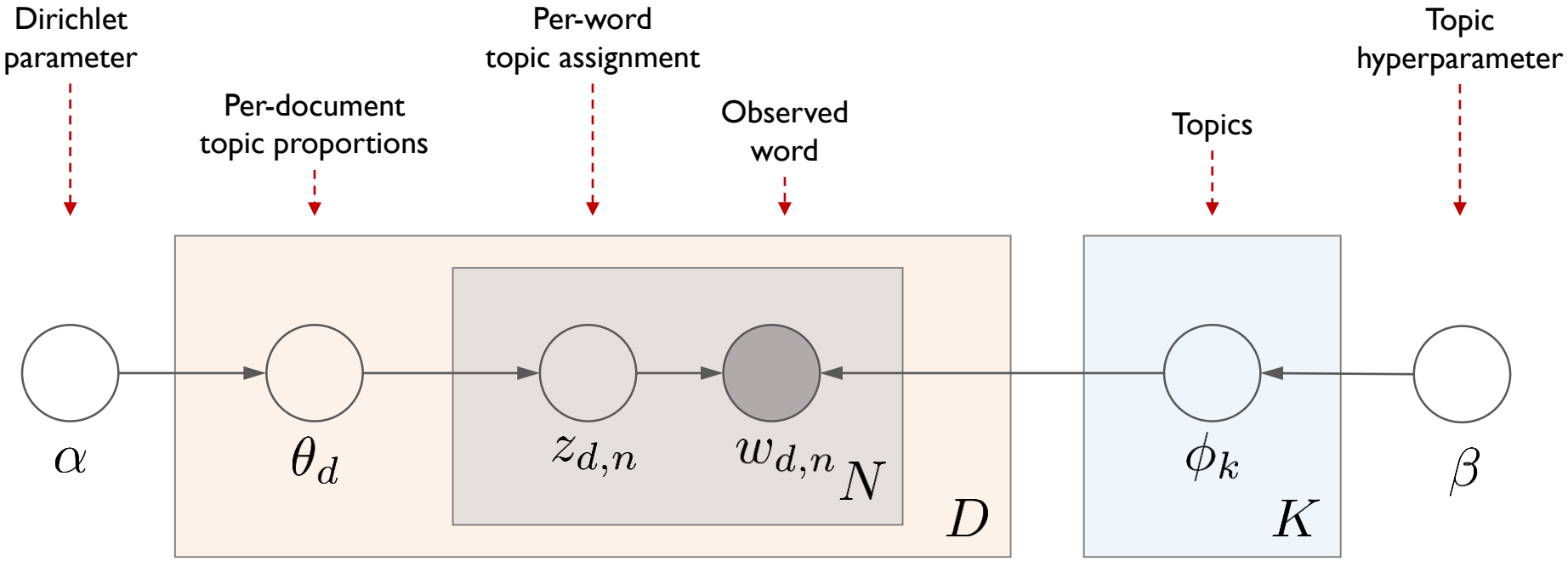
✓ The goal of LDA is to infer the hidden variables

✓ i.e. compute their distribution conditioned on the document

➔ p(topics, proportions, assignments | documents)

LDA Overview

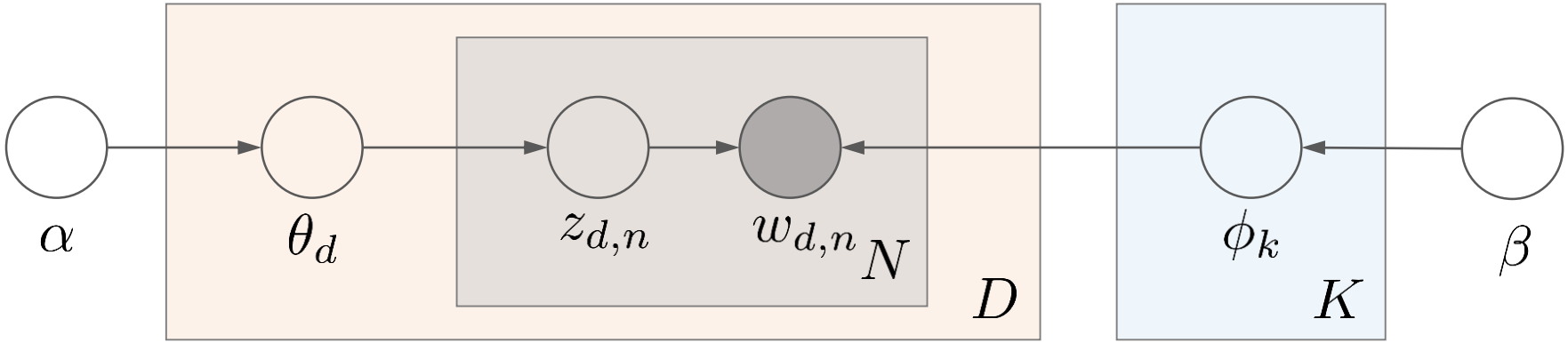
- Documents exhibit multiple topics



- ✓ Encode **assumptions**
- ✓ Define a **factorization** of the joint distribution
- ✓ Connect to **algorithms** for computing with data

LDA Overview

- LDA structure



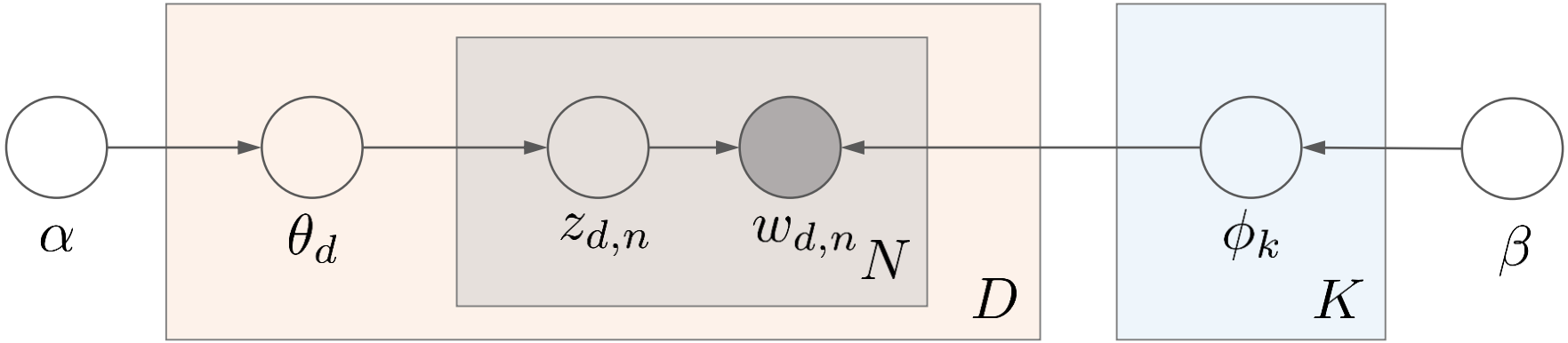
- ✓ Nodes are random variables while edges indicate dependence
- ✓ Shaded nodes are observed
- ✓ Plates indicate replicated variables

$$p(\phi_{1:K}, \theta_{1:D}, z_{1:D}, w_{1:D}) =$$

$$\prod_{i=1}^{K} p(\phi_i|\beta) \prod_{d=1}^{D} p(\theta_d|\alpha) \Big(\prod_{n=1}^{N} p(z_{d,n}|\theta_d) p(w_{d,n}|\phi_{1:K}, z_{d,n}) \Big)$$

LDA: Document generation process

- Document generation process



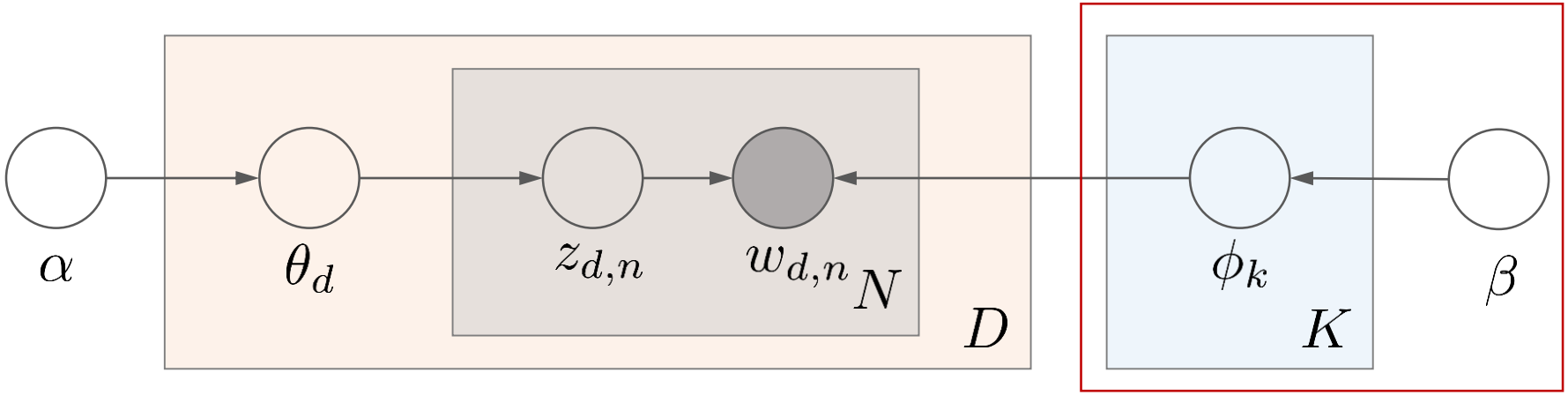
✓ Draw each topic $\phi_k \sim Dir(\beta)$ for $i \in \{1, ..., K\}$

✓ For each document

- Draw topic proportions $\theta_d \sim Dir(\alpha)$
- For each word
  - Draw $z_{d,n} \sim Multi(\theta_d)$
  - Draw $w_{d,n} \sim Multi(\phi_{z_{d,n},n})$

LDA: Document generation process

- Document generation process



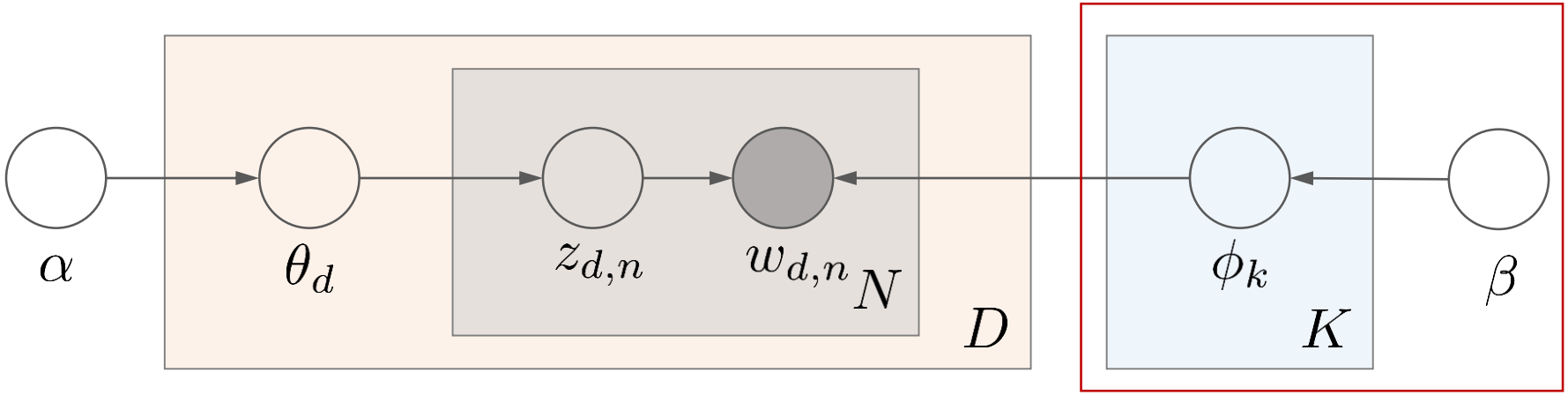
✓ Term distribution per topic

- Drawn from the Dirichlet distribution, given the Dirichlet parameter β, which is a V-vector with component $\beta_v > 0$

$$p(\phi|\beta) = \prod_{k=1}^{K} \frac{\Gamma(\beta_{k,.})}{\prod_{v=1}^{V} \Gamma(\beta_{k,v})} \prod_{v=1}^{V} \phi_{k,v}^{\beta_{k,v}-1}$$

LDA: Document generation process

- Document generation process

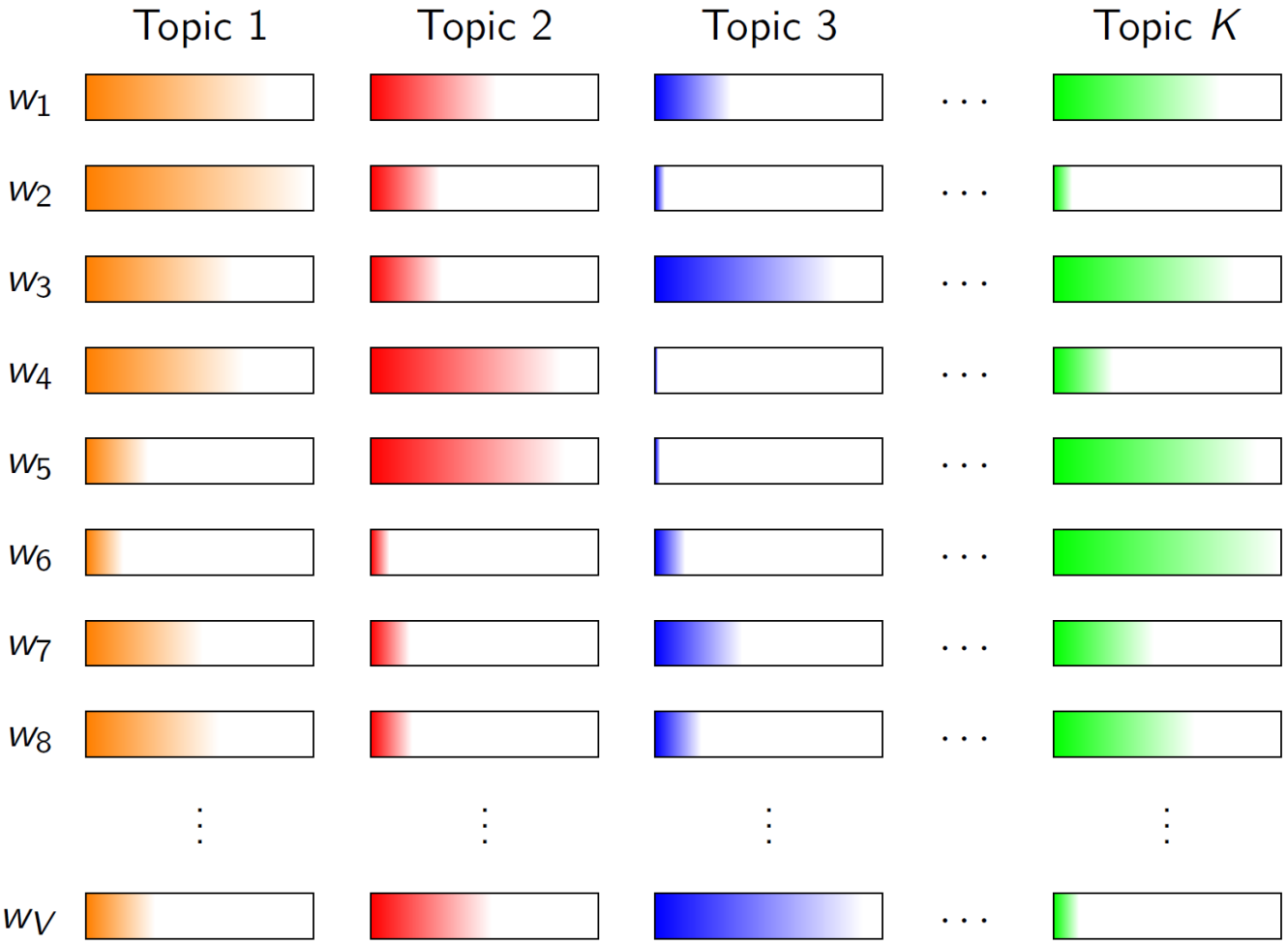


✓ Term distribution per topic

		Term 1	Term 2	Term 3	Term 4	Term 5=V
Topic 1	$\phi_{k=1}$	0.1	0.1	0	0.7	0.1
Topic 2	$\phi_{k=2}$	0.2	0.1	0.2	0.2	0.3
Topic 3	$\phi_{k=3}$	0.01	0.2	0.39	0.3	0.1
Topic 4	$\phi_{k=4=K}$	0.0	0.0	0.5	0.3	0.2

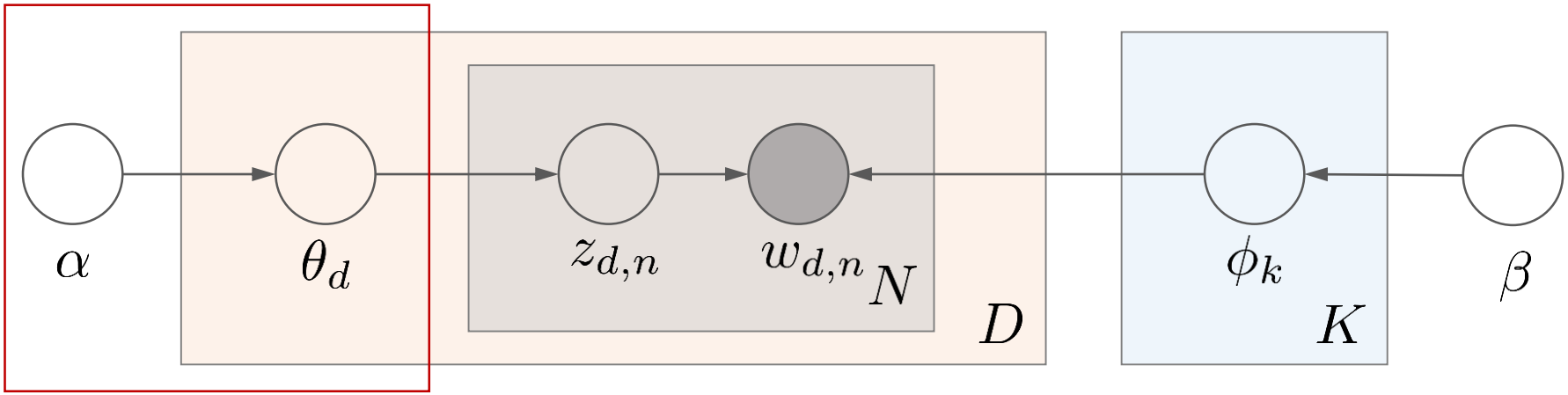
LDA: Document generation process

- Document generation process
  - ✓ Term distribution per topic



LDA: Document generation process

- Document generation process



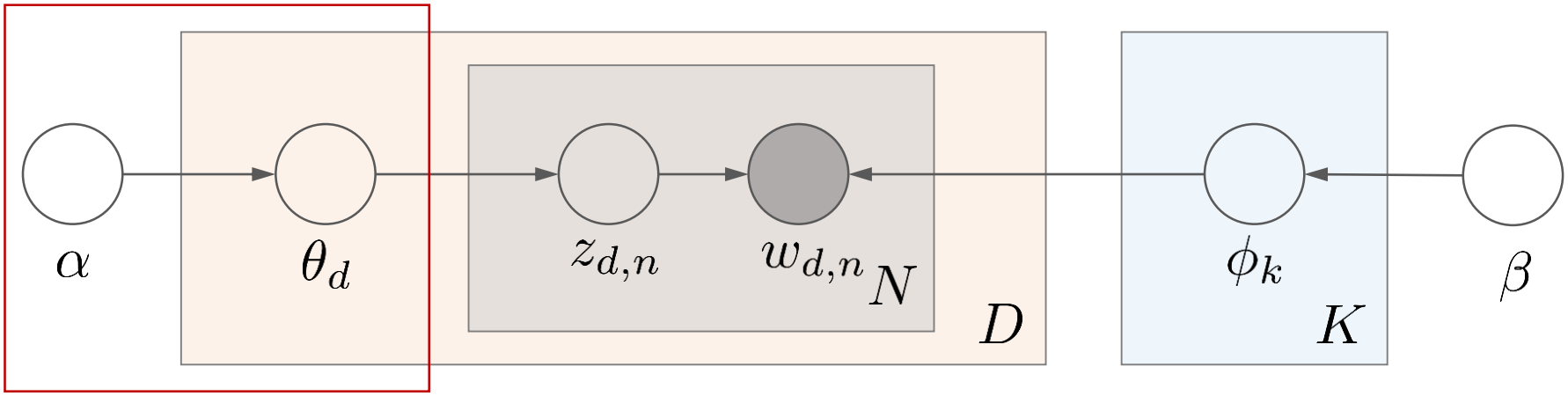
✓ Topic distribution per document

- Drawn from the Dirichlet distribution, given the Dirichlet parameter α, which is a K-vector with components $\alpha_k > 0$

$$p(\theta|\alpha) = \frac{\Gamma(\sum_{k=1}^{K} \alpha_k)}{\prod_{k=1}^{K} \Gamma(\alpha_k)} \theta_1^{\alpha_k - 1} \cdots \theta_K^{\alpha_K - 1} = \frac{\Gamma(\alpha_\cdot)}{\prod_{k=1}^{K} \Gamma(\alpha_k)} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1}$$

LDA: Document generation process

- Document generation process

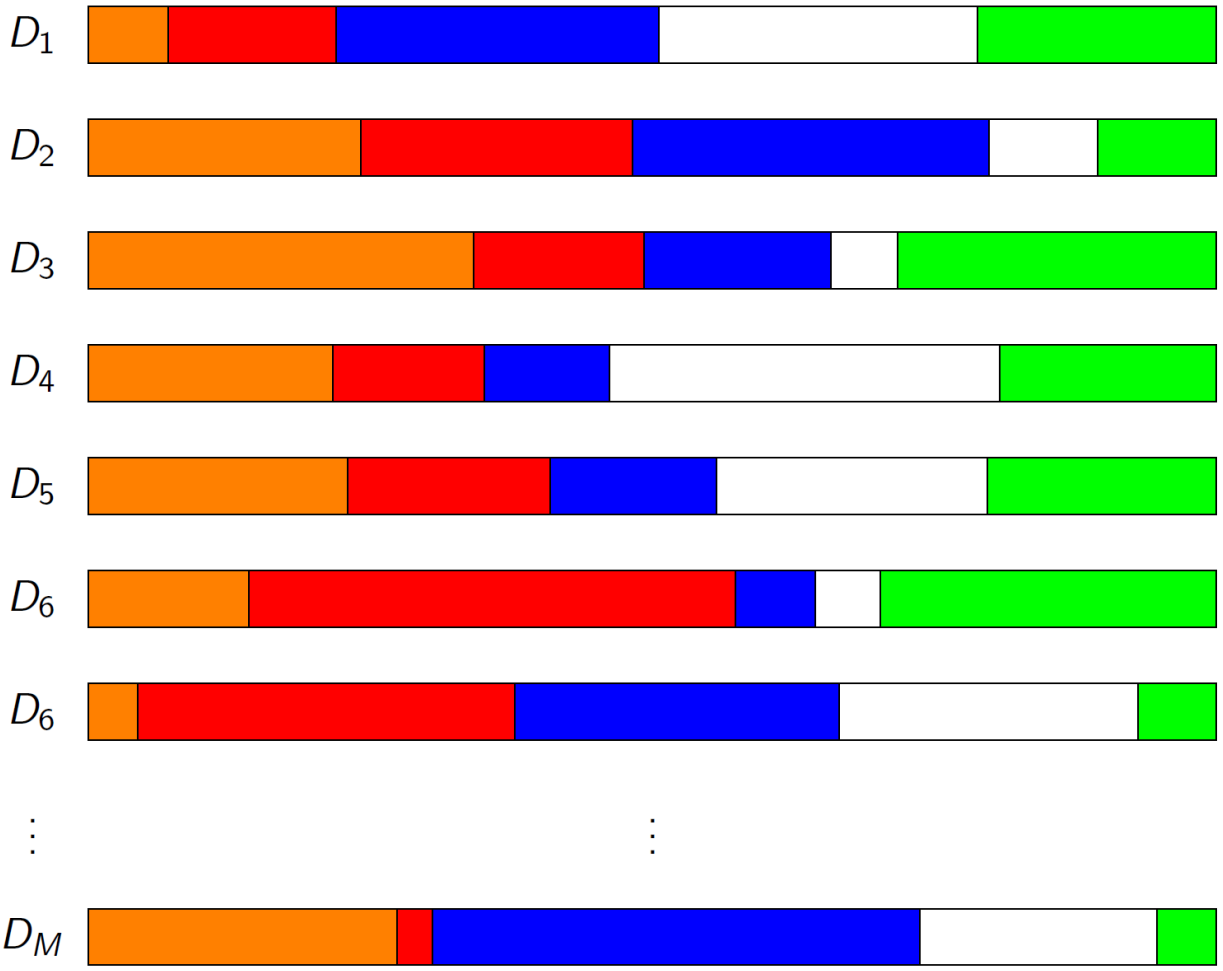


✓ Topic distribution per document

		Topic 1	Topic 2	Topic 3	Topic 4
Document 1	$\theta_{d=1}$	0.5	0.1	0.3	0.1
Document 2	$\theta_{d=2}$	0.0	0.9	0.1	0.0
Document 3	$\theta_{d=3=D}$	0.02	0.48	0.25	0.25

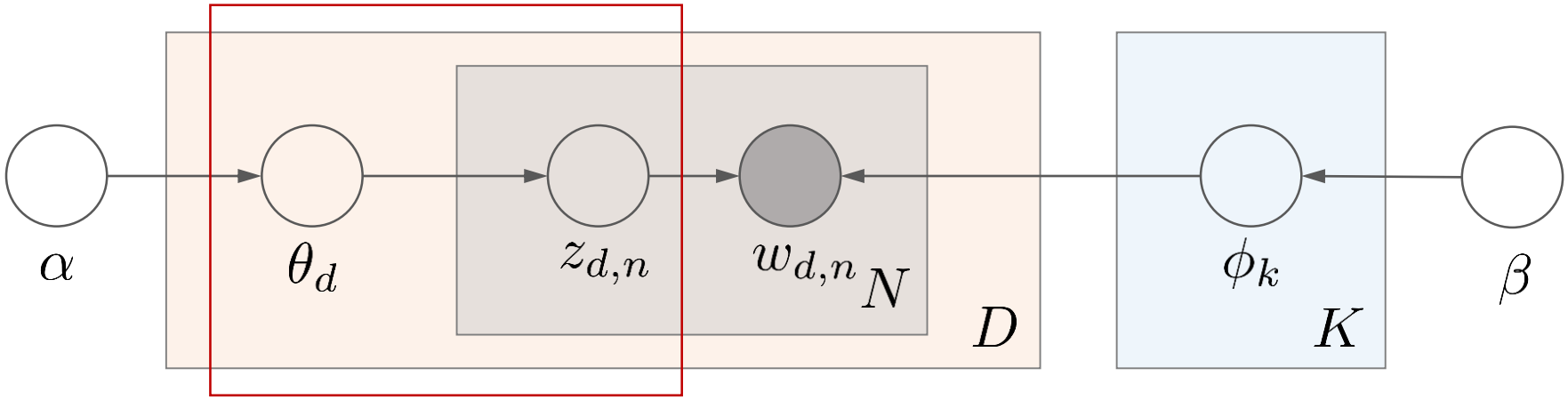
LDA: Document generation process

- Document generation process
  - ✓ Topic distribution per document



LDA: Document generation process

- Document generation process



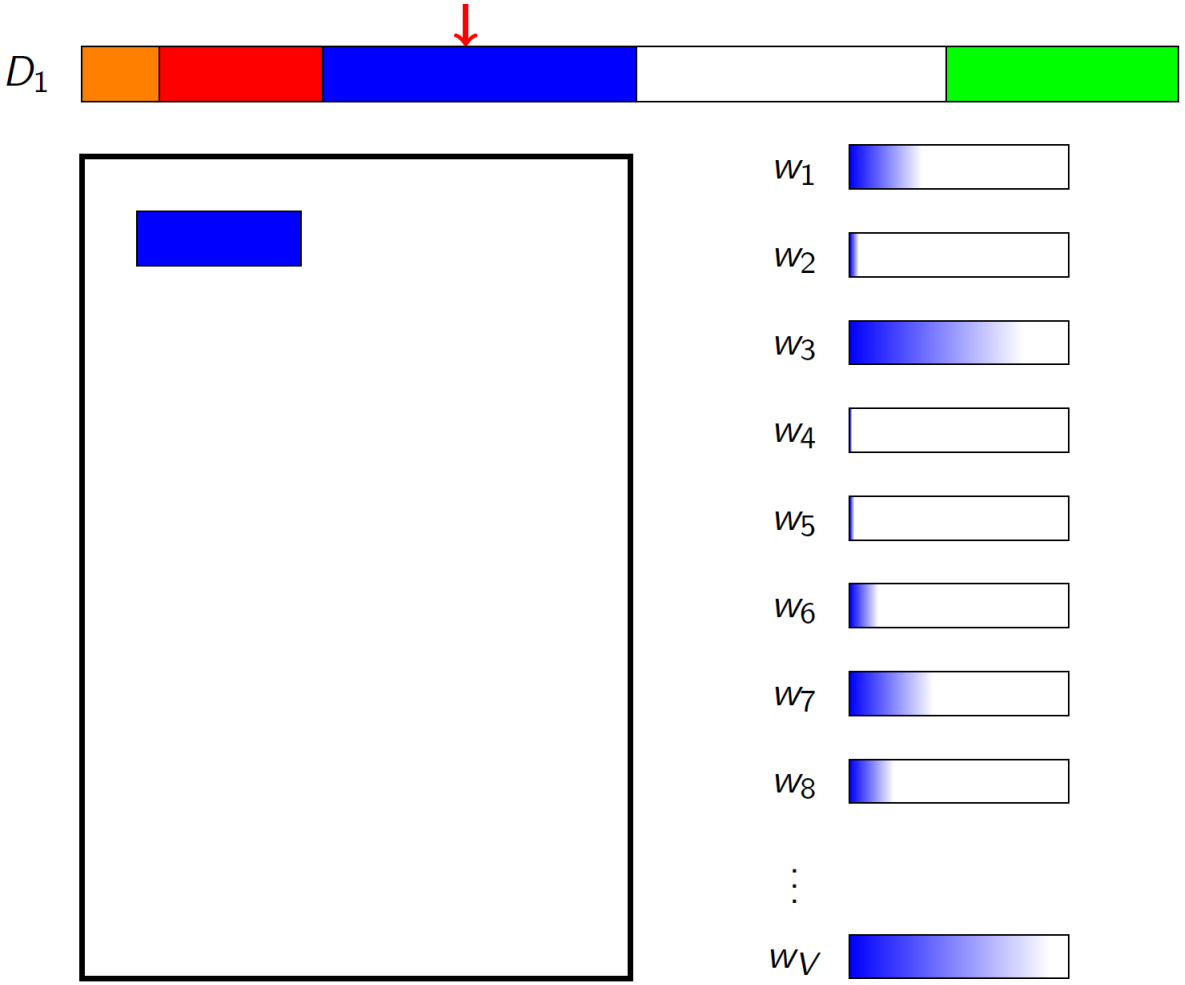
✓ Topic to words assignments

		Word w_1	Word w_2	Word w_3	Word w_4	Word w_5	Word w_6
Document 1	$z_{d=1}$	Topic k=2	Topic k=1	Topic k=1	Topic k=4	Topic k=3	Topic k=3
Document 2	$z_{d=2}$	Topic k=2	Topic k=3	Topic k=2	Topic k=2		
Document 3	$z_{d=3=D}$	Topic k=4	Topic k=2	Topic k=2	Topic k=4	Topic k=3	

$$p(z|\theta) = \prod_{d=1}^{D} \prod_{k=1}^{K} \theta_{d,k}^{n_{d,k,.}}$$

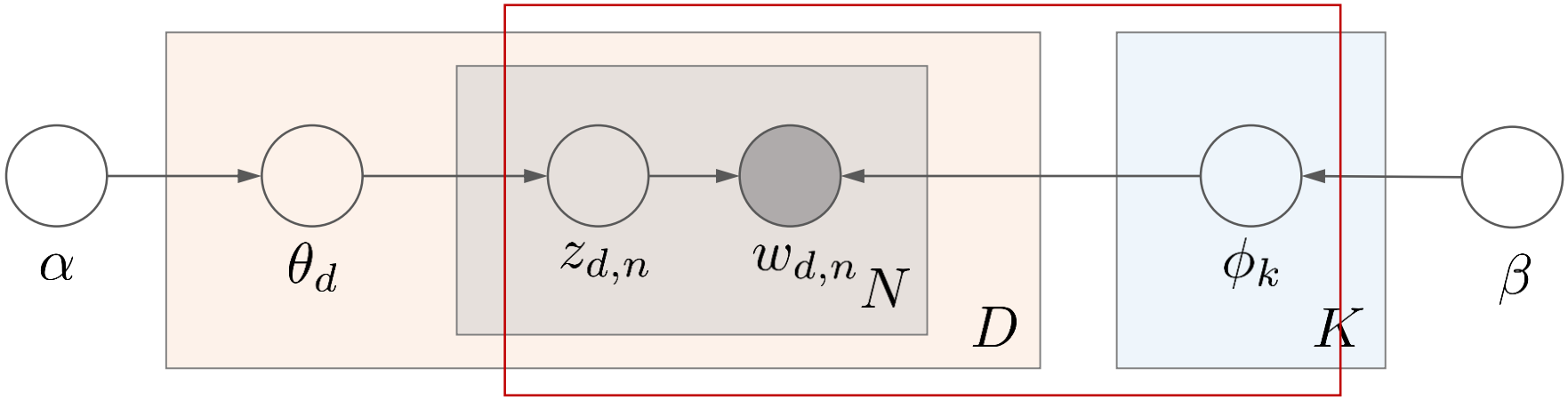
LDA: Document generation process

- Document generation process
  - ✓ Topic to words assignments



LDA: Document generation process

- Document generation process

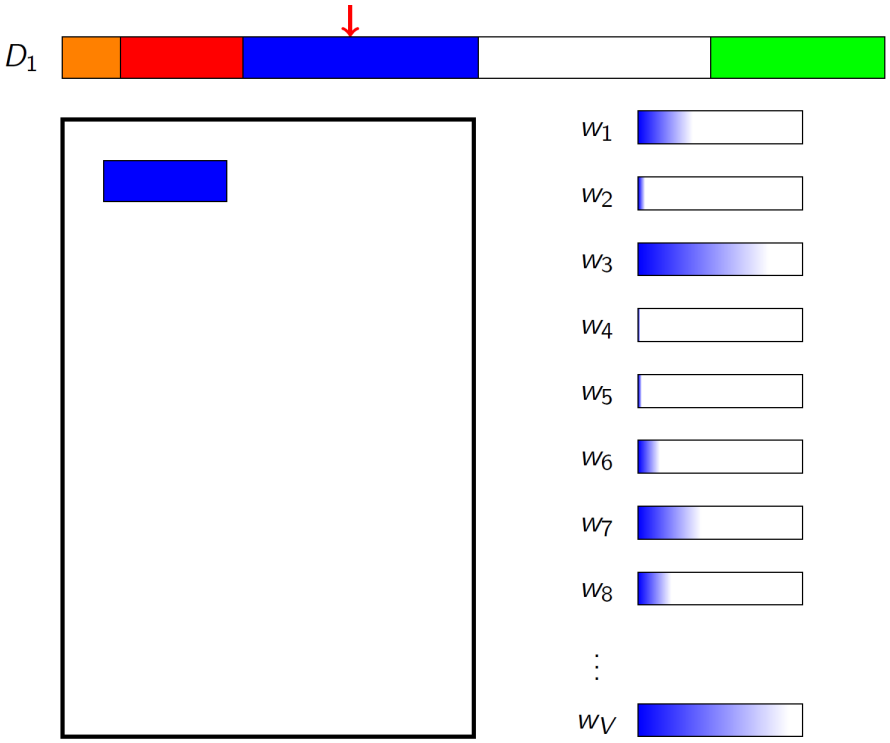


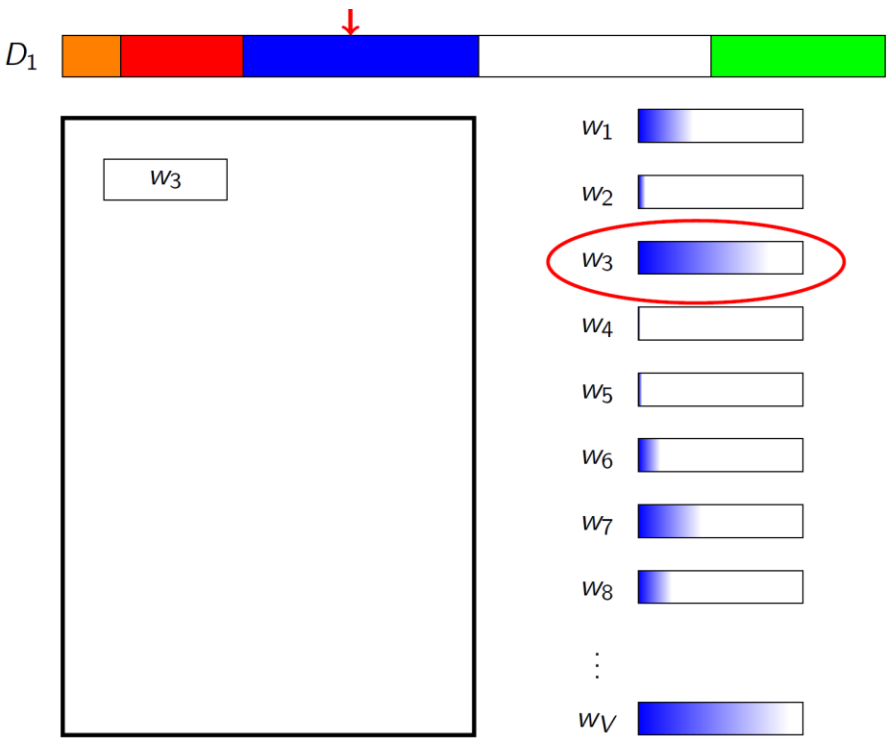
✓ Probability of a corpus

$$p(w|z,\phi) = \prod_{k=1}^{K} \prod_{v=1}^{V} \phi_{k,v}^{n_{.,k,v}}$$

LDA: Document generation process

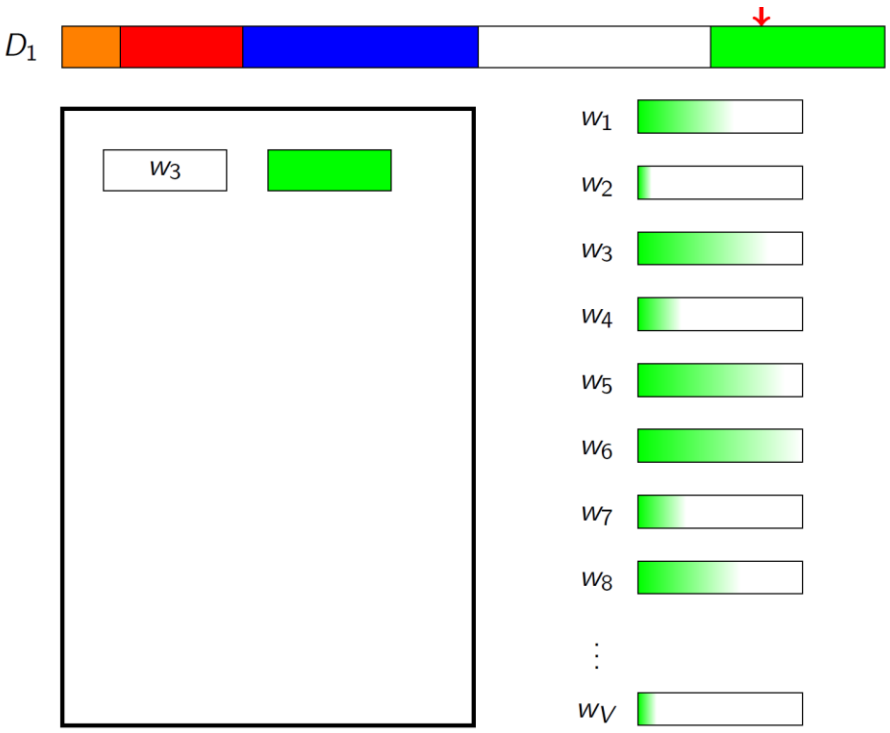
- Document generation process
  - ✓ Word selection

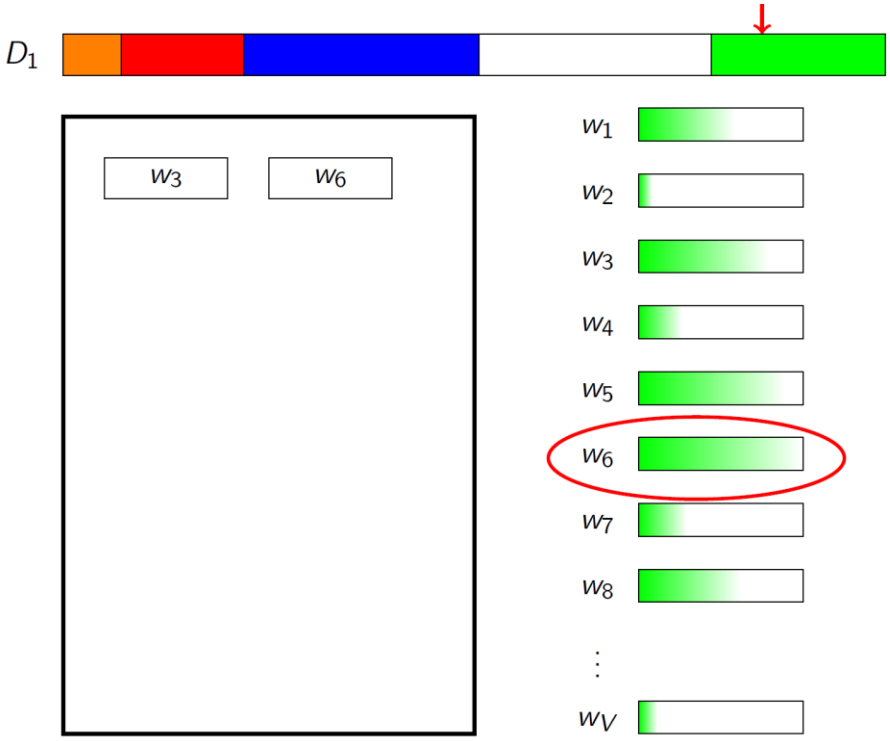




LDA: Document generation process

- Document generation process
  - ✓ Word selection

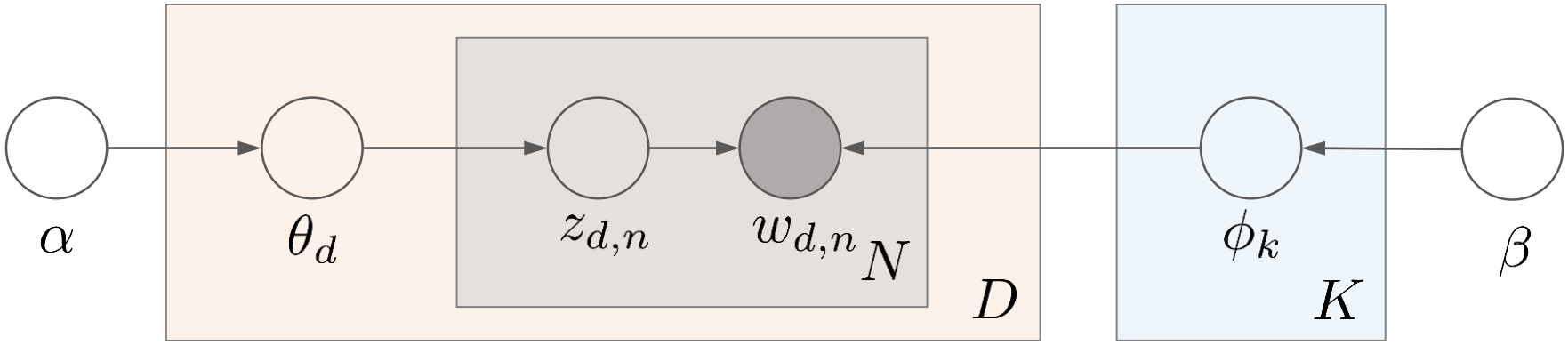




AGENDA

LDA Inference

- LDA structure



$$p(\phi_{1:K}, \theta_{1:D}, z_{1:D}, w_{1:D}) = \prod_{i=1}^{K} p(\phi_i|\beta) \prod_{d=1}^{D} p(\theta_d|\alpha) \Big(\prod_{n=1}^{N} p(z_{d,n}|\theta_d) p(w_{d,n}|\phi_{1:K}, z_{d,n}) \Big)$$

✓ From a collection of documents, we infer

- Per-word topic assignment $z_{d,n}$
- Per-document topic proportions θ_d
- Per-corpus topic distributions ϕ_k

LDA Inference

- Inference
  - ✓ The posterior of the latent variables given the document is

$$p(\phi, \theta, \mathbf{z}|\mathbf{w}) = \frac{p(\phi, \theta, \mathbf{z}, \mathbf{w})}{\int_\phi \int_\theta \sum_{\mathbf{z}} p(\phi, \theta, \mathbf{z}, \mathbf{w})}$$

  - ✓ Computing the posterior is intractable (we cannot compute the denominator, the marginal p(w))
  - ✓ Approximate posterior inference algorithms
    - Mean field variational methods
    - Expectation propagation
    - Collapsed Gibbs sampling
    - Collapsed variational inference
    - Online variational inference

LDA: Dirichlet Distribution

- Binomial & Multinomial
  - ✓ Binomial distribution: the number of successes in a sequence of independent yes/no experiments (Bernoulli trials)

$$p(X = x|n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

  - ✓ Multinomial distribution: suppose that each experiment results in one of k possible outcomes with probabilities $p_1, \ldots, p_k$, multinomial models the distribution of the histogram vector which indicates how many time each outcome was observed over N trials of experiments.

$$p(x_1, ..., x_k|n, p_1, ..., p_k) = \frac{N!}{\prod_{i=1}^{k} x_i!} p_i^{x_i}, \quad \sum_i x_i = N, \quad x_i \geq 0$$

LDA: Dirichlet Distribution

- Beta distribution

$$p(p|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}$$

✓ $p \in [0,1]$: considering p as the parameter of a Binomial distribution, we can think of Beta is a "distribution over distributions" (binomials)

$$B(\alpha,\beta) = \int_0^1 t_{\alpha-1}(1-t)^{\beta-1}dt \qquad B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$\Gamma(x) = \int_0^\infty \frac{t^{x-1}dt}{exp(t)}, \quad (x>0) \qquad \Gamma(n) = (n-1)!$$

LDA: Dirichlet Distribution

- Dirichlet distribution

$$p(P = \{p_i\}|\alpha_i) = \frac{\Gamma(\sum_i \alpha_i)}{\prod_i \Gamma(\alpha_i)} \prod_i p_i^{\alpha_i - 1}$$

✓ $\sum_i p_i = 1, \quad p_i \geq 0$

✓ Two parameters

- the scale (or concentration): $\sigma = \sum_i \alpha_i$
- the base measure: $(\alpha'_1, ..., \alpha'_k), \ \alpha'_i = \alpha_i / \sigma$

✓ A generalization of Beta

- Beta is a distribution over binomials (in an interval $p \in [0, 1]$)
- Dirichlet is a distribution over Multinomials (in the so-called simplex $\sum_i p_i = 1, \ p_i \geq 0$)

✓ Dirichlet is the conjugate prior of multinomial

LDA: Dirichlet Distribution

- Important properties of Dirichlet distribution
  - ✓ Posterior is also Dirichlet

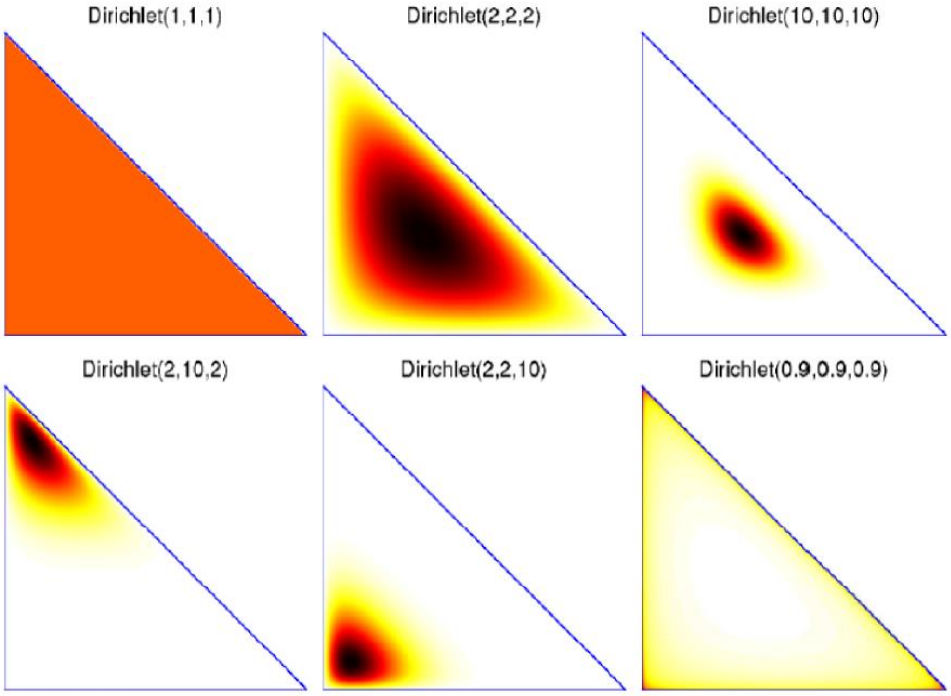
$$p(P = \{p_i\}|\alpha_i) = \frac{\Gamma(\sum_i \alpha_i)}{\prod_i \Gamma(\alpha_i)} \prod_i p_i^{\alpha_i - 1}$$

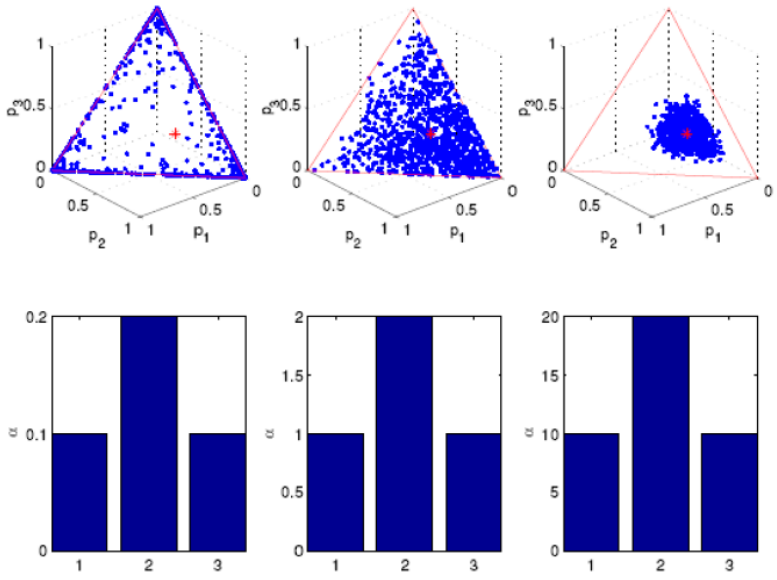
$$p(x_1, ..., x_k|n, p_1, ..., p_k) = \frac{n!}{\prod_i^k x_i!} p_i^{x_i}$$

$$p(\{p_i\}|x_1, ..., x_k) = \frac{\Gamma(N + \sum_i \alpha_i)}{\prod_i^k \Gamma(\alpha_i + x_i)} \prod_i p_i^{\alpha_i + x_i - 1}$$

LDA: Dirichlet Distribution

- Important properties of Dirichlet distribution
  - ✓ The parameter α controls the mean shape and sparsity of θ
  - ✓ A Dirichlet with $\alpha_i < 1$ favors extreme distribution





LDA: Dirichlet Distribution

- Important properties of Dirichlet distribution
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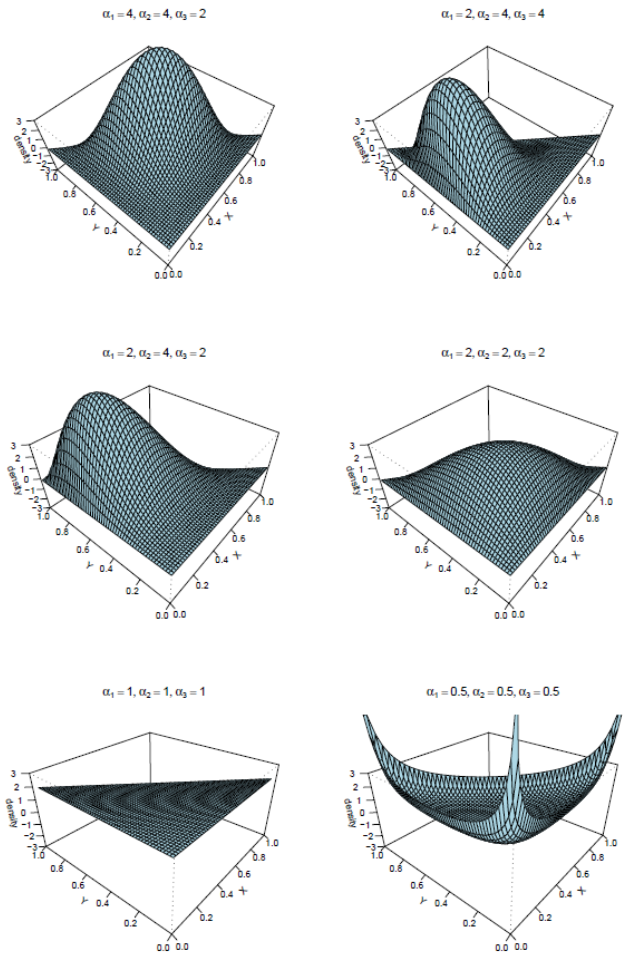
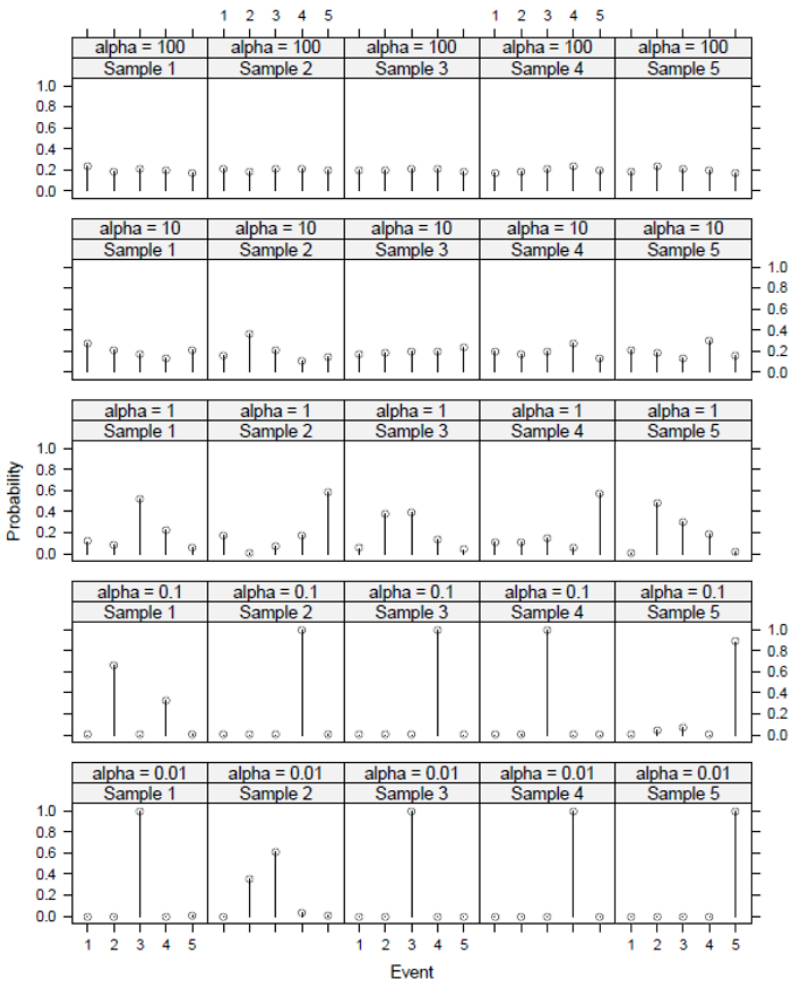
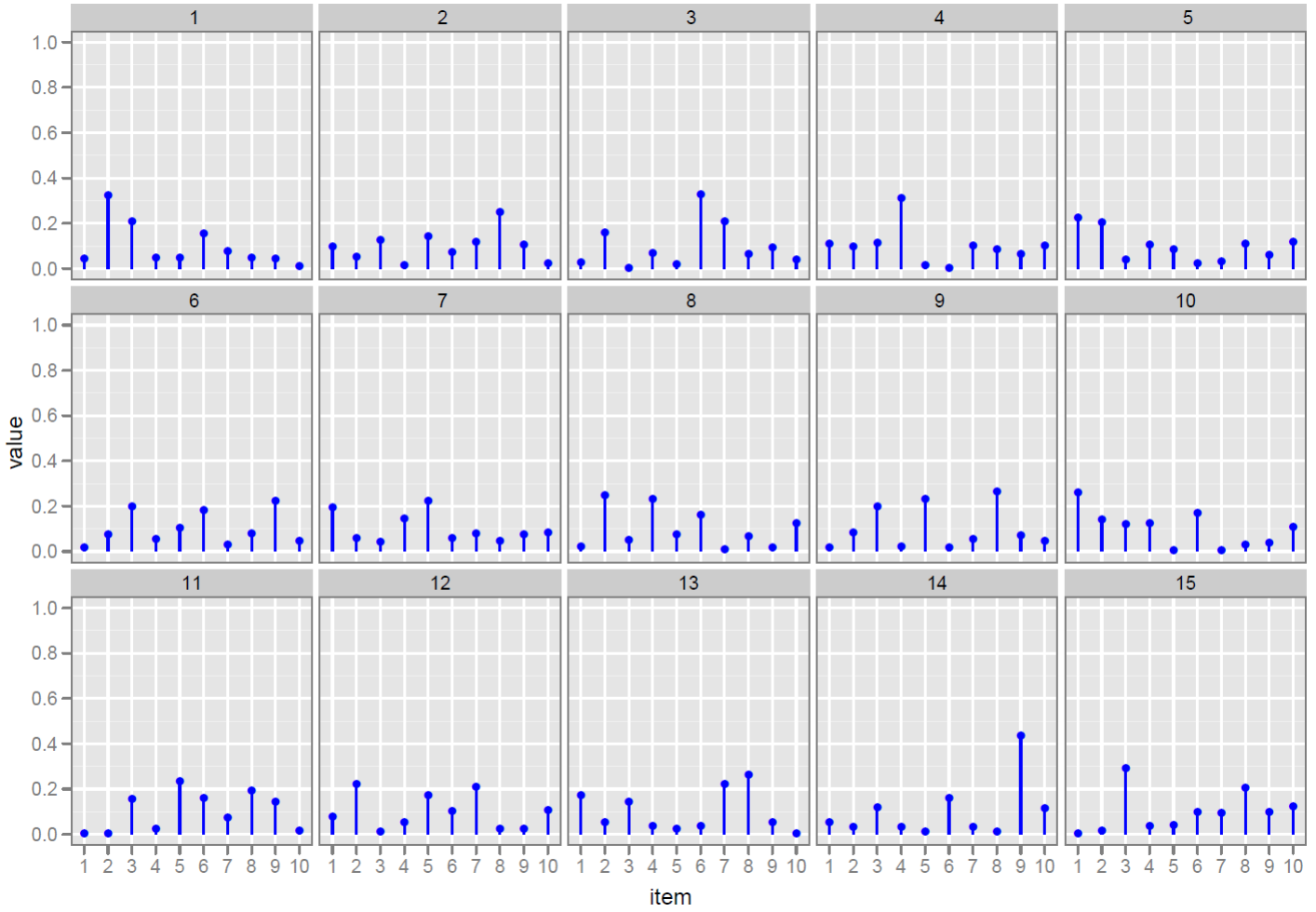




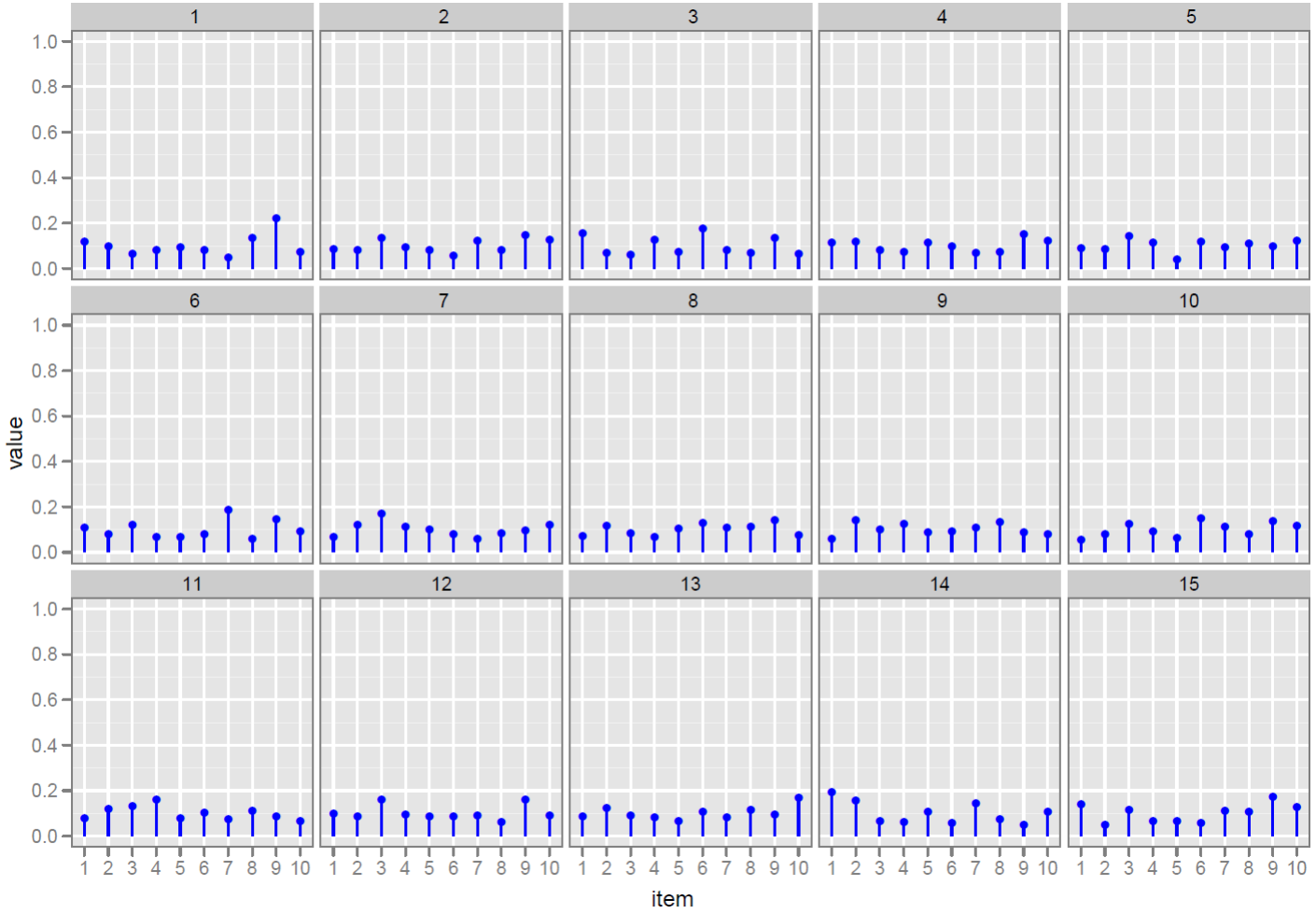
LDA: Dirichlet Distribution

- Sampling results from Dirichlet with different α
  - ✓ α = 1



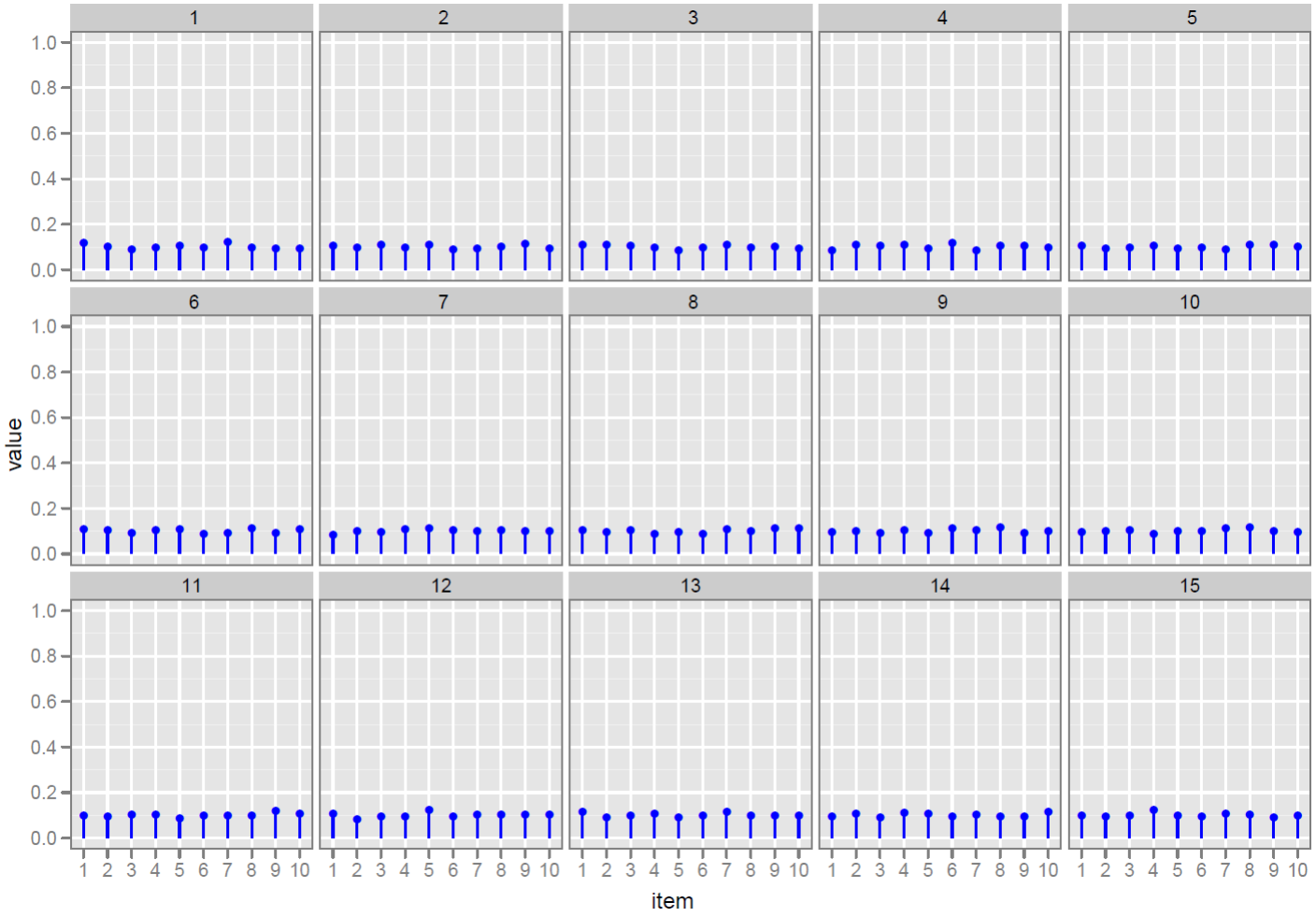
LDA: Dirichlet Distribution

- Sampling results from Dirichlet with different α
  - ✓ α = 10



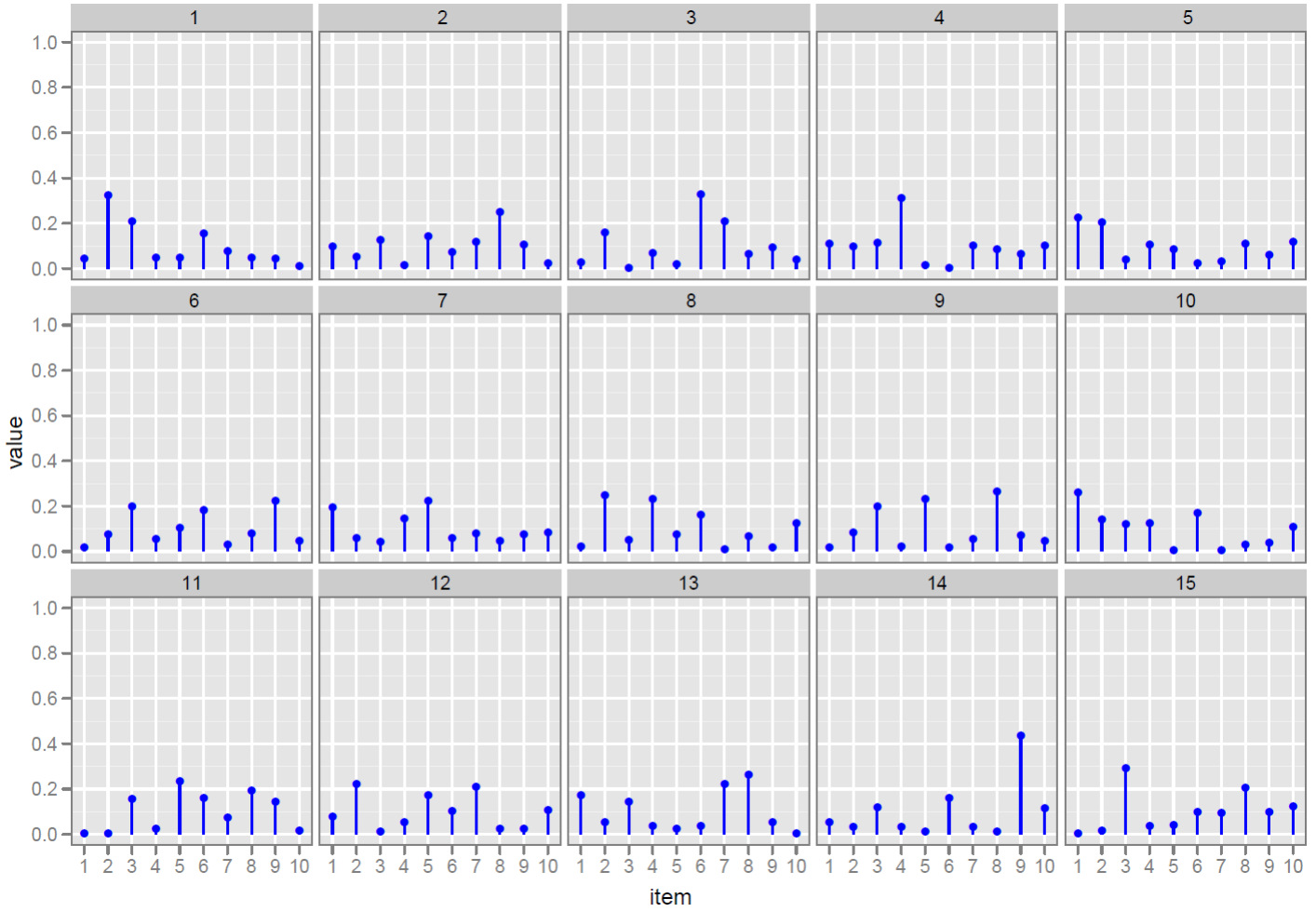
LDA: Dirichlet Distribution

- Sampling results from Dirichlet with different α
  - ✓ α = 100



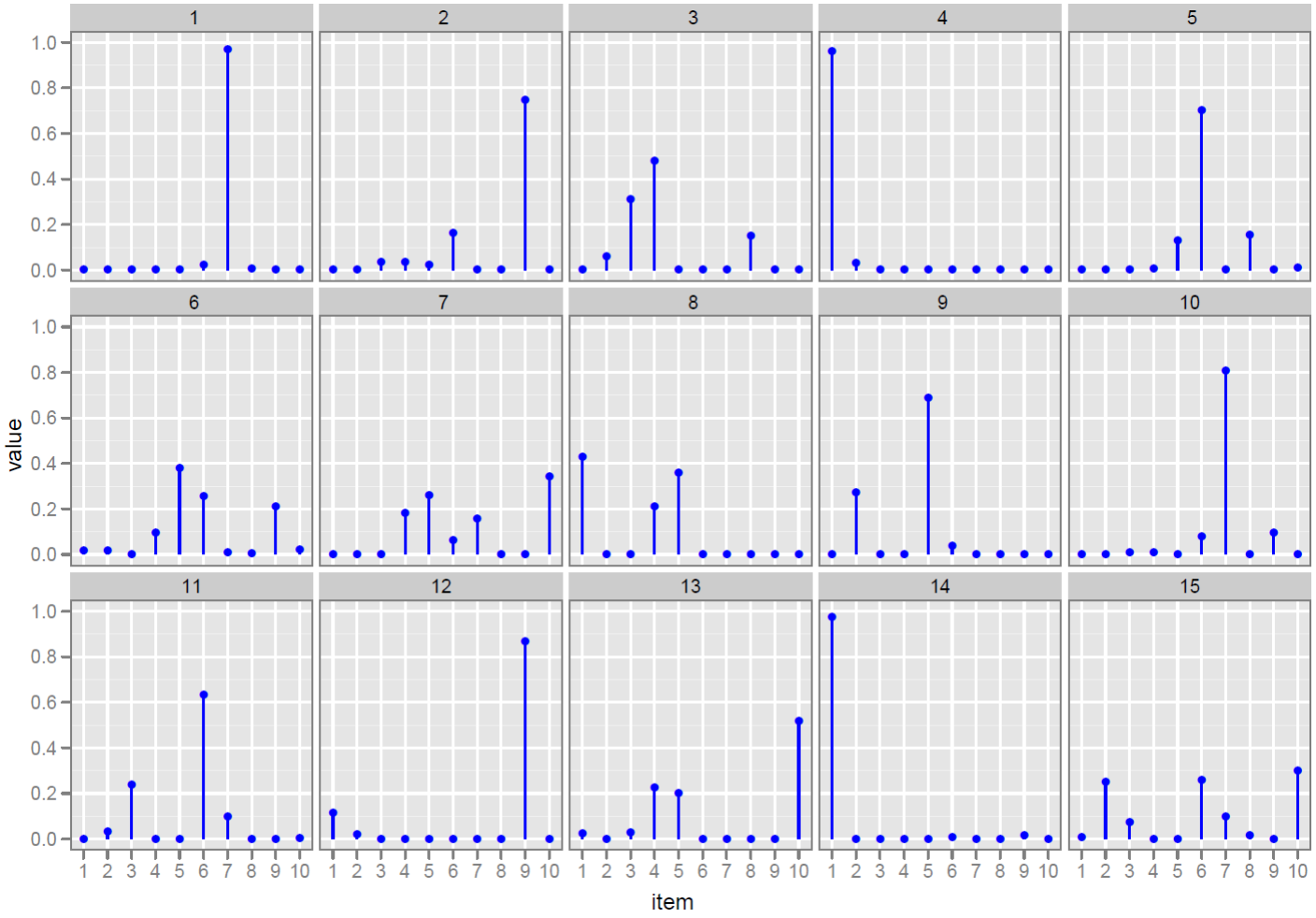
LDA: Dirichlet Distribution

- Sampling results from Dirichlet with different α
  - ✓ α = 1



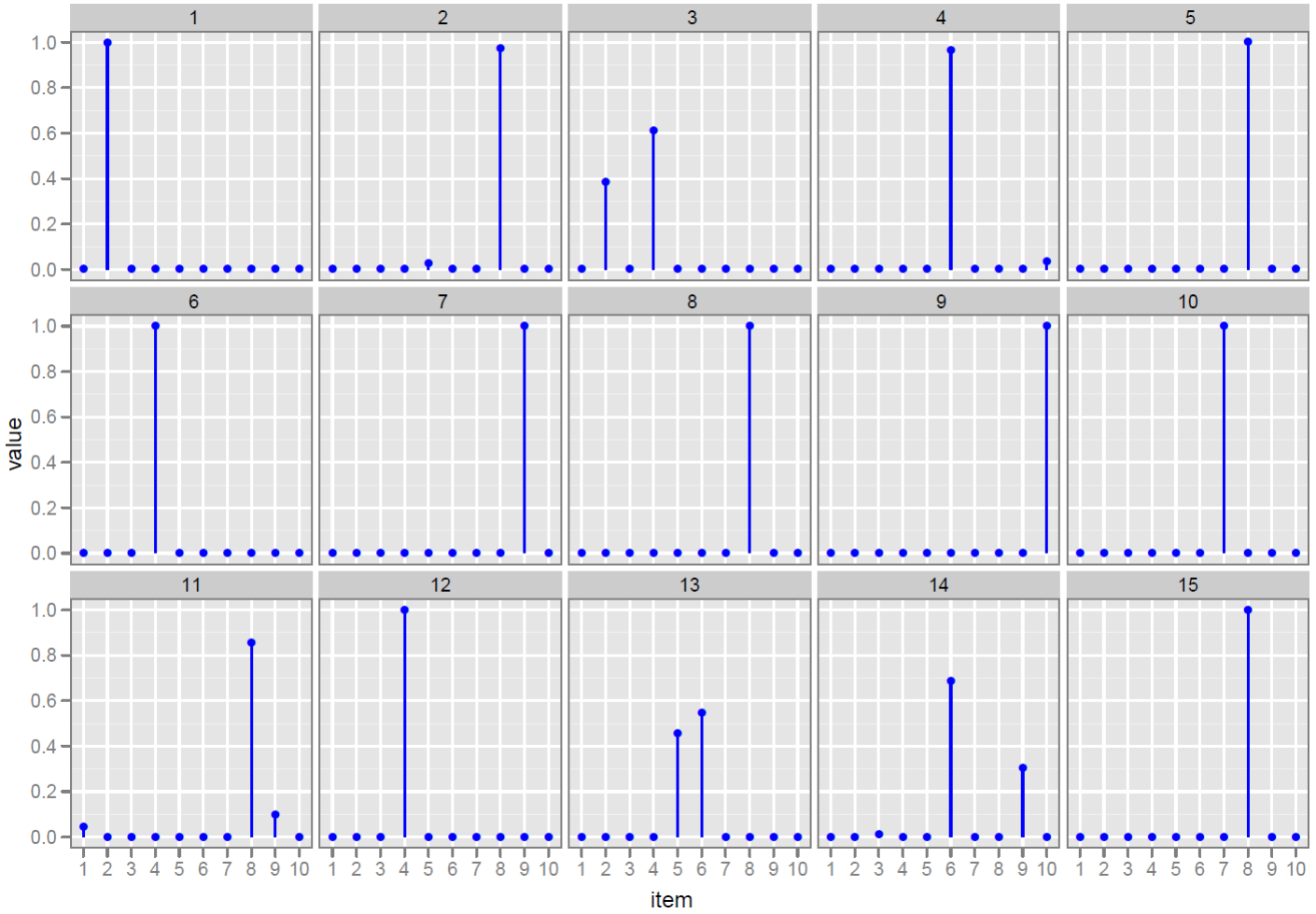
LDA: Dirichlet Distribution

- Sampling results from Dirichlet with different α
  - ✓ α = 0.1



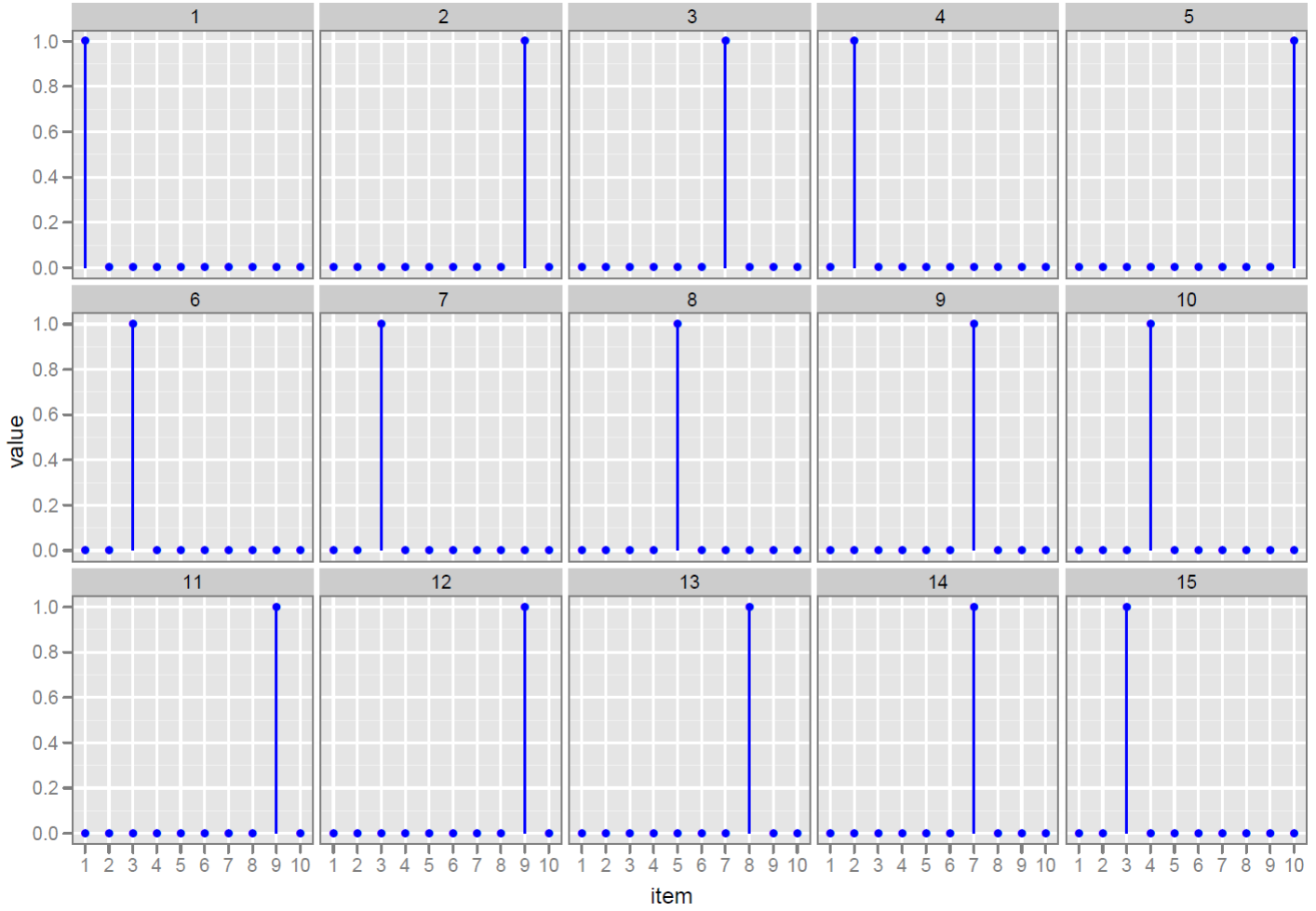
LDA: Dirichlet Distribution

- Sampling results from Dirichlet with different α
  - ✓ α = 0.01



LDA: Dirichlet Distribution

- Sampling results from Dirichlet with different α
  - ✓ α = 0.001



LDA Inference

- We are interested in posterior distribution

$$p(Z|X,\Theta)$$

  - ✓ Here, latent variables are topic assignments z and topics θ. X is the words (divided into documents), and Θ are hyper-parameters to Dirichlet distributions: α for topic proportions, β for topics

$$p(\mathbf{z},\phi,\theta|\mathbf{w},\alpha,\beta)$$

$$p(\phi_{1:K},\theta_{1:D},z_{1:D},w_{1:D}) =$$

$$\prod_{i=1}^{K} p(\phi_i|\beta) \prod_{d=1}^{D} p(\theta_d|\alpha) \Big(\prod_{n=1}^{N} p(z_{d,n}|\theta_d) p(w_{d,n}|\phi_{1:K},z_{d,n}) \Big)$$

LDA Inference

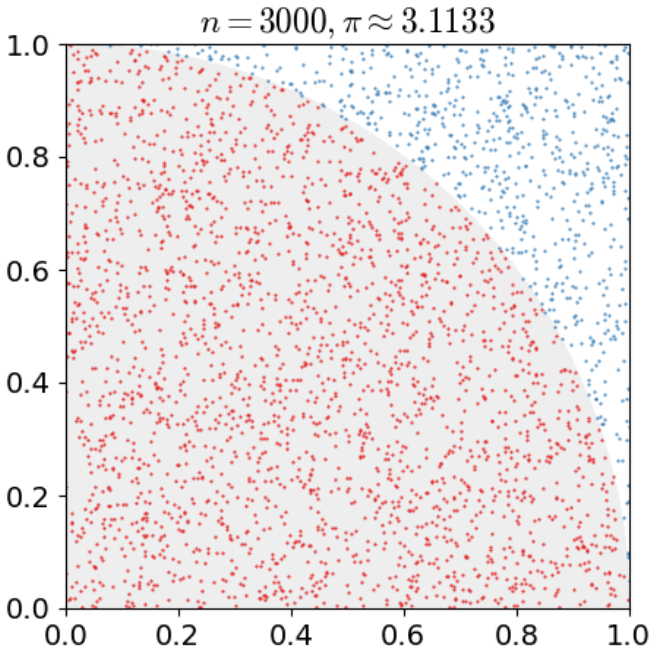
- Gibbs Sampling
  - ✓ A form of Markov Chain Monte Carlo
  - ✓ Chain is a sequence of random variable states
  - ✓ Given a state $\{z_1, ..., z_N\}$ given certain technical conditions,

    drawing $z_k \sim p(z_1, ..., z_{k-1}, z_{k+1}, ..., z_N | X, \Theta)$ for all k (repeatedly) results in a

    Markov Chain whose stationary distribution is the posterior
  - ✓ For notational call $\mathbf{z}$ with $z_{d,n}$ removed $\mathbf{z}_{-d,n}$

LDA Inference

- Monte Carlo method
  - ✓ Computational algorithms that rely on repeated random sampling to obtain numerical results
  - ✓ Use randomness to solve problems that might be deterministic in principle
  - ✓ Example: approximating the value of π



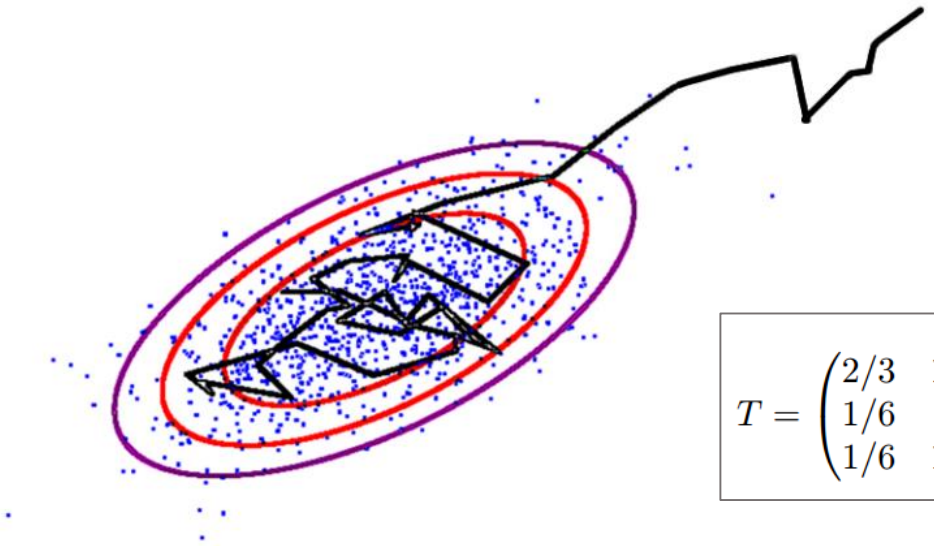
LDA Inference

Murray (2009)

- Markov Chain Monte Carlo sampling
  - ✓ Sampling from a probability distribution based on constructing a Markov Chain that has the desired distribution as its equilibrium distribution
  - ✓ Use the local information rather than a complete randomness

Construct a biased random walk that explores target dist $P^\star(x)$

Markov steps, $x_t \sim T(x_t \leftarrow x_{t-1})$



$$T = \begin{pmatrix} 2/3 & 1/2 & 1/2 \\ 1/6 & 0 & 1/2 \\ 1/6 & 1/2 & 0 \end{pmatrix} \qquad T_{ij} = T(x_i \leftarrow x_j)$$

MCMC gives approximate, correlated samples from $P^\star(x)$

LDA Inference

Murray (2009)

- Gibbs Sampling

A method with no rejections:

- Initialize $\mathbf{x}$ to some value
- Pick each variable in turn or randomly and resample $P(x_i|\mathbf{x}_{j\neq i})$

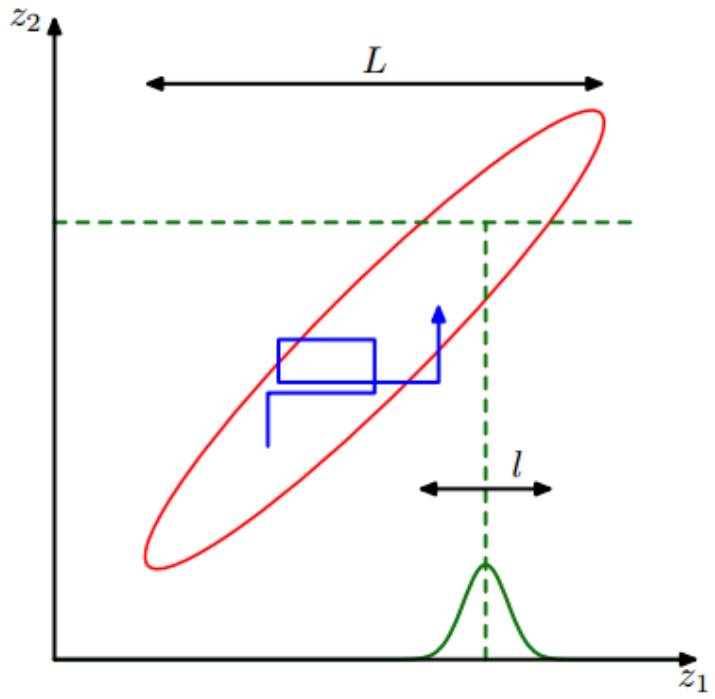
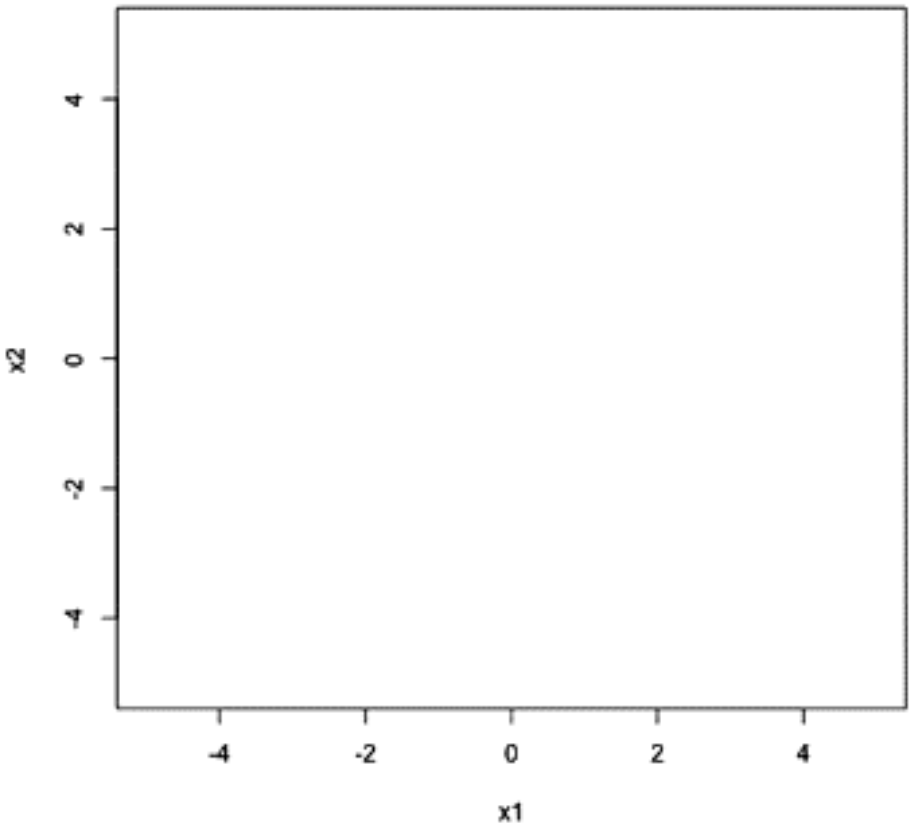


Figure from PRML, Bishop (2006)

LDA Inference

- Gibbs Sampling



https://www.youtube.com/watch?v=ZaKwpVgmKTY

LDA Inference

Tang (2008)

- Gibbs Variants
  - ✓ Gibbs Sampling
    - Draw a conditioned on b, c
    - Draw b conditioned on a, c
    - Draw c conditioned on a, b
  - ✓ Block Gibbs Sampling
    - Draw a, b conditioned on c
    - Draw c, conditioned on a, b
  - ✓ Collapsed Gibbs Sampling
    - Draw a conditioned on c
    - Draw c conditioned on a
    - b is collapsed out during the sampling process

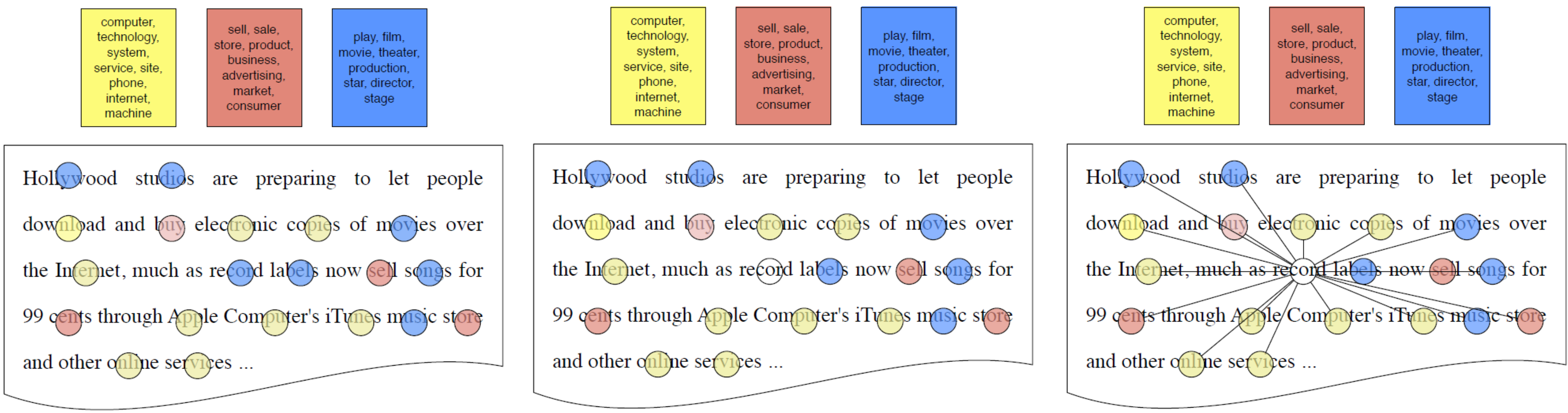
LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Gibbs sampling procedure boils down to estimate

$$p(z_i = j | \mathbf{z}_{-i}, \mathbf{w})$$

✓ θ and ϕ are integrated out



LDA Inference: Collapsed Gibbs Sampling

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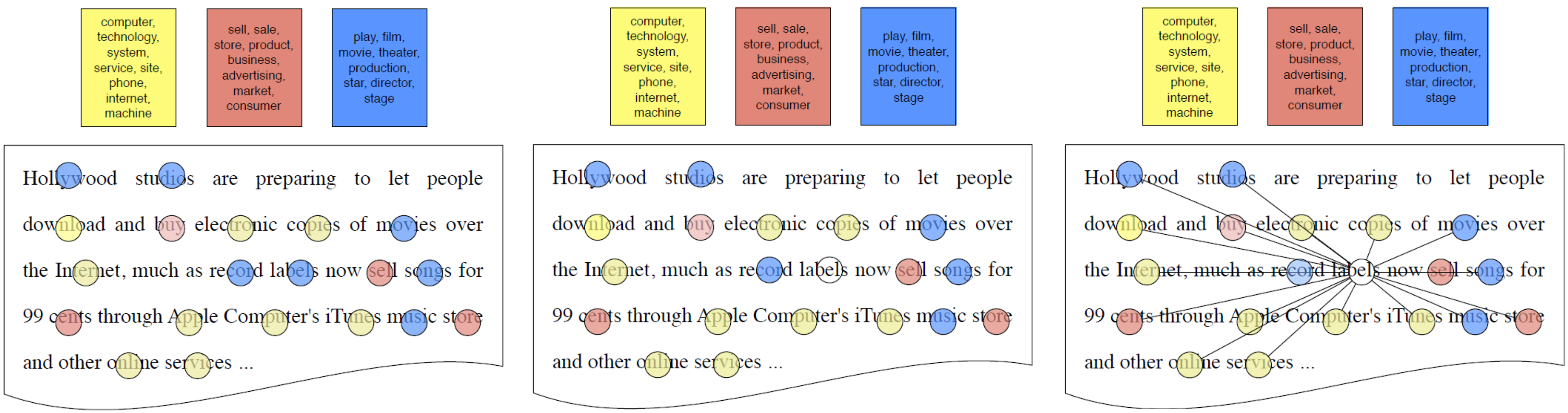
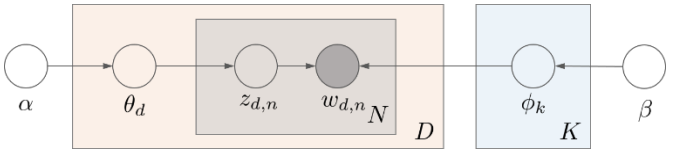


LDA Inference: Collapsed Gibbs Sampling

Tang (2008)

- Conditional posterior distribution for z_i is given by



$$p(z_i = j|\mathbf{z}_{-i}, \mathbf{w}) \propto p(z_i = j, \mathbf{z}_{-i}, \mathbf{w})$$

$$= p(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i})p(z_i = j|\mathbf{z}_{-i}, \mathbf{w}_{-i})$$

$$= p(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i})p(z_i = j|\mathbf{z}_{-i})$$

✓ The first term is the likelihood and the second term acts like a prior

$$p(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i}) = \int p(w_i|z_i = j, \phi^{(j)})p(\phi^{(j)}|\mathbf{z}_{-i}, \mathbf{w}_{-i})d\phi^{(j)}$$

$$= \int \phi^{(j)}_{w_i} p(\phi^{(j)}|\mathbf{z}_{-i}, \mathbf{w}_{-i})d\phi^{(j)}$$

$$p(\phi^{(j)}|\mathbf{z}_{-i}, \mathbf{w}_{-i}) \propto p(\mathbf{w}_{-i}|\phi^{(j)}, \mathbf{z}_{-i})p(\phi^j) \sim Dirichlet(\beta + n^{(w)}_{-i,j})$$

LDA Inference: Collapsed Gibbs Sampling

Tang (2008)

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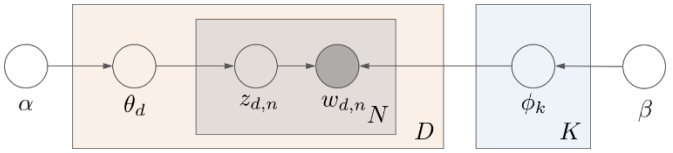

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$$= p(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i})p(z_i = j|\mathbf{z}_{-i})$$

✓ Here, $n_{-i,j}^{(w)}$ is the number of instances of word w assigned to topic j excluding the current one.

✓ Using the property of expectation of Dirichlet distribution, we have

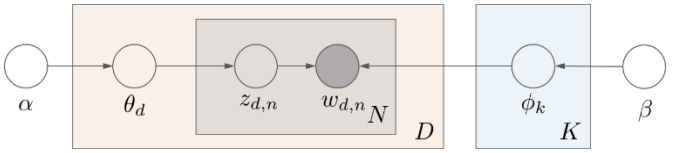
$$p(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i}) = \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + V\beta}$$

✓ where $n_{-i,j}^{(\cdot)}$ is the total number of words assigned to topic j excluding the current one in the corpus

LDA Inference: Collapsed Gibbs Sampling

Tang (2008)

- Conditional posterior distribution for z_i is given by



$$p(z_i = j|\mathbf{z}_{-i}, \mathbf{w}) \propto p(z_i = j, \mathbf{z}_{-i}, \mathbf{w})$$

$$= p(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i})p(z_i = j|\mathbf{z}_{-i}, \mathbf{w}_{-i})$$

$$= p(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i})p(z_i = j|\mathbf{z}_{-i})$$

$$p(z_i = j|\mathbf{z}_{-i}) = \int p(z_i = j|\theta^{(d)})p(\theta^{(d)}|\mathbf{z}_{-i})d\theta^{(d)}$$

$$p(\theta^{(d)}|\mathbf{z}_{-i}) \propto p(\mathbf{z}_{-i}|\theta^{(d)})p(\theta^{(d)}) \sim Dirichlet(n^{(d)}_{-i,j} + \alpha)$$

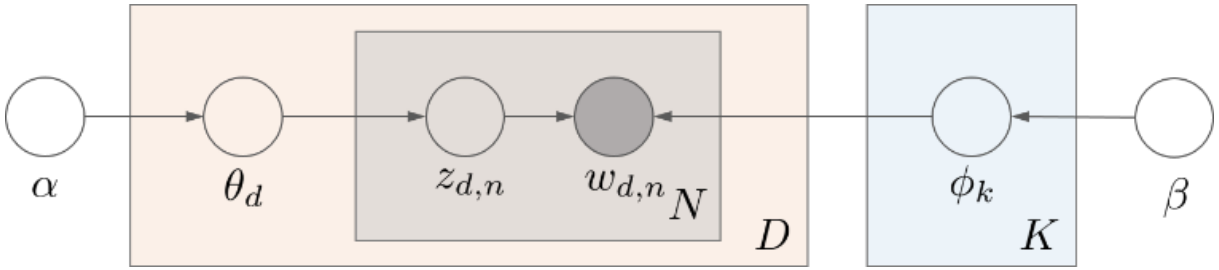
✓ where $n^{(d)}_{-i,j}$ is the number of words assigned to topic j excluding current one in the document d.

$$p(z_i = j|\mathbf{z}_{-i}) = \frac{n^{(d)}_{-i,j} + \alpha}{n^{(d)}_{-i,\cdot} + K\alpha}$$

LDA Inference: Collapsed Gibbs Sampling

Tang (2008)

- Gibbs Sampling Equation



$$p(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + V\beta} \times \frac{n_{-i,j}^{(d)} + \alpha}{n_{-i,\cdot}^{(d)} + K\alpha}$$

✓ Need to record four count variables

- Document-topic count $n_{-i,j}^{(d)}$
- Document-topic sum $n_{-i,\cdot}^{(d)}$
- Topic-term count $n_{-i,j}^{(w_i)}$
- Topic-term sum $n_{-i,j}^{(\cdot)}$

LDA Inference: Collapsed Gibbs Sampling

Tang (2008)

- Parameter Estimation
  - ✓ To obtain θ and ϕ, two ways are possible (draw one sample of z or draw multiple samples of z to calculate the average)

$$\phi_{j,w} = \frac{n_w^{(j)} + \beta}{\sum_{w=1}^{V} n_w^{(j)} + V\beta} \qquad \theta_j^{(d)} = \frac{n_j^{(d)} + \alpha}{\sum_{z=1}^{K} n_z^{(d)} + K\alpha}$$

  - ✓ where $n_w^{(j)}$ is the frequency of word assigned to topic j, and $n_z^{(d)}$ is the number of word assigned to topic z in the document d

LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Gibbs Sampling Equation: Another Form

$$p(z_i = j|\mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n^{(w_i)}_{-i,j} + \beta}{n^{(\cdot)}_{-i,j} + V\beta} \times \frac{n^{(d)}_{-i,j} + \alpha}{n^{(d)}_{-i,\cdot} + K\alpha}$$

$$\frac{n_{d,k} + \alpha_k}{\sum_{i=1}^{K}(n_{d,i} + \alpha_i)} \times \frac{v_{k,w_{d,n}} + \beta_{w_{d,n}}}{\sum_{j=1}^{V}(v_{k,j} + \beta_j)}$$

- ✓ Number of times document d uses topic k
- ✓ Number of times topic k uses word type $w_{d,n}$
- ✓ Dirichlet parameter for document topic distribution
- ✓ Dirichlet parameter for topic to word distribution
- ✓ How much this document likes topic k
- ✓ How much this topic likes word $w_{d,n}$

LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Gibbs Sampling Equation: Another Form

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$$\frac{n_{d,k} + \alpha_k}{\sum_{i=1}^{K}(n_{d,i} + \alpha_i)} \times \frac{v_{k,w_{d,n}} + \beta_{w_{d,n}}}{\sum_{j=1}^{V}(v_{k,j} + \beta_j)}$$

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LDA Inference: Collapsed Gibbs Sampling

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$$\frac{n_{d,k} + \alpha_k}{\sum_{i=1}^{K}(n_{d,i} + \alpha_i)} \times \frac{v_{k,w_{d,n}} + \beta_{w_{d,n}}}{\sum_{j=1}^{V}(v_{k,j} + \beta_j)}$$

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- ✓ How much this document likes topic k
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LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Gibbs Sampling Equation: Another Form

$$p(z_i = j|\mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n^{(w_i)}_{-i,j} + \beta}{n^{(\cdot)}_{-i,j} + V\beta} \times \frac{n^{(d)}_{-i,j} + \alpha}{n^{(d)}_{-i,\cdot} + K\alpha}$$

$$\frac{n_{d,k} + \alpha_k}{\sum_{i=1}^{K}(n_{d,i} + \alpha_i)} \times \frac{v_{k,w_{d,n}} + \beta_{w_{d,n}}}{\sum_{j=1}^{V}(v_{k,j} + \beta_j)}$$

- ✓ Number of times document d uses topic k
- ✓ Number of times topic k uses word type $w_{d,n}$
- ✓ Dirichlet parameter for document topic distribution
- ✓ Dirichlet parameter for topic to word distribution
- ✓ How much this document likes topic k
- ✓ How much this topic likes word $w_{d,n}$

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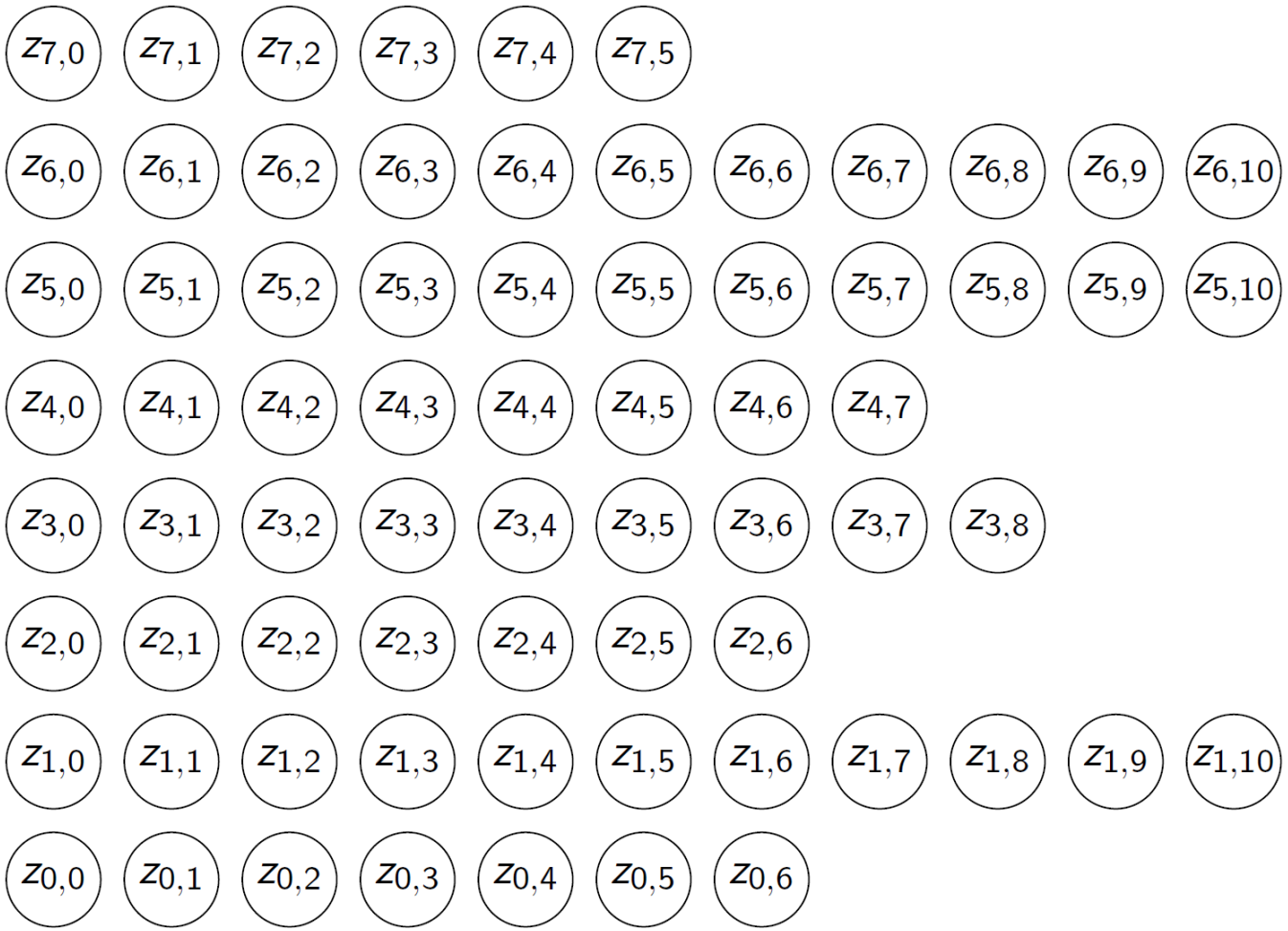
LDA Inference: Collapsed Gibbs Sampling

Speh et al. (2013)

- Collapsed Gibbs Sampling
  - ✓ Illustrative procedure



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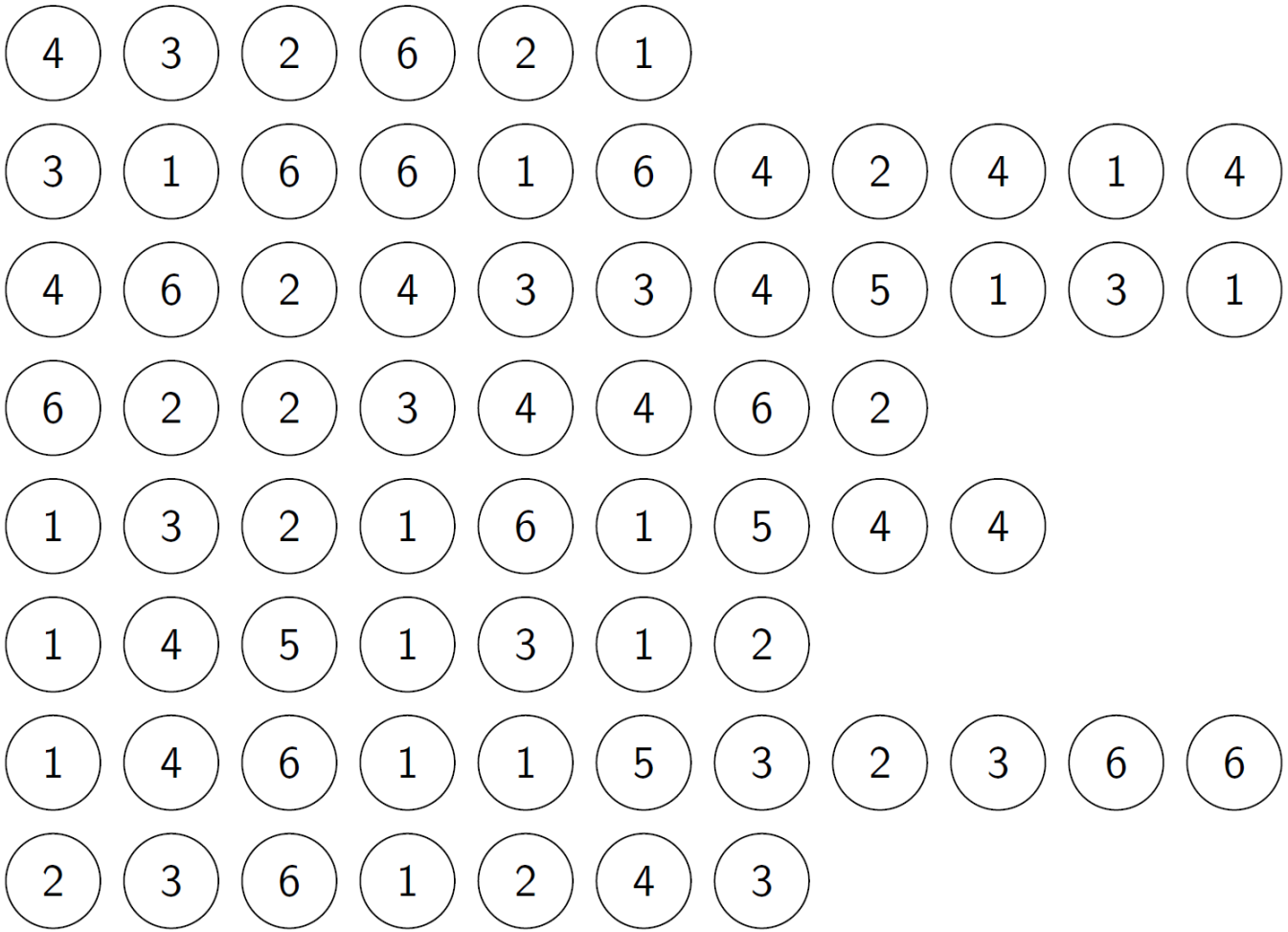

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Speh et al. (2013)

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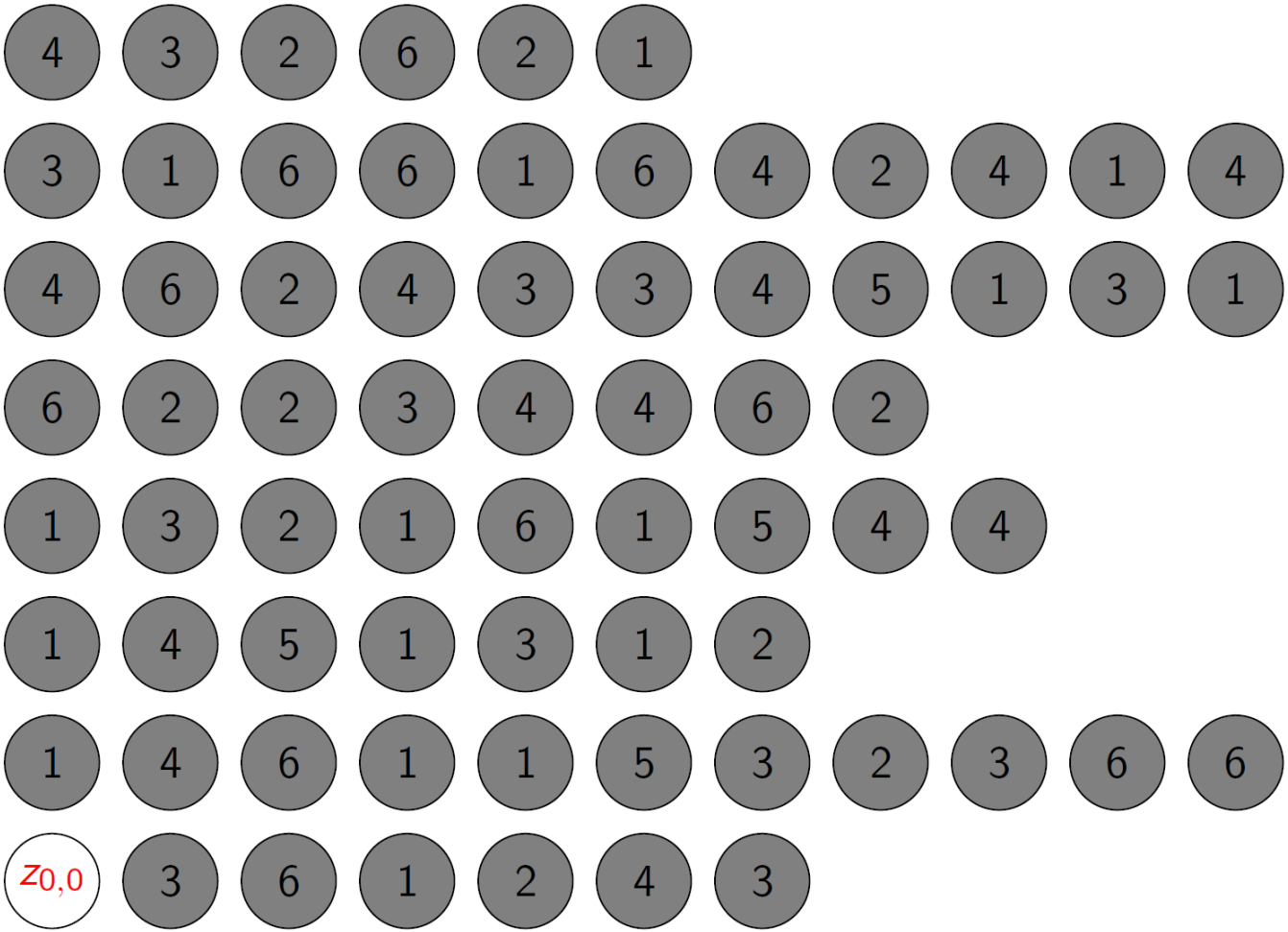

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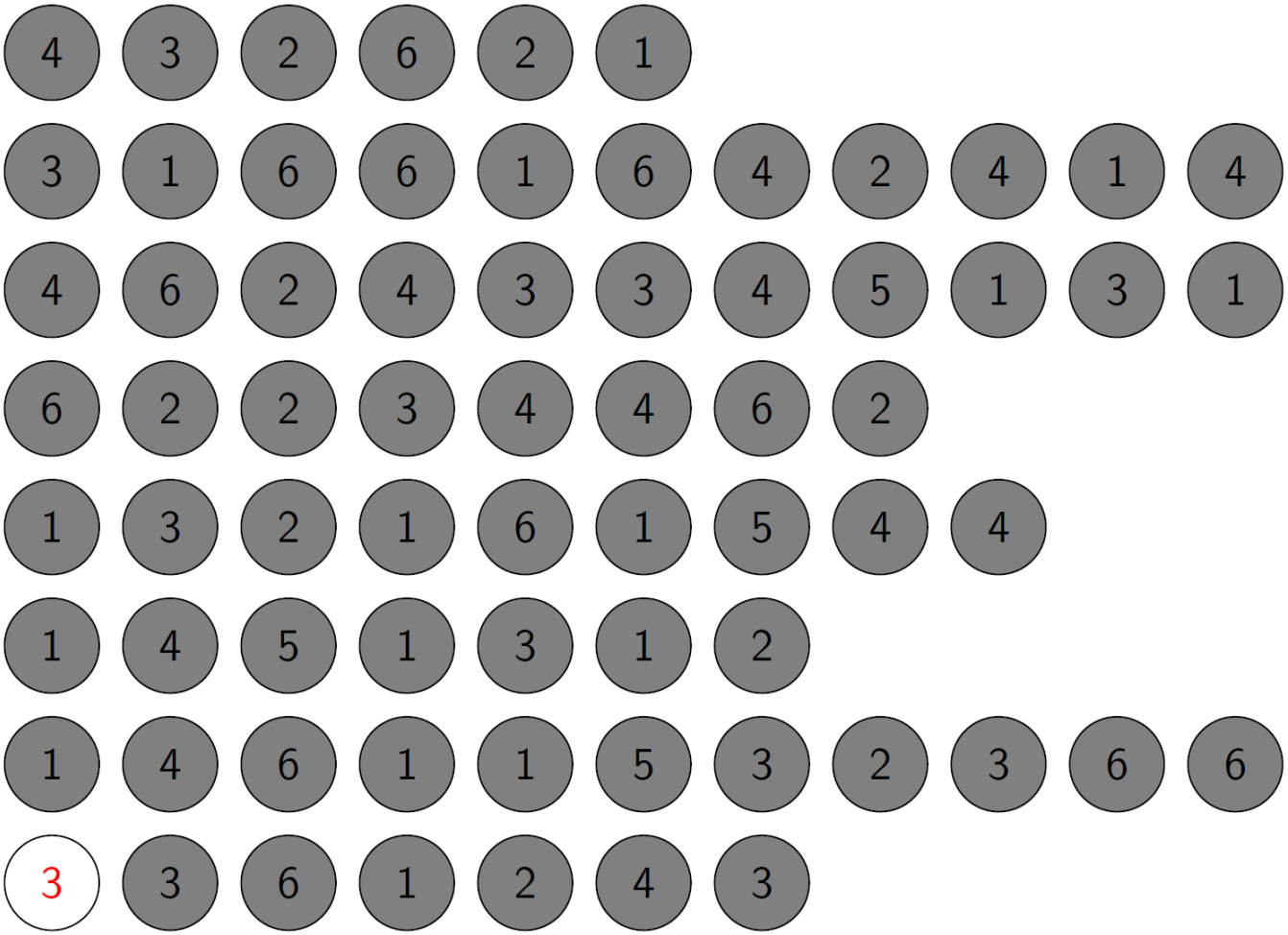
  - ✓ Illustrative procedure



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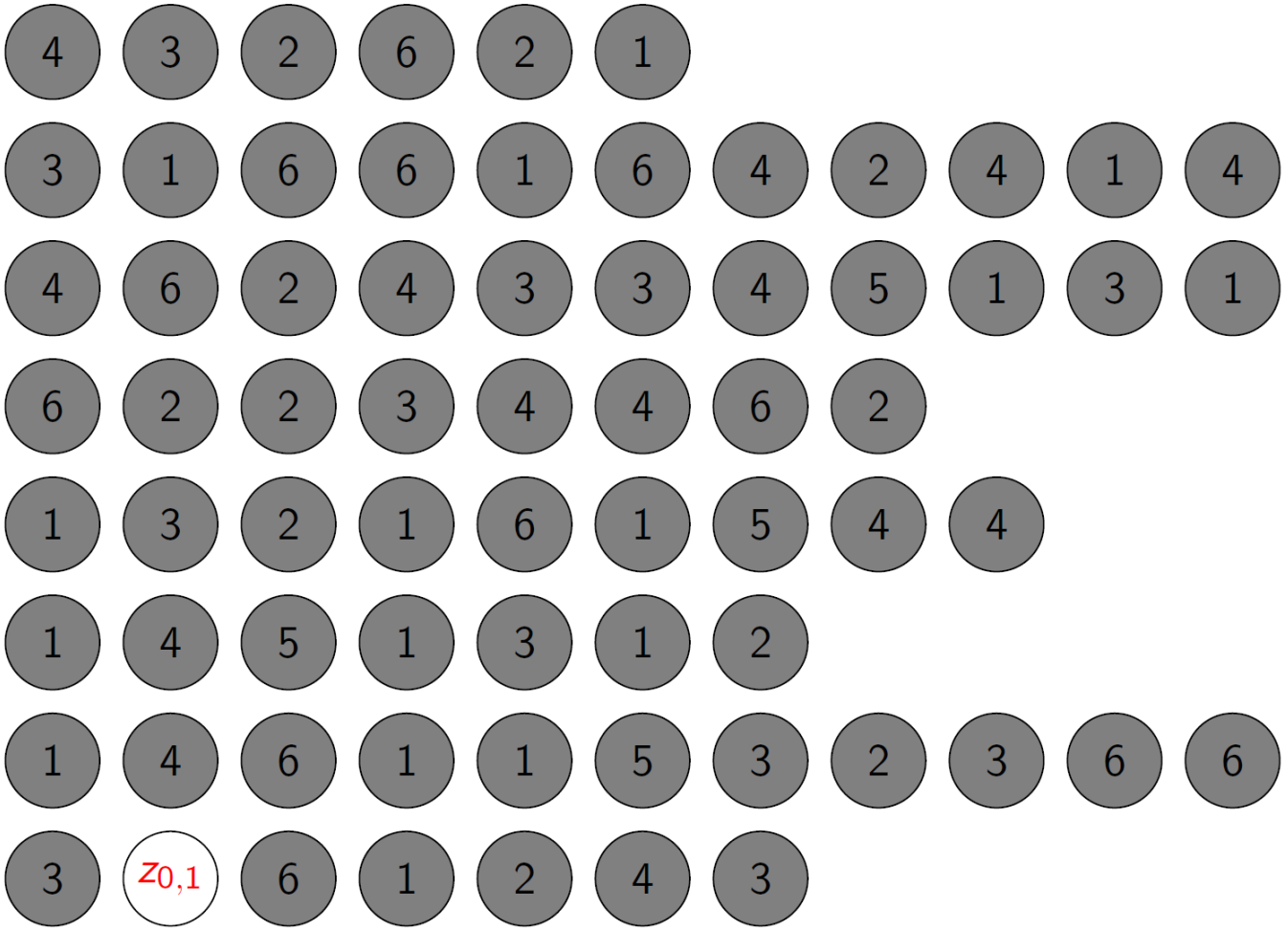
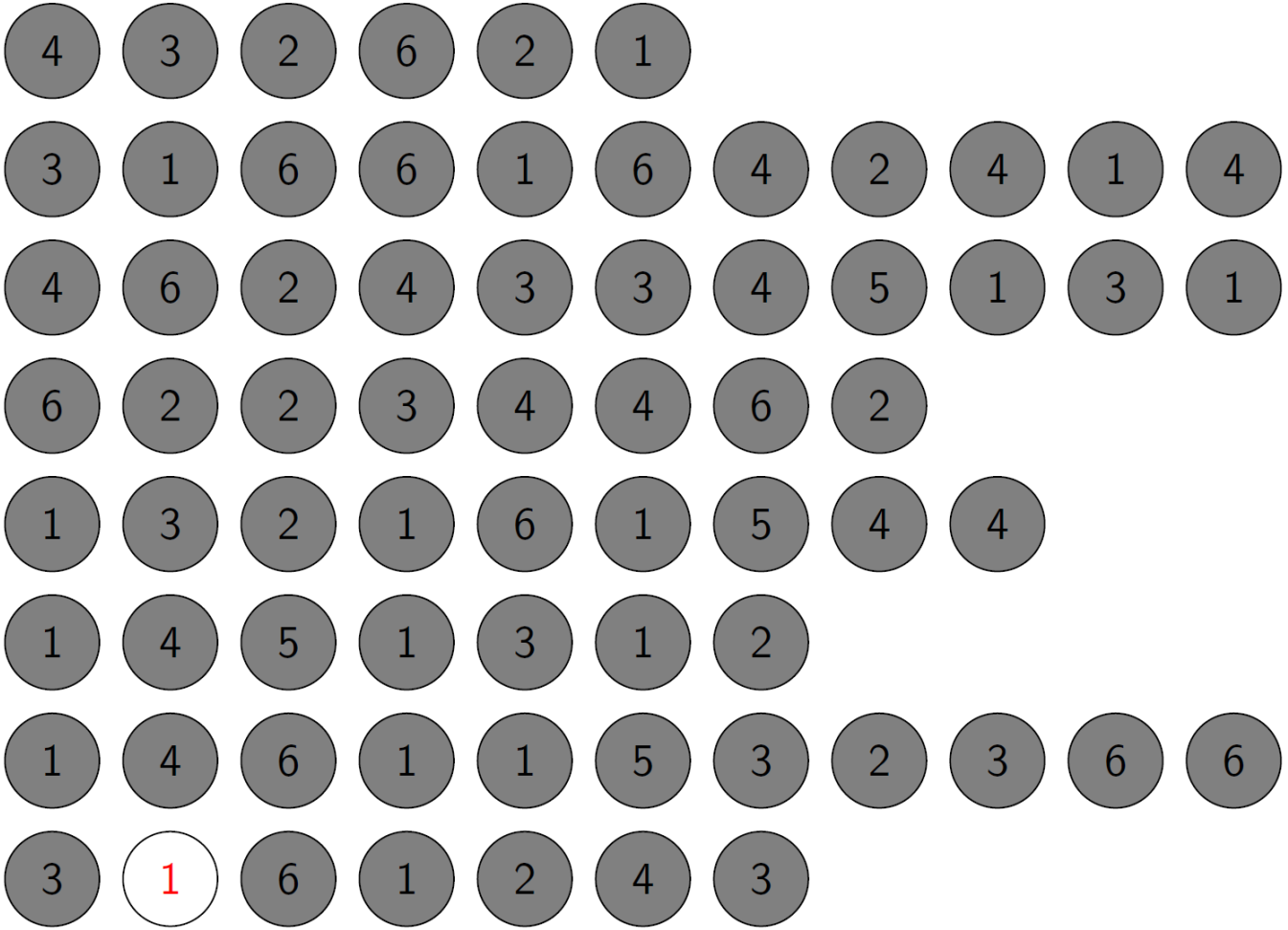


LDA Inference: Collapsed Gibbs Sampling

Speh et al. (2013)

- Collapsed Gibbs Sampling
  - ✓ Illustrative procedure



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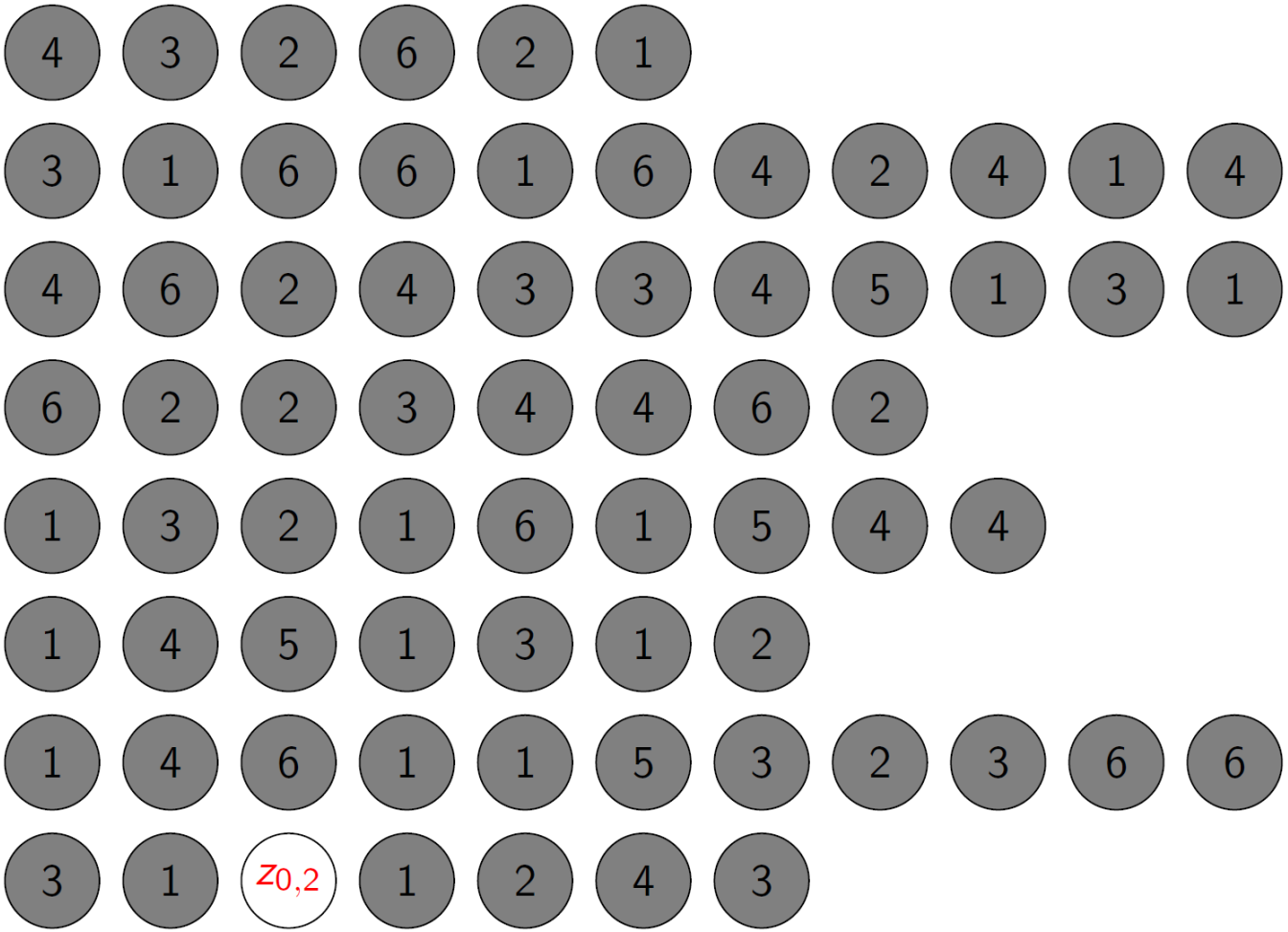
  - ✓ Illustrative procedure

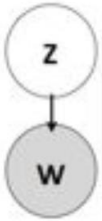


LDA Inference: Collapsed Gibbs Sampling

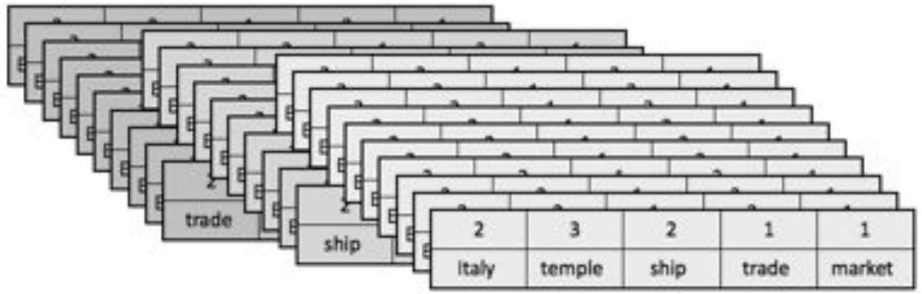
Boyd-Graber (2014)

- Randomly assign topics

Etruscan	trade	price	temple	market



3	2	1	3	1
Etruscan	trade	price	temple	market



LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Randomly assign topics

3	2	1	3	1
Etruscan	trade	price	temple	market

Total counts from all docs →

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	8	1
...			

$$\frac{n_{d,k} + \alpha_k}{\sum_{i=1}^{K}(n_{d,i} + \alpha_i)} \times \frac{v_{k,w_{d,n}} + \beta_{w_{d,n}}}{\sum_{j=1}^{V}(v_{k,j} + \beta_j)}$$

LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Sampling

3	**2**	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	**8**	1
...			

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	**7**	1
...			

- What is the conditional distribution for this topic
  - ✓ Part 1: How much does this document like each topic?
  - ✓ Part 2: How much does each topic like the word?

LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- What is the conditional distribution for this topic
  - ✓ Part 1: How much does this document like each topic?

$$\frac{n_{d,k} + \alpha_k}{\sum_{i=1}^{K}(n_{d,i} + \alpha_i)} \times \frac{v_{k,w_{d,n}} + \beta_{w_{d,n}}}{\sum_{j=1}^{V}(v_{k,j} + \beta_j)}$$

3	?	1	3	1
Etruscan	trade	price	temple	market

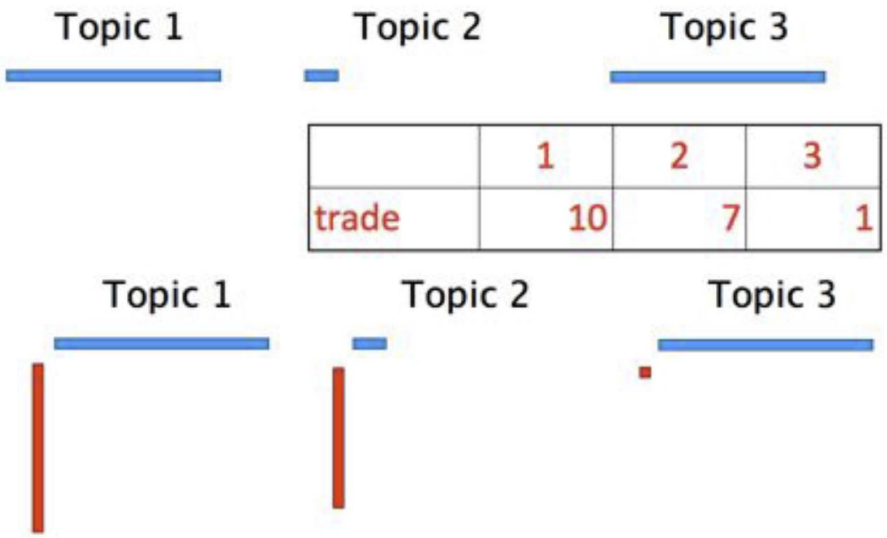
LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- What is the conditional distribution for this topic
  - ✓ Part 2: How much does each topic like the word?

$$\frac{n_{d,k} + \alpha_k}{\sum_{i=1}^{K}(n_{d,i} + \alpha_i)} \times \frac{v_{k,w_{d,n}} + \beta_{w_{d,n}}}{\sum_{j=1}^{V}(v_{k,j} + \beta_j)}$$

3	?	1	3	1
Etruscan	trade	price	temple	market

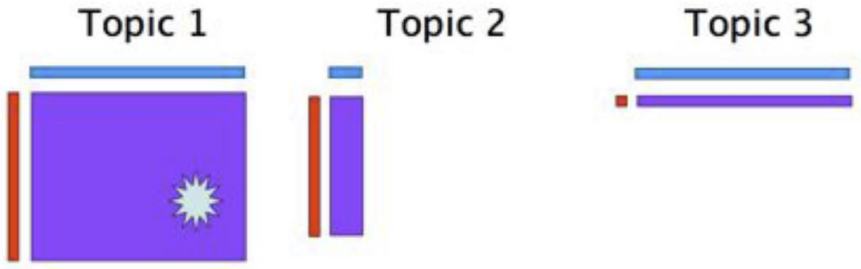


LDA Inference: Collapsed Gibbs Sampling

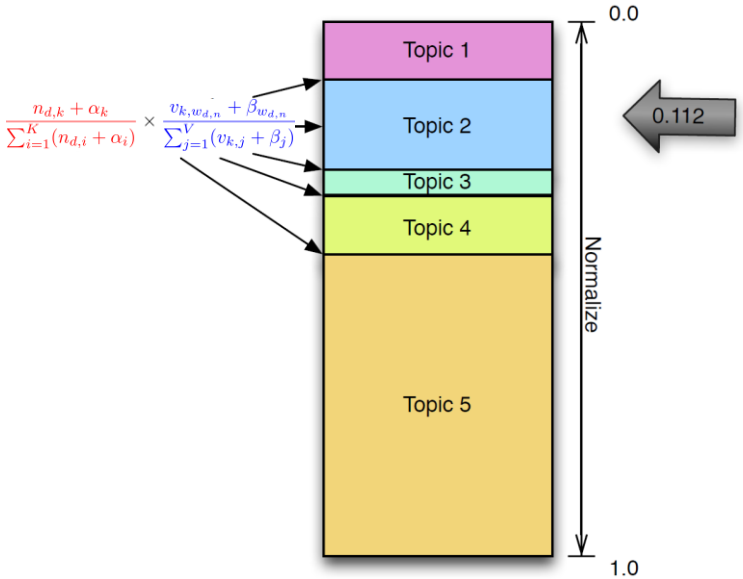
Boyd-Graber (2014)

- What is the conditional distribution for this topic
  - ✓ Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market



Normalization



LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Update count

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	**10**	7	1
...			

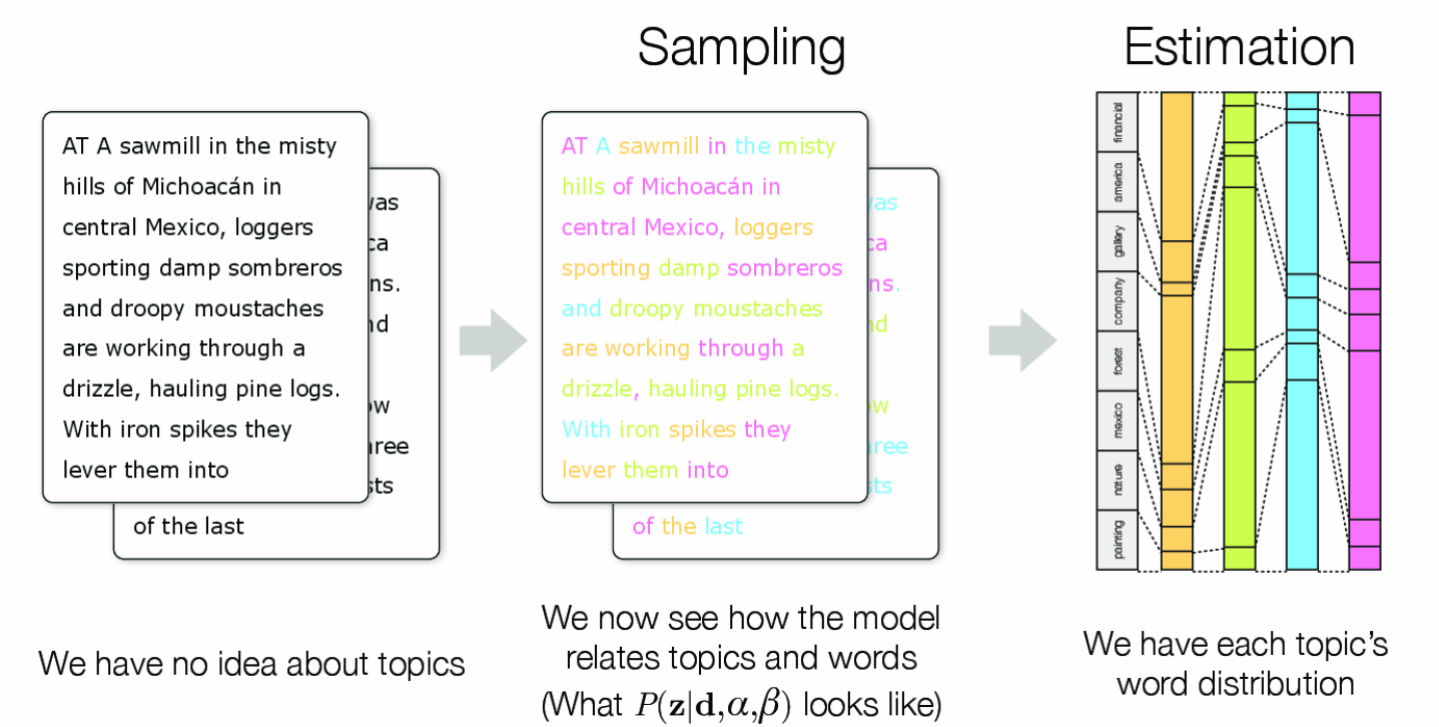
3	**1**	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	**11**	7	1
...			

LDA Inference: Collapsed Gibbs Sampling

Boyd-Graber (2014)

- Gibbs Sampling



AGENDA

LDA Evaluation & Model Selection

Qui et al. (2014)

- How many topics are optimal?
  - ✓ Split the data into training and test data sets
    - Log-likelihood for Gibbs sampling

$$\log(p(w|z)) = k \log\left(\frac{\Gamma(V\delta)}{\Gamma(\delta)^V}\right) + \sum_{K=1}^{k}\left\{\left[\sum_{j=1}^{V}\log(\Gamma(n_K^{(j)}+\delta))\right] - \log(\Gamma(n_K^{(\cdot)}+V\delta))\right\}$$

    - Perplexity (the geometric mean per-word likelihood) is often used

$$\text{Perplexity}(w) = \exp\left\{-\frac{\log(p(w))}{\sum_{d=1}^{D}\sum_{j=1}^{V} n^{(jd)}}\right\} \qquad \log(p(w)) = \sum_{d=1}^{D}\sum_{j=1}^{V} n^{(jd)} \log\left[\sum_{K=1}^{k}\theta_K^{(d)}\beta_K^{(j)}\right]$$

    - Topic weights is determined for the new data (hold0ut data set) using Gibbs sampling
    - Term distributions for topics are kept fixed from the training corpus

LDA Evaluation & Model Selection

- Model Selection based on Perplexity

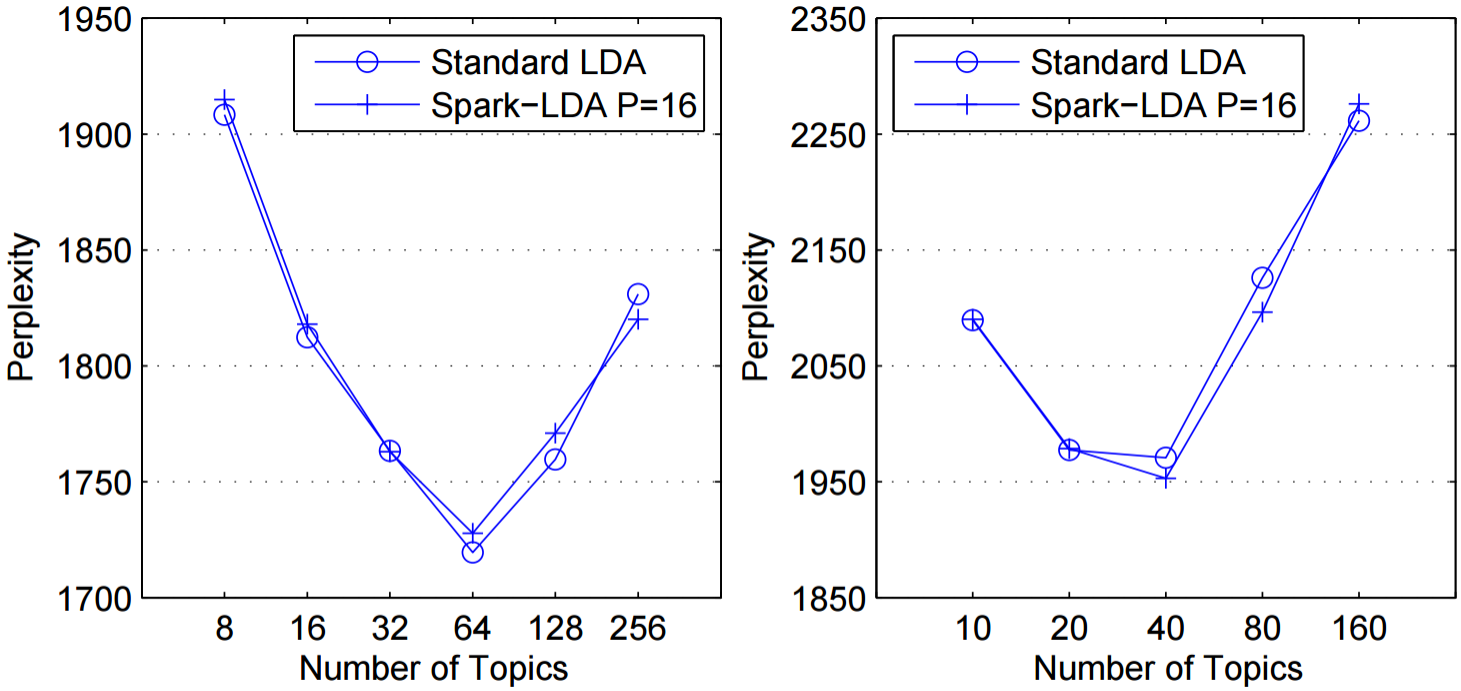
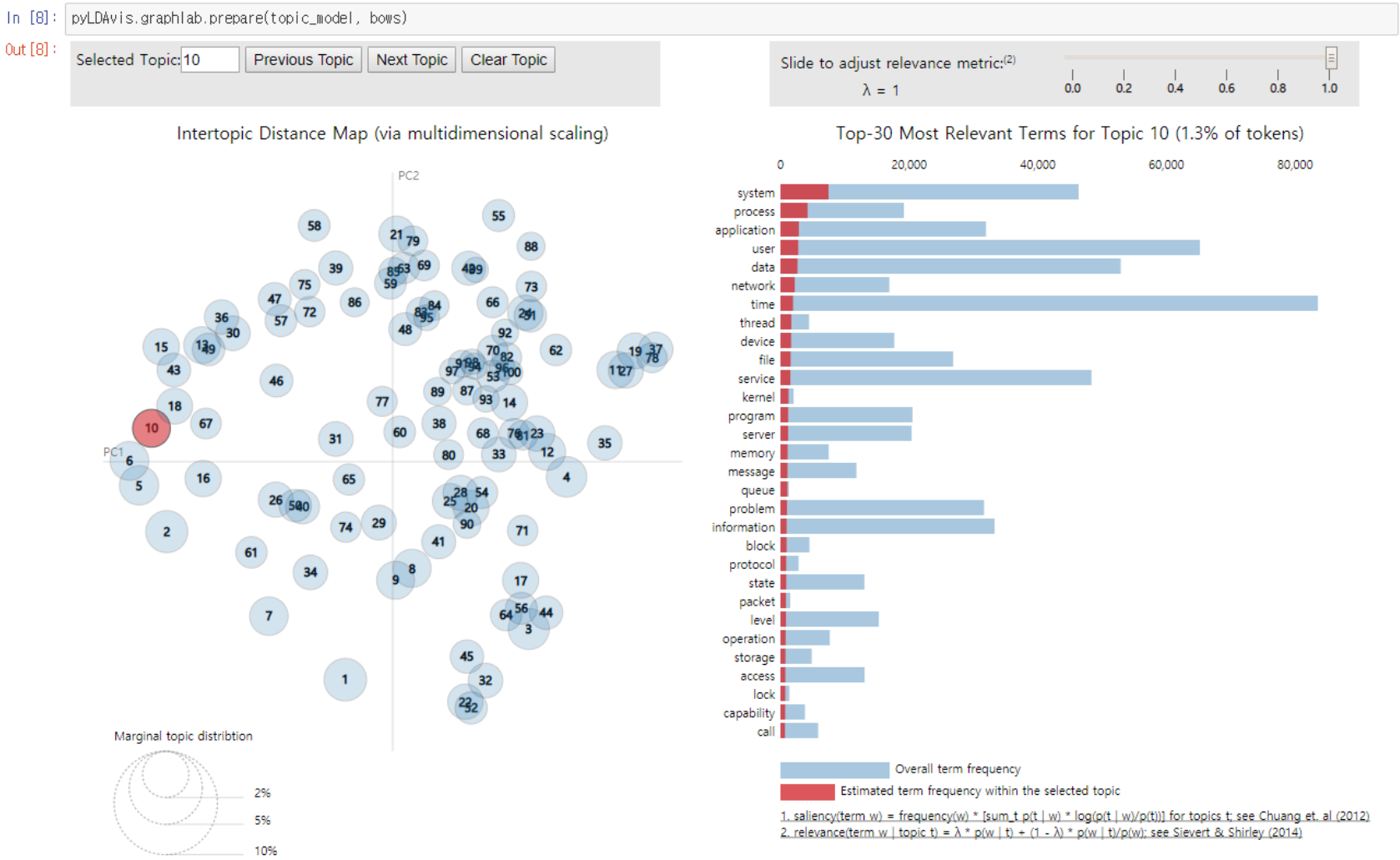


Figure 5: Test set perplexity versus number of topics K for KOS (left) and NIPS (right).

LDA Visualization

- LDAviz




ANY
questions?

AGENDA

01 Topic Modeling

02 Probabilistic Latent Semantic Analysis

03 LDA: Document Generation Process

04 LDA Inference: Gibbs Sampling

05 LDA Evaluation

References

Research Papers

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Other Materials

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