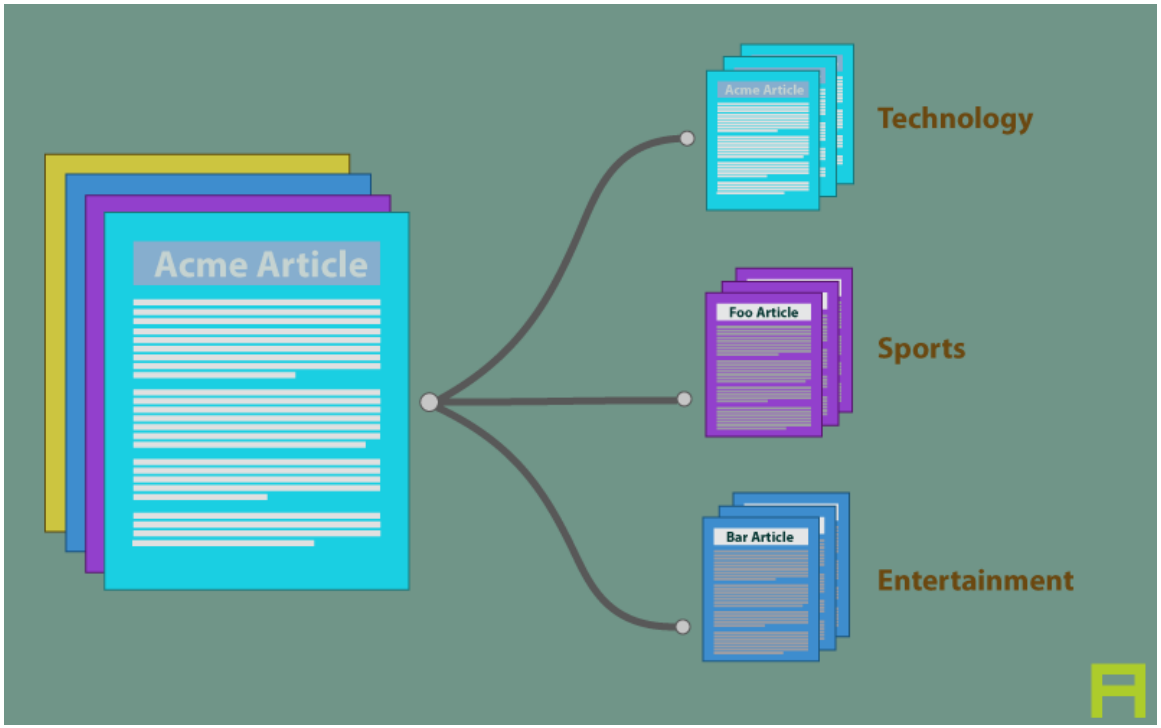

Lecture 10: Document Classification I

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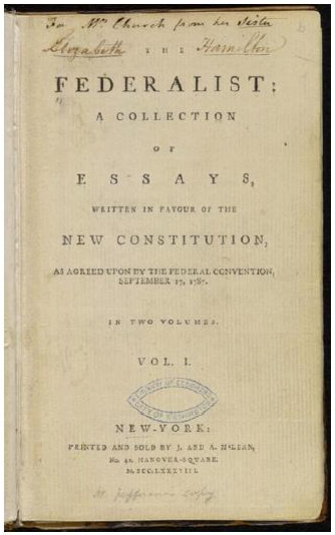
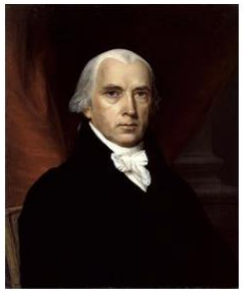
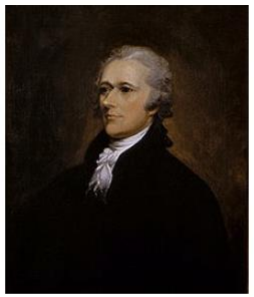
AGENDA

Document Classification: Examples

- Who wrote which federalist papers?
  - ✓ 1787-8: anonymous essays try to convince New York to ratify U.S Constitution: Jay, Madison, Hamilton
  - ✓ Authorship of 12 of the letters in dispute
  - ✓ 1963: solved by Mosteller and Wallace using Bayesian methods


FEDERALIST:
A COLLECTION
OF
ESSAYS,
WRITTEN IN FAVOUR OF THE
NEW CONSTITUTION,
IN TWO VOLUMES.
VOL. I.
NEW-YORK:


James Madison


Alexander Hamilton

Document Classification: Examples

- Positive or Negative reviews?

Tied for the best movie I have ever seen
★★★★★★★★★★
Author: carflo from Texas

A classic piece of unforgettable film-making.
★★★★★★★★★★
Author: Justin M (kaspen12) from Vancouver, Canada

Simply amazing. The best film of the 90's.
★★★★★★★★★★
Author: Thomas Peluso (tpeluso@gmail.com) from Long Island, NY

The best story ever told on film
★★★★★★★½☆☆
Author: Si Cole

It lacks surprises or excitement.
★★★★☆☆☆☆☆☆
Author: Comeuppance Reviews from United States Minor Outlying Islands
3 December 2014

The Last Samurai: Hacked to Death
★☆☆☆☆☆☆☆☆☆
Author: Jon-Jokerjon from County Durham

Document Classification: Examples

- Spam or not?

From: "" <takworlld@hotmail.com>
Subject: real estate is the only way... gem oalvgkay

Anyone can buy real estate with no money down

Stop paying rent TODAY !

There is no need to spend hundreds or even thousands for similar courses

I am 22 years old and I have already purchased 6 properties using the methods outlined in this truly INCREDIBLE ebook.

Change your life NOW !

==========
Click Below to order:
http://www.wholesaledaily.com/sales/nmd.htm
==========

Document Classification: Examples

- What is the subject of this article?

MEDLINE Article

Syntactic frame and verb bias in aphasia: Plausibility judgments of undergoer-subject sentences

?

MeSH Subject Category Hierarchy

- Antogonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- …

Document Classification: Applications

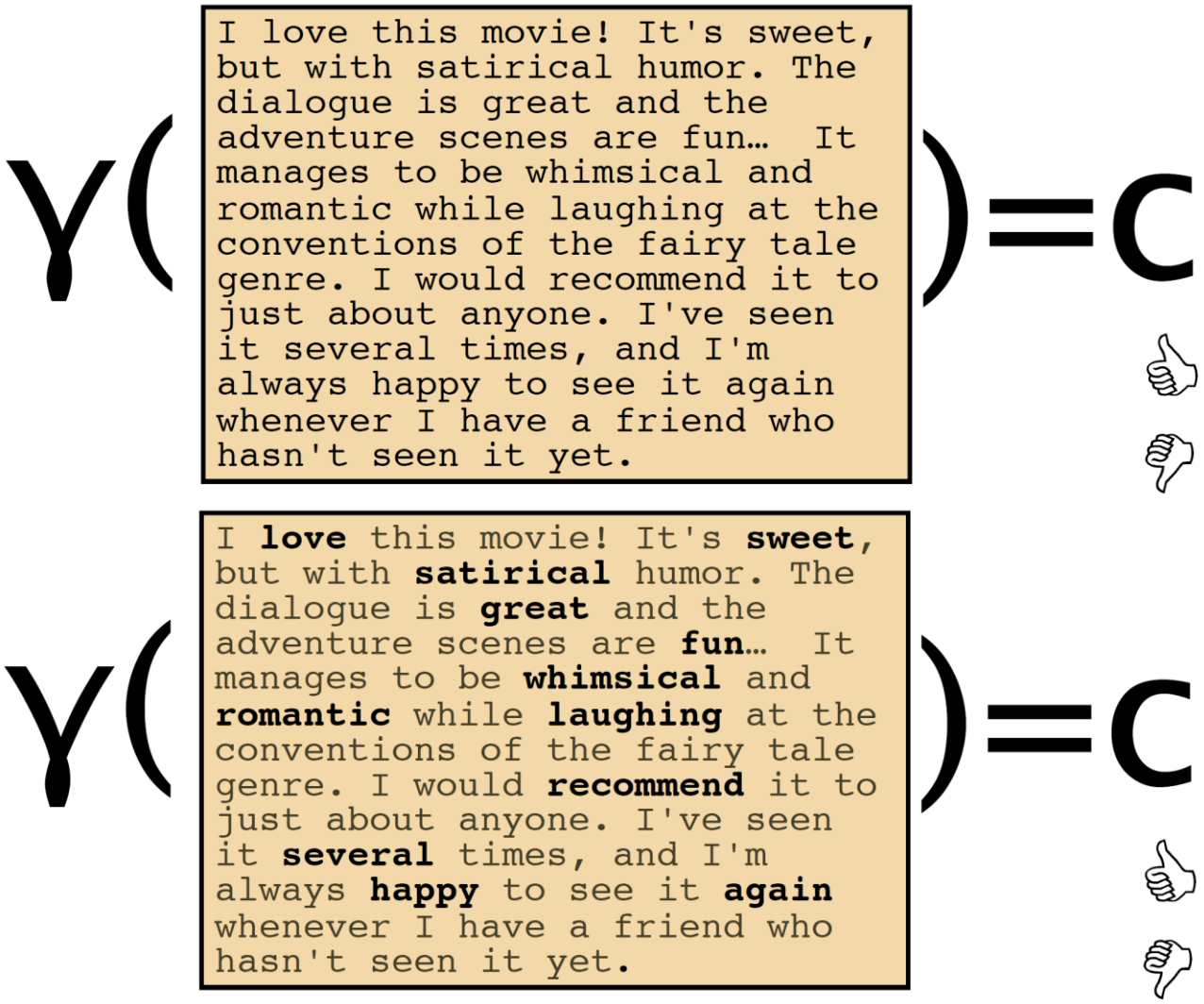
- Document Classification/Categorization
  - ✓ Assigning subject categories, topics, or genres
  - ✓ Spam detection
  - ✓ Authorship identification
  - ✓ Age/gender identification
  - ✓ Language identification
  - ✓ Sentiment analysis
  - ✓ …

Document Classification: Definition

- Document Classification
  - ✓ Input (I)
    - A document d
    - A fixed set of classes C = $\{c_1, c_2, \ldots, c_j\}$
  - ✓ Output (O)
    - A predicted class c in C
- Supervised Machine Learning
  - ✓ Input: I + a training set of m hand-labeled documents: $(d_1, c_1), \ldots, (d_m, c_m)$
  - ✓ Output: a learned classifier y: f(d) = c

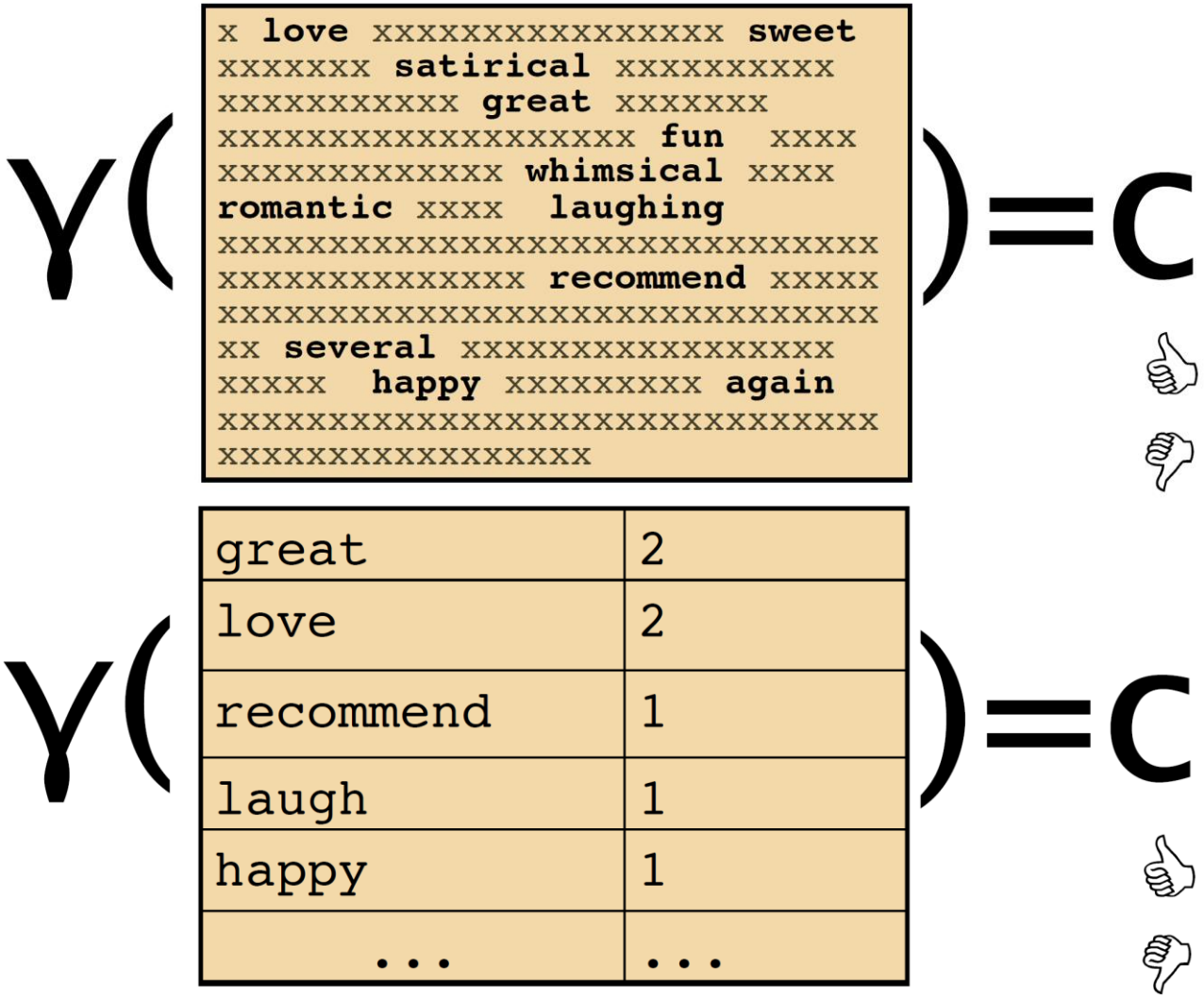
Document Classification: Definition

- Supervised Machine Learning with Bag-of-Words representation



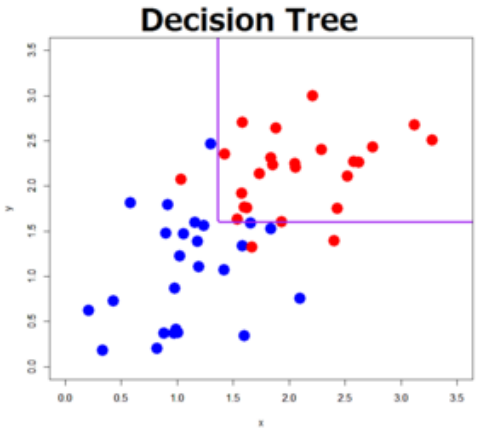
Document Classification: Definition

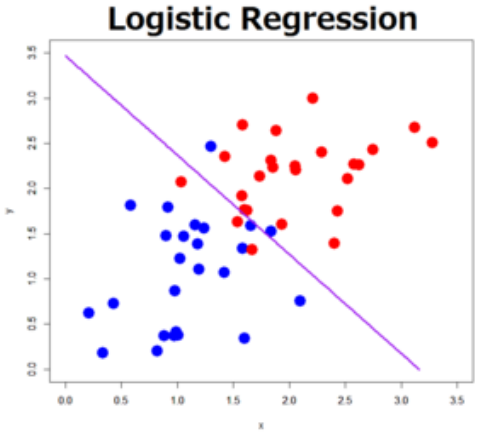
- Supervised Machine Learning with Bag-of-Words representation

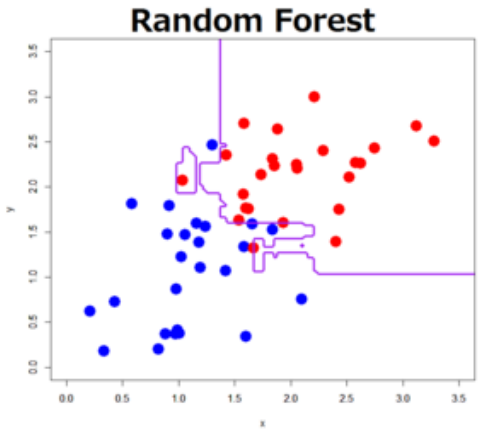


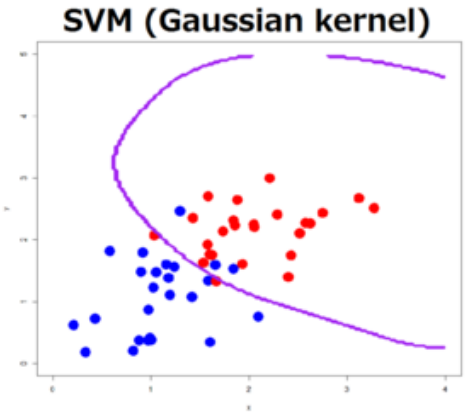
Why Are There So Many Classifiers?

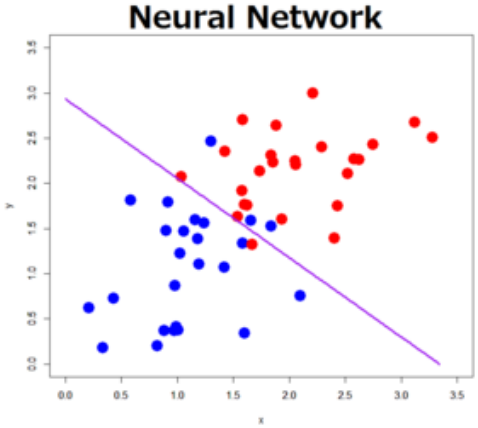
- We cannot guarantee that a single classifier is always better than the others











AGENDA

Naive Bayesian Classifier

- Baye's Rule (one of the most important rules in statistics)

$$P(C_i|x_1, x_2) = \frac{P(x_1, x_2|C_i) \cdot P(C_i)}{P(x_1, x_2)}$$

- Naive: Let's assume that all variables are statistically independent to each other

$$= \frac{P(x_1|C_i) \cdot P(x_2|C_i) \cdot P(C_i)}{P(x_1, x_2)}$$

Naive Bayesian Classifier

- Maximum likelihood estimates
  - ✓ Simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{N.Doc(C = c_j)}{Total\ number\ of\ documents}$$

$$\hat{P}(w_i|c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

Fraction of times word w_i appears among all words in documents of class c_j

- Limitation
  - ✓ What if we have seen no training document with the word "fantastic" and classified in in the class "positive"?

$$\hat{P}(\text{fantastic}|\text{positive}) = \frac{count(\text{fantastic}, \text{positive})}{\sum_{w \in V} count(w, \text{positive})} = 0$$

  - ✓ Zero probabilities cannot be conditioned away, no matter the other evidence!

Naive Bayesian Classifier

- Example: Classifying movie reviews into Positive/Negative class
  - ✓ Review 1: This movie was awesome! I really enjoyed it.
  - ✓ Review 2: This movie was boring and waste of time.
- Step 1: Estimate the conditional probabilities for each class

Words	P(Word/Positive)	P(Word/Negative)
This	0.1	0.1
Movie	0.1	0.1
Was	0.1	0.1
Awesome	0.4	0.01
I	0.2	0.2
Really	0.3	0.05
enjoyed	0.5	0.05
It	0.1	0.1
Boring	0.02	0.3
And	0.1	0.1
Waste	0.02	0.35
Of	0.02	0.02
Time	0.15	0.15

Naive Bayesian Classifier

- For the Review 1
  - ✓ Review 1: This movie was awesome! I really enjoyed it.

$$\prod_i P(Word_i|Pos) = 120 \times 10^{-8} > \prod_i P(Word_i|Neg) = 0.5 \times 10^{-8}$$

Words	P(Word/Positive)	P(Word/Negative)
This	0.1	0.1
Movie	0.1	0.1
Was	0.1	0.1
Awesome	0.4	0.01
I	0.2	0.2
Really	0.3	0.05
enjoyed	0.5	0.05
It	0.1	0.1
Boring	0.02	0.3
And	0.1	0.1
Waste	0.02	0.35
Of	0.02	0.02
Time	0.15	0.15

Naive Bayesian Classifier

- For the Review 1
  - ✓ Review 2: This movie was boring and waste of time.

$$\prod_i P(Word_i|Pos) = 0.012 \times 10^{-8} \; < \; \prod_i P(Word_i|Neg) = 3.15 \times 10^{-8}$$

Words	P(Word/Positive)	P(Word/Negative)
This	0.1	0.1
Movie	0.1	0.1
Was	0.1	0.1
Awesome	0.4	0.01
I	0.2	0.2
Really	0.3	0.05
enjoyed	0.5	0.05
It	0.1	0.1
Boring	0.02	0.3
And	0.1	0.1
Waste	0.02	0.35
Of	0.02	0.02
Time	0.15	0.15

Naive Bayesian Classifier

- Smoothing Techniques
  - ✓ Laplace (add-1) smoothing

$$\hat{P}(w_i|c_j) = \frac{count(w_i, c_j) + 1}{\sum_{w \in V}(count(w, c_j) + 1)} = \frac{count(w_i, c_j) + 1}{\sum_{w \in V} count(w, c_j) + |V|}$$

- Example

Model pos	
0.1	I
0.1	love
0.01	this
0.05	fun
0.1	film

Model neg	
0.2	I
0.001	love
0.01	this
0.005	fun
0.1	film

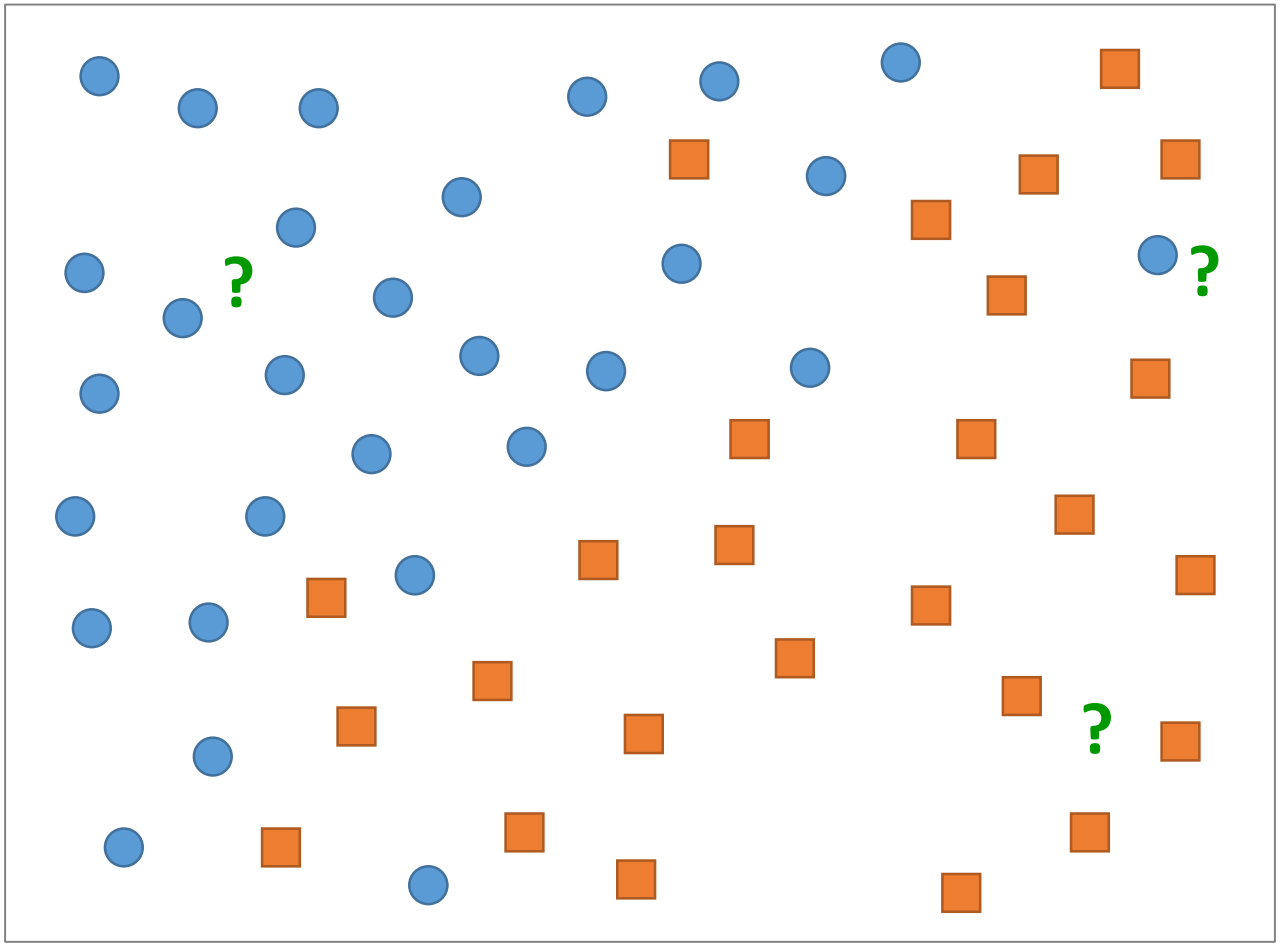
I	love	this	fun	film
0.1	0.1	0.01	0.05	0.1
0.2	0.001	0.01	0.005	0.1

P(s|pos) > P(s|neg)

AGENDA

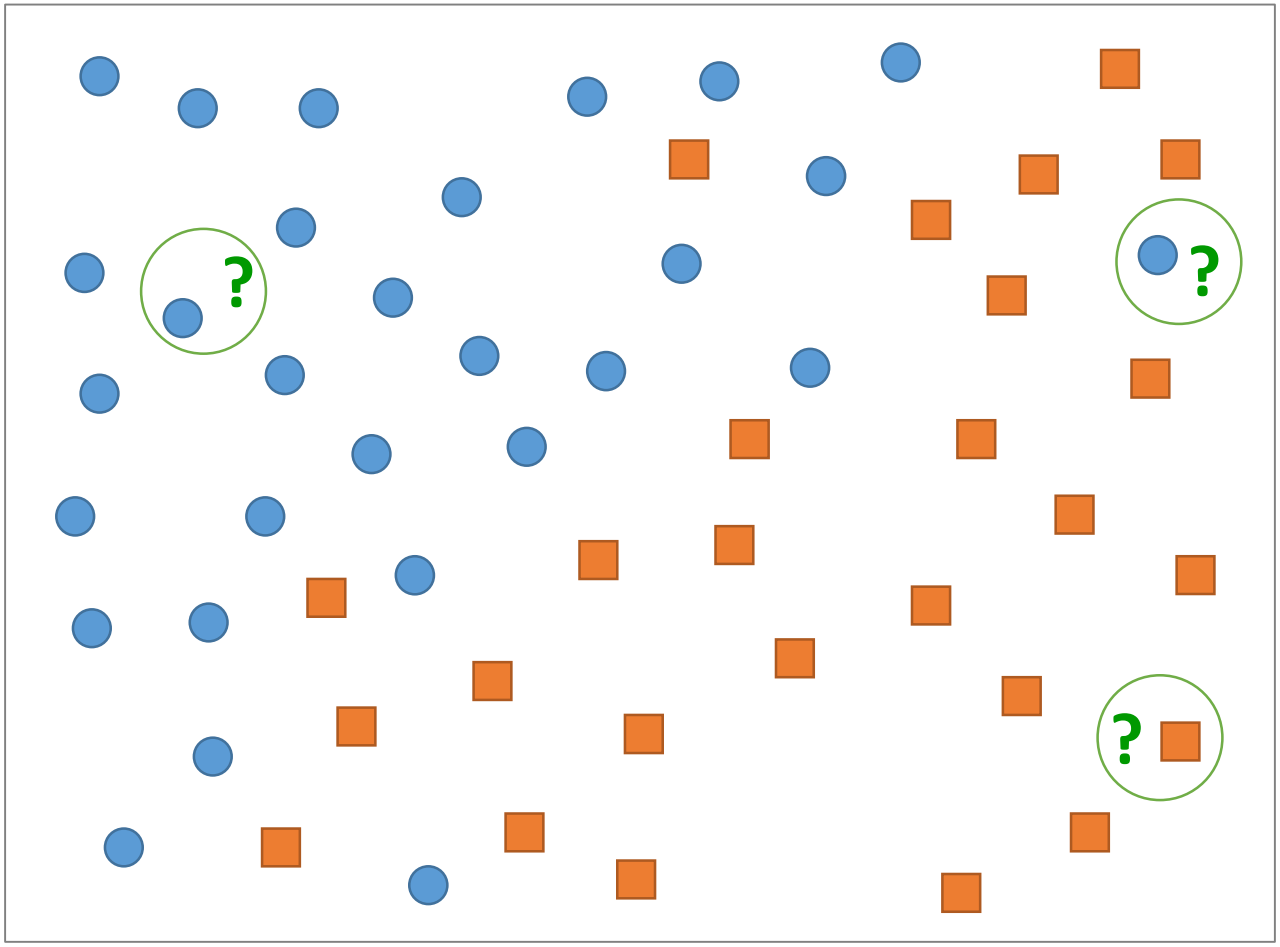
k-Nearest Neighbor Classification

- Which class does the question mark belong to?



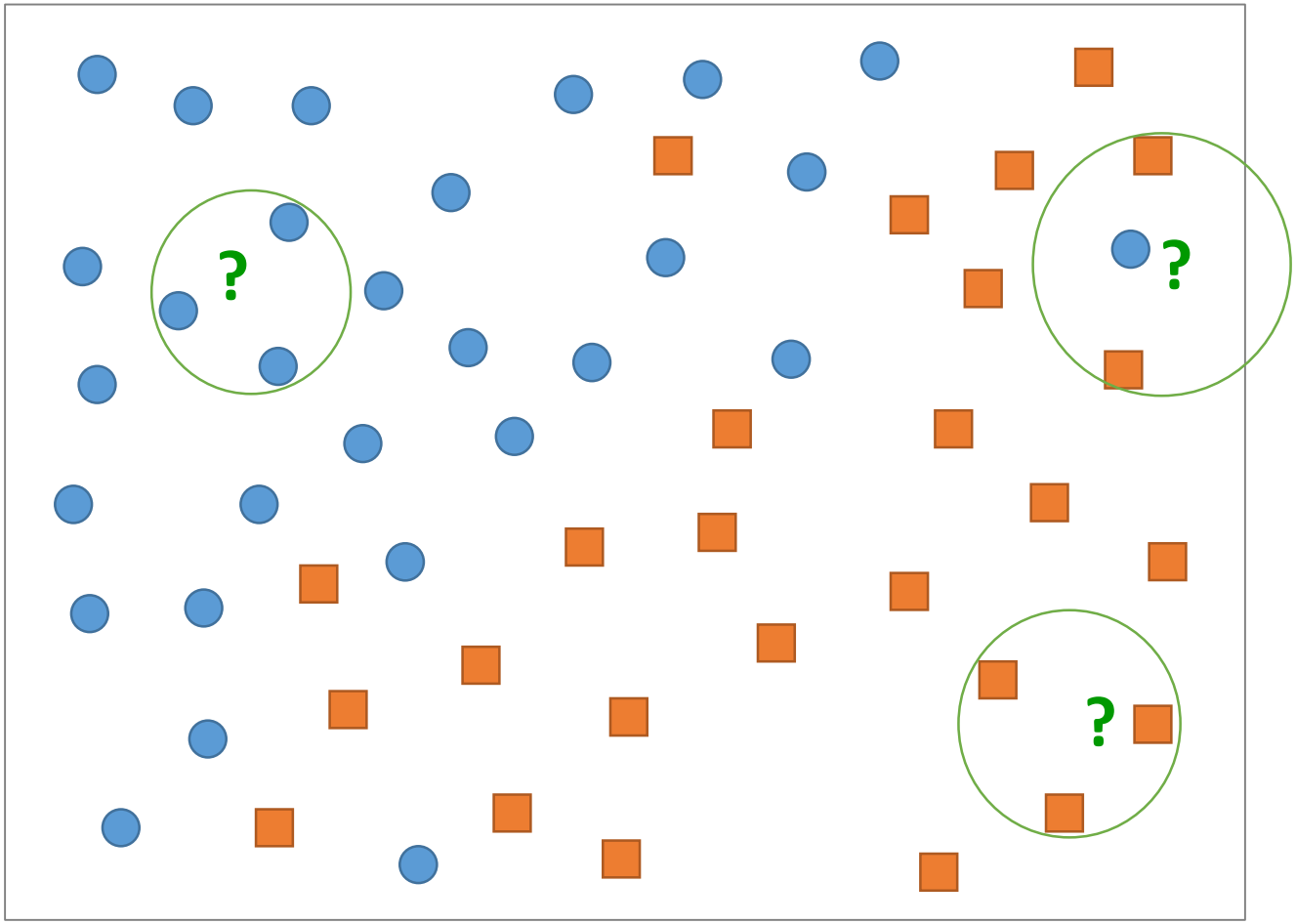
k-Nearest Neighbor Classification

- Which class does the question mark belong to?



k-Nearest Neighbor Classification

- Which class does the question mark belong to?



k-Nearest Neighbor Classification

- Motivation

類類相從　近墨者黑

"Birds of a feather flock together"

k-Nearest Neighbor Classification

k-NN Classification Process

- Step 1: Prepare the reference data
  - ✓ Define attributes
    - BoW representation or distributed representation
  - ✓ Collect sufficient number of records from each class

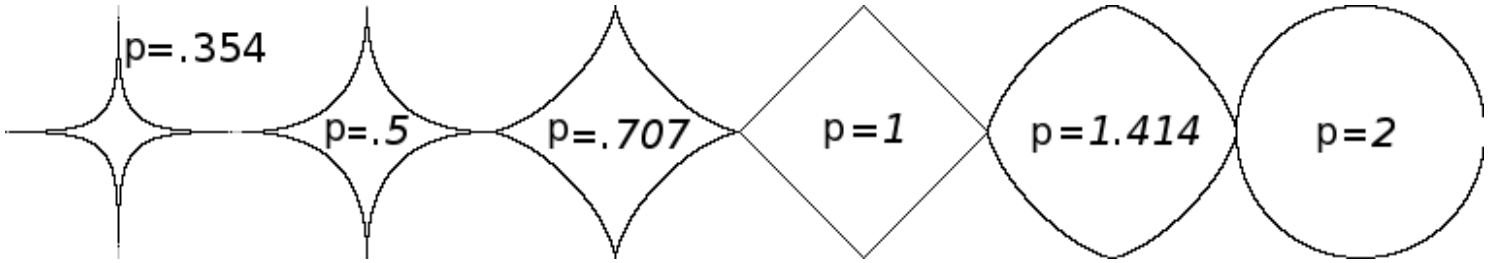
Doc.	Vision	Finance	…	Class
1	2.54	0.15	…	TPAMI
2	1.78	0	…	TPAMI
3	0.12	2.65	…	JoF
4	0.25	3.52	…	JoF
…	…	…	…	…
N	3.12	0.14	…	TPAMI

k-Nearest Neighbor Classification

k-NN Classification Process

- Step 2: Define the similarity measure
  - ✓ Similarity $\propto$ 1/distance
  - ✓ Minkovski distance with order p

$$\text{distance}\left(P=(x_1,x_2,...,x_n),\ Q(y_1,y_2,...,y_n)\right)=\left(\sum_{i=1}^{n}|\ x_i-y_i\ |^p\right)^{\frac{1}{p}}$$



  - ✓ p=2: Euclidean distance
  - ✓ p=1: Mahattan distance

k-Nearest Neighbor Classification

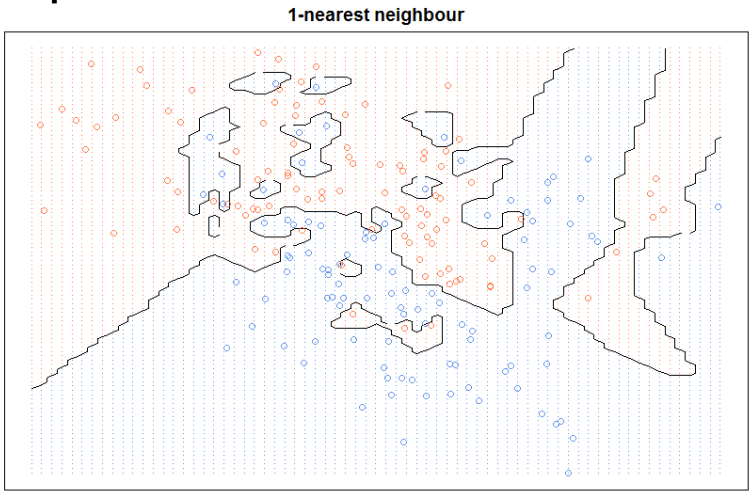
k-NN Classification Process

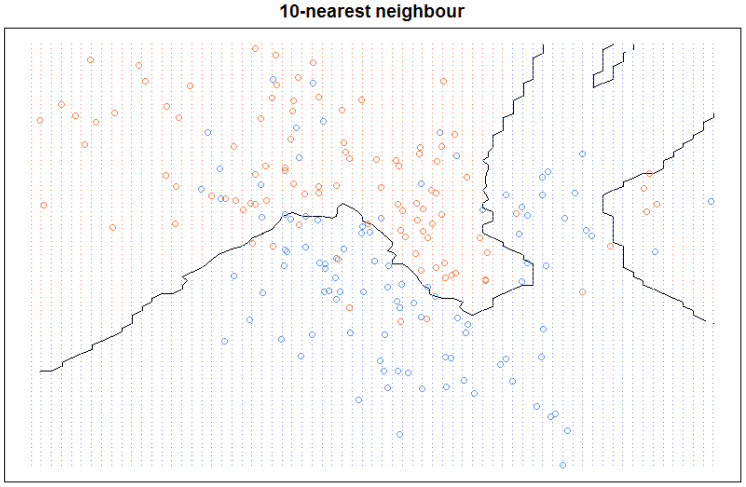
- Step 3: Initialize the set of candidate values for k
  - ✓ If k is too small, then the classification will be highly locally sensitive (over-fitting).
  - ✓ If k is too large, then it will lose the ability to capture the local structure (under-fitting).
  - ✓ A proper k should be chosen among a set of candidates.
  - ✓ Use the validation data.

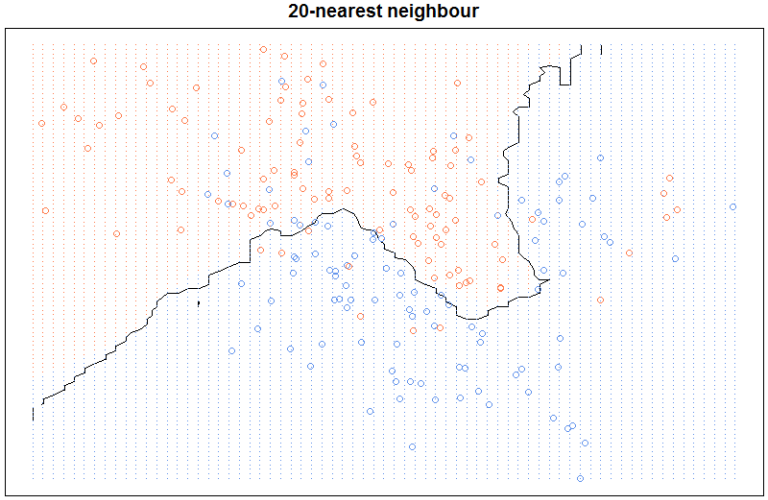
k-Nearest Neighbor Classification

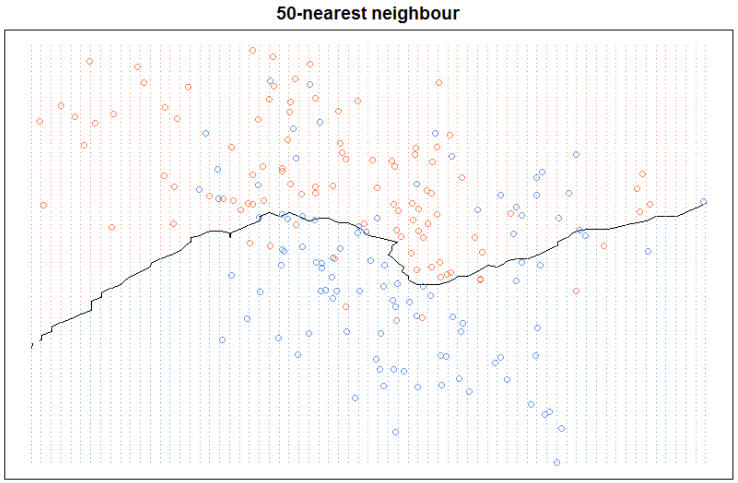
k-NN Classification Process

- Step 3: Initialize the set of candidate values for k









k-Nearest Neighbor Classification

k-NN Classification Process

- Step 4: Determine the combining rule
  - ✓ Majority voting vs. Weighted voting

For a new data **X**

Neighbor	Class	Distance	1/distance	Weight
N1	TPAMI	1	1.00	0.44
N2	JoF	2	0.50	0.22
N3	TPAMI	3	0.33	0.15
N4	JoF	4	0.25	0.11
N5	JoF	5	0.20	0.08

  - ✓ Majority voting: P(X in TPAMI) = 2/5 = 0.4
  - ✓ Weighted voting: P(X in TPAMI) = 0.59
  - ✓ If the cut-off is set to 0.5 X is classified as JoF by the majority voting while classified as TPAMI by the weighted voting

k-Nearest Neighbor Classification

k-NN Classification Process

- Step 5: Find the best k using the validation dataset

Value of k	% Error Training	% Error Validation	
1	0.00	33.33	
2	16.67	33.33	
3	11.11	33.33	
4	22.22	33.33	
5	11.11	33.33	
6	27.78	33.33	
7	22.22	33.33	
8	22.22	16.67	<--- Best k
9	22.22	16.67	
10	22.22	16.67	
11	16.67	33.33	
12	16.67	16.67	
13	11.11	33.33	
14	11.11	16.67	
15	5.56	33.33	
16	16.67	33.33	
17	11.11	33.33	
18	50.00	50.00	

k-Nearest Neighbor Classification

- k-NN Issue 1: Normalization
  - ✓ Normalization or scaling must be done before finding k-nearest neighbors
  - ✓ If not, variables with large measuring units are over-emphasized while variables with small measuring units are under-evaluated

[Before Normalization]

No.	Height	Weight	BFS	Gender
1	187	93	15	M
2	165	51	25	F
3	174	68	14	M
4	156	48	29	F
…	…	…	…	…
N	168	59	12	M
Avg.	165	65	20	-
Stdev.	15	10	5	-

[After Normalization]

No.	Height	Weight	BFS	Gender
1	1.47	2.80	-1.00	M
2	0.00	-1.40	1.00	F
3	0.60	0.30	-1.20	M
4	-0.60	-1.70	1.80	F
…	…	…	…	…
N	0.20	-0.60	-1.60	M

k-Nearest Neighbor Classification

- k-NN Issue 2: Cut-off
  - ✓ Consider the prior probability of each class
  - ✓ Assume that N(M) = 100, N(F) = 400

For a new data **X**

Neighbor	Class
N1	M
N2	F
N3	M
N4	F
N5	F

Majority voting

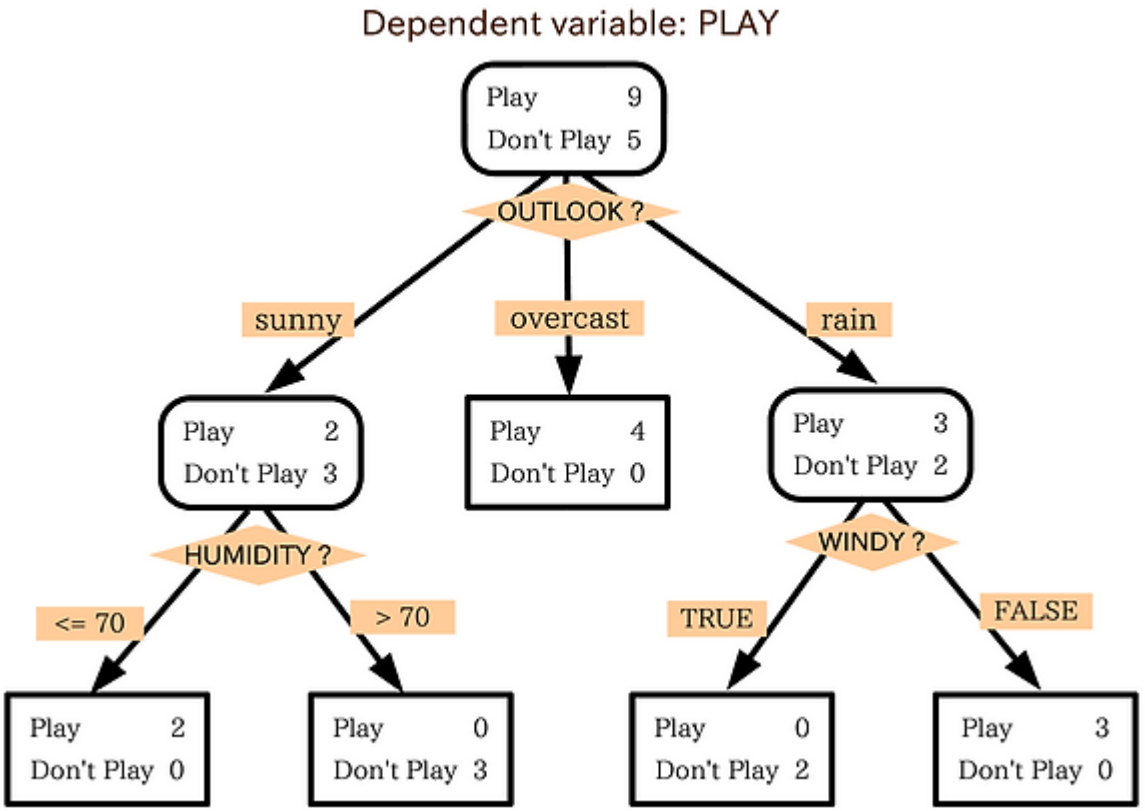
P(X=M)=0.4

  - ✓ If the cut-off is set to 0.5 (assuming equal class distribution), then X is classified as F.
  - ✓ If the cut-off is set to 0.2 (proportion of M among the people), then X is classified as M.

AGENDA

Classification Tree

- Goal
  - ✓ Classify or predict an outcome based on a set of predictors.
  - ✓ The output is a set of rules.



Rule example

If outlook is sunny

and if humidity > 70

then he does not play

or

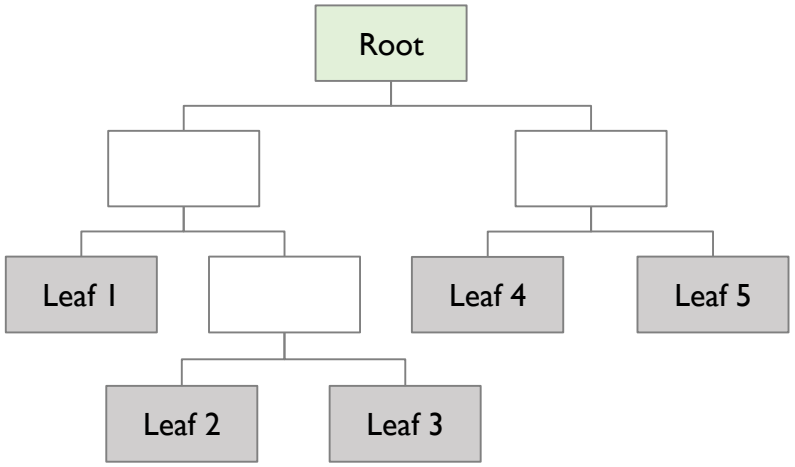
If outlook is rainy

and it is not windy

then he does play

Classification Tree

- Terminologies
  - ✓ Parent node: node before split
  - ✓ Child node: node after split
  - ✓ Split criterion: a certain variable value used for split a node
  - ✓ Root node: node that only has child nodes but no parent node
  - ✓ Leaf nodes: nodes that only have a parent node but no child nodes

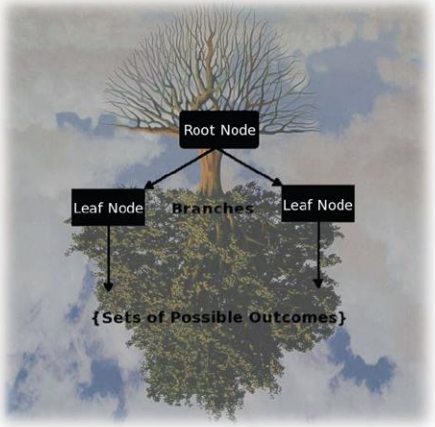


Classification Tree

- Why CART?
  - ✓ Simple to understand and interpret.
  - ✓ Requires little data preparation (normalization, missing value treatments, etc.)
  - ✓ Able to handle both numerical and categorical data.

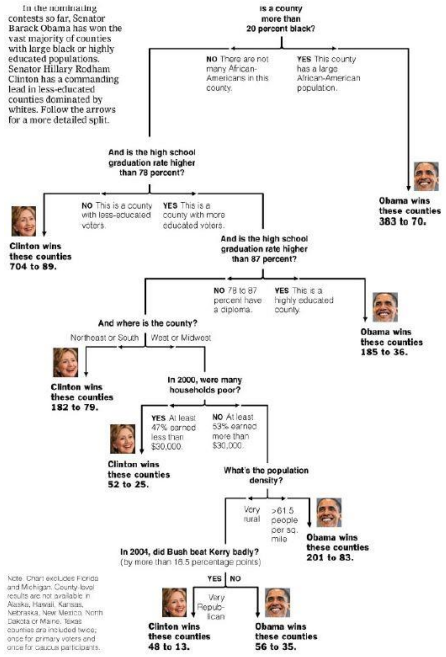
- Key Ideas
  - ✓ Recursive Partitioning
    - Repeatedly split the records into two parts so as to achieve maximum homogeneity within the new parts.
  - ✓ Pruning the Tree
    - Simplify the tree by pruning peripheral branches to avoid over-fitting.

Classification Tree



Classification and Regression Tree (CART)

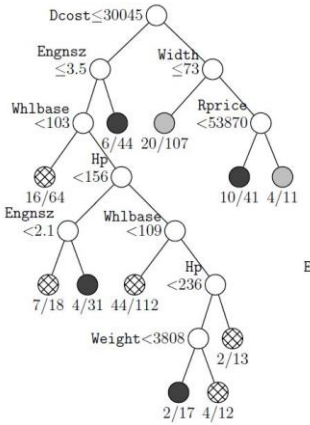
- Generate a set of rules by recursively partitioning the entire datasets to increase the purity of the partitioned area (Breiman, 1984)
- Being able to explain the reason of the prediction result by following the rules to the target leaf node
- Can handle categorical and numerical variables simultaneously

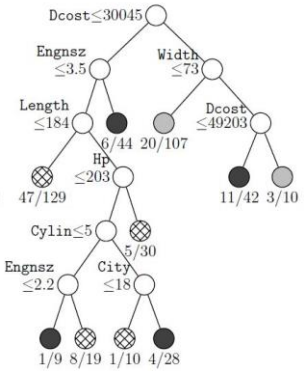


Recursive Partitioning

Pruning

- Partition the data in a parent node into two child nodes using a certain value of a certain variable
- Select the split point to maximize the purity of the child nodes
- Gini-index (for categorical variable) and the variance (for numerical variable) are used to measure the impurity of a node



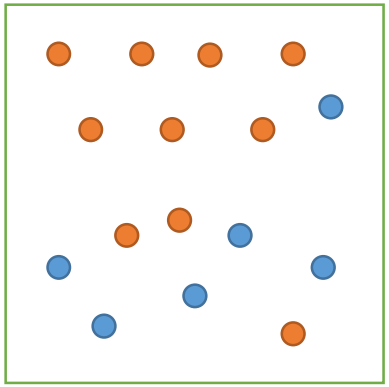


Classification Tree: Impurity Measure

- Measuring Impurity 1: Gini Index
  - ✓ Gini Index for rectangle A containing m records

$$I(A) = 1 - \sum_{k=1}^{m} p_k^2$$

  - p = proportion of cases in rectangle A that belong to class k.



$$I(A) = 1 - \sum_{k=1}^{m} p_k^2$$

$$= 1 - \left(\frac{6}{16}\right)^2 - \left(\frac{10}{16}\right)^2$$

$$\approx 0.47$$

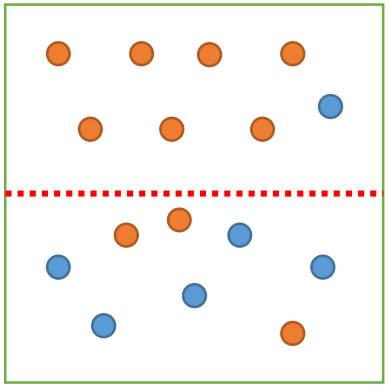
  - I(A) = 0 when all cases belong to the same class.
  - Max value when all classes are equal represented (=0.5 in binary case)

Classification Tree: Impurity Measure

- Measuring Impurity 1: Gini Index
  - ✓ When their more than two rectangles

$$I(A) = \sum_{i=1}^{d} \left(R_i \left(1 - \sum_{k=1}^{m} p_{ik}^2 \right) \right)$$

  - R_i = proportion of cases in rectangle Ri among the training data.



$$I(A)$$

$$= 0.5 \times \left(1 - \left(\frac{7}{8} \right)^2 - \left(\frac{1}{8} \right)^2 \right) + 0.5 \times \left(1 - \left(\frac{3}{8} \right)^2 - \left(\frac{5}{8} \right)^2 \right)$$

$$= 0.34$$

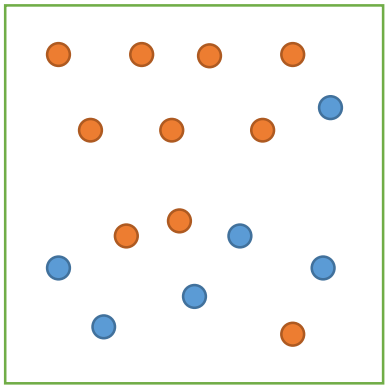
  - "Information gain" after splitting: 0.47-0.34=0.13

Classification Tree: Impurity Measure

- Measuring Impurity 2: Deviance

$$D_i = -2 \sum_k n_{ik} log(p_{ik})$$

✓ i: node index, k: class index, p_{ik}: probability of class k in node I



$$D_i = -2 \times \left(10 \times log\left(\frac{10}{16}\right) + 6 \times log\left(\frac{6}{16}\right) \right)$$

$$= 21.17$$

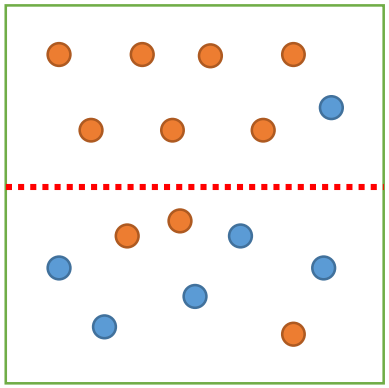
✓ Deviance = 0 if and only if all nodes contain instances from the same class

Classification Tree: Impurity Measure

- Measuring Impurity 2: Deviance

$$D_i = -2 \sum_k n_{ik} log(p_{ik})$$

✓ i: node index, k: class index, p_{ik}: probability of class k in node I



$$D_1 = -2 \times \left(7 \times log\left(\frac{7}{8}\right) + 1 \times log\left(\frac{1}{8}\right) \right) = 6.03$$

$$D_2 = -2 \times \left(3 \times log\left(\frac{3}{8}\right) + 5 \times log\left(\frac{5}{8}\right) \right) = 10.59$$

$$D_1 + D_2 = 16.62$$

✓ Information gain = 21.17 – 16.62 = 4.55

Classification Tree: Recursive Partitioning

- Example: Riding Mowers
  - ✓ Goal: Classify 24 households as owning or not owning riding mowers
  - ✓ Predictors: Income, Lot size

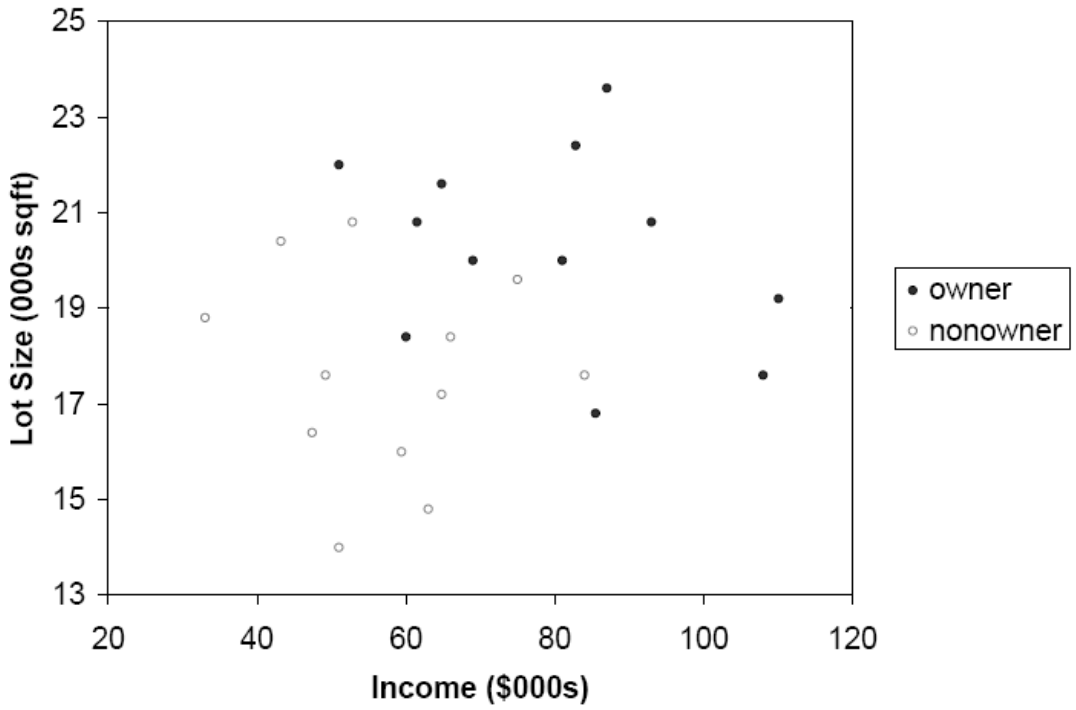
Income	Lot size	Ownership	Income	Lot size	Ownership
60.0	18.4	Owner	75.0	19.6	Non-owner
85.5	16.8	Owner	52.8	20.8	Non-owner
64.8	21.6	Owner	64.8	17.2	Non-owner
61.5	20.8	Owner	43.2	20.4	Non-owner
87.0	23.6	Owner	84.0	17.6	Non-owner
110.1	19.2	Owner	49.2	17.6	Non-owner
108.0	17.6	Owner	59.4	16.0	Non-owner
82.8	22.4	Owner	66.0	18.4	Non-owner
69.0	20.0	Owner	47.4	16.4	Non-owner
93.0	20.8	Owner	33.0	18.8	Non-owner
51.0	22.0	Owner	51.0	14.0	Non-owner
81.0	20.0	Owner	63.0	14.8	Non-owner

Classification Tree: Recursive Partitioning

I

Order records according to one variable

- Order the data with regard to lot size



Income	Lot size	Ownership
51.0	14.0	Non-owner
63.0	14.8	Non-owner
59.4	16.0	Non-owner
47.4	16.4	Non-owner
85.5	16.8	Owner
64.8	17.2	Non-owner
108.0	17.6	Owner
84.0	17.6	Non-owner
49.2	17.6	Non-owner
60.0	18.4	Owner
66.0	18.4	Non-owner
33.0	18.8	Non-owner
110.1	19.2	Owner
75.0	19.6	Non-owner
69.0	20.0	Owner
81.0	20.0	Owner
43.2	20.4	Non-owner
61.5	20.8	Owner
93.0	20.8	Owner
52.8	20.8	Non-owner
64.8	21.6	Owner
51.0	22.0	Owner
82.8	22.4	Owner
87.0	23.6	Owner

Classification Tree: Recursive Partitioning

2

Find midpoints between successive values

- First midpoint = 14.4 (0.5*(14.0+14.8))
- Divide records into those with Lot size > 14.4 and those < 14.4
- Compute the impurity: Gini index
  - ✓ Before splitting:

$$1-\left(\frac{12}{24}\right)^2-\left(\frac{12}{24}\right)^2=0.5$$

  - ✓ After splitting:

$$\frac{1}{24}\left(1-\left(\frac{1}{1}\right)^2\right)+\frac{23}{24}\left(1-\left(\frac{12}{23}\right)^2-\left(\frac{11}{23}\right)^2\right)\approx 0.48$$

  - ✓ Information gain: 0.50-0.48=0.02

Income	Lot size	Ownership
51.0	14.0	Non-owner
63.0	14.8	Non-owner
59.4	16.0	Non-owner
47.4	16.4	Non-owner
85.5	16.8	Owner
64.8	17.2	Non-owner
108.0	17.6	Owner
84.0	17.6	Non-owner
49.2	17.6	Non-owner
60.0	18.4	Owner
66.0	18.4	Non-owner
33.0	18.8	Non-owner
110.1	19.2	Owner
75.0	19.6	Non-owner
69.0	20.0	Owner
81.0	20.0	Owner
43.2	20.4	Non-owner
61.5	20.8	Owner
93.0	20.8	Owner
52.8	20.8	Non-owner
64.8	21.6	Owner
51.0	22.0	Owner
82.8	22.4	Owner
87.0	23.6	Owner

Classification Tree: Recursive Partitioning

2

Find midpoints between successive values

- First midpoint = 14.4 (0.5*(14.0+14.8))
- Divide records into those with Lot size > 14.4 and those < 14.4
- Compute the impurity: Entropy
  - ✓ Before splitting:

$$-\frac{1}{2}\log_2\left(\frac{1}{2}\right)-\frac{1}{2}\log_2\left(\frac{1}{2}\right)=1$$

  - ✓ After splitting:

$$\frac{1}{24}\left(-\log(1)\right)+\frac{23}{24}\left(-\frac{12}{23}\log_2\left(\frac{12}{23}\right)-\frac{11}{23}\log_2\left(\frac{11}{23}\right)\right)\approx 0.96$$

  - ✓ Information gain: 1-0.96=0.04

Income	Lot size	Ownership
51.0	14.0	Non-owner
63.0	14.8	Non-owner
59.4	16.0	Non-owner
47.4	16.4	Non-owner
85.5	16.8	Owner
64.8	17.2	Non-owner
108.0	17.6	Owner
84.0	17.6	Non-owner
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66.0	18.4	Non-owner
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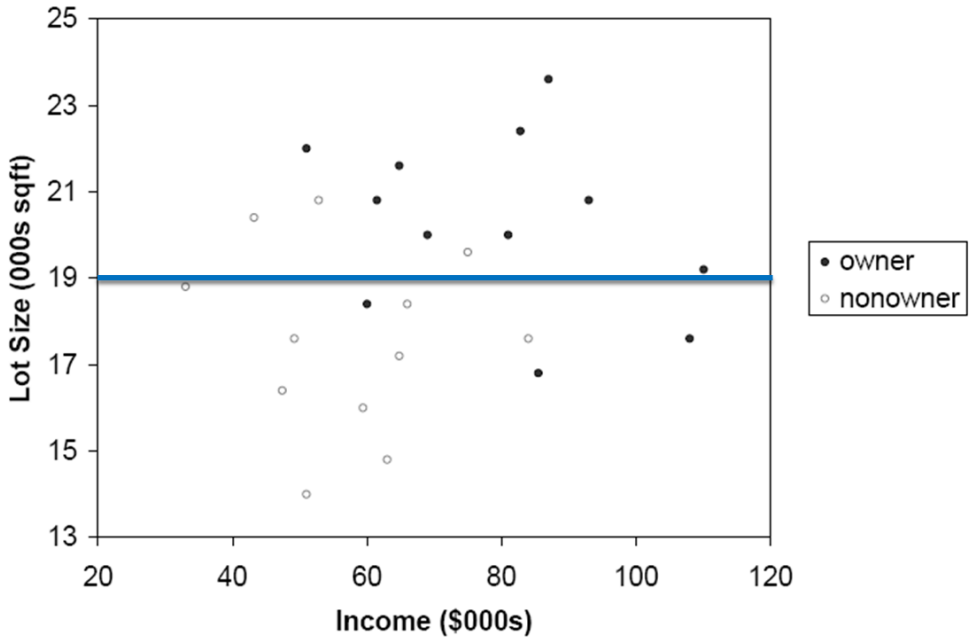
110.1	19.2	Owner
75.0	19.6	Non-owner
69.0	20.0	Owner
81.0	20.0	Owner
43.2	20.4	Non-owner
61.5	20.8	Owner
93.0	20.8	Owner
52.8	20.8	Non-owner
64.8	21.6	Owner
51.0	22.0	Owner
82.8	22.4	Owner
87.0	23.6	Owner

Classification Tree: Recursive Partitioning

3

Find the best split

- Find the best split which maximize the (Gini or information gain)



Income	Lot size	Ownership
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52.8	20.8	Non-owner
64.8	21.6	Owner
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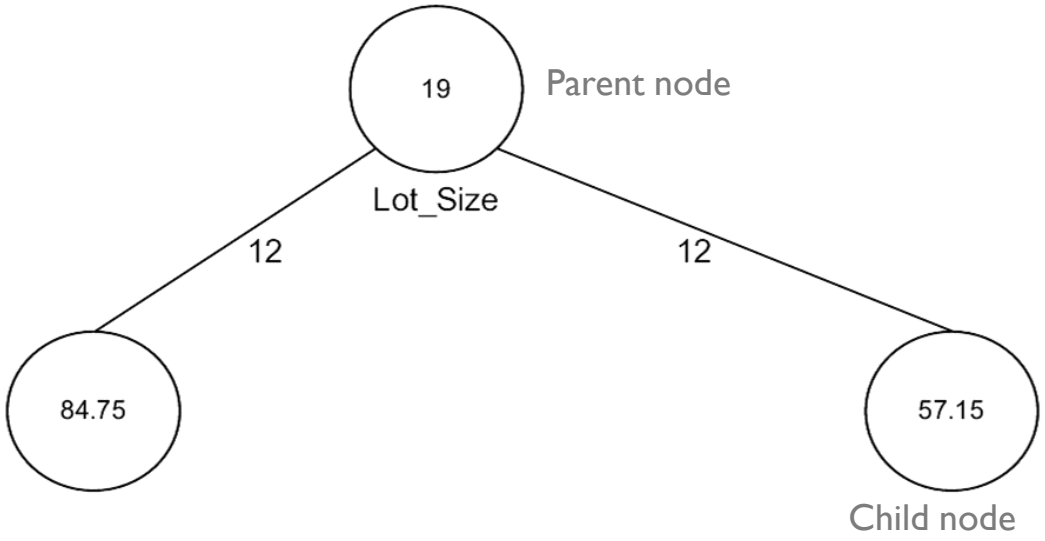
Classification Tree: Recursive Partitioning

3

Tree structure



- Split point become nodes on tree (circles with split value in center)
- Rectangles represent "leaves" (terminal points, no future splits, classification value noted)
- Numbers on lines between nodes indicate # cases.

Income	Lot size	Ownership
51.0	14.0	Non-owner
63.0	14.8	Non-owner
59.4	16.0	Non-owner
47.4	16.4	Non-owner
85.5	16.8	Owner
64.8	17.2	Non-owner
108.0	17.6	Owner
84.0	17.6	Non-owner
49.2	17.6	Non-owner
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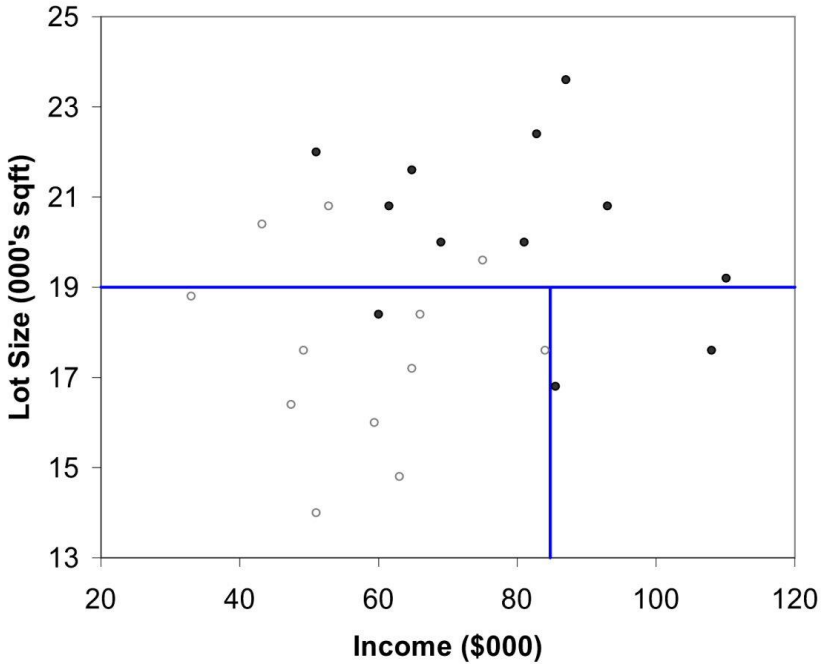
82.8	22.4	Owner
87.0	23.6	Owner

Classification Tree: Recursive Partitioning

Repeat the splitting for each node

- Repeat the splitting until there is no gain.
- E.g., second split = income = 84.75



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Classification Tree: Recursive Partitioning

Repeat the splitting for each node

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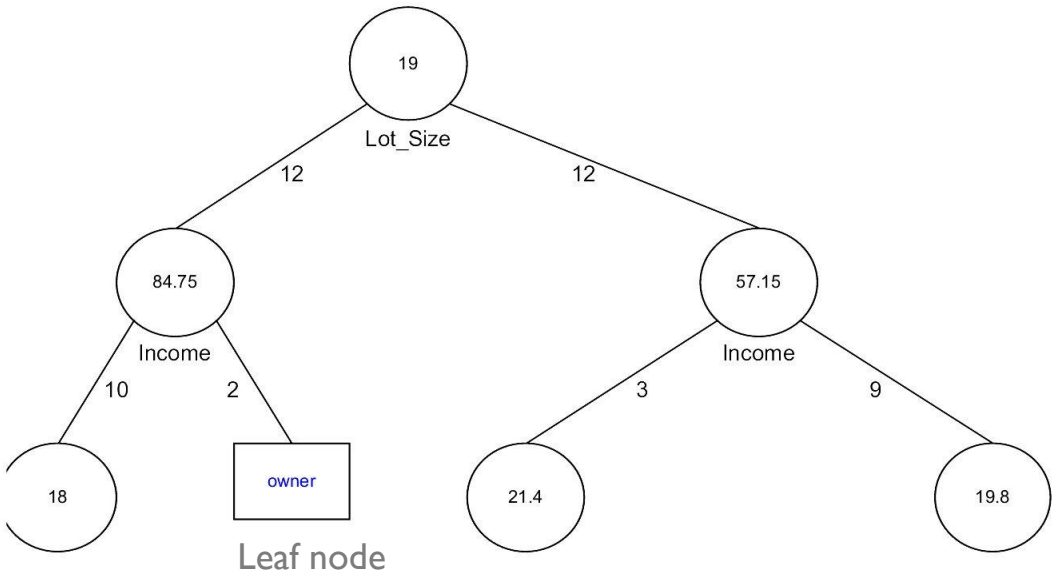

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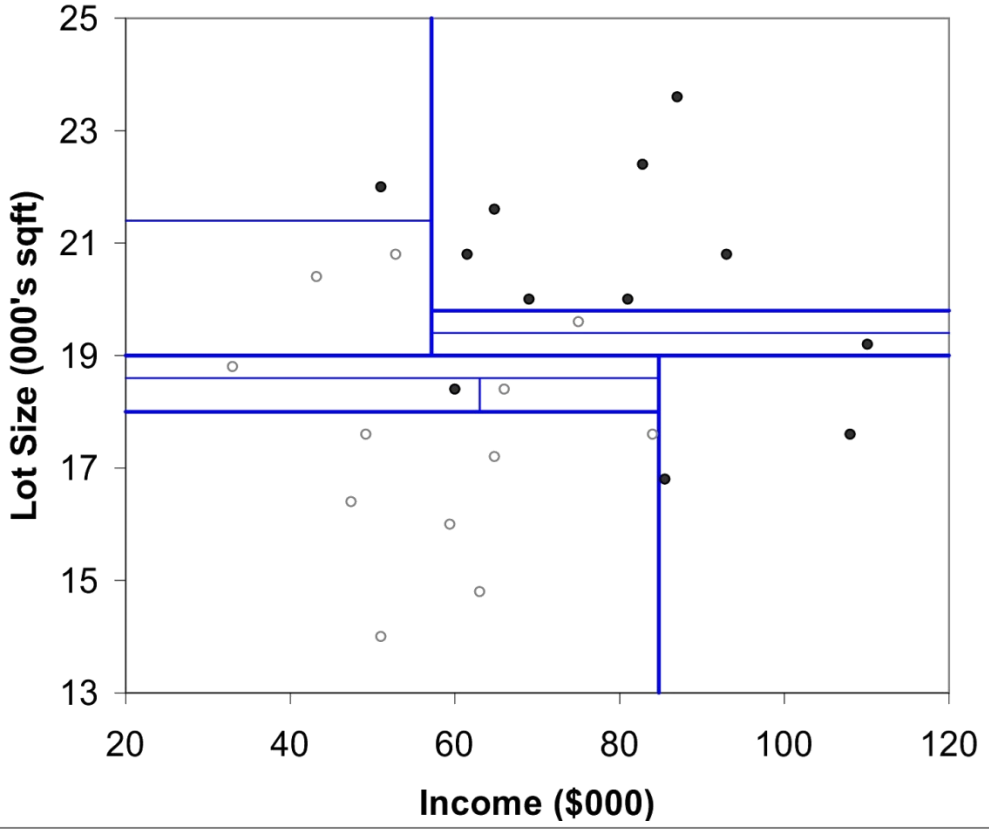
59.4	16.0	Non-owner
60.0	18.4	Owner
63.0	14.8	Non-owner
64.8	17.2	Non-owner
66.0	18.4	Non-owner
84.0	17.6	Non-owner
85.5	16.8	Owner
108.0	17.6	Owner

Classification Tree: Recursive Partitioning

Repeat the splitting for each node

- Repeat the splitting until there is no gain.
- Final splitting



4

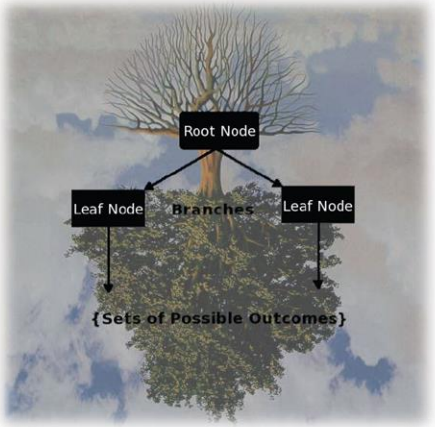
Classification Tree: Recursive Partitioning

Repeat the splitting for each node

- Each leaf node label is determined by "voting" of the records within it, and by the cutoff value.
- Records within each leaf node are from the training data.
- Default cutoff=0.5 means that the leaf node's label is the majority class.
- Cutoff = 0.75 requires majority of 75% of more "1" records in the leaf to label it a "1" node.

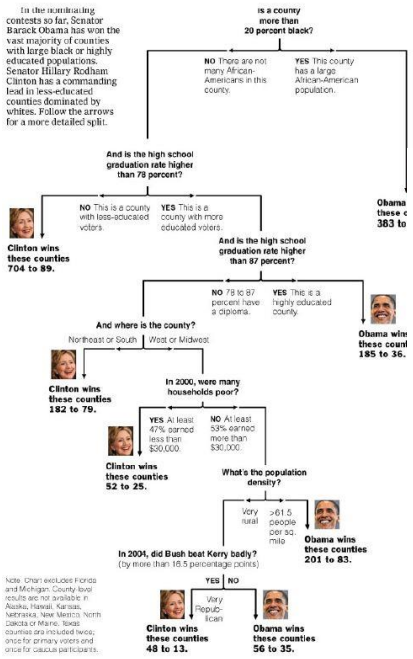
4

Classification Tree: Pruning



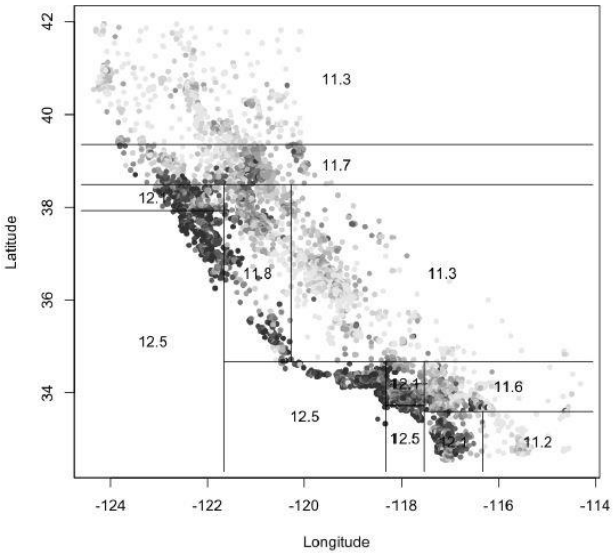
Classification and Regression Tree (CART)

- Generate a set of rules by recursively partitioning the entire datasets to increase the purity of the partitioned area (Breiman, 1984)
- Being able to explain the reason of the prediction result by following the rules to the target leaf node
- Can handle categorical and numerical variables simultaneously



Recursive Partitioning

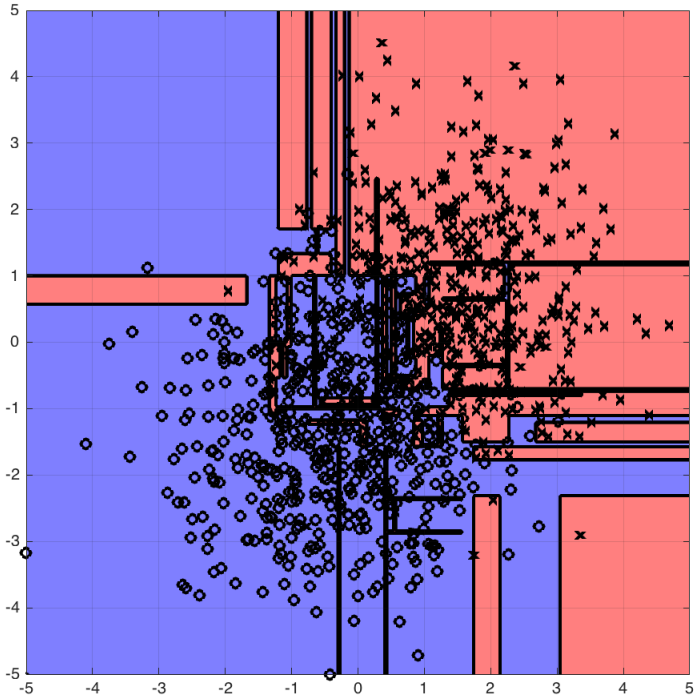
Pruning

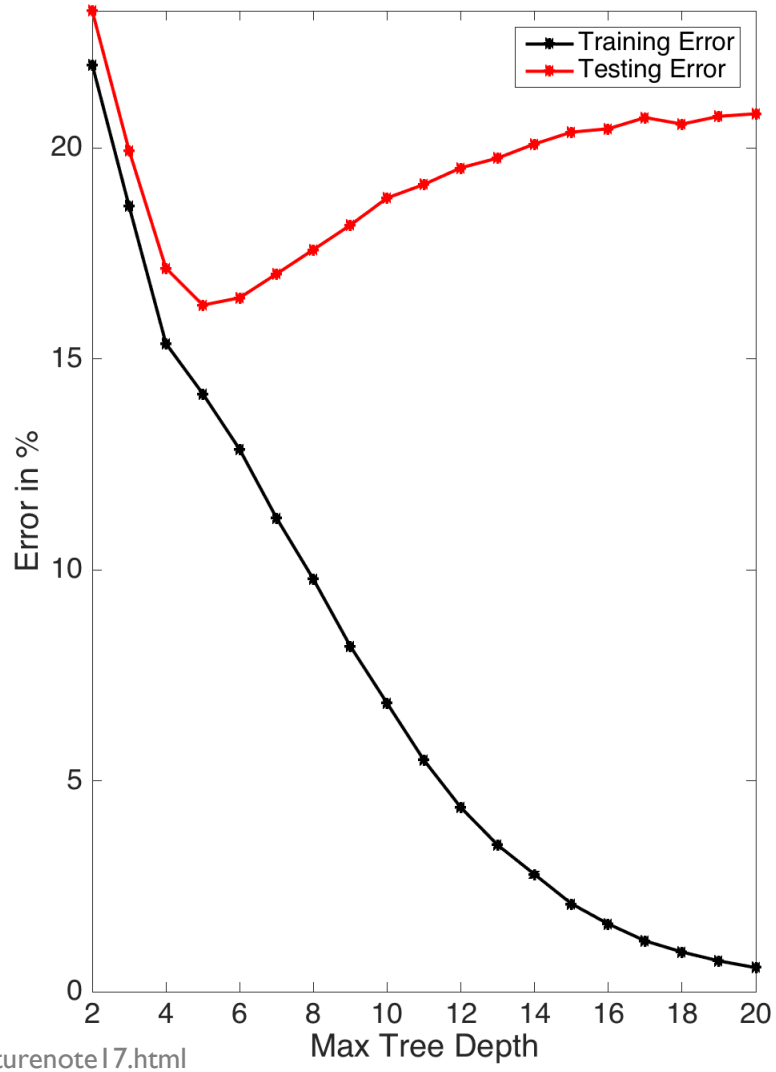


- Aggregate some child node into a parent node to avoid over-fitting
- Pre-pruning: pruning is done during the tree construction
- Post-pruning: Once a full-tree is constructed, nodes are pruned by taking the validation error and tree complexity

Classification Tree: Pruning

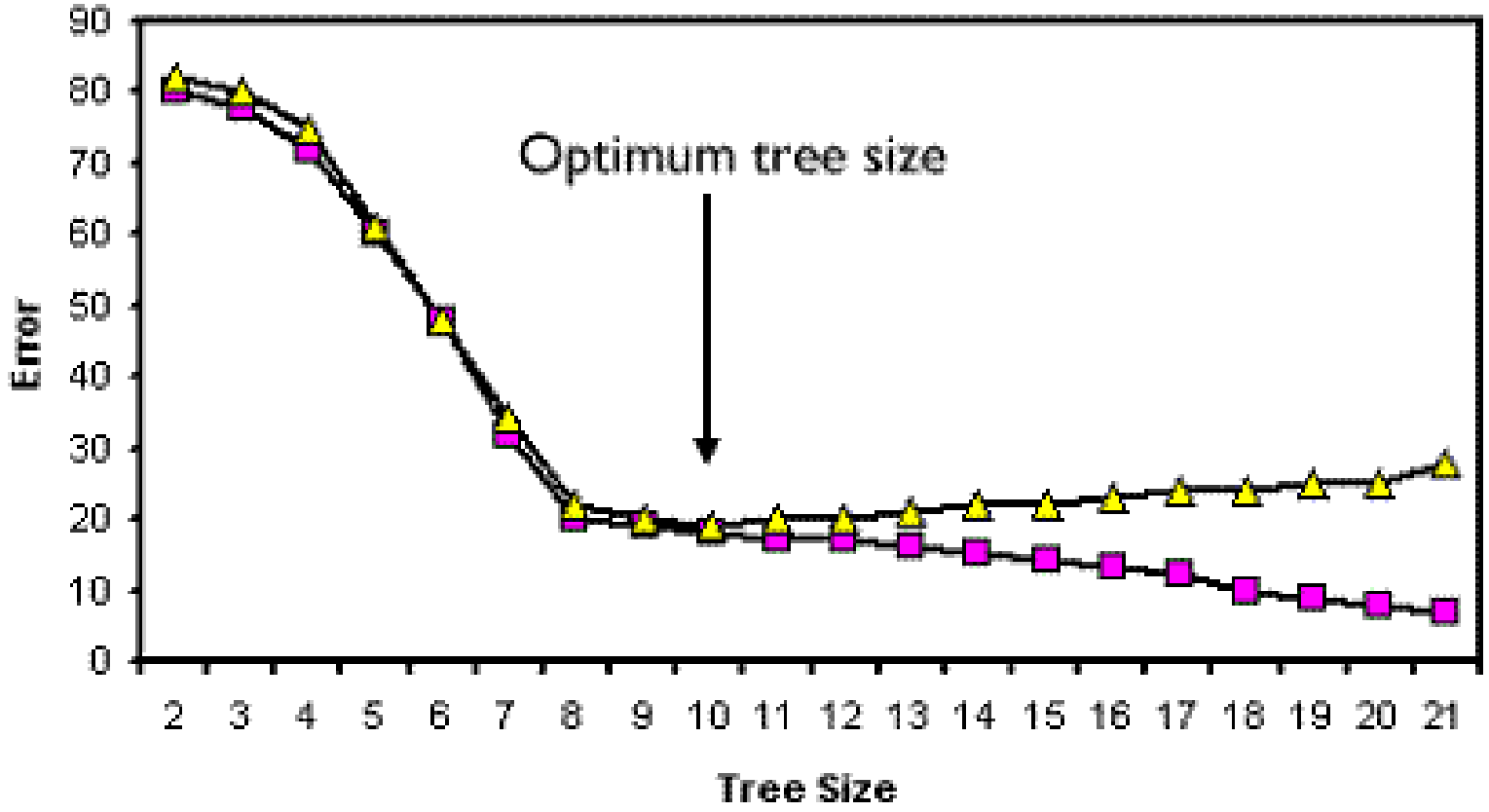
- Recursive partitioning is completed when every leaf node has 100% purity





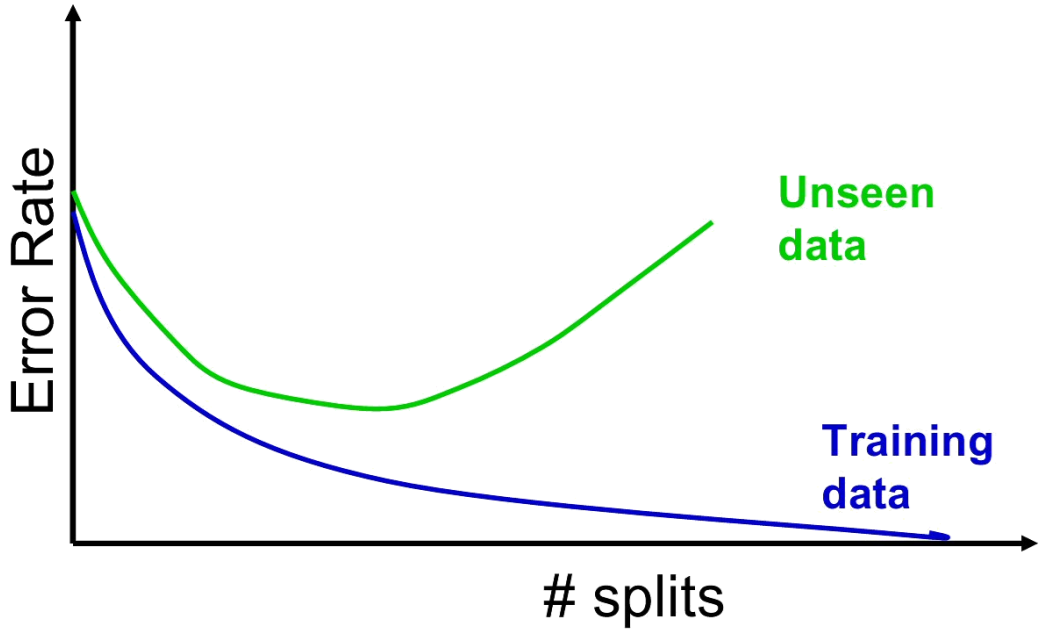
Classification Tree: Pruning

- Full tree which is a result of recursive partitioning has a risk of overfitting, which in turn, results in poor generalization ability
  - ✓ It tends to memorize the training dataset, rather than discovering significant patterns



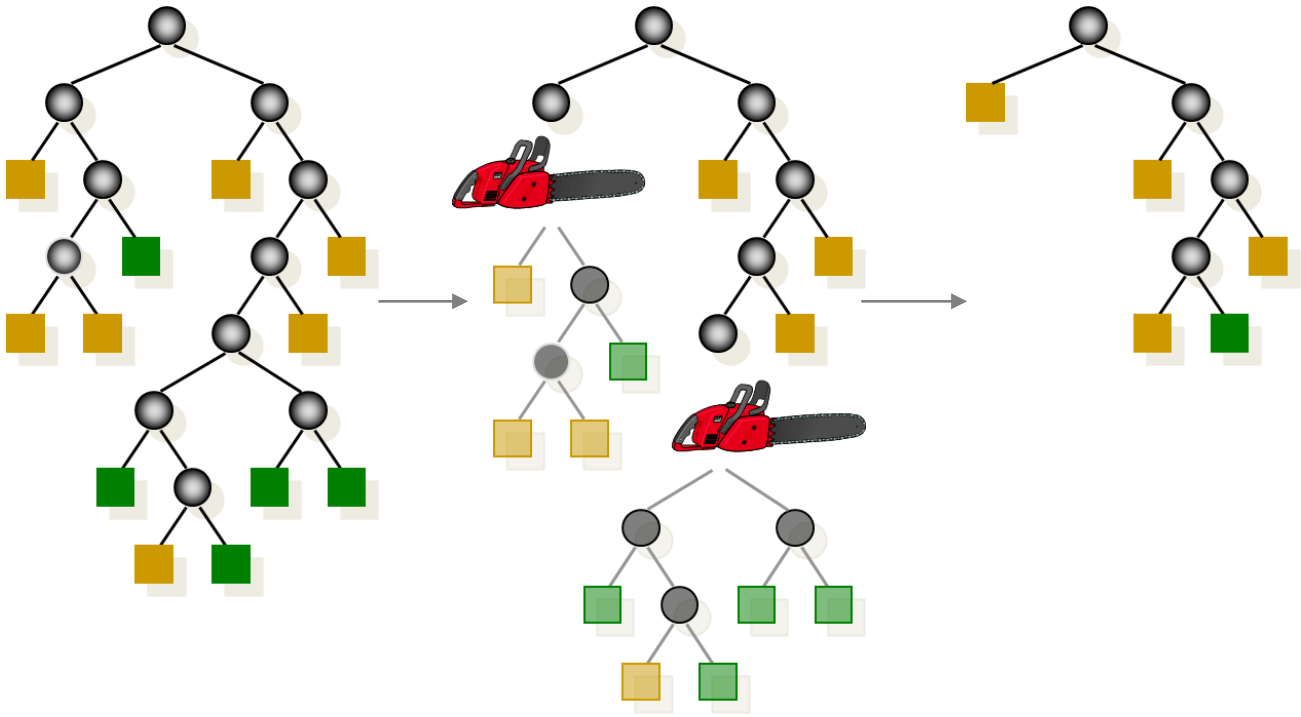
Classification Tree: Pruning

- Overfitting problem
  - ✓ The end of recursive partitioning process is 100% purity in each leaf
  - ✓ It over-fits the data, ending up fitting noise in the data and leading to low predictive accuracy of new data
  - ✓ Past a certain point, the error rate for the validation data starts to increase



Classification Tree: Pruning

- Pruning



- ✓ CART lets tree grow to full extent, then prunes it back.
- ✓ Idea is to find that point at which the validation error begins to rise.
- ✓ Generate successively smaller trees by pruning leaves.
- ✓ At each pruning stage, multiple trees are possible.
- ✓ Use "cost complexity" to choose the best tree at that stage.

Classification Tree: Pruning

- Cost complexity

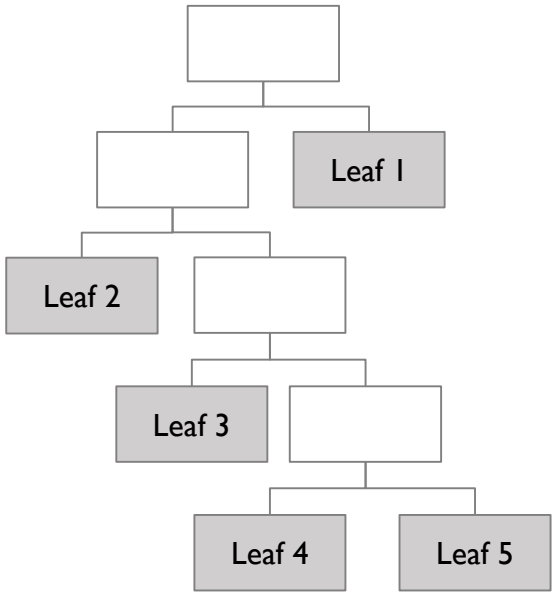
$$CC(T) = Err(T) + \alpha \times L(T)$$

- ✓ CC(T) = cost complexity of a tree
- ✓ ERR(T) = proportion of misclassified records in the validation data
- ✓ Alpha = penalty factor attached to the tree size (set by the user)

Classification Tree: Pruning

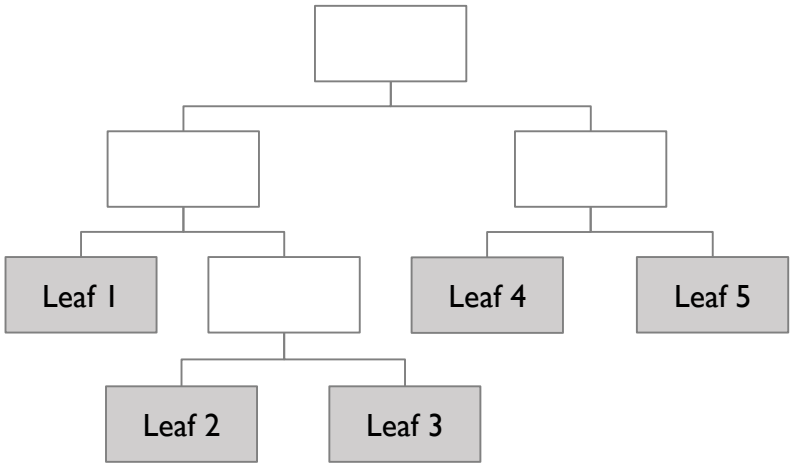
- Cost Complexity Example 1

Tree 1



Validation error = 10%

Tree 2



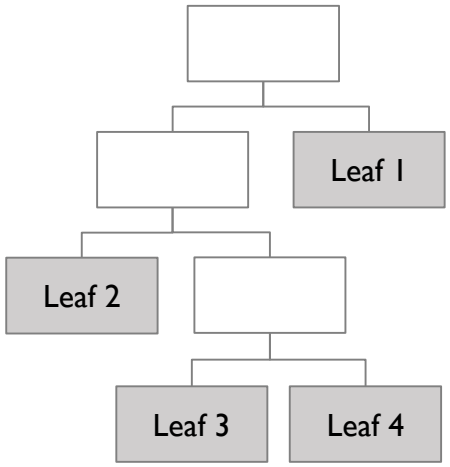
Validation error = 15%

- Two trees have the same number of leaf nodes but Tree 1 yields lower validation error → Tree 1 should be preferred

Classification Tree: Pruning

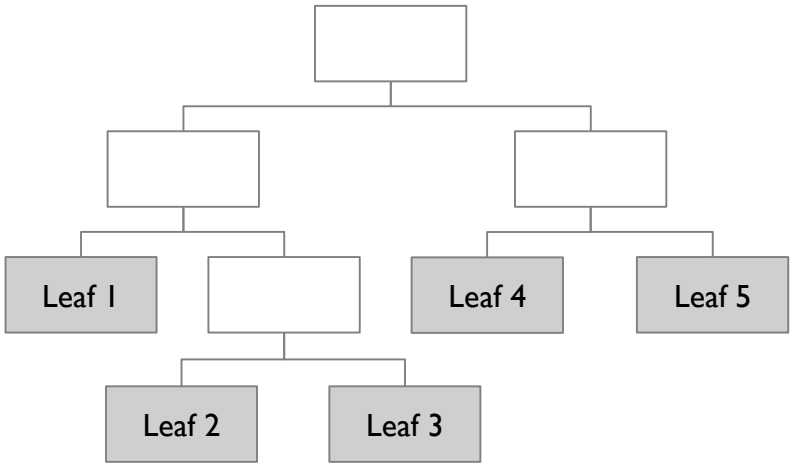
- Cost Complexity Example 2

Tree A



Validation error = 15%

Tree B

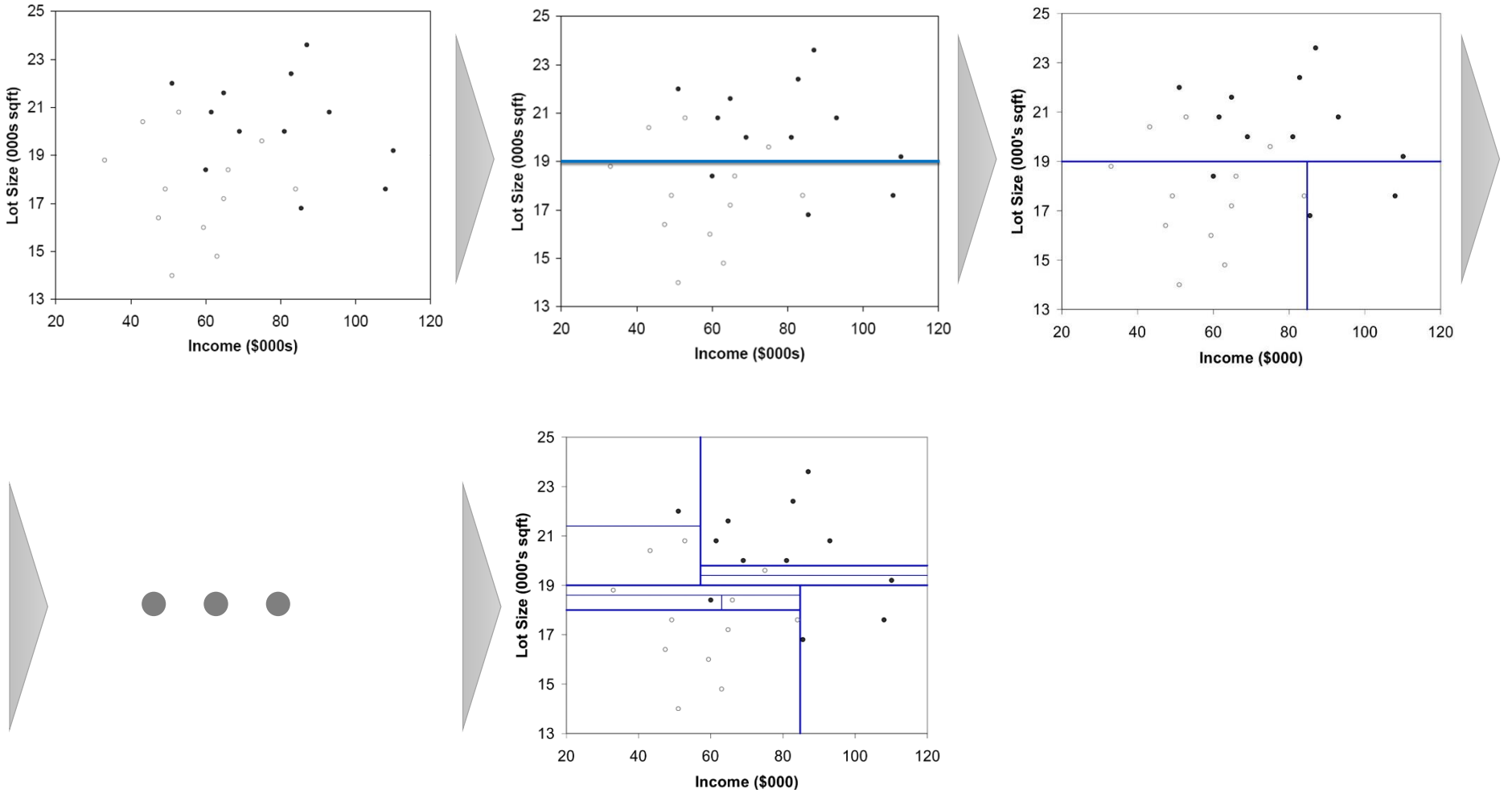


Validation error = 15%

✓ Two trees yield the same validation error, but Tree A has fewer leaf nodes (simpler tree structure) → Tree A should be preferred

Classification Tree: Pruning

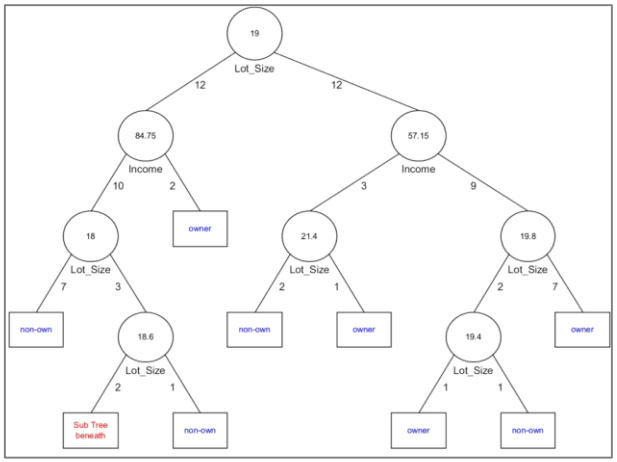
- Full tree constructed by recursive partitioning



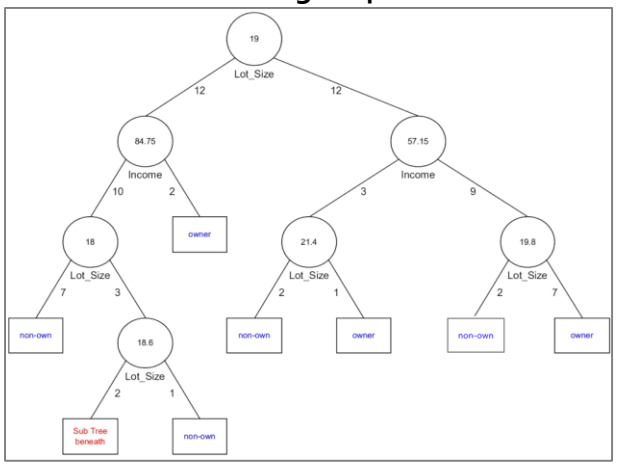
Classification Tree: Pruning

- Pruning

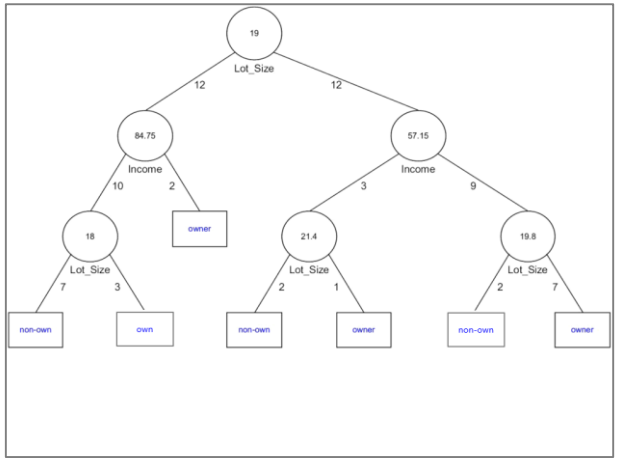
Full Tree

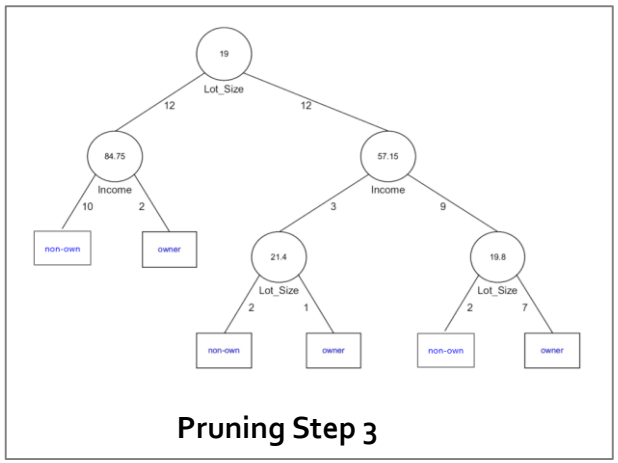


Pruning Step 1

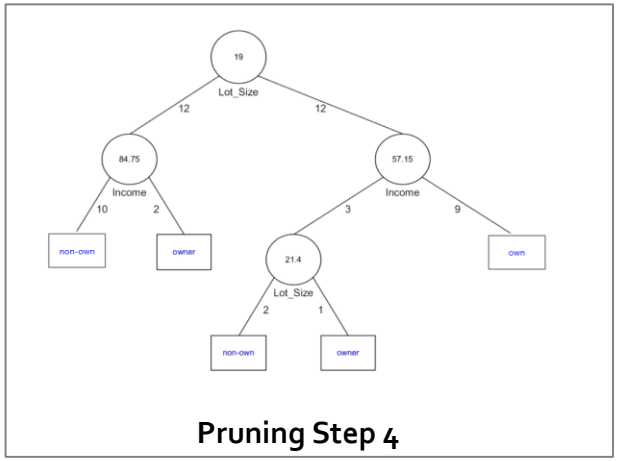


Pruning Step 2

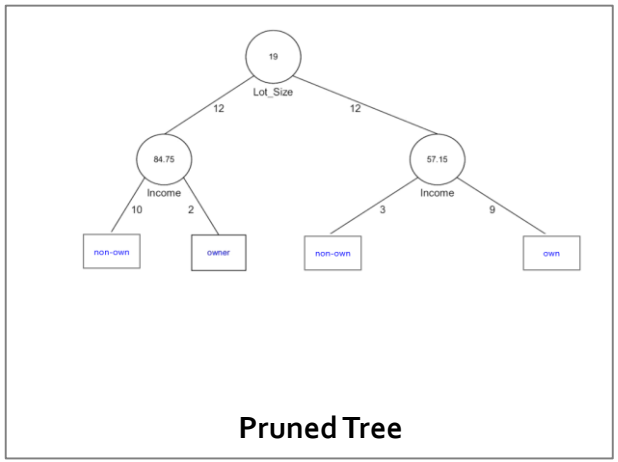




Pruning Step 3



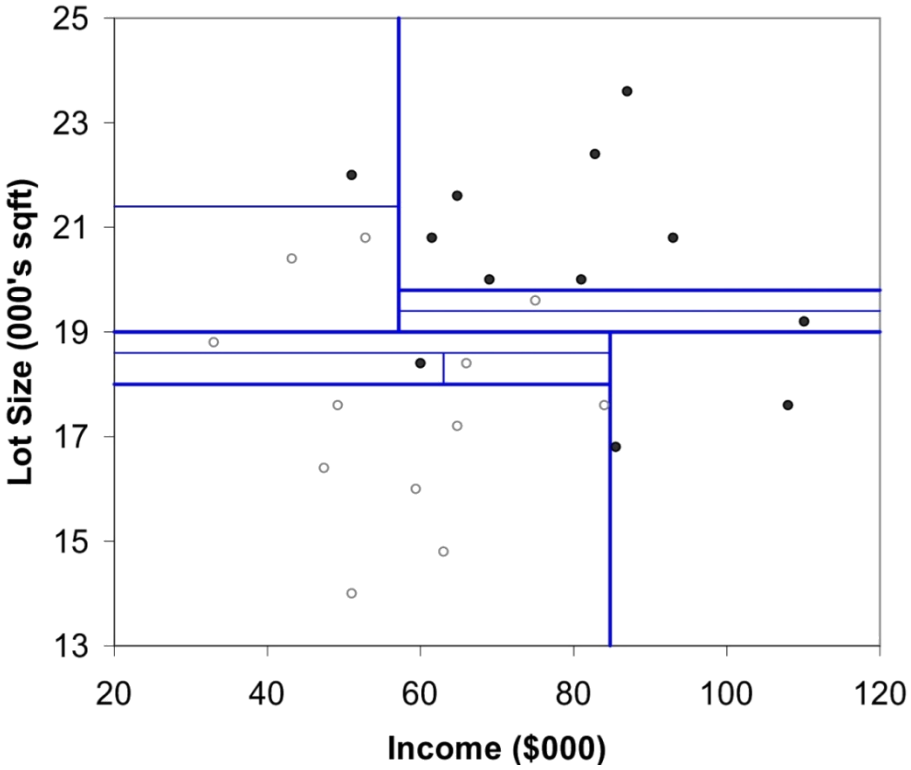
Pruning Step 4

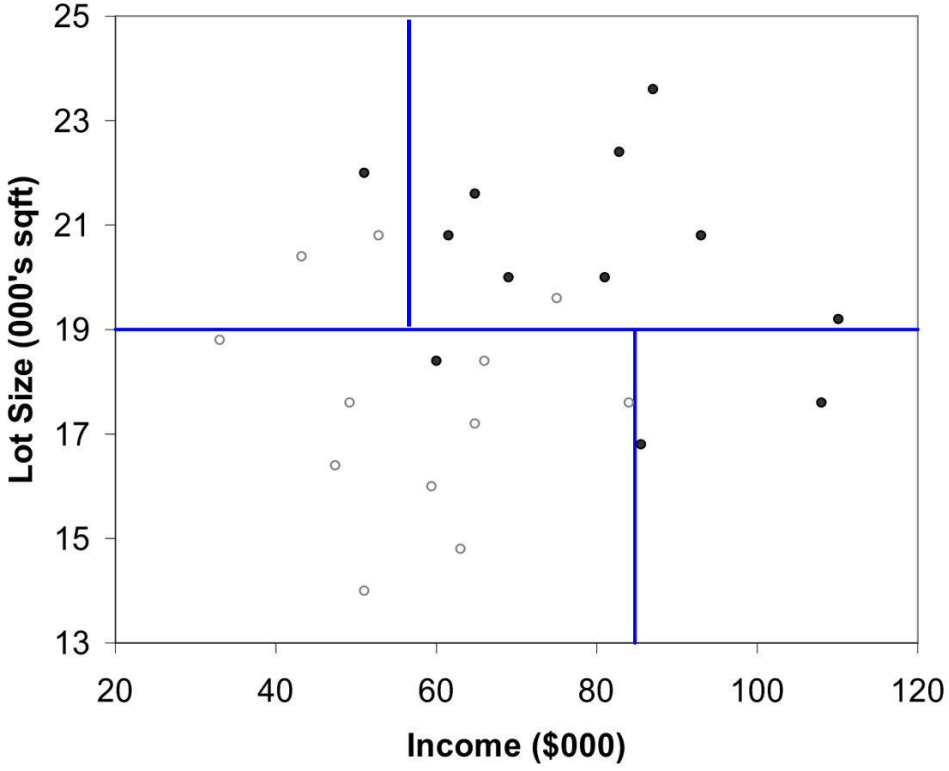


Pruned Tree

Classification Tree: Pruning

- Full tree vs. Pruned tree

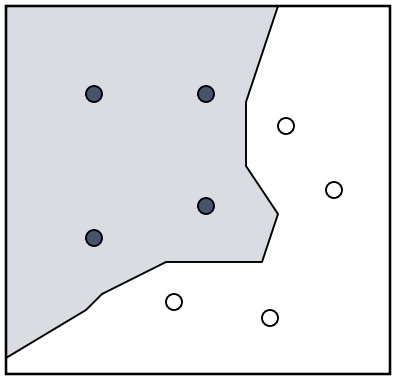




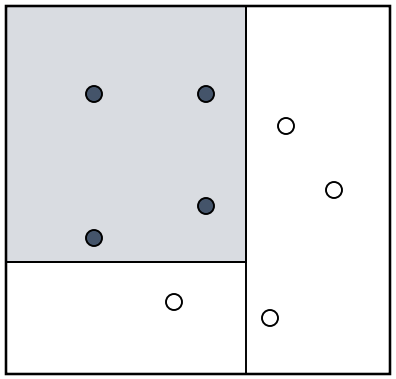
AGENDA

Support Vector Machine

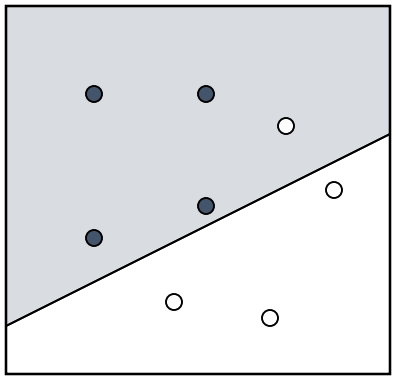
- Discriminant functions in classification



Nearest Neighbor

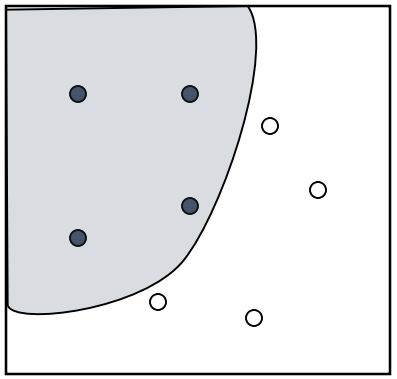


Decision Tree



Linear Functions

$$g(\mathbf{x}) = \mathbf{w}^T\mathbf{x} + b$$

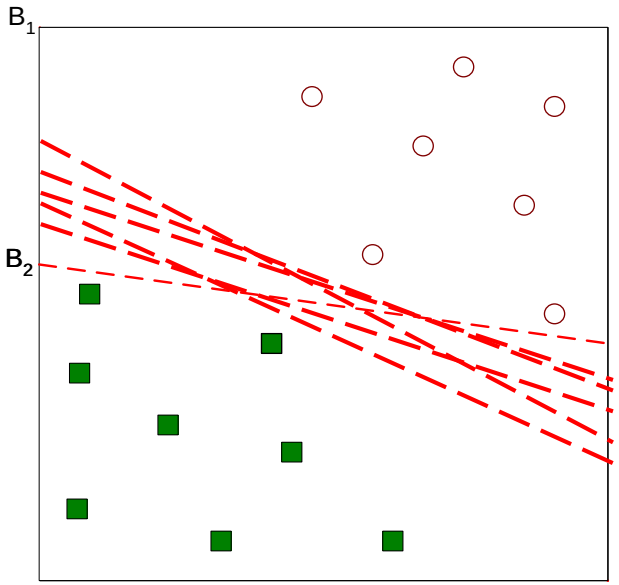


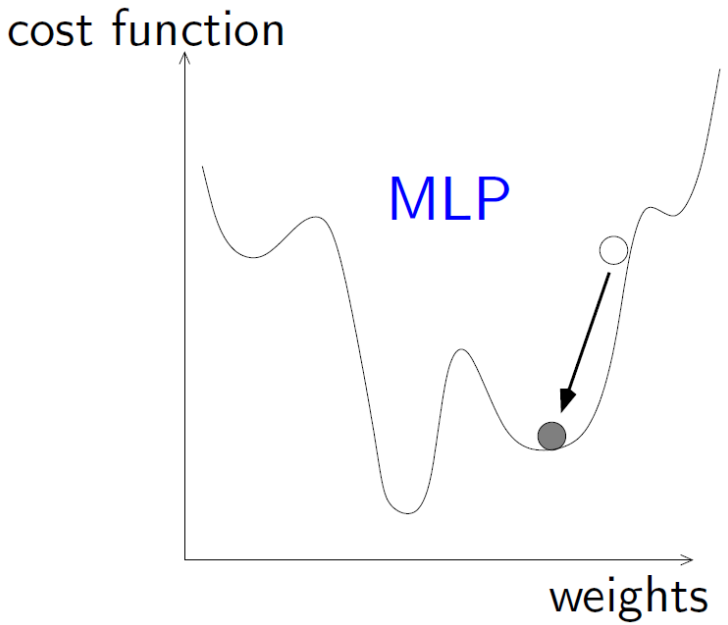
Nonlinear Functions

Support Vector Machine

Local optimum? Global optimum!

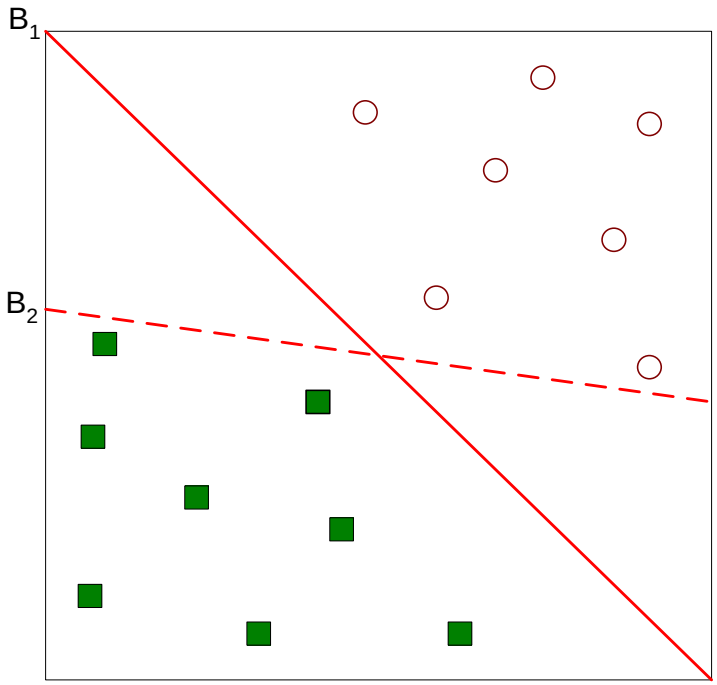
- Many non-linear classification algorithms:
  - ✓ Universal approximation of continuous nonlinear functions
  - ✓ Learning from input-output patterns
  - ✓ Parallel network architecture, multiple inputs and outputs
  - ✓ But, existence of many local optimums!

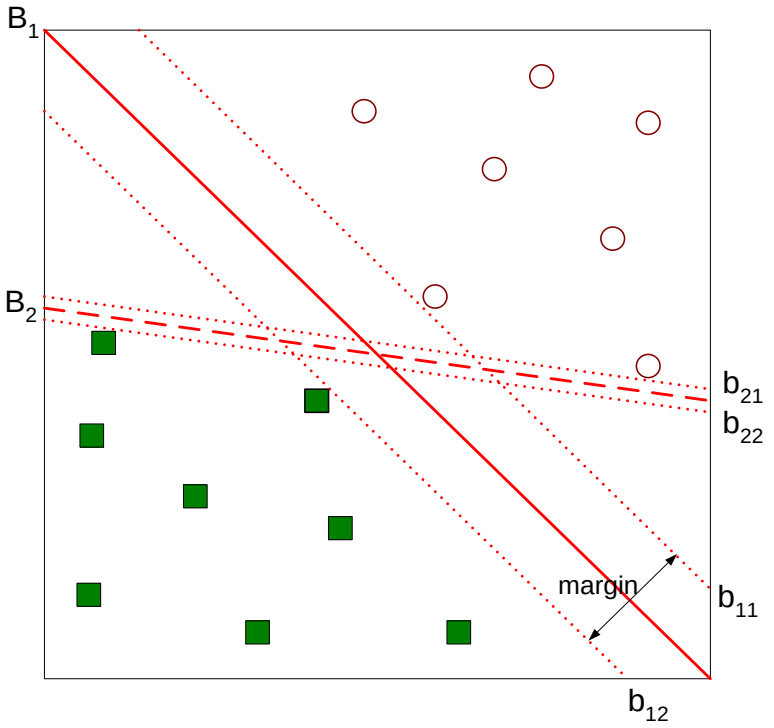




Support Vector Machine

- Which like is better? B_1 or B_2
  - ✓ How do you define "better"

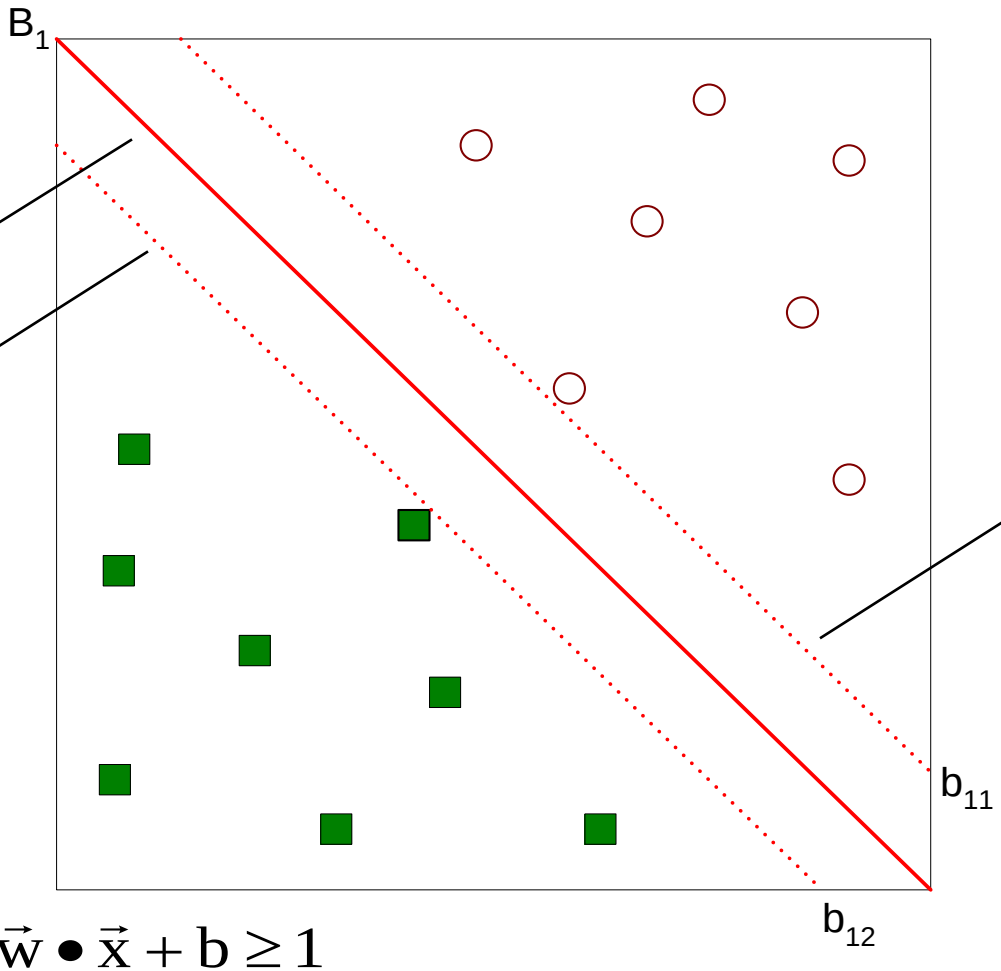




- Find the hyperplane that maximizes the margin!

Support Vector Machine

- Support Vector Machine Formulation



$\vec{w} \bullet \vec{x} + b = 0$

$\vec{w} \bullet \vec{x} + b = -1$

$\vec{w} \bullet \vec{x} + b = +1$

$$f(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \bullet \vec{x} + b \geq 1 \\ -1 & \text{if } \vec{w} \bullet \vec{x} + b \leq -1 \end{cases}$$

$$\text{Margin} = \frac{2}{\| \vec{w} \|^2}$$

Support Vector Machine

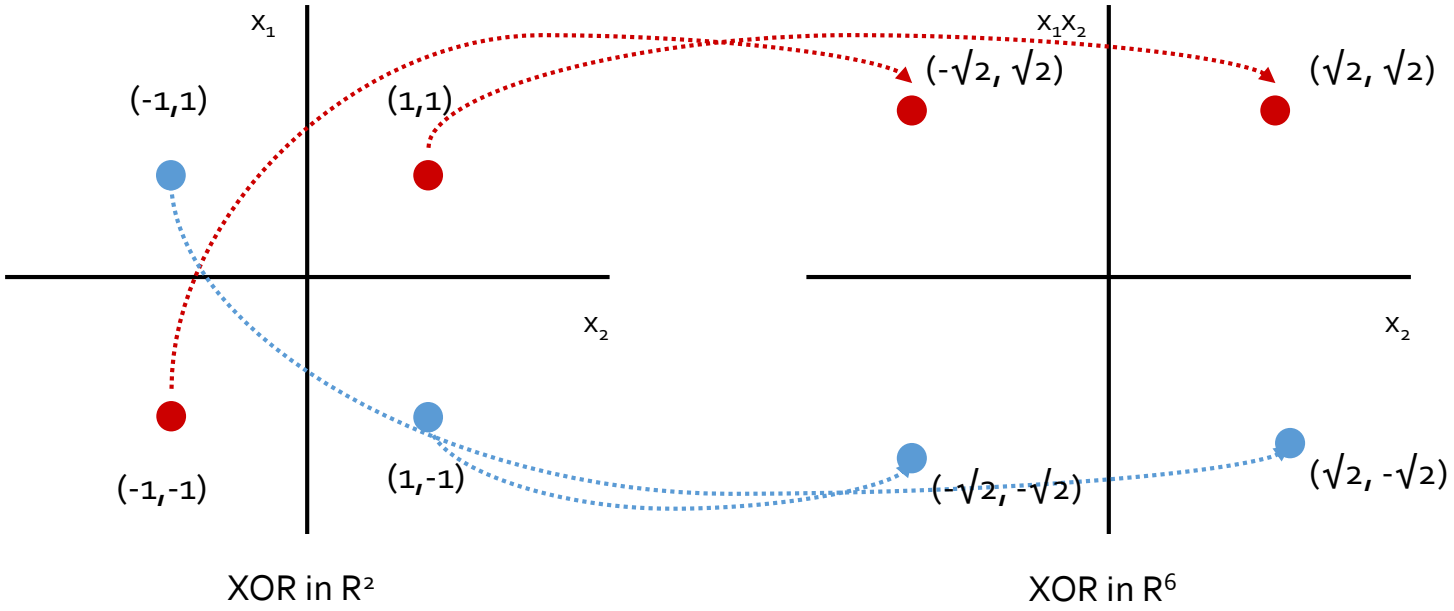
- Support Vector Machine Formulation

	Hard margin?	**Soft margin?**
Linearly separable?	Basic form	Introduce penalty terms
Linearly non-separable?	Utilize Kernel Trick	**Introduce penalty terms** **Utilize Kernel Trick**

Support Vector Machine

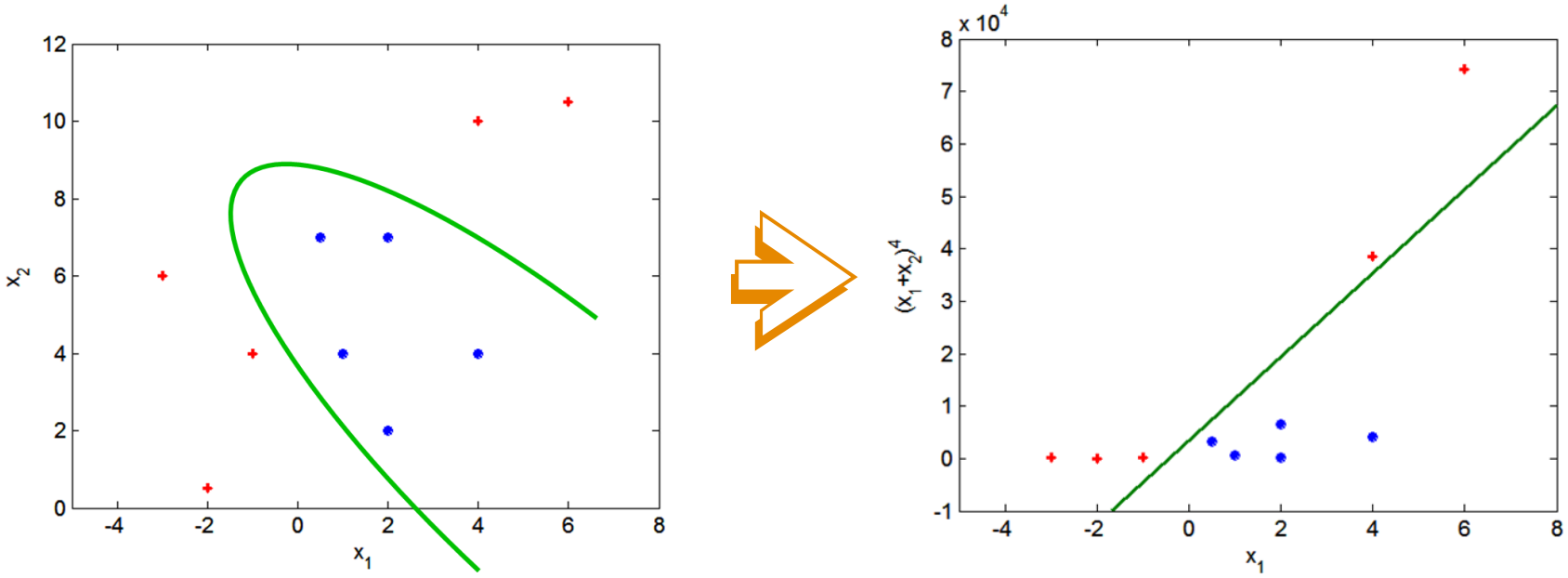
- What if the decision boundary is not linear?
  - ✓ Use a mapping function $\Phi(\mathrm{x})$ to transform an input vector from a lower dimensional space into a higher dimensional space

$$\Phi : (\mathbf{x}_1, \mathbf{x}_2) \rightarrow (\mathbf{x}_1^2, \mathbf{x}_2^2, \sqrt{2}\mathbf{x}_1, \sqrt{2}\mathbf{x}_2, \sqrt{2}\mathbf{x}_1\mathbf{x}_2, 1)$$



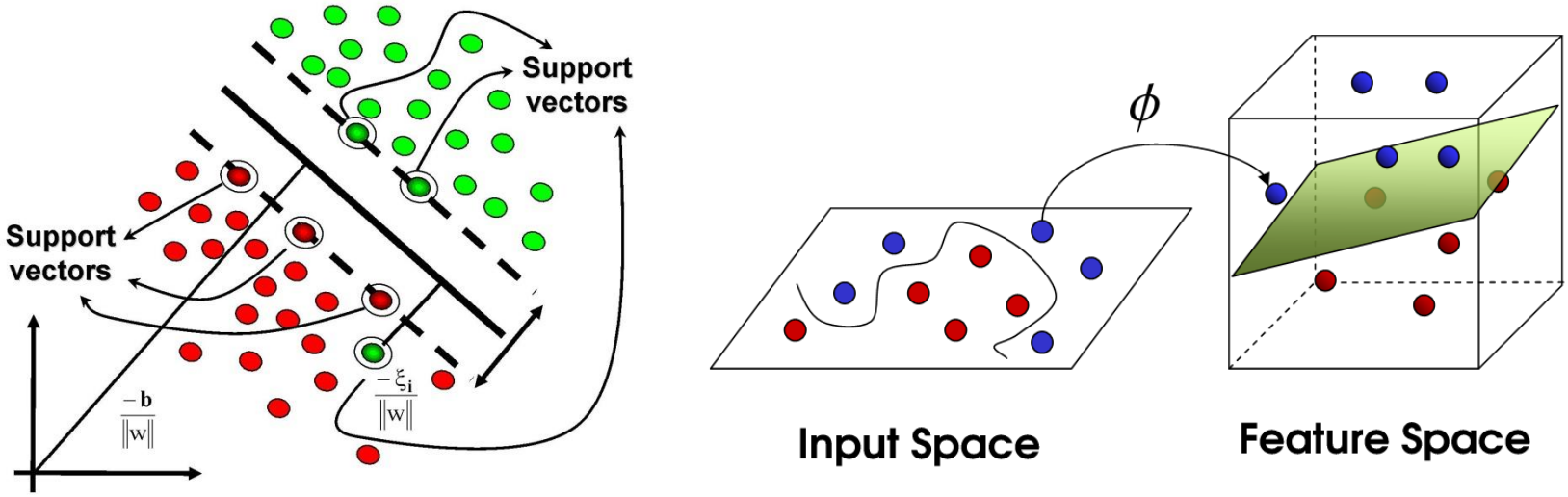
Support Vector Machine

- What if the decision boundary is not linear?
  - ✓ Transform an input vector into a higher feature vector



Support Vector Machine

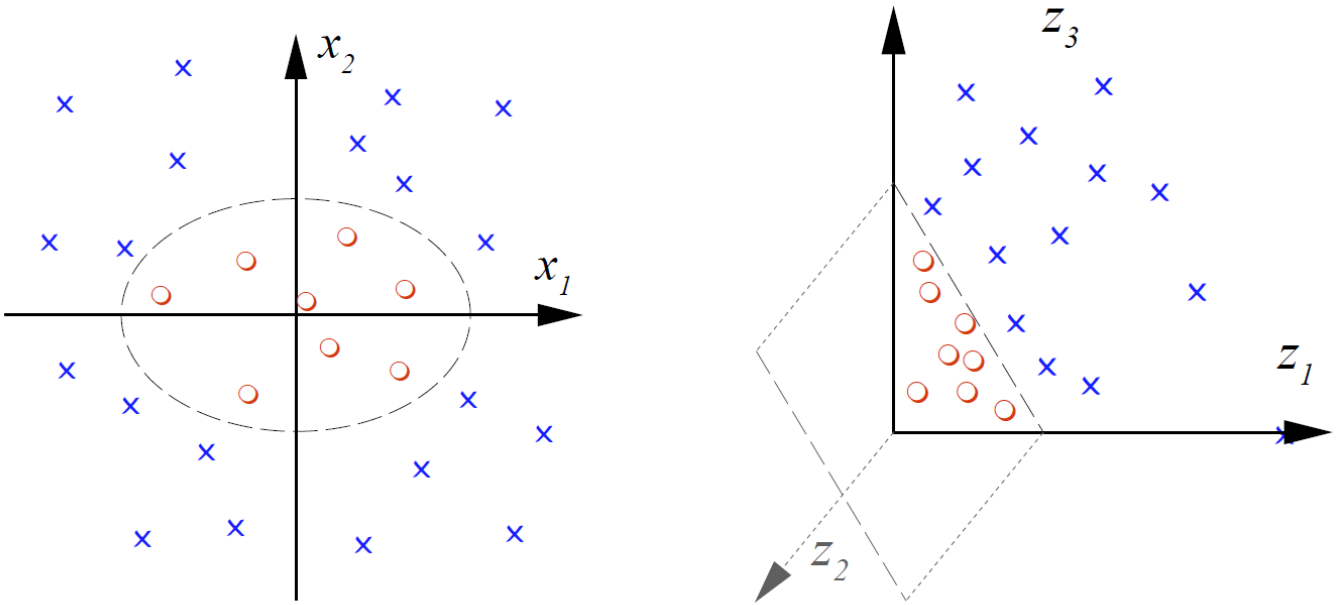
- Goal: To find the best hyperplane that maximizes the margin and minimize the misclassification error
  - ✓ Large margin: preserve the generalization ability
  - ✓ Flexible: can generate an arbitrary shape of boundary in the input space



Support Vector Machine

- Goal: To find the best hyperplane that maximizes the margin and minimize the misclassification error
  - ✓ Large margin: preserve the generalization ability
  - ✓ Flexible: can generate an arbitrary shape of boundary in the input space

$$\Phi : \mathbb{R}^2 \rightarrow \mathbb{R}^3$$
$$(x_1, x_2) \mapsto (z_1, z_2, z_3) := (x_1^2, \sqrt{2}\, x_1 x_2, x_2^2)$$

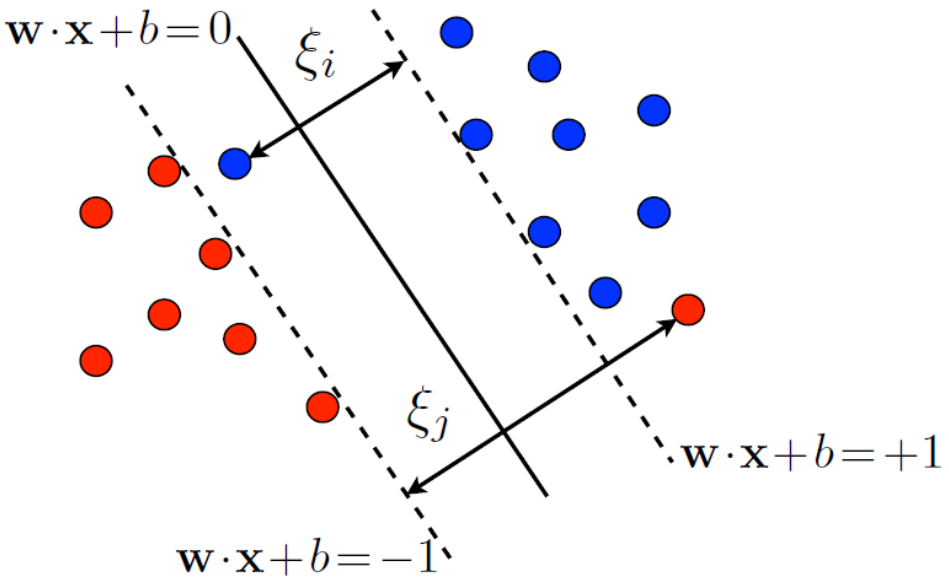


Support Vector Machine

- SVM as an optimization problem

$$\min \quad \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{N}\xi_i$$

$$s.t. \quad y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1 - \xi_i, \quad \xi_i \geq 0, \quad \forall i$$



Support Vector Machine

- Lagrangian Problem

$$\min \quad L_P(\mathbf{w}, b, \alpha_i) = \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n}\xi_i$$

$$-\sum_{i=1}^{n}\alpha_i\left(y_i(\mathbf{w}^T\Phi(\mathbf{x}_i)+b)-1+\xi_i\right)-\sum_{i=1}^{n}\mu_i\xi_i$$

- KKT condition

$$\frac{\partial L_P}{\partial \mathbf{w}} = 0 \Rightarrow \mathbf{w} = \sum_{i=1}^{n}\alpha_i y_i \Phi(\mathbf{x}_i) \qquad \frac{\partial L_P}{\partial b} = 0 \Rightarrow \sum_{i=1}^{n}\alpha_i y_i = 0$$

$$\frac{\partial L_P}{\partial \xi_i} = 0 \Rightarrow C - \alpha_i - \mu_i = 0$$

Support Vector Machine

- Lagrangian Problem (Primal)

$$\min \quad L_P(\mathbf{w}, b, \alpha_i) = \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n}\xi_i$$

$$-\sum_{i=1}^{n}\alpha_i\left(y_i(\mathbf{w}^T\Phi(\mathbf{x}_i)+b)-1+\xi_i\right)-\sum_{i=1}^{n}\mu_i\xi_i$$

- Lagrangian Problem (Dual)

$$\max \quad L_D = \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j \Phi(\mathbf{x}_i)^T\Phi(\mathbf{x}_j)$$

$$s.t. \quad \sum_{i=1}^{n}\alpha_i y_i = 0 \quad and \quad 0 \le \alpha_i \le C$$

Support Vector Machine

- How to find a mapping function Φ?
  - ✓ Introduce a Kernel Trick

$$\max \quad L_D = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j \Phi(\mathbf{x}_i)^T \Phi(\mathbf{x}_j)$$

$$s.t. \quad \sum_{i=1}^{n} \alpha_i y_i = 0 \quad and \quad 0 \leq \alpha_i \leq C$$

$$\max \quad L_D = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j K(\mathbf{x}_i, \mathbf{x}_j)$$

$$s.t. \quad \sum_{i=1}^{n} \alpha_i y_i = 0 \quad and \quad 0 \leq \alpha_i \leq C$$

Support Vector Machine

- Conditions on Kernel functions

$K(\mathbf{x}, \mathbf{x}')$ is a valid kernel iff

1. It is symmetric and **2.** The matrix:

$$\begin{bmatrix} K(\mathbf{x}_1, \mathbf{x}_1) & K(\mathbf{x}_1, \mathbf{x}_2) & \dots & K(\mathbf{x}_1, \mathbf{x}_N) \\ K(\mathbf{x}_2, \mathbf{x}_1) & K(\mathbf{x}_2, \mathbf{x}_2) & \dots & K(\mathbf{x}_2, \mathbf{x}_N) \\ \dots & \dots & \dots & \dots \\ K(\mathbf{x}_N, \mathbf{x}_1) & K(\mathbf{x}_N, \mathbf{x}_2) & \dots & K(\mathbf{x}_N, \mathbf{x}_N) \end{bmatrix}$$

is **positive semi-definite**

for any $\mathbf{x}_1, \cdots, \mathbf{x}_N$ (Mercer's condition)

Support Vector Machine

- Type of Kernel Functions
  - ✓ Polynomial

$$K(x, y) = (x \cdot y + c)^d, \quad c > 0$$

  - ✓ Gaussian (RBF)

$$K(x, y) = \exp\Big(- \frac{||x - y||^2}{2\sigma^2} \Big), \quad \sigma \neq 0$$

  - ✓ Sigmoid

$$K(x, y) = \tanh(a(x \cdot y) + b), \quad a, b \geq 0$$

Support Vector Machine

- From the KKT condition, we know that

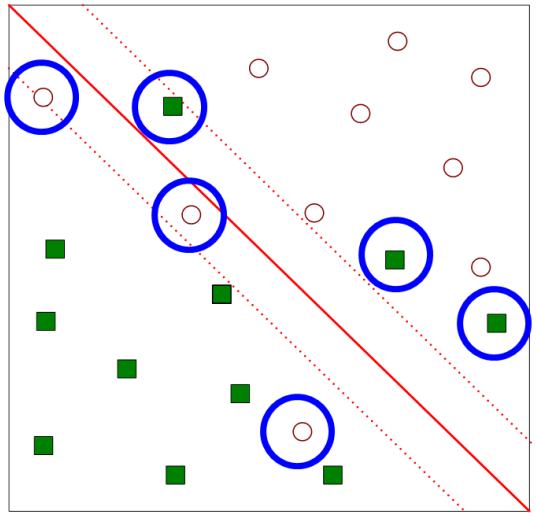
$$\alpha_i \left(y_i (\mathbf{w}^T \mathbf{x}_i + b) - 1 + \xi_i \right) = 0$$

✓ Thus, the only support vectors have $\alpha_i \neq 0$

✓ In addition,

$$C - \alpha_i - \mu_i = 0 \quad \& \quad \mu_i \xi_i = 0$$

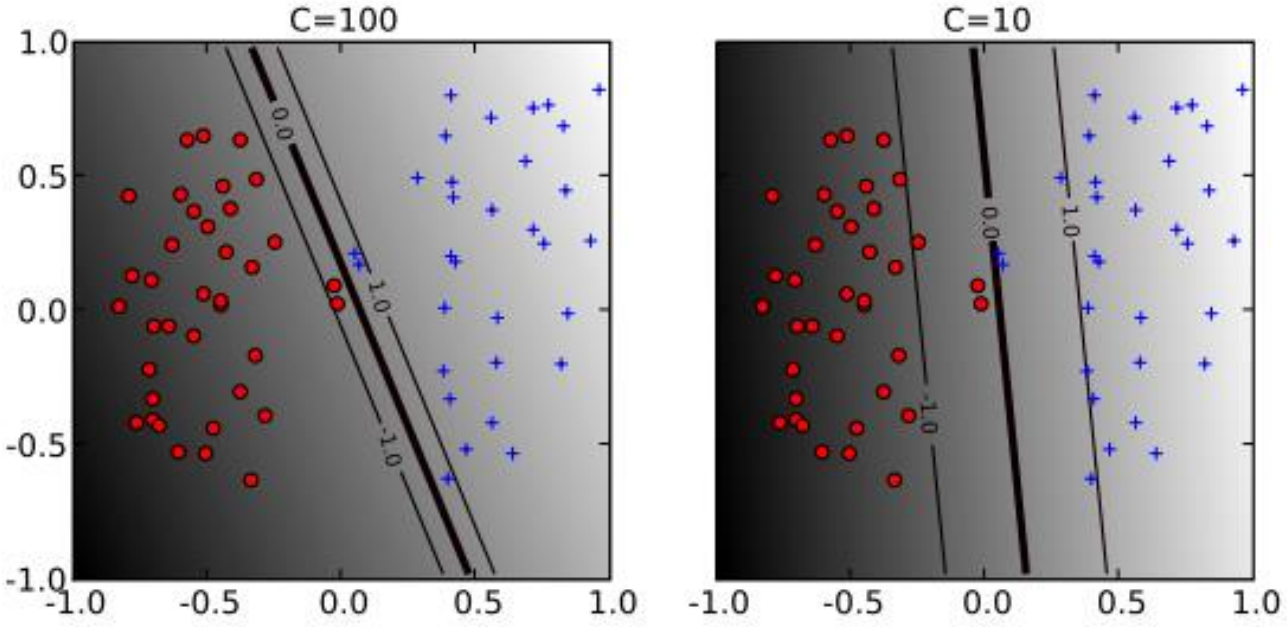
- Case 1: $\alpha_i = 0$ → non-support vectors
- Case 2: $0 < \alpha_i < C$ → support vectors on the margin
- Case 3: $\alpha_i = C$ → support vectors outside the margin



Support Vector Machine

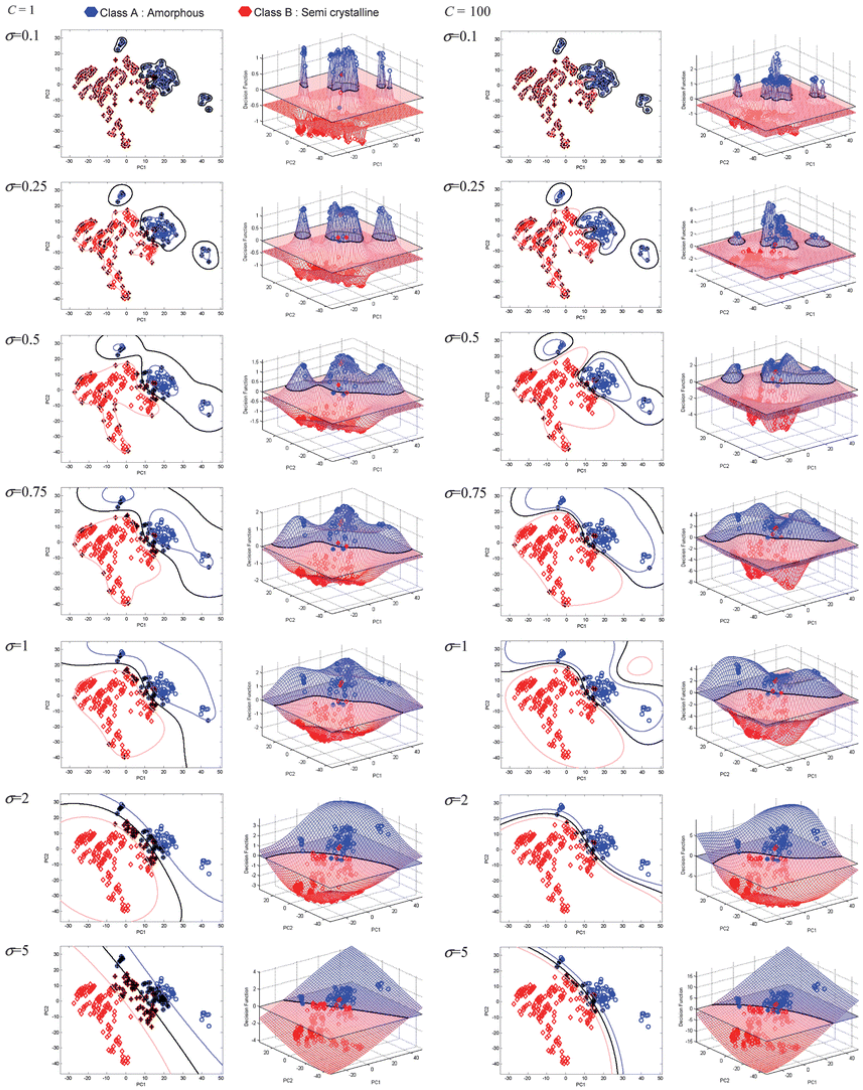
- Regularization cost C
  - ✓ Large C: narrow margin with a few support vectors
  - ✓ Small C: wide margin with many support vectors

$$\min \quad \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{N}\xi_i$$



Support Vector Machine

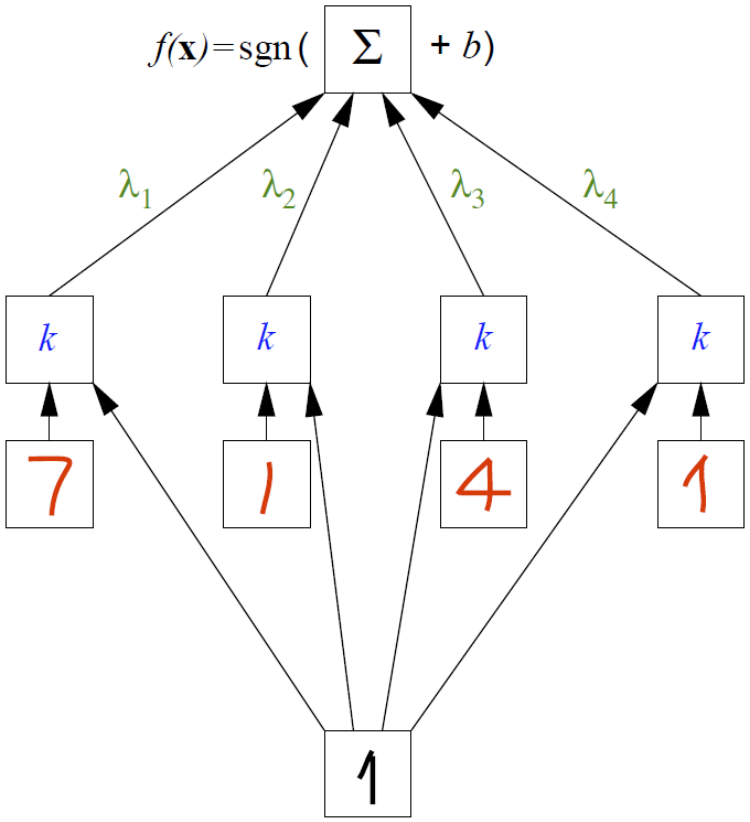
- Decision boundaries according to different σ and C combinations



Support Vector Machine

- Class decision for a new instance **x**

$$f(\mathbf{x}) = sign\left(\sum_{i=1}^{N} \alpha_i y_i \, K(\mathbf{x}_i, \mathbf{x}) + b\right)$$



classification $f(\mathbf{x})=\text{sgn}\left(\sum \lambda_i k(\mathbf{x},\mathbf{x}_i) + b\right)$

weights

comparison: $k(\mathbf{x},\mathbf{x}_i)$, e.g. $k(\mathbf{x},\mathbf{x}_i)=(\mathbf{x}\cdot\mathbf{x}_i)^d$

$k(\mathbf{x},\mathbf{x}_i)=\exp(-\|\mathbf{x}-\mathbf{x}_i\|^2 / c)$

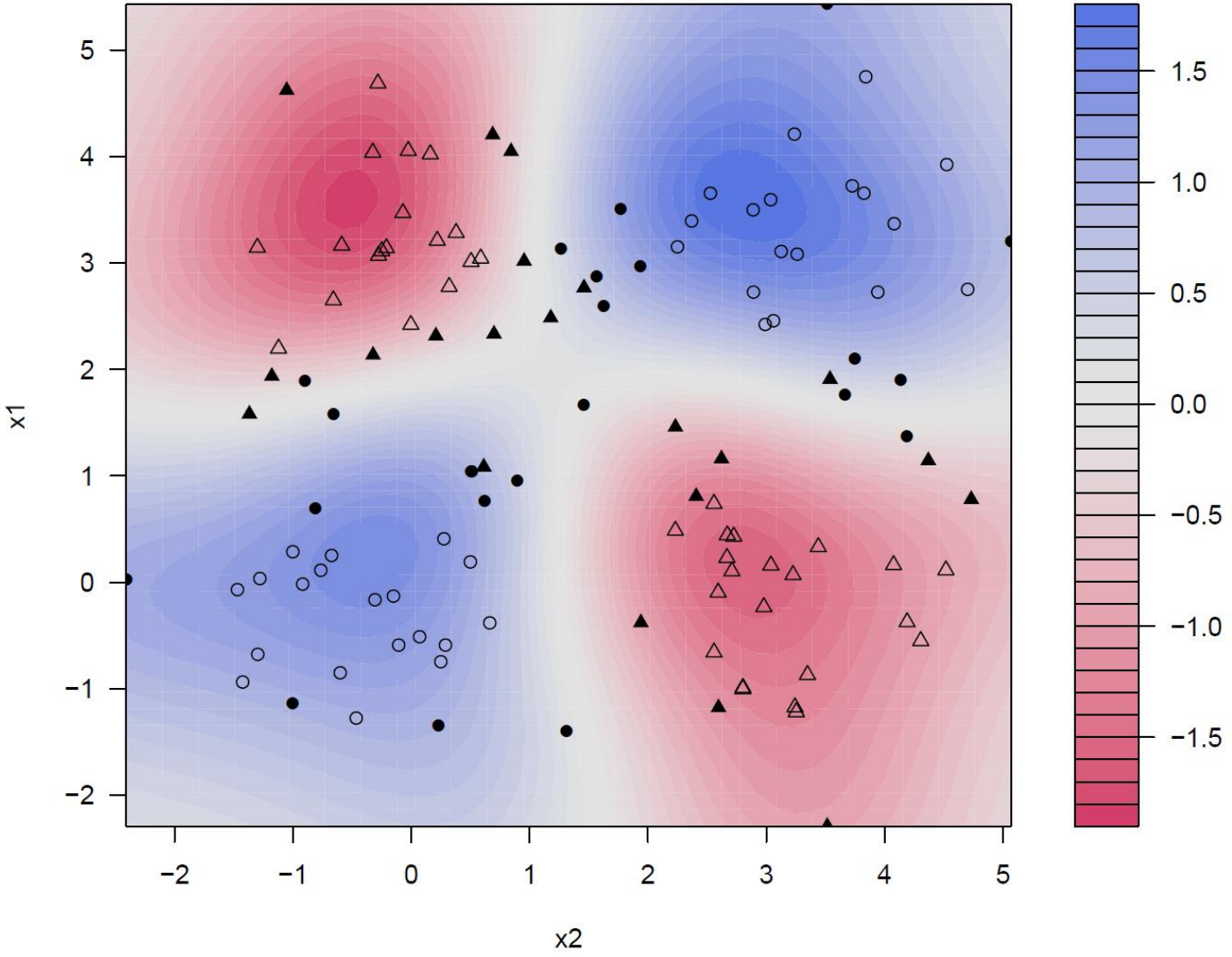
support vectors $\mathbf{x}_1 \ldots \mathbf{x}_4$

$k(\mathbf{x},\mathbf{x}_i)=\tanh(\kappa(\mathbf{x}\cdot\mathbf{x}_i)+\theta)$

input vector **x**

Support Vector Machine

- XOR problem




ANY
questions?

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References

Other Materials

- Image on the first page: http://aylien.com/press/aylien-announces-text-analysis-api-suite-nlp-information-retrieval-machine-learning-apis-extract-insights-documents/
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